Grocery List of Definitions

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CHAPTER 1

Algebra

One of the basic algebraic tools:

Definition 1. Let S be some set. A **Law of Composition** $f : S \times S \to S$ consists of

- a pair of sets $\mathrm{dom}(f) = S \times S$, and $\mathrm{cod}((f) = S$

equipped with

- a set $f \subseteq \mathrm{dom}(f) \times \mathrm{cod}(f)$

such that

- for each $x \times y \in \mathrm{dom}(f)$, there is a corresponding $z \in \mathrm{cod}(f)$ such that $(x \times y, z) \in f$.

We can use it to introduce various mathematical objects, e.g.

Definition 2. A **Monoid** consists of

- a set M

equipped with

- a law of composition $* : M \times M \to M$
- an identity element $e \in M$

such that

- the law of composition is associative, i.e. $(x * y) * z = x * (y * z)$ for all $x, y, z \in M$;
- closure under the law of composition, i.e. $(x * y) \in M$ for all $x, y \in M$;
- the identity satisfies $e * x = x * e = x$ for all $x \in M$.

Definition 3. A **Group** G consists of

- a monoid G

equipped with

- an inversion operator $(\cdot)^{-1} : G \to G$

such that

- $x^{-1} * x = x * x^{-1} = e$ for all $x \in G$.

CHAPTER 2

Measure Theory

DEFINITION 4. Let X be a set. A σ**-Algebra** over X consists of

- a collection Σ of subsets of X

such that

- Σ is nonempty,
- if $x \in \Sigma$, the $x^C \in \Sigma$,
- let I be a finite indexing set, then

$$\left(\bigcup_{i \in I} E_i \right) \in \Sigma$$

  for countably many $E_i \in \Sigma$.

It allows us to introduce the notion of a measure:

DEFINITION 5. Let X be some set, Σ be a σ-algebra over X. A **measure** μ consists of

- a function $\mu : \Sigma \to [-\infty, \infty]$

such that

- $\mu(E) \geq 0$ for all $E \in \Sigma$;
- $\mu(\emptyset) = 0$;
- if $\{E_i\}_{i \in I}$ is a countable collection of pairwise disjoint sets in Σ, then

$$\mu \left(\bigcup_{i \in I} E_i \right) = \sum_{i \in I} \mu(E_i).$$

Bibliography

[1] W. Rudin, *Principles of mathematical analysis*. McGraw-Hill New York, 1964.