Eigen value analysis of 2D structure with reduced degree of freedom system

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4/12/2020

Pledge

I pledge on my honour that I have not received answer sheets and codes from any of my course mates for this project. I also pledge on my honour that I have not sent my answer sheets and codes to any of my course mates for this project. In addition, I have not indulged in any other unfair means while doing this project.

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Abstract

The presented work is based on the characteristic constraint modes for the component mode synthesis[2] where the size of the model generated by Craig Bampton method[1] is reduced further using the constraint modes. Component mode synthesis (CMS) techniques are widely used for dynamic analyses of complex structures. Significant computational savings can be achieved by using CMS, since a modal analysis is performed on each component structure (substructure). The Craig–Bampton (CB) method of CMS first reduced the size of the model and then the eigenvalue analysis is performed on partition of reduced mass and stiffness matrices that correspond to the constraint mode. When the characteristic constraint modes are truncated, a CMS model with a highly reduced number of degrees of freedom (dof) is obtained.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

At the time of modeling the dynamics of a complex structure, it is often impractical to perform a finite element analysis of the entire structure. In some cases, the finite element model (FEM) of the full structure has so many degrees of freedom (dof) that a global finite element analysis is impossible due to computer memory constraints or node limits in the software. So component structures to be designed or redesigned independently,which makes it more convenient to perform a separate finite element analysis for each component.

Component mode synthesis(CMS) was developed for modeling and analyzing the dynamics of the global structure in such cases. In CMS, the dynamics of a structure are described by selected sets of normal modes of the individual component structures and a set of static vectors that account for the coupling at each interface where component structures are connected. Thus, each component structure is dynamically reduced by a separate modal analysis before being coupled at the system level, yielding savings in finite element costs as well as providing significant order reduction for the CMS model relative to the FEM of the complex structure.

The work presented in this report is for reducing further the size of a CMS model by performing an eigen value analysis on the constraint mode partitions of the reduced mass and stiffness matrices. The resultant eigen vectors are called the characteristic constraint modes.

These modes are truncated to yield a highly reduced order model.

1.1 Objectives

- Performing the component mode synthesis using the Craig-Bampton method.
- Obtaining the assembled mass and stiffness matrices of the substructures with reduction in the size of the model.
- Performing the eigen value analysis on partition of reduced mass and stiffness matrices that correspond to the constraint mode.
- Selected set of eigen vectors are used for transforming the mass and stiffness matrices of CMS model to yield the Reduced-Order-Model(ROM).
- Comparing the selected set of lowest natural frequencies with the CB-Method and full length model.

1.2 Outlines of Report

The work presented in this report is distributed in five chapters, In which chapter 1 briefly discusses the problem and it's background. chapter 2 discusses the Methodology used in the problem solving. Then the next chapter represents the results that is obtained by applying those methodologies and in the subsequent Chapter, those results are analyzed and interpreted and conclusions are drawn based on those interpretation.

Chapter 2

Methodology

The problem of 2D structure is represented in figure2.1. Solution of the problem is provided in following steps:

2.1 Craig-Bampton method For CMS

This method is based on the assumption that the dynamic behaviour of the subsystem can be described in terms of two types of information:[1]

- Static constraint modes resulting by applying unit displacements on the boundary degrees of freedom and
- Internal vibration modes found by fixing the boundary degrees of freedom. it means that

$$\mathbf{u} = \mathbf{u_{static}} + \mathbf{u_{dynamic}}$$

Considering the substructures, the undamped equation of motion with partitioning of the substructure is as follows:

$$\mathbf{M\ddot{u}} + \mathbf{Ku} = \mathbf{g} \tag{2.1}$$

$$\begin{bmatrix} \mathbf{M_{ii}} & \mathbf{M_{ib}} \\ \mathbf{M_{bi}} & \mathbf{M_{bb}} \end{bmatrix} \begin{bmatrix} \mathbf{\ddot{u}_i} \\ \mathbf{\ddot{u}_b} \end{bmatrix} + \begin{bmatrix} \mathbf{K_{ii}} & \mathbf{K_{ib}} \\ \mathbf{K_{bi}} & \mathbf{K_{bb}} \end{bmatrix} \begin{bmatrix} \mathbf{u_i} \\ \mathbf{u_b} \end{bmatrix} = \begin{bmatrix} \mathbf{o} \\ \mathbf{g_b} \end{bmatrix} \tag{2.2}$$

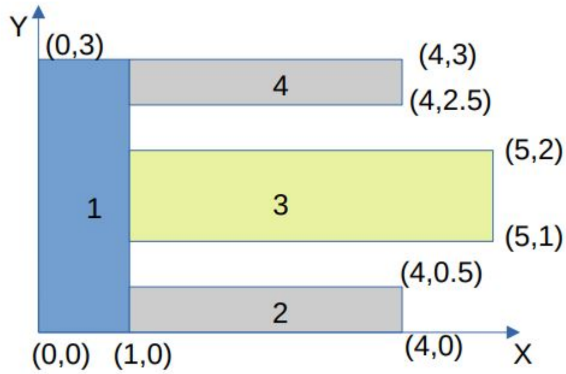

Figure 2.1: Full length structure

Here i denote the internal dof's and b denote the boundary interface dof's. It is assumed that there are no external forces acting on the internal dof's.

- Statically condensation of the internal dof's to the boundary dof's by ignoring the dynamic effects as follows:
- From equation 2.2 we have,

$$\mathbf{M_{ii}\ddot{u}_i + M_{ib}\ddot{u}_b + K_{ii}u_i + K_{ib}u_b = 0} \tag{2.3}$$

- neglecting the inertia terms we get:

$$\mathbf{K_{ii}u_i + K_{ib}u_b = 0}$$
$$\mathbf{u_i} = -\mathbf{k_{ii}^{-1}K_{ib}u_b}$$
$$\mathbf{u_i} = \Psi_C \mathbf{u_b}$$
$$Where \quad \mathbf{\Psi_C} = -\mathbf{k_{ii}^{-1}K_{ib}}$$

Here $\mathbf{\Psi_C}$ is static Condensation Matrix

- Now the static part of the response is as follows:

$$\mathbf{u_{static}} = \begin{bmatrix} \mathbf{u_i} \\ \mathbf{u_b} \end{bmatrix} = \begin{bmatrix} \mathbf{\Psi_C} \\ \mathbf{I} \end{bmatrix} \mathbf{u_b} \tag{2.4}$$

To account for the dynamic response, the static modes are augmented by the fixed interface vibration modes.

- The fixed interface modes are obtained by setting $\mathbf{u_b} = 0$ in Equation 2.2. Then we obtain:

$$\mathbf{M_{ii}\ddot{u}_i + K_{ii}u_i = 0}$$

Solving the above eigenvalue problem to get the mass normalized eigenvector ϕ_i,$i = 1, 2...n'$ where where n' is much smaller than the internal dof's The displacement of the internal dof's is then represented as

$$\mathbf{u_i} = \sum_{i=1}^{n'} \phi_i \eta_i = \mathbf{\Phi}\eta$$

Where $\mathbf{\Phi} = [\phi_1, \phi_2...\phi_{n'}]$ So, In this way the Dynamic part of the Response is as follows:

$$\mathbf{u_{Dynamic}} = \begin{bmatrix} \mathbf{u_i} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{0} \end{bmatrix} \eta \tag{2.5}$$

Using the above equations 2.4 and 2.5, the total response of the system is written as:

$$\mathbf{u} = \begin{bmatrix} \mathbf{u_i} \\ \mathbf{u_b} \end{bmatrix} = \mathbf{u_{static}} + \mathbf{u_{dynamic}} = \begin{bmatrix} \mathbf{\Psi_C} \\ \mathbf{I} \end{bmatrix} \mathbf{u_b} + \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{0} \end{bmatrix} \eta$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{\Phi} & \mathbf{\Psi_C} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \eta \\ \mathbf{u_b} \end{bmatrix}$$

$$\mathbf{u} = \mathbf{R_{CB}q} \tag{2.6}$$

where

$$\mathbf{R_{CB}} = \begin{bmatrix} \mathbf{\Phi} & \mathbf{\Psi_C} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

and

$$\mathbf{q} = \begin{bmatrix} \eta & \mathbf{u_b} \end{bmatrix}^T$$

Substituting $\mathbf{u} = \mathbf{R_{CB}q}$ in equation 2.2 and pre-multiplying by $\mathbf{R_{CB}}^\mathbf{T}$ We obtain

$$\mathbf{\hat{M}\ddot{q} + \hat{K}q = \hat{g}} \tag{2.7}$$

Following the above procedure, we can get the reduced equation for the substructure 1, 2, 3 & 4. The equation 2.7 can be written in partitioned form as follows:

$$\begin{bmatrix} \hat{\mathbf{M}}_{\eta\eta} & \hat{\mathbf{M}}_{\eta\mathbf{b}} \\ \hat{\mathbf{M}}_{\mathbf{b}\eta} & \hat{\mathbf{M}}_{\mathbf{bb}} \end{bmatrix} \begin{bmatrix} \ddot{\eta} \\ \ddot{\mathbf{u}}_{\mathbf{b}} \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{K}}_{\eta\eta} & \hat{\mathbf{K}}_{\eta\mathbf{b}} \\ \hat{\mathbf{K}}_{\mathbf{b}\eta} & \hat{\mathbf{K}}_{\mathbf{bb}} \end{bmatrix} \begin{bmatrix} \eta \\ \mathbf{u}_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{o} \\ \mathbf{g}_{\mathbf{b}} \end{bmatrix} \tag{2.8}$$

Here,

$$\begin{aligned} \hat{\mathbf{M}}_{\eta\eta} &= \mathbf{\Phi}^{\mathbf{T}}\mathbf{M}_{\mathbf{ii}}\mathbf{\Phi} = \mathbf{I} \\ \hat{\mathbf{M}}_{\eta\mathbf{b}} &= \mathbf{\Phi}^{\mathbf{T}}\mathbf{M}_{\mathbf{ii}}\mathbf{\Psi} + \mathbf{\Phi}^{\mathbf{T}}\mathbf{M}_{\mathbf{ib}} \\ \hat{\mathbf{M}}_{\mathbf{b}\eta} &= \hat{\mathbf{M}}_{\eta\mathbf{b}}^{\mathbf{T}} \\ \hat{\mathbf{M}}_{\mathbf{bb}} &= \mathbf{\Psi}^{\mathbf{T}}\mathbf{M}_{\mathbf{ii}}\mathbf{\Psi} + \mathbf{\Psi}^{\mathbf{T}}\mathbf{M}_{\mathbf{ib}} + \mathbf{M}_{\mathbf{bi}}\mathbf{\Psi} + \mathbf{M}_{\mathbf{bb}} \end{aligned}$$

Similarly,

$$\begin{aligned} \hat{\mathbf{K}}_{\eta\eta} &= \mathbf{\Phi}^{\mathbf{T}}\mathbf{K}_{\mathbf{ii}}\mathbf{\Phi} = \wedge \\ \hat{\mathbf{K}}_{\eta\mathbf{b}} &= \mathbf{\Phi}^{\mathbf{T}}\mathbf{K}_{\mathbf{ii}}\mathbf{\Psi} + \mathbf{\Phi}^{\mathbf{T}}\mathbf{K}_{\mathbf{ib}} = \mathbf{0} \\ \hat{\mathbf{K}}_{\mathbf{b}\eta} &= \hat{\mathbf{K}}_{\eta\mathbf{b}}^{\mathbf{T}} = \mathbf{0}^{\mathbf{T}} \\ \hat{\mathbf{K}}_{\mathbf{bb}} &= \mathbf{\Psi}^{\mathbf{T}}\mathbf{K}_{\mathbf{ib}} + \mathbf{K}_{\mathbf{bb}} \end{aligned}$$

A similar set of reduced equations are also derived for all substructures as follows:

$$\begin{bmatrix} \mathbf{I} & \hat{\mathbf{M}}_{\eta\mathbf{b}} \\ \hat{\mathbf{M}}_{\mathbf{b}\eta} & \hat{\mathbf{M}}_{\mathbf{bb}} \end{bmatrix} \begin{bmatrix} \ddot{\eta} \\ \ddot{\mathbf{u}}_{\mathbf{b}} \end{bmatrix} + \begin{bmatrix} \wedge & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{K}}_{\mathbf{bb}} \end{bmatrix} \begin{bmatrix} \eta \\ \mathbf{u}_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{g}_{\mathbf{b}} \end{bmatrix} \tag{2.9}$$

After getting the reduced matrices for all substructures with reference to equation 2.9, We assemble the substructure 2, 3 & 4 in the following manner where,

$$\hat{\mathbf{M}}^{\beta} = \begin{bmatrix} \mathbf{I^2} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{2}}_{\eta\mathbf{b}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I^3} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{3}}_{\eta\mathbf{b}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I^4} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{4}}_{\eta\mathbf{b}} \\ \hat{\mathbf{M}}^{\mathbf{2}}_{\mathbf{b}\eta} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{2}}_{\mathbf{bb}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{3}}_{\mathbf{b}\eta} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{3}}_{\mathbf{bb}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{4}}_{\mathbf{b}\eta} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}^{\mathbf{4}}_{\mathbf{bb}} \end{bmatrix}$$

$$\hat{\mathbf{K}^{\beta}} = \begin{bmatrix} \wedge^{\mathbf{2}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \wedge^{\mathbf{3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \wedge^{\mathbf{4}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}^{\mathbf{2}}_{\mathbf{bb}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}^{\mathbf{3}}_{\mathbf{bb}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}^{\mathbf{4}}_{\mathbf{bb}}} \end{bmatrix}$$

And

$$\mathbf{q}^{\beta} = \begin{bmatrix} \eta^{\mathbf{2}} & \eta^{\mathbf{3}} & \eta^{\mathbf{4}} & \mathbf{u}^{\mathbf{2}}_{\mathbf{b}} & \mathbf{u}^{\mathbf{3}}_{\mathbf{b}} & \mathbf{u}^{\mathbf{4}}_{\mathbf{b}} \end{bmatrix}^{\mathbf{T}}$$

Now we will try to assemble the substructure 1 with the assembled substructures in β where α represents the substructure 1 and β represents the combination of substructure 2, 3 and 4. So on assembly we get;

$$\begin{bmatrix} \mathbf{I}^{\alpha} & \hat{\mathbf{M}_{\eta\mathbf{b}}}^{\alpha} & \mathbf{0} & \mathbf{0} \\ \hat{\mathbf{M}_{\mathbf{b}\eta}}^{\alpha} & \hat{\mathbf{M}_{\mathbf{bb}}}^{\alpha} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}^{\beta} & \hat{\mathbf{M}_{\eta\mathbf{b}}}^{\beta} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}_{\mathbf{b}\eta}}^{\beta} & \hat{\mathbf{M}_{\mathbf{bb}}}^{\beta} \end{bmatrix} \begin{bmatrix} \ddot{\eta}^{\alpha} \\ \ddot{\mathbf{u}_{\mathbf{b}}}^{\alpha} \\ \ddot{\eta}^{\beta} \\ \ddot{\mathbf{u}_{\mathbf{b}}}^{\beta} \end{bmatrix} + \begin{bmatrix} \wedge^{\alpha} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{K}_{\mathbf{bb}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \wedge^{\beta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}_{\mathbf{bb}}} \end{bmatrix} \begin{bmatrix} \eta^{\alpha} \\ \mathbf{u}^{\alpha}_{\mathbf{b}} \\ \eta^{\beta} \\ \mathbf{u}^{\beta}_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}^{\alpha} \\ \mathbf{g}^{\alpha}_{\mathbf{b}} \\ \mathbf{0}^{\beta} \\ \mathbf{g}^{\beta}_{\mathbf{b}} \end{bmatrix}$$

Or we can say

$$\mathbf{M}^{\alpha\beta}\ddot{\mathbf{q}^{\alpha\beta}} + \mathbf{K}^{\alpha\beta}\mathbf{q}^{\alpha\beta} = \mathbf{g}^{\alpha\beta} \tag{2.10}$$

We now impose the continuity of displacements across the interface, where $\mathbf{u}_{\alpha} = \mathbf{u}_{\beta} = \mathbf{u}_{\mathbf{b}}$ So let

$$\begin{bmatrix} \eta^{\alpha} \\ \mathbf{u}^{\alpha}_{\mathbf{b}} \\ \eta^{\beta} \\ \mathbf{u}^{\beta}_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \eta^{\alpha} \\ \eta^{\beta} \\ \mathbf{u}_{\mathbf{b}} \end{bmatrix}$$

$$\mathbf{q}^{\alpha\beta} = \mathbf{T}\hat{\mathbf{q}^{\alpha\beta}} \tag{2.11}$$

Substituting equation 2.11 in equation 2.10 and pre-multiplying by $\mathbf{T}^{\mathbf{T}}$ we get:

$$\hat{\mathbf{M}^{\alpha\beta}}\ddot{\mathbf{q}^{\alpha\beta}} + \hat{\mathbf{K}^{\alpha\beta}}\mathbf{q}^{\alpha\beta} = \mathbf{0} \tag{2.12}$$

Where

$$\hat{\mathbf{M}}^{\alpha\beta} = \begin{bmatrix} \mathbf{I}^{\alpha} & \mathbf{0} & \hat{\mathbf{M}}_{\eta\mathbf{b}}^{\alpha} \\ \mathbf{0} & \mathbf{I}^{\beta} & \hat{\mathbf{M}}_{\eta\mathbf{b}}^{\beta} \\ \hat{\mathbf{M}}_{\mathbf{b}\eta}^{\alpha} & \hat{\mathbf{M}}_{\mathbf{b}\eta}^{\beta} & \hat{\mathbf{M}}_{\mathbf{bb}}^{\alpha} + \hat{\mathbf{M}}_{\mathbf{bb}}^{\beta} \end{bmatrix} \tag{2.13}$$

And

$$\hat{\mathbf{K}}^{\alpha\beta} = \begin{bmatrix} \wedge^{\alpha} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \wedge^{\beta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}}_{\mathbf{bb}}^{\alpha} + \hat{\mathbf{K}}_{\mathbf{bb}}^{\beta} \end{bmatrix} \tag{2.14}$$

Note that $\mathbf{T}^{\mathbf{T}}\mathbf{g}^{\alpha\beta} = \mathbf{0}$ where we have used the fact that $\mathbf{g}^{\alpha} + \mathbf{g}^{\beta} = \mathbf{0}$

2.2 Reduced order model using constraint modes

The number of necessary constraint modes may be reduced by seeking a new set of modes that correspond to more natural physical motion at the interface. This is posed as an eigenvalue problem for the constraint-mode partitions of the CMS matrices:[2]
Here lets take

$$\mathbf{K_C} = \hat{\mathbf{K}}_{\mathbf{bb}}^{\alpha} + \hat{\mathbf{K}}_{\mathbf{bb}}^{\beta}$$
$$\mathbf{M_C} = \hat{\mathbf{M}}_{\mathbf{bb}}^{\alpha} + \hat{\mathbf{M}}_{\mathbf{bb}}^{\beta}$$

Applying the eigen value analysis

$$\mathbf{K_C}\mathbf{\Phi}^{\mathbf{CC}} = \lambda\mathbf{M_C}\mathbf{\Phi}^{\mathbf{CC}} \tag{2.15}$$

The selected set of eigen vectors $\hat{\mathbf{\Phi}}^{\mathbf{CC}}$ from $\mathbf{\Phi}^{\mathbf{CC}}$ is used to transform the mass and stiffness matrices to yield a reduced-order model (ROM). The transformation from CMS generalized coordinates $\mathbf{q}^{\alpha\beta}$ to ROM generalized coordinates $\mathbf{q}^{\gamma}$ is defined as follows:

$$\mathbf{q}^{\alpha\beta} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{\Phi}}^{\mathbf{CC^T}} \end{bmatrix} \begin{bmatrix} \mathbf{q}^{\gamma} \end{bmatrix} \tag{2.16}$$

And

$$\mathbf{T_{ROM}^T} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\hat{\Phi}^{CC^T}} \end{bmatrix} \tag{2.17}$$

Substituting equation 2.16 in equation 2.12 and pre-multiplying by $\mathbf{T_{ROM}^T}$the mass and stiffness matrices in ROM generalized coordinates are, thus

$$\mathbf{M_{ROM}} = \begin{bmatrix} \mathbf{I}^\alpha & \mathbf{0} & \hat{\mathbf{M_{\eta b}}}^\alpha \mathbf{\hat{\Phi}^{CC}} \\ \mathbf{0} & \mathbf{I}^\beta & \hat{\mathbf{M_{\eta b}}}^\beta \mathbf{\hat{\Phi}^{CC}} \\ \mathbf{\hat{\Phi}^{CC^T}} \hat{\mathbf{M_{b\eta}}}^\alpha & \mathbf{\hat{\Phi}^{CC^T}} \hat{\mathbf{M_{b\eta}}}^\beta & \mathbf{\hat{\Phi}^{CC^T}} (\hat{\mathbf{M_{bb}}}^\alpha + \hat{\mathbf{M_{bb}}}^\beta) \mathbf{\hat{\Phi}^{CC}} \end{bmatrix} \tag{2.18}$$

And

$$\mathbf{K_{ROM}} = \begin{bmatrix} \wedge^\alpha & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \wedge^\beta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\hat{\Phi}^{CC^T}} (\hat{\mathbf{K_{bb}}}^\alpha + \hat{\mathbf{K_{bb}}}^\beta) \mathbf{\hat{\Phi}^{CC}} \end{bmatrix} \tag{2.19}$$

Now, compared to the mass and stiffness matrices of assembled substructures, the size of matrix partition has been reduced. The above methodology is implemented using the OCTAVE and lowest 10 natural frequencies are obtained by eigenvalue analysis of $\mathbf{M_{ROM}}$ and $\mathbf{K_{ROM}}$,thus

$$K_{\mathbf{ROM}} \phi_{\mathbf{ROM}} = \lambda_{ROM} M_{\mathbf{ROM}} \phi_{\mathbf{ROM}} \tag{2.20}$$

Where the λ_{ROM} is eigen frequencies of ROM model

Chapter 3

Results

After applying the above mentioned methodology the following results were obtained

Table 3.1: Lowest 10 natural frequencies after applying CB method with different set of selected modes

Craig Bampton Method					
ω^2	With 6 $\hat{\Phi}$	With 10 $\hat{\Phi}$	With 14 $\hat{\Phi}$	With 20 $\hat{\Phi}$	With 24 $\hat{\Phi}$
1	0.000277	0.000277	0.000277	0.000277	0.000277
2	0.0022477	0.0022477	0.002247	0.0022479	0.00224
3	163.47104	163.4709	163.47087	163.4708	163.4708
4	175.0530	175.04866	175.04747	175.04671	175.0466
5	180.59997	180.5963	180.59524	180.58909	180.5874
6	520.977	520.9536	520.9456	520.9401	520.9383
7	1586.7818	1586.2619	1586.1094	1585.4209	1585.195
8	1831.0909	1830.9600	1830.9520	1830.9393	1830.936
9	2006.6280	2006.5391	2006.4537	2006.4202	2006.414
10	4012.4785	4011.5521	4011.2624	4009.4046	4009.214

3.1 Craig-Bampton Method

The Craig-bampton method is applied for component mode synthesis with considering the different set of selected eigen modes $\hat{\boldsymbol{\Phi}}$ for the substructures, the reduced model is obtained with less degree of freedom. In the above table 3.1, the lowest 10 natural frequencies is extracted for every selected set of eigen modes.

3.2 Reduced order model(ROM) using the constraint modes

Table 3.2: Lowest 10 natural frequencies for ROM model with different set of selected modes

ROM model using Constraint Mode method					
ω^2	With 6 $\hat{\Phi}$	With 10 $\hat{\Phi}$	With 14 $\hat{\Phi}$	With 20 $\hat{\Phi}$	With 24 $\hat{\Phi}$
1	0.0002774	0.0002774	0.0002774	0.0002774	0.000277
2	0.0022479	0.0022479	0.0022479	0.0022479	0.002247
3	163.55696	163.47198	163.47140	163.47099	163.4708
4	175.07967	175.05001	175.04806	175.04684	175.0466
5	180.61829	180.60275	180.59588	180.58923	180.5874
6	521.22483	520.97205	520.95273	520.94204	520.9389
7	1640.4435	1617.9635	1586.4532	1585.5413	1585.206
8	2008.7885	1834.6886	1831.1257	1830.9725	1830.9521
9	2180.3496	2010.8227	2006.8656	2006.5195	2006.442
10	4060.4483	4038.8915	4014.5816	4010.2723	4009.358

After obtaining the CMS model by Craig-Bampton Method, The number of necessary constraint modes may be reduced by seeking a new set of modes that correspond to more natural physical motion at the interface. Now considering the different set of selected constraint mode $\hat{\mathbf{\Phi}}^{\mathbf{CC}}$, the Reduced-Order-Model is obtained as explained in the methodology section 2.2. The lowest 10 natural frequencies are extracted which are represented in table 3.2 for ROM model

3.3 Eigenvalue analysis on assembled substructures with full degree of freedom System

Table 3.3: Lowest 10 natural frequencies for full degree of freedom system

Full dof system	
$\omega^{\mathbf{2}}$	With 10 $\hat{\Phi}$
1	0
2	0
3	163.44
4	175.004
5	180.5865
6	520.7361
7	1585.154
8	1827.736
9	2004.940
10	4009.087

The eigen value analysis on full degree of freedom system is done and the the lowest 10 natural frequencies are obtained which are represented in the table 3.3

3.4 Error analysis

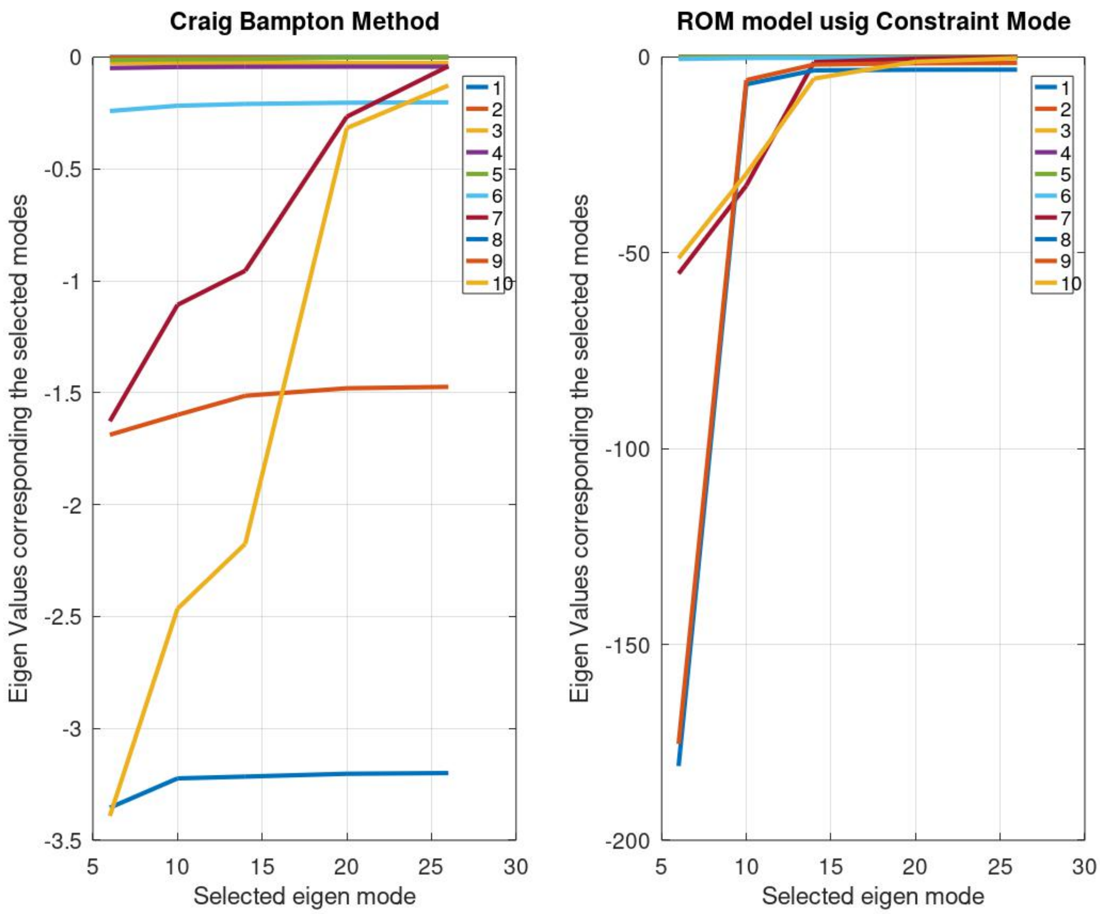

Figure 3.1: Eigen values error w.r.t full dof system

The error analysis is done between the eigen frequencies obtained through Craig-Bampton method and ROM reduced model using constraint modes w.r.t to full degree of freedom system. The eigen values Error analysis for the Craig-Bampton method and ROM model using constraint mode is shown in the Figure 3.1 where x-axis represents the different set of eigen modes and y-axis represents the error w.r.t full degree of freedom system for CB method and ROM model.

Chapter 4

Discussions

According to table 3.1, when the component mode synthesis using Craig-Bampton method is done for the different set of selected eigen modes, approximately consistent values of the 10 lowest natural frequencies is obtained for 24 selected set of eigen mode $\hat{\mathbf{\Phi}}$. Also analyzing the table 3.1, it clear that the lowest 6 natural frequencies are approximately consistent with 10 selected set of eigen mode. $\hat{\mathbf{\Phi}}$.

In the similar way, according to the table 3.2,when the reduced order model is obtained using constraint modes for the different set of selected eigen modes, approximately Consistent values of the 10 lowest natural frequencies is obtained for 24 Selected set of eigen mode $\hat{\mathbf{\Phi}}^{\mathbf{CC}}$. Here also it clear that the lowest 6 natural frequencies are approximately consistent with 10 selected set of eigen mode $\hat{\mathbf{\Phi}}^{\mathbf{CC}}$.

The figure 3.1 represented in section 3.4 clearly represents that as the selected set of eigen mode increases the error between the lowest 10 natural frequencies of the full dof system and lowest natural frequencies obtained through the CB and ROM method decreases.

When the results shown in table 3.1 and 3.2 is compared with the table 3.3, which represents the lowest 10 natural frequencies of the full degree of freedom system. The mean percentage error of $\mathbf{0.033421}$ is obtained between the selected set of 24 eigen mode $\hat{\mathbf{\Phi}}$ in CB method and full dof system for the lowest 10 natural frequencies and in the similar manner, the mean percentage error of $\mathbf{0.034097}$ is obtained between the selected set of 24 eigen mode $\hat{\mathbf{\Phi}}$ in Reduced-Order-Model using CC modes method and full dof system for the lowest 10 natural frequencies.

Chapter 5

Original Work

The described methodology in the chapter 2 is implemented using the OCTAVE software. In which the mass and stiffness matrices of the substructures is provided as the input. The output results are represented in the table 3.1,3.2 and 3.3 as the lowest 10 natural frequencies for CB, ROM and full degree of freedom system. As the comparison between the eigen frequencies obtained through Craig-Bampton Method and ROM reduced model using constraint modes w.r.t full degree of freedom system, the eigen values error analysis for the Craig-Bampton method and ROM model using constraint mode is also shown in the figure 3.1.

Chapter 6

Conclusions

Based on the results in chapter 3 and discussions on the results, it is calculated that the mean percentage error of $\mathbf{0.033421}$ is obtained between the selected set of 24 eigen mode $\hat{\boldsymbol{\Phi}}$ in CB method and full dof system for the lowest 10 natural frequencies and in the similar manner, the mean percentage error of $\mathbf{0.034097}$ is obtained between the selected set of 24 eigen mode $\hat{\boldsymbol{\Phi}}$ in Reduced-Order-Model using CC modes method and full dof system for the lowest 10 natural frequencies. So it can be concluded that with further reduction in the CMS model using constraint modes give the approximately good results with less degree of freedom.

Bibliography

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[2] Matthew P. Castanier, Yung-Chang Tan, and Christophe Pierre *Characteristic Constraint Modes for Component Mode Synthesis*. AIAA Journal, Vol. 39, No. 6, June 2001