二元函数的极限

$$\left.\begin{array}{l}\sin x\\ \arcsin x\\ \tan x\\ \arctan x\\ \ln(1+x)\\ e^x-1\end{array}\right\}\Rightarrow x$$

$$a^x-1 \}\Rightarrow x\ln a$$

$$\sec x-1 \Leftrightarrow 1-\cos x \Leftrightarrow \frac{x^2}{2}$$

$$(1+x)^{\frac{1}{n}}-1 \Leftrightarrow \frac{x}{n}$$

偏导数
- 求法　其他的都当成常数
- 高阶偏导 $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$

全微分
- 求法 ①求偏导 $\frac{\partial z}{\partial x}=\cdots$
  ②代入公式 $dz=\frac{\partial z}{\partial x}dx\ \cdots\cdots$
  (③代值进入)
- 近似计算 $f(x,y)\approx f(x_0,y_0)+\frac{\partial z}{\partial x}\cdot\Delta x+\frac{\partial z}{\partial y}\cdot\Delta y$
- 可微的判定 #

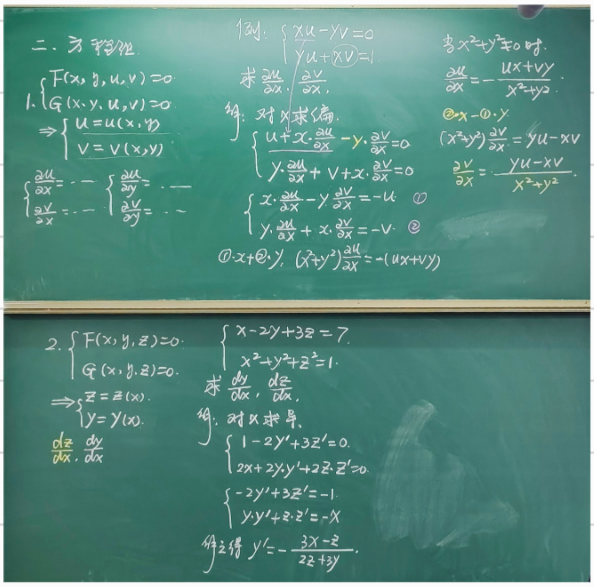
复合求导
- 一元　$y=f[U(x)] \Rightarrow y'=f'(u)\,U'(x)$
- 二元与一元复合：链式法则，链上求积，链间求和
  $\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$
- 多元与多元复合　$z=f(xy^2,y)$，求 $\frac{\partial^2 z}{\partial x\partial y}$
  $\frac{\partial z}{\partial x}=f_1'\cdot y^2$
  $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=y^2[f_{11}''\,2xy+f_{12}'']+2y\cdot f_1'$

隐函数求导
- 一元隐函数 $\frac{dy}{dx}=-\frac{F_x}{F_y}$

二元隐函数　$x+z=yf(x^2-z^2)$

$F(x,y,z)=x+z-yf(x^2-z^2)$

方程组中隐函数



偏导数的应用

空间曲线的切线

①对参数求导
$$\begin{cases} x' = \varphi'(t) \\ y' = \psi'(t) \\ z' = w'(t) \end{cases}$$

②算导数值，得 $\vec{s} = \{\varphi'(t_0), \psi'(t_0), w'(t_0)\}$

③由点向式得切线方程　$\frac{x-x_0}{\varphi'(t_0)} = \frac{y-y_0}{\psi'(t_0)} = \frac{z-z_0}{w'(t_0)}$

④法平面方程　$\varphi'(t_0)(x-x_0)+\psi'(t_0)(y-y_0)+w'(t_0)(z-z_0)=0$

空间曲面的切平面与法线

①偏导
$$\begin{cases} F_x' \\ F_y' \\ F_z' \end{cases}$$

②算偏导值，得 $\vec{n} = \{F_x'(x_0,y_0,z_0), F_y'(x_0,y_0,z_0), F_z'(x_0,y_0,z_0)\}$

③求切平面方程　$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$

④求法线方程　$\frac{x-x_0}{F_x(x_0,y_0,z_0)} = \frac{y-y_0}{F_y(x_0,y_0,z_0)} = \frac{z-z_0}{F_z(x_0,y_0,z_0)}$