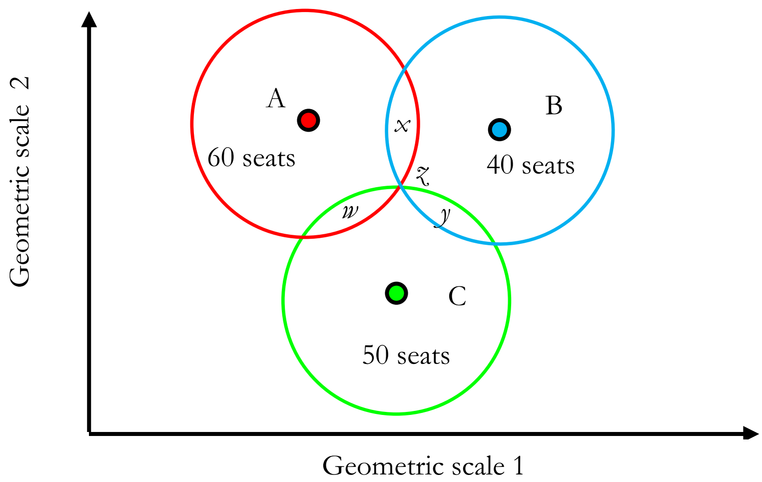
Sheets Chapter 9 Spatial Model of Coalition Formation

The minimum winning coalition theory of Von Neumann and Morgenstern declares that any combination of two players that forms a majority is the solution of a three-person coalition game.

William Riker developed in *The Theory of Political Coalition* (1962) the coalition theory further by increasing the three-person game into an *n*-person game and the coalition will be a minimal winning coalition.

The solution of Riker became known as the 'size principle' and the underlying reason for forming a majority coalition with a minimum number of persons is to maximize the benefits for the involved persons.

Figure 1 The size principle



The size principle defines the rational choice for a party, namely to choose the smallest partner to form a minimal winning coalition. The reason is that the benefits of being a member of the coalition are distributed according to political weights of its members.

Table 1 Distribution of gains

	A - B (x)	A - C (w)	B - C (y)	A - B - C (z)
gain A	$^6/_{10}$	$^6/_{11}$	0	$^6/_{15}$
gain B	$^4/_{10}$	0	$^4/_9$	$^4/_{15}$
gain C	0	$^5/_{11}$	$^5/_9$	$^5/_{15}$

The order of preference for the four possible coalitions w, x, y, and z are now based on the distribution of the benefits in Table 1.

party A	$x = {}^{6}/_{10}$,	$w = {}^{6}/_{11}$,	$z = {}^{6}/_{15}$,	$y = 0$	$x > w > z > y$
party B	$y = {}^{4}/_{9}$,	$x = {}^{4}/_{10}$,	$z = {}^{4}/_{15}$,	$w = 0$	$y > x > z > w$
party C	$\mathrm{y} = {}^{5}/_{9}$,	$w = {}^{5}/_{11}$,	$z = {}^{5}/_{15}$,	$x = 0$	$y > w > z > x$

If we interpret the case as a majority vote, than the size principle of Riker predicts that party B and C will form a majority and alternative y is the winner.

If the four alternative coalitions were put forward in a majority vote, than alternative y would be the majority winner. The voting matrix shows that *y* is the majority winner.

Figure 1 The size principle

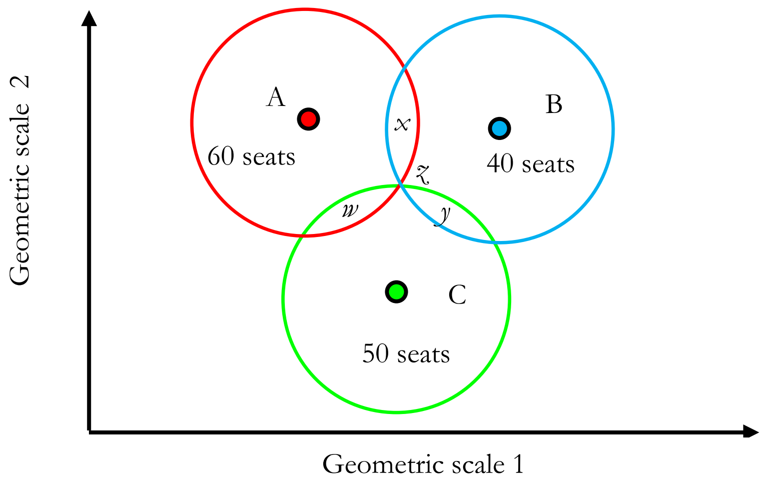


Figure 2 Voting matrix

	x	*y*	*w*	*z*	*Win*	
x	*	1 - 2	2 - 1	2 - 1	2	
y	2 - 1	*	2 - 1	2 - 1	3	maximum
w	1 - 2	1 - 2	*	2 - 1	1	
z	1 - 2	1 - 2	1 - 2	*	0	
Lose	1	0	1	3		
		minimum				

With the introduction of the size principle the policy model is no longer undetermined, but this conclusion has consequences for the formal model itself. The spatial voting game assumes that all three parties have no specific features that will change the preferences of the players. Each player is indifferent between the other two players, and if the distance to one player is shorter than the distance to the other he will prefer the player that is the closest. The same is true for the coalition points x, *y*, and *z*. The Euclidean distance between the ideal point of B and the alternatives *x* and *y* is the same in Figure 1 and B should be indifferent between *x* and *y*. However, the size component of the parties has changed the characteristics of these parties, and more importantly the size component has changed the deals between the parties. The three coalitions *x*, *y*, and *z* are no longer similar in the sense that in each coalition two parties will get ½ and the third parties gets nothing (assuming that it is a positive-sum game and not a three-person zero sum game). The spoils of forming a coalition together are divided according to size if we include the size of the parties in the equation. The same will happen if we include another feature in the model, namely if the dimensions with geometric scales are replaced by axes that measure money spend on specific policy issues. A policy space with substantive dimensions makes it possible to investigate deals between parties to form a coalition.

The only concern of players in a non-cooperative three-person zero-sum game is to be a member of a coalition. Players will maximize their utility by forming a *minimum winning* coalition (Von Neumann and Morgenstern 1944).

Riker pointed out that winning and losing is not just about numbers, what matters is the size or weights of the members of a coalition. The *size principle* implies that players will form a coalition just as large to ensure winning and no larger (Riker 1962).

Theories based on minimum winning and the size principle is office-seeking or policy-blind theories. With the integration of game theory in political science, the political reality, such as policy position of parties, was integrated in the coalition formation theories (Axelrod 1970; Brown and Franklin 1973; De Swaan 1973).

The features of parties themselves, such as the size and the location in the political spectrum, were considered as improvements of the coalition theories (Peleg 1981; Budge and Laver 1985; Van Deemen 1991). Most office-seeking and policy-oriented theories use a single dimension for their analysis (Schofield 1986; McKelvey and Schofield 1986). The next step in the evolution of the coalition formation theories is the spatial theory of party competition, where parties are located in a political space (Laver and Schofield 1990; Schofield 1993; Grofman 1996; Schofield 1997; De Vries 1999).

We present a new two-level coalition game that explains why a party makes an optimal action by forming a certain coalition.

Michael Laver and Kenneth Shepsle introduce a new coalition formation theory in which parties not only has a preference for another party, but they also have a preference for certain policy departments (portfolios). The portfolio-allocation model is an example of the thick-rationality theory that combines the searching for a partner and the making of a government into one single formal model.

The portfolio-allocation model

Laver and Shepsle demonstrate that the policy of a coalition is not simply defined by the positions of parties in some policy space; one has to take into account the division of policy departments in the process of the formation.

Figure 1 shows the original portfolio-allocation model of Laver and Shepsle with three parties A, B and C and two policy dimensions.

The horizontal axis in Figure 1 represents the financial policy of the Department of Finance and the vertical axis stands for foreign policy of the Department of Foreign Affairs.

The two dimensions Finance and Foreign Affairs define the policy space of the portfolio-allocation model.

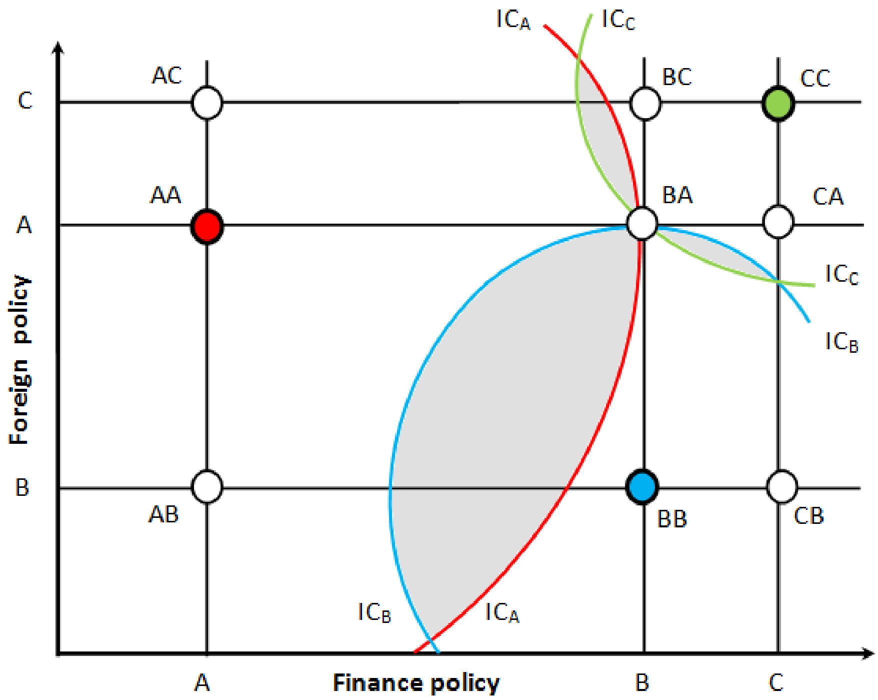
The nine points AA, AB, AC, BA, BB, BC, CA, CB, and CC represent the credible policy positions that are defined by three horizontal and three vertical lines, the so-called orthogonal 'lattice'.

The lattice is shaped by the preferred position of parties on the horizontal finance policy dimension - from left to right by the letters A, B, and C - and the preferred position on the vertical foreign policy dimension - from top to base by C, A, and B (Laver and Shepsle 1996: 64).

The ideal point of each party, the red dot labelled AA, the blue spot BB, and the green dot CC, is the situation in which a party has control over both policy dimensions.

The six other portfolio proposals, AB, AC, BA, BC, CA, and CB, are illustrated by the white dots in Figure 1.

Figure 1 Portfolio allocation model



For example, proposal AB stands for allocating the Finance policy to party A and the Foreign policy to party B, and in proposal BA the policy departments are turned around. In both proposals, AB and BA, the government is formed by the parties A and B, but the two portfolios will generate different government policy outputs (Laver and Shepsle 1996: 34).

The portfolio model assumes "that the actors are trying to move government policy outputs as close as possible to their own preferred policies" (Laver and Shepsle 1990: 874). In order to apply this maximizing rule Laver and Shepsle have to "assume Euclidean preferences throughout, so that preferences are measured by the Euclidean distance from an actor's ideal point" (Laver and Shepsle 1990: 888).

The smaller the Euclidean distance between the ideal point and the policy proposal the higher the preference for that proposal. Table 1 shows the order of preference of the three parties over the credible policy proposals in which utility value 'µ9' stands for the highest utility and 'µ1' for the lowest.

Table 1 The utility values for all nine proposals

credible policy proposal	party A	party B	party C
AA	μ9	μ1	μ2
AC	μ8	μ2	μ3
AB	μ7	μ4	μ1
BA	μ6	μ7	μ6
BC	μ5	μ5	μ7
BB	μ4	μ9	μ4
CA	μ3	μ6	μ8
CC	μ2	μ3	μ9
CB	μ1	μ8	μ5

In conventional spatial models any point in the policy space—and any point in the policy win-set—is a feasible policy position (Schofield 1986; McKelvey and Schofield 1987).

However, Laver and Shepsle have restricted the set of feasible policy positions by selecting nine credible proposals - the so-called lattice win-set (L), none of which is located inside the win set.

The result of this restriction is an empty policy win-set. Thus, "while there are *policies* preferred to those forecast for the BA government, there is no *cabinet* (point in L) that is forecast to implement such a policy" (Laver and Shepsle 1996: 65).

The schedule of preference that matches the utility values in Table 1 is as follows:

A: AA > AC > AB > BA > BC > BB > CA > CC > CB
B: BB > CB > BA > CA > BC > AB > CC > AC > AA
C: CC > CA > BC > BA > CB > BB > AC > AA > AB

Table 2 The Condorcet winner

alternatives	party A support	party B support	party C support	winner
BA - BC	BA	BA	BC	BA
BA - BB	BA	BB	BA	BA
BA- CA	BA	BA	CA	BA
BA - CC	BA	BA	CC	BA
BA - CB	BA	CB	BA	BA
BA - AA	AA	BA	BA	BA
BA - AC	AC	BA	BA	BA
BA - AB	AB	BA	BA	BA

Figure 2 The policy proposals in a geometric policy space

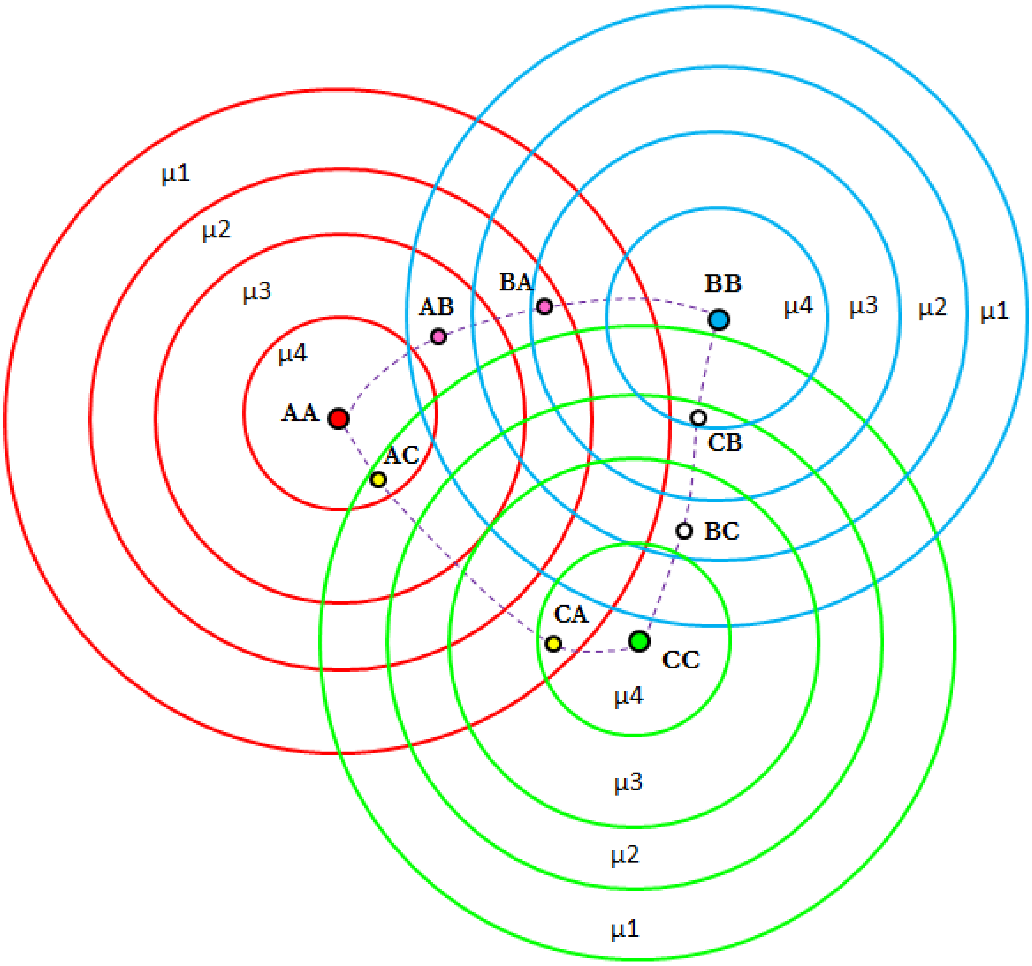


Figure 2 presents the same policy proposals as Figure 1, but now we have a proper space with geometric dimensions. The circles in the policy space are utility contours, with the highest utility in the inner circle and the lowest utility in the outer circle.

The Euclidean distance between an ideal point {AA, BB, CC} and the six coalition's points {AC, CA, AB, BA, BC, CB} measures the utility level of the six possible coalitions for each party.

A: AA > AC > AB > BA > BC > BB > CA > CC > CB
B: BB > CB > BA > CA > BC > AB > CC > AC > AA
C: CC > CA > BC > BA > CB > BB > AC > AA > AB

it is not possible to integrate proposals BB, CC and CB somewhere in the red circles of party A, without distressing the circles of party B and C. The maps of circles, the three contours of utility in Figure 2, match the following preference ordering relations.

A: AA > AC > AB > BA > CA
B: BB > CB > BA > BC > AB
C: CC > CA > BC > CB > AC

The ideal point of a party is the basis of the preference ordering relation, but it is not a feasible majority coalition and this means that the preference ordering relations for a majority coalition are as follows:

A: AC > AB > BA > CA
B: CB > BA > BC > AB
C: CA > BC > CB > AC

Table 3 Utility values

relevant policy proposal	party A	party B	party C
AC	$\mu4$	0	$\mu1$
AB	$\mu3$	$\mu1$	0
BA	$\mu2$	$\mu3$	0
BC	0	$\mu2$	$\mu3$
CA	$\mu1$	0	$\mu4$
CB	0	$\mu4$	$\mu2$

Table 4 Imputation dominated by another division of policy proposals

imputation	dominated by
AC	BC, CB
AB	AC, BC, CB
BA	AC, CB
BC	BA, CA,
CA	AB, BA
CB	CA,

All six proposals, i.e. the imputations in Table 4, are dominated by another proposal, which means that whatever policy proposal we consider, there is always one party that can make a better deal by making an unilateral move and break up the coalition.

In other words, there is not one coalition that dominates all others and there is no *core* of the political space and without a core there is no equilibrium, where no player has an incentive to deviate unilaterally. The set of proposals in Figure 2 generates a cyclical nature and no single imputation dominates all others.

The difference between the portfolio-allocation model of Laver and Shepsle and our alternative model, the so-called two-level coalition game, is the assignment of utility values to the policy proposals. In the setting of the portfolio-allocation model each player gets a utility value for all nine proposals, even for the coalitions in which he is not a member. As a result no one has an incentive in BA to make a unilateral move, which makes BA the Condorcet winner of the game and the core of the political space. We demonstrated that there is no Condorcet winner and no equilibrium. Each policy proposal between two parties is dominated by a deal of another coalition.

The cyclical nature of the proposals in Figure 2 is a normal phenomenon in politics "this lack of transitivity is a most typical phenomenon in social organizations" (Rosenblith 2004: 672).

The coalition game as a non-cooperative game in extensive form

Figure 4A The complete tree diagram with player A on top

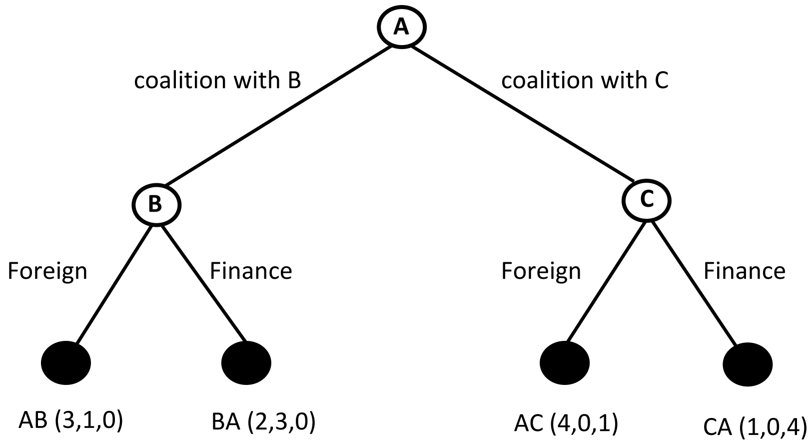


Figure 4B Backward induction: choice of player A

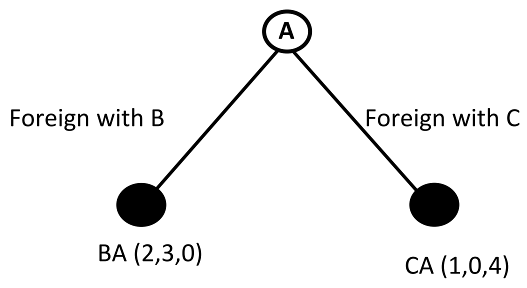


Figure 4C The subgame of player A when the others make a mistake

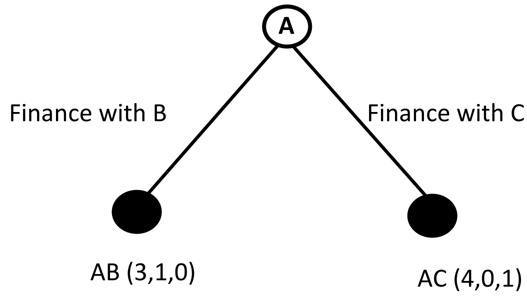


Figure 5A The complete tree diagram with player B on top

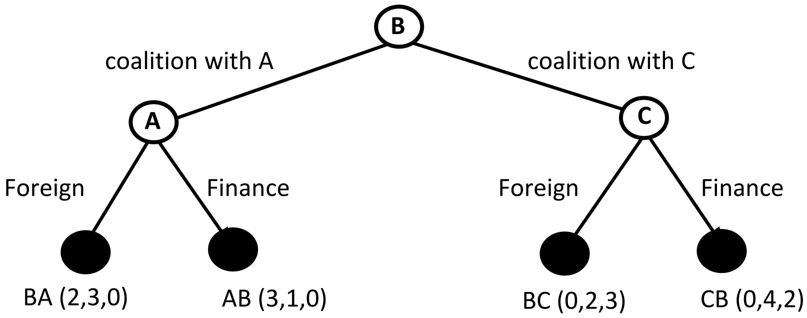


Figure 5B Backward induction: choice of player B

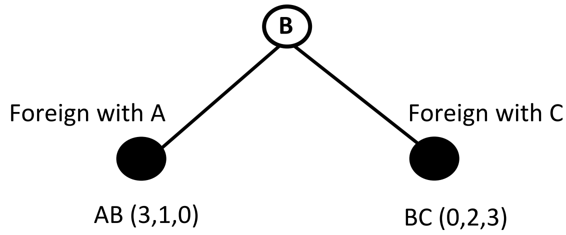


Figure 6A The complete tree diagram with player C on top

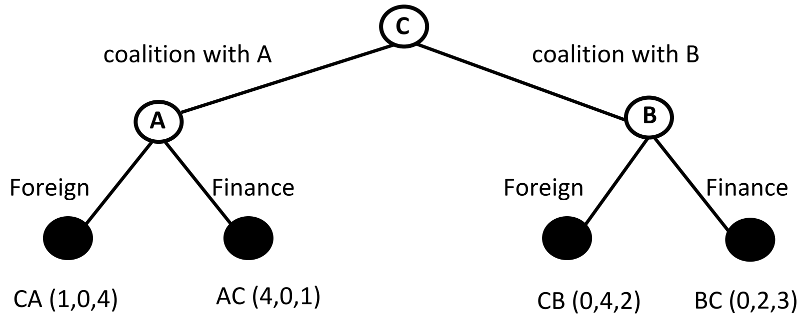
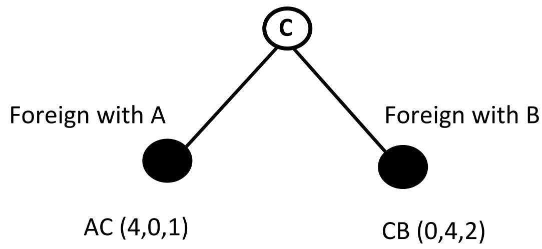
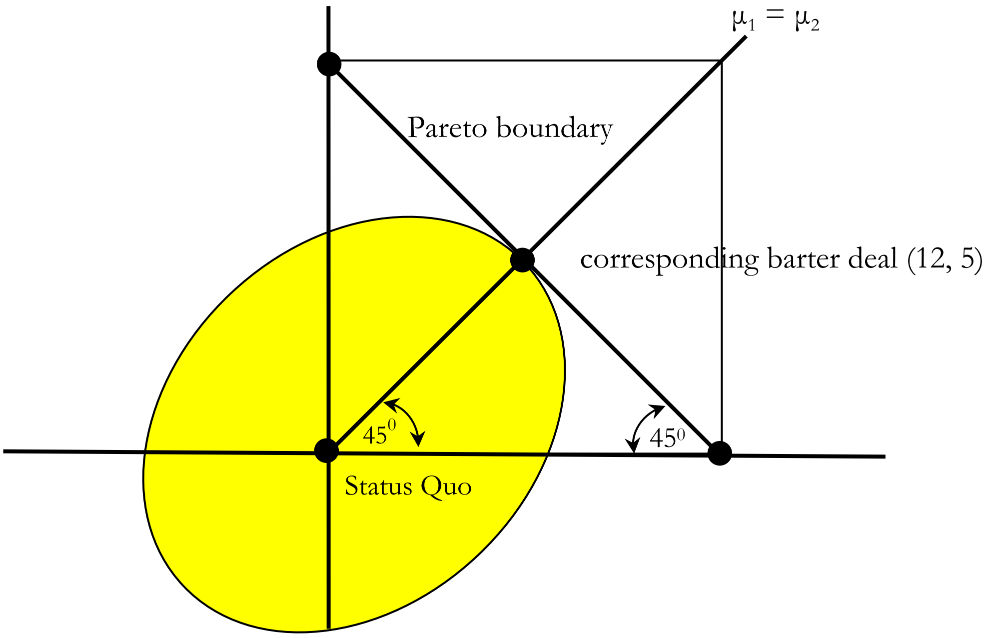


Figure 6B Backward induction: choice of player C



Assignment Chapter 10

Figure 7 The Nash solution



Explain that the Nash solution of the bargaining problem satisfies the condition of

1) *Maximizing total utility*
2) *Equality*
3) the '*split-the-difference*' solution

max 250 words

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