$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^{x} = e$$

$$\left.\begin{array}{l} \sin x \\ \arcsin x \\ \tan x \\ \arctan x \\ \ln(x+1) \\ e^{x}-1 \end{array}\right\} \Rightarrow x$$

$$a^{x}-1 \Rightarrow x\cdot \ln a$$

$$\sec x-1 \Rightarrow 1-\cos x \Rightarrow \frac{x^2}{2}$$

$$(1+x)^{\frac{1}{n}}-1 \Rightarrow \frac{1}{n}x$$

$$\sin x' = \cos x$$

$$\cos x' = -\sin x$$

$$\tan x' = \sec^2 x \qquad \sec x' = \sec x \tan x$$

$$\cot x' = -\csc^2 x \qquad \csc x' = -\csc x \cot x$$

$$\arcsin x' = \frac{1}{\sqrt{1-x^2}}$$

$$\arccos x' = -\frac{1}{\sqrt{1-x^2}}$$

$$\arctan x' = \frac{1}{1+x^2}$$

$$\operatorname{arccot} x' = -\frac{1}{1+x^2}$$

$$\int k\,dx = kx + c \qquad \int x^{\mu}dx = \frac{x^{\mu+1}}{\mu+1}+c \qquad \int \frac{1}{x}dx = \ln|x| + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c \qquad \int e^x dx = e^x + c \qquad \int \cos x dx = \sin x +$$

$$\int \sin x dx = -\cos x + c \qquad \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + c \qquad \int \frac{1}{\cos x} dx = \ln\left|\frac{1}{\cos x} + \tan x\right| + c$$

$$\int \frac{1}{\sin^2 x} = \int \csc^2 x dx = -\cot x + c \qquad \int \frac{1}{1+x^2} dx = \arctan x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int \tan x dx = -\ln|\cos x| + c \qquad \int \cot x dx = \ln|\sin x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c \qquad \int \csc x dx = \ln|\csc x - \cot x| + c$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\arctan\frac{x}{a} + c \qquad \int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\frac{x}{a} + c \qquad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$$

$$\int u dv = uv - \int v du$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

这3个常用于第二类换元积分法

$$\left.\begin{aligned} &\sin 2x = 2\sin x\cos x \\ &\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \end{aligned}\right\} \xrightarrow{\text{变形}} \begin{aligned} &\sin^2\frac{x}{2} = \frac{1-\cos x}{2} \\ &\cos^2\frac{x}{2} = \frac{1+\cos x}{2} \end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \qquad \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \qquad \tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\sin^2\alpha = \frac{1}{2} - \frac{\cos 2\alpha}{2}$

$\cos^2\alpha = \frac{1}{2} + \frac{\cos 2\alpha}{2}$

$\int_a^b f(x)dx = F(b) - F(a)$

$\int_a^b f(x)dx = \int_\alpha^\beta f[\varphi(t)]\varphi(t)'dx = \int_\alpha^\beta f[\varphi(t)]\, d[\varphi(t)]$

瑕积分先找瑕点

可导 $f'_-(x) = f'_+(x)$

连续 $f(x^-) = f(x) = f(x^+)$