Exercise Sheet 2

1. Apply the Newton's method with step size 1 to the following problem:

$$\min_{x=(t_1,t_2)^T\in\mathbb{R}^2} \quad t_1^2 + t_2^2 + t_1^4 \tag{1}$$

with starting point $x_0 = (\epsilon, \epsilon)^T$ where $\epsilon > 0$ is very small, calculate the next iterate and you should find that $\|x_1\|_2 = O(\epsilon^3)$.

2. Write a program on quasi-Newton method with exact line search to solve the problem:

$$\min_{x=(t_1,t_2)^T\in\mathbb{R}^2} \quad t_1^2 + 2t_2^2 \tag{2}$$

starting from $x_0 = (1,1)^T$. Use BFGS and DFP update formula, respectively. Set the initial matrix $H_0 = I$ for both methods.

3. Prove that if $J_k^T r_k \neq 0$, the Cauchy point of the trust region subproblem for nonlinear least-squares problem

$$\min \ q_k(d) = \frac{1}{2}\|J_k d + r_k\|_2^2 \quad s.t.\ \|d\|_2 \leq \Delta_k$$

satisfies

$$q_k(0) - q_k(s_k^c) \geq \frac{1}{2}\|J_k^T r_k\| \min\left\{\frac{\|J_k^T r_k\|}{\|J_k^T J_k\|}, \Delta_k\right\}.$$

4. Find the KKT point(s) of the following problem:

$$\begin{aligned} \min_{x=(t_1,t_2)^T\in\mathbb{R}^2} \quad & t_1 + t_2 \\ \text{s. t.} \quad & 2 - 2t_1^2 - t_2^2 \geq 0, \quad t_2 \geq 0. \end{aligned}$$

5. Write a program to apply a penalty function method to solve

$$\begin{aligned} \min_{x=(t_1,t_2)^T\in\mathbb{R}^2} \quad & t_1 + t_2 \\ \text{s. t.} \quad & t_1^2 + t_2^2 - 2 = 0. \end{aligned}$$

Calculate the iterates generated in the first two iterations, namely, x_1 and x_2.