Projectile Lab: Deviations from Kinematics

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# 1 Goal

The goal of this project was to develop a kinematic model to describe the range of a projectile launched from a constant height at any given angle and velocity. This kinematic model was compared to real-world data and the deviations between theoretical and experimental were statistically modeled and discussed.

# 2 Kinematic Model

The motion of our projectile was modeled in two dimensions assuming constant gravational acceleration. Figure 1 shows the motion of the projectile as depicted by this model.

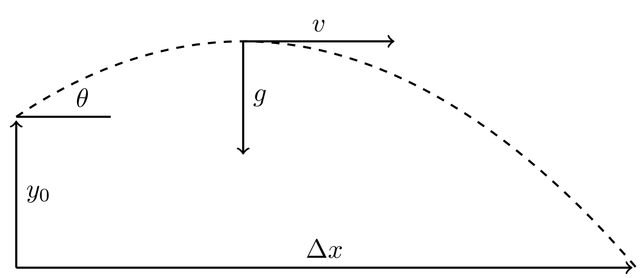


Figure 1: Diagram of kinematic model

Over time, the y-coordinate of the projectile can be described as

$$y = y_0 + v_{y_0}t + \frac{1}{2}gt^2. \quad (1)$$

Setting $y = 0$ and using the quadratic equation, equation 1 becomes

$$t_{land} = \frac{-v_{y0} \pm \sqrt{{v_{y0}}^2 - 2gy_0}}{g}. \tag{2}$$

Now, the analogue of equation 1 for the x-coordinate of the projectile is

$$x = x_0 + x_{y0}t \tag{3}$$

As there is no acceleration in the x direction. Finally, plugging equation 2 into equation 3, we obtain

$$x_{land} = x_0 + \frac{v_{x_0}}{g}\left( - v_{y0} \pm \sqrt{{v_{y0}}^2 - 2gy_0}\right). \tag{4}$$

Tidying up, we will let $\Delta x = x_{land} - x_0$, $v_{x0} = v_0 \cos\theta$, $v_{y0} = v_0 \sin\theta$. Selecting only the positive root of the quadratic equation yields the final

$$\Delta x = -\frac{v_0 \cos\theta}{g}\left(v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 - 2gy_0}\right). \tag{5}$$

# 3 Methods

A basic ramp was set up with the starting end being higher than the launch end. Gravatational propulsion launched a small metal ball off the launch end. The projectile started at varying heights - from 55 cm to 75 cm in 5 cm increments - measured from the lowest point of the launch ramp. For each of these heights, the ball was launched at angles from 0° to 40° in 10° increments. The horizontal distance from the 75 cm meter starting point to the lowest point of the ramp and end of the ramp were 97 cm and 134 cm respectively. Furthermore, the horizontal distance from the launch point to the ground was 91 cm with approximately 3 cm being added for every 10° additional increment.

During data collection, the metal ball's range was recorded for each drop height and angle. The five different values for each variable times three three trials totaled 75 individual data points. The data collection started with a 0° ramp cycling through the five different starting heights. This process was repeated for the subsequent four 10° increments. The impact location of the metal ball was measured with a combination of three sheets of carbon paper and a measuring tape. In order to compare these results to the kinematic model, takeoff velocity was measured by two photogates placed a fixed 2cm apart. The collection of this information was relatively straightforward as LoggerPro automatically calculated the corresponding velocities from the time interval betwen the two photogate activations.

## 4 Plot

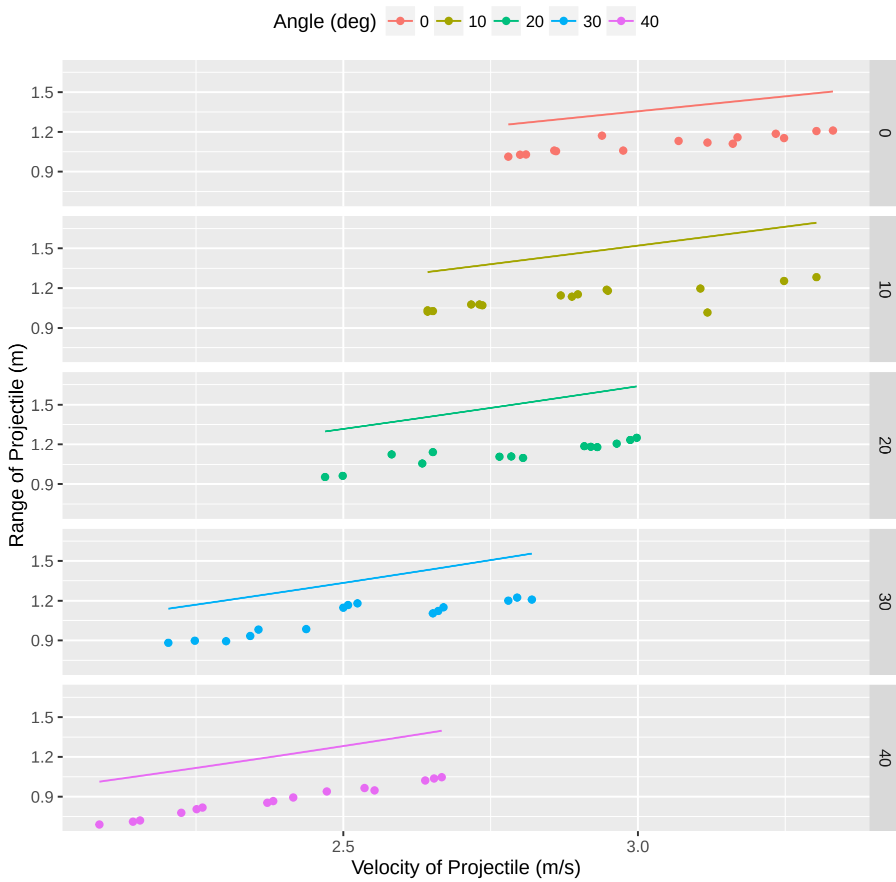


Figure 2: A plot of takeoff velocity vs. range for various angles. Experimentally observed data are shown as points, and theoretical valuesa are shown as lines.

## 5 Analysis

One immediate observation is that, regardless of the angle, the range appeared to be linear with the takeoff velocity. Indeed, substituting $\theta = 45°$ and $y_0 = 1m$ into equation 5, we obtain

$$\Delta x = 0.0510 v_0 \sqrt{v_0^2 + 39.2} \tag{6}$$

which is clearly quadratic in the limit. A plot of this function is shown in Figure 5, demonstrating this behavior.

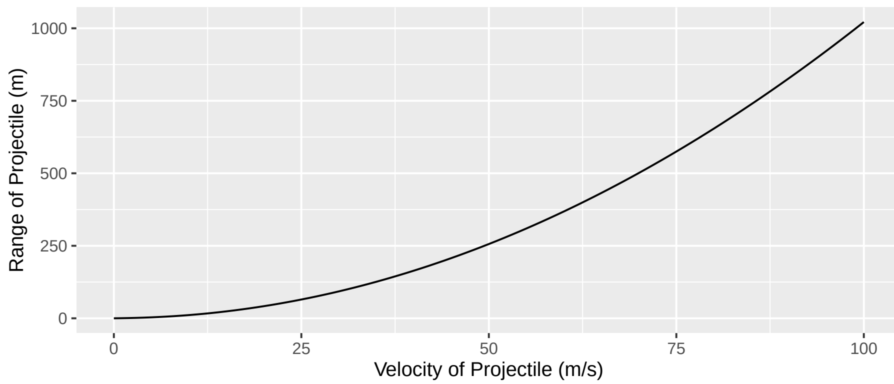


Figure 3: Theoretical of a projectile launched at 45° from a height of 1 m

Furthermore, there appeared to be a constant error between the theoretical and actual range of the projectile. Table 1 gives the average error and standard deviation of error for each launch angle.

| Angle (deg) | Error (m) | SD (m) |
|---|---|---|
| 0.000 | 0.263 | 0.041 |
| 10.000 | 0.340 | 0.077 |
| 20.000 | 0.367 | 0.052 |
| 30.000 | 0.278 | 0.059 |
| 40.000 | 0.336 | 0.017 |

Table 1: A table showing mean error and standard devation for velocities measured from different launch angles to three decimal places

This substantial difference cannot be simply attributed to air resistance - the metal ball had substantial inertia and certainly would not have lost 30 cm by that cause. Most likely, the distance between the photogates wasn't measured to a high enough degree of precision, causing the takeoff velocity to be miscalculated consistently by a roughly constant factor. Furthermore, the rough estimation of height increase per degree incline could have contributed, but certainly only a few centimeters of height at the most.

Final observations include that the error was roughly constant for each angle, but the standard devation of velocity within each angle was near zero. This indicates a precise yet inaccurate experimental setup. Further work could improve these inaccuracies (likely by focusing on finer calibration of the photogates) or measure on a larger scale how hyperbolic geometries improve errors in models that assume constant downward acceleration.

## 6 Appendix: Collected Data

| Drop | Angle | Takeoff | Range |
|---|---|---|---|
| 0.550 | 0.000 | 2.780 | 1.013 |
| 0.550 | 0.000 | 2.800 | 1.028 |
| 0.550 | 0.000 | 2.810 | 1.029 |
| 0.600 | 0.000 | 2.939 | 1.171 |
| 0.600 | 0.000 | 2.858 | 1.059 |
| 0.600 | 0.000 | 2.861 | 1.054 |
| 0.650 | 0.000 | 3.118 | 1.119 |
| 0.650 | 0.000 | 2.975 | 1.059 |
| 0.650 | 0.000 | 3.161 | 1.110 |
| 0.700 | 0.000 | 3.169 | 1.159 |
| 0.700 | 0.000 | 3.069 | 1.131 |
| 0.700 | 0.000 | 3.248 | 1.153 |
| 0.750 | 0.000 | 3.234 | 1.186 |
| 0.750 | 0.000 | 3.303 | 1.206 |
| 0.750 | 0.000 | 3.331 | 1.210 |
| 0.550 | 10.000 | 2.643 | 1.023 |
| 0.550 | 10.000 | 2.652 | 1.027 |
| 0.550 | 10.000 | 2.643 | 1.032 |
| 0.600 | 10.000 | 2.717 | 1.076 |
| 0.600 | 10.000 | 2.731 | 1.076 |
| 0.600 | 10.000 | 2.736 | 1.070 |
| 0.650 | 10.000 | 2.869 | 1.145 |
| 0.650 | 10.000 | 2.898 | 1.153 |
| 0.650 | 10.000 | 2.888 | 1.136 |
| 0.700 | 10.000 | 3.118 | 1.016 |
| 0.700 | 10.000 | 2.949 | 1.180 |
| 0.700 | 10.000 | 2.947 | 1.188 |
| 0.750 | 10.000 | 3.106 | 1.197 |
| 0.750 | 10.000 | 3.248 | 1.254 |
| 0.750 | 10.000 | 3.303 | 1.283 |
| 0.550 | 20.000 | 2.499 | 0.963 |
| 0.550 | 20.000 | 2.469 | 0.954 |
| 0.600 | 20.000 | 2.652 | 1.141 |
| 0.600 | 20.000 | 2.582 | 1.124 |
| 0.600 | 20.000 | 2.634 | 1.056 |
| 0.650 | 20.000 | 2.765 | 1.107 |
| 0.650 | 20.000 | 2.785 | 1.109 |

| Drop | Angle | Takeoff | Range |
|---|---|---|---|
| 0.650 | 20.000 | 2.805 | 1.098 |
| 0.700 | 20.000 | 2.920 | 1.182 |
| 0.700 | 20.000 | 2.909 | 1.186 |
| 0.700 | 20.000 | 2.931 | 1.179 |
| 0.750 | 20.000 | 2.998 | 1.250 |
| 0.750 | 20.000 | 2.987 | 1.234 |
| 0.750 | 20.000 | 2.964 | 1.206 |
| 0.550 | 30.000 | 2.301 | 0.894 |
| 0.550 | 30.000 | 2.248 | 0.898 |
| 0.550 | 30.000 | 2.203 | 0.882 |
| 0.600 | 30.000 | 2.437 | 0.985 |
| 0.600 | 30.000 | 2.356 | 0.982 |
| 0.600 | 30.000 | 2.342 | 0.933 |
| 0.650 | 30.000 | 2.508 | 1.167 |
| 0.650 | 30.000 | 2.500 | 1.147 |
| 0.650 | 30.000 | 2.524 | 1.180 |
| 0.700 | 30.000 | 2.652 | 1.104 |
| 0.700 | 30.000 | 2.661 | 1.122 |
| 0.700 | 30.000 | 2.670 | 1.150 |
| 0.750 | 30.000 | 2.780 | 1.200 |
| 0.750 | 30.000 | 2.820 | 1.208 |
| 0.750 | 30.000 | 2.795 | 1.223 |
| 0.550 | 40.000 | 2.155 | 0.720 |
| 0.550 | 40.000 | 2.086 | 0.690 |
| 0.550 | 40.000 | 2.143 | 0.711 |
| 0.600 | 40.000 | 2.251 | 0.805 |
| 0.600 | 40.000 | 2.225 | 0.778 |
| 0.600 | 40.000 | 2.261 | 0.818 |
| 0.650 | 40.000 | 2.381 | 0.867 |
| 0.650 | 40.000 | 2.415 | 0.894 |
| 0.650 | 40.000 | 2.371 | 0.854 |
| 0.700 | 40.000 | 2.553 | 0.947 |
| 0.700 | 40.000 | 2.472 | 0.939 |
| 0.700 | 40.000 | 2.536 | 0.965 |
| 0.750 | 40.000 | 2.654 | 1.037 |
| 0.750 | 40.000 | 2.667 | 1.047 |
| 0.750 | 40.000 | 2.639 | 1.022 |

(units are meters, degrees, meters per second, and meters, respectively)