Lab Report

EXPERIMENT 1: Coulomb's Law

Group 10
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We hereby declare that we are the sole authors of this Lab Report and that we have not used any sources other than those listed in the bibliography and identified as references. We further declare that We have not submitted this thesis at any other institution in order to obtain a degree.

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2 Introduction

In this experiment, our main goal is concerned with measuring the force of electrostatic attraction between 2 charges, which would make us, in turn, attempt to verify Coulomb's law.

Coulomb's law of electrostatic attraction describes how electric charges exert forces on one another. Much like the gravitational force, electrostatic forces follow an inverse square relationship with distance. However, unlike gravity,which always attracts, electrostatic forces can either attract or repel, depending on whether the charges are of opposite or similar sign. Specifically, Coulomb found that the magnitude of the electrostatic force between two point charges, Q_1 and Q_2 separated by a distance d, is given by Equation (2.1)

$$F_e = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{Q_1 Q_2}{d^2} \tag{2.1}$$

Where ϵ_0 is the permittivity of free space (which sets the scale of electrostatic interactions in a vacuum), and ϵ_r is the relative permittivity of a medium such as air.

Despite this simple mathematical form, verifying Coulomb's law in a real lab requires several considerations. When conducting an experiment on electrostatic forces, one typically uses spherical conductors, since according to Gauss' law, a charged sphere at sufficiently large distances from its surface, acts like a point charge located at its center. However, practical complications arise if the spheres are brought too close together, since non-uniform charge distributions can appear, distorting the inverse-square law.

A Faraday cup is essentially a conductive enclosure designed to capture and measure the net charge on an incoming object. For a conductor in electrostatic equilibrium, any excess charge resides on the outer surface, and the interior is shielded from external fields. When a charged object (such as a sphere) is inserted into the Faraday cup, the metal walls of the cup rearrange their free charges so as to cancel the electric field inside the conducting material. This redistribution of charge on the cup's inner and outer surfaces ensures that the net charge transferred to the cup is precisely equal to the net charge on the object.

Practically, the Faraday cup is connected to an electrometer or a similar high-impedance measuring circuit. As the charged object moves fully inside the cup, an equal and opposite charge is induced on the inner surface, and the original charge that was on the cup or circuit is displaced to its outer surface or to an attached capacitor. That displacement creates an observable voltage difference across the capacitor, which is then read by the electrometer. Because the circuit's capacitance C is known, the measured voltage U can be used to compute the enclosed charger by Equation (2.2)

$$Q = CU \tag{2.2}$$

Another key aspect is that large electric charges can be difficult to store in open air, because of leakage effects due to humidity, dust, or imperfect insulation can cause the charge on a conductor to diminish over time. Cleaning and carefully handling the insulating rods and surfaces can mitigate some of these losses, but in most real laboratories, a small amount of charge leakage still occurs.

Method and Procedure

2.1 Safety Precautions

In this experiment, we will be dealing with high voltage supplies when charging the spheres, a high voltage supply that can deliver up to $30,000V$ at a current of $0.5mA$. Naturally, this creates a level of hazardousness in the experiment, since this high voltage is enough to pose a health risk if handled incorrectly. For safety, one must always wear the metal wristband that connects to ground, in particular, on the same hand that might come into contact with high voltage. Additionally, before changing any connections, it is essential that one switches off the high-voltage power supply and ensure its output is turned fully to zero. Moreover, the equipment should be arranged so that no one can accidentally touch any exposed or uninsulated high-voltage parts. Finally, the plug pin carrying high voltage must not be allowed near the experimenter or other conductive objects, and extra care is needed to keep the force sensor and CASSY interface (which are unprotected against high voltage) well clear of any high voltage cables.

In order to verify Coulomb's law, the following investigation centers on generating a data set which produces a linear correlation by plotting the force F experienced by a charged ball against the quantity $\frac{1}{d^2}$ with gradient m to derive the following expression for the relative permittivity ϵ_r:

$$F(\frac{1}{d^2}) = \frac{Q_1Q_2}{4\pi\epsilon_0\epsilon_r} \cdot \frac{1}{d^2} = m\frac{1}{d^2} \implies m = \frac{Q_1Q_2}{4\pi\epsilon_0\epsilon_r}$$

$$\implies \epsilon_r = \frac{Q_1Q_2}{4m\pi\epsilon_0} \tag{2.3}$$

To consider the error for ϵ_r in this method, one must first consider the error for the slope of the graph. The equation for the error in a slope [1] is given by in Equation (2.4) for a scatterplot of size n and correlation coefficient R^2:

$$|\frac{\Delta m}{m}| = \sqrt{\frac{1}{n-2} \cdot \frac{(1-R^2)}{R^2}} \tag{2.4}$$

Method

As stated above, there are experimental inefficiencies for which a method must be developed in order to be able to consider the uncertainty generated by the setup. The following section will concern these methods.

Equipment List

- Metalized spheres with holder

- Cassy force sensor
- Cassy displacement transducer - Precision metal rail with trolley
- High power supply
- Charging device and amplifier - Electrometer circuitry
- Multimeter
- Cassy and Laptop
- Grounding wristband

The setup for the experiment included two metalized spheres: one of which will be charged to apply the force and is held by the trolley in a Precision metal rail with a string attached to it which connects the trolley to the Cassy displacement transducer and uses a small weight and a pulley to create tension and measure the displacement of the trolley, as seen in Figure 2.1. Separately, the setup also includes a Clamp Stand holding the Cassy force sensor, with a metalized sphere held precisely used to capture the repulsive force that will be felt by the other sphere when charged; the transducer also had to be calibrated with respect to the force sensor. To do so, first position the trolley at its maximum distance and adjust the displacement sensor to 40 cm, ensuring a 0 to 45 cm range in the Cassy settings. Verify that the laptop consistently displays the correct distance. Define the zero point by letting the spheres slightly touch and correcting the offset. Move the trolley back to create a 4 cm gap to ensure Coulombs Law is preserved, adjusting the clamp rider to keep this as the minimum distance. The center-to-center minimum distance to be 8.02 cm, and the maximum distance to be 33.68cm.

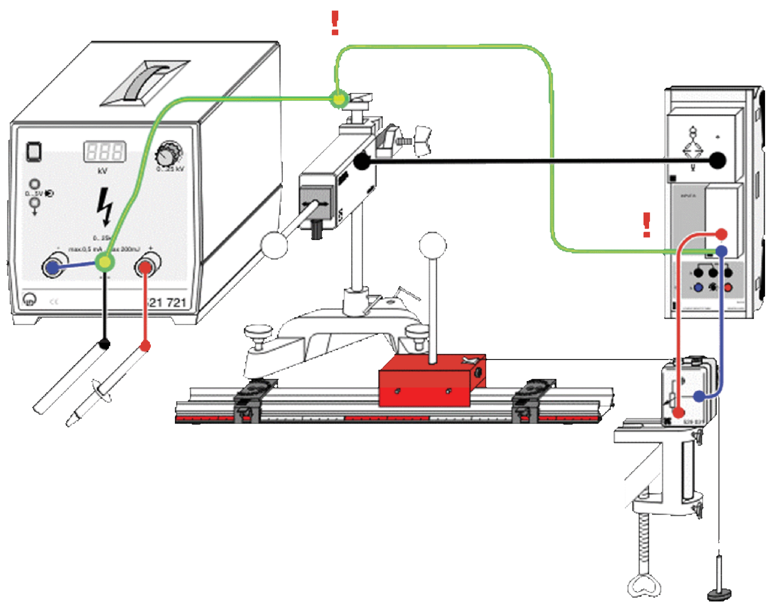


Figure 2.1: Diagram of experimental setup

Furthermore, the high power supply is connected to a plug pin in the positive channel and the negative channel is connected to the ground channel, the clamp stand, and the

Cassy connection with the transducer to allow discharge and not risk damaging any equipment not protected against high voltages. A voltage of $14.6kV$ was used to charge the sphere for all the measurements performed in this report. Due to the high voltage a grounding wristband was always utilized when handling charged spheres or any dangerous equipment for safety reasons.

Additionally, there is a charging device and amplifier that use: a Faraday cup to capture and measure the voltage in a charged sphere, a capacitor which is charged due to the voltage (as explained in the introduction) and a Multimeter to measure the readings as seen in Figure 2.2.

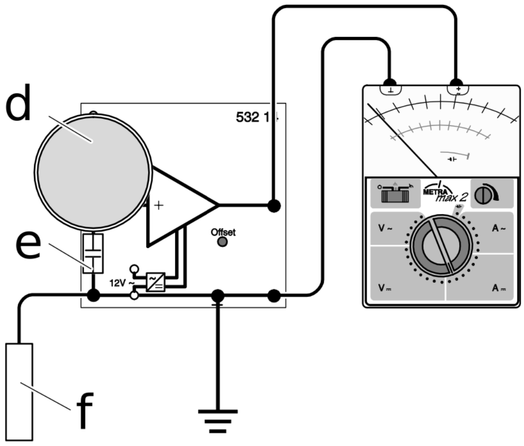


Figure 2.2: Electrometer Circuitry Diagram with labeling for capactior (e), Faraday cup (d) and discharge rod (f)

Reproducibility of Sphere Charging

The following procedure was done for both spheres in attempt to check the reproducibility of the results from the method and to calculate the charges Q_1 and Q_2 for the spheres. To do so, two methods for discharging the spheres was used to capture the Voltage with the Capacitor and Faraday Cup and then calculate the charge: through direct contact between the cup and the charged sphere to get voltage V^d and through electric induction to get V^i. Using Equation (2.2) and taking a value of C = 10nF [2] for capacitance, the charges can be computed. In each measurement, the sphere was charged for approximately 2 seconds to ensure a reliable charge distribution on it. After the charging of the sphere was done, the power supply was immediately turned off without the sphere being in contact with the rod to not affect the charge it retains, and to not let the electric field produced by the very high voltage source affect/distort our measurements. The time between this step and the discharge into the faraday cup was minimized (almost instant). A total of $n = 5$ measurements were made for both methods and for both spheres, with an additional trial of 5 measurements for each method to test their reproducibility, resulting in a total of 20 measurements. After each individual measurement, the discharge rod was used to allow the voltage to drop to zero.

Moreover the mean for all charges $\overline{Q_1^i}, \overline{Q_1^d}, \overline{Q_2^i}, \overline{Q_2^d}$ and error of mean is calculated from Equation (2.5) [1]:

$$\overline{Q} = \frac{\sum_{k=1}^{n} Q_i}{n} \qquad \Delta\overline{Q} = \frac{\sigma_Q}{\sqrt{n}} \tag{2.5}$$

such that $\forall i \in \{1, 2, ..., n\}$, Q_i denotes each individual measurement and σ_Q is the standard deviation for the set of averaged values of variable Q_i given by: [1]

$$\sigma_Q = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Q_i - \overline{Q})^2} \tag{2.6}$$

Finally the values for Q_1, Q_2 can be found with appropriate error.

Test for Loss of Charge

The test for loss of charge was conducted to determine whether the spheres retained sufficient charge over time, ensuring reliable force measurements. If charge dissipated too quickly, it could lead to inaccuracies in the experiment, requiring corrective measures before proceeding with force-distance measurements. The settings for the CASSY readings were the following:

Ranges	**Averaging**	**Measurement interval**	**Number of measurements**
$\pm$10 mN and 45 cm	1 s	5 s	50

To perform the test, both spheres were charged to 14.6kV with same polarity. Sphere 2 was positioned to the minimum distance of (8.02 ± 0.01) cm away from sphere 1 and the electrostatic force was measured. The force was recorded over time and normalized to its initial maximum value. The normalization was done with respect to the initial maximum value to allow for a direct comparison of charge retention over time, independent of the absolute force measurement and ensures that variations due to initial charge differences do not affect the assessment of charge loss.

Our initial results showed a charge loss exceeding 10% per minute, requiring us to repeat the test. To minimize charge dissipation, we cleaned the insulated rods with DI water, carefully dried the components using a hair dryer, and ensured that external disturbances from the room were minimized by keeping an appropriate distance from all apparatus during each trial. After implementing these measures, we repeated the test, achieving a significantly lower rate of charge loss, allowing us to proceed with the force-distance measurements. The percentage loss $L\%$ was calculated by taking two arbitrary values for the force $F_1(\tilde{t})$, $F_2(\tilde{t}+60)$ from the 60 second interval that is covers the span closest to the mean of the data and computing the percentage loss of charge by using (2.7):

$$L\% = \frac{F_1 - F_2}{F_1} \cdot 100 \tag{2.7}$$

Force - Distance Measurements

The objective of the procedure was to measure the electrostatic force between the two charged spheres as a function of distance (As dictated by Coulomb's Law), ensuring that charge loss is minimized and accurate data is collected. To start, we discharged the spheres using the grounded connection rod of the electrometer. Sphere 2 was positioned at maximum distance, and the force sensor was reset to zero. Both spheres were charged. The Cassy was then used to quickly measure the force while moving Sphere 2 steadily towards Sphere 1, ensuring the measurement was completed within 5 seconds. To avoid averaging over long distances, we configured the system for fast measurements with the following settings:

Ranges	**Averaging**	**Measurement interval**	**Number of measurements**
±10 mN and 45 cm	15 ms	50 ms	manually

Five full sets of force-distance measurements were conducted over the entire range. The data was plotted as Force F vs. distance d, followed by a transformation to Force vs. $\frac{1}{d^2}$ for all five sets. A linear regression fit was applied to the $F(\frac{1}{d^2})$ plots, to determine the gradient m_i and its uncertainty for each set using Equation (2.4). The slopes were then used to determine the relative permittivity of the medium ϵ_r using Equation (2.3).A value of $\epsilon_0 = 8.8541878 \cdot 10^{-12}\mathrm{AsV}^{-1}\mathrm{m}^{-1}$ was used for permittivity of free space [2].

For ϵ_r's associated error $\overline{\epsilon_r}$, the five different values of ϵ_r were used to find the mean and error of mean to obtain the final best value and its error. The result was then compared to the accepted literature value of ϵ_r, and any deviations were analyzed in terms of potential sources of systematic error and experimental limitations.

To calculate the error of ϵ_r one must consider the statistical error of each one of the individual gradients m_i, as well as the error of mean from the averaging all the gradients together.

Assuming we have a variable y is a simple function of independent variables $x_1, x_2, ..., x_n$, the formula for the averaged error in y is given in (2.8) as:

$$\Delta y_{avg} = \sqrt{{\Delta x_1}^2 + {\Delta x_2}^2 + ... + {\Delta x_n}^2} \tag{2.8}$$

3 Results and Error Analysis

Charge Measurements

The measurements for the charges $Q_1^i, Q_1^d, Q_2^i, Q_2^d$ in Coulombs C can be found in Figure 3.1 and Figure 3.2. The value for the mean and the error of the mean of each charge is calculated using Equation (2.5) and the standard deviation using Equation (2.6) as follows:

$$\overline{Q_1^i} = \frac{\sum_{k=1}^{n} Q_k}{n} = \frac{2.61 + 2.61 + 2.63 + 2.63 + 2.64}{5} \cdot 10^{-8} = 2.624 \cdot 10^{-8}C$$

$$\overline{Q_2^i} = \frac{2.40 + 2.50 + \cdots + 2.52}{5} \cdot 10^{-8} = 2.438 \cdot 10^{-8}C$$

$$\overline{Q_1^d} = \frac{2.75 + 2.75 + \cdots + 2.76}{5} \cdot 10^{-8} = 2.734 \cdot 10^{-8}C \tag{3.1}$$

$$\overline{Q_2^d} = \frac{2.22 + 2.20 + \cdots + 2.25}{5} \cdot 10^{-8} = 2.224 \cdot 10^{-8}C \tag{3.2}$$

$$\Delta\overline{Q_1^i} = \frac{\sigma_{Q_1^i}}{\sqrt{n}} = \frac{1.34164 \cdot 10^{-10}}{\sqrt{5}} = 6.00 \cdot 10^{-11} C$$

$$\Delta\overline{Q_2^i} = \frac{8.4971 \cdot 10^{-10}}{\sqrt{5}} = 3.80 \cdot 10^{-10} C$$

$$\Delta\overline{Q_1^d} = \frac{4.15933 \cdot 10^{-10}}{\sqrt{5}} = 1.86011 \cdot 10^{-10} C \tag{3.3}$$

$$\Delta\overline{Q_2^d} = \frac{3.39116 \cdot 10^{-10}}{\sqrt{5}} = 1.51658 \cdot 10^{-10} C \tag{3.4}$$

Since the measurements done for all other data are done through direct discharge of the current form the spheres, the measurements taken are the ones for Q_1^d and Q_2^d for the calculation of ϵ_r from Equations (3.1), (3.3), (3.2), (3.4) that is:

$$Q_1 = \overline{Q_1^d} \pm \Delta\overline{Q_1^d} = (2.734 \cdot 10^{-8} \pm 1.86011 \cdot 10^{-10}) C \tag{3.5}$$

$$Q_2 = \overline{Q_2^d} \pm \Delta\overline{Q_2^d} = (2.224 \cdot 10^{-8} \pm 1.51658 \cdot 10^{-10}) C \tag{3.6}$$

Figure 3.1: Sphere 1 - Inductive and Direct Contact Charge Tables

Voltage (V_1^i)	Charge (Q_1^i)
2.61	2.61E-08
2.61	2.61E-08
2.63	2.63E-08
2.63	2.63E-08
2.64	2.64E-08
Voltage (V_1^d)	Charge (Q_1^d)
2.75	2.75E-08
2.75	2.75E-08
2.66	2.66E-08
2.75	2.75E-08
2.76	2.76E-08

Figure 3.2: Sphere 1 - Inductive and Direct Contact Charge Tables

Voltage (V_2^i)	Charge (Q_2^i)
2.4	2.4E-08
2.5	2.5E-08
2.46	2.46E-08
2.31	2.31E-08
2.52	2.52E-08
Voltage (V_2^d)	Charge (Q_2^d)
2.22	2.22E-08
2.2	2.2E-08
2.24	2.24E-08
2.29	2.29E-08
2.25	2.25E-08

Loss of Charge Plots and Tables

The scatter-plots seen in Figure 3.3 and Figure 3.4 are derived from the table of values for the Force measured in mN from the repulsion force between the spheres against time in seconds:

From Figure 3.3, the values for F_1, F_2 taken are $t_1 = 100$ and $t_2 = 160$ which give values of $F_1 = 1.42$ mN and $F_2 = 1.30$ mN. From Equation (2.7):

$$L\% = \frac{F_1 - F_2}{F_1} \cdot 100 = \frac{1.42 - 1.30}{1.42} \cdot 100 = 8.450704\%$$

Figure 3.3: Force (mN) vs. Time (s)

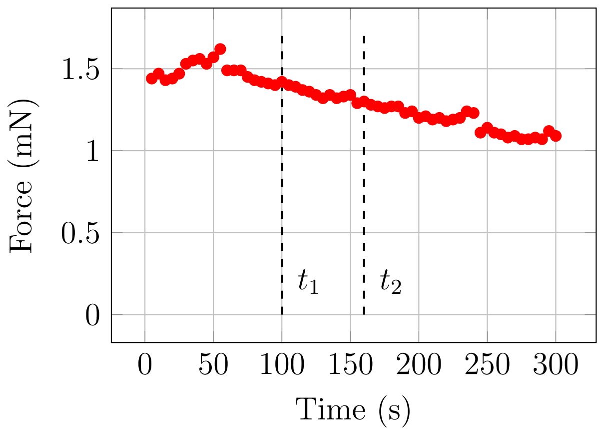
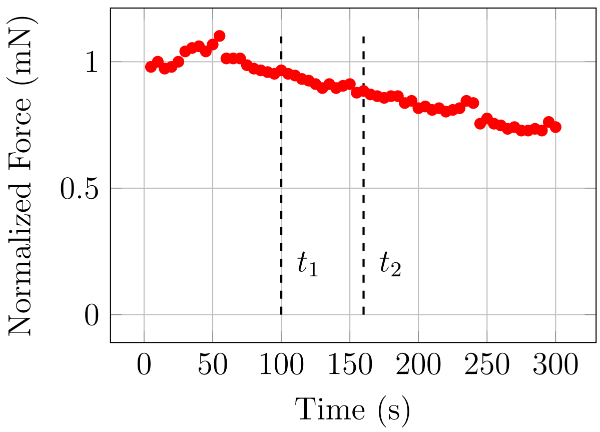


Figure 3.4: Normalized Force (mN) vs. Time (s)



Force vs Distance and Calculation of Relative Permittivity

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[1] Jürgen Fritz Dr. Donfack Patrice. *Modern Physics Lab Manual*. Constructor University, Spring 2023.

[2] Tim Jesko Söcker Dr. Donfack Patrice. *Modern Physics Lab Manual*. Constructor University, Spring 2025, pp. 3–11.