$$\Phi(\boldsymbol{x},t) = \Big(\phi_{\boldsymbol{x},t} * K\Big)(\boldsymbol{x},t) \quad .$$

$$\Phi(\boldsymbol{x},t) = \Big(\phi\ \delta_{\boldsymbol{x}_s(t)} * f\Big)(\boldsymbol{x},t) \quad .$$

$$\Phi(\boldsymbol{x},t) = \Big(\phi\ \delta_{\boldsymbol{x}_s} * K\Big)(\boldsymbol{x},t)$$

$$\Phi(\boldsymbol{x},t) = \Big(\phi\ \delta_{\boldsymbol{x}_s} * K\Big)(\boldsymbol{x},t)$$

$$\mu(\boldsymbol{x}_q) = \int_{\mathcal{P}\times[0,T]} \Phi(\mathrm{d}\boldsymbol{x} \times \mathrm{d}t) \quad .$$

$$\boldsymbol{m}^* \ = \ \arg\max_{\boldsymbol{m}\in\boldsymbol{M}} P(?|\boldsymbol{m}) \quad .$$

$$\boldsymbol{m}^* \ = \ \arg\inf_{\boldsymbol{m}\in\boldsymbol{M}} \big(\,\mathrm{nll}(\boldsymbol{y}|\boldsymbol{m})\ - \log(P(\boldsymbol{m}))\big)$$

$$\mu(\boldsymbol{x}_q) = \phi\, A\, T\, K(\boldsymbol{x}_q - \boldsymbol{x}_s) \quad , \tag{1}$$

$$\mu(\boldsymbol{x}_q) = \phi\, A\, T \sum_{k=1}^{N} w_k\, f_k(\boldsymbol{x}_q - \boldsymbol{x}_s) \tag{2}$$

$$P(y_q) = \frac{(\mu(\boldsymbol{x}_q))^{y_q}}{y_q!} \exp(-\mu(\boldsymbol{x}_q)) \quad ,$$

$$\mathrm{nll}(\boldsymbol{y}|\boldsymbol{m}) = \sum_{q\in\mathcal{P}} \big(\,\mu_q(\boldsymbol{m})\ -\ y_q\ \log(\mu_q(\boldsymbol{m}))\big) \tag{3}$$

$$K_N(\boldsymbol{x})\ =\ \sum_{k=1}^{N} w_k\, f_k(\boldsymbol{x}) \quad , \tag{4}$$

$$\begin{aligned} \mu(\boldsymbol{x}_q)\ &= C_0\, \phi\ PSF_N(\boldsymbol{x}_q - \boldsymbol{x}_s) \quad \text{for} \quad C_0\ =\ T\, A \,\text{and}\ \ q\ \in\ 1, 2....M \\ &= C_0\, \phi \sum_{k=1}^{N} w_k\, f_k(\boldsymbol{x}_q - \boldsymbol{x}_s) \quad . \end{aligned}$$

$$\begin{aligned} \mathrm{nll}(.) &= \sum_{\boldsymbol{x}_q}\ \big(\mu(\boldsymbol{x}_q) - y_q\ \log(\mu(\boldsymbol{x}_q))\big] \\ &= \sum_{\boldsymbol{x}_q}\ \Big(C_0\, \phi \sum_{k=1}^{N} w_k\, f_k(\boldsymbol{x}_q - \boldsymbol{x}_s)\ -\ y_q\ \log\big(C_0\, \phi \sum_{k=1}^{N} w_k\, f_k(\boldsymbol{x}_q - \boldsymbol{x}_s)\big)\Big] \quad . \end{aligned} \tag{5}$$

$$\mathrm{nll}(\phi,\, \boldsymbol{w}) \ = \sum_{x_q} \Big(\phi \boldsymbol{D} \boldsymbol{w} - \boldsymbol{y} log(\phi \boldsymbol{D} \boldsymbol{w}) \Big) \quad . \tag{6}$$

$$\inf_{\substack{\boldsymbol{w} \in \Delta \\ \phi \in \mathbb{R}^+}} \mathrm{nll}(\phi,\, \boldsymbol{w}) \ + \ \lambda \, \mathbf{card}(\boldsymbol{w}) \quad .$$

$$\inf_{\substack{\boldsymbol{w} \in \Delta \\ \phi \in \mathbb{R}^+}} \mathrm{nll}(\phi,\, \boldsymbol{w}) \ + \ \lambda \, \|\boldsymbol{w}\|_1$$

$$\inf_{\boldsymbol{w} \in \mathbb{R}^+} \mathrm{nll}(\boldsymbol{w}') \ + \ \lambda \, \|\boldsymbol{w}'\|_1$$

$$\mathrm{S}(\phi,\, \boldsymbol{w}) = \mathrm{nll}(\phi,\, \boldsymbol{w}) + \lambda \, \|\boldsymbol{w}\|_1 \quad . \tag{7}$$

$$\mathrm{nll}(\boldsymbol{w}') \ = \sum_{x_q} \Big(\boldsymbol{D} \boldsymbol{w}' - \boldsymbol{I} . \log(\boldsymbol{D} \boldsymbol{w}') \Big) \quad . \tag{8}$$

$$\inf_{\boldsymbol{w} \in \Delta} \mathrm{S}(\boldsymbol{w}) \ = \ \inf_{\boldsymbol{w}} \mathrm{nll}(\boldsymbol{w}) + \lambda \, \|\boldsymbol{w}\|_1 + \mathrm{i}_\Delta(\boldsymbol{w})$$

$$\inf_{\substack{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3 \\ \boldsymbol{v}_1 = \mathbf{D}\mathbf{w} \\ \boldsymbol{v}_2 = \boldsymbol{w},\, \boldsymbol{v}_3 = \boldsymbol{w}}} \mathrm{S}_1(\boldsymbol{v}_1) + \mathrm{S}_2(\boldsymbol{v}_2) + \mathrm{S}_3(\boldsymbol{v}_3) \quad .$$

$$\begin{aligned} \mathrm{AS}(\boldsymbol{w},\, \boldsymbol{v},\, \boldsymbol{b}) = {} & \mathrm{S}_1(\boldsymbol{v}_1) + \mathrm{S}_2(\boldsymbol{v}_2) + \mathrm{S}_3(\boldsymbol{v}_3) + \\ & \frac{1}{2\gamma} \|\boldsymbol{b}_1 + \boldsymbol{D}\boldsymbol{w} - \boldsymbol{v}_1\|_2^2 + \frac{1}{2\gamma} \|\boldsymbol{b}_2 + \boldsymbol{w} - \boldsymbol{v}_2\|_2^2 + \frac{1}{2\gamma} \|\boldsymbol{b}_3 + \boldsymbol{w} - \boldsymbol{v}_3\|_2^2 \quad . \end{aligned}$$

Primal updates:

$$\boldsymbol{w}^{k+1} \ = \ \arg\inf_{\boldsymbol{w}} \mathrm{AS}(\boldsymbol{w}, \boldsymbol{v}_1^k, \boldsymbol{v}_2^k, \boldsymbol{v}_3^k, \boldsymbol{b}^k) \tag{9}$$

$$\boldsymbol{v}_1^{k+1} \ = \ \arg\inf_{\boldsymbol{v}_1} \mathrm{AS}(\boldsymbol{w}^{k+1}, \boldsymbol{v}_1, \boldsymbol{v}_2^k, \boldsymbol{v}_3^k, \boldsymbol{b}^k) \tag{10}$$

$$\boldsymbol{v}_2^{k+1} \ = \ \arg\inf_{\boldsymbol{v}_2} \mathrm{AS}(\boldsymbol{w}^{k+1}, \boldsymbol{v}_1^{k+1}, \boldsymbol{v}_2, \boldsymbol{v}_3^k, \boldsymbol{b}^k) \tag{11}$$

$$\boldsymbol{v}_3^{k+1} \ = \ \arg\inf_{\boldsymbol{v}_3} \mathrm{AS}(\boldsymbol{w}^{k+1}, \boldsymbol{v}_1^{k+1}, \boldsymbol{v}_2^{k+1}, \boldsymbol{v}_3, \boldsymbol{b}^k) \tag{12}$$

Dual updates:

$$
\begin{aligned}
\boldsymbol{b}_1^{k+1} &= \boldsymbol{b}_1^k + \boldsymbol{D}\boldsymbol{w}^{k+1} - \boldsymbol{v}_1^{k+1} \\
\boldsymbol{b}_2^{k+1} &= \boldsymbol{b}_2^k + \boldsymbol{w}^{k+1} - \boldsymbol{v}_2^{k+1} \\
\boldsymbol{b}_3^{k+1} &= \boldsymbol{b}_3^k + \boldsymbol{w}^{k+1} - \boldsymbol{v}_3^{k+1}
\end{aligned}
$$

$$
\boldsymbol{w}^{k+1} = \arg\inf_{\boldsymbol{w}} \frac{1}{2\gamma} \|{\boldsymbol{b}_1}^k + \boldsymbol{D}\boldsymbol{w} - {\boldsymbol{v}_1}^k\|_2^2 + \frac{1}{2\gamma} \|{\boldsymbol{b}_2}^k + \boldsymbol{w} - {\boldsymbol{v}_2}^k\|_2^2 + \frac{1}{2\gamma} \|{\boldsymbol{b}_3}^k + \boldsymbol{w} - {\boldsymbol{v}_3}^k\|_2^2
$$

$$
\begin{aligned}
&\boldsymbol{D}^T \left({\boldsymbol{b}_1}^k + \boldsymbol{D}\boldsymbol{w} - {\boldsymbol{v}_1}^k\right) + \left({\boldsymbol{b}_2}^k + \boldsymbol{w} - {\boldsymbol{v}_2}^k\right) + \left({\boldsymbol{b}_3}^k + \boldsymbol{w} - {\boldsymbol{v}_3}^k\right) = 0 \\
\implies\ & \boldsymbol{w}^{k+1} = \left[\boldsymbol{D}^T\boldsymbol{D} + 2\boldsymbol{I}\right]^{-1} \left[\boldsymbol{D}^T \left({\boldsymbol{b}_1}^k - {\boldsymbol{v}_1}^k\right) + \left({\boldsymbol{b}_2}^k - {\boldsymbol{v}_2}^k\right) + \left({\boldsymbol{b}_3}^k - {\boldsymbol{v}_3}^k\right)\right]
\end{aligned}
$$

$$
\boldsymbol{v}_1^{k+1} = \arg\inf_{\boldsymbol{v}_1} \mathrm{S}_1(\boldsymbol{v}_1) + \frac{1}{2\gamma} \|{\boldsymbol{b}_1}^k + \boldsymbol{D}\boldsymbol{w}^{k+1} - \boldsymbol{v}_1\|_2^2
$$

$$
\arg\inf_{v_q} \mathrm{nll}(v_q) + \frac{1}{2\gamma} \left[({\boldsymbol{b}_1}^k)_q + (\boldsymbol{D}\boldsymbol{w}^{k+1})_q - v_q\right]^2
$$

$$
(v_q)^2 + (\gamma - ({\boldsymbol{b}_1}^k)_q - (\boldsymbol{D}\boldsymbol{w}^{k+1})_q).v_q - \gamma.i_q = 0
$$

$$
\boldsymbol{v}_2^{k+1} = \arg\inf_{\boldsymbol{v}_2} \mathrm{S}_2(\boldsymbol{v}_2) + \frac{1}{2\gamma} \|{\boldsymbol{b}_2}^k + \boldsymbol{w}^{k+1} - \boldsymbol{v}_2\|_2^2
$$

$$
\arg\inf_{v_n} \lambda \|v_n\| + \frac{1}{2\gamma} \left[({\boldsymbol{b}_2}^k)_n + (\boldsymbol{w}^{k+1})_n - v_n\right]^2
$$

$$
v_n = \begin{cases} \left(1 - \frac{\lambda'}{\|x_n\|}\right), & \forall\ \|x_n\| \leqslant \lambda' \\ 0, & \text{else} \end{cases} \quad ,
$$

where, x_n equals $({\boldsymbol{b}_2}^k)_n + (\boldsymbol{w}^{k+1})_n$ and λ' is equal to $(\lambda\,\gamma)$.

$$
\boldsymbol{v}_3^{k+1} = \arg\inf_{\boldsymbol{v}_3} \mathrm{S}_3(\boldsymbol{v}_3) + \frac{1}{2\gamma} \|{\boldsymbol{b}_3}^k + \boldsymbol{w}^{k+1} - \boldsymbol{v}_3\|_2^2
$$

$$
\arg\inf_{\boldsymbol{v}_3 \in \Delta} \|{\boldsymbol{b}_3}^k + \boldsymbol{w}^{k+1} - \boldsymbol{v}_3\|_2^2
$$