$$f(x) = F'(x)$$

$$\int f(x)\,dx = F(x) + C$$

$f(x)$在区间上的原函数

基本积分公式

$\int k\,dx = kx + C$　　$\int x^{\mu}dx = \frac{x^{\mu+1}}{\mu+1} + C$　　$\int \frac{1}{x}dx = \ln|x| + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$　　$\int e^x dx = e^x + C$　　$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$　　$\int \frac{1}{\cos^2 x}dx = \int \sec^2 x\,dx = \tan x + C$

$\int \frac{1}{\sin^2 x} = \int \csc^2 x\,dx = -\cot x + C$　　$\int \frac{1}{1+x^2}dx = \arctan x + C$

$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$　　$2\int \frac{1}{\sqrt{x}}dx = \sqrt{x} + C$

补充积分公式

$\int \tan x\,dx = -\ln|\cos x| + C$　　$\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$　　$\int \csc x\,dx = \ln|\csc x - \cot x| + C$

$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\frac{x}{a} + C$　　$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C$　　$\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

$$\int [f(x) \pm g(x)]dx = f(x)dx \pm g(x)dx$$

$\int k f(x)dx = k\int f(x)dx$（$k$与$x$无关就可以提出来

→可以套娃，凑完一次再凑一次，直到可以化简

第一换元积分法（凑微分法）　$\int 1\,d(x^2+2) = x^2+2+C$

$$\int f[\varphi(x)]\varphi'(x)\,dx = \int f[\varphi(x)]\,d[\varphi(x)] = F[\varphi(x)] + C$$

把d前面的一部分求原函数，拿d里面去

d里面可以任意加减常数

就是让dx成du，这样更好的套公式

九、常用凑微分公式

积分型	换元公式
$\int f(ax+b)dx=\frac{1}{a}\int f(ax+b)d(ax+b)$	$u=ax+b$
$\int f(x^{\mu})x^{\mu-1}dx=\frac{1}{\mu}\int f(x^{\mu})d(x^{\mu})$	$u=x^{\mu}$
$\int f(\ln x)\cdot\frac{1}{x}dx=\int f(\ln x)d(\ln x)$	$u=\ln x$
$\int f(e^{x})\cdot e^{x}dx=\int f(e^{x})d(e^{x})$	$u=e^{x}$
$\int f(a^{x})\cdot a^{x}dx=\frac{1}{\ln a}\int f(a^{x})d(a^{x})$	$u=a^{x}$
$\int f(\sin x)\cdot\cos xdx=\int f(\sin x)d(\sin x)$	$u=\sin x$
$\int f(\cos x)\cdot\sin xdx=-\int f(\cos x)d(\cos x)$	$u=\cos x$
$\int f(\tan x)\cdot\sec^{2}xdx=\int f(\tan x)d(\tan x)$	$u=\tan x$
$\int f(\cot x)\cdot\csc^{2}xdx=\int f(\cot x)d(\cot x)$	$u=\cot x$
$\int f(\arctan x)\cdot\frac{1}{1+x^{2}}dx=\int f(\arctan x)d(\arctan x)$	$u=\arctan x$
$\int f(\arcsin x)\cdot\frac{1}{\sqrt{1-x^{2}}}dx=\int f(\arcsin x)d(\arcsin x)$	$u=\arcsin x$

第二换元积分法

(1) $x=\varphi(t)$ 积出来

(2) 换回x

把d里面的向外拿（一般用于解决外面有根号的情况）

eg: $\int\frac{1}{1+\sqrt{x}}dx=\int\frac{1}{1+t}dt^{2}=\int\frac{1}{1+t}2tdt$

$=2\int(1-\frac{1}{1+t})dt$

$$=2\int\left(1-\frac{1}{1+t}\right)dx$$

$$=2(t-\ln|t+1|)+C$$

代回 $x=t^2$

$$=2\sqrt{x}-2\ln(\sqrt{x}+1)+C$$

eg: $\int\sqrt{a^2-x^2}\,dx\ (a>0)$ $\quad x=a\sin t$

分布积分法

$$\int u\,dv=uv-\int v\,du$$

$$\int u'v\,dx=uv-\int uv'\,dx$$