# Robust Bases for Spectrum Pooling Systems on Wavelet Packet Multi-carrier Modulation MIMO Architecture

M.K.Lakshmanan, I. Budiarjo and H. Nikookar
*International Research Centre for Telecommunications and Radar*
*Department of Electrical Engineering, Mathematics and Computer Science*
*Delft University of Technology, Mekelweg 4 2628CD, Delft,*
*The Netherlands*

## 1. Introduction

### 1.1 Background

The growing popularity of wireless applications has placed enormous burden on valuable resources such as spectral bandwidth. This has brought about a major revamp of traditional resource allocation policies culminating in an explosion of research activity in the field of Cognitive Radio (CR). In this chapter we demonstrate the operation of a spectrum pooling system built from a wavelet packet Multi-carrier modulation Multiple Input Multiple Output (MIMO) scheme. The objective is to combine the promise of optimum utilization of radio resources by Cognitive Radio, the high data throughput without additional bandwidth/ power requirements offered by MIMO and the flexibility of wavelets, to develop intelligent wireless systems. Taking the example of effective spectrum utilization, we demonstrate the feasibility and efficiency of the proposed system through simulation studies. To enable the WP-MCM system to co-exist with other licensed users wavelet packet (WP) carriers in and near the region of the licensed user spectrum are dynamically deactivated. Various wavelets including the well-known families Daubechies, Coiflet, Symlet are applied and studied. The emphasis is on the design and development of optimal wavelet bases that have narrow and well confined spectral footprints. To this end filter banks that are maximally frequency selective are derived through a modified Remez exchange algorithm. Through simulation results the operation of the proposed system is demonstrated.

### 1.2 Theme of Work

The paradox of non-availability of spectrum even when large swathes of licensed spectrum is underutilized most of the time has incited an explosion of research activity in the field of Cognitive Radio CR) (Mitola, 2000); (Haykin, 2005) - wireless systems that intelligently adapt their transmission parameters in accordance with changing environments and opportunistically utilize radio resources. CR enables public access to spectral ranges of licensed frequency bands which are seldom used by overlaying a secondary rental user (RU) to an existing licensed user (LU).

Multi-carrier modulation (MCM) has been mooted as a strong candidate for CR system design (Weiss and Jondral, 2004). By merely vacating a set of subcarriers, the spectrum of a MCM based CR can be easily and flexibly shaped to occupy spectral gaps without interfering with the LU. It has been shown that adaptive MCM based CR is a robust method to achieve good quality of communication and efficient use of the spectrum.

A further enhancement to the system performance can be brought about by building MIMO (Multiple Input - Multiple Output) systems (De Lima E.R. et al, 2004) using multiple antennas both at the transmitter and receiver with adaptive MCM arms. Through clever spatial processing, the MIMO offers significant increases in data throughput and link range without additional bandwidth or transmit power requirements.

A third dimension of system optimization can be gained by deriving the MCM carriers using wavelet bases instead of conventional Fourier bases (Bouwel et al, 2000); (Jamin and Mahonen, 2005); (Lindsey, 1997); (Negash and Nikookar, 2000). Unlike Fourier bases which are static sines/cosines, wavelets (Lakshmanan and Nikookar, 2006) offer flexibility and adaptability which can be tailored to satisfy an engineering demand.

In this work we attempt to combine the desirable features of CR, MIMO and wavelet features and demonstrate the operation of a Wavelet packet Multi-carrier modulation (WPMCM) based MIMO system in the context of a spectrum pooling setup. The wavelet packet subcarriers are derived from multistage tree-structured paraunitary filter banks (Lakshmanan et al, 2008); (Vaidyanathan, 1993); (Vetterli, 1995); (Daubechies, 1992).

MIMO systems are possible because of space-time coding (STC) algorithms. In this chapter a promising STC realization for MIMO systems called vertical Bell-labs Layered Space-Time (VBLAST) (Wolniansky P.W. et al, 1998) coding technique is applied. The effectiveness of the proposed system is demonstrated through simulation results.

Various wavelets including the well-known Daubechies, Coiflet, Symlet families are considered. A key goal in this process is to ensure that the carriers have narrow and well-confined spectral footprints that don’t spill over to neighboring regions. This way the affected carriers can be easily identified and isolated to facilitate efficient spectrum notching. In this regard filter banks that are maximally frequency selective are derived through a modified Remez exchange algorithm (Rioul and Duhamel, 1994).

### 1.3 Organization of the Chapter

The chapter is organized as follows. In Section 2 the major elements of the proposed MIMO system and its implementation are laid out. Section 3 delves into the procedure to estimate the radio environment and accordingly shape the spectrum of the transmission waveform to utilize spectrum holes without hindering LU operation. The procedure to derive the wavelet packet signal waveforms for the MCM is detailed in Section 4. Section 5 explains the process to derive the best wavelet bases for WP-MCM. Section 6 gives the spatial processing technique used to detect data at the receiver. The simulation setup and results are discussed in Sections 7 and 8, respectively. Finally, the conclusions are drawn in Section 9.

## 2. System Blocks And Operation

The system model is illustrated in Figure 1. The major blocks of the system are the spectrum estimator, wavelet packet based adaptive multicarrier modulator and the MIMO setup.

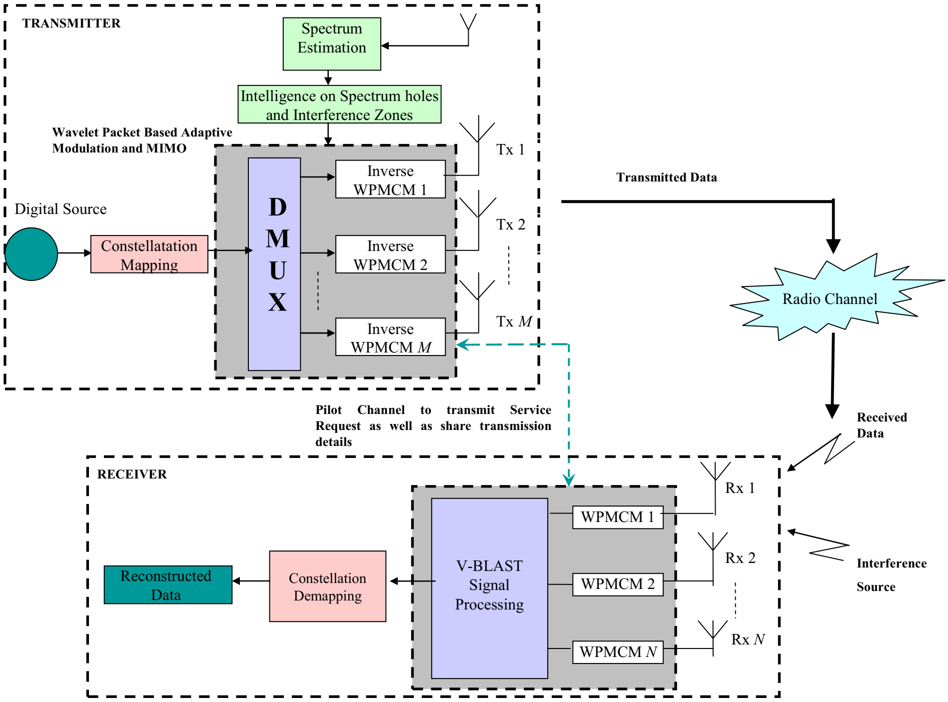



Fig. 1. System model of the Cognitive Radio. Transmitter: The major blocks include the spectrum and channel state estimator, adaptive pulse shaper (Transmission waveform shaping) and modulator. Receiver: The main components are the Signal base band processor and detector.

An incoming high-rate serial data stream is mapped and split into lower-rate parallel streams. The data in each parallel branch is upsampled and modulated with wavelet packet subcarriers. The modulated subcarriers are then added to obtain a single WPMCM symbol. This constitutes an inverse WPMCM (equivalent of IFFT in Orthogonal Frequency Division Multiplexing or OFDM) operation. The spectrum estimator gauges the radio spectrum scenario and performs radio scene analysis to detect the presence of interference regions and spectrum holes. This information is coded in the form of a spectrum vector containing ones (representing free bands) and zeros (representing occupied bands). Based on this vector the transmitter adapts the signal spectrum by activating or vacating WPMCM carriers and a transmission waveform that has little or no energy in the interference domains is derived.
A collection of such shaped WPMCM symbols is then demultiplexed into $M$ substreams, and each substream is fed to a different transmit antenna in the same frequency band. The

transmitters' assemblage forms a vector-valued transmitter with the transmit power in each arm being proportional to $1/M$ so that the total radiated power is constant and independent of the number of transmitter arms $M$ (De Lima E.R. et al, 2004).
The receiver consists of an array of $N$ antennas each of which pick up the signals from all $M$ transmit antennas. The received signals at each receiver antenna $n$ from each of the $m$ transmit antenna is be given as:

$$Y_n = \sum_{m=1}^{M} C_{nm} X_m + \eta_n \tag{1}$$

This can be represented in matrix form as:

$$\mathbf{Y} = \mathbf{CX} + \eta \tag{2}$$

where $\mathbf{Y}=[Y_1, Y_2,\ldots Y_N]$ is the received signal vector, $\mathbf{X}=[X_1, X_2,\ldots X_M]^T$ is the transmitted signal vector, $\boldsymbol{\eta}=[\eta_1, \eta_2,\ldots \eta_N]^T$ is the noise vector and the channel matrix $\mathbf{C}$ and $C_{nm}$ is the channel link between transmit antenna $m$ and receive antenna $n$ (Figure 2).
To detect the transmitted data, each of the substreams of the received signal vector is first demodulated by WPMCM (equivalent of FFT operation in OFDM). WPMCM involves de-convolving the signal substream with a sieve of receiver sub-carrier waveforms which are orthogonal duals of the sub-carriers used at the transmitter end. Estimation of information symbols through a VBLAST signal processing algorithm (described in Section 6) follows WPMCM demodulation.
The transmitter and receiver are kept cognizant at all times on the radio environment and the transmission signal characteristics through a pilot channel.

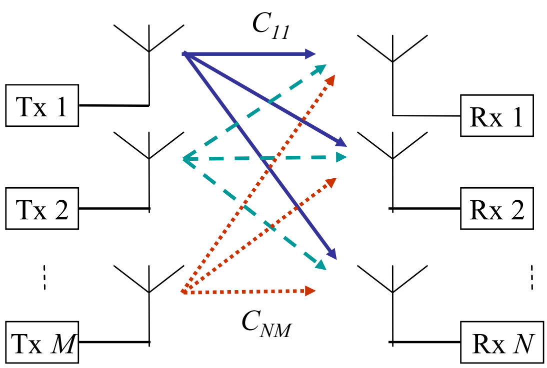



Fig. 2. MIMO Channel model. Tx1-TxM are the $M$ transmitter antennas and Rx1-RxN are the $N$ receiver antennas. $C_{nm}$ is the channel link between transmit antenna $m$ and receive antenna $n$.

## 3. Spectrum Estimation, Notching And Spectrum Shaping

### 3.1 Spectral Estimation.

The first and foremost task of any cognitive radio system is to gauge the wireless environments over wide frequency bands and identify spectrum holes and occupied bands. This is done so that the CR system can opportunistically claim unused bands and operate invisibly without causing any distortion to other primary and licensed users. The challenge of spectrum sensing module is in identification and detection of primary user signals amidst harsh and noisy environs.

Spectral estimation in the proposed WPMCM based MIMO system is accomplished by a wavelet packet transformation involving filtering and decimating the samples of the radio environment (Lee et al., 2002). The first iteration of the signal decomposition (filtering and decimating) process divides the data into two sub-bands, the detailed and coarse sub-bands. Detailed subband coefficients are the result of passing the data through a highpass filter and decimating, or down-sampling, the filter output by a factor of two. Coarse sub-band coefficients are the result of lowpass filtering the data and decimating the filter output by a factor of two. The wavelet packet decomposition process continues by subsequently splitting and down sampling the low and high pass sub-components. This iterative decomposition is repeated until the wavelet packet tree structure has been fully expanded. Following iterative decomposition, through suitable threshold, a “notched” spectral magnitude vector is generated by setting the retained wavelet packet sub-band coefficients to one and those discarded to zero. The final output of the iterative decomposition process represents the magnitude of the spectral estimates.

### 3.2 Thresholding and Spectral Notching

The spectral information is coded in the form of a spectrum vector containing ones and zeros. The zeros correspond to bands which are occupied and the ones represent bands that are free (spectrum holes). The pattern of ones and zeros effectively characterizes the desired magnitude of the spectral estimate. The threshold is performed on a sub-band-by-sub-band basis whereby the power contained in each sub-band is independently compared to a predetermined threshold. Following the recommendations of (Lee et al., 2002) the threshold value is defined in terms of the noise power. When sub-band power exceeds the noise power by 20%, interference is declared present and all of the sub-band coefficients are set to a value of zero. If sub-band power does not exceed the threshold, all of the sub-band coefficients are retained (set to a value of one).

### 3.3 Transmission Waveform Shaping By Identification of Affected WPMCM Carriers

Based on the spectrum vector, sub-channels of the WPMCM system that lie in and around the spectrum of the LU are vacated to facilitate coexistence. This way the CR transmission signal is dynamically sculpted such that it has no or very little time-frequency components competing with the LU and the CR operation is made invisible to the LU.

## 4. Signal Waveforms for MCM

### 4.1 Background

The subcarrier signal waveforms in traditional MCM implementations, such as OFDM, are sine/cosine basis functions. In WPMCM the sub-carrier waveforms are derived from poly-channel tree structures built by cascading multiple two-channel filter banks like the one shown in Figure 3.

A two-channel filter bank consists of a set of 4 perfect reconstruction filters (2 high pass and 2 low pass) which allow the decomposition and reconstruction of a signal without amplitude or phase or aliasing distortion (Vaidyanathan, 1993). The two-channel filter bank has the property of splitting the signal into two lower resolution versions – namely the coarse (low pass) and the detail (high pass). When the decomposition into coarse and detail components is continued iteratively, it leads to the generation of wavelet packet bases. When the perfect reconstruction filters used satisfy an additional property known as paraunitary condition (explained later), they lead to wavelet packet bases with impulse responses that are mutually orthogonal to one-another and to their duals.

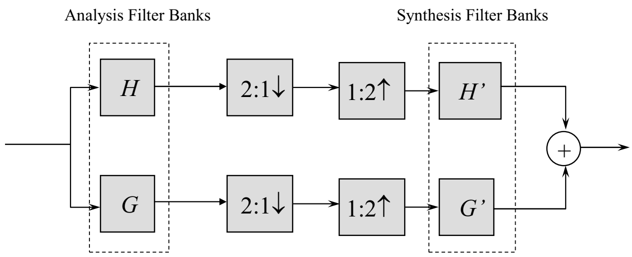



Fig. 3. Two channel filter bank analysis with analysis filters *H* and *G* (low and high pass, respectively) and synthesis filters *H'* and *G'* (low and high pass, respectively).

### 4.2 Generation of Wavelet Packet Sub-Carrier Bases

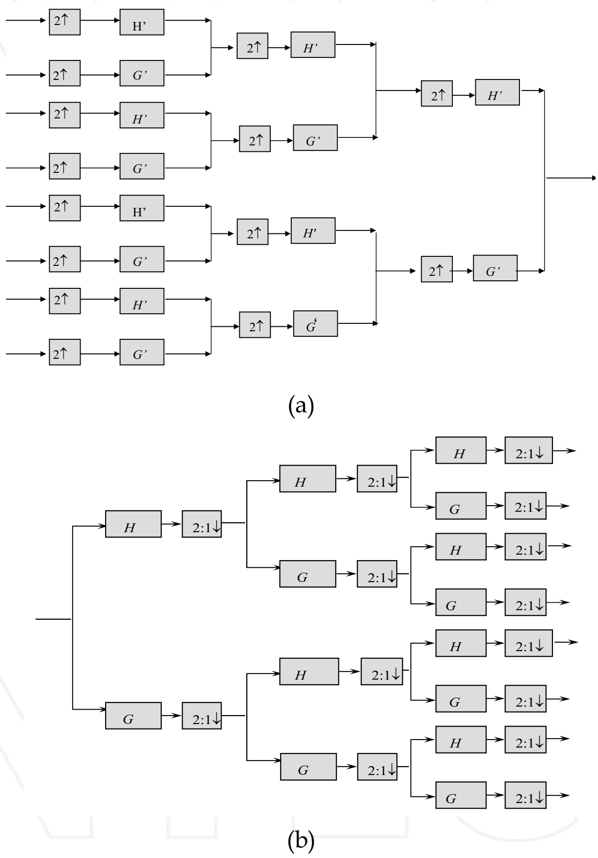
The wavelet packet sub-carriers (to be used at the transmitter end) are generated through a multichannel filterbank consisting of cascaded two-channel filters applying the synthesis filters ($H'$ and $G'$). This represents an inverse discrete wavelet packet transformation or IDWPT and consists of binary interpolation (up-sampling) by 2, filtering and recombination at each level. The number of iterations $J$ determines the number of subcarriers $M$ generated and the relationship is given as $M \leq 2^J$ .The time domain representation of the wavelet packet bases $\psi_i[k]$ is obtained through a simple convolution rule as given in (3).

$$\begin{aligned}
&\psi_i[k] = f(k) * f(k/2) * \ldots * f(k/2^{J-2}) * f(k/2^{J-1}); \\
&\text{where}, 0 \leq i \leq 2^J - 1 \\
&\text{and}, f(k) = \begin{cases} h'(k), \text{for lowpass branches} \\ g'(k), \text{for highpass branches} \end{cases}
\end{aligned} \tag{3}$$

Here $h'$ and $g'$ stand for the impulse responses of the low and high pass synthesis filters, respectively.

### 4.3 Generation of Wavelet Packet Dual Bases

The wavelet packet duals (to be used at the receiver end) are obtained from multichannel filter bank analysis too, though the processes are reversed. The duals are obtained from the analysis filters ($H$ and $G$) through the analysis procedure which consists of filtering, decimation (down sampling) by 2 and decomposition at each stage. This process is called discrete wavelet packet transformation or DWPT. First the signal is passed through a half-band high and low pass filter. The half-band low pass filter removes all frequencies that are above half of the highest frequency, while the half-band high pass filter removes all frequencies that are below half of the highest frequency of the signal. Such a half-band filtering halves the resolution, but leaves the scale unchanged. The signal is then sub-sampled by two since half of the number of samples is redundant, according to the Nyquist's rule. This decomposition halves the time resolution since only half the number of samples then comes to characterize the entire signal. Conversely, it doubles the frequency resolution, since the frequency band of the signal spans only half the previous frequency band effectively reducing the uncertainty by half. This procedure is iteratively repeated till the desired degree of resolution. The duals $\tilde{\psi}_i[k]$ are derived as:




(a)

(b)

Fig. 4. (a) Generation of wavelets. A level-3 tree gives 8 wavelet packet bases. The up arrows represent interpolation by 2. $H'$ and $G'$ denote the frequency responses of the low and high pass reconstruction filters, respectively; (b) Generation of wavelet Duals. A level-3 tree gives 8 wavelet packet dual bases. The down arrows represent decimation by 2. $H$ and $G$ denote the frequency responses of the low and high pass decomposition filters, respectively.

$$\tilde{\psi}_i[k] = f(k) * f(2k) * .... * f(2^{J-2}k) * f(2^{J-1}k);$$
$$\text{where}, 0 \le i \le 2^J - 1 \quad (4)$$
$$\text{and}, f(k) = \begin{cases} h(k), \text{for lowpass branches} \\ g(k), \text{for highpass branches} \end{cases}$$

In the equation (4), $h$ and $g$ denote the impulse responses of the low and high pass analysis filters, respectively. Figure 4 illustrates the derivation of 8 wavelet packet bases and their duals from a cascaded level-3 tree structure.

## 5. Best Wavelet Packet Bases for WPMCM

### 5.1 Wavelet Theory

The attributes of a multicarrier modulation system greatly depends on the set of waveforms used. The property of the waveforms in turn depends on underlying filter banks used. Many considerations go into the design of a wavelet system including properties such as orthogonality, compact support, symmetry, and smoothness. Here we shall discuss a few important ones.

#### 5.1.1 Paraunitary Condition

The paraunitary condition is essential for many reasons. Firstly, it is a prerequisite for generating orthonormal wavelets (Vaidyanathan, 1993). Second, it automatically ensures perfect reconstruction of the decomposed signal (Vaidyanathan, 1993) i.e., the original signal can be reconstructed without amplitude or phase or aliasing distortion, if the filter banks used satisfy the paraunitary condition. Only paraunitary filters are considered in this article and for such solution pairs the high pass and low pass filters share the relationship (Vaidyanathan, 1993) ; (Vetterli and Kovacevic, 1995); (Daubechies, 1992):

$$g[L-1-n] = (-1)^n h[n] \quad (5)$$

where $L$ is the length of the filters. Further, paraunitary filters automatically satisfy the perfect reconstruction criterion (Vetterli and Kovacevic, 1995) with the decomposition and reconstruction filters being complex conjugate time reversed versions of one another i.e.

$$h'[n] = h^{\dagger}[-n] \text{ and } g'[n] = g^{\dagger}[-n] \quad (6)$$

Filters satisfying this condition are commonly used in signal processing, and are known as the Quadrature Mirror Filters (QMF). A nice import of these relations is that it is enough to design a single filter, either the low or high pass filter alone.

### 5.1.2 Compact support

This property ensures that the wavelet is of finite duration and the filter banks used to derive the wavelets have a finite number of non-zero coefficients (Burrus et al, 1998).

### 5.1.3 Regularity

This property is a measure of smoothness of the wavelet. The regularity condition requires that the wavelet be locally smooth and concentrated in both the time and frequency domains. It is normally quantified by the number of times a wavelet is continuously differentiable. The simplest regularity condition is the “flatness” constraint which is stated on the low pass filter. A LPF is said to satisfy *K*th order flatness if its transfer function $H(z)$ contains $K$ zeroes located at the Nyquist frequency ( $z = -1$ or $\omega = \pi$ ). Parameter $K$ is called the regularity order and for a filter of length $L$ it satisfies the relation $0 \le K \le L/2$.
Wavelets are defined by the wavelet function $\psi(t)$ (i.e. the mother wavelet) and scaling function $\varphi(t)$ (also called father wavelet) in the time domain. Another way to determine the regularity of the wavelets is in terms of the number of vanishing moments of the wavelet and scaling functions (Burrus et al, 1998) and used the dual vanishing moments to determine the convergence rate of the multiresolution projections. The $j$th moments of the wavelet and scaling functions, $m_w(j)$ and $m_s(j)$, respectively, are defined in continuous time domain as follows:

$$m_w(j) = \int t^j \psi(t) dt \quad \text{and} \quad m_s(j) = \int t^j \varphi(t) dt \tag{7}$$

### 5.2 Wavelet Families

In this work we shall largely deal with the Daubechies family and its variants.

### 5.2.1 Daubechies

The Daubechies are a family of orthonormal wavelets with compact support with highest degree of smoothness. It was derived by Ingrid Daubechies (Daubechies, 1992) who used all the degrees of freedom $K$ to generate a wavelet family of maximum regularity for a given filter length $L$, or minimum $L$ for a given regularity (Daubechies, 1992). This she did by imposing the maximum number of zero moments to the wavelet function in the vanishing moments’ condition (equation 7).

### 5.2.2 Coiflet

Coiflets are a variation of the Daubechies wavelets. They are so named because it was derived by I. Daubechies at the behest of R. Coifman who suggested the construction of orthonormal wavelet basis with vanishing moment conditions for both wavelet and scaling functions (unlike Daubechies where only the wavelet functions have zero moments). The wavelet function has $2L$ moments equal to 0 and the scaling function has $2L$ -1 moments equal to 0.

### 5.2.3 Symlet
The symlet family of wavelets is another variant of the Daubechies family which are nearly-symmetrical (as opposed to being symmetrical). These modifications were also proposed by I. Daubechies and the properties of the two wavelet families are similar.

## 5.3 Choosing the Right Wavelet
In theory any time and frequency limited function can be utilized. However in practice, the wavelet bases cannot be arbitrarily chosen and instead have to satisfy a number of requirements. In general the choices to make can be with regard to the system of representation (continuous or discrete), properties of the wavelets desired (orthogonality/ biorthogonality, regularity/smoothness, frequency selectivity), the application in hand and the context of use (Burke, 1998). A framework that accounts for these requirements must first be defined and the wavelet selected in a principled approach through optimization of the wavelet design parameters.

## 5.4 Wavelet Design Considerations for WP-MCM application
With regard to the applicability to WP-MCM systems, the desirable properties may be listed as follows:

- The wavelet bases must be time-limited
- The bases must be well confined in frequency.
- The wavelet packet bases and their duals must be orthogonal (or at least linearly independent) to one another to enable perfect reconstruction.
- The bases must be orthogonal (or at least linearly independent) to one another in order to have unique demodulation.
- The bases must enable the system to handle channel effects and other distortions.
- The system must be easily realizable and must permit application of fast algorithms.

And in the filter bank domain the objective of the design procedure translates to construction of filters with the characteristics that they:

- have finite impulse response (FIR)
- are maximally frequency selective
- allow orthonormal expansion and perfect reconstruction of discrete-time signals
- satisfy the paraunitary condition
- satisfy a desired flatness/regularity condition

Amongst these properties the paraunitary and regularity properties are mandatory to the design of the filter banks. In addition to these properties the criterion that needs specific focus is the property of frequency selectivity.

## 5.5 Need for Maximally Frequency Selective Filter banks
In an ideal scenario the filter banks used to generate the wavelets have zero transition bands $B$, i.e., difference between pass and stop band frequencies (refer Figure 5). Under such an ideal scenario the wavelet packet bases derived from a level-$i$ decomposition have confined spectral footprints with bandwidth $(1/2^i)$ times that of the Nyquist frequency. However, available wavelet families are derived from filter banks that have a wide transition band and

hence the resultant wavelet sub-carriers have a dispersed spectrum with footprints spilling into neighboring regions. The wider the transition bandwidth the greater the dispersion of the carrier's spectral footprint and therefore the greater the difficulty in isolating those sub-carriers that fall in the region of the licensed user. This greatly reduces the efficiency of the system. It is therefore important to design filter banks that have narrow transition bands.

### 5.6 Design of Maximally Frequency SelectiveFilter banks

The design procedure comprises of defining a low pass FIR filter, satisfying the regularity, paraunitary and frequency selectivity conditions, expressed in the form of an impulse response $h(n)$ or a transfer function $H(z)$ or a difference equation. For a filter of length L this is essentially solving $L$ unknown filter coefficients from $L$ linear equations. Of these $L$ linear equations, $L/2$ equations come from the paraunitary constraint, $K$ equations come from the regularity or flatness constraint and the remaining $L/2 - K$ conditions offer the room for maneuverability to establish the desired wavelet property such as frequency selectivity. The larger the value of $L/2 - K$, the greater the degree of freedom for frequency selectivity and the greater the loss in regularity. There is therefore a trade-off between frequency selectivity and regularity. Wavelets such as the Daubechies family are maximally flat with regularity order $K$=$L/2$ and hence they are not frequency selective.

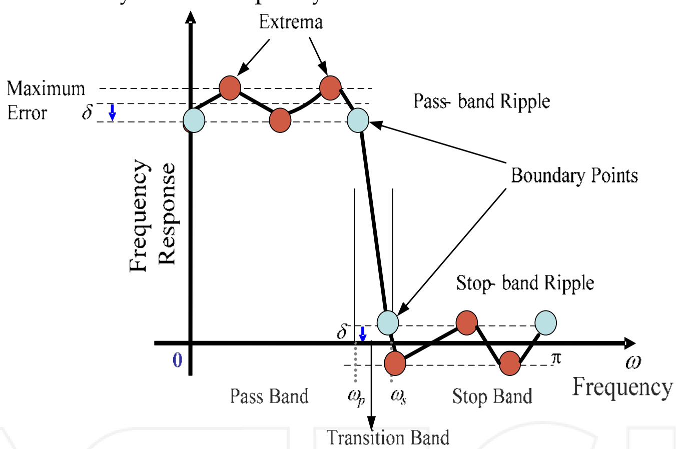



Fig. 5. Filter Characteristics in frequency. In the figure $\omega_p$ and $\omega_s$, denote pass and stop band frequencies, respectively, [0, $\omega_p$] is called the pass-band, [$\omega_s$,$\pi$] is called the stop-band and [$\omega_p$, $\omega_s$] is the transition band. $\delta$ is the tolerance or ripple.

To implement the frequency selective filters, the design parameters are stated in the frequency domain in terms of the desired magnitude response of the LPF as shown in Figure 5. In the figure $\omega_p$ and $\omega_s$ denote pass and stop band frequencies, respectively, [0, $\omega_p$] is called the pass-band, [$\omega_s$,$\pi$] is called the stop-band and [$\omega_p$, $\omega_s$] is the transition band. $\delta$ is the tolerance or ripple. The design goal is to generate the filter with a desired transition band even while the maximum error $\delta$ in the pass/stop-band is minimized.

The fundamental theory on the design of frequency selective filter banks was developed by Rioul and Duhamel (Rioul and Duhamel, 1994). They devised the procedure to design maximally frequency selective filter banks under a given set of constraints using the Remez exchange algorithm. The Remez exchange algorithm is an optimization algorithm that is commonly used in the design of FIR filters. It is popular because of its flexibility and computational efficiency. Also known as the Parks-McClellan algorithm, it works by converting the filter design problem into a problem of polynomial approximation (Oppenheim and Schafer, 1989). The algorithm is an application of the Chebyshev alternation theorem that constructs the polynomial of best approximation to a desired function under a set of constraints. Through a minimax approximation the scheme seeks to arrive at a $l$th order approximation polynomial function $A(x)$ that best approximates a desired filter polynomial function $D(x)$ (in our case the LPF $H(z)$ that satisfies the design specifications) in the given interval such that the absolute maximum error is minimized. The error is defined here as the weighted difference between the desired filter polynomial function and the approximation polynomial function and is given as

$$E(x) = W(x)(D(x) - A(x)) \tag{8}$$

$E(x)$ and $W(x)$ are respectively the error and weighting polynomial functions. All polynomial functions are of the form $\sum_{i=0}^{l} p_i x^i$, with coefficients $p_i$ and degree of the polynomial function $l$. Chebyshev proved that such a polynomial $A(x)$ exists and that it is unique. He also gave the criteria for a polynomial to be a minimax polynomial. The algorithm states that in the interval of consideration, the necessary and sufficient condition that $A(x)$ is the unique mini-max polynomial solution of degree l is that there are at least ($l$ + 2) points at which the error function $E(x)$ attains the absolute maximum value $\delta$ with alternating sign i.e.

$$E(x_i) = -E(x_{i+1}) = \pm \max_{x \in I} \{| E(x) |\} = (-1)^i \delta \tag{9}$$

for $x_1 < x_2 < \ldots < x_{l+2}$ in the desired interval I. Parks and McClellan proved that this approach could be used to derive a filter of a given length with minimal ripple. The right set of extremal points $x_i$ is arrived through an iterative procedure. In each iteration an interpolation problem is solved and the reference set of extremal points is updated. Rioul and Duhamel deduced that $L/2 - K + 1$ extremal points in the pass-band are necessary and sufficient to characterise a unique and optimal solution (Rioul and Duhamel, 1994).

The procedure starts by choosing an arbitrary set of $L/2 - K + 1$ points in the given interval. These $L/2 - K + 1$ points help form $L/2 - K + 1$ linear equations. The filter coefficients are obtained by solving the $L/2 - K + 1$ linear equations in a way that the error at the $L/2 - K + 1$ points considered is equal in magnitude and alternating in sign. It cannot be guaranteed after the first step that solution satisfies the minimax condition for the error function. That is the magnitude of the error need not be the absolute maximum magnitude in the interval of consideration. In order to find the minimax solution, the second step of the algorithm seeks new set of $L/2 - K + 1$ points that approach the $L/2 - K + 1$ points of the minimax solution. The new set of is determined by locating those points where the slope of the error function $E(x)$ is zero. Once these points are identified, the old set of

$L/2-K+1$ points is exchanged with the new points. This process is iteratively performed till the desired set of points that satisfy the minimax solution is obtained. The algorithm is said to have converged when the set of extremal points remains unchanged. Once the right set of extremal points is identified, the optimum error and the filter can be obtained. The exact details of how these equations are solved to obtain the low pass filter H(z) using the modified Remez exchange algorithm can be found in (Rioul and Duhamel, 1994). From the low pass filter $H(z)$, the high pass filter $G(z)$ and the reconstruction filters ($H'(z)$ and $G'(z)$) can be obtained by applying (5) and (6).

## 6. VBLAST and Spatial Processing to Estimate Data

The transmitted signals arrive at the receiver antenna array as multiple streams with different spatial signatures and to estimate the data a suitable detection algorithm is necessary. In this work we consider VBLAST, a promising and elegant spatial coding technique for MIMO realizations (Wolniansky P.W. et al, 1998), for detection of transmitted data.
The V-BLAST algorithm detects data by a technique known as linear nulling or symbol cancellation. In linear nulling, each received sub-stream is considered to be the desired signal with the remaining sub-streams taken as interferers. Using symbol cancellation, interference from already-detected components of the transmitted vector is subtracted from the received signal vector. This results in a modified received vector with fewer interferers. To this end the received data streams are ordered on the basis of their strength. The substream with the strongest signal is detected and its contribution subtracted from the total received vector signal. The process is continued till all other substreams are identified. The entire process may be likened to decision feedback equalization (Wolniansky P.W. et al, 1998).
Assuming that the receiver has complete knowledge of the channel matrix **C**, the V-BLAST algorithm is implemented as follows:
1. Build a Moore pseudo-inverse matrix **P** of the channel matrix **C**,

$$\mathbf{P} = (\mathbf{C}^*\mathbf{C})^{-1}\mathbf{C}^* \tag{10}$$

2. Find the row of P where its Euclidean norm is the smallest one,

$$k = \arg\min_j \| \mathbf{P}_j \| \tag{11}$$

and $j$ is the column of matrix **P**.
3. Take the row $k$ of **P** as the nulling vector w,

$$\mathbf{w} = (\mathbf{P})_k \tag{12}$$

4. Obtain the strongest transmit signal,

$$\mathrm{r}_k = \mathbf{w}^*\mathbf{y} \tag{13}$$

5. Estimate the transmitted symbol $\hat{s}_k$ by demapping $r_k$.

6. After detection of the strongest transmitted signal, its effect is cancelled from the received signal vector to reduce the detection complexity of the remaining transmit signals.

$$\mathbf{y} = \mathbf{y} - (\mathbf{C})_k * \{\text{Mapping}\}(\hat{s}_k) \tag{14}$$

here $k$ is the column index. The $k$th column of channel matrix $\mathbf{C}$ is then zeroed for the purpose of detection of the strongest transmitted signal on the next layer.
The procedure is continued until transmitted symbols in all layers are detected.

## 7. Simulation Setup

The CR is a WPMCM based MIMO system which operates with 128 equally spaced carriers that can be adaptively deactivated to shape the transmission spectrum. The 128 wavelet packet carriers are derived from a level-7 cascaded tree. The filter banks considered are daubechies, symlet, coiflet and the maximally frequency selective filters with parameters $L$=30, $K$=12, $B$=0.1. The LU is taken to occupy two equal bands with the cumulative bandwidth comparable to 32 carriers (1/4th) of the CR system and located centered on the 32nd and 96th carrier (each equal to around 16 CR carriers' bandwidth) of the CR spectral band, as shown in Figure 6.

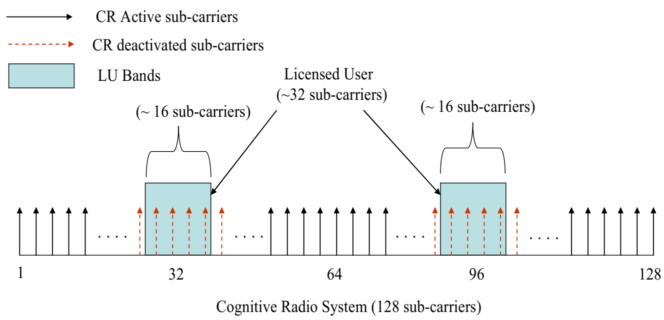



Fig. 6. Cognitive Radio (CR) and Licensed User (LU) Characteristics.

As stated earlier, the carriers of the CR system in and around the LU spectrum are dynamically deactivated to facilitate co-existence. Various MIMO configurations are considered to enhance the system functioning with all the MIMO arms transmitting the same data. The modulation scheme considered is binary phase shift keying (BPSK).To measure the effectiveness of the CR system, the Bit-error rate (BER) versus the Signal-to-Noise Ratio (SNR) is plotted. To simplify the evaluation of the system, the communication channel is assumed to be Additive White Gaussian Noise (AWGN). No multi-path interference is considered. The transceivers are taken to operate in unison maintaining full time/phase/frequency synchronization. Moreover, they are assumed to be stationary with negligible Doppler effects.

## 8. Simulation Results

### 8.1 Performance of CR Setup

To understand the performance of the CR setup, two set of results are evaluated – operation of CR under LU and operation of LU under CR. A well designed CR setup must allow both the CR and LU systems to coexist seamlessly without any compromise in performance of either system.

Figure 7(a) shows the BER performance curves of the WP-MCM MIMO based CR system in the scenarios: absence of LU (only AWGN), presence of LU without any carrier deactivation and presence of LU with carrier deactivation. The MIMO configuration considered is 2x2. The carriers were derived from maximally frequency selective filters with $L$=30, $K$=12 and $B$=0.1. From the plots it is quite clear that the presence of LU affects the CR performance. And when the CR transmission is communicated around the LU with carriers in and around the region of interference removed, the CR system recovers. Best results are obtained when a total of 36 (18 each on and around the two LU bands) or more of the CR carriers are vacated. Figure 7(b) shows the corresponding BER curves for the LU co-existing with CR. The results of the WPMCM based CR system with carrier deactivation are equally encouraging.

### 8.2 Influence of MIMO Configuration on WP-MCM MIMO Setup

We now present the performance of the CR system under various WPMCM MIMO configurations. The WPMCM MIMO based CR system operates under a LU with 40 of its 128 carriers in and around the region of LU deactivated. Figure 8 shows the related plots. To better understand the operation, we classify the MIMO configurations as follows:

a) First we consider the case with the same of number of antennas in the both the transmitter and receiver ends (Figure 8a). The formations considered are 1x1, 2x2, 3x3 and 4x4. The pattern of the BER curves is unambiguous. With increase in the number of transmit-receive pairs, the BER system functioning improves. This is due to the array gain.

b) Effect of receiver antennas – Increasing the number of receiver antennas significantly improves the system performance. This is illustrated in Figure 8b where the number of transmit antennas is kept constant at 2 while increasing the number of receive antennas to 3, 4, 5 and 6 respectively. For a BER of $10^{-4}$ the improvements with respect to the 2x2 configuration are 2 dB for 2x3, 3dB for 2x4, 4dB for 2x5 and 5dB for 2x6.

c) Figure 8c shows the curves for a few other MIMO configurations. In all MIMO configurations the performance of the CR setup is efficient and allows co-existence with LU.

### 8.3 Comparison of performance of WP-MCM and OFDM based CR systems

It will be interesting to see as to how the WP-MCM construction devised in this paper matches up to traditional OFDM implementations. Both the OFDM and WP-MCM scheme use the same set of transmission parameters – 128 carriers each with 40 carriers in the interference band removed. The wavelet used belongs to the family of maximally frequency selective filters. The curves (Figure 9) are for the MIMO configurations 1x1, 2x2 and 3x3. For all the cases considered the WPMCM setup performs comparably well with its OFDM counterpart.

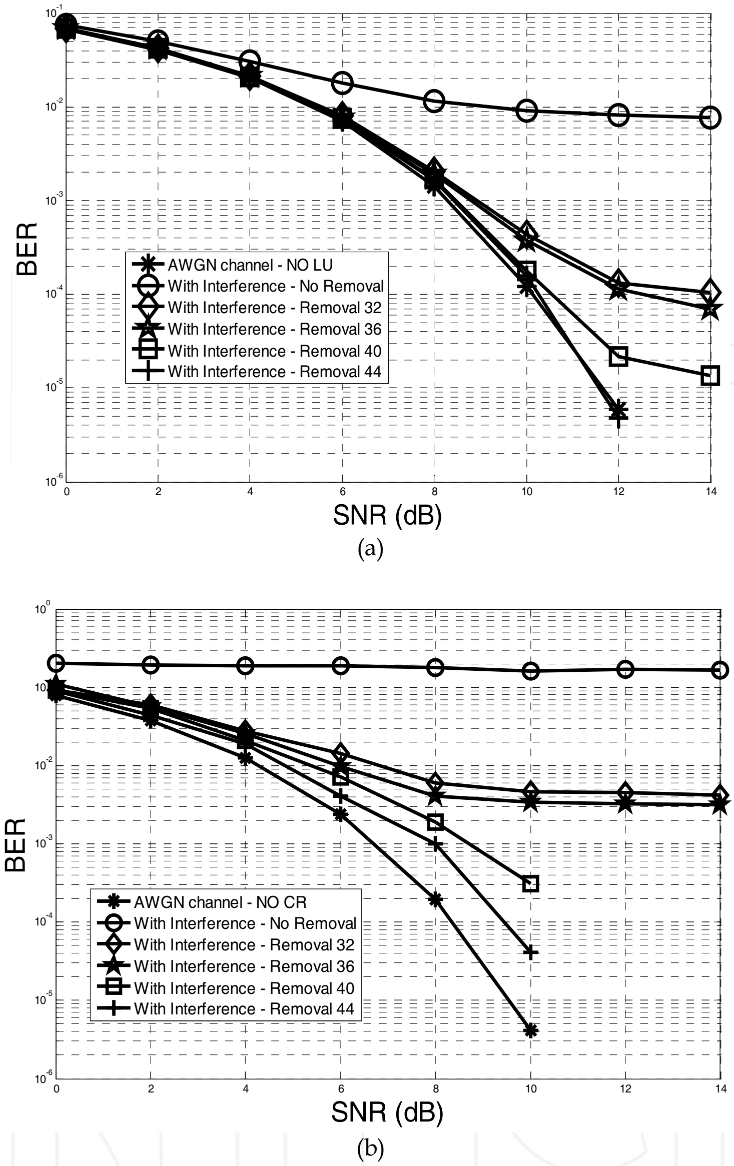
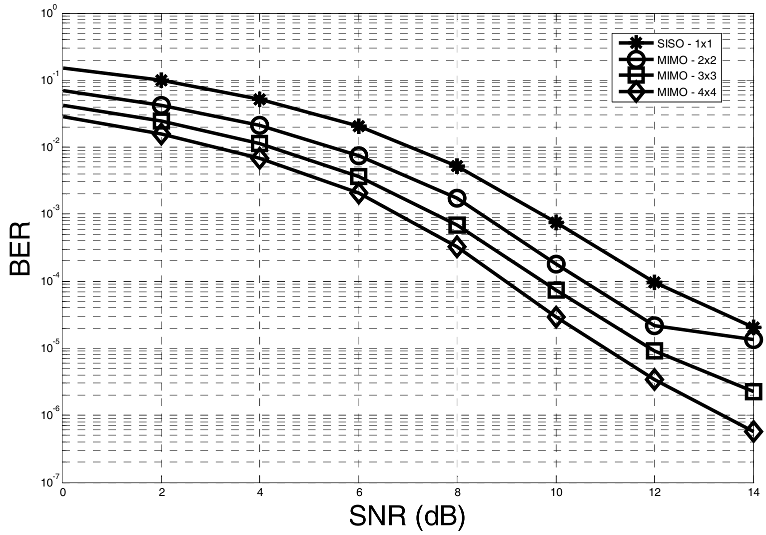
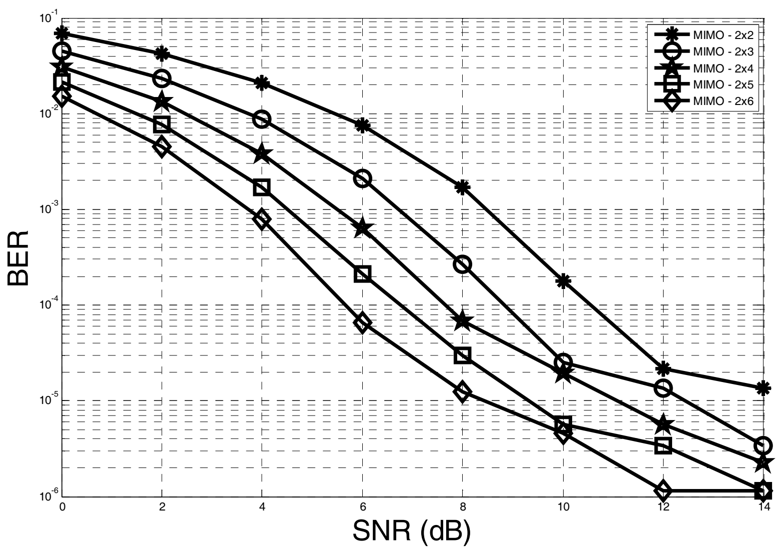


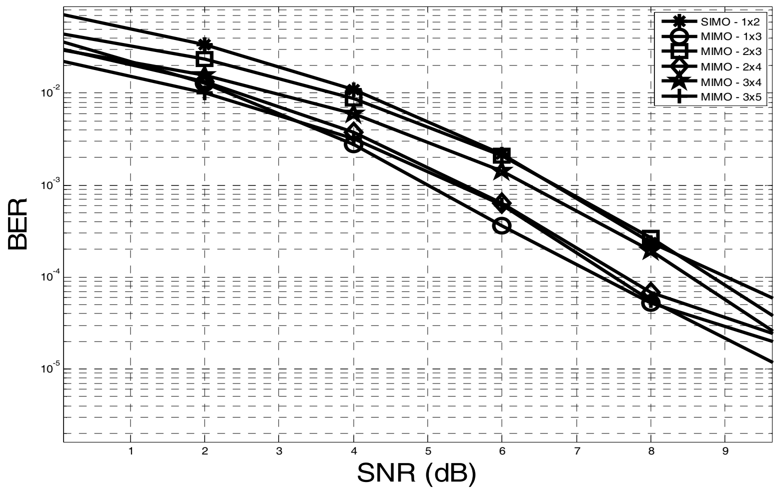

Fig. 7. Performance of the proposed WP-MCM MIMO based CR system. a) CR under LU; b) LU under CR. The carriers of CR in the region of the LU are removed to promote co-existence of CR and LU. The carriers were derived from Frequency selective filters with $L$=30, $K$=12 and $B$=0.1. The CR is 2x2 MIMO.


BER
SNR (dB)
SISO - 1x1
MIMO - 2x2
MIMO - 3x3
MIMO - 4x4

(a)


BER
SNR (dB)
MIMO - 2x2
MIMO - 2x3
MIMO - 2x4
MIMO - 2x5
MIMO - 2x6

(b)




(c)

Fig. 8. Performance of WPMCM MIMO based CR system for various combinations of transmit-receive antennas. a) Balanced Case when the number of transmit and receive antennas are the same; b) Effect of receiver configuration. c) A few other MIMO configurations; MIMO stands for Multiple input-multiple output, SISO for single input-single output and SIMO for single input-multiple output. The carriers were derived from Frequency selective filters with $L$=30, $K$=12 and $B$=0.1. The CR is 2x2 MIMO.

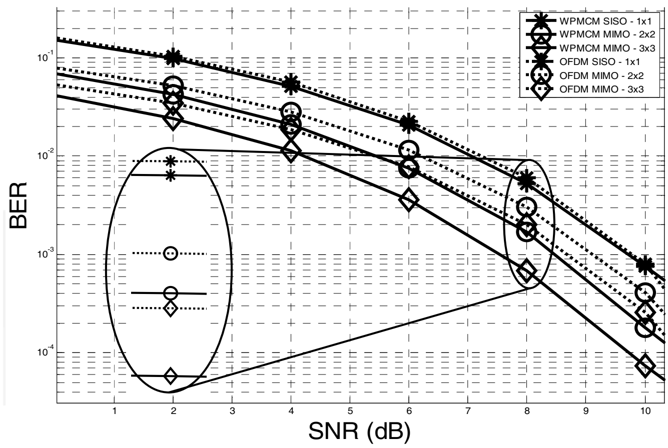



Fig. 9. OFDM versus WP-MCM based CR. BER performance comparison of OFDM and WP-MCM based CR systems in the presence of LU. MIMO stands for Multiple input-multiple output, SISO for single input-single output.

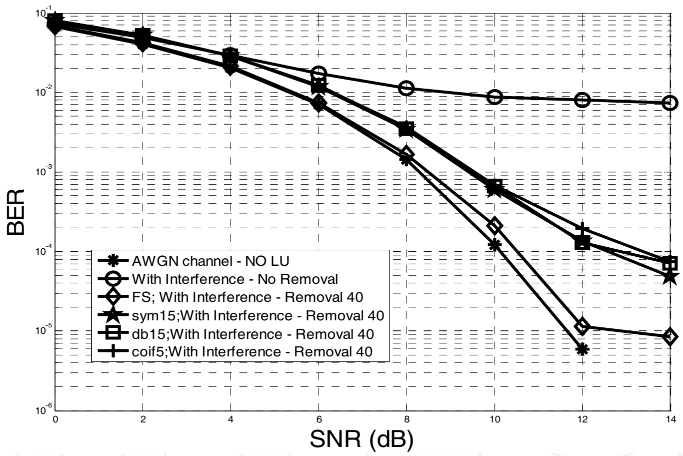



Fig. 10. Comparison of different wavelets: BER performance of the WPMCM MIMO based CR system in the presence of LU. The carriers of CR in the region of the LU are removed to enable it to co-exist with LU. FS denotes the maximally frequency selective wavelet, db denotes Daubechies, sym denotes symlet and coif stands for coiflet. The MIMO used is 2x2.

| Name | Orthonormal | Length | Regularity |
|---|---|---|---|
| Daubechies | Yes | 30 | 15 |
| Coiflet | Yes | 30 | 15 |
| Symlet | Yes | 30 | 15 |
| Frequency Selective | Yes | 30 | 12 |

Table 1. Filter Bank Characteristics

### 8.4 Performance comparison of the system when using the maximally frequency selective filter bank with respect to conventional wavelets of similar lengths.

Lastly we equate the performance improvements brought by maximally frequency selective filter bank with a few well known wavelets. The wavelets considered are Daubechies-15, Coiflet-5 and Symlet-15. All of these filters satisfy the paraunitary condition and hence give orthonormal bases. More on these wavelets can be found in table 1. For the fairness of comparison, all the filter banks are taken to be of the same length ($L$=30). The system performance has a direct correlation to the frequency selectivity of the filter bank used to derive the carriers. This effect is exemplified in Figure 10 where the BER performance curves of the CR under LU systems have been plotted. The MIMO used is 2x2.

The maximally frequency selective filter banks have the narrowest transition band and hence they easily surpass the other wavelet families (by up to 3dB). This essentially translates to less carrier removals and better SNR gain.

## 9. Summary

A novel wavelet packet based multi-carrier modulation system that attempts to blend the benefits of Cognitive Radio, and MIMO was presented. Cohabitation of the WP-MCM based CR system with LU was made possible by dynamically activating/deactivating the CR carriers in a way that the CR and LU systems don't have any competing time-frequency components. The carriers of the WP-MCM system were generated by multistage tree-structured paraunitary filter banks. Wavelets including the Daubechies, Coiflet and Symlet families were used for the study. The emphasis was on deriving optimal maximally frequency selective wavelet packet bases that best suit applicability to spectrum shaping in CR systems. Through simulation studies the usefulness and potential of WP-MCM for developing CR systems was demonstrated.

In addition to the CR and MIMO features, using the wavelet packet bases provided a third dimension of system optimization. Unlike conventional Fourier bases which are static in nature, wavelets offer multitude of variations which can be customised to the requirement in hand. In this article, the design criterion is set to avoid interference to licensed primary users. This can be easily furthered by altering the design criterion to include other requirements such as reduction of ISI/ICI or PAPR.

Comparing with traditional OFDM implementations, the performance of the proposed system in the presence of a LU matches quite well.

In conclusion, the performance results of the simulation studies make us to conclude that the novel proposed system can be fruitfully used to construct adaptive intelligent cognitive systems.

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**Cognitive Radio Systems**
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Cognitive radio is a hot research area for future wireless communications in the recent years. In order to increase the spectrum utilization, cognitive radio makes it possible for unlicensed users to access the spectrum unoccupied by licensed users. Cognitive radio let the equipments more intelligent to communicate with each other in a spectrum-aware manner and provide a new approach for the co-existence of multiple wireless systems. The goal of this book is to provide highlights of the current research topics in the field of cognitive radio systems. The book consists of 17 chapters, addressing various problems in cognitive radio systems.