Exercise Sheet

1. Compute the gradient and Hessian of the function $q(x) = \frac{1}{2}x^T Ax + b^T x$, where A is symmetric.
2. Compute the gradient and Hessian of

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

   Show that $x^* = (1, 1)^T$ is the only local minimizer of this function, and that the Hessian at this point is positive definite. Write a program on trust region method with subproblems solved by the Dogleg method. Apply it to minimize this function. Choose $B_k = \nabla^2 f(x_k)$. Experiment with the update rule of trust region. Give the first two iterates.

3. Apply Steepest Descent method with exact line search to the problem:

$$\min \quad f(x) = 5x_1^2 + \frac{1}{2}x_2^2.$$

   Carry out two iterations, starting from $x^0 = (0.1, 1)^T$. Think about how $\{x^k\}$ will behave.

4. Assume that there exsits $M > 0$ such that $\|B_k\|_2 \|B_k^{-1}\|_2 \leq M$ where B_k is symmetric and positive definite. Show that if $d_k = -B_k^{-1}\nabla f_k$,

$$\frac{-d_k^{\nabla} f_k}{\|d_k\|\|\nabla f_k\|} \geq \frac{1}{M}.$$

5. Apply CG method with exact line search to solve

$$\min \quad \frac{1}{2}x^T Ax + b^T x,$$

   starting from $x_0 = (2, 1)^T$. Here $A = [4, 1; 1, 3]$ and $b = -(1, 2)^T$.