

Assignment-1



1. i) a) $\Omega = \{HH, TT, HT, TH\}$

b) $P(\Omega) = \{ \phi, \{HH\}, \{TT\}, \{HT\}, \{TH\}, \{HH, TT\}, \{HT, TH\}, \{HH, TH\}, \{HT, TT\}, \{TH, HT\}, \{HH, TT, TH\}, \{HH, TT, HT\}, \{HT, TH, TT\}, \{TH, HT, TT\}, \{HH, TH, TT\} \}$

i) $\frac{1}{4}$

iii) $\{HT, TH\}$
 $= \frac{2}{4} = \frac{1}{2}$

ii) $\{HT, HH, TH\} = \frac{3}{4}$

2.1) $n=50, k=45, p=0.9, 1-p=0.1$

$$f(k, n, p) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$f(45) = \frac{50!}{45!5!} (0.9)^{45} (0.1)^5$$

$$\approx 0.185$$

2.2) $\lambda=10, k, f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$

a) $k=0$

$$f(0) = e^{-10}$$

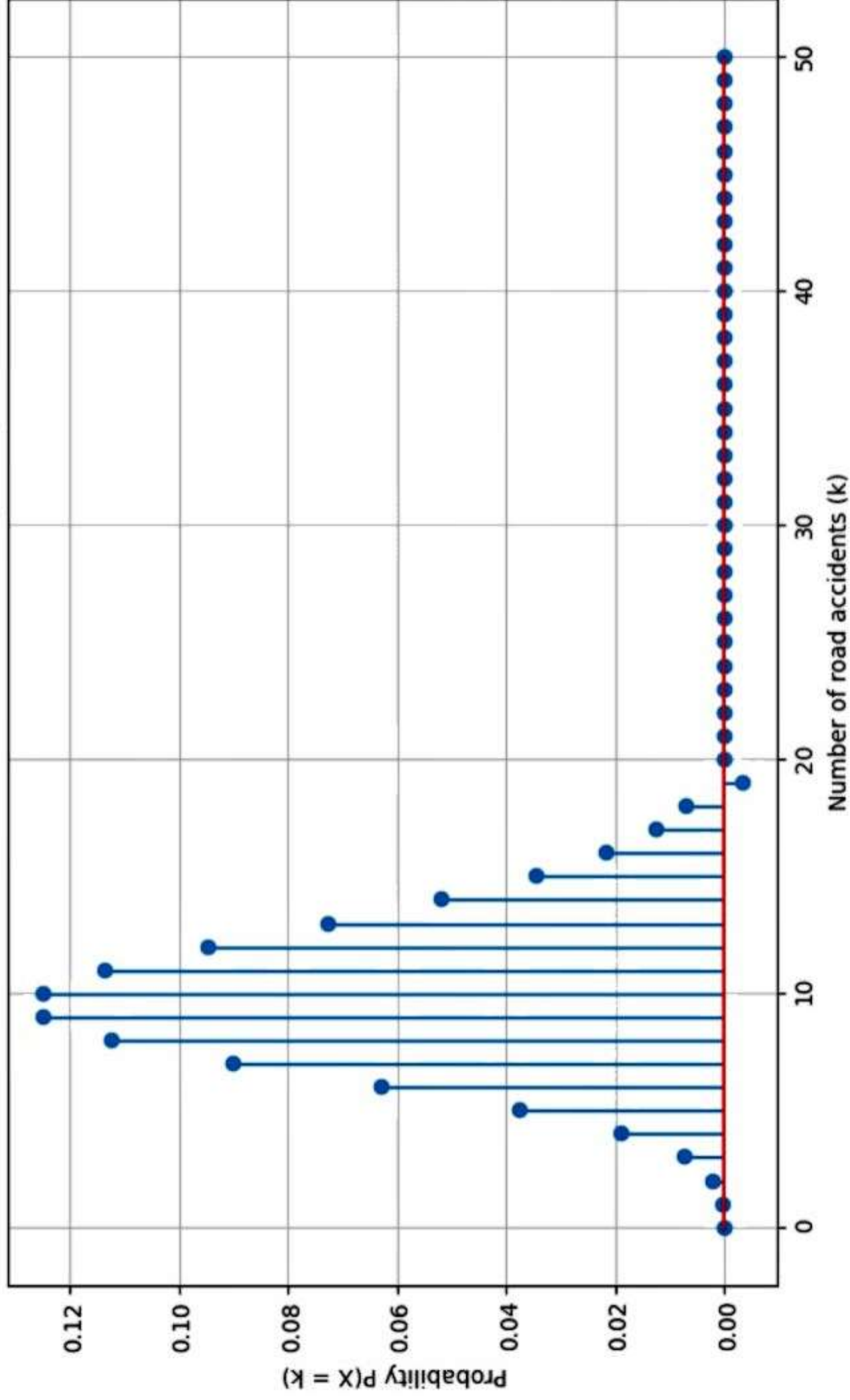
b) $k=8 \rightarrow f(8) = \frac{10^8 e^{-10}}{8!}$

$k=9 \rightarrow f(9) = \frac{10^9 e^{-10}}{9!}$

$$P(7 < X < 10) = f(8) + f(9) = \frac{10^8 e^{-10}}{8!} \left(\frac{10}{9} + e^{-1} \right)$$

$$\approx 0.287$$

Poisson Distribution PMF ($\lambda=10$)



$$b.1) f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$a) x=0, \mu=1, \sigma=1$$

$$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \approx 0.242$$

z

$$b) x=1, \mu=0, \sigma=1$$

$$f(1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \approx 0.242$$

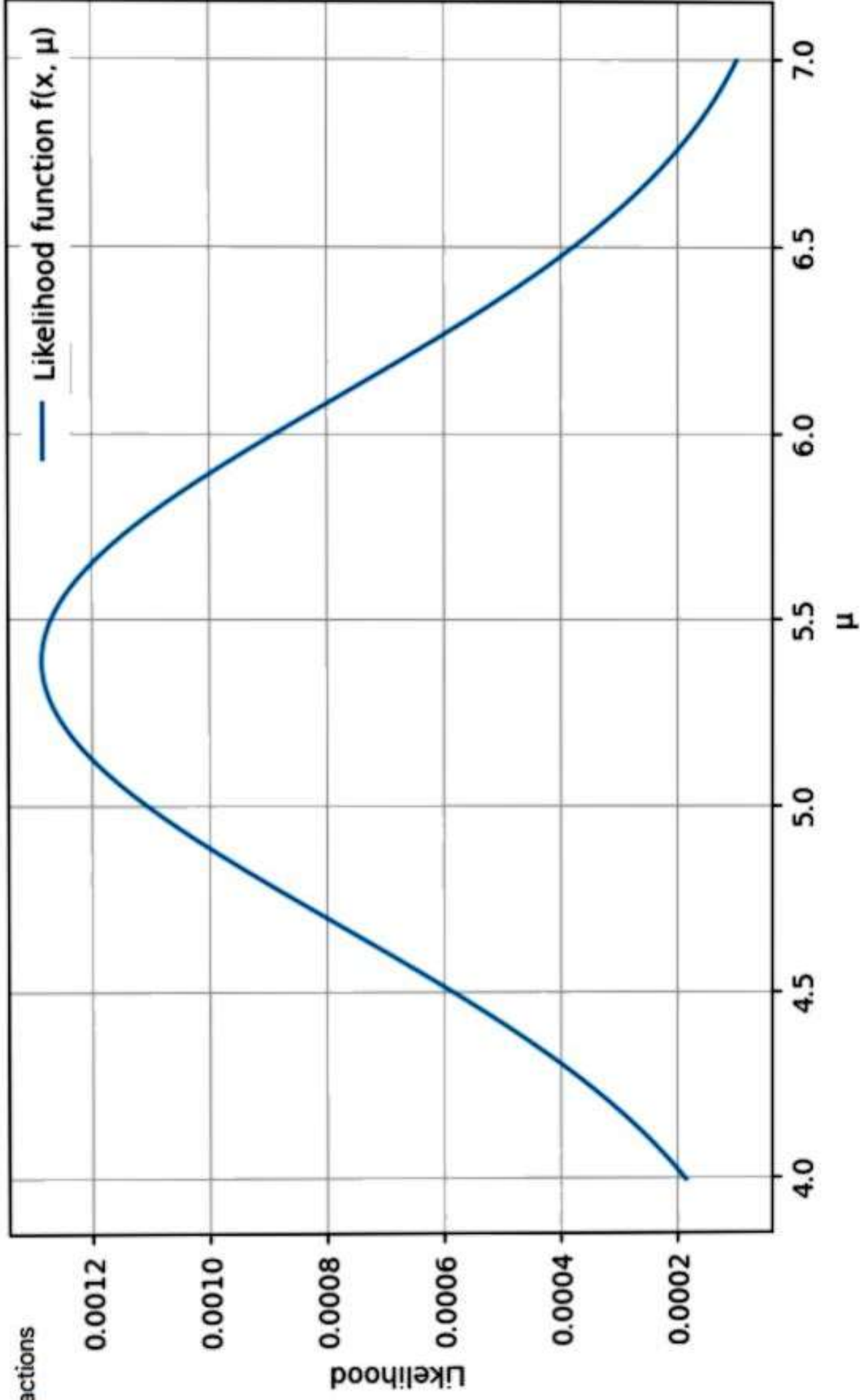
$$c) P(x_1 < X < x_2) \approx 0.3, P(x_2 < X < x_3) \approx 0.45$$

$$P(x_2 < X < x_3) = P(x_1 < X < x_3) - P(x_1 < X < x_2)$$

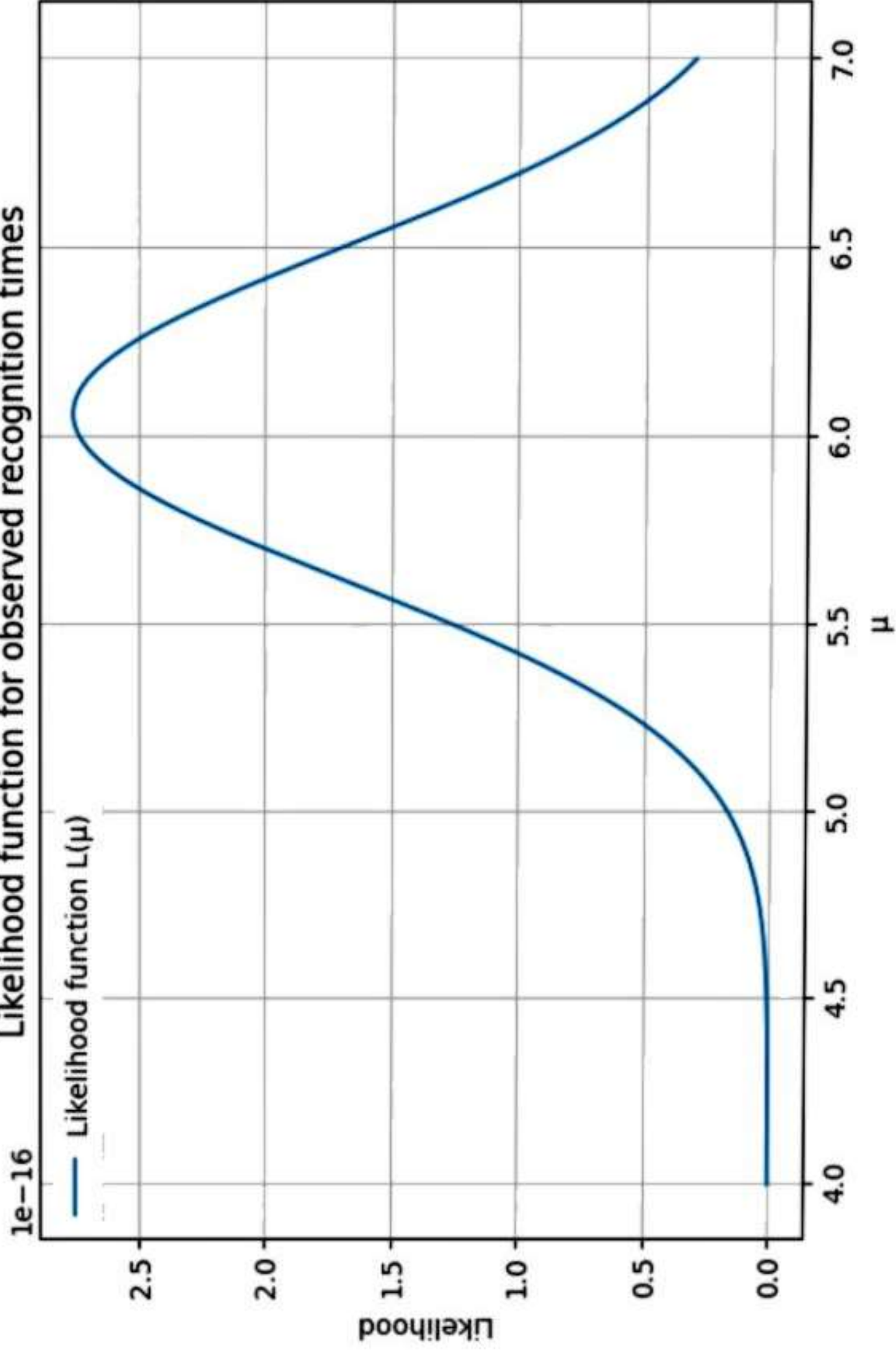
$$= 0.45 - 0.3$$

$$= 0.15 \quad \underline{\underline{ans}}$$

Likelihood function for $x = 220$



Likelihood function for observed recognition times



c))

$$x_i = 303, 280, 443, 220, 560, 880$$

$$f = \frac{1}{\prod_{i=1}^n x_i (\sqrt{2\pi})} e^{-\frac{\sum_{i=1}^n (\log(x_i) - \mu)^2}{2}}$$

for max f

$$\rightarrow \sum_{i=1}^n \frac{(\log x_i - \mu)^2}{2}$$

minimum

must be

$\sum_{i=1}^n \frac{(\log x_i - \mu)^2}{n^2}$ is minimum when

$\mu = \text{mean}(\log x_i)$

$$\mu = \frac{\log 303 + \log 443 + \log 220 + \log 560 + \log 880}{5}$$

$$\mu = \frac{\log 303 + \log 443 + \log 220 + \log 560 + \log 880}{5}$$

$$\mu = \frac{\log (303 \times 443 \times 220 \times 560 \times 880)}{5} = 0.06$$