Exercises

1.7

A home computer is tied to a mainframe computer. The computer will try to log on to the mainframe until a connection is established. Let c denote that a connection was established and n denote that no connection was established. (a) Construct a tree diagram to represent the dialling process.

```
 c
/   c
\  /
  n   c
   \ /
    n
     \
      :
```

(b) Are the paths through the tree equally likely?

No, all paths end with c but are all of different lengths.

(c) List the sample points generated by the tree. Can this list ever be completed?

$\{c, nc, nnc, nnnc, ...\}$, and infinite list that cannot be completed.

(d) List the sample points that constitute event A; connection is established in at most four attempts.

$\{c, nc, nnc, nnnc\}$

(e) Give an example of two events that are not impossible but are mutually exclusive.

A_1 = connection established on the first try= $\{c\}$

A_2 = connection established on the second try= $\{nc\}$

1.9

(see exercise solutions 1 in repo)

1.13

The Apollo missoin made use of a system with the following basic structure:

Main engine $\longrightarrow$ Service propulsion system $\longrightarrow$ Command service module

$\longrightarrow$ Lunar excursion module $\longrightarrow$ LEM engine

For successful operation, all five components must function. Each component is either operable (O) or inoperable (i), for example OOOOi is the state that all components except the LEM engine is operable.

(a) How many states are possible?

$2^5 = 32$

(b) How many states are possible in which the LEM engine is inoperable?

$2^4 = 16$

(c) The mission is deemed partially successful is the first three components are operable. How many states represent a partial success?

$2^2 = 4$

(d) The mission is a total success if and only if all five components are operable. How many states represent total success?

1

1.17

(see exercise solutions 1)

1.21

(see exercise solutions 1)

1.32

(see exercise solutions 1)

2.9

Assume that in a particular military exercise involving two units, Red and Blue, there is a 60% chance that Red will successfully meet its objective and a 70% chance that Blue will do so. There is an 18% chance that only Red will be successful.

a) What is the probability that both succeed?

$P[R] = .6, P[B] = .7, P[R \cap B'] = 0.18 \Longrightarrow P[R \cap B] = P[R] - P(R \cap B') = .42$

b) What is the probability that one or the other, but not both, of the units will be successful?

$P[R \cup B] - P[R \cap B] = P[R] + P[B] - 2P[R \cap B] = .46$

2.15

In a study of waters near power plants it was found that 5% showed signs of chemical and thermal pollution, 40% showed signs of chemical pollution and 35% showed signs of thermal pollution. a) What is the probability that water that shows signs of thermal pollution will also show signs of chemical pollution? $.05/.35 \simeq 0.14$ b) What is the probability that water that shows signs of chemical pollution will also show signs of thermal pollution? $35/40 \simeq .88$

2.17

In studying the cause of power failure, these data have been generated:

5% are due to transformer damage

80% are due to line damage

1% involve both problems.

Approximate the probability that the power failure involves

a) line damage, given that there is transformer damage

1/5

b) transformer damage, given that there is line damage

1/80

c) transformer damage but not line damage

.04

d) transformer damage given that there is no line damage

4/20

e) transformer damage or line damage

.84

2.19

Let A_1 and A_2 be events such that $P[A_1] = .6, P[A_2] = .4$ and $P[A_1 \cup A_2] = .8$. Are A_1 and A_2 independent?

No. $P[A_1 \cap A_2] = .2 \neq P[A_1]P[A_2]$

2.39

In a simulation program, three random two-digit numbers will be generated independently of one another. These number assume values 00, 01, ..., 99 with equal probability.

a) What is the probability that a given number will be less than 50? 1/2

b) What is the probability that all three numbers will be less than 50? 1/8

3.7

When grafting oranges, the density for X, the number of grafts that fail in a series of five trials is given by

x	0	1	2	3	4	5
f(x)	.7	.2	.05	.03	.01	?

a) Find f(5)

$$f(5) = 1 - \sum_{x=0}^{4} f(x) = .01$$

b) Find the table for F

x	0	1	2	3	4	5
F(x)	.7	.9	.95	.98	.99	1.00

c) Use F to find the probability that at most three grafts fail.

$p = .98$

d) Use F to find the probability that at least three grafts fail.

$p = .1$

3.13

Explain why the cumulative distribution function for a discrete random variable can never decrease in value.

It's a sum of numbers that are all greater than or equal to 0.

3.24

The probability that a wildcat well will be productive is 1/13. Assume that drilling wells is an independent activity. Let X denote the number of wells drilled before finding the first productive well.

a) Verify that X is geometric and identify the value of the parameter p.

$p = 1/13, f(1) = 1/13, f(2) = (1-1/13)(1/13), f(3) = (1-1/13)^2(1/13) \to f(x) = (1-p)^{x-1}p$

b) What is the exact expression for the density of X?

$f(x) = (12/13)^{x-1}(1/13)$

c) - (skip this one)

d) What are the numerical values of $E[X], E[X^2], \sigma^2, \sigma$?

$E[X] = 1/p = 13$

$E[X^2] = \frac{1+q}{p} = \frac{25/13}{(1/13)} = 325$, raw moment isn't easy to interpret!

$\sigma^2 = q/p^2 = \frac{12/13}{(1/13)^2} = 156$, variance isn't all that much easier!

$\sigma = \sqrt{\sigma^2} = 12.5$, there we go – on average it takes 13 ± 12.5 attempts, ie from 1 to 25 attempts.

e) Find $P[X \geq 2]$

$P[X \geq 2] = 1 - P[X < 2]$,

$P[X = 1] = (1-p)^0 p = p = 1/13$,

$P[X \geq 2] = 12/13 \simeq 0.92$

3.41

A brand of printers has been found to operate correctly at the time of installation in 80% of all cases. The rest require som adjustment. A dealer sells 10 units during a given month.

a) Find the probability that at least nine printers operate correctly upon installation.

$p = .3758$

b) Consider 5 months in which 10 units are sold per month. What is the probability that at least 9 units operate correctly at the end of the 5 months?

$p = 0.3758^5$