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A Robust and Sparse Process Fault Detection Method Based on RSPCA

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ABSTRACT As a method widely used in fault detection, principal component analysis (PCA) still has challenges in applicability due to its sensitivity to outliers and its difficulty in principal components (PCs) interpretation. In this paper, a robust sparse PCA (RSPCA) model is proposed to improve the robustness and interpretability of the PCA-based fault detection methods. Specifically, better robustness is achieved through capturing the maximal L1-norm variance while the sparse non-zero loadings are given for the PCs to achieve improved interpretability. A fault detection method is developed based on the RSPCA model, which uses the Genetic Algorithm (GA) to determine the number of non-zero loadings in each PC via the variance-sparsity tradeoff and defines T^2 and SPE statistics based on the selected PCs. In addition, an outlier removing strategy is proposed based on the SPE statistic, which can achieve further improvement to the robustness of the proposed method. The effectiveness and efficacy of the proposed method is evaluated by applying it to both two numerical simulation examples and the Tennessee Eastman (TE) process.

INDEX TERMS Fault detection, robust sparse PCA (RSPCA), genetic algorithm (GA), Tennessee Eastman (TE) process.

I. INTRODUCTION

In the modern complex industrial processes, fault detection is playing an increasingly critical role in ensuring product quality. Existing methods for fault detection and diagnosis mainly include the model-based methods and the data-driven methods. For the processes comprising many subsystems with complex relationships between each other, it is hard to obtain an accurate model for the whole system. In this context, data-driven fault detection methods have become very popular recently, which is partly ascribed to the development of data analysis algorithms.

Amongst existing methods, principal component analysis (PCA) is the most popular statistical data-driven process monitoring technique [1]–[5]. It divides the solution space into two orthogonal subspaces, i.e., the principal component subspace and the residual subspace. Specifically, the principal component subspace captures the most variation of the normal data while the residual subspace collects the unexplained variations inversely. Then T^2 statistic in the principal component subspace and SPE statistic in the residual subspacecan be derived for fault detection. Online data samples are projected into both the subspaces and the corresponding T^2 statistic and SPE statistic are calculated accordingly. If the statistics are out of their upper limit, then an abnormal situation is detected.

Although PCA gains great success in process fault detection, there are still two major shortcomings in practical applications: (1) the standard PCA is sensitive to outliers and (2) the principal components (PCs) obtained by PCA are difficult to interpret.

Many researchers have made efforts to address the first drawback, and it has been found out that this problem can be solved by the robust PCA (RPCA). Various robust alternatives have been proposed in recent studies [6], [7]. For example, in [8] and [9], L1-PCA was proposed and heuristic estimation method and convex programming method were applied to solve the problem. However, as a rotational variant, L1-PCA is quite computationally expensive. In [10], another rotational variant, i.e. R1-PCA, was proposed, which unfortunately was

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still hard to converge when the dimension of input space is relatively large. In [11], PCA-L1 was developed and it achieve rotational invariant PCA and the locally maximal solution is easy to find. Other discussions about RPCA can be found in [6].

The second drawback also can be solved by the sparse PCA (SPCA). For conventional PCA, PCs are combinations of all variables. SPCA can obtain sparse loading for each PC by adding L0-norm penalty. Numerous methods are developed to realize sparsity,such as lasso based sparse PCA (SPCA) [12], direct sparse PCA (DSPCA) [13], Greedy sparse PCA(GSPCA) [14] and sparse PCA by d.c. programming (DC-SPCA) [15]. Generally, the increase of sparsity means the decrease of explained variance. So, it is crucial to specify the number of non-zero loadings for each sparse PC. To tackle this numerically-hard combinatorial optimization problem, Genetic algorithms (GA) is employed in this paper. GA is a powerful evolutionary algorithm that has been successfully used in many applications [16]–[19] and can be be easily extended to solve such problems.

In order to enhance both robustness and sparsity at the same time, robust sparse PCA (RSPCA) is developed in [20]. Unlike conventional SPCA, RSPCA tries to maximize the L1-norm variance of the data. In this paper, a robust and sparse PCA process fault detection model based on RSPCA is proposed to address the two major drawbacks of the standard PCA. A new approach based on GA is proposed to select the robust sparse PCs. The selected robust sparse PCs are used to define T^2 and SPE statistics for process fault detection. It is noteworthy that mentioning that a outliers removing strategy based on SPE statistic is proposed in this paper to further increase the robustness of the proposed fault detection method. The proposed method achieves three advantages as follows: (1) the proposed fault detection method is less sensitive to outliers; (2) the obtained fault detection model is easier to interpret; (3) online process fault detection takes less computing time as the loadings for each PC are mostly zero, which has also been addressed by recently proposed distributed process monitoring framework. The effectiveness of the proposed method is tested on two simulation examples and Tennessee Eastman(TE) process.

The rest of this manuscript is as follows. Section II gives a brief review of the PCA and RSPCA models as well as an introduction to the evaluation index, and the evaluation index is also introduced. In Section III, the proposed robust and sparse fault detection model based on RSPCA is explained in detail. In Section IV, the proposed method is demonstrated and tested using both two numerical experiments and the TE process. The conclusions of this work are summarized in Section V.

II. ROBUST SPARSE PRINCIPAL COMPONENT ANALYSIS

A. PROBLEM FORMULATION

Let $X_{m\times n}$ represents the centered data matrix with n samples of m process variables. In conventional principal component analysis (PCA), also known as L2-PCA, one struggles to find a set of loading vectors w by solving the following optimization problem:

$$\begin{aligned} W^* = \underset{W}{argmax}\ & \left\| W^T X \right\|_2 \\ subject\ to:\ & W^T W = I_m \end{aligned} \tag{1}$$

However, the L2-norm is sensitive to outliers and several methods based on L1-norm are proposed to resolve this problem. Among them, the PCA-L1 proposed by Kwak is the most popular method [11]. The matching optimization problem is is given below:

$$\begin{aligned} W^* = \underset{W}{argmax}\ & \left\| W^T X \right\|_1 \\ subject\ to:\ & W^T W = I_m \end{aligned} \tag{2}$$

Another obvious drawback of PCA is the loading of each principal component (PC) are typically non-zero. This makes it difficult to explain the derived PCs. To address this difficulty, the sparse principal component analysis (SPCA) is developed. To control the sparsity, a L0-norm constraint is usually added, then the optimization problem becomes:

$$\begin{aligned} W^* = \underset{W}{argmax}\ & \left\| W^T X \right\|_2 \\ subject\ to:\ & W^T W = I_m \\ & \|w_i\|_0 \leqslant k, \quad i = 1, 2, \ldots, m \end{aligned} \tag{3}$$

where k is the tuning parameter to control the sparsity and may be different for each PC.

Combining the advantages of the above two approaches, robust sparse PCA (RSPCA) is proposed [20]. The mathematical representation is as follows:

$$\begin{aligned} W^* = \underset{W}{argmax}\ & \left\| w^T X \right\|_1 \\ subject\ to:\ & W^T W = I_m \\ & \|w_i\|_0 \leqslant k, \quad i = 1, 2, \ldots, m \end{aligned} \tag{4}$$

The RSPCA method not only can enhance the interpretability of extracted PCs but also is intrinsically less sensitive to outliers.

B. SOLUTION

The global optimal solution of (4) is hard to find when $m > 1$. To improve the problem, (4) is simplified to a series of $m = 1$ problems using a greedy search method. Although the solution obtained by successive greedy search may differ from optimal solution, an good approximation to the optimal solution has been provided. When $m = 1$, then (4) becomes the following optimization problem:

$$\begin{aligned} w^* = \underset{w}{argmax}\ & \left\| w^T X \right\|_1 \\ subject\ to:\ & w^T w = 1 \\ & w_0 \leqslant k \end{aligned} \tag{5}$$

For the L0-norm constraint problem, the non-convex L0 penalty will be relaxed to convex L1 penalty. As such, (5) can be reformulated as:

$$w^* = \underset{w}{argmax} \left\| w^T X \right\|_1 \\ subject\ to:\ w^T w = 1 \\ w_1 \leqslant t \tag{6}$$

Then we can solve (5) using the algorithm proposed by Meng et. al. [20]. It is noteworthy that the algorithm can only converge to a locally optimal solution. In this paper, we run the algorithm repeatedly with different random initial values and choose the largest solution corresponding to a specific objective function. The details of the algorithm can be found in [20].

After showing that one best sparse PC that maximizes the L1 objective function can be found(6), the algorithm can be easily extended to extract remaining features by applying the same procedure using a greedy strategy. The general procedure is listed as follows.

Algorithm 1 Robust Sparse PCA

Input: data matrix X, sparsity k, desired PC number p

Output: p k-sparse PCs $\{w_i\}_{i=1}^{p}$

set $w_0 = \vec{0} \in R^m$; denote $X^0 = X$

for $j = 1, 2, \ldots, p$ **do**

for all $i \in \{1, \ldots, n\}$, $x_i^j = x_i^{j-1} - w_{j-1}(w_{j-1}^T x_i^{j-1})$

find k-sparse PC vector w_j for X_j using algorithm in [20].

end for

Once w_i is obtained, the variance of the *ith* robust sparse PC is defined as [11]:

$$\lambda_i = \frac{\sum_{j=1}^{n} \left(w_i^T x_j\right)^2}{n} \tag{7}$$

In PCA, the eigenvalue of the associated eigenvector is equal to the variance. Similarly, λ_i can be viewed as the pseudoeigenvalue of w_i. Again, It is noteworthy that an enough good approximation of the optimal solution of RSPCA can be get through Algorithm 1 and the sparsity k for each PC may be different and should be carefully chosen.

C. THE SELECTION OF SPARSITY

Currently, the mainstream methods for selecting the sparsity k is in a sequential manner. For example, Yu et. al. proposed a method with minimum information loss to select the sparsity k in [21]. When increasing the value of k does not yield a significant increase in the total amount of variance for the first PC, an appropriate value of k is determined and the value of k is the same for all pseudoeigenvectors. Obviously, these methods search the sequence of k in a greedy way and the resulting solution is often locally optimal. When the monitored variables are large, the obtained local optimal solution will seriously affect the fault detection effect. In this paper, we propose a two-stage method to solve this problem from a global perspective. In the first stage, the cumulative percent variance (CPV) method is used to select p PCs for PCA-L1 [22]. In the second stage, the GA algorithm is used to specify the value of k for each PC of RSPCA as the RSPCA can be viewed as a sparse approximation of PCA-L1. The function to be maximized is the sparsity criterion (SC) below:

$$SC = \frac{V_a}{V_o} + \lambda \frac{\#_0}{p^2} \tag{8}$$

where V_a is the adjusted variance of RSPCA, V_o is the original variance of PCA-L1. RSPCA can be viewed as an approximation of RPCA, so $\frac{V_a}{V_o}$ indicates the goodness-of-fit. $\#_0$ is the total number of zero loadings in the RSPCA loadings matrix, so $\frac{\#_0}{m^2}$ indicates the sparsity. And λ controls the trade-off between goodness-of-fit and sparseness.

III. ROBUST SPARSE FAULT DETECTION

A. ROBUST SPARE MONITORING MODEL

When loading matrix $W = [w_1, w_2, \ldots, w_p] \in R_{m\times p}$ containing p PCs are obtained, the sample matrix X can be decomposed as follows:

$$X = TW^{\mathrm{T}} + \tilde{X} \tag{9}$$

where $T \in R_{n\times p}$ denotes the score matrix:

$$T = XW \tag{10}$$

For a new sample $x_{m\times 1}$, the projection of x on the monitoring model is:

$$t = xW \tag{11}$$

$$x = tW^{\mathrm{T}} + \tilde{x} \tag{12}$$

B. FAULT DETECTION STATISTICS

The RSPCA model uses the SPE and T^2 statistics to monitoring the process [23]. The SPE statistic is given by:

$$SPE = x^{\mathrm{T}} \left(I - WW^{\mathrm{T}}\right) x \tag{13}$$

The T^2 statistics is given by:

$$T^2 = x^{\mathrm{T}} W D^{-1} W^{\mathrm{T}} x \tag{14}$$

where x is a new observation sample to be monitored, D refers to the diagonal matrix of the l largest pseudoeigenvalues. The upper limits of SPE and T^2 statistics can be obtained through kernel density estimation (KDE) [24], [25]. KDE can be used to estimate the probability function of SPE and T^2 statistics. Then the 95% and 99% quantiles can be used as the control limits.

C. OUTLIERS REMOVING

In this paper, we propose an outliers removing strategy. In other words, the monitoring model is robust to outliers, and more importantly can find the outliers and remove them. The main idea of this strategy is that the outliers have more residuals than normal data, which indicates that the sample with high SPE statistic is more likely to be an outlier.

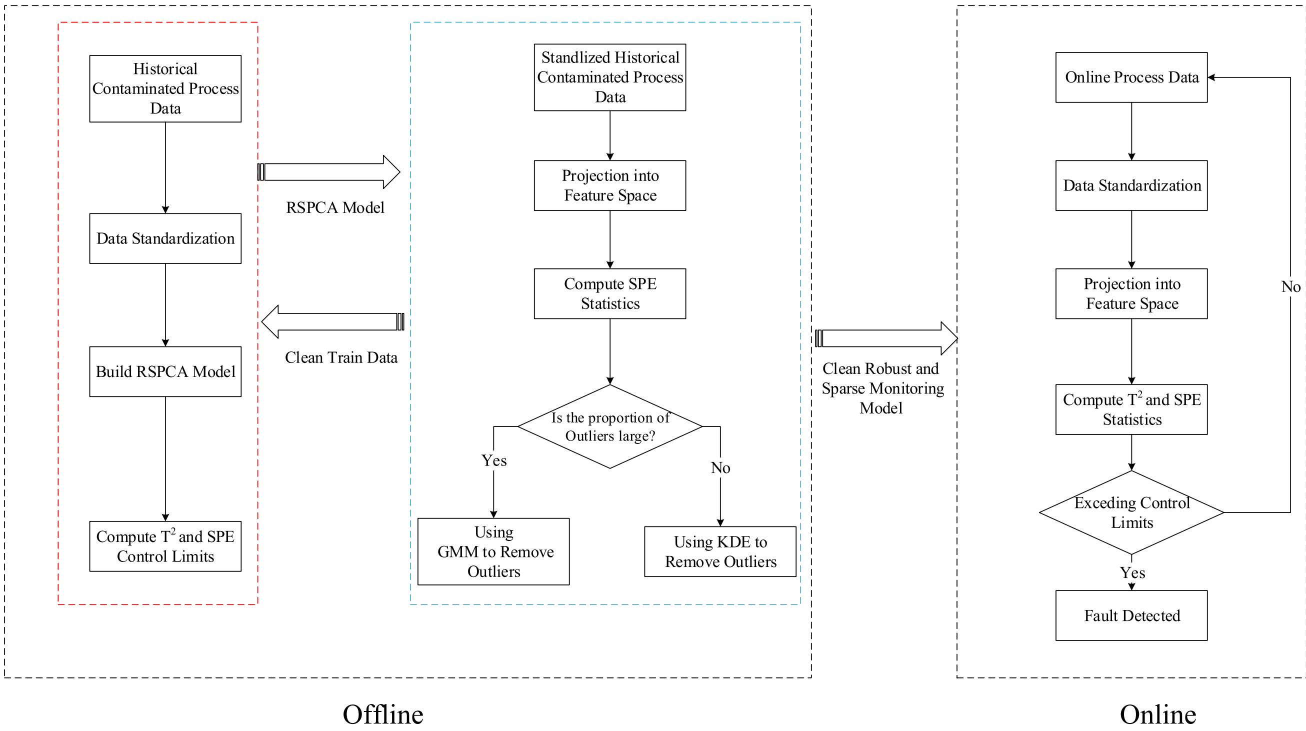

FIGURE 1. Illustrative diagram of the proposed RSPCA-based fault detection approach.

Common novelty detection methods can be used [26]–[30]. In this paper, if the mixture of outliers is small, a non-parametric approach based on kernel density estimation is recommended [31]–[33] . Otherwise, a parametric approach based on the Gaussian mixture model (GMM) is recommended to remove the outliers [26], [28], [34]. The details of this strategy will be demonstrated and discussed in the testing of Numerical Examples I and II.

D. EVALUATION INDEX

In order to evaluate the monitoring performance, the conventional way is to check three index: fault alarm rate(FAR), fault detection rate(FDR) and the fault detection delay(FDD). Let J and J_{th} be the test statistic and corresponding upper limit. Then FAR, FDR and FDD are given by [35]:

$$FAR = pro\,(J > J_{th}|f = 0) \tag{15}$$

$$FDR = pro\,(J > J_{th}|f \neq 0) \tag{16}$$

$$FDD = T\,(J > J_{th}) - T\,(fault\ occur) \tag{17}$$

IV. MONITORING PROCEDURE

The detailed schematic diagram of complete monitoring procedure is shown in Figure 1. The procedure consists of two stages.

Stage I: Offline modeling. For offline modeling, the first substage is to build an RSPCA model, and the second substage is to remove the outliers.

Step I-1-1: Normalizing the contaminated training data set to zero mean and unit variance.

Step I-1-2: Using CPV to choose the p PCs, and GA is applied to choose a set of k values: $\{k_1, \ldots, k_p\}$, bulid RSPCA model based on the set of p robust sparse PCs generated. Go to step I-1-3 and step I-2-1.

Step I-1-3: Determining the control limits of T^2 and SPE statistics.

Step I-2-1: Projecting the normalized historical contaminated process data into the feature space.

Step I-2-2: Computing the corresponding SPE statistic for each sample.

Step I-2-3: Using novelty detection methods to remove outliers. When the proportion of outliers is large, GMM is recommended to remove the outliers. Otherwise, KDE is recommended to remove the outliers.

After removing the outliers found in this substage, re-executing the steps in the first substage. The algorithm flow can be executed multiple times. In this paper, it is executed only once.

Stage II: Online modeling

Step II-1: Normalizing the online process data using the mean and variance of clean train data.

Step II-2: Projecting the normalized sample into the feature space.

Step II-3: Computing the corresponding T^2 and SPE statistics.

Step II-4: Checking whether computed T^2 and SPE statistics exceed their control limits. If yes, detect a fault; otherwise, go to step II-1 to monitor the next sample.

V. CASE STUDY

In this section, three cases are utilized to evaluate the proposed method, which include two numerical examples and the Tennessee Eastman (TE) process. Three methods, namely

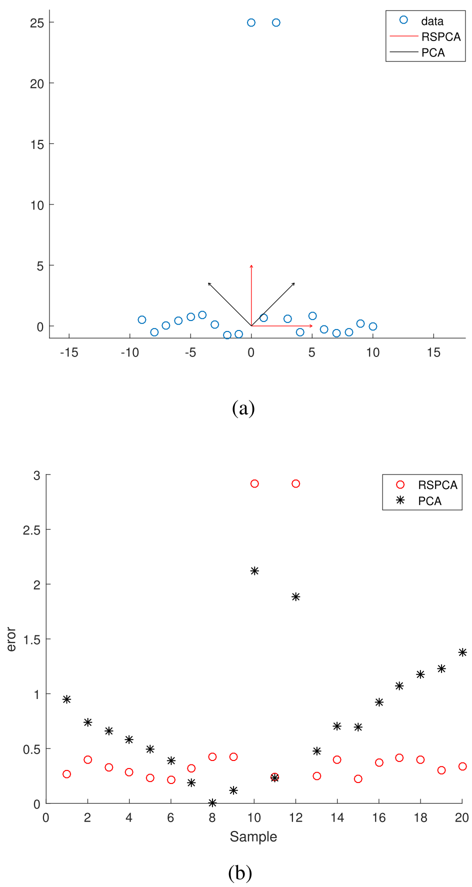

FIGURE 2. **(a)the toy data points with two outliers, and the PCs obtained by PCA and RSPCA (b)the reconstruction errors of PCA and RSPCA by using the first PC.**

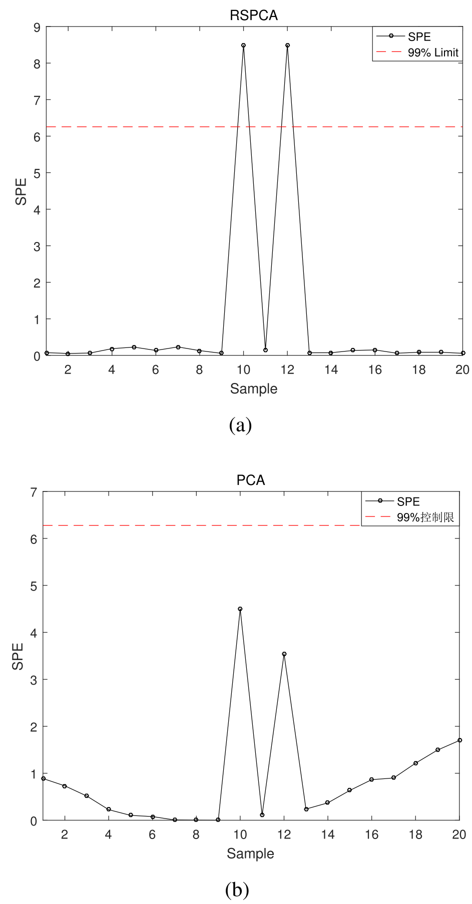

FIGURE 3. **(a) SPE statistic based monitoring chart for PCA (b)SPE statistic based outliers removing strategy for RSPCA.**

traditional PCA, robust nonlinear sparse PCA (RNPCA) [21] and the proposed RSPCA method, are used for process fault detection. The first numerical example is used to illustrate the proposed outliers removing strategy based on SPE statistic. The second numerical example is used to show the overall fault detection procedure based on RSPCA. TE process is a real industrial process used to demonstrate the advantages of the RSPCA based fault detection model over PCA based model and RNPCA based model. It is noteworthy that for the monitoring statistics, 99% confidence limits are adopted as the fault detection threshold.

A. A TOY PROBLEM WITH OUTLIERS

We first demonstrate the robustness of the proposed RSPCA process monitoring model and the outliers removing strategy based on SPE statistic in a 2D toy data set. The first dimension of the data is generated by picking x_i from the interval $[-9, 1]$ with a same step size 1. The second dimension of the data are generated by sampling y_i from the uniform distribution $[-1, 1]$, except at $x_i = 0$ and $x_i = 1$, and the value of $5y_i$sis set around 25. The generated data are depicted in Figure 2(a). It is noteworthy that if the outliers are removed, the first principal component of the data should be the sparse vector (1,0). It can be seen that the first principal obtained by RSPCA is (1,0) while the first principal obtained by PCA is (0.707,0.707). This indicates that PCA is more sensitive to outliers and the PCs are seriously affected. In Figure 2(b), the reconstruction errors of PCA and RSPCA are shown. It can be seen that the errors of RSPCA are larger than that of PCA at $x_i = 0$ and $x_i = 1$ while the errors of RSPCA are smaller than that of PCA at most of the other points. As RSPCA captures the real characteristics of the data, the reconstruction errors of outliers are larger and most of the information of outliers is left in the residual subspace. This prompts us to see if the SPE statistics of outliers exceed the upper limit.

In this experiment, the percentage of outliers is small, so kernel density estimation is applied remove outliers. That is to say the monitoring chart of SPE statistic for RSPCA can be used to remove outliers. The monitoring charts of SPE statistic for RSPCA and PCA when keeping the first principal component are shown in Figure 3. For standard PCA based models, outliers are treated as the normal data, son the monitoring chart of SPE statistic, the SPE statistics of the outliers do not exceed upper limit due to the fact that information about them has been captured by the model. However,for the RSPCA based model, the model is robust

TABLE 1. The loadings and variance of PCA and RSPCA for the simulation example.

	PCA			RSPCA			
	PC1	*PC2*	*PC3*	*PC1*	*PC2*	*Adjusted PC1*	*Adjusted PC2*
x_1	0.3541	-0.3530	-0.003	0.5004	0	0	0.5006
x_2	0.3523	-0.3549	-0.003	0.4990	0	0	0.4990
x_3	0.3543	-0.3526	-0.0029	0.5008	0	0	0.5007
x_4	0.3520	-0.3551	-0.0031	0.4997	0	0	0.4997
x_5	0.3548	0.3522	0.0075	0	0.5012	0.4883	0
x_6	0.3532	0.3539	0.0076	0	0.4999	0.4813	0
x_7	0.3544	0.3526	0.0076	0	0.5003	0.5155	0
x_8	0.3532	0.3537	0.0076	0	0.4986	0.5140	0
x_9	0.0051	0.0112	-0.7068	0	0	0	0
x_{10}	0.0040	-0.7072	0.593	0	0	0	0
Variance	4.78%	39.04%	11.18%	39.37%	39.37%	34.41%	34.38%

to outliers and does not capture the information of outliers. As such, the SPE statistics exceed the upper bound. This means that the SPE statistic based monitoring chart can be used to find outliers and remove them.

B. A SIMPLE SIMULATION EXAMPLE

In this subsection, a simulated sparse multivariate process proposed in [12] is first considered, and its mathematical model is given below:

$$\begin{cases} X_i = V_1 + \epsilon_i^1, & \epsilon_i^1 \sim N(0,1),\ i = 1,2,3,4 \\ X_i = V_2 + \epsilon_i^1, & \epsilon_i^2 \sim N(0,1),\ i = 5,6,7,8 \\ X_i = V_3 + \epsilon_i^1, & \epsilon_i^3 \sim N(0,1),\ i = 9,10 \\ \epsilon_i^1 \ \textit{are independent}, & j = 1,2,3,\ i = 1,\ldots,10 \end{cases} \tag{18}$$

where

$$\begin{cases} V1 \sim N(0,290) \\ V2 \sim N(0,300) \\ V3 = -0.3 * V1 + 0.925 * V2 + \epsilon,\ \epsilon \sim N(0,1) \\ V1, V2 \ \textit{and}\ \epsilon \ \textit{are independent} \end{cases} \tag{19}$$

Note that V1, V2, V3 correspond to three hidden factors. In this simulation example, we can explain both the robustness and sparsity and, and on this basis demonstrate the complete fault detection procedure based on RSPCA. In this study, a train data collection contains 360 normal observations and 40 abnormal observations are generated. The abnormal observations are obtained by letting $x_i = 0$, $i = 1, 2, \ldots, 8$ and $x_i = \zeta_i, i = 9, 10$, where $\zeta_i \sim N(0, 3000)$. Two fault validating datasets with 400 samples are simulated to test the proposed algorithm. The fault dataset is designed as follows:

- The system is running under normal mode, and V_1 is imposed with a step bias -400 from the 161st sample.
- The system is running under normal mode, and V_2 has a ramp change with the slope of 20 from the 161st sample.

First, CPV is used to select the first p PCs of PCA-L1, the cumulative contribution rate of robust principal components is set as 75%. Then we use the GA algorithm to determine the number of non-zero loading for the selected PCs when building the RSPCA model. In this paper, the real

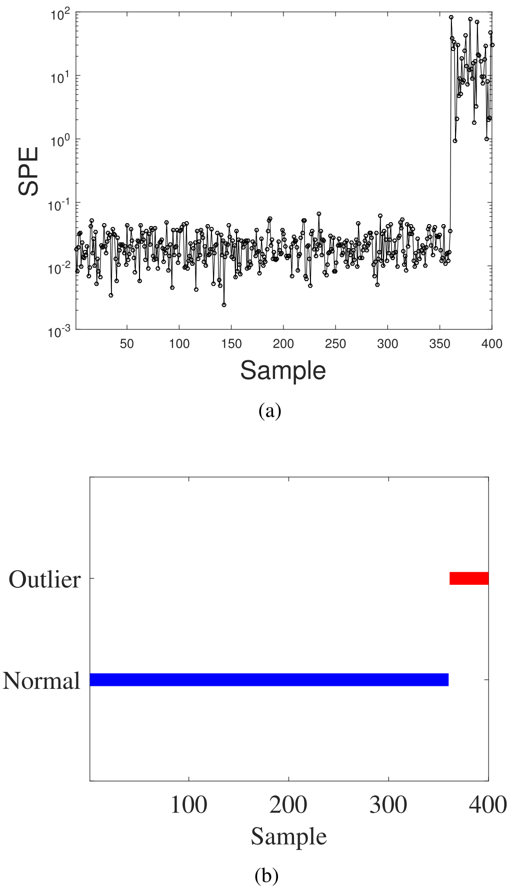

FIGURE 4. **(a)The SPE monitoring chart of RSPCA (b)The novelty detection results obtained by GMM.**

coded GA for solving integer optimization problems proposed in [36] is used. The GA toolbox of MATLAB is used in the experiment. SC is adopted as the fitness function and λ is specified as 1.4, The population size is set as 50 and the generations are set as 10 in detail. After 10 iterations, then the obtained result is (4, 4), this means that RSPCA correctly identifies the set of important variables in although there are outliers. As is shown in Figure 4, for the RSPCA fault detection model, the proposed outliers removing strategy based on SPE statistic using GMM can find these outliers. After

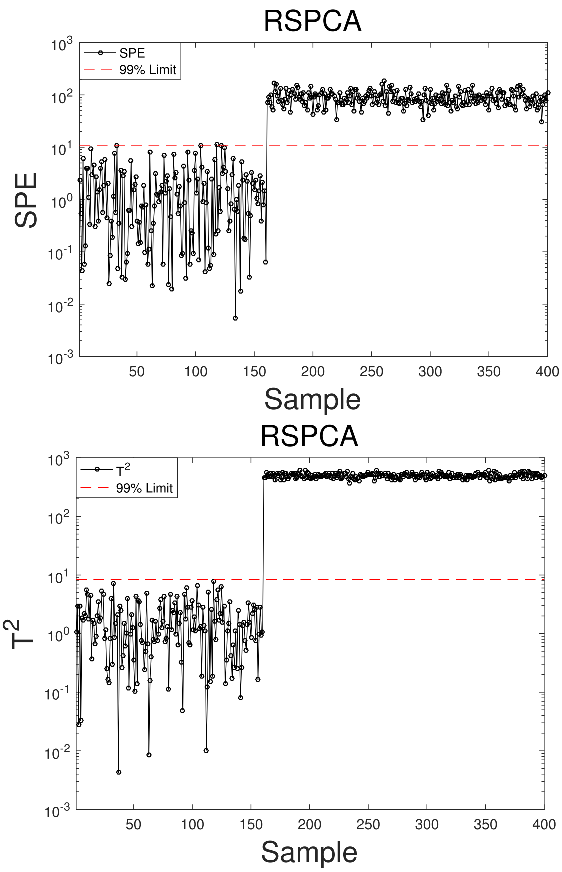

FIGURE 5. RSPCA based monitoring for fault 1.

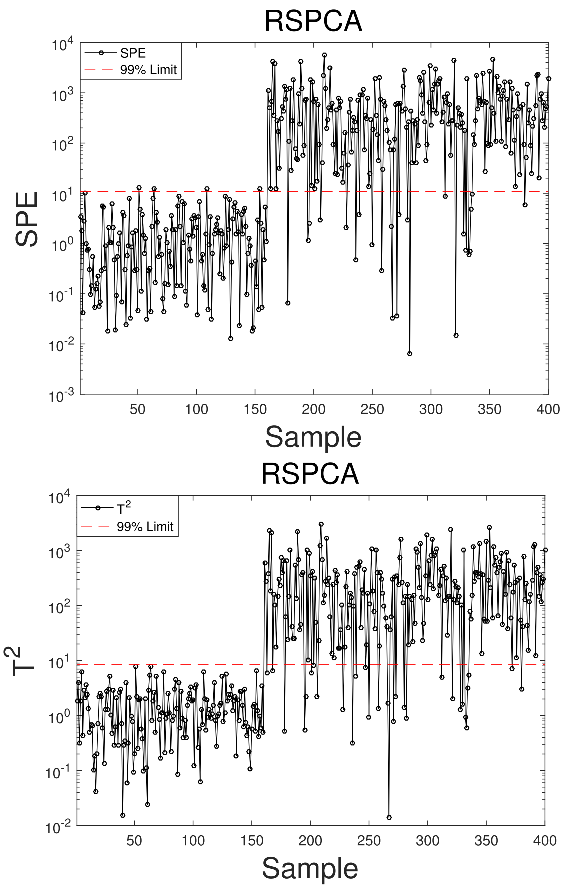

FIGURE 6. RSPCA based monitoring for fault 2.

TABLE 2. FDR(%) of designed faults obtained by RSPCA and PCA.

fault	RSPCA		PCA	
	SPE	T^2	SPE	T^2
1	100	100	11.25	100
2	91.67	87.5	23.33	85.83
Average	95.84	93.75	17.29	92.92

removing the outliers, a new model can be built on the clean dataset. In this study, the modified model is used for fault detection.

The loadings of the principal components of PCA and RSPCA are shown in Table 2. Clearly, RSPCA identifies the sets of underlying variables correctly when the training set consists of 10% outliers. However, the first PC and the second PC are in reverse order. After removing the outliers, adjusted PC1 and PC2 are totally correct.

The monitoring charts of RSPCA are shown in Figures 5 and 6. The monitoring charts of PCA are shown in Figures 7 and 8 for comparison. The false detection rates of RSPCA and PCA are listed in Table 2. For the fault 1 validating dataset, the false detection rates for PCA T^2 and SPE are 11.25% and 100%, respectively. Compared to PCA, the corresponding false detection rates of RSPCA are 100% and 100%, which shows a significant improvement. The PCs obtained by PCA are seriously affected by the outliers. The monitoring performance of the SPE statistic is poor as PCA tries to reduce the reconstruction errors of all input data. Similar phenomenon can be found when monitoring fault 2. The average false detection rates of RSPCA are 95.84% and 93.75% for SPE and T^2 statistic, superior to the SPE and T^2 statistic of PCA. The RSPCA achieves obvious performance improvement in process monitoring with the designed faults when outliers are removed.

C. TENNESSEE EASTMAN PROCESS

In this subsection, the Tennessee Eastman (TE) process is used to test the performance of the proposed fault detection method based on RSPCA. Specifically, the TE process is proposed by Downs and Vogel in 1993 and, which has since then been widely used in process control and process monitoring [37]. There are five major operation units in the process: a reactor, a product condenser, a recycle compressor, a flash separator and a product stripper. The flowchart of the TE process is shown in Figure 9. There are 21 faults in the TE process, as listed in Table 4. For each fault, there are 960 samples and the fault is introduced after the 161st sample. The data set can be downloaded from the website: http://depts.washington.edu/control/LARRY/TE/download.html. A detailed description of the faults can also be found on the website. In total,

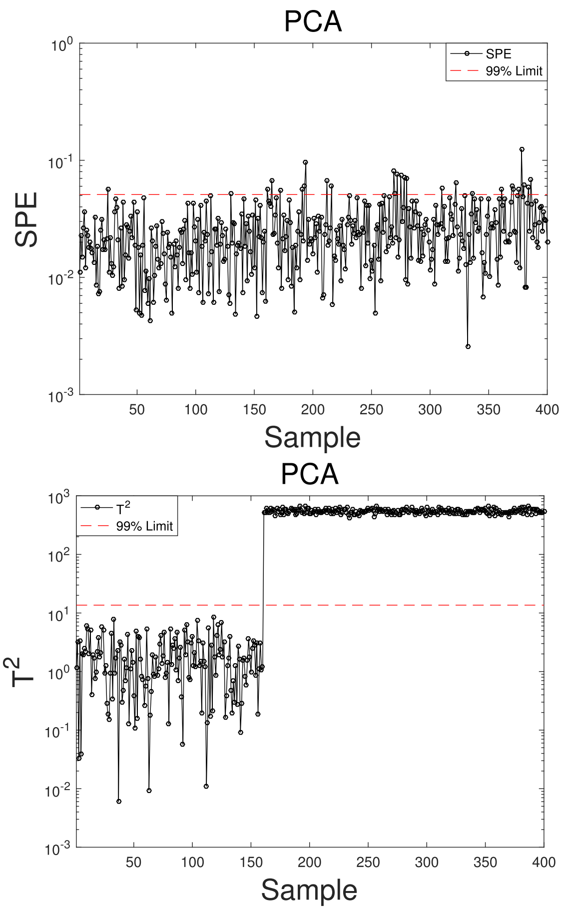

FIGURE 7. PCA based monitoring for fault 1.

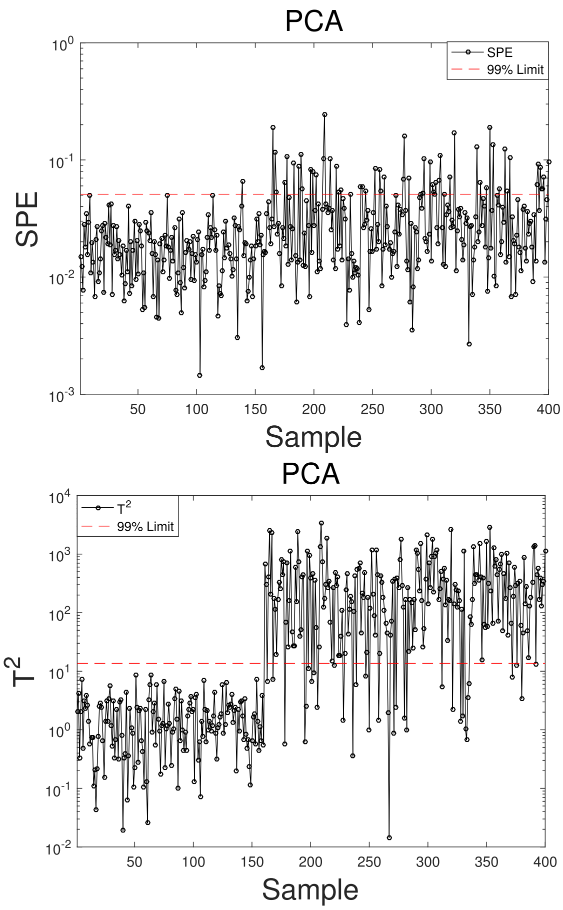

FIGURE 8. PCA based monitoring for fault 2.

TABLE 3. Monitored variables of TE process.

Variable	Description	Unit	Variable	Description	Unit
v_1	A feed (Stream 1)	kscmh	v_{18}	stripper temperature	^0C
v_2	D feed (Stream 2)	kgh^{-1}	v_{19}	Stripper steam flow	kgh^{-1}
v_3	E feed (Stream 3)	kgh^{-1}	v_{20}	Compressor work	kW
v_4	A and C feed (Stream 4)	kscmh	v_{21}	Reactor cooling water outlet temperature	^0C
v_5	Recycle (Stream 8)	kscmh	v_{22}	Separator cooling water outlet temperature	^0C
v_6	Reactor feed rate (Stream 6)	kscmh	v_{23}	D feed flow (stream 2)	kgh^{-1}
v_7	Reactor pressure	kPa	v_{24}	E feed flow (stream 3)	kgh^{-1}
v_8	Reactor level	%	v_{25}	A feed flow (stream 1)	kscmh
v_9	Reactor temperature	^0C	v_{26}	A and C feed flow (stream 4)	kscmh
v_{10}	Purge rate (Stream 9)	kscmb	v_{27}	Compressor recycle valve	%
v_{11}	Product separator temperature	^0C	v_{28}	Purge valve (stream 9)	%
v_{12}	Product separator level	%	v_{29}	Separator pot liquid flow(stream 10)	m$^3h^{-1}$
v_{13}	Product separator pressure	kPa gauge	v_{30}	Stripper liquid product flow(stream 11)	m$^3h^{-1}$
v_{14}	Product separator underflow (stream 10)	m$^3h^{-1}$	v_{31}	Stripper steam valve	%
v_{15}	stripper level	%	v_{32}	Reactor cooling water flow	m$^3h^{-1}$
v_{16}	stripper pressure	kPa gauge	v_{33}	Condenser cooling water flow	m$^3h^{-1}$
v_{17}	Stripper underflow (stream 11)	m$^3h^{-1}$			

33 process variables including 22 continuously monitored variables and 11 manipulated process variables are monitored for online fault detection, as listed in Table 3.

TABLE 4. The descriptions of faults in the TE process.

Fault	Description	Type
1	A/C feed ratio, B Composition constant(Stream 4)	Step
2	B composition, A/C Ratio constant(Stream 4)	Step
3	D Feed temperature(Stream 2)	Step
4	Reactor cooling water inlet temperature	Step
5	Condenser cooling water inlet temperature	Step
6	A Feed loss(Stream 1)	Step
7	C Header pressure loss âĂŞ reduced availability(Stream 4)	Step
8	A,B,C Feed composition(Stream 4)	Random
9	D Feed temperature(Stream 2)	Random
10	C Feed temperature(Stream 4)	Random
11	Reactor cooling water inlet temperature	Random
12	Condenser cooling water inlet temperature	Random
13	Reaction kinetics	Slow Drift
14	Reactor cooling water valve	Sticking
15	Condenser cooling water valve	Sticking
16-20	Unknown	Unknown
21	The valve for stream 4 was fixed at the steady-state position	Constant

In the first part of this experiment, 500 normal samples with 0% contamination is used to build the fault detection model for RSPCA, PCA and RNPCA. Similar to example 2, we first use GA to find robust sparse PCs(RSPCs) for RSPCA. λ is set as 1.4, the population size is set as 100 and the generations are set as 50. The cumulative contribution rate of robust principal components is set as 95%. A total of 20 RSPCs are extracted from the process data. The non-zero elements of each RSPC are listed in Table 6.

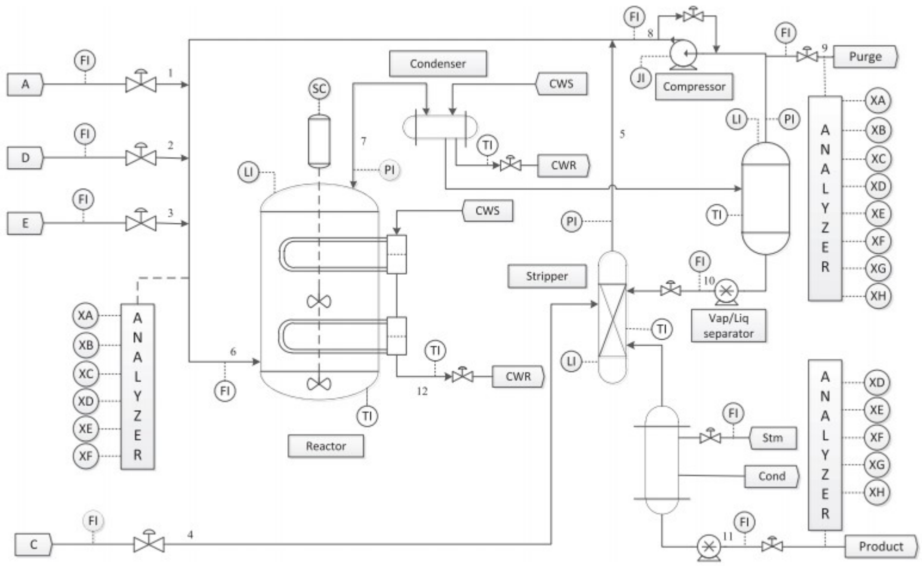

FIGURE 9. Tennessee eastman process flow diagram.

TABLE 5. Non-zero variables for each RSPC.

RSPC	Non-zero Variables	RSPC	Non-zero Variables
1	v_{18}, v_{19}, v_{31}	11	v_{22}
2	v_7, v_{13}, v_{16}	12	v_6
3	v_{12}, v_{14}	13	v_2
4	v_{15},v_{30}	14	v_{24}
5	v_{17},v_{33}	15	v_8
6	v_1,v_{25}	16	v_4
7	v_{10},v_{28}	17	v_5
8	v_9,v_{26}	18	v_{23}
9	v_{20}	19	v_3
10	v_{21}	20	v_{26}

The extracted 20 RSPCs contains almost all process variables. The sparse characteristics of RSPCs reveal the physical links between process variables. For example, RSPC1 indicates the effect of stripper on the whole process (v_{18}, v_{19}, v_{31}), RSPC2 includes all the variables associated with pressure (v_7, v_{13}, v_{16}) and RSPC3 reveals the interaction between underflow of product separator and product level (v_{12}, v_{14}).

After getting the RSPC, we can get the robust sparse loading matrix. The RSPCA-based fault detection model is built on the sparse loading matrix. The first 15 fault are used to test the performance. PCA and RNPCA are used for comparison. Fault detection rate (FDR) is introduced as the evaluation index. The false detection rates of RSPCA, PCA and RNPCA are given in detail in Table 4. The fault detection results of PCA are from [38] and the fault detection results of RNPCA are from [21]. We can see that RSPCA is superior to PCA and RNPCA when there are no confounding outliers in the training set. This indicates that RSPCA can obtain true relations among variables compared

TABLE 6. Fault detection rates(%) for PCA, RNSPCA and RSPCA (for each fault condition, the highest FDR is marked bold).

Fault	**RSPCA**		**PCA**		**RNSPCA(Spearman)**		**RNSPCA(Kendall)**	
	T^2	SPE	T^2	SPE	T^2	SPE	T^2	SPE
1	99.5	**99.8**	99.5	99.1	99.3	99.5	99.8	99.6
2	95.6	98.3	98.4	98.5	**98.8**	98.5	98.8	98.6
3	**10.8**	0.5	0.6	3.6	8.6	1.1	7.0	1.6
4	31.4	**100**	98	21.8	5.1	1.5	7.3	1.3
5	30.6	32	21.7	25.7	29.5	21.5	**32.3**	19.9
6	99.5	**100**	99.9	98.9	100	100	100	100
7	**100**	100	99.9	99.9	50	35.3	51.5	32.6
8	97.8	96.4	96.8	97.4	97.3	**98.5**	97.4	97.8
9	**6.6**	4.4	1	3.4	3.0	3.8	3.6	3.0
10	50.9	62.1	15.4	36.7	58.1	50.6	**62.9**	45.5
11	50.5	**80.1**	63.8	41.4	25.8	35.8	27.1	32.9
12	99	**99.4**	92.5	98.5	98.5	98.1	98.1	97.4
13	94.9	93.5	95	94.3	**96.0**	94.6	96	93.8
14	100	**86.4**	99.9	98.8	91.3	100	85.4	100
15	12.1	8.4	0.7	3.5	12.0	6.0	**16.1**	2.5
Average	65.3	**70.8**	61.28	57.62	58.2	56.3	58.9	55.1

TABLE 7. Fault detection results for RSPCA and PCA (for each fault condition, the highest FDR, the lowest FAR and the minimum FDD are marked as bold).

Fault	RSPCA						PCA					
	T^2			SPE			T^2			SPE		
	FDR(%)	FAR(%)	FDD(ns)	FDR(%)	FAR(%)	FDD(ns)	FDR(%)	FAR(%)	FDD(ns)	FDR(%)	FAR(%)	FDD(ns)
1	**83.5**	**0**	7	31.5	1.8	**0**	31.4	0	8	26.5	1.8	0
2	**98.5**	**5**	5	98.1	**0**	12	26.4	0	22	12.5	2.5	5
3	**25.3**	13.8	**15**	7.3	5.6	27	4.8	**0.6**	38	7.9	5	55
4	19.8	2.5	1	**20.5**	3.8	**0**	14.6	**1.3**	1	7.5	1.9	69
5	**30.4**	2.5	**0**	22.9	2.5	1	22.4	**0.6**	1	16.8	1.9	0
6	65.8	1.8	15	**100**	**0**	1	63.75	1.25	15	100	0	**0**
7	45.3	**0**	1	**54.6**	**0**	0	29.4	1.25	1	15.5	1.9	0
8	**97.9**	1.9	**15**	69	1.88	15	81.1	**0**	31	47.5	0.6	20
9	**15.8**	24.3	**1**	7.8	4.3	1	5.8	**6.9**	1	6.5	8.8	18
10	**47.9**	1.9	6	43.3	5	**0**	36.3	**0.6**	8	35.3	1.3	0
11	38.3	3.8	7	**59.1**	1.9	**6**	18.5	**0.6**	11	38.8	3.1	6
12	**98.5**	**0**	3	85.8	0.6	**2**	76.3	0	8	46.3	0	2
13	94.8	0.6	26	**95.1**	0.6	37	79.1	0	49	51.1	**0**	**25**
14	98.1	5.6	1	**100**	5	**0**	53.8	**0.6**	3	3.5	1.9	51
15	**20.9**	5	**63**	9.9	1.9	63	5.5	**0**	242	11.8	0.6	124
16	**46.3**	32.5	**1**	45.8	6.9	1	34.4	8.2	2	30.8	**1.9**	14
17	45.3	**0**	38	**45.5**	0.6	40	37.6	1.3	**2**	13.1	4.4	10
18	73	0	114	88.8	0	88	85.1	0	98	85.8	0	101
19	31.5	4.3	95	94.3	**96.0**	94.6	12	0	11	30.9	0	1
20	43.8	0.6	**8**	**47.9**	0.6	8	33.5	**0**	90	8.3	**0.6**	102
21	50.5	12.5	21	**55.5**	6.9	**1.9**	8	2.5	251	5.3	**0.6**	40

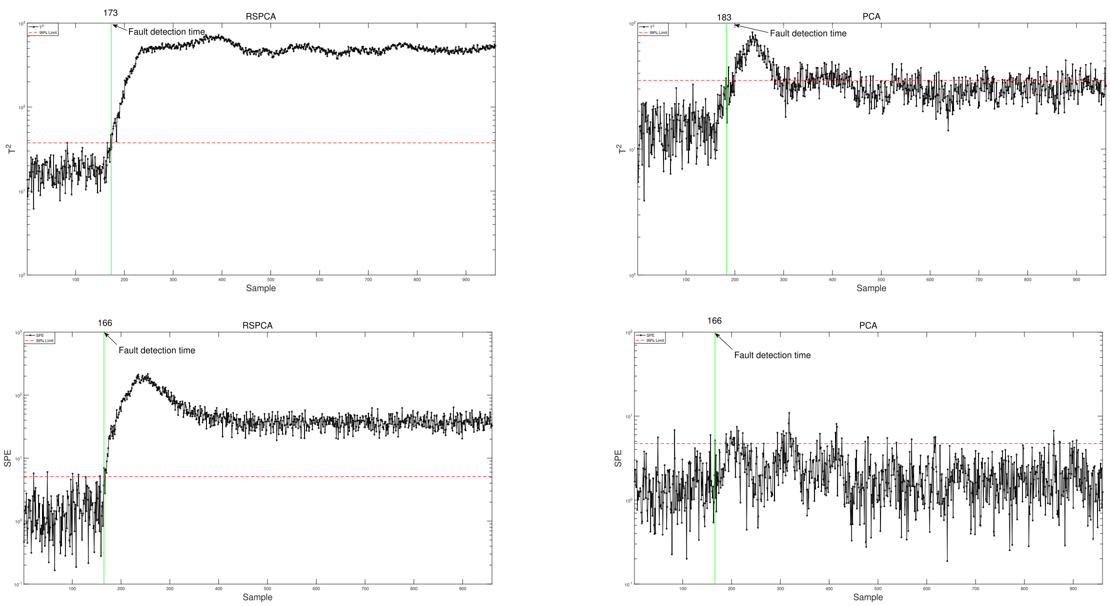

FIGURE 10. RSPCA and PCA based monitoring for fault 2.

to PCA and RNPCA. As shown in Table 4, three methods have low FDRs for fault 3, 9 and 15 as these faults have little influence on the process. However, RSPCA has the highest FDRs for fault 3 and 9 among the three methods. For fault 1, 2, 6, 8, 12 and 14, the RSPCA-based method, PCA-based method and RNPCA all have high FDRs of about 100%. The RSPCA-based method has much better monitoring performance than the PCA-based method and RNPCA for faults 4, 5 and 11. In comparison with the PCA-based method, RNPCA(Spearman) and RNPCA(Kendall), the mean FDR of T^2 statistic of the RSPCA-based method is increased by 6.6%, 12.2% and 10.9%, respectively. The mean FDR of SPE statistic is increased by 22.9%, 25.8% and 28.5%, respectively. In conclusion, the mean FDRs of RSPCA are higher than those of PCA and RNPCA for both T^2 and SPE statistic.

In the second part of this experiment, the robustness of the proposed method is tested on the contaminated data. The percentage of the contamination is set as 10% to verify the robustness by by making comparison to PCA. For each fault,

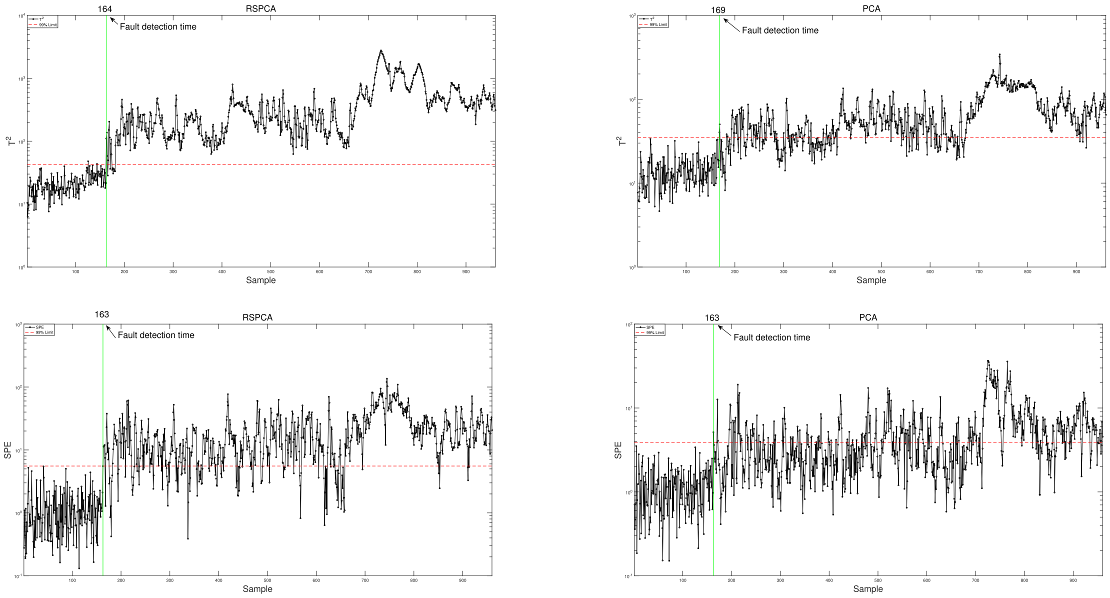

FIGURE 11. **RSPCA and PCA based monitoring for fault 12.**

the training data set will be mixed with 10% corresponding fault data. The parameters of GA for the second part are the same as those for the first part. FDR, FAR and FDD are adopted as the evaluation index. The detailed fault detection results are shown in Table 6.

As is shown in Table 6, both RSPCA and PCA have low FARs and FDDs for T^2 and SPE statistics. However, the FDRs of RSPCA are much higher than those of PCA. For faults 2, 8 and 12, RSPCA-based method can detect the fault with almost 100% FDRs.The RSPCA-based method has higher FDRs than the PCA-based method for other faults. The mean FDR of T^2 statistic for RSPCA-based method is 55.8%, 43.8% higher than the the PCA-based method. The mean FDR of SPE statistic for RSPCA-based method is 56.3%, 86.4% higher than the PCA-based method. The results show that RSPCA is much better than PCA when the outliers exist.

The monitoring charts of fault 2 and fault 12 are selected to demonstrate the monitoring performance of RSPCA and PCA. In Figure 10, The T^2 statistic of RSPCA detects the fault after the 186th sample, resulting in a low FDD of 5 and a high FDR of 98.5%. On the contracy, both T^2 statistic and SPE statistic of PCA detect the fault with high FDD (22 and 5). The faulty samples after the 400th are missed, so the FDRs(26.4% and 12.5%) are very low. In Figure 11, the T^2 and SPE statistics of the RSPCA-based method can detect the fault in time for fault 12. For the PCA-based method, the SPE statistic detects the fault only with a FDR of 46.3%. The results indicate that the fault detection performance of PCA-based method is heavily affected by the outliers while the RSPCA-based method has better fault detection performance than the PCA-based method.

VI. CONCLUSIONS

In this paper, a robust and sparse fault detection model based on RSPCA is proposed. The robustness is achieved by using L1-norm maximization while L0-norm penalty is added to realize sparsity. A sparsity criterion which balances the sparsity and variance is proposed and the GA algorithm is adopted to obtain a sequence of sparsity k. Based on the selected RSPCs, an outliers removing strategy based on SPE statistic is proposed. For the outliers, the SPE statistics are significantly higher than those of normal data. As such, novelty detection methods can be used to remove the outliers. KDE and GMM based methods are applied to find the outliers.

Three computational experiments are designed to demonstrate the outliers removing strategy and show the superior performance of the proposed method. Specifically, the first experiment is designed to illustrate the outliers removing strategy. The second experiment is designed to show the overall procedure of the RSPCA-based fault detection method. The third experiment uses the TE process to show the advantages of the proposed method. The results show that the RSPCA-based method has much better fault detection performance that PCA-based methods and recently popular RNPCA-based methods.

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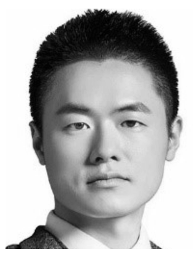

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