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Optimal Control for the Active Above-Knee Prosthesis

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Control of an active above-knee prosthesis has been simulated for a selected gait activity using a hierarchical closed-loop method. An extension of finite-state control, referred to as artificial reflex control, was adopted at the strategic level of control. At the actuator level of control an optimal tracking method, based on dynamic programming, is applied. This deals mainly with the actuator level of control, but considers the interaction of the leg dynamics and the switching effects of artificial reflex control. Optimal tracking at the actuator level of the above-knee prosthesis reduces the on-off effects of finite-state methods, such as artificial reflex control. The proposed method can also be used for the design of prosthetic elements. Specific attention is paid to the limited torque and power in the prosthetic joint actuator, which are imposed by the principle of self-containment in the artificial leg. The hierarchical structure, integrating artificial reflex control and optimal tracking, can be used in real time, as estimated from the number of computer operations required for the suggested method.

Keywords *— Optimal tracking, Dynamic programming, Above-knee prosthesis.*

INTRODUCTION

The first control algorithms for gait restoration of handicapped humans were openloop. Open-loop control assumes a knowledge of the system and its behavior under all environmental conditions. The simplified models available for the human body and the incomplete data about its behavior have made the application of open-loop controllers unsuccessful in rehabilitation.

In introducing feedback for closed-loop control, important issues arise concerning the type of control, the nature and quality of sensors, and their applicability in real time. Two different closed-loop control methods can be used: one relies on analytic, so-called state-space, modelling of the plant and its behavior, while the other involves a finite-state representation. Furthermore, when dealing with analytic control methods for man-machine systems, two different feedback approaches exist,

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based on either natural sensors (e.g., myoelectric activity) or artificial sensors as a source of control signals.

By application of pattern recognition methods or correlation techniques, functional motions can be created if an adequate interface to natural sensors is used (23,24). Use of multiple electrodes and special filtering techniques is required for regulation of several degrees of freedom (e.g., knee flexion and extension, ankle dorsiflexion). The use of artificial sensors in analytic, closed-loop systems requires specific sensors (e.g., for touch, position, pressure, and slippage) in prosthetic and orthotic devices of all kinds (33). Such man-machine systems really need sensors distributed in a matrix with high resolution. This sensory information must also be adequately pre-processed before it is used for control purposes.

Finite-state modelling of locomotion enables the use of nonnumerical methods of control. Pattern-driven and pattern-matching control of active assistive systems is a highly desirable feature of man-machine interaction in the execution of functional motions. To apply this approach to man-machine control required development of general methods for the synthesis of nonnumerical, pattern-matching controllers. Inspired by the role of reflexes in the execution of natural functional motions, a new control method was proposed, called Artificial Reflex Control (ARC). An outline of this nonnumerical control method is given below.

ARC of assistive systems avoids model representation in state-space and control does not, therefore, depend on state-space variables directly. It also avoids solving complex equations describing the human-machine model. Thus, *a priori* knowledge about plant dynamics is not available in the form of space equations, but the output space data are used for ARC. Initially, the method was suggested as a finite-state control (25,26), but evolved to a logical control (4,27,29) and further to a rule-based control (1–3,12,14,15,28,30).

The formalism of production rules was used in several fields well before the advent of artificial intelligence (6,20). A production rule is a situation–action pair, meaning that whenever a certain situation is encountered, given as the left side of the rule, the action on the right side of the rule is performed. The form of the situation or of the action are not constrained *a priori*. Production rules were found to be ideal for the design of controllers, based on the method of expert systems, to assist human gait (17,18,27). The control problem is divided into a number of distinct states and the rules which determine the choice of state at each moment in time must be specified. Within each state, the specification of local control actions should be determined for either open-loop or closed-loop performance. The resulting controller is sometimes known as a rule-based controller or as a finite-state automation.

Rule-based control can be implemented directly in hardware, but it is more usually done in software. There are some difficulties in converting rules and actions into an appropriate program in assembly language or a higher level language such as *C*. A major problem is that thinking about what the controller should do often becomes confused with thinking about how the controller is to do it. A second problem arises with errors at run time (such as unending loops) that are liable to cause incorrect or potentially dangerous situations.

The design process can be facilitated most commonly by the use of state diagrams. State diagrams generally indicate only the transitions between the states, not the actions within the states. If the actions were included, the diagrams would become cluttered and difficult to understand. No satisfactory method has been found for representing on a diagram the "global" information which in programs is declared in symbolic constants, procedures, functions, etc. (15). ARC is rule-based and uses production rules.

The actual ARC system for an above-knee prosthesis (AKP) consists of three blocks: a general rule base, an operating rule base, and a data acquisition block. The general rule base has three subsets of rules: regular, mode, and hazard rules. Regular rules deal with the strategy of motor execution in regular, cyclic motor activities. Mode rules deal with environmental recognition and adaptation within a specified modality of the gait. Hazard rules deal with conflict situations when the hardware is unable to produce a desired outcome or unexpected situations arise within an actual gait pattern. Execution or "firing" of a regular or hazard rule changes the actuator state (locking, damping, or extension-flexion of the knee joint). The operating rule base is a selected set of rules from all three types of rules, corresponding to the gait mode, expected failures, expected environmental changes, and general hazards. Firing a mode rule initiates a transition of an associated set of rules from the general rule base to the operating rule base. Details about the organization of ARC control are published elsewhere (17,18,19).

ARC is the control method operating in the output space and it considers finite states recognized in the form of coded sensory information. The control algorithms activated are of an on-off nature. To introduce smooth movements and to reduce jerking, ARC must be combined with adequate control algorithms at the actuator level. Optimal tracking at the actuator level was chosen because it effectively takes care of the dynamics of the system as required above. It must also be consistent with the principle of self-containment on the body, which is the essential condition for effective rehabilitation. This principle necessitates size limits to the energy source and actuator unit and hence to the maximal joint torque and maximal power.

DYNAMICAL MODEL OF THE AKP: FORMULATION OF THE OPTIMAL TRACKING PROBLEM

The AKP dynamical model is developed using the Lagrange method. Kinetic energy ε of the AKP may be expressed in the form

$$\begin{aligned} 2\varepsilon = {} & 2\varepsilon_{BP} + J_{CT}\dot{\phi}_T^2 + J_{CS}\dot{\phi}_S^2 \\ & + m_T[(\dot{x}_H + d_T\dot{\phi}_T\cos\phi_T)^2 + (\dot{y}_H + d_T\dot{\phi}_T\sin\phi_T)^2] \\ & + m_S[(\dot{x}_H + L_T\dot{\phi}_T\cos\phi_T + d_S\dot{\phi}_S\cos\phi_S)^2 \\ & \quad + (\dot{y}_H + L_T\dot{\phi}_T\sin\phi_T + d_S\dot{\phi}_S\sin\phi_s)^2] \ , \end{aligned} \tag{1}$$

where dots denote time derivatives, BP is the biological part of the system, H is for the hip, T is for the thigh, K is for the knee, S is for the shank and the foot, CS and CT are the center of mass of thigh and shank, m is the mass, L the length, d the distance from the proximal joint to the mass center, and J is the inertia moment of the segment regarding the corresponding axes perpendicular to the plane of movement (Fig. 1). The ankle joint is considered fixed.

Let M be an external torque and M^r a resistive torque. The resistive torque is the

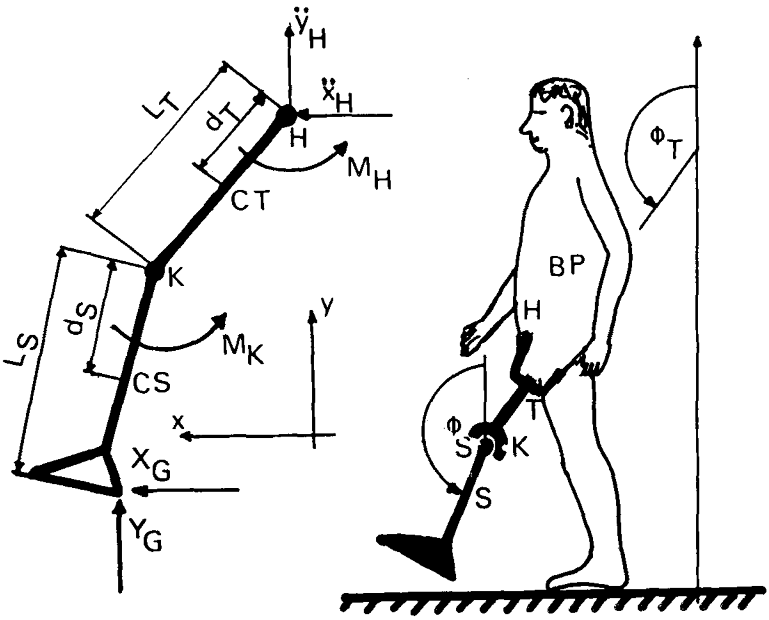


FIGURE 1. The model of the above-knee prosthesis. *H* is for the hip, *K* for the knee, *T* for thigh, *S* for shank, and *F* for foot. *L* is the length of the segment, *d* is the proximal distance from the joint, X_G and Y_G are horizontal and vertical ground reaction forces, *x* and *y* are Cartesian coordinates, double dot is for the second time derivative (acceleration). *C* is centre of mass, ϕ is the angle from the vertical axis, *M* is the torque around a joint. *BP* is the biological part of the system.

result of the air resistance, damping, and elastic properties of the joint and the actuator. The resistive torque includes the ground reaction in the stance phase. We assume that there is no sliding of the foot during the stance phase of the gait. We have the following expressions:

$$\begin{cases} M_K^r = -\kappa_1(\phi_S - \phi_T) - \kappa_2(\dot{\phi}_S - \dot{\phi}_T) - \alpha L_S(X_G \sin\phi_S - Y_G \cos\phi_S) \\ M_H^r = \kappa_3\dot{\phi}_T + \kappa_4\left(\phi_T - \dfrac{\pi}{2}\right) + \alpha L_T(X_G \sin\phi_T - Y_G \cos\phi_T) \ , \end{cases} \tag{2}$$

where K is for the knee, H is for the hip, X_G and Y_G are for the components of the ground reaction, κ_i $(i = 1, \ldots, 4)$ are parameters of the actuator unit, and

$$\alpha = \begin{cases} 1 \text{ for } t_0 \le t \le T_S \text{ (Stance phase)} \\ 0 \text{ for } T_S < t \le t_0 + T \text{ (Swing phase)} \ , \end{cases} \tag{3}$$

where t is time in seconds, T is the duration of the gait cycle and t_0 is the initiation instant.

The Langrange principle yields the equations of motion in the form

$$
\begin{aligned}
&A_1\ddot{\phi}_S + A_2\ddot{\phi}_T\cos(\phi_T - \phi_S) + A_3\dot{\phi}_T^2\sin(\phi_S - \phi_T) \\
&\qquad - A_4\ddot{x}_H\sin\phi_S - A_5(\ddot{y}_H + g)\cos\phi_S = M_K - M_K^r \\
&B_1\ddot{\phi}_T + B_2\ddot{\phi}_S\cos(\phi_T - \phi_S) + B_3\dot{\phi}_S^2\sin(\phi_T - \phi_S) \\
&\qquad - B_4\ddot{x}_H\sin\phi_T - B_5(\ddot{y}_H + g)\cos\phi_T = -M_K + M_K^r + M_H - M_H^r\ ,
\end{aligned}
\tag{4}
$$

where g is the gravitational constant, and

$$
\begin{aligned}
&A_1 = J_{CS} + m_S d_S^2, &&B_1 = J_{CT} + m_S L_T^2 + m_T d_T^2 \\
&A_2 = m_S d_S L_T, &&B_2 = A_2 \\
&A_3 = A_2, &&B_3 = -B_2 \\
&A_4 = m_S d_S, &&B_4 = m_S L_T + m_T d_T \\
&A_5 = -A_4, &&B_5 = -B_4\ .
\end{aligned}
\tag{5}
$$

We now introduce the following notation:

$$
\begin{aligned}
&x_1 = \phi_S, \quad x_2 = \dot{x}_1, \quad x_3 = \phi_T, \quad x_4 = \dot{x}_3, \quad \boldsymbol{x} = (x_1, x_2, x_3, x_4) \\
&u_1 = M_K, \quad u_2 = M_H, \quad \boldsymbol{u} = (u_1, u_2) \\
&\varphi_1 = \ddot{x}_H, \quad \varphi_2 = \ddot{y}_H, \quad \boldsymbol{\varphi} = (\varphi_1, \varphi_2) \\
&\psi_1 = -M_K^r, \quad \psi_2 = M_H^r, \quad \boldsymbol{\psi} = (\psi_1, \psi_2)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&F_1 = F_1(\boldsymbol{x}) \equiv A_3 x_4^2\sin(x_3 - x_1) + A_4\varphi_1\sin x_1 + A_5(\varphi_2 + g)\cos x_1 + \psi_1 \\
&F_2 = F_2(\boldsymbol{x}) \equiv B_3 x_s^2\sin(x_3 - x_1) + B_4\varphi_1\sin x_1 + B_5(\varphi_2 + g)\cos x_3 + \psi_2\ .
\end{aligned}
\tag{7}
$$

Note that, by virtue of (2), we have

$$
\begin{aligned}
&\psi_1 = \psi_1(\boldsymbol{x}) \equiv -\kappa_1(x_2 - x_4) - \kappa_2(x_1 - x_3) - \alpha L_S(X_G\sin x_1 - Y_G\cos x_1) \\
&\psi_2 = \psi_2(\boldsymbol{x}) \equiv \kappa_3 x_4 + \kappa_5\left(x_3 - \frac{\pi}{2}\right) + \alpha L_T(X_G\sin x_3 - Y_G\cos x_3)
\end{aligned}
\tag{8}
$$

and the equations of motion, Eqs. 3 and 4, can be written in the form:

$$
\begin{aligned}
&A_1\dot{x}_2 + A_2\cos(x_3 - x_1)\dot{x}_4 = F_1 + u_1 \\
&B_1\dot{x}_4 + B_2\cos(x_3 - x_1)\dot{x}_2 = F_2 + u_2 - u_1\ .
\end{aligned}
\tag{9}
$$

Considering the relationships $\dot{x}_1 = x_2$ and $\dot{x}_3 = x_4$, and solving the system (Eq. 9) with respect to $\dot{x}_2$ and $\dot{x}_4$, we find

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= P_1 + G_1 u_1 + G_2 u_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= P_2 + G_3 u_1 + G_4 u_2 \ , \end{aligned} \tag{10}$$

where

$$\begin{aligned} P_1 &= P_1(\boldsymbol{x}) \equiv [B_1 F_1 - A_2 F_2 \cos(x_3 - x_1)]/C \\ P_2 &= P_2(\boldsymbol{x}) \equiv [A_1 F_2 - B_2 F_1 \cos(x_3 - x_1)]/C \\ G_1 &= G_1(\boldsymbol{x}) \equiv [B_1 + A_2 \cos(x_3 - x_1)]/C \\ G_2 &= G_2(\boldsymbol{x}) \equiv A_2 \cos(x_3 - x_1)/C \\ G_3 &= G_3(\boldsymbol{x}) \equiv -[A_1 + B_2 \cos(x_3 - x_1)]/C \\ G_4 &= G_4(\boldsymbol{x}) \equiv A_1/C \end{aligned} \tag{11}$$

with

$$C = C(\boldsymbol{x}) \equiv A_1 B_1 - A_2 B_2 \cos^2(x_3 - x_1) \ . \tag{12}$$

In the following, the components of the ground reaction, $(X_G(t), Y_G(t))$, and of the hip acceleration $\boldsymbol{\varphi} = \boldsymbol{\varphi}(t)$, $(\varphi_1(t), \varphi_2(t))$, will be assumed as known for $t_0 \le t \le t_0 + T$. Then, for a given pair of torque actions, $\boldsymbol{u}(t) = (u_1(t), u_2(t))$, in the gait cycle, the corresponding solution of the nonlinear and nonstationary dynamical system (Eq. 10) satisfying the associated boundary conditions at $t = t_0$ and $t = t_0 + T$, describes the whole motion of the AKP in the gait cycle. As is customary, $\boldsymbol{x} = \boldsymbol{x}(t)$ and $\boldsymbol{u} = \boldsymbol{u}(t)$ will be called the **state** and **control variables**, respectively.

Considering the limitations in the actions of the torques used in AKP, the set

$$\boldsymbol{\Omega} = \{(u_1, u_2) | M_1 \le u_k \le M_2, \ k = 1,2\} \tag{13}$$

will be designated as the **control region**, where M_1 and M_2 are two constants. A control $\boldsymbol{u} = \boldsymbol{u}(t)$ will be called **admissible** if $\boldsymbol{u}(t) \in \boldsymbol{\Omega}$ and is piecewise continuous for $t_0 \le t \le t_0 + T$. The **set of all admissible controls** will be denoted by U:

$$U = \{\boldsymbol{u}(t) | \boldsymbol{u} = \boldsymbol{u}(t) \text{ is admissible}\} \ . \tag{14}$$

Clearly, the motion of the AKP is completely determined by the angles $\phi_S \equiv x_1$ and $\phi_T \equiv x_3$. Accordingly, we will call the pair $\boldsymbol{X} = \boldsymbol{X}(t) \equiv (x_1(t), x_3(t))$, $t_0 \leq t \leq t_0 + T$, the **trajectory of the AKP** for the gait cycle. Let $\boldsymbol{Z} = \boldsymbol{Z}(t) \equiv (z_1(t), z_3(t))$, $t_0 \leq t \leq t_0 + T$, be the **trajectory of the desired movement** of the AKP during the gait cycle. Usually, $\boldsymbol{Z}(t)$ is determined experimentally (16). We wish to choose an admissible control $\boldsymbol{u} = \boldsymbol{u}(t)$ in such a way that the actual trajectory $\boldsymbol{X}(t)$ with this control will be as close as possible to the desired trajectory $\boldsymbol{Z}(t)$ for $t_0 \leq t \leq t_0 + T$. To measure the closeness of the trajectory $\boldsymbol{X}(t)$, under the admissible control $\boldsymbol{u}(t)$, to the trajectory $\boldsymbol{Z}(t)$ we will use the following **performance index**:

$$R(\boldsymbol{u}) = \int_{t_0}^{t_0+T} \{[x_1(t) - z_1(t)]^2 + [x_3(t) - z_3(t)]^2\} dt \qquad (\boldsymbol{u} \in U) \ . \tag{15}$$

Then, we can formulate the **optimal tracking problem** as follows:

Find an admissible control $\boldsymbol{u}^* = \boldsymbol{u}^*(t)$ in such a way that the functional $R(\boldsymbol{u})$ will assume its minimal value:

$$\min_{\boldsymbol{u} \in U} R(\boldsymbol{u}) = R(\boldsymbol{u}^*) \qquad (\boldsymbol{u}^* \in U) \ . \tag{16}$$

The solution of this optimization problem will be discussed in Appendix A.

IMPLEMENTATION

Current AKPs are limited in use by the fact that knee function is preprogrammed (13). High metabolic energy consumption with an AKP is a consequence of nonpowered knee and ankle joints and insufficient symmetry of the gait (21). Collision shock due to hyperextension during heel strike leads to high forces at the socket-stump interface and at the hip joint. A short stump, hip disarticulation or muscle weakness at the thigh joint all make the use of AKP even more difficult and less effective. Valuable criteria for using an AKP were stated in general form (32). Attempts to improve the AKP have been directed to knee actuator design, computer-aided socket design and control. Control strategies for AKPs have been tested mainly on laboratory prototypes, since no AKPs with externally powered knee or ankle joints are in practical use.

An MIT group proposed use of interactive computer simulations. For this purpose they developed a high performance electrohydraulic knee with umbilical connections to a hydraulic power supply (8–10,22). With the active torque capability of the MIT device knee bounce, which is typical of normal leg kinetics during stance phase, could be accomplished. Gait symmetry was also obtained by constraining the artificial leg to follow the same dynamic pattern as the normal leg (11).

They assumed that myoelectrical control could be applied, but the complex musculature around the hip joint, and unpredictable relocations of muscle mass after amputation made myoelectric control unsatisfactory. Difficulties associated with myoelectric approach led them to study what they called "mirror" control. A goniometer fixed to the normal leg provided the reference, which was then phase-shifted to control the prosthetic knee. This strategy proved to be successful, but extremely complex. Additional information on the dynamics of the sound and the prosthetic leg, as well as the state of foot switches on the normal and the amputated side, permitted automatic detection of gait transitions from level walking to stair or ramp climbing. This approach led to development of a finite-state controller. The powered MIT knee was limited to a laboratory environment, but the control strategy has been applied to an externally controlled but nonpowered knee mechanism (10).

To design a practical system we were interested in objective criteria of performance for an externally controlled and powered above-knee prosthesis. Objective criteria for the efficacy of an active AKP include energy consumption, gait symmetry, and the interface forces between stump and socket. We have explored these using computer simulations of an active AKP. The input data were angles ϕ_S and ϕ_T, ground reactions X_G and Y_G and hip acceleration $\ddot{x}_H$ and $\ddot{y}_H$ in addition to body parameters (Table 1 and Fig. 2). Input data were collected using the Watsmart system (Northern Digital Ltd., Waterloo, Canada) with two cameras. The ground reaction forces were recorded using specially designed shoes (Institute Jožef Štefan, Ljubljana, Yugoslavia) equipped with eight force sensors based on strain gauges. This specific set of data was recorded on a normal individual using a cast (ankle foot orthosis) on his ankle. The ankle casting was included because the model of an AKP does not have an ankle joint. Data were collected on the casted leg, as well as on the contralateral side. The same individual also performed the same gait with the same speed without the cast (16).

This control method optimizes the trajectory tracking, while taking into account the limitations in power and torques. Limitations in power and torque, as explained earlier in this paper, are the consequences of the principle that the prosthesis must be self-contained. It is obvious that a mechanical system with unlimited torques and power will have ideal tracking, as can be proved by a simple simulation of the dynamical equations for the model. The mathematical model has a unique stable solution, so a desired motion is obtained with a unique external power and vice versa. However, if the torques are limited, a given trajectory cannot be tracked ideally but the proposed method provides control to optimized tracking. Figures 3 and 4 present a set of trajectories and corresponding torques for given output. Note that the hip torque is a measure of the bending torque at the stump-socket interface and that this computer simulation has lower torques and power consumption that some other published results (11). We considered a modified gait pattern suitable for the above-knee amputee, since a "normal" gait pattern may require higher energy and power than can be accommodated in the actuators of a self-contained artificial leg.

TABLE 1. Body parameters, defined as follows (see also Fig. 1 and the text): m = mass, S = shank, T = thigh, L = length, d = the distance from the proximal joint to the center of mass (C), J = the moment of inertia of the segment, and k_i ($i = 1, \ldots, 4$) are parameters of the actuator unit. All values are in standard MKS units.

m_S [kg]	m_T [kg]	L_S [m]	L_T [m]	d_S [m]	d_T [m]
5.05	7.10	0.54	0.44	0.25	0.25
J_{CS} [kgm^2]	J_{CT} [kgm^2]	k_1 [Nm]	k_2 [Nms]	k_3 [Nms]	k_4 [Nm]
0.062	0.084	0.01	0.01	0.01	0.01

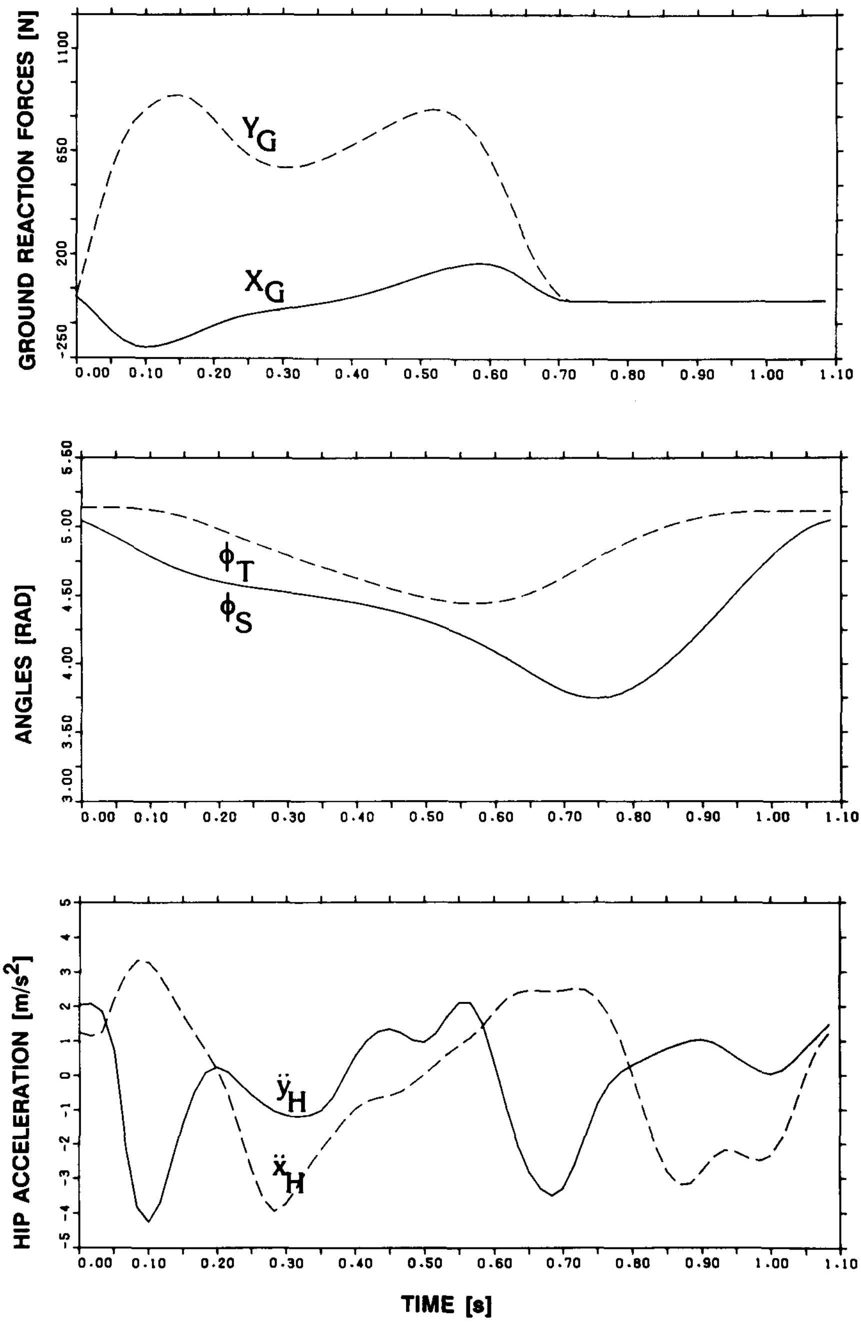


FIGURE 2. Input data for the simulation. ϕ_S and ϕ_T are desired angular values of the thigh and shank measured relative to the vertical line, X_G and Y_G are the ground reactions, and $\ddot{x}_H$ and $\ddot{y}_H$ are the accelerations of the hip. The data present one full gait cycle (one complete step) and were obtained from averaging three minutes of locomotion. Vertical axes are in radians for the angles (ϕ_S and ϕ_T), in Newtons for forces (X_G and Y_G), and in m/s^2 for hip accelerations ($\ddot{x}_H$ and $\ddot{y}_H$). The gait cycle is $T = 1.08$ seconds and the step length 1.39 m.

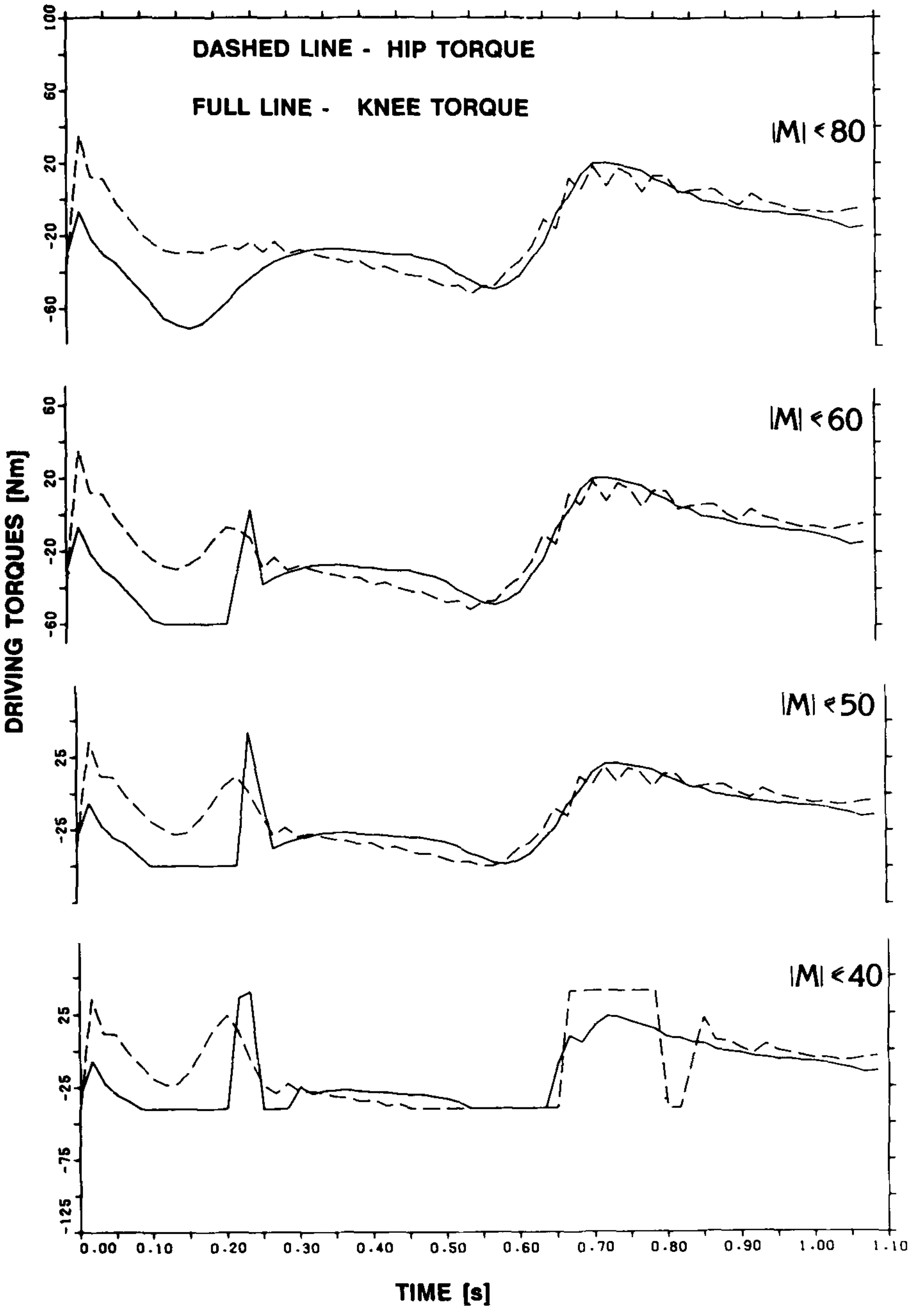


FIGURE 3. M_i are driving torques obtained using the optimal tracking control method. The dashed curve is the torque in the hip joint, and the full line presents the knee torque. Torque limitation was included in simulation and 80 Nm, 60 Nm, 50 Nm, and 40 Nm limits are presented from the top to the bottom of this figure. It is clear that torque limitation involves some torque saturation. The 80 Nm limitation does not saturate, meaning that the torques required for the motor task are less than 80 Nm.

Clearly, the results of the simulation allow several alternatives. One can:

1. choose the actuator for the powered leg;
2. calculate the jerking effects on the stump and hip;
3. optimize energy consumption; or
4. calculate allowable mismatching of desired and actual motions preventing some conflict situations for the gait (foot clearance, etc.).

Figure 5 presents forces acting at the hip joint. These forces are acting on the stump from the socket, assuming that the socket-stump interface has been designed in a way that allows the modelling of these two bodies as a single rigid body (32). If this assumption is not justified, the dynamics of the socket-stump interface will have to be modelled as a visco-elastic connection. The softness of the interface will decrease the hip forces, but will also change the dynamic properties of the overall system. In this paper we restrict our attention to the control of the powered knee joint when the stump and socket can be treated as a single rigid body. Our results show that the optimal tracking control reduces the jerk of the leg and allows reliable feedback from the ground contact. The reliability is imposed by the excellent tracking of the hip forces to the ground force in the stance phase.

Clearly, this model is restricted to the dynamics of the prostheses and does not include the dynamics of the actuator. Here, the actuator is assumed to be ideal, having no inertia, friction, backlash, and rise and fall time. The actuator dynamics may be included in the model, but would just complicate the explanation of the methodology presented here.

How the optimal tracking can be incorporated in ARC is a separate issue. The ARC, as described earlier, provides discrete changes elicited by separate sensory patterns. A logical continuation of sensory patterns is expected in regular locomotor activities. Having this in mind, the gait cycle may be presented as a sequence of separate transitions with known finite states, but the transition itself is not determined by the ARC. It is highly desirable that a smooth movement is achieved in each transition. This paper shows that optimal tracking is a procedure which can be applied in an efficient way for the complete gait cycle. Of course, it can be applied to any specific sequence within the gait cycle, for example to provide smooth transitions and eliminate side effects of the switching introduced by the rule-based control. Such an exchange of the control level requires the flow of information only from the higher level to the actuator level and allows complete autonomy of each of the local controllers.

The main problem for application of the optimal tracking method in humans is connected with the real time operation. The autonomy allows parallel processing, meaning that we have to consider only one joint controller. In our recent application, the time required for the processing is a product of the number of time samples and the sum of number of multiplications, additions, input/output functions and calculations of trigonometric functions. The use of symbolic presentation may be extremely effective here (31). The number of operations required for one time interval is 98 multiplications, 51 additions, 6 trigonometric functions and 16 input/output operations. The time required for these operations, when using a 68HC11 based microcontroller, is less than 3 ms. The sampling period for the ARC control is selected to be 20 ms. An average artificial reflex has a duration of 120 ms. Thus, the time constraints im-

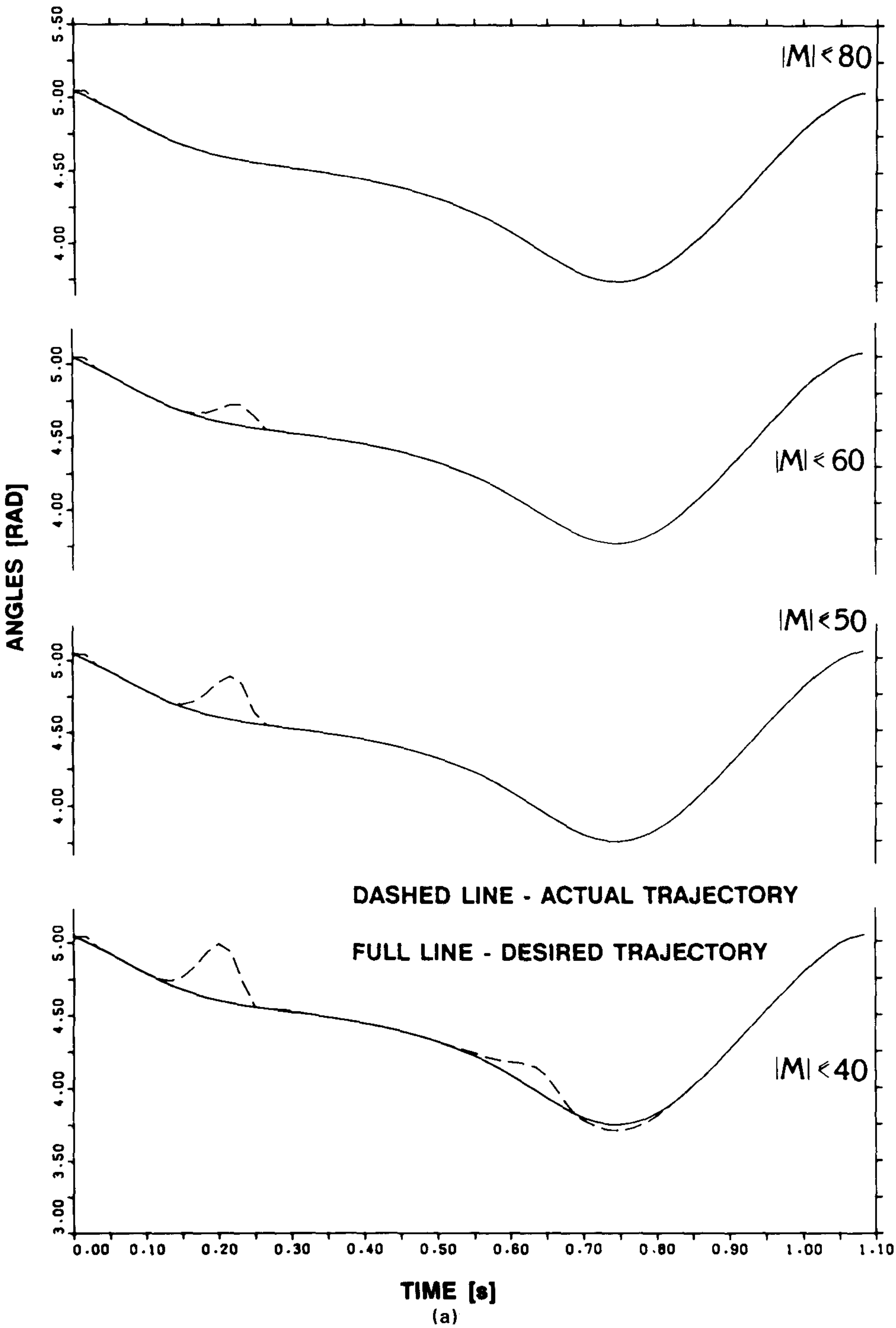
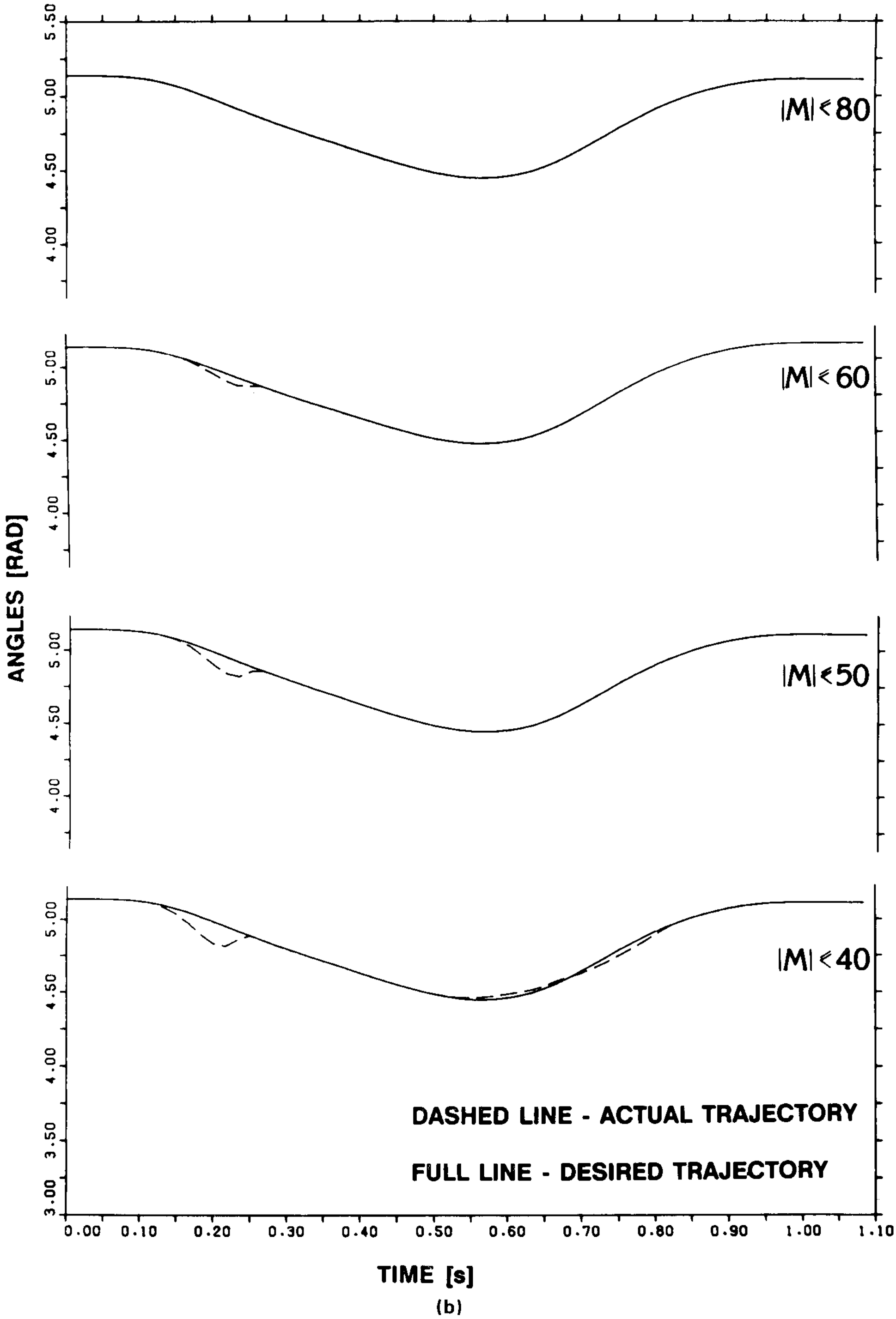


FIGURE 4. (a) Angular changes at the shank (ϕ_S) for the same torque limitations given in Figure 3 are shown. As expected, the 80 Nm limit provides ideal tracking, and decreased limits for torque result in bigger differences between desired and actual angular changes. (*Figure continued on facing page.*)



(b)

FIGURE 4 continued. (b) Angular changes at the thigh (ϕ_T) for the same torque limitations given in Figure 3 are shown. As expected, the 80 Nm limit provides ideal tracking, and decreased limits for torque result in bigger differences between desired and actual angular changes.

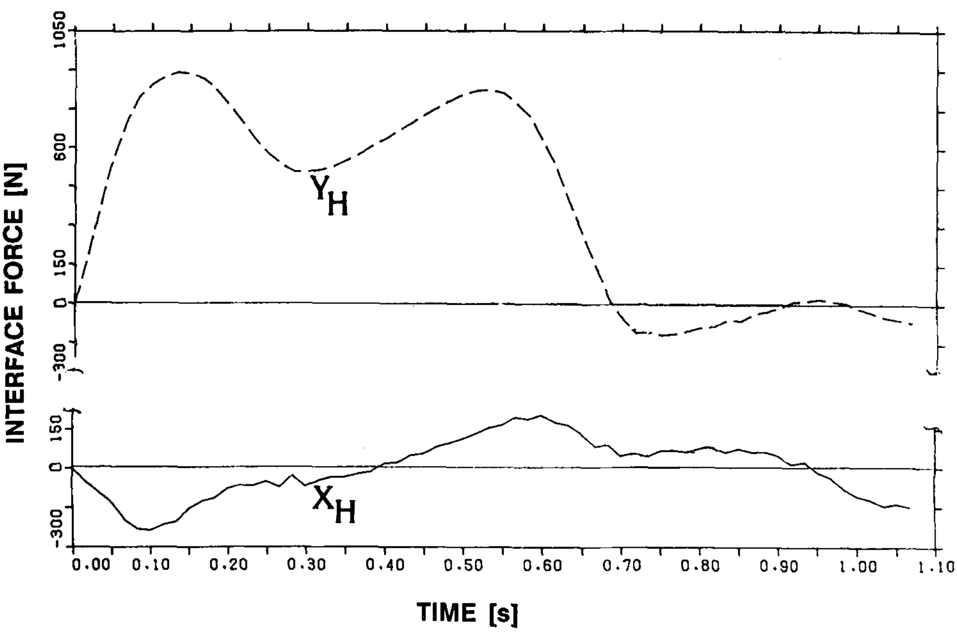


FIGURE 5. Y_H and X_H are calculated forces at the hip joint, i.e. at the interface between the stump and the socket. The results obtained show that the ground reactions are transferred to the socket. The results impose a pattern such that the optimal control provides minimal jerks and smooth changes of the force.

ply that optimal tracking control can be applied and that it is a suitable algorithm at the lower level of control.

The use of optimal tracking demonstrated another interesting feature. The trajectory tracking differs very much from the optimization of energy consumption. Optimal energy consumption maximizes the length of free states (no external power is applied) where the movement is a result of the gravity force and inertial forces. The optimization of energy, however, is very inconvenient for the systems where a torque limitation is included.

The energy consumption E of the artificial leg can be calculated using the formula

TABLE 2. Energy consumption for the different torque limitations. Energy consumption is in Joules, torque limitation is in Nm and the limitation applies to either positive or negative torques.

Torque limitation	35	40	45	50	55
Energy consumption – hip	113.1	89.7	79.2	70.5	80.1
Energy consumption – knee	957.1	205.3	128.5	93.8	83.5
Torque limitation	60	65	70	75	80
Energy consumption – hip	82.1	84.6	86.0	86.1	86.1
Energy consumption – knee	82.3	82.6	85.2	85.2	85.2

$$E = \sum_{i,j} |M_i| \, |\beta_i(t_{j+1}) - \beta_i(t_j)| \qquad (i = H,K; j = 1,2,3,\ldots,n) \tag{17}$$

where M_i are hip and knee torques, β_i are the relative joint angles at the hip and knee, t_{j+1} and t_j are consecutive time instants. The results obtained imply that the use of adequate motor torques is crucial for the efficiency of the whole system. A weak torque motor will consume much greater power and thereby increase energy consumption in the hip joint. Since the hip joint is controlled by the amputee, the metabolic energy consumption will be increased. On the other hand, motors with higher capacity than 60 Nm are not needed because they will only increase thermal losses (Table 2).

CONCLUSION

This paper describes a hierarchical control method for a powered, above-knee prosthesis. The higher, strategic level of the controller is based on the artificial reflex method and is presented in other publications in detail. Artificial reflex control has some advantages, but also some disadvantages, the major one being on-off dynamic behavior. However, the structure of artificial reflex control allows easy integration of optimal tracking methods for the control at the lower, actuator level of the above-knee prosthesis. Optimal tracking methods, based on dynamic programming, may be used in two ways: simulation prior to design of the artificial leg, and control of the actuators of the leg in real time. Simulation of the leg behavior provides information concerning the effects of the prosthesis on the gait pattern when different limitations in torque and power are included. The use of dynamic programming at a local controller level can improve the dynamic performance of the externally powered leg. In combination with the artificial reflex control, optimal tracking may provide smoother motion and reduce collision effects and jerks. This specific hierarchical control may provide a reliable control method for practical, externally powered artificial legs.

APPENDIX A. SOLUTION OF THE OPTIMAL TRACKING PROBLEM

To solve the optimal problem formulated above, we shall use the techniques of the discrete dynamic programming (5,7).

Let N be a sufficiently large integer. Put

$$h = T/N, \quad t_n = t_0 + nh \qquad (n = 0,1,\ldots,N) \; . \tag{A1}$$

We will denote the value of a function $\theta = \theta(t)$ at $t = t_n$ by $\theta_n : \theta_n = \theta(t_n)$. Let $\boldsymbol{u}_n = (u_{1,n}, u_{2,n})$ be the control applied at $t = t_n$. Then

$$\boldsymbol{u}(t) = \boldsymbol{u}_n(u_1(t) = u_{1,n}, u_2(t) = u_{2,n}), \quad t_n \le t < t_{n+1}, \quad u_n \in \boldsymbol{\Omega} \qquad (n = 0,1,\ldots,N-1) \tag{A2}$$

is an admissible control: $\boldsymbol{u}(t) \in U$. For large N,

$$\dot{\theta}_n \approx (\theta_{n+1} - \theta_n)/h \tag{A3}$$

and, according to Eqs. (10), we have

$$\begin{aligned} x_{1,n+1} &\approx x_{1,n} + hx_{2,n} \\ x_{2,n+1} &\approx x_{2,n} + h[P_{1,n} + G_{1,n}u_{1,n} + G_{2,n}u_{2,n}] \\ x_{3,n+1} &\approx x_{3,n} + hx_{4,n} \\ x_{4,n+1} &\approx x_{4,n} + h[P_{2,n} + G_{3,n}u_{1,n} + G_{4,n}u_{4,n}] \ . \end{aligned} \tag{A4}$$

We can easily show that

$$\begin{aligned} x_{1,n+2} &\approx y_{1,n} + Q_{1,n}u_{1,n} + Q_{2,n}u_{2,n} \\ x_{3,n+2} &\approx y_{3,n} + Q_{3,n}u_{1,n} + Q_{4,n}u_{2,n} \ , \end{aligned} \tag{A5}$$

where

$$\begin{aligned} y_{1,n} &= x_{1,n} + 2hx_{2,n} + h^2P_{1,n} \\ y_{3,n} &= x_{3,n} + 2hx_{4,n} + h^2P_{2,n} \end{aligned} \tag{A6}$$

and

$$Q_{j,n} = h^2G_{j,n} \qquad (j = 1,2,3,4) \ . \tag{A7}$$

We assume that the actual and the desired trajectories of AKP coincide at $t = t_0$ and $t = t_0 + T$:

$$x_{1,0} = z_{1,0}, \quad x_{1,N} = z_{1,N}, \quad x_{3,0} = z_{3,0}, \quad x_{3,N} = z_{3,N} \ . \tag{A8}$$

We can easily show that

$$\begin{aligned} R(\boldsymbol{u}) &= \int_{t_0}^{t_0+T} \{[x_1(t) - z_1(t)]^2 + [x_3(t) - z_3(t)]^2\}dt \\ &= \sum_{n=0}^{N-1} \int_{t_n}^{t_{n+1}} \{[x_1(t) - z_1(t)]^2 + [x_3(t) - z_3(t)]^2\}dt \\ &= \frac{1}{2}\sum_{n=0}^{N-2} \int_{t_n}^{t_{n+2}} \{[x_1(t) - z_1(t)]^2 + [x_3(t) - z_3(t)]^2\}dt \\ &\approx \frac{h}{2}\sum_{n=0}^{N-2} \boldsymbol{r}_n(\boldsymbol{X}_n, \boldsymbol{u}_n) \ , \end{aligned} \tag{A9}$$

where

$$\begin{aligned} \boldsymbol{r}_n &= \boldsymbol{r}_n(\boldsymbol{X}_n, \boldsymbol{u}_n) \\ &\equiv [(x_{1,n} - z_{1,n})^2 + (x_{3,n} - z_{3,n})^2] \\ &\quad + [(x_{1,n+2} - z_{1,n+2})^2 + (x_{3,n+2} - z_{3,n+2})^2] \ . \end{aligned} \tag{A10}$$

Combining (A5) and (A10), we find

$$\begin{aligned} \boldsymbol{r}_n(\boldsymbol{X}_n,\boldsymbol{u}_n) &= (x_{1,n} - z_{1,n})^2 + (x_{3,n} - z_{3,n})^2 \\ &\quad + (y_{1,n} - z_{1,n+2} + Q_{1,n}u_{1,n} + Q_{2,n}u_{2,n})^2 \\ &= (y_{3,n} - z_{3,n+2} + Q_{3,n}u_{1,n} + Q_{4,n}u_{2,n})^2 \ . \end{aligned} \tag{A11}$$

Note that the first two terms in (A11) do not depend on $\boldsymbol{u}_n$ explicitly.

We now define

$$\begin{gathered} \boldsymbol{R}_n = \boldsymbol{R}_n(\boldsymbol{X}_n) \equiv \min_{\{\boldsymbol{u}_n,\ldots,\boldsymbol{u}_{N-2}\}} \sum_{k=n}^{N-2} \boldsymbol{r}_k(\boldsymbol{X}_k,\boldsymbol{u}_k) \\ (\boldsymbol{u}_i \in \boldsymbol{\Omega},\ i = 0,1,\ldots,N-2) \ . \end{gathered} \tag{A12}$$

Then, using the principle of optimality, we can write

$$\begin{gathered} \boldsymbol{R}_n(\boldsymbol{X}_n) = \min_{\boldsymbol{u}_n \in \boldsymbol{\Omega}} \{\boldsymbol{r}_n(\boldsymbol{X}_n,\boldsymbol{u}_n) + \boldsymbol{R}_{n+1}(\boldsymbol{X}_{n+1})\} \\ (n = 0,1,\ldots,N-2) \ , \end{gathered} \tag{A13}$$

which is the Bellmann's functional equation for our optimization problem. Clearly,

$$\begin{cases} \boldsymbol{R}_0(\boldsymbol{X}_0) = \boldsymbol{R}_0 = \min\limits_{u \in \boldsymbol{\Omega}} R(u) = R(\boldsymbol{u}^\circ), \quad u^\circ \in \boldsymbol{\Omega} \\ \boldsymbol{R}_{N-1}(\boldsymbol{X}_{N-1}) = 0 \end{cases} \tag{A14}$$

where $\boldsymbol{u}^\circ = \boldsymbol{u}^\circ(t)$ is the optimal control for the discrete problem. Thus, we have the following relationships:

$$\begin{aligned} \boldsymbol{R}_{N-1}(\boldsymbol{X}_{N-1}) &= 0 \\ \boldsymbol{R}_{N-2}(\boldsymbol{X}_{N-2}) &= \min_{\boldsymbol{u}_{N-2} \in \Omega} \boldsymbol{r}_{N-2}(\boldsymbol{X}_{N-2},\boldsymbol{u}_{N-2}) \\ &\ldots \\ \boldsymbol{R}_n(\boldsymbol{X}_n) &= \min_{\boldsymbol{u}_n \in \Omega} [\boldsymbol{r}_n(\boldsymbol{X}_n,\boldsymbol{u}_n) + \boldsymbol{R}_{n+1}(\boldsymbol{X}_{n+1})] \\ &\qquad (n = N-3, N-4, \ldots, 1) \\ &\ldots \\ \boldsymbol{R}_0(\boldsymbol{X}_0) &= \min_{\boldsymbol{u}_0 \in \Omega} [\boldsymbol{r}_0(\boldsymbol{X}_0,\boldsymbol{u}_0) + \boldsymbol{R}_1(\boldsymbol{X}_1)] \\ &= R(\boldsymbol{u}^\circ) \ . \end{aligned} \tag{A15}$$

Note that $\boldsymbol{R}_{n+1}(\boldsymbol{X}_{n+1})$ does not depend on $\boldsymbol{u}_n$ explicitly. Let $\boldsymbol{u}_n = \boldsymbol{u}_n^0$ be the admissible control which minimizes $\boldsymbol{r}_n(\boldsymbol{X}_n, \boldsymbol{u}_n)$:

$$\min_{\boldsymbol{u}_n \in \Omega} \boldsymbol{r}_n(\boldsymbol{X}_n, \boldsymbol{u}_n) = \boldsymbol{r}_n(\boldsymbol{X}_n, \boldsymbol{u}_n^0), \quad \boldsymbol{u}_n^0 \in \boldsymbol{\Omega} \ . \tag{A16}$$

Thus, we can easily show that

$$u_{k,n}^0 = \begin{cases} m_1 & \text{if } V_{k,n} < M_1 \\ V_{k,n} & \text{if } M_1 \leq V_{k,n} \leq M_2 \\ M_2 & \text{if } V_{k,n} > M_2 \end{cases} \tag{A17}$$

for $k = 1,2$, where

$$V_{k,n} = W_{k,n}/W_{3,n} \quad (k = 1,2) \tag{A18}$$

with

$$\begin{aligned} W_{1,n} &= E_{5,n}E_{4,n} - E_{6,n}E_{2,n} \\ W_{2,n} &= E_{1,n}E_{6,n} - E_{3,n}E_{5,n} \\ W_{3,n} &= E_{1,n}E_{4,n} - E_{2,n}E_{3,n} \end{aligned} \tag{A19}$$

where

$$\begin{aligned} E_{1,n} &= Q_{1,n}^2 + Q_{3,n}^2 \\ E_{2,n} &= Q_{1,n}Q_{2,n} + Q_{3,n}Q_{4,n} \\ E_{3,n} &= E_{2,n} \\ E_{4,n} &= Q_{2,n}^2 + Q_{4,n}^2 \\ E_{5,n} &= Q_{1,n}(z_{1,n+2} - y_{1,n}) + Q_{3,n}(z_{3,n+2} - y_{3,n}) \\ E_{6,n} &= Q_{2,n}(z_{1,n+2} - y_{1,n}) + Q_{4,n}(z_{3,n+2} - y_{3,y}) \ . \end{aligned} \tag{A20}$$

According to the above analysis, the control

$$\boldsymbol{u}^0 = \boldsymbol{u}^0(t) = (u_{1,n}^0, u_{2,n}^0) \qquad \text{for } t_n \leq t < t_{n+1} \tag{21}$$

$$(n = 0,1,\ldots,N-2)$$

is admissible and is the solution of the discrete optimization problem. Because of the smoothness of the state equations (10), for sufficiently large N, $\boldsymbol{u}^0(t)$ approximates well the solution $\boldsymbol{u}^*(t)$ of the original continuous optimal tracking problem. We omit the details here.

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