Week 9: Hierarchical GLM

Quynh Vu

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Lip cancer

Here is the lip cancer data given to you in terribly unreproducible and error-prone format.

- `aff.i` is proportion of male population working outside in each region
- `observe.i` is observed deaths in each region
- `expect.i` is expected deaths, based on region-specific age distribution and national-level age-specific mortality rates.

Question 1: Explain a bit more what the `expect.i` variable is. For example, if a particular area has an expected deaths of 6, what does this mean?

Expected deaths is the implied number of lip cancer deaths for a particular region given that region's age structure and the national level age-specific mortality rates for lip cancer. For example, an expected number of deaths of 6 would mean that particular region, we would expected 6 lip cancer deaths if this region were to experience the same age specific mortality rates as at the national level.

Question 2: Run three different models in Stan with three different set-up's for estimating θ_i, that is the relative risk of lip cancer in each region:

1. Intercept α_i is same in each region $= \alpha$
2. α_i is different in each region and modeled separately (with covariate)
3. α_i is different in each region and the intercept is modeled hierarchically (with covariate)

$$y_i | \theta_i \sim \text{Poisson}(\theta_i \cdot e_i)$$

Model 1: $\log \theta_i = \alpha + \beta x_i$

```
##                 mean       se_mean         sd         2.5%          25%           50%
## alpha  -0.008328024 0.0009762687 0.02148674 -0.05082467 -0.02155265 -0.008823333
## beta    2.437492394 0.0062636984 0.17192935  2.12701859  2.32254512  2.436858919
##                  75%       97.5%     n_eff       Rhat
## alpha 0.005358399 0.03611338 484.3980 1.000973
## beta  2.557332984 2.77275258 753.4222 1.000182
```

Model 2: $\log \theta_i = \alpha_i + \beta x_i$

```
##                    mean      se_mean         sd          2.5%           25%
## alpha[1]   -0.330727032 0.008374375 0.3888719 -1.042145738 -0.6014377837
## alpha[2]    0.272842806 0.007144313 0.2658334 -0.286031568  0.0940786875
## alpha[3]    0.534540129 0.014240386 0.2745777 -0.026169621  0.3570136217
## alpha[4]   -0.338869560 0.011706420 0.3905615 -1.207273587 -0.5823591899
## alpha[5]    0.534437393 0.012770909 0.3234877 -0.124573171  0.3280985020
## alpha[6]   -0.710283845 0.010878972 0.2489426 -1.220642439 -0.8633357094
## alpha[7]    0.516123588 0.014572559 0.2392248  0.070204623  0.3553166047
## alpha[8]   -0.561447425 0.016842931 0.3358376 -1.317107575 -0.7611270789
## alpha[9]    0.732322046 0.007340603 0.2757100  0.195178548  0.5611662819
## alpha[10]   0.779008543 0.005359288 0.2209610  0.335329640  0.6303092074
## alpha[11]  -0.108939423 0.018030515 0.4797071 -1.043334562 -0.4130187505
## alpha[12]   0.834791104 0.011785379 0.3266132  0.101340049  0.6451147249
## alpha[13]   0.005298794 0.008327627 0.2897427 -0.599223019 -0.1927093991
## alpha[14]   0.663427438 0.006838376 0.2206089  0.209451946  0.5280854545
## alpha[15]   0.343827923 0.006117409 0.2840676 -0.246466042  0.1564983189
## alpha[16]   0.912685799 0.008562029 0.2710050  0.362358783  0.7347452316
## alpha[17]   1.036467423 0.006978944 0.2426208  0.553673444  0.8914017717
## alpha[18]   0.593021555 0.005850189 0.2090693  0.166357860  0.4521418316
## alpha[19]   0.063048530 0.013583293 0.2363191 -0.424472302 -0.0937319720
## alpha[20]   0.431579352 0.010702610 0.4257397 -0.480434521  0.1599429877
## alpha[21]   0.878751679 0.012739520 0.3269456  0.191322867  0.6809585592
## alpha[22]   1.075643946 0.008952848 0.2251533  0.607061643  0.9391202838
## alpha[23]   0.449132097 0.011647176 0.2766096 -0.146324976  0.2647855542
## alpha[24]  -0.131701911 0.010062284 0.4181539 -1.039042043 -0.4115695010
## alpha[25]   0.318987755 0.013938215 0.2505535 -0.182761508  0.1503699568
## alpha[26]   0.852354121 0.005599518 0.2111411  0.412208131  0.7114785882
## alpha[27]   0.674467838 0.007345115 0.2592638  0.165625676  0.4925059991
## alpha[28]   0.819034530 0.012124693 0.2670932  0.257442190  0.6454779658
## alpha[29]  -0.075386417 0.016596299 0.5391619 -1.231411010 -0.4072678668
## alpha[30]   0.594622036 0.019163586 0.3448550 -0.080903495  0.3778255546
## alpha[31]   0.213918932 0.010260583 0.3564372 -0.524846108 -0.0135003912
## alpha[32]  -0.513675793 0.014441250 0.4989930 -1.645697920 -0.8316971944
## alpha[33]   0.718731191 0.007098639 0.2770574  0.140316819  0.5397032788
## alpha[34]   0.405095309 0.008038031 0.2810470 -0.175832708  0.2194617745
## alpha[35]  -0.080957552 0.010951369 0.4034396 -0.962058398 -0.3381466452
## alpha[36]  -0.434116287 0.019784224 0.3016541 -1.003259631 -0.6428559914
## alpha[37]   0.588967397 0.008341562 0.3017438 -0.065149601  0.3828042979
## alpha[38]  -0.130365899 0.014258025 0.3925457 -0.897248094 -0.3843936507
## alpha[39]  -0.187386213 0.008754608 0.3399911 -0.871662985 -0.4104599422
## alpha[40]   0.458094467 0.010946756 0.3464658 -0.223432459  0.2311242407
## alpha[41]  -0.426547662 0.009812626 0.4209139 -1.311693825 -0.7175550645
## alpha[42]  -0.232161127 0.007801795 0.2930521 -0.874188492 -0.4185288981
## alpha[43]   0.100113250 0.007264253 0.3168193 -0.584329723 -0.1196761041
## alpha[44]  -0.172283430 0.007526905 0.2908654 -0.791496678 -0.3594105488
## alpha[45]   0.869763757 0.005503002 0.2013505  0.445307200  0.7365318990
## alpha[46]  -0.160233421 0.009745324 0.2335522 -0.657512490 -0.2968123810
## alpha[47]   0.409181285 0.009435661 0.2985294 -0.188805841  0.1949022822
## alpha[48]   0.744576853 0.008988522 0.3586832  0.023477538  0.5114439864
## alpha[49]   0.158153427 0.008175270 0.2659795 -0.376965968 -0.0089605132
## alpha[50]   0.012228218 0.009093599 0.2773758 -0.555353577 -0.1654092248
## alpha[51]  -0.349063146 0.013894362 0.2935885 -0.957898533 -0.5492257883
## alpha[52]   0.068386855 0.010891613 0.2774881 -0.503668405 -0.1061029443
## alpha[53]  -0.476138095 0.009054489 0.3739424 -1.246820087 -0.7347261560
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## alpha[54]     0.999630570 0.006875078 0.2720988  0.436574565  0.8291029631
## alpha[55]     0.520700520 0.009288467 0.3684114 -0.248945723  0.2932346938
## alpha[56]    -0.072787844 0.007538952 0.2447973 -0.563080701 -0.2349502902
## alpha[57]     0.876561847 0.008212783 0.2846794  0.269588839  0.6963526302
## alpha[58]    -0.498779525 0.012907732 0.3425528 -1.175489220 -0.7198162908
## alpha[59]    -0.274193475 0.011898889 0.3990862 -1.190496985 -0.5301042909
## alpha[60]     0.529672473 0.013571806 0.3575766 -0.233474482  0.3087748776
## alpha[61]    -0.065316823 0.007636120 0.3530700 -0.805939458 -0.2866403831
## alpha[62]    -0.113414783 0.010368913 0.3947567 -0.991712900 -0.3660690948
## alpha[63]    -0.152177989 0.006959474 0.2589180 -0.705243314 -0.3194878753
## alpha[64]    -0.308791395 0.010053096 0.4279754 -1.162465139 -0.5910581903
## alpha[65]     0.234121949 0.009770714 0.3842635 -0.577738849  0.0049058212
## alpha[66]     0.086874132 0.007690757 0.2850394 -0.503023830 -0.1001594572
## alpha[67]     0.168212342 0.013222964 0.4228280 -0.711903451 -0.0942588988
## alpha[68]    -0.039362005 0.011519435 0.4425738 -0.958612025 -0.3169154666
## alpha[69]    -0.179734538 0.008406209 0.2587606 -0.699472399 -0.3278598149
## alpha[70]    -0.072342442 0.012982805 0.4411061 -0.976470993 -0.3635391535
## alpha[71]     0.141112226 0.008011912 0.3528976 -0.542041845 -0.1025078126
## alpha[72]    -0.649671003 0.012383286 0.5386529 -1.789468085 -1.0145333274
## alpha[73]     0.435868177 0.009999339 0.3534353 -0.293558377  0.2141933542
## alpha[74]    -0.678068528 0.009621614 0.3474683 -1.382647258 -0.8906376466
## alpha[75]     0.263710573 0.010958769 0.3916539 -0.543174709  0.0089414640
## alpha[76]     0.579166169 0.007644494 0.2800225  0.001987991  0.3969824620
## alpha[77]    -0.090994385 0.006418132 0.2746487 -0.654486923 -0.2714988848
## alpha[78]     0.627551236 0.006223576 0.2417313  0.110059602  0.4764279180
## alpha[79]     0.326842842 0.009483474 0.3871140 -0.434473030  0.0778657101
## alpha[80]     0.550295549 0.012456460 0.4579238 -0.407848097  0.2899057792
## alpha[81]     0.016384997 0.012809186 0.4086255 -0.882642549 -0.2396487592
## alpha[82]     0.642981786 0.007212106 0.2910845  0.040614603  0.4559889725
## alpha[83]     0.360403007 0.006852941 0.3049053 -0.289921372  0.1635345081
## alpha[84]     0.627791908 0.015197649 0.2761127  0.063605462  0.4383603487
## alpha[85]     0.313556553 0.009002934 0.3064352 -0.311195769  0.1093564412
## alpha[86]     0.334324927 0.007682249 0.2445322 -0.194010487  0.1836221002
## alpha[87]     0.393641623 0.006681672 0.2690675 -0.208998310  0.2356292491
## alpha[88]     0.521318828 0.004980888 0.2184188  0.087037402  0.3831549467
## alpha[89]     0.714176955 0.010654384 0.4172191 -0.132945464  0.4592542216
## alpha[90]     0.467516074 0.008988915 0.2889787 -0.107529818  0.2850838689
## alpha[91]    -0.695661400 0.007854641 0.3647376 -1.443182962 -0.9197791325
## alpha[92]     0.111998506 0.010978379 0.3591856 -0.720649451 -0.1026668013
## alpha[93]     0.062905659 0.010187588 0.2025095 -0.321912010 -0.0790323806
## alpha[94]     0.617319479 0.008179871 0.2739952  0.040337268  0.4390412219
## alpha[95]     0.338167708 0.006182111 0.2795295 -0.224289191  0.1557474310
## alpha[96]     0.468413044 0.006700558 0.2380149 -0.020884355  0.3119717814
## alpha[97]     0.423251742 0.009016578 0.2224922 -0.026378691  0.2693801618
## alpha[98]     0.171424734 0.006636021 0.2639897 -0.400697971  0.0139947578
## alpha[99]    -0.760746257 0.013201567 0.5676733 -2.043234711 -1.1414888534
## alpha[100]   -0.048952754 0.010701976 0.1537563 -0.339585089 -0.1567140040
## alpha[101]    0.403322225 0.007054055 0.2777020 -0.148905130  0.2229382957
## alpha[102]    0.170532398 0.007423288 0.2609208 -0.345724949  0.0050781754
## alpha[103]    0.173557424 0.011093916 0.4021387 -0.698150926 -0.0857400066
## alpha[104]    0.140569552 0.008453903 0.2397969 -0.335834754 -0.0135390437
## alpha[105]   -0.022377879 0.008297173 0.3370514 -0.668386031 -0.2356872651
## alpha[106]    0.010788284 0.007390749 0.2851996 -0.566388688 -0.1732688926
## alpha[107]    0.503274696 0.009076508 0.3225267 -0.119349996  0.2767068135
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## alpha[108] -0.534183094 0.013105644 0.5103484 -1.618543620 -0.8581185508
## alpha[109]  0.015900157 0.006256554 0.2825182 -0.539899146 -0.1594835731
## alpha[110] -0.357396742 0.012205157 0.4587622 -1.379077558 -0.6434156323
## alpha[111]  0.135544962 0.008549269 0.2678984 -0.412516023 -0.0438990550
## alpha[112]  0.044811834 0.006340625 0.2741451 -0.506344697 -0.1453546832
## alpha[113] -0.075557972 0.010139037 0.2835378 -0.638859585 -0.2682619515
## alpha[114]  0.049182431 0.010312460 0.2968555 -0.596190982 -0.1407603304
## alpha[115]  0.124359831 0.008472481 0.3262435 -0.569374745 -0.0996265257
## alpha[116] -0.262013318 0.009890473 0.3961676 -1.096734263 -0.5004733270
## alpha[117]  0.153963124 0.007177109 0.2355385 -0.326117650  0.0025616027
## alpha[118] -1.302754623 0.008918940 0.3012165 -1.880319866 -1.5021720843
## alpha[119]  0.187396793 0.007611326 0.2140306 -0.237911107  0.0494412784
## alpha[120] -0.087874840 0.009648358 0.4249454 -0.976877160 -0.3433583003
## alpha[121]  0.127981350 0.008407643 0.3086955 -0.530274669 -0.0723743666
## alpha[122] -1.054611033 0.013260365 0.4978761 -2.059377535 -1.3496825476
## alpha[123] -0.125548710 0.007095632 0.2943066 -0.694984945 -0.3391040421
## alpha[124] -0.481866612 0.007900940 0.3102983 -1.108507143 -0.6955535352
## alpha[125]  0.181484101 0.007457107 0.2959297 -0.458145261 -0.0155019695
## alpha[126] -0.317326353 0.010925820 0.3706837 -1.161320004 -0.5356265908
## alpha[127] -0.442044878 0.010830048 0.2912603 -1.052534254 -0.6141189198
## alpha[128] -0.062869249 0.011936777 0.4068733 -0.956843317 -0.3157441868
## alpha[129] -0.282326133 0.005774971 0.2267378 -0.754828123 -0.4134525537
## alpha[130] -0.207733589 0.005994584 0.2461531 -0.737346631 -0.3545039159
## alpha[131] -0.071896835 0.010865225 0.4716582 -1.070722803 -0.3598394817
## alpha[132]  0.003066423 0.008689208 0.3509682 -0.743813096 -0.2200644740
## alpha[133] -0.017350303 0.008850223 0.3298179 -0.671261922 -0.2529244325
## alpha[134] -0.245070752 0.012056704 0.3889365 -1.072740428 -0.4919496876
## alpha[135] -0.299633404 0.012370945 0.4341242 -1.235827498 -0.5725303117
## alpha[136] -0.105089371 0.010793877 0.4619213 -1.085446231 -0.4250634070
## alpha[137] -0.850184690 0.012444929 0.5438321 -2.038174429 -1.1923526903
## alpha[138]  0.347209260 0.008186136 0.2858009 -0.222904773  0.1666889084
## alpha[139]  0.015809824 0.007936960 0.2846448 -0.612182926 -0.1725533122
## alpha[140]  0.230281229 0.006798745 0.2991605 -0.465801717  0.0391461301
## alpha[141]  0.065948917 0.011103479 0.3368030 -0.585643805 -0.1669483468
## alpha[142]  0.260591436 0.012843094 0.3872214 -0.492732794 -0.0007889347
## alpha[143] -0.268724501 0.007872266 0.3655560 -1.040740930 -0.5168015934
## alpha[144] -0.079877352 0.012944367 0.3820320 -0.873667508 -0.3331080752
## alpha[145]  0.375218398 0.010360127 0.2947116 -0.230553520  0.1867844936
## alpha[146] -0.055352726 0.009028144 0.2921340 -0.631027758 -0.2582821420
## alpha[147]  0.332483462 0.006427565 0.2008185 -0.062388298  0.1980746981
## alpha[148] -0.277981704 0.009643495 0.3689078 -1.007495927 -0.5254395361
## alpha[149]  0.761404798 0.006212912 0.2407085  0.269384012  0.6044630585
## alpha[150] -0.598020755 0.005527908 0.1552899 -0.891021797 -0.7019294521
## alpha[151] -0.337343429 0.008895092 0.3350186 -1.077075424 -0.5420847416
## alpha[152] -0.053310400 0.009155281 0.2429850 -0.573857067 -0.2087266974
## alpha[153]  0.517797964 0.008114861 0.2859477 -0.072783226  0.3271915303
## alpha[154]  0.386733834 0.006192767 0.2875669 -0.222108615  0.2001442976
## alpha[155] -0.007756365 0.007138617 0.2391821 -0.504487821 -0.1578277640
## alpha[156] -0.228483817 0.007182665 0.2515339 -0.724496525 -0.3847106337
## alpha[157] -0.607501452 0.012569811 0.4920830 -1.644429001 -0.9123606512
## alpha[158] -0.548096673 0.009461708 0.3190435 -1.249422359 -0.7448663961
## alpha[159]  0.216877042 0.007549615 0.2772927 -0.366046611  0.0609264126
## alpha[160] -0.188058555 0.011439164 0.3941843 -1.003489339 -0.4561026170
## alpha[161] -0.004430264 0.008624208 0.2909624 -0.659064590 -0.1647541789
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## alpha[162] -0.087900969 0.009812284 0.2814031 -0.664389830 -0.2645054466
## alpha[163]  0.033673368 0.007009385 0.3060532 -0.576639024 -0.1608517708
## alpha[164]  0.022098284 0.008453169 0.3179815 -0.598143600 -0.1769301283
## alpha[165] -0.259659486 0.006774925 0.3145999 -0.894099379 -0.4747217199
## alpha[166] -0.360038314 0.009503740 0.4174229 -1.172200156 -0.6282318654
## alpha[167]  0.377225032 0.007755848 0.3203162 -0.239059851  0.1482299216
## alpha[168] -0.180106391 0.007589290 0.3325932 -0.875486182 -0.3921795102
## alpha[169] -1.379387250 0.006236759 0.2229303 -1.827932106 -1.5365009639
## alpha[170]  0.042422520 0.010715483 0.3095906 -0.625840936 -0.1506159854
## alpha[171] -0.763725773 0.012354670 0.5394812 -1.913611925 -1.1269371363
## alpha[172]  0.166062276 0.007729914 0.3006706 -0.446690353 -0.0353675018
## alpha[173] -0.254874152 0.010726368 0.3916592 -1.135362743 -0.5203612326
## alpha[174] -0.117861546 0.010913041 0.3100630 -0.780920119 -0.2915434795
## alpha[175] -0.262444706 0.010880781 0.2815269 -0.835229982 -0.4541196791
## alpha[176] -0.176625871 0.010347357 0.3237661 -0.851304356 -0.3968107130
## alpha[177]  0.535794051 0.007223114 0.2858754 -0.048097401  0.3514900675
## alpha[178] -0.567515799 0.006540299 0.1907945 -0.948896582 -0.6955103968
## alpha[179] -0.464179936 0.009461592 0.4258526 -1.325737610 -0.7495517559
## alpha[180] -0.009997898 0.010505894 0.2985845 -0.643325312 -0.2040956922
## alpha[181] -0.092907425 0.009725838 0.3142747 -0.715302930 -0.2910569659
## alpha[182] -0.105025869 0.010725738 0.2819826 -0.701501995 -0.3007810420
## alpha[183]  0.087913179 0.008526660 0.3025382 -0.543519685 -0.0893287271
## alpha[184] -1.527220226 0.017017361 0.6561767 -2.809973708 -1.9616547050
## alpha[185] -0.045669190 0.009374552 0.3680245 -0.897510135 -0.2689521455
## alpha[186] -0.658423656 0.012376119 0.4902072 -1.674815132 -0.9757183915
## alpha[187] -0.441548996 0.006982341 0.2839297 -1.023180625 -0.6280748525
## alpha[188]  0.251588592 0.009741684 0.3256067 -0.412279948  0.0324502477
## alpha[189]  0.218855429 0.006486581 0.2864317 -0.369766973  0.0542023512
## alpha[190] -0.337449941 0.010114600 0.3997960 -1.185736870 -0.6022771689
## alpha[191] -0.269126914 0.012319571 0.4605789 -1.222393369 -0.5510531237
## alpha[192] -0.302315865 0.008508244 0.3252470 -0.964774773 -0.5181971666
## alpha[193]  0.108250825 0.008292900 0.3098338 -0.573836925 -0.1009619779
## alpha[194] -0.408461235 0.004892569 0.1931880 -0.792515451 -0.5383371145
## alpha[195] -0.739911200 0.010229216 0.1378542 -1.005838136 -0.8359603506
## beta         1.423367716 0.064345341 0.6408844  0.192814467  0.9792312148
##                        50%          75%         97.5%        n_eff        Rhat
## alpha[1]   -0.3185113760 -0.033682793  0.4204763078 2156.29595 0.9974746
## alpha[2]    0.2882609177  0.459376897  0.7610756778 1384.51664 0.9989434
## alpha[3]    0.5489569901  0.711131174  1.0657766024  371.78074 1.0009664
## alpha[4]   -0.3240301126 -0.078434798  0.4174387231 1113.09081 1.0000204
## alpha[5]    0.5410606552  0.756398411  1.1568521414  641.61105 0.9968639
## alpha[6]   -0.7060682892 -0.551668322 -0.2583222010  523.62795 0.9986231
## alpha[7]    0.5125403959  0.680986793  0.9670529539  269.48887 0.9988643
## alpha[8]   -0.5340233805 -0.351941906  0.0625110430  397.57892 1.0035359
## alpha[9]    0.7356435191  0.930854071  1.2553685994 1410.72281 1.0024783
## alpha[10]   0.7869582200  0.931107510  1.1751586084 1699.87560 0.9964269
## alpha[11]  -0.0763190752  0.241019063  0.6837706715  707.84166 1.0018044
## alpha[12]   0.8676500320  1.057930378  1.4140499661  768.03390 0.9984562
## alpha[13]   0.0151441321  0.203395714  0.5284999347 1210.54906 0.9999779
## alpha[14]   0.6666222916  0.810438925  1.0809586677 1040.73544 0.9975804
## alpha[15]   0.3627642356  0.531433213  0.8751068042 2156.29595 0.9963262
## alpha[16]   0.9245663005  1.087974176  1.4335441493 1001.84682 1.0007588
## alpha[17]   1.0404363481  1.211052194  1.4696476772 1208.58392 0.9986852
## alpha[18]   0.6013423055  0.734646177  0.9943586327 1277.14692 0.9985388
```

```
## alpha[19]   0.0645455443  0.220562307  0.5108381974   302.68259 0.9998227
## alpha[20]   0.4631271458  0.727757846  1.1957428701  1582.37275 0.9966892
## alpha[21]   0.8828752808  1.091082711  1.5143962424   658.63513 0.9972478
## alpha[22]   1.0757701666  1.223178877  1.5038827537   632.46176 0.9983605
## alpha[23]   0.4616371602  0.643997332  0.9668327879   564.01839 0.9977768
## alpha[24]  -0.0960073078  0.163781920  0.6019413771  1726.94770 0.9993273
## alpha[25]   0.3155472940  0.485087466  0.8149992376   323.13689 1.0006624
## alpha[26]   0.8584540187  0.998340981  1.2462086798  1421.81830 0.9982073
## alpha[27]   0.6841399379  0.872498654  1.1182370129  1245.91007 0.9987781
## alpha[28]   0.8328747308  1.013946387  1.2886616381   485.27074 0.9988948
## alpha[29]  -0.0023435474  0.310161455  0.8397658066  1055.39691 0.9977625
## alpha[30]   0.5755092048  0.823532625  1.2852313497   323.83174 0.9993956
## alpha[31]   0.2361744687  0.463995293  0.8587568044  1206.76325 0.9978231
## alpha[32]  -0.4721008082 -0.150519083  0.3551974035  1193.93122 0.9965782
## alpha[33]   0.7337768649  0.913658182  1.2178702623  1523.31330 0.9989666
## alpha[34]   0.4165742823  0.604441069  0.9124045420  1222.52708 0.9970066
## alpha[35]  -0.0610411725  0.188051779  0.6807982614  1357.12600 0.9979770
## alpha[36]  -0.4228407194 -0.225109349  0.1739560610   232.47717 0.9997913
## alpha[37]   0.6039288846  0.805749058  1.1264967912  1308.52514 0.9966451
## alpha[38]  -0.1168880437  0.126968955  0.5872362409   757.98684 0.9996416
## alpha[39]  -0.1627999366  0.057866605  0.3771497429  1508.20994 0.9970006
## alpha[40]   0.4783434239  0.697214618  1.1023746905  1001.72795 0.9983254
## alpha[41]  -0.4051652038 -0.120341650  0.3443499893  1839.99204 0.9989190
## alpha[42]  -0.1977319688 -0.028465723  0.3073693909  1410.91405 0.9980560
## alpha[43]   0.1212895178  0.313518276  0.6788562751  1902.13489 0.9963411
## alpha[44]  -0.1564398152  0.023249036  0.3535380953  1493.31476 0.9981569
## alpha[45]   0.8761531486  1.016650660  1.2332010116  1338.76974 0.9991562
## alpha[46]  -0.1521012769 -0.001476914  0.2775673807   574.34829 0.9985771
## alpha[47]   0.4200661342  0.630167507  0.9542108724  1000.98991 0.9992078
## alpha[48]   0.7637464884  0.993959283  1.3679809116  1592.37577 0.9991246
## alpha[49]   0.1701086429  0.337737029  0.6866619443  1058.50306 1.0018354
## alpha[50]   0.0265691090  0.202748512  0.5287093036   930.39128 1.0040848
## alpha[51]  -0.3396374426 -0.140226240  0.1715731437   446.47890 0.9997753
## alpha[52]   0.0772532994  0.254320685  0.5731411037   649.08900 0.9971525
## alpha[53]  -0.4586659412 -0.203209144  0.1965039183  1705.61700 1.0001173
## alpha[54]   1.0230049809  1.169713586  1.5096081889  1566.38337 1.0001565
## alpha[55]   0.5487717254  0.778171949  1.1649139832  1573.17871 0.9994845
## alpha[56]  -0.0593298845  0.090120805  0.3775540558  1054.36572 0.9997872
## alpha[57]   0.8989031376  1.069293199  1.3863653274  1201.52137 0.9978532
## alpha[58]  -0.4907426157 -0.264112922  0.1041783964   704.29572 0.9979231
## alpha[59]  -0.2524489403  0.014465892  0.4049335257  1124.91711 0.9990310
## alpha[60]   0.5409639777  0.773859753  1.1707065663   694.16535 1.0018712
## alpha[61]  -0.0486661544  0.181429852  0.5804831879  2137.84484 0.9976701
## alpha[62]  -0.0886235789  0.162486926  0.6075883889  1449.41411 0.9969357
## alpha[63]  -0.1415171438  0.022332899  0.3287962592  1384.11384 0.9979915
## alpha[64]  -0.2854806293 -0.009511106  0.4810770662  1812.33298 0.9970127
## alpha[65]   0.2492850536  0.496612985  0.9121294496  1546.69820 0.9966325
## alpha[66]   0.0897866492  0.283937440  0.6158560907  1373.63572 0.9980675
## alpha[67]   0.1893234688  0.467326752  0.9195743931  1022.51551 0.9970381
## alpha[68]  -0.0030127211  0.265267727  0.7503576614  1476.07671 0.9965930
## alpha[69]  -0.1780389065  0.003027785  0.2599343391   947.53645 0.9993349
## alpha[70]  -0.0493104196  0.249283000  0.7430302616  1154.38044 0.9977950
## alpha[71]   0.1490689594  0.377570181  0.7985750389  1940.10474 0.9970653
## alpha[72]  -0.6132936119 -0.253636634  0.2983001444  1892.10927 0.9985042
```

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## alpha[73]    0.4580079969    0.670935475    1.0846190204  1249.32996  0.9995478
## alpha[74]   -0.6561909070   -0.447312977   -0.0482597886  1304.17089  0.9964872
## alpha[75]    0.2950809891    0.520992275    1.0012658121  1277.26625  0.9981008
## alpha[76]    0.5860754241    0.785723059    1.0882806606  1341.80139  0.9971292
## alpha[77]   -0.0703670018    0.094293201    0.4169848072  1831.20831  0.9982301
## alpha[78]    0.6418214308    0.790190627    1.0794810996  1508.64027  0.9974661
## alpha[79]    0.3331425375    0.580422945    1.0246205573  1666.26010  0.9968116
## alpha[80]    0.5918506163    0.843445471    1.3889874790  1351.44114  0.9973270
## alpha[81]    0.0366469868    0.287970962    0.7400424831  1017.67216  0.9985745
## alpha[82]    0.6552196931    0.821736312    1.1895538399  1628.97347  0.9978353
## alpha[83]    0.3746103723    0.568363985    0.8898762299  1979.59363  0.9974435
## alpha[84]    0.6353276405    0.816527416    1.1379360798   330.08059  1.0008083
## alpha[85]    0.3345143433    0.522268219    0.8981061567  1158.53491  1.0004082
## alpha[86]    0.3437281504    0.480550467    0.8042287990  1013.20031  0.9977887
## alpha[87]    0.4049446452    0.561223312    0.9291696536  1621.63140  0.9969145
## alpha[88]    0.5277255403    0.669106985    0.9317837512  1922.94344  0.9980080
## alpha[89]    0.7264868818    1.006075617    1.4270106885  1533.45737  0.9985406
## alpha[90]    0.4778575573    0.661923216    1.0193520456  1033.51555  0.9984800
## alpha[91]   -0.6831420875   -0.449356132    0.0008642181  2156.29595  0.9971392
## alpha[92]    0.1344689946    0.345119315    0.7778831628  1070.43766  0.9983500
## alpha[93]    0.0699861971    0.206863498    0.4451629896   395.13745  0.9989312
## alpha[94]    0.6328790065    0.797987621    1.1193293858  1122.00045  0.9979658
## alpha[95]    0.3476300232    0.537270396    0.8285324813  2044.47436  0.9984507
## alpha[96]    0.4840032555    0.636644299    0.9273952226  1261.78745  0.9986920
## alpha[97]    0.4313601633    0.585585372    0.8362462444   608.90020  0.9988737
## alpha[98]    0.1845315777    0.344272305    0.6748848371  1582.55317  0.9991247
## alpha[99]   -0.7223847818   -0.361238499    0.2356840669  1849.03840  0.9991790
## alpha[100]  -0.0505039679    0.058722893    0.2505414506   206.41336  1.0014171
## alpha[101]   0.4207721152    0.592199524    0.9193334052  1549.81602  0.9978690
## alpha[102]   0.1828116190    0.352249838    0.6331120760  1235.44856  0.9972648
## alpha[103]   0.1904017966    0.455208345    0.8763330218  1313.95936  0.9986234
## alpha[104]   0.1497906585    0.295916087    0.5942587964   804.58646  0.9981630
## alpha[105]  -0.0164205263    0.203556900    0.5906881798  1650.18240  0.9999200
## alpha[106]   0.0193262556    0.221954203    0.5049190896  1489.09016  0.9988814
## alpha[107]   0.5212824093    0.739938325    1.0358341590  1262.68151  0.9980463
## alpha[108]  -0.4912812117   -0.193601295    0.4064563185  1516.41064  0.9970027
## alpha[109]   0.0239838600    0.215680479    0.5378132743  2039.02434  0.9980234
## alpha[110]  -0.3292166617   -0.047136984    0.4722389122  1412.82524  0.9970090
## alpha[111]   0.1480102220    0.318379313    0.6343342829   981.93366  1.0000343
## alpha[112]   0.0582098456    0.235771716    0.5532114338  1869.37654  0.9987134
## alpha[113]  -0.0567755735    0.137562124    0.4236170746   782.03935  0.9970257
## alpha[114]   0.0684216878    0.250857881    0.6031525295   828.63957  0.9989511
## alpha[115]   0.1359855092    0.343347113    0.7358547072  1482.73150  0.9979076
## alpha[116]  -0.2445179205    0.018091223    0.4598597298  1604.44100  0.9967526
## alpha[117]   0.1616389913    0.317308050    0.5880357469  1077.02233  1.0021624
## alpha[118]  -1.2961347829   -1.098578249   -0.7488616926  1140.59349  1.0010282
## alpha[119]   0.1940057101    0.334413438    0.5988737249   790.73492  0.9969461
## alpha[120]  -0.0476313548    0.188576250    0.6726720214  1939.81079  1.0004298
## alpha[121]   0.1421824659    0.342842866    0.6755590027  1348.06880  0.9993730
## alpha[122]  -1.0069595098   -0.734022994   -0.1729058514  1409.71591  0.9969297
## alpha[123]  -0.1145816026    0.076315546    0.4273315576  1720.35393  0.9981357
## alpha[124]  -0.4798744098   -0.266881284    0.0688319395  1542.41540  0.9967800
## alpha[125]   0.1957646117    0.378753232    0.7237966889  1574.84023  0.9969893
## alpha[126]  -0.2817063604   -0.064539181    0.3794694414  1151.06228  0.9974285
```

```
## alpha[127] -0.4247809559 -0.251330372  0.0869661816  723.27242 0.9969153
## alpha[128] -0.0387803122  0.216842846  0.6736375001 1161.83439 0.9986706
## alpha[129] -0.2762619679 -0.129452195  0.1278697459 1541.51645 0.9979447
## alpha[130] -0.2008789767 -0.035518609  0.2435265584 1686.13584 0.9972150
## alpha[131] -0.0385083702  0.257130490  0.7369533650 1884.41804 0.9961392
## alpha[132]  0.0252452685  0.244993220  0.6558045014 1631.45538 0.9968013
## alpha[133]  0.0118933479  0.210312600  0.5900306958 1388.80117 1.0011596
## alpha[134] -0.2016490728  0.030880563  0.4424729038 1040.63950 0.9977417
## alpha[135] -0.2708325406 -0.013502260  0.4915234284 1231.46548 0.9968170
## alpha[136] -0.0707773612  0.226396095  0.7037862411 1831.39103 0.9964528
## alpha[137] -0.8328940348 -0.457946601  0.1010440517 1909.61063 0.9966921
## alpha[138]  0.3604132674  0.522936143  0.9177350891 1218.90301 0.9968201
## alpha[139]  0.0351970043  0.218748363  0.5224499850 1286.16898 0.9964890
## alpha[140]  0.2514866349  0.429757833  0.8108588716 1936.20420 0.9976107
## alpha[141]  0.0709612296  0.312883785  0.7051076653  920.09749 0.9968126
## alpha[142]  0.2900546863  0.535831519  0.9302795876  909.03243 0.9982367
## alpha[143] -0.2523598940 -0.006723866  0.4176765455 2156.29595 0.9973976
## alpha[144] -0.0752560542  0.204381617  0.5904895892  871.03941 1.0010161
## alpha[145]  0.3844064531  0.572235596  0.9272807063  809.21553 0.9970666
## alpha[146] -0.0357700846  0.155685040  0.4669430643 1047.04944 0.9984667
## alpha[147]  0.3297815766  0.462745475  0.7073043073  976.14537 0.9979068
## alpha[148] -0.2588120536 -0.018895703  0.3856807005 1463.41256 0.9981926
## alpha[149]  0.7640788868  0.926851539  1.2000100190 1501.04086 0.9967566
## alpha[150] -0.5962522037 -0.490153585 -0.3131680420  789.15930 0.9985647
## alpha[151] -0.3245825185 -0.111027768  0.2707177406 1418.52496 0.9969580
## alpha[152] -0.0402150327  0.118656662  0.3719788390  704.39389 0.9990915
## alpha[153]  0.5332050301  0.712825044  1.0367914135 1241.68392 0.9978551
## alpha[154]  0.3987694857  0.593572945  0.9173049819 2156.29595 0.9962666
## alpha[155]  0.0008644997  0.152999124  0.4280534892 1122.61048 0.9963877
## alpha[156] -0.2250074037 -0.056074013  0.2467002931 1226.37065 0.9981398
## alpha[157] -0.5690386844 -0.257061924  0.2839799022 1532.56585 0.9969242
## alpha[158] -0.5398716492 -0.338752803  0.0589255112 1137.00107 0.9970065
## alpha[159]  0.2241262467  0.407896455  0.7949799400 1349.04741 0.9970250
## alpha[160] -0.1738600312  0.111593613  0.5004729411 1187.43584 0.9974275
## alpha[161]  0.0051916645  0.180543235  0.5703056856 1138.24407 0.9973325
## alpha[162] -0.0829906041  0.098362055  0.4498074134  822.46496 1.0008795
## alpha[163]  0.0355338365  0.232323273  0.6256515057 1906.48804 0.9971328
## alpha[164]  0.0401817958  0.240838062  0.5725068872 1415.02612 0.9979949
## alpha[165] -0.2379204757 -0.030697106  0.3113585013 2156.29595 0.9969800
## alpha[166] -0.3409735164 -0.063363041  0.3870566474 1929.13842 0.9965104
## alpha[167]  0.3896773586  0.613499495  0.9483192888 1705.68623 0.9984730
## alpha[168] -0.1589543430  0.051925942  0.3822555664 1920.54550 0.9965948
## alpha[169] -1.3678121343 -1.220946834 -0.9755806471 1277.67430 0.9983000
## alpha[170]  0.0551337369  0.278224923  0.5906318640  834.74178 1.0018197
## alpha[171] -0.7078367147 -0.398811557  0.2002284790 1906.73519 0.9987462
## alpha[172]  0.1740634423  0.393168634  0.7290456616 1512.97750 0.9967367
## alpha[173] -0.2324198120  0.028295291  0.4191538783 1333.24919 0.9995397
## alpha[174] -0.0975556988  0.084740682  0.4337824975  807.25061 0.9979047
## alpha[175] -0.2436618125 -0.076875749  0.2845402814  669.45238 0.9971297
## alpha[176] -0.1725666485  0.039908355  0.4370626423  979.04780 0.9991390
## alpha[177]  0.5566563280  0.735502532  1.0460194962 1566.40734 0.9975239
## alpha[178] -0.5670289873 -0.427494576 -0.2212589449  851.01380 0.9989072
## alpha[179] -0.4366659453 -0.174358867  0.2735274847 2025.76997 0.9964002
## alpha[180]  0.0060708809  0.198335445  0.5286816807  807.73429 1.0002971
```

```
## alpha[181]  -0.0962173039   0.136996728   0.4974257259  1044.15429  0.9971842
## alpha[182]  -0.1030908701   0.100036417   0.4025524794   691.17838  0.9978626
## alpha[183]   0.1021842518   0.289834083   0.6418045160  1258.93315  0.9984883
## alpha[184]  -1.4759064691  -1.045361605  -0.3485278596  1486.81583  0.9971995
## alpha[185]  -0.0271927363   0.200726483   0.6109555181  1541.17673  0.9972476
## alpha[186]  -0.6236789314  -0.307075427   0.2536931685  1568.88215  0.9974447
## alpha[187]  -0.4346619282  -0.249451712   0.0870637847  1653.55830  0.9972633
## alpha[188]   0.2795697766   0.478067009   0.8429347242  1117.16824  0.9967410
## alpha[189]   0.2215616095   0.406087990   0.7602685882  1949.89191  0.9981432
## alpha[190]  -0.3218440115  -0.048863032   0.3983250137  1562.35399  0.9964382
## alpha[191]  -0.2451233688   0.042491773   0.5812882853  1397.70937  0.9978834
## alpha[192]  -0.2861031849  -0.074435589   0.3011697275  1461.32474  0.9971667
## alpha[193]   0.1233332093   0.327295136   0.6690163672  1395.86875  0.9987995
## alpha[194]  -0.4023074100  -0.282195262  -0.0280885315  1559.14468  0.9963271
## alpha[195]  -0.7400382224  -0.644839401  -0.4756590785   181.61645  1.0009185
## beta          1.4082627048   1.882656920   2.7096757455    99.20309  1.0041447
```

Model 3: $\log \theta_i = \alpha_i + \beta x_i$ with $\alpha_i \; + \beta x_i$

```
##                    mean      se_mean         sd          2.5%           25%
## alpha[1]    -0.132166758 0.0030460716 0.25905328 -0.661077391 -0.298210642
## alpha[2]     0.211218970 0.0030880451 0.23814321 -0.283684619  0.060113724
## alpha[3]     0.332089549 0.0033887884 0.22347251 -0.130812140  0.189902081
## alpha[4]    -0.143076219 0.0033706993 0.28457714 -0.693596013 -0.328033597
## alpha[5]     0.338040968 0.0034191641 0.26901611 -0.180826998  0.156817061
## alpha[6]    -0.607620628 0.0028223192 0.20097613 -1.004183571 -0.745395477
## alpha[7]     0.337002686 0.0026921241 0.18072094 -0.013655086  0.215195375
## alpha[8]    -0.432385364 0.0039004800 0.22385713 -0.897721132 -0.573493107
## alpha[9]     0.576416154 0.0029294947 0.23570150  0.087696120  0.418245560
## alpha[10]    0.629058499 0.0027649261 0.20456143  0.226870652  0.496798961
## alpha[11]   -0.006438384 0.0040751744 0.30504757 -0.621481364 -0.208475440
## alpha[12]    0.515181716 0.0035845313 0.26296835 -0.022628298  0.345944835
## alpha[13]    0.021531381 0.0029866608 0.23832444 -0.450301661 -0.133852183
## alpha[14]    0.501628839 0.0030744478 0.20544782  0.073778894  0.371252912
## alpha[15]    0.261864770 0.0030580094 0.23286237 -0.207453389  0.112749138
## alpha[16]    0.621990630 0.0031913489 0.24481055  0.126749735  0.459347746
## alpha[17]    0.745101031 0.0032591841 0.23630208  0.268981559  0.593532514
## alpha[18]    0.480862773 0.0023714016 0.20187178  0.086435402  0.344329790
## alpha[19]   -0.011747721 0.0026361975 0.18449782 -0.374930000 -0.132694676
## alpha[20]    0.277809993 0.0040082951 0.29960364 -0.318439823  0.076280223
## alpha[21]    0.530886601 0.0034592704 0.26817154  0.007164146  0.351199294
## alpha[22]    0.795286457 0.0030882820 0.21999209  0.344903683  0.645839218
## alpha[23]    0.305490022 0.0031667693 0.23151061 -0.148951547  0.152996529
## alpha[24]   -0.017853766 0.0034593312 0.28481985 -0.595936902 -0.206383590
## alpha[25]    0.188304963 0.0030482831 0.20184546 -0.209595299  0.051488457
## alpha[26]    0.650465441 0.0028096362 0.21973143  0.210652708  0.502013288
## alpha[27]    0.467791070 0.0030629743 0.24442709 -0.027094413  0.307992718
## alpha[28]    0.588010812 0.0029924261 0.22817650  0.126976996  0.442518559
## alpha[29]    0.019969739 0.0045937455 0.31674221 -0.597051093 -0.195100179
## alpha[30]    0.337235724 0.0035420012 0.27302737 -0.211465924  0.162823373
## alpha[31]    0.139601088 0.0038615393 0.26610188 -0.398845031 -0.035347019
## alpha[32]   -0.198816644 0.0040854022 0.29825892 -0.822148202 -0.383373503
## alpha[33]    0.521159213 0.0032556297 0.25110064  0.024951732  0.357234721
## alpha[34]    0.289287415 0.0036085519 0.25087780 -0.237198868  0.128066246
```

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## alpha[35]   -0.008261080 0.0035150635 0.26698112 -0.538465215 -0.180275299
## alpha[36]   -0.410149746 0.0029745586 0.21230267 -0.839760307 -0.551546149
## alpha[37]    0.428369013 0.0033468332 0.24776090 -0.081718904  0.272731369
## alpha[38]   -0.033645682 0.0036868629 0.27042426 -0.615562045 -0.207532843
## alpha[39]   -0.041286024 0.0033817382 0.25093572 -0.552681536 -0.199129236
## alpha[40]    0.276806609 0.0037518895 0.26378995 -0.263155839  0.108458430
## alpha[41]   -0.163808430 0.0039145800 0.28424635 -0.719957512 -0.349366885
## alpha[42]   -0.108223530 0.0028432956 0.25216637 -0.632283713 -0.275546579
## alpha[43]    0.087024806 0.0031691418 0.23033860 -0.391599956 -0.060643164
## alpha[44]   -0.091465612 0.0029805893 0.22326814 -0.543600411 -0.242411774
## alpha[45]    0.712241045 0.0027010646 0.19882980  0.306382236  0.584946828
## alpha[46]   -0.163929736 0.0028264425 0.20695671 -0.581062974 -0.301817359
## alpha[47]    0.352675767 0.0031108473 0.24774042 -0.158651858  0.195516408
## alpha[48]    0.445515673 0.0040071911 0.30597395 -0.203526702  0.247475258
## alpha[49]    0.126569199 0.0029737206 0.22414460 -0.305493114 -0.032483841
## alpha[50]    0.069193251 0.0030726139 0.23208421 -0.399216963 -0.085398006
## alpha[51]   -0.273146331 0.0030677664 0.21988832 -0.701840769 -0.425159706
## alpha[52]    0.036956000 0.0032369663 0.23530476 -0.437928618 -0.117070277
## alpha[53]   -0.219890954 0.0036429386 0.27209405 -0.783053681 -0.401669902
## alpha[54]    0.738990674 0.0032888479 0.25528756  0.228164517  0.572326518
## alpha[55]    0.321082885 0.0039988271 0.29773887 -0.276236626  0.117873011
## alpha[56]   -0.063368323 0.0024598285 0.20389861 -0.484214294 -0.195668387
## alpha[57]    0.651094108 0.0034795489 0.25406013  0.115219239  0.484257557
## alpha[58]   -0.349539250 0.0030406625 0.23157178 -0.809054160 -0.502700202
## alpha[59]   -0.070584527 0.0034170679 0.25962619 -0.609906916 -0.236204001
## alpha[60]    0.398948623 0.0039009172 0.29803414 -0.183209217  0.187710563
## alpha[61]    0.009298314 0.0033556229 0.26873755 -0.503799062 -0.177021963
## alpha[62]    0.009738562 0.0036339897 0.28399233 -0.573495266 -0.168651435
## alpha[63]   -0.105919013 0.0025673028 0.20546333 -0.521666189 -0.238503739
## alpha[64]   -0.090474525 0.0040525471 0.29543992 -0.669638367 -0.289470933
## alpha[65]    0.200196028 0.0036961728 0.28192549 -0.363679634  0.010886288
## alpha[66]    0.131828810 0.0030379121 0.23398820 -0.336468467 -0.018794835
## alpha[67]    0.182153713 0.0036638103 0.30636484 -0.412669688 -0.013359200
## alpha[68]    0.035491921 0.0039116163 0.29242224 -0.563232675 -0.157371269
## alpha[69]   -0.164753883 0.0030155250 0.20225194 -0.572932892 -0.302971203
## alpha[70]    0.059999093 0.0037139052 0.29722629 -0.534184817 -0.133503933
## alpha[71]    0.157914454 0.0032902505 0.26424859 -0.372817332 -0.021679343
## alpha[72]   -0.176639753 0.0044191456 0.33416224 -0.842571490 -0.394884137
## alpha[73]    0.293200076 0.0035238658 0.26647962 -0.234817348  0.110675233
## alpha[74]   -0.413859936 0.0035441974 0.25839300 -0.962267621 -0.586459060
## alpha[75]    0.208026228 0.0033542438 0.28616086 -0.390652356  0.027896642
## alpha[76]    0.399361049 0.0029539828 0.23917615 -0.095577306  0.241101496
## alpha[77]   -0.037376323 0.0027830967 0.22026012 -0.506576802 -0.174270444
## alpha[78]    0.500447017 0.0029201636 0.21701713  0.055260142  0.356434013
## alpha[79]    0.204050428 0.0039507658 0.30337279 -0.422575319  0.002093740
## alpha[80]    0.306996287 0.0040766068 0.32705974 -0.359929551  0.089829127
## alpha[81]    0.026626815 0.0036678054 0.28795161 -0.569128636 -0.167346388
## alpha[82]    0.471818387 0.0030874764 0.24689795 -0.030080096  0.308199243
## alpha[83]    0.268900271 0.0033582892 0.25286800 -0.227426503  0.098008983
## alpha[84]    0.420917698 0.0032302270 0.22401295 -0.020360912  0.273638174
## alpha[85]    0.216448052 0.0036251382 0.25438453 -0.297844973  0.046683235
## alpha[86]    0.303021985 0.0025857407 0.21909438 -0.141158840  0.157938700
## alpha[87]    0.338135439 0.0032436888 0.22627610 -0.117949479  0.186601885
## alpha[88]    0.418034484 0.0031403348 0.22189845 -0.047542738  0.274689838
```

```
## alpha[89]    0.421958361 0.0035895301 0.30599687 -0.190947588  0.214917328
## alpha[90]    0.341008782 0.0035846034 0.24454313 -0.154757181  0.181456078
## alpha[91]   -0.338528120 0.0035595716 0.25183160 -0.854975354 -0.504679039
## alpha[92]    0.084575996 0.0036440043 0.27555361 -0.479546043 -0.095419351
## alpha[93]    0.140551233 0.0022320944 0.15971239 -0.180842880  0.033936255
## alpha[94]    0.483666033 0.0029930166 0.22923815  0.029799373  0.320188939
## alpha[95]    0.287695429 0.0029167708 0.22153774 -0.143819116  0.136800985
## alpha[96]    0.404690555 0.0028477801 0.21271440 -0.014819995  0.265088207
## alpha[97]    0.410212256 0.0024765817 0.19079037  0.037272081  0.284340645
## alpha[98]    0.186552248 0.0027902766 0.22621231 -0.278003746  0.046537171
## alpha[99]   -0.235147714 0.0036802815 0.30190033 -0.826325561 -0.433066610
## alpha[100]  -0.112774928 0.0016875287 0.11918457 -0.360362010 -0.190197357
## alpha[101]   0.323789762 0.0033481682 0.23778753 -0.160423437  0.164164805
## alpha[102]   0.166361449 0.0033541634 0.22025276 -0.278787386  0.028300368
## alpha[103]   0.148386850 0.0037208000 0.28408656 -0.415025908 -0.043569620
## alpha[104]   0.181623828 0.0026173344 0.19956853 -0.219669514  0.044226450
## alpha[105]   0.042921012 0.0032206519 0.26333956 -0.507098737 -0.132060291
## alpha[106]   0.070336597 0.0028267920 0.23368472 -0.421772822 -0.080370101
## alpha[107]   0.335948467 0.0030088409 0.25052809 -0.177096701  0.162242037
## alpha[108]  -0.162480510 0.0037882921 0.29721001 -0.739710570 -0.365859502
## alpha[109]   0.058119701 0.0030719307 0.24504670 -0.434304844 -0.097009631
## alpha[110]  -0.068859528 0.0038204799 0.28956105 -0.666467491 -0.263938075
## alpha[111]   0.164217067 0.0030415032 0.23725588 -0.311102051  0.002389946
## alpha[112]   0.098129337 0.0026643737 0.24283754 -0.395606225 -0.064030613
## alpha[113]   0.020923824 0.0033916983 0.24483600 -0.468675712 -0.144715022
## alpha[114]   0.121384353 0.0031803002 0.22906412 -0.320448118 -0.034442252
## alpha[115]   0.141038986 0.0031028570 0.26314763 -0.387427232 -0.030169689
## alpha[116]  -0.053970708 0.0037101819 0.28639722 -0.660235203 -0.240375717
## alpha[117]   0.176029179 0.0025121864 0.20532439 -0.247267322  0.037784941
## alpha[118]  -0.953332287 0.0031502662 0.21672803 -1.404601486 -1.090949393
## alpha[119]   0.211292205 0.0023856773 0.19001214 -0.168584686  0.092299314
## alpha[120]   0.027824959 0.0037312496 0.28064749 -0.537283137 -0.158455760
## alpha[121]   0.093315116 0.0033670509 0.25349334 -0.402543174 -0.077049109
## alpha[122]  -0.407066671 0.0043389787 0.30025712 -1.029713243 -0.596354808
## alpha[123]  -0.025399535 0.0036828785 0.23649728 -0.484693127 -0.183226684
## alpha[124]  -0.257574428 0.0034127059 0.24483954 -0.747219253 -0.408901001
## alpha[125]   0.158039594 0.0026148158 0.24002640 -0.334826366 -0.003450222
## alpha[126]  -0.130128782 0.0034652242 0.26276329 -0.671374323 -0.307522307
## alpha[127]  -0.304206735 0.0032357188 0.22375045 -0.742668923 -0.457306362
## alpha[128]   0.042308827 0.0038321735 0.29473353 -0.548656964 -0.154194945
## alpha[129]  -0.195220405 0.0027039779 0.19274237 -0.592439768 -0.317596516
## alpha[130]  -0.120120439 0.0032534455 0.20700021 -0.536321187 -0.256281243
## alpha[131]   0.045203701 0.0037662056 0.30040723 -0.555454002 -0.163686436
## alpha[132]   0.055137327 0.0032463665 0.27758584 -0.522829701 -0.125393929
## alpha[133]   0.059089507 0.0035047096 0.25932398 -0.462031485 -0.107340986
## alpha[134]  -0.043006805 0.0033430865 0.27457747 -0.602133097 -0.211560655
## alpha[135]  -0.085739277 0.0033889252 0.29051517 -0.666116200 -0.280283670
## alpha[136]   0.010700981 0.0038323693 0.29296846 -0.577681554 -0.181341984
## alpha[137]  -0.275445858 0.0044454571 0.31056597 -0.903764182 -0.483518657
## alpha[138]   0.301181367 0.0035793133 0.24477369 -0.195644970  0.136480823
## alpha[139]   0.084871903 0.0032502832 0.23013861 -0.375691436 -0.073346840
## alpha[140]   0.210999082 0.0030622520 0.24319363 -0.265366654  0.051600679
## alpha[141]   0.125325510 0.0032085510 0.26115274 -0.398517292 -0.047123813
## alpha[142]   0.234661325 0.0035646138 0.27989018 -0.344251072  0.048848500
```

```
## alpha[143]  -0.096714529 0.0032274632 0.24739385 -0.604373462 -0.249822854
## alpha[144]   0.045744852 0.0035504879 0.26034670 -0.481389889 -0.125184303
## alpha[145]   0.328964667 0.0031425356 0.25348866 -0.186495172  0.158225160
## alpha[146]   0.033481961 0.0030929473 0.23165148 -0.429234688 -0.126016581
## alpha[147]   0.320074544 0.0025383089 0.19064394 -0.071010999  0.193417236
## alpha[148]  -0.102812723 0.0039169864 0.26308656 -0.613747798 -0.276855015
## alpha[149]   0.581325926 0.0031137510 0.23291536  0.107194404  0.427530318
## alpha[150]  -0.488491353 0.0017940067 0.13581698 -0.762521433 -0.577849744
## alpha[151]  -0.145463430 0.0028333297 0.24362279 -0.654568872 -0.301690069
## alpha[152]  -0.073210938 0.0028785598 0.20501994 -0.493024122 -0.203157174
## alpha[153]   0.374107994 0.0031339570 0.25149778 -0.134946929  0.210037946
## alpha[154]   0.321074076 0.0026987066 0.23951581 -0.147026039  0.157618927
## alpha[155]   0.057984077 0.0028213526 0.21682352 -0.371705479 -0.085559631
## alpha[156]  -0.120274233 0.0030575439 0.21705019 -0.561954614 -0.261742767
## alpha[157]  -0.204890539 0.0039621420 0.30090743 -0.816307892 -0.401778527
## alpha[158]  -0.285518304 0.0028515919 0.24127740 -0.770255113 -0.447063737
## alpha[159]   0.222891067 0.0025100261 0.23134286 -0.242123939  0.069237334
## alpha[160]  -0.031767950 0.0034510637 0.27093288 -0.597909656 -0.201239864
## alpha[161]   0.074184685 0.0031701792 0.23419119 -0.412231990 -0.079658587
## alpha[162]   0.007117137 0.0027808623 0.22282359 -0.443935021 -0.141922557
## alpha[163]   0.104439489 0.0031513686 0.23433591 -0.374432717 -0.050067120
## alpha[164]   0.056134282 0.0036278210 0.26946921 -0.485439785 -0.122483301
## alpha[165]  -0.118629945 0.0033514592 0.24118520 -0.602182845 -0.283634192
## alpha[166]  -0.132107321 0.0036350734 0.28262745 -0.716098634 -0.318403079
## alpha[167]   0.274733994 0.0032410512 0.26277030 -0.256401165  0.098434557
## alpha[168]  -0.064226550 0.0032969020 0.25071610 -0.570275010 -0.234272873
## alpha[169]  -1.077146770 0.0025159038 0.17237552 -1.435684266 -1.187113469
## alpha[170]   0.051214394 0.0029338933 0.24693113 -0.433060194 -0.120335833
## alpha[171]  -0.244710688 0.0043730187 0.30895338 -0.863867525 -0.459591083
## alpha[172]   0.142773267 0.0036260107 0.25650271 -0.373246490 -0.030311714
## alpha[173]  -0.085221469 0.0033855218 0.27434866 -0.624820475 -0.267426875
## alpha[174]   0.004882498 0.0034637122 0.23556378 -0.474701867 -0.150448667
## alpha[175]  -0.104179215 0.0031654506 0.23575772 -0.591688875 -0.259711022
## alpha[176]  -0.021296572 0.0029058833 0.25011316 -0.512814029 -0.190498214
## alpha[177]   0.399905779 0.0038196692 0.25312974 -0.109999014  0.231407202
## alpha[178]  -0.427787384 0.0022061302 0.15267508 -0.735635811 -0.530642101
## alpha[179]  -0.168313516 0.0033558738 0.27619217 -0.722721210 -0.351117448
## alpha[180]   0.064718333 0.0035239989 0.24293357 -0.413534065 -0.111598199
## alpha[181]   0.026550765 0.0036077759 0.25165600 -0.508572248 -0.137574176
## alpha[182]   0.005664565 0.0031764786 0.24126824 -0.451059629 -0.153780148
## alpha[183]   0.127498758 0.0032990618 0.24128198 -0.348762897 -0.033747261
## alpha[184]  -0.458849298 0.0047458848 0.33120741 -1.130930707 -0.666521216
## alpha[185]   0.035378082 0.0032272352 0.25642787 -0.501759074 -0.130149343
## alpha[186]  -0.230933821 0.0035235553 0.29676311 -0.816967627 -0.425636880
## alpha[187]  -0.291180553 0.0027687553 0.21855580 -0.746745097 -0.433268486
## alpha[188]   0.239999120 0.0029632894 0.24455805 -0.248997203  0.072952172
## alpha[189]   0.198301457 0.0032903034 0.25302422 -0.323917089  0.033216340
## alpha[190]  -0.082278089 0.0034109095 0.29274832 -0.668061047 -0.277064609
## alpha[191]  -0.028264519 0.0038685502 0.31474771 -0.675344538 -0.240099994
## alpha[192]  -0.109739641 0.0032711499 0.26499189 -0.642724613 -0.281384585
## alpha[193]   0.136143229 0.0036602325 0.25722500 -0.377192579 -0.035655043
## alpha[194]  -0.296878680 0.0019972149 0.15680834 -0.597631774 -0.403355680
## alpha[195]  -0.612107741 0.0016447463 0.10184253 -0.813316358 -0.679024769
## beta         1.968384733 0.0078029809 0.33422981  1.276299213  1.755302709
```

```
## mu          0.085220364 0.0006756028 0.03778452  0.010434068  0.059613625
## sigma       0.387521809 0.0007912054 0.03043245  0.330849840  0.366089383
##                     50%          75%          97.5%      n_eff       Rhat
## alpha[1]   -0.1262004805  0.041776357  0.3590139270 7232.659 0.9993705
## alpha[2]    0.2154350172  0.367062375  0.6686320881 5947.154 0.9994507
## alpha[3]    0.3410695973  0.482475549  0.7511930254 4348.699 0.9988279
## alpha[4]   -0.1349061079  0.051955496  0.3930898396 7127.874 0.9991870
## alpha[5]    0.3379724153  0.528348737  0.8394904103 6190.371 0.9990808
## alpha[6]   -0.6002264566 -0.467369182 -0.2344492174 5070.802 0.9993773
## alpha[7]    0.3403352975  0.466185129  0.6719064108 4506.369 0.9984940
## alpha[8]   -0.4272763304 -0.279765643 -0.0063509849 3293.865 0.9996451
## alpha[9]    0.5865741781  0.742361197  1.0313327273 6473.502 0.9990012
## alpha[10]   0.6315164173  0.767781257  1.0144136624 5473.693 0.9990016
## alpha[11]  -0.0008644995  0.201330399  0.5729716654 5603.286 0.9986994
## alpha[12]   0.5173385198  0.688523571  1.0257131478 5381.982 0.9986605
## alpha[13]   0.0250543767  0.187392616  0.4769247642 6367.447 0.9985042
## alpha[14]   0.5020796576  0.643040784  0.8931570824 4465.487 0.9985956
## alpha[15]   0.2690690379  0.419075385  0.6829629962 5798.571 0.9989241
## alpha[16]   0.6312269759  0.783972588  1.1055579090 5884.529 0.9984794
## alpha[17]   0.7504469182  0.905702776  1.1992143340 5256.750 0.9987391
## alpha[18]   0.4831993842  0.618955660  0.8638695411 7246.712 0.9987836
## alpha[19]  -0.0073190069  0.114982302  0.3356100553 4898.088 0.9982953
## alpha[20]   0.2841942296  0.482550759  0.8618165758 5586.950 0.9985179
## alpha[21]   0.5274547685  0.712494084  1.0546246299 6009.749 0.9986148
## alpha[22]   0.8011741751  0.940951706  1.1987334999 5074.348 0.9984537
## alpha[23]   0.3092756184  0.464561203  0.7507946860 5344.523 0.9988432
## alpha[24]  -0.0076560491  0.176578630  0.5269397111 6778.853 0.9983524
## alpha[25]   0.1904614354  0.327707342  0.5717642826 4384.574 0.9998584
## alpha[26]   0.6553355908  0.801709395  1.0703014606 6116.235 0.9985278
## alpha[27]   0.4768379292  0.634576894  0.9371897786 6368.130 0.9986126
## alpha[28]   0.5902838204  0.736471732  1.0411442555 5814.267 0.9984456
## alpha[29]   0.0189241566  0.246389607  0.6125466578 4754.206 0.9983229
## alpha[30]   0.3424092666  0.519630248  0.8431597933 5941.758 0.9986438
## alpha[31]   0.1437056580  0.326966542  0.6261522004 4748.702 0.9988576
## alpha[32]  -0.1942764712  0.004023386  0.3653562804 5329.878 0.9991501
## alpha[33]   0.5181164054  0.694387244  1.0098183179 5948.749 0.9992660
## alpha[34]   0.2955461284  0.458625033  0.7764300940 4833.465 0.9987280
## alpha[35]  -0.0082346821  0.172598813  0.5077770626 5768.923 0.9991007
## alpha[36]  -0.4049098747 -0.270390579 -0.0043115304 5094.081 0.9991652
## alpha[37]   0.4343438237  0.591577896  0.8985278211 5480.213 0.9985517
## alpha[38]  -0.0344306207  0.156036364  0.4761754601 5379.939 0.9985425
## alpha[39]  -0.0359146199  0.127257810  0.4190262924 5506.112 0.9987995
## alpha[40]   0.2829393319  0.454066096  0.7736709525 4943.294 1.0005414
## alpha[41]  -0.1530883837  0.034173446  0.3527150416 5272.534 0.9992089
## alpha[42]  -0.0994140712  0.063291978  0.3709896857 7865.572 0.9992058
## alpha[43]   0.0939358574  0.240700120  0.5197720659 5282.629 0.9991219
## alpha[44]  -0.0865750888  0.062632746  0.3295689745 5611.116 0.9981005
## alpha[45]   0.7151127978  0.844785921  1.1059371714 5418.674 0.9984869
## alpha[46]  -0.1589449942 -0.025373594  0.2348266341 5361.406 0.9992074
## alpha[47]   0.3575275390  0.513067394  0.8354933256 6342.148 0.9987837
## alpha[48]   0.4496211846  0.646406675  1.0504441557 5830.272 0.9986493
## alpha[49]   0.1243560470  0.292043246  0.5387814760 5681.411 0.9989361
## alpha[50]   0.0764301500  0.221251145  0.5155017082 5705.257 0.9995221
## alpha[51]  -0.2704327770 -0.127812449  0.1479759548 5137.594 0.9988815
```

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## alpha[52]    0.0408470080   0.199895197  0.4794847265 5284.271 0.9991265
## alpha[53]   -0.2116875424  -0.024261716  0.2812579933 5578.718 0.9985364
## alpha[54]    0.7414816466   0.912103332  1.2287251291 6025.204 0.9991093
## alpha[55]    0.3246432232   0.530239415  0.8824307356 5543.778 0.9986476
## alpha[56]   -0.0597941601   0.076563337  0.3227514868 6870.984 0.9985892
## alpha[57]    0.6576986382   0.820574143  1.1454267252 5331.227 0.9987016
## alpha[58]   -0.3511661875  -0.189618806  0.0908028099 5800.091 0.9985924
## alpha[59]   -0.0657856646   0.103959083  0.4270079012 5772.844 0.9988663
## alpha[60]    0.4040498044   0.611974572  0.9667259041 5837.119 0.9985973
## alpha[61]    0.0149231078   0.190722227  0.5184691077 6413.726 0.9988494
## alpha[62]    0.0175643845   0.197903950  0.5454964765 6107.252 0.9985400
## alpha[63]   -0.1033363161   0.036282305  0.2834893107 6404.931 0.9987181
## alpha[64]   -0.0845945914   0.110831029  0.4745380925 5314.742 0.9991842
## alpha[65]    0.2083168378   0.392927671  0.7253220951 5817.872 0.9991526
## alpha[66]    0.1357596363   0.292855753  0.5669632882 5932.496 0.9992804
## alpha[67]    0.1761636560   0.388069704  0.7749388191 6992.169 0.9989016
## alpha[68]    0.0388868343   0.243079735  0.5904407949 5588.668 0.9988436
## alpha[69]   -0.1549855400  -0.025695412  0.2168224413 4498.415 1.0001111
## alpha[70]    0.0678152595   0.266486321  0.6163118963 6404.907 0.9999448
## alpha[71]    0.1685098353   0.342103949  0.6513891759 6450.115 0.9991469
## alpha[72]   -0.1810955053   0.055865059  0.4669481846 5717.921 0.9984293
## alpha[73]    0.2932922525   0.484264619  0.7905587564 5718.594 0.9986341
## alpha[74]   -0.4003917164  -0.239142289  0.0728767091 5315.275 0.9986366
## alpha[75]    0.2162534183   0.401510621  0.7567055546 7278.321 0.9989046
## alpha[76]    0.4022868153   0.562774592  0.8485857331 6555.711 0.9982357
## alpha[77]   -0.0266976165   0.106850673  0.3737829625 6263.472 0.9993377
## alpha[78]    0.5113751359   0.650593129  0.9000654433 5522.982 0.9984378
## alpha[79]    0.2096606565   0.409339767  0.7880713651 5896.451 0.9989335
## alpha[80]    0.3161483682   0.526251736  0.9309489728 6436.600 0.9987364
## alpha[81]    0.0308876034   0.224894153  0.5691441473 6163.486 0.9984950
## alpha[82]    0.4795841436   0.640362209  0.9276314320 6394.810 0.9990264
## alpha[83]    0.2789626847   0.443818677  0.7445442325 5669.589 0.9986098
## alpha[84]    0.4286125423   0.573622759  0.8425745379 4809.281 0.9991584
## alpha[85]    0.2121563813   0.389453080  0.7040481111 4924.161 0.9994878
## alpha[86]    0.3052605659   0.453528055  0.7194233930 7179.473 0.9984098
## alpha[87]    0.3448138933   0.491915069  0.7727626396 4866.301 0.9996379
## alpha[88]    0.4186742107   0.572872409  0.8495837728 4992.943 0.9991238
## alpha[89]    0.4311178394   0.636405121  0.9840207534 7267.060 0.9985537
## alpha[90]    0.3453822033   0.505267355  0.8092828284 4654.025 0.9994040
## alpha[91]   -0.3293228766  -0.168772708  0.1456119901 5005.241 0.9985118
## alpha[92]    0.0865636626   0.274248103  0.5990808068 5718.136 0.9986730
## alpha[93]    0.1414950956   0.251416765  0.4415118489 5119.789 0.9989040
## alpha[94]    0.4945998952   0.643990234  0.9206493103 5866.182 0.9988512
## alpha[95]    0.2927493257   0.438737400  0.7052613461 5768.871 0.9985969
## alpha[96]    0.4094639206   0.547710369  0.8051610444 5579.315 0.9985650
## alpha[97]    0.4176854927   0.538909488  0.7646976872 5934.820 0.9986797
## alpha[98]    0.1900128431   0.336565023  0.6212841844 6572.611 0.9987709
## alpha[99]   -0.2257995947  -0.028715884  0.3338930071 6729.226 0.9989411
## alpha[100]  -0.1107955380  -0.032203528  0.1159649347 4988.130 0.9986605
## alpha[101]   0.3342992398   0.487762637  0.7900596395 5043.867 0.9985020
## alpha[102]   0.1727636009   0.313337570  0.5839033578 4311.956 0.9992998
## alpha[103]   0.1487822480   0.343789014  0.6971438316 5829.466 0.9989325
## alpha[104]   0.1872968381   0.320458959  0.5685429736 5813.875 0.9988477
## alpha[105]   0.0531374743   0.223631697  0.5375884617 6685.665 0.9989206
```

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## alpha[106]   0.0735721323   0.230231916   0.5165661412 6833.968 0.9990708
## alpha[107]   0.3439879825   0.514628675   0.7972543204 6932.891 0.9984914
## alpha[108]  -0.1583729974   0.038257928   0.3923856747 6155.169 0.9987245
## alpha[109]   0.0625443075   0.223873556   0.5066544781 6363.190 0.9984242
## alpha[110]  -0.0583473193   0.129388655   0.4786249160 5744.397 0.9990016
## alpha[111]   0.1685045797   0.328227771   0.6233711835 6084.955 0.9992829
## alpha[112]   0.1030695332   0.266013277   0.5599056310 8306.946 0.9986966
## alpha[113]   0.0269031138   0.191923993   0.4714061063 5210.941 0.9992627
## alpha[114]   0.1248668398   0.277401117   0.5591784753 5187.736 0.9987817
## alpha[115]   0.1456485922   0.317363214   0.6318923806 7192.426 0.9990630
## alpha[116]  -0.0451485521   0.143396755   0.4812777113 5958.640 0.9987167
## alpha[117]   0.1842541602   0.317114449   0.5645071947 6680.013 0.9986243
## alpha[118]  -0.9451255028  -0.808507027  -0.5490649973 4732.991 0.9996725
## alpha[119]   0.2128364879   0.334392944   0.5746346804 6343.651 0.9987054
## alpha[120]   0.0336997251   0.218797123   0.5615547547 5657.359 0.9992153
## alpha[121]   0.0938863032   0.266403448   0.5784036034 5668.051 0.9992944
## alpha[122]  -0.3952857697  -0.194728038   0.1552744499 4788.633 0.9987998
## alpha[123]  -0.0142138923   0.139421494   0.4084798890 4123.610 0.9988334
## alpha[124]  -0.2501868394  -0.087921603   0.1977291689 5147.133 0.9985679
## alpha[125]   0.1668404208   0.319532102   0.6049165278 8426.279 0.9985235
## alpha[126]  -0.1221354983   0.045921404   0.3641129012 5749.985 0.9983878
## alpha[127]  -0.3020567035  -0.142803944   0.1128320770 4781.744 0.9989065
## alpha[128]   0.0432878275   0.243172921   0.6090884872 5915.191 0.9987674
## alpha[129]  -0.1893538631  -0.063450765   0.1707193679 5080.987 0.9991130
## alpha[130]  -0.1157243034   0.018564398   0.2795809483 4048.131 0.9990377
## alpha[131]   0.0495764269   0.251011763   0.6201201080 6362.279 0.9983928
## alpha[132]   0.0592443139   0.241715837   0.5944421019 7311.383 0.9986675
## alpha[133]   0.0685561045   0.237671706   0.5407116984 5474.964 0.9982856
## alpha[134]  -0.0315547621   0.139225771   0.4659249914 6745.817 0.9985404
## alpha[135]  -0.0734977115   0.116372890   0.4587592567 7348.753 0.9987858
## alpha[136]   0.0098462856   0.210238014   0.5707399990 5843.957 0.9983659
## alpha[137]  -0.2678847443  -0.066176431   0.3057260253 4880.619 0.9991639
## alpha[138]   0.3084679634   0.468401531   0.7590717883 4676.598 0.9989983
## alpha[139]   0.0898737692   0.244452364   0.5085319193 5013.448 0.9996443
## alpha[140]   0.2168068967   0.374671468   0.6783623338 6306.996 0.9997800
## alpha[141]   0.1306139658   0.308125863   0.6143780574 6624.777 0.9993367
## alpha[142]   0.2365503018   0.434776541   0.7739750738 6165.245 0.9985081
## alpha[143]  -0.0957837301   0.070553138   0.3736178455 5875.640 0.9985766
## alpha[144]   0.0566904483   0.223009337   0.5319590473 5376.852 0.9988868
## alpha[145]   0.3322761851   0.501013880   0.8047894768 6506.638 0.9990175
## alpha[146]   0.0458497984   0.196533202   0.4501097951 5609.513 0.9988148
## alpha[147]   0.3258007670   0.449888257   0.6791408682 5641.012 0.9989492
## alpha[148]  -0.0886614599   0.079173541   0.4003839170 4511.211 0.9996476
## alpha[149]   0.5878875684   0.742888790   1.0267354831 5595.366 0.9990455
## alpha[150]  -0.4862661104  -0.397644552  -0.2301249597 5731.390 0.9985725
## alpha[151]  -0.1395233005   0.014690354   0.3371033699 7393.355 0.9990256
## alpha[152]  -0.0638333608   0.064117129   0.3207565138 5072.729 0.9992402
## alpha[153]   0.3733942239   0.549727258   0.8609568331 6439.946 0.9988003
## alpha[154]   0.3241818487   0.488253680   0.7743212418 7876.931 0.9993047
## alpha[155]   0.0626069476   0.201485571   0.4741676195 5906.063 0.9991183
## alpha[156]  -0.1150557088   0.031308390   0.2794139089 5039.355 0.9984517
## alpha[157]  -0.1939351284   0.003429718   0.3717359883 5767.741 0.9984055
## alpha[158]  -0.2746426858  -0.118272243   0.1727663737 7159.102 0.9983855
## alpha[159]   0.2294055698   0.381419334   0.6496971542 8494.850 0.9986989
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## alpha[160] -0.0230847720  0.150466887   0.4871885216 6163.359 0.9989898
## alpha[161]  0.0863392304  0.227207974   0.5142411366 5457.245 0.9991150
## alpha[162]  0.0102564142  0.159755214   0.4282301242 6420.419 0.9990774
## alpha[163]  0.1116922296  0.269250541   0.5419755979 5529.416 0.9986344
## alpha[164]  0.0645172630  0.245550029   0.5554669208 5517.300 0.9983532
## alpha[165] -0.1078015435  0.049934510   0.3258910645 5178.851 0.9991728
## alpha[166] -0.1236155885  0.062887595   0.3892179392 6045.083 0.9992569
## alpha[167]  0.2788320658  0.460371627   0.7825888041 6573.259 0.9988485
## alpha[168] -0.0631691180  0.109430524   0.4111215388 5782.989 0.9990444
## alpha[169] -1.0761716368 -0.958089227  -0.7601895357 4694.217 0.9986516
## alpha[170]  0.0588666113  0.222325220   0.5104850372 7083.747 0.9983942
## alpha[171] -0.2366391747 -0.027973677   0.3267186500 4991.410 0.9988178
## alpha[172]  0.1483840257  0.322604253   0.6125308436 5004.098 0.9985982
## alpha[173] -0.0752871651  0.100115185   0.4240630784 6566.810 0.9988604
## alpha[174]  0.0061484997  0.163481723   0.4515618144 4625.231 0.9985680
## alpha[175] -0.0976127187  0.051395257   0.3653486288 5547.033 0.9984868
## alpha[176] -0.0207250466  0.156343652   0.4393859257 7408.269 0.9986945
## alpha[177]  0.4102818191  0.570051785   0.8801951883 4391.722 0.9988041
## alpha[178] -0.4258810827 -0.324003466  -0.1289415616 4789.322 0.9984569
## alpha[179] -0.1546489188  0.026173690   0.3498430765 6773.474 0.9987658
## alpha[180]  0.0794328426  0.233794725   0.5302639556 4752.296 0.9991533
## alpha[181]  0.0362585342  0.196621343   0.5028253341 4865.589 0.9994256
## alpha[182]  0.0111990731  0.173271158   0.4641887778 5769.105 0.9993113
## alpha[183]  0.1359993410  0.295393668   0.5736986214 5348.954 0.9987608
## alpha[184] -0.4502900986 -0.229054088   0.1718024404 4870.412 0.9990247
## alpha[185]  0.0414665882  0.209050845   0.5249571039 6313.486 0.9986216
## alpha[186] -0.2227273376 -0.025836133   0.3143628704 7093.452 0.9985939
## alpha[187] -0.2833738667 -0.149045828   0.1370028940 6230.968 0.9988189
## alpha[188]  0.2402584916  0.412115125   0.7087666940 6811.077 0.9986765
## alpha[189]  0.2060734359  0.369077222   0.6755779207 5913.605 0.9983579
## alpha[190] -0.0785835168  0.124459788   0.4873256970 7366.284 0.9985756
## alpha[191] -0.0189677934  0.185461339   0.5806112017 6619.553 0.9993718
## alpha[192] -0.1011293258  0.081496557   0.3817857715 6562.424 0.9992937
## alpha[193]  0.1404918420  0.315249708   0.6093166184 4938.659 0.9987523
## alpha[194] -0.2943725445 -0.186997369   0.0009824821 6164.370 0.9985097
## alpha[195] -0.6111659303 -0.540970931  -0.4168484359 3834.074 0.9986499
## beta        1.9679193461  2.194616748   2.6372177848 1834.718 1.0002183
## mu          0.0855675477  0.110220227   0.1598473268 3127.842 0.9996519
## sigma       0.3866669899  0.408332635   0.4491145745 1479.433 1.0008316
```

Question 3: Make two plots (appropriately labeled and described) that illustrate the differences in estimated θ_i's across regions and the differences in θs across models.