

二元函数的极限

$$\left. \begin{array}{l} \sin x \\ \arcsin x \\ \tan x \\ \arctan x \\ \ln(1+x) \\ e^x - 1 \end{array} \right\} \Rightarrow x$$

$$a^x - 1 \} \Rightarrow x \ln a$$

$$\sec x - 1 \Leftrightarrow 1 - \cos x \Leftrightarrow \frac{x^2}{2}$$

$$(1+x)^{\frac{1}{n}} - 1 \Leftrightarrow \frac{x}{n}$$

偏导数

$$\left\{ \begin{array}{l} \text{求法} \quad \text{其他的都当成常数} \\ \text{高阶偏导} \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) \end{array} \right.$$

全微分

$$\left\{ \begin{array}{l} \text{求法} \quad \text{①求偏导} \quad \frac{\partial z}{\partial x} = \dots \\ \quad \text{②代入公式} \quad dz = \frac{\partial z}{\partial x} dx + \dots \\ \quad \text{③代值进入} \\ \text{近似计算} \quad f(x, y) \approx f(x_0, y_0) + \frac{\partial z}{\partial x} \cdot \Delta x + \frac{\partial z}{\partial y} \cdot \Delta y \\ \text{可微的判定} \quad \# \end{array} \right.$$

复合求导

$$\left\{ \begin{array}{l} \text{一元} \quad y = f[u(x)] \Rightarrow y' = f'(u) \cdot u'(x) \\ \text{二元与一元复合: 链式法则, 链上求和, 链间求和} \\ \quad \frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} \\ \text{多元与多元复合} \quad z = f(xy^2, y), \text{求 } \frac{\partial^2 z}{\partial x \partial y} \\ \quad \frac{\partial z}{\partial x} = f'_1 \cdot y^2 \\ \quad \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) = y^2 [f''_{11} \cdot 2xy + f''_{12}] + 2y f'_1 \end{array} \right.$$

隐函数求导

$$\int \text{一元隐函数} \quad \frac{dy}{dx} = -\frac{F_x}{F_y}$$

二元隐函数  $x+z = y f(x^2-z')$

$$F(x, y, z) = x+z - y f(x^2-z')$$

方程组中隐函数

二. 方程组

1.  $\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$   
 $\Rightarrow \begin{cases} u = u(x, y) \\ v = v(x, y) \end{cases}$

求  $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}$

解: 对  $x$  求偏导

$$\begin{cases} u + x \frac{\partial u}{\partial x} - y \frac{\partial v}{\partial x} = 0 \\ y \frac{\partial u}{\partial x} + v + x \frac{\partial v}{\partial x} = 0 \end{cases}$$

$$\begin{cases} x \frac{\partial u}{\partial x} - y \frac{\partial v}{\partial x} = -u \\ y \frac{\partial u}{\partial x} + x \frac{\partial v}{\partial x} = -v \end{cases}$$

①  $x+0 \cdot y, (x^2+y^2) \frac{\partial u}{\partial x} = -(u+xv)$

②  $x^2+y^2 \neq 0$  时

$$\frac{\partial u}{\partial x} = -\frac{u+xv}{x^2+y^2}$$

③  $x=0, y$

$$(x^2+y^2) \frac{\partial v}{\partial x} = yu-xv$$

$$\frac{\partial v}{\partial x} = -\frac{yu-xv}{x^2+y^2}$$

2.  $\begin{cases} F(x, y, z) = 0 \\ G(x, y, z) = 0 \end{cases}$   
 $\Rightarrow \begin{cases} z = z(x, y) \\ y = y(x) \end{cases}$

求  $\frac{dz}{dx}, \frac{dy}{dx}$

解: 对  $x$  求导

$$\begin{cases} 1 - 2y' + 3z' = 0 \\ 2x + 2y'y' + 2z'z' = 0 \\ -2y' + 3z' = -1 \\ y'y' + z'z' = -x \end{cases}$$

解得  $y' = -\frac{3x-z}{2z+3y}$

偏导数的应用

空间曲线的切线

① 对参数求导

$$\begin{cases} x' = \varphi'(t) \\ y' = \psi'(t) \\ z' = w'(t) \end{cases}$$

② 算导数值, 得  $\vec{s} = \{\varphi'(t_0), \psi'(t_0), w'(t_0)\}$

③ 由点向式得切线方程  $\frac{x-x_0}{\varphi'(t_0)} = \frac{y-y_0}{\psi'(t_0)} = \frac{z-z_0}{w'(t_0)}$

④ 法平面方程  $\varphi'(t_0)(x-x_0) + \psi'(t_0)(y-y_0) + w'(t_0)(z-z_0) = 0$

空间曲面的切平面与法线

① 偏导

$$\begin{cases} F'_x \\ F'_y \\ F'_z \end{cases}$$

② 算偏导值, 得  $\vec{n} = \{F'_x(x_0, y_0, z_0), F'_y(x_0, y_0, z_0), F'_z(x_0, y_0, z_0)\}$

③ 求切平面方程  $F'_x(x_0, y_0, z_0)(x-x_0) + F'_y(x_0, y_0, z_0)(y-y_0) + F'_z(x_0, y_0, z_0)(z-z_0) = 0$

④ 求法线方程  $\frac{x-x_0}{F'_x(x_0, y_0, z_0)} = \frac{y-y_0}{F'_y(x_0, y_0, z_0)} = \frac{z-z_0}{F'_z(x_0, y_0, z_0)}$

