Asymptotic Analysis:

The efficiency of an algorithm depends on the amount of time, storage and other resources required to execute the algorithm. The efficiency is measured with the help of asymptotic notations.

An algorithm may not have the same performance for different types of inputs. With the increase in the input size, the performance will change.

The study of change in performance of the algorithm with the change in the order of the input size is defined as asymptotic analysis.

Asymptotic Notations:

Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

For example: In bubble sort, when the input array is already sorted, the time taken by the algorithm is linear i.e. the best case.

But, when the input array is in reverse condition, the algorithm takes the maximum time (quadratic) to sort the elements i.e. the worst case.

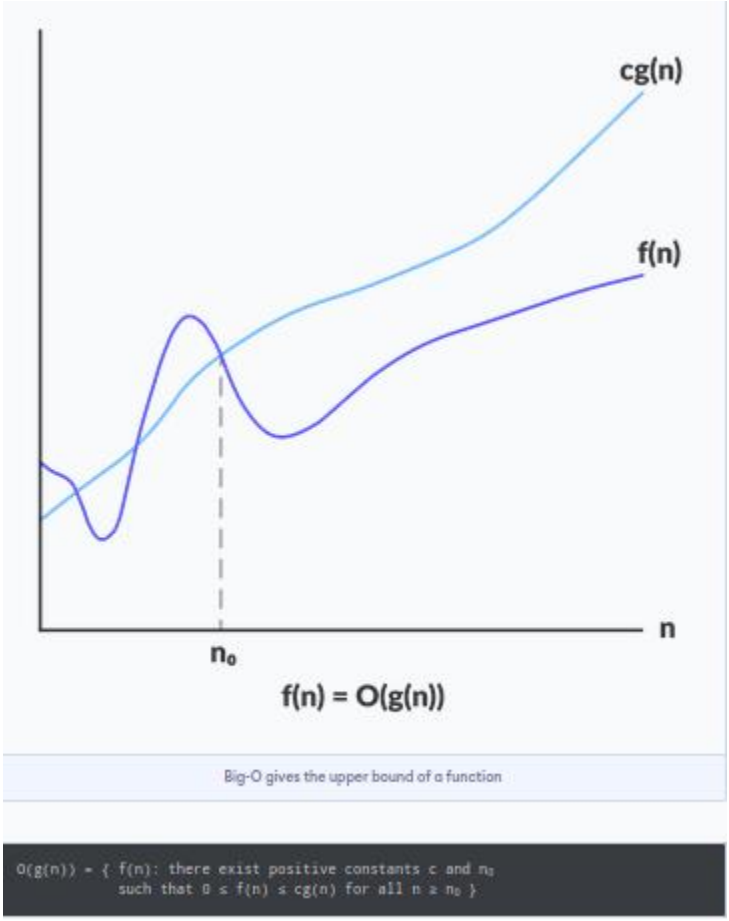
When the input array is neither sorted nor in reverse order, then it takes average time. These durations are denoted using asymptotic notations.

There are mainly three asymptotic notations:

- Big-O notation
- Omega notation
- Theta notation

Big-O Notation (0-notation):

Big-O notation represents the upper bound of the running time of an algorithm. Thus, it gives the worst-case complexity of an algorithm.



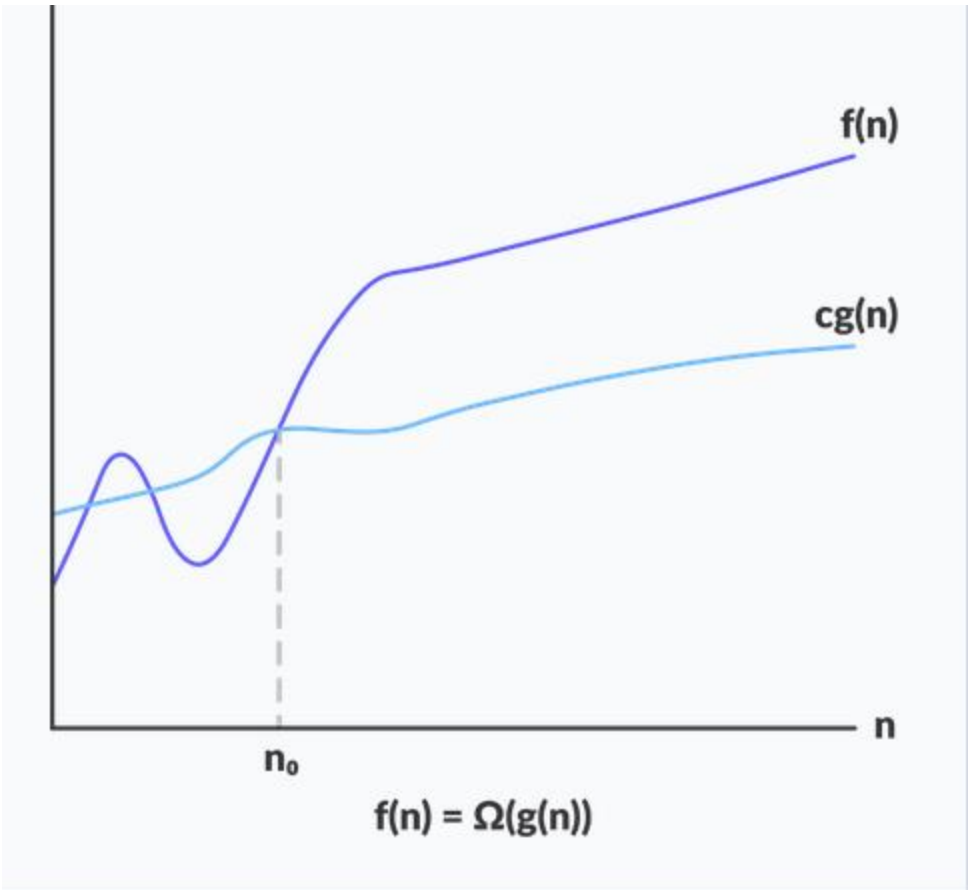
The above expression can be described as a function f(n) belongs to the set O(g(n)) if there exists a positive constant c such that it lies between 0 and cg(n), for sufficiently large n.

For any value of n, the running time of an algorithm does not cross the time provided by O(g(n)).

Since it gives the worst-case running time of an algorithm, it is widely used to analyze an algorithm as we are always interested in the worst-case scenario.

Omega Notation (Ω- notation):

Omega notation represents the lower bound of the running time of an algorithm. Thus, it provides the best case complexity of an algorithm.



Omega gives the lower bound of a function

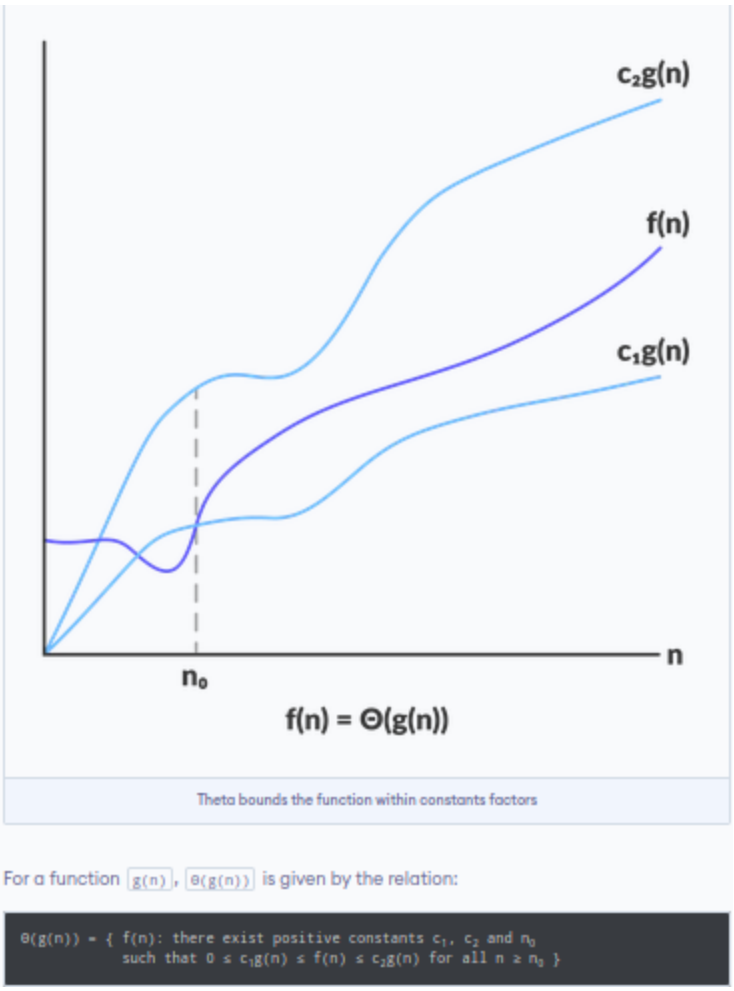
$\Omega(g(n)) = \{ f(n)$: there exist positive constants c and n_0 such that $0 \le cg(n) \le f(n)$ for all $n \ge n_0 \}$

The above expression can be described as a function f(n) belongs to the set Ω(g(n)) if there exists a positive constant c such that it lies above cg(n), for sufficiently large n.

For any value of n, the minimum time required by the algorithm is given by Omega Ω(g(n)).

Theta Notation (Θ-notation):

Theta notation encloses the function from above and below. Since it represents the upper and the lower bound of the running time of an algorithm, it is used for analyzing the average-case complexity of an algorithm.



The above expression can be described as a function f(n) belongs to the set Θ(g(n)) if there exist positive constants c_1 and c_2 such that it can be sandwiched between $c_1g(n)$ and $c_2g(n)$, for sufficiently large n.

If a function f(n) lies anywhere in between $c_1g(n)$ and $c_2g(n)$ for all $n \geq n0$, then f(n) is said to be asymptotically tight bound.

Divide and Conquer Algorithm:

A **divide and conquer algorithm** is a strategy of solving a large problem by:

1. breaking the problem into smaller sub-problems
2. solving the sub-problems, and
3. combining them to get the desired output.

How Divide and Conquer Algorithm Work?

Here are the steps involved:

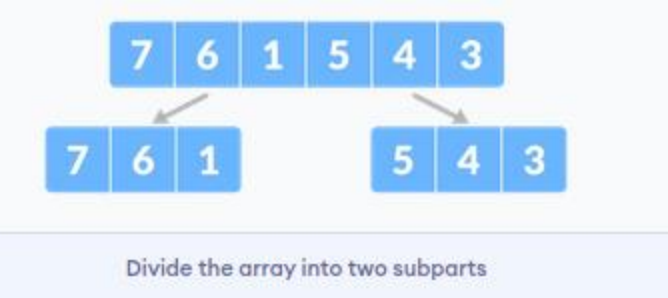
1. **Divide**: Divide the given problem into sub-problems using recursion.
2. **Conquer**: Solve the smaller sub-problems recursively. If the subproblem is small enough, then solve it directly.
3. **Combine:** Combine the solutions of the sub-problems that are part of the recursive process to solve the actual problem.

Let us understand this concept with the help of an example.

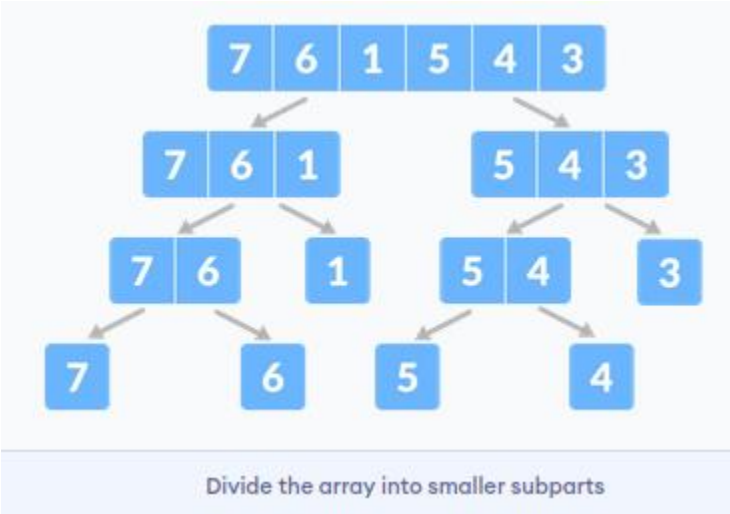
Here, we will sort an array using the divide and conquer approach (ie. merge sort).

1. Let the given array be:

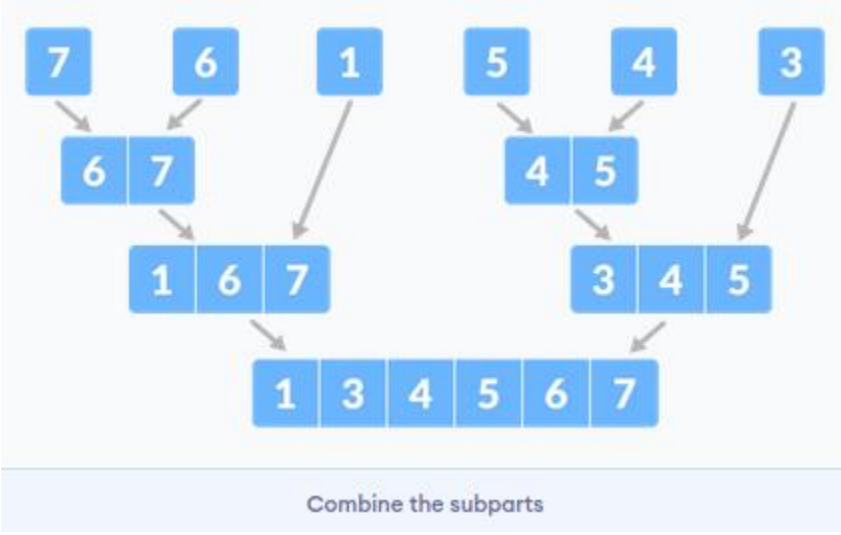
2. **Divide** the array into two halves.



Again, divide each subpart recursively into two halves until you get individual elements.



3. Now, combine the individual elements in a sorted manner. Here, **conquer** and **combine** steps go side by side.



Divide and Conquer Vs Dynamic approach:

The divide and conquer approach divides a problem into smaller subproblems; these subproblems are further solved recursively. The result of each subproblem is not stored for future reference, whereas, in a dynamic approach, the result of each subproblem is stored for future reference.

Use the divide and conquer approach when the same subproblem is not solved multiple times. Use the dynamic approach when the result of a subproblem is to be used multiple times in the future.

Let us understand this with an example. Suppose we are trying to find the Fibonacci series. Then,

Divide and Conquer approach:

```
fib(n)
    If n < 2, return 1
    Else , return f(n - 1) + f(n -2)
```

Dynamic approach:

```
mem = []
fib(n)
    If n in mem: return mem[n]
    else,
        If n < 2, f = 1
        else , f = f(n - 1) + f(n -2)
        mem[n] = f
        return f
```

In a dynamic approach, *mem* stores the result of each subproblem.

Advantages of Divide and Conquer Algorithm:

- The complexity for the multiplication of two matrices using the naive method is $O(n^3)$, whereas using the divide and conquer approach (i.e. Strassens's matrix multiplication) is $O(n^{2.8074})$. This approach also simplifies other problems, such as the Tower of Hanoi.
- This approach is suitable for multiprocessing systems.
- It makes efficient use of memory caches.

Divide and Conquer Applications:

- Binary Search
- Merge Sort
- Quick Sort
- Strassen's Matrix multiplication