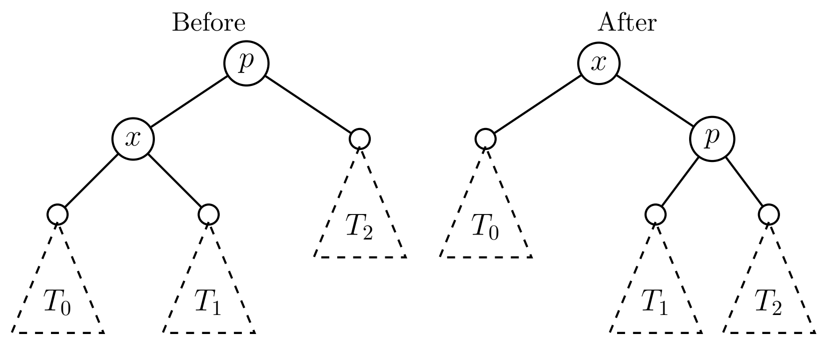
CSCI 104L Lecture 25 : Splay Trees

Splay Trees

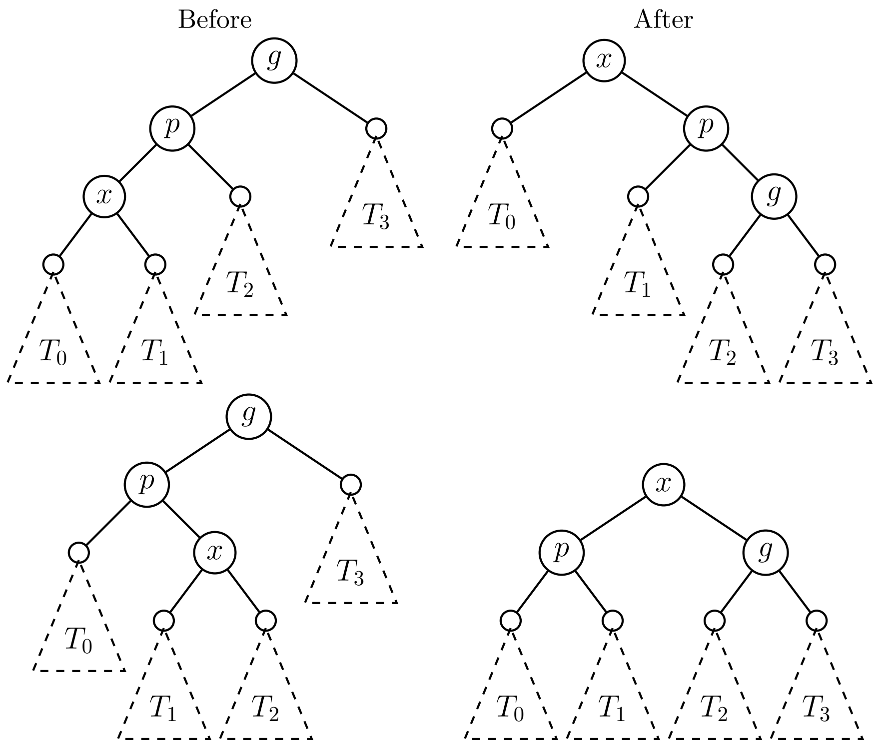
Question 1. We saw AVL trees, which guaranteed a search time of $O(\log n)$, where n is the number of nodes in the tree. Is it reasonable to optimize to guarantee the worst-case lookup time is $O(\log n)$, or are there other things we should be worried about?

In a *splay tree*, our goal is that recently-used data should be near the top. We add and search as per a normal binary search tree, except when we're done, we're going to splay it to the root.

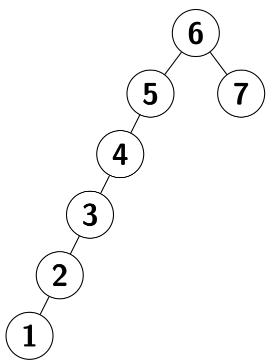
If we just inserted x and it is now a child of the root (whether because we inserted it there or because it splayed up to that location), the adjustment is easy:



Otherwise, if x is not a child of the root (and is also not a child of the root), we're going to bring it up two steps. There are two possibilities (four if you count the mirrors):



Question 2. Consider the following splay tree; what does it look like after `find(1)`?



Question 3. From the tree that resulted, what does it look like after `find(3)`?

Question 4. Suppose we start with an initially empty splay tree and then insert the values $1, 2, \ldots n$ in order and then call `find(1)`. What is the total running time? What is the amortized time per operation?

The "Splay Trees Are Awesome" Conjecture: there is a conjecture that, for any sequence of binary search tree operations, splay trees are asymptotically as fast as any other implementation (basic, AVL, Red/Black, etc). Some sets can be done in less than $O(\log n)$ time, and the conjecture is that if one binary search tree implementation can do it, so can splay trees.