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BIG DATA ANALYTICS

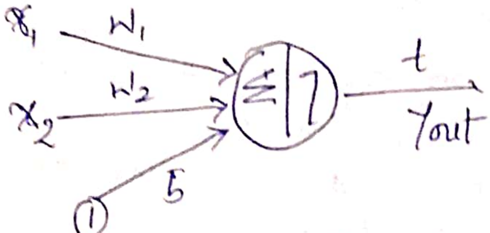
ASSIGNMENT - 2

PERCEPTRON MODELS

Develop a perceptron for the following logic functions with Bipolar inputs and targets (initial weights are 0; learning rate = 1; and threshold = 1)

(i) OR
(ii) NAND
(iii) NOR
(iv) NOT

(1) OR



Y_{in} (net input) $= x_1 w_1 + x_2 w_2 + b$

Given
$w_1 = w_2 = b = 0, \quad \alpha = 1, \quad \theta = 1$

x_1	x_2	$t = x_1$ OR x_2	Y_{out} = ?
1	1	1	?
1	-1	1	
-1	1	1	
-1	-1	-1	

Activation function

$$Y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$$

(1) For First inputs :-

(i) $x_1 = 1 \quad x_2 = 1 \quad t = 1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$

$1 \times 0 + 1 \times 0 + 0$

$\boxed{Y_{in} = 0} = f(0)$

$Y_{out} = f(y_{in})$

$\boxed{Y_{out} = -1}$

$\boxed{t = 1 \neq Y_{out} = -1}$

$\therefore$ update the weights

$\omega_1(n) = \omega_1(0) + \alpha t x_1$

$= 0 + 1 \cancel{*} 1 \times 1$

$\boxed{\omega_1(n) = 1}$

$\omega_2(n) = \omega_2(0) + \alpha t x_2$

$= 0 + 1 * 1 * 1$

$\boxed{\omega_{2n} = 1}$

$b_{(new)} = b_{(0)} + \alpha t$

$= 0 + 1 * 1$

$\boxed{b_n = 1}$

(ii) $x_1 = 1 \quad x_2 = 1 \quad t = 1$

$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$

$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$

$= 1 * 1 + 1 * 1 + 1 * 1$

$\boxed{Y_{in} = 3}$

$Y_{out} = f(y_{in}) = f(3)$

$\boxed{Y_{out} = 1}$

$\boxed{t = 1 == Y_{out} = 1}$

Stop the process

For $\underline{2^{nd} \text{ set of inputs}}$

$x_1 = 1 \quad x_2 = -1 \quad t = 1$

$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$

$$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$$
$$= 1 * 1 + (-1) * 1 + 1$$
$$= 1 - 1 + 1$$
$$\boxed{y_{in} = 1}$$

$$Y_{out} = f(y_{in}) => f(1)$$
$$\boxed{Y_{out} = 1}$$

$$\boxed{t = 1 \;==\; Y_{out} == 1}$$

Stop the process

For $\underline{3^{rd} \text{ set of inputs}}$

$x_1 = -1 \quad x_2 = 1 \quad t = 1$

$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$

$$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$$
$$= (-1) \times 1 + 1 \times 1 + 1$$
$$= -1 + 1 + 1$$
$$\boxed{Y_{in} = 1}$$

$$Y_{out} = f(y_{in}) \quad f(1)$$
$$\boxed{Y_{out} = 1}$$

$$\boxed{t = 1 \;==\; Y_{out} == 1}$$

Stop the process

For $\underline{4^{th} \text{ set of inputs}}$:

$x_1 = -1 \quad x_2 = -1 \quad t = -1$

$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$

$$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$$

$$= (-1) \times 1 + (-1) \times 1 + 1$$

$$Y_{in} = -1 - 1 + 1$$

$$\boxed{Y_{in} = -1}$$

$$\boxed{t = -1 == Y_{out} == -1}$$

Stop

For the linear class

$$\omega_1 x_1 + \omega_2 x_2 + b = 0$$

$\omega_1 = 1$ $\omega_2 = 1$ $b = 1$

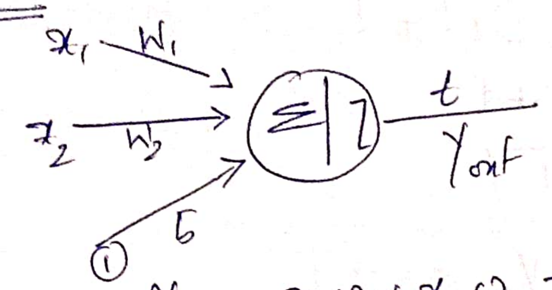
$$x_1 * 1 + x_2 * 1 + 1 = 0$$

$$\boxed{x_1 = -x_2 - 1}$$

x_1	x_2	$t = x_1$ OR x_2	Y_{out}
1	1	1	1
1	-1	1	1
-1	1	1	1
-1	-1	-1	-1

—— X ——

(2) NAND :



$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$

Given $W_1 = W_2 = b = 0$, $\alpha = 1$, $\theta = 1$

x_1	x_2	$t = X_1$ NAND X_2	Yout = ?
1	1	-1	?
1	-1	1	?
-1	1	1	?
-1	-1	1	?

Activation function

$$Y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$$

For first inputs:

$x_1 = 1 \quad x_2 = 1 \quad t = -1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= 1*0 + 1*0 + (-1)*0$$
$$\boxed{Y_{in} = 0}$$

$$Y_{out} = f(y_{in}) = f(0)$$
$$\boxed{Y_{out} = -1}$$

$$\boxed{t = -1 == Y_{out} == -1}$$

Stop the process

For second set of inputs

(1) $x_1 = 1 \quad x_2 = -1 \quad t = 1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$

$= 1 \times 0 + (-1) \times 0 + 0$

$\boxed{Y_{in} = 0}$

$Y_{out} = f(y_{in}) = f(0)$

$\boxed{Y_{out} = -1}$

$\boxed{t = 1 \neq Y_{out} = -1}$

update the weights

$\omega_{1(n)} = \omega_{1(0)} + \alpha t x_1$

$= 0 + (1)(1)(1)$

$\boxed{\omega_1 = 1}$

$\omega_{2(n)} = \omega_{2(0)} + \alpha t x_2$

$= 0 + (1)(1)(-1)$

$\boxed{\omega_2 = -1}$

$b_{(n)} = b_{(0)} + \alpha t$

$= 0 + 1 \times 1$

$\boxed{b = 1}$

(ii) $x_1 = 1 \quad x_2 = -1 \quad t = 1$

$\omega_1 = 1 \quad \omega_2 = -1 \quad b = 1$

$Y_{in} = x_1 \omega_1 + x_2 \omega_2 + b$

$= 1 \times 1 + (-1) * (-1) + 1$

$= 1 + 1 + 1$

$\boxed{Y_{in} = 3}$

$Y_{out} = f(y_{in}) = f(3)$

$\boxed{Y_{out} = 1}$

$\boxed{t == 1 \;==\; Y_{out} = 1}$

Stop the process

For 3rd set of inputs

(i) $x_1 = -1$ $x_2 = 1$ $t = 1$

$w_1 = 1$ $w_2 = -1$ $b = 1$

$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= (-1) \times (1) + (1)(-1) + 1$$
$$= -1 - 1 + 1$$
$$\boxed{Y_{in} = -1}$$

$$Y_{out} = f(y_{in}) = f(-1)$$
$$\boxed{Y_{out} = -1}$$

$$\boxed{t = 1 \neq Y_{out} = -1}$$

update the weights

$$w_{1(new)} = w_{1(0)} + \alpha t(x_1)$$
$$= 1 + 1 * 1 * (-1)$$
$$= 1 - 1$$
$$\boxed{w_1 = 0}$$

$$w_{2(n)} = w_{2(0)} + \alpha t(x_2)$$
$$= (-1) + 1 * 1 * 1$$
$$\boxed{w_2 = 0}$$

$$b_{(n)} = b_0 + \alpha t$$
$$= 1 + 1 * 1$$
$$\boxed{b = 2}$$

(ii) $x_1 = -1$ $x_2 = 1$ $t = 1$

$w_1 = 0$ $w_2 = 0$ $b = 2$

$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= (-1) \times 0 + 1 \times 0 + 2$$
$$\boxed{Y_{in} = 2}$$

$$Y_{out} = f(y_{in}) = f(2)$$

$\boxed{Y_{out} = 21}$

$\boxed{t = 1 \;==\; Y_{out} = 1}$

Stop the process

For 4th set of inputs :

(i) $x_1 = -1 \quad x_2 = -1 \quad t = 1$

$w_1 = 0 \quad w_2 = 0 \quad b = 2$

$Y_{in} = x_1 w_1 + x_2 w_2 + b$

$= (-1) * 0 + (-1) \times 0 + 2$

$\boxed{Y_{in} = 2}$

$Y_{out} = f(Y_{in}) = f(2)$

$\boxed{Y_{out} = 1}$

$\boxed{t = 1 \;==\; Y_{out} = 1}$

Stop the process

x_1	x_2	$t = x_1$ NAND x_2	Y_{out}
1	1	-1	-1
1	-1	1	1
-1	1	1	1
-1	-1	1	1

For the linear class

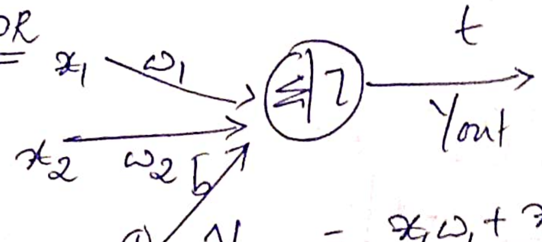
$w_1 x_1 + w_2 x_2 + b = 0$

$w_1 = 0 \quad w_2 = 0 \quad b = 2$

$x_1 * 0 + 0 * \cancel{x_2} + 2 = 0$

(B) NOR

Ans



$Y_{in} = x_1 w_1 + x_2 w_2 + b$

Given

$w_1 = w_2 = b = 0, \quad \alpha = 1 \quad \theta = 1$

x_1	x_2	$t = x_1$ NOR x_2	$Y_{out} = ?$
1	1	-1	?
1	-1	-1	
-1	1	-1	
-1	-1	1	

Activation Function :-

$$Y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$$

(i) For first set of inputs

$x_1 = 1 \quad x_2 = 1 \quad t = -1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= 1 * 0 + 1 * 0 + 0$$
$$\boxed{Y_{in} = 0}$$

$$Y_{out} = f(Y_{in}) = f(0)$$
$$\boxed{Y_{out} = -1}$$

$$\therefore \boxed{t = -1 \; == \; Y_{out} == -1}$$

Stop the process

(ii) For second set of inputs

$x_1 = 1 \quad x_2 = -1 \quad t = -1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$$Y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= 1 * 0 + (-1) * 0 + 0$$
$$\boxed{Y_{in} = 0}$$

$$Y_{out} = f(Y_{in}) = f(0)$$
$$\boxed{Y_{out} = -1}$$

$$\therefore \boxed{t == -1 \; == \; Y_{out} == -1}$$

Stop the process

(iii) For third set of inputs

$x_1 = -1 \quad x_2 = 1 \quad t = -1$

$w_1 = 0 \quad w_2 = 0 \quad b = 0$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= (-1) * 0 + (1)(0) + 0$$
$$\boxed{y_{in} = 0}$$
$$y_{out} = f(y_{in}) = f(0)$$
$$\boxed{y_{out} = -1}$$
$$\boxed{t == -1 == y_{out} == -1}$$

Stop the process.

(iv) For Fourth set of inputs

(i) $x_1 = -1$ $x_2 = -1$ $t = 1$

$w_1 = 0$ $w_2 = 0$ $b = 0$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$
$$= (-1) * 0 + (-1) * 0 + 0$$
$$\boxed{y_{in} = 0}$$
$$y_{out} = f(y_{in}) = f(0)$$
$$\boxed{y_{out} = -1}$$
$$\boxed{t = 1 \neq y_{out} = -1}$$

update the weights

$$w_{1(n)} = w_{1(o)} + \alpha t x_1$$
$$= 0 + (1)(1)(-1)$$
$$\boxed{w_1 = -1}$$
$$w_{2(n)} = w_{2(o)} + \alpha t x_2$$
$$= 0 + (1)(1)(-1)$$
$$\boxed{w_2 = -1}$$
$$b_{(n)} = b_{(o)} + \alpha t$$
$$= 0 + 1 * 1$$
$$\boxed{b = 1}$$

(ii) $x_1 = -1$ $x_2 = -1$ $t = 1$

$w_1 = -1$ $w_2 = -1$ $b = 1$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$= (-1)\times(-1) + (-1)\times(-1) + 1$

$= 1 + 1 + 1$

$\boxed{Y_{in} = 3}$

$Y_{out} = f(y_{in}) = f(3)$

$\boxed{Y_{out} = 1}$

$\boxed{t = 1 == Y_{out} = 1}$

Stop the process

For the linear class

$w_1 x_1 + w_2 x_2 + b = 0$

$w_1 = -1 \quad w_2 = -1 \quad b = 1$

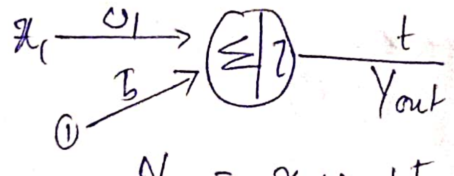
$-x_1 - x_2 + 1 = 0$

$-x_1 - x_2 = -1$

$\boxed{x_1 + x_2 = 1}$

x_1	x_2	$t = x_1$ NOR x_2	Y_{out}
1	1	-1	-1
1	-1	-1	-1
-1	1	-1	-1
-1	-1	1	1

(4) NOT



$Y_{in} = x_1 w_1 + b$

Given $w_1 = w_2 = b = 0$, $\alpha = 1$, $\theta = 2$

x_1	$t =$ NOT x_1	$Y_{out} = ?$
1	-1	
-1	1	

Activation function

$Y_{out} = f(y_{in}) \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$

(i) For the first set of inputs

$x_1 = 1 \qquad t = -1$

$w_1 = 0 \qquad b = 0$

$Y_{in} = x_1 w_1 + b$

$= 1 * 0 + 0$

$\boxed{Y_{in} = 0}$

$Y_{out} = f(Y_{in}) \; f(0)$

$\boxed{Y_{out} = -1}$

$\therefore \boxed{t == -1 \quad == Y_{out} == -1}$

Stop the process

(ii) For the second set of inputs

(i) $x_1 = -1 \qquad t = 1$

$w_1 = 0 \qquad b = 0$

$Y_{in} = x_1 w_1 + b$

$= -1 * 0 + 0$

$\boxed{Y_{in} = 0}$

$Y_{out} = f(Y_{in}) = f(0)$

$\boxed{Y_{out} = -1}$

$\therefore \boxed{t = 1 \neq Y_{out} == -1}$

$\therefore$ update the weight

$w_{1(n)} = w_{1(0)} + \alpha t x_1$

$= 0 + 1(1) * (-1)$

$\boxed{w_1 = -1}$

$b = b_{(0)} + \alpha t$

$= 0 + 1 * 1$

$\boxed{b = 1}$

(ii) $x_1 = -1 \quad t = 1$
$w_1 = -1 \quad b = 1$

$$Y_{in} = x_1 w_1 + b$$
$$= (-1) * (-1) + 1$$
$$= 1 + 1$$
$$\boxed{Y_{in} = 2}$$

$$Y_{out} = f(Y_{in}) = f(2)$$
$$\boxed{Y_{out} = 1}$$

$$\therefore \boxed{t = 1 == Y_{out} == 1}$$

Stop the process

~~For the~~ Linear class

x_1	$t = \text{NOT}(x_1)$	Y_{out}
1	-1	-1
-1	1	1