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CECS 503
ASSIGNMENT 1
DATE SUBMITTED:

1.7

a. PROVE $\log X < X$ FOR ALL $X > 0$.

① FOR $0 < x \leq 1$,

$\log(1) = 0$, $\therefore$ IF $x < 1 \Rightarrow \underline{\log x < x}$

② FOR $x > 1$,

$\log(2) = 1$ $\therefore$ IF $1 < x \leq 2 \Rightarrow \underline{\log x < x}$

$\Rightarrow$ LET $P =$ POSITIVE INTEGER

$P < x \leq 2P$

$\log AB = \log A + \log B$ FOR $A, B > 0$

$\Rightarrow \log 2P = \log 2 + \log P$

IF $P = 1$, $\log 2P = 1$ $\Rightarrow$ $\log 2P < 2P$
$2P = 2$

$\log P = 0 \Rightarrow \log P < P$

X WILL ALWAYS BE GREATER THAN 1 FOR THIS CASE AND THUS $\log x < x$ WILL HOLD TRUE FOR ALL POSITIVE INTEGERS.

COMBINING BOTH CASES $\Rightarrow$ $\underline{\log x < x \text{ FOR ALL } x > 0}$.

b. PROVE $\log(A^B) = B \log A$

LET $A = 2^x$

$A^B = (2^x)^B \Rightarrow A^B = 2^{xB}$, $x^{yz} = x^{yz}$

$\log A^B = xB$, SINCE $A = 2^x \Rightarrow \log A = x$

$\Rightarrow \log A^B = (\log A) \cdot B$

$\underline{\log(A^B) = B \log A}$

1.10

WHAT IS $2^{100} \pmod 5$?

x	SOLVE FOR 2^x	
0	$2^0 = 1$, $1 \bmod 5$	$= 1$
1	$2^1 = 2$, $2 \bmod 5$	$= 2$
2	$2^2 = 4$, $4 \bmod 5$	$= 4$
3	$2^3 = 8$, $8 \bmod 5$	$= 3$
4	$2^4 = 16$, $16 \bmod 5$	$= 1$
5	$2^5 = 32$, $32 \bmod 5$	$= 2$

$\Rightarrow$ CONTINUES IN CYCLE OF FOUR DIFFERENT OUTCOMES.

WHEN $x = 100$, CYCLE WILL BE SAME AS 2^0 OR 2^4

$$\therefore \quad 2^{100} \pmod 5 = 2^4 \pmod 5 = \boxed{1}$$

2.1

ORDER THE FUNCTIONS BY GROWTH RATE, INDICATE WHICH ARE THE SAME.

N, $\sqrt{N}$, $N^{1.5}$, N^2, $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log (N^2)$, $2/N$, 2^N, $2^{N/2}$, 37, $N^2 \log N$, N^3.

SORTED FROM SLOWEST TO FASTEST:

1. $2/N$
2. 37
3. $\sqrt{N}$
4. N
5. $N \log \log N$
6. $N \log N = N \log (N^2)$
7. $N \log^2 N$
8. $N^{1.5}$
9. N^2
10. $N^2 \log N$
11. N^3
12. $2^{N/2}$
13. 2^N

2.3

WHICH GROWS FASTER, $N \log N$ OR $N^{1+\epsilon/\sqrt{\log N}}$, $\epsilon > 0$?

INITIALLY ASSUME THAT $N \cdot \log N > N^{1+\epsilon/\sqrt{\log N}}$

$\Rightarrow \quad N \cdot \log N > N \cdot N^{\epsilon/\sqrt{\log N}}$

$\Rightarrow \quad \log N > N^{\epsilon/\sqrt{\log N}}$

$\Rightarrow \quad \log \log N > \frac{\epsilon}{\sqrt{\log N}} \cdot (\sqrt{\log N})^2$

$\Rightarrow \quad \log \log N > \epsilon \sqrt{\log N}$

LET $X = \log N$

$\Rightarrow \log X > \epsilon \sqrt{X}$

$\Rightarrow \log^2 X > \epsilon^2 X$,

WHICH IS NOT TRUE

$\Rightarrow$ $\boxed{N^{1+\epsilon/\sqrt{\log N}} \text{ GROWS FASTER}}$

2.4

PROVE THAT FOR ANY CONSTANT K, $\log^K N = o(N)$

- BY SMALL-OH DEFINITION, IF $f(N) = o(g(N))$

THEN $\lim_{N \to \infty} \frac{f(N)}{g(N)} = 0 \Rightarrow \lim_{N \to \infty} \frac{\log^K N}{N}$

L'HÔPITAL'S
$\Rightarrow \lim_{N \to \infty} \frac{d/dN \log^K N}{d/dN \, N} = \lim_{N \to \infty} \frac{K \cdot \log^{K-1}(N) \, 1/N}{1} = K \cdot \lim_{N \to \infty} \frac{\log^{K-1} N}{N}$

IF WE CONTINUE TO APPLY L'HÔPITAL'S, THE $\log^{K-}$ EXPONENT EVENTUALLY REACHES 0.

$\Rightarrow K \cdot \lim_{N \to \infty} \frac{\log^{K-1} N}{N} \Rightarrow K \cdot (K-1) \cdot \ldots \cdot 1 \cdot \lim_{N \to \infty} \frac{1}{N}$

$\lim_{N \to \infty} 1/N = 0$, $\therefore$ $\boxed{\log^K N = o(N) \text{ FOR ANY CONSTANT } K}$

2.6

a)

N	FINE	EQN
1	2	2^(2^(1-1))
2	4	2^(2^(2-1))
3	16	2^(2^(3-1))
4	256	2^(2^(4-1))
5	65,536	2^(2^(5-1))

$\Rightarrow$ ON DAY N,

$\boxed{\text{FINE} = 2^{2^{N-1}}}$

b) $2^{2^{N-1}} = D$

$\Rightarrow 2^{N-1} = \log D$

$\Rightarrow N-1 = \log(\log D)$

$\Rightarrow \boxed{N = \log(\log D) + 1}$ OR $\boxed{O(\log \log D)}$

2.7

a) ① $O(n)$, LOOP ITERATES n TIMES

② $O(n^2)$, LOOP ITERATES n^2 TIMES

③ $O(n^3)$, LOOP ITERATES $n*n$ FOR n TIMES $= n^3$

④ $O(n^2)$, LOOP ITERATES $\sim n^2$ TIMES

⑤ $O(n^5)$, $j \simeq i^2 \Rightarrow n^2$, $k \simeq j \Rightarrow n^2$, $i \Rightarrow n$, $n^2 \cdot n^2 \cdot n = n^5$

⑥ $O(n^4)$, $j \simeq i^2 \Rightarrow n^2$, $i \Rightarrow n$, IF STATEMENT CAN ITERATE n^3 TIMES MAX BUT, IS ONLY TAKE i TIMES FOR EACH i $\Rightarrow n^2$. IF IT REACHES THE DEEPEST LOOP, IT IS EXECUTED n^2 TIMES $\Rightarrow n^2 \cdot n^2 = n^4$.

PART B AND C ATTACHED AS LAST PAGE!

2.22

SHOW THAT x^{62} CAN BE COMPUTED WITH ONLY 8 MULTIPLICATIONS.

STARTING WITH x

$\Rightarrow$ $x \cdot x = \underline{x^2}$ ①

$x^2 \cdot x^2 = \underline{x^4}$ ②

$x^4 \cdot x^4 = \underline{x^8}$ ③

$x^8 \cdot x^2 = \underline{x^{10}}$ ④

$x^{10} \cdot x^{10} = \underline{x^{20}}$ ⑤

$x^{20} \cdot x^{20} = \underline{x^{40}}$ ⑥

$x^2 \cdot x^{20} \cdot x^{40} = \underline{x^{62}}$ ⑦, ⑧ 2.7 Parts b) and c)

Segment 1:

```
Ace:CECS503 A$ g++ 7.cpp -o 7
Ace:CECS503 A$ ./7
N: 64
Runtime: 446
N: 128
Runtime: 463
N: 256
Runtime: 748
N: 512
Runtime: 1414
N: 1024
Runtime: 2754
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be linear, increasing by an approximate rate of 2. The runtimes gathered appear to do that, which confirms the big oh notation.

Segment 2:

```
Ace:CECS503 A$ ./7
N: 64
Runtime: 11842
N: 128
Runtime: 44622
N: 256
Runtime: 174831
N: 512
Runtime: 692200
N: 1024
Runtime: 2.75868e+06
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be quadratic, increasing by an approximate rate of 2^2(4). The runtimes gathered appear to do that, which confirms the big oh notation.

Segment 3:

```
Ace:CECS503 A$ ./7
N: 64
Runtime: 908568
N: 128
Runtime: 6.65761e+06
N: 256
Runtime: 4.12558e+07
N: 512
Runtime: 3.16618e+08
N: 1024
Runtime: 2.41021e+09
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be cubic, increasing by an approximate rate of 2^3(8). The runtimes gathered appear to do that, which confirms the big oh notation.

Segment 4:

```
[Ace:CECS503 A$ g++ 7.cpp -o 7
[Ace:CECS503 A$ ./7
N: 64
Runtime: 6880
N: 128
Runtime: 24091
N: 256
Runtime: 92649
N: 512
Runtime: 398045
N: 1024
Runtime: 1.46392e+06
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be quadratic, increasing by an approximate rate of 2^2(4). The runtimes gathered appear to do that, which confirms the big oh notation.

Segment 5:

```
[Ace:CECS503 A$ ./7
N: 2
Runtime: 372
N: 4
Runtime: 619
N: 8
Runtime: 7888
N: 16
Runtime: 262053
N: 32
Runtime: 7.89511e+06
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be quantic, increasing by an approximate rate of 2^5(32). The runtimes gathered don't initially provide feedback confirming the suspected big oh notation, but start to demonstrate expected behavior as N grows larger. Big oh notation can be confirmed.

Segment 6:

```
[Ace:CECS503 A$ ./7
N: 2
Runtime: 409
N: 4
Runtime: 524
N: 8
Runtime: 2943
N: 16
Runtime: 28645
N: 32
Runtime: 387683
```

Analysis: Runtime shown in nanoseconds. The rate of growth for runtime should be quartic, increasing by an approximate rate of 2^4(16). The runtimes gathered don't initially provide feedback confirming the suspected big oh notation, but start to demonstrate expected behavior as N grows larger. Big oh notation can be confirmed.