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Homework 3

Example 1.1. A patient takes a lab test and a result comes back positive. The test returns a correct postive result in 98% of the cases in which the disease is actually present, and a correct negative result in only 97% of the cases in which the disease is not present. Furthermore,0.008 of the entire population have this cancer

Note 1.2. $P(+|C) = 1 - P(-|C)$ is the probability of positive test result over the entire population that has the same cancer, and $P(C)$ the entirety of population which has this cancer. In contrast, $P(+|-C) = 1 - P(-|-C)$, is the positive test result over the entire population that does not have the same cancer. $P(-C)$ the entirety of population which does not have this cancer.

Definition 1.3. $P(+) = P(+|C) * P(C) + P(+|-C) * P(-C)$ and $P(C|+) = \frac{P(+|C)P(C)}{P(+)}$

Solution 1.4. The following yield

$$P(+) = P(+|C) * P(C) + (1 - P(-|-C)) * P(-C) =$$

$$P(+) = (0.98) * (0.008) + (1 - 0.97) * (1 - 0.002)$$

and so

$$P(C|+) = \frac{(0.98) * (0.008)}{(0.98) * (0.008) + (0.03) * (0.992)}$$

which amount to $P(C|+) = 2.09 * 10^{-1}$

Definition 1.5. $P(-) = P(-|C) * P(C) + P(-|-C) * P(-C)$ and $P(C|-) = \frac{P(-|C)P(C)}{P(-)}$

Solution 1.6. The following yield

$$P(-) = P(-|C) * P(C) + P(-|-C) * P(-C)$$

$$P(-) = (1 - P(+|C)) * P(C) + P(-|-C) * P(-C)$$

$$P(-) = (1 - .98) * (0.008) + (0.97) * (0.992)$$

and so

$$P(C|-) = \frac{(0.02) * (0.008)}{(0.02) * (0.008) + (0.97) * (0.992)}$$

which amount to $P(C|-) = 1.66 * 10^{-4}$

Note 1.7. Interestingly enough there's a higher chance of the population for this cancer to be tested positive than the population for this cancer to be tested negative.

Definition 1.8. $P(++) = P(++|C) * P(C) + P(++|-C) * P(-C)$ and $P(C|++) = \frac{P(++|C)P(C)}{P(++)}$

Solution 1.9. The following yield

$$P(++) = P(++|C) * P(C) + (1 - P(--|-C)) * P(-C) =$$

$$P(++) = (0.98)^2 * (0.008) + (1 - 0.97)^2 * (1 - 0.002)$$

and so

$$P(C|++) = \frac{(0.98)^2 * (0.008)}{(0.98)^2 * (0.008) + (0.03)^2 * (0.992)}$$

which amount to $P(C|++) = 7.92 * 10^{-2}$

Solution 1.6. The following yield

$$P(--) = P(--|C) * P(C) + P(--|-C) * P(-C)$$

$$P(--) = (1 - P(++|C)) * P(C) + P(--|-C) * P(-C)$$

$$P(--) = (1 - .98)^2 * (0.008) + (0.97)^2 * (0.992)$$

and so

$$P(C|--) = \frac{(0.02)^2 * (0.008)}{(0.02)^2 * (0.008) + (0.97)^2 * (0.992)}$$

which amount to $P(C|--) = 3.48 * 10^{-6}$

Note 1.8. The patient is very likely have cancer after the second retest since the chance of testing double negative is small or tiny.