For the n_k subjects belonging to cluster k we have (1)

$$\underset{n_k\times J}{\underset{\sim}{Y}} \sim \mathrm{MATSN}_{n_k, J}\left(\underset{n_k\times J}{M}, \underset{J\times 1}{\underset{\sim}{\alpha}_k}, I_{n_k}, \underset{J\times J}{\Sigma_k}\right)$$

where $n_k = \sum_{i=1}^{n} (Z_i = k)$

$$\mathrm{Vec}(Y) = \underset{n_k J\times 1}{\underset{\sim}{y}} = (\underset{J\times 1}{y_1^T}, \dots \underset{J\times 1}{y_{n_k}^T})^T$$

$$\mathrm{Vec}(M) = \underset{nJ\times 1}{\underset{\sim}{\mu}} = (\underset{J\times 1}{\mu_1^T}, \dots \underset{J\times 1}{\mu_{n_k}^T})^T$$

with $\underset{J\times 1}{\mu_i} = X_i \beta_k$

and $\underset{J\times 1}{\underset{\sim}{\alpha}_k} = (\alpha_{k1}, \dots \alpha_{kJ})^T$

This implies that the i^{th} row of Y is

$$\underset{J\times 1}{y_i} = \mathrm{MSN}_J(\underset{J\times 1}{\mu_i}, \underset{J\times 1}{\alpha_k}, \underset{J\times J}{\Sigma_k}) = \mathrm{MSN}_J(X_i\beta_k, \alpha_k, \Sigma_k)$$

Now, if we condition on T_i as well as $Z_i = k$

$$Y_i \mid Z_i = k, T_i \sim N_J(X_i \beta_k + \underset{J\times 1}{\psi_k} T_i, \Sigma_k),$$

Implying that, combining all the responses for cluster k,

$$\underset{n_k \times J}{Y} \mid \underset{n_k \times 1}{T} = t \sim \mathrm{MatNorm}_{n_k, J}(M, I_n, \Sigma_k),$$

where, here,

$$\underset{n_k J \times 1}{\mathrm{vec}(M)} = \underset{n_k J \times p}{X} \, \underset{p \times 1}{\beta_k} + \underset{n_k \times 1}{t} \otimes \underset{J \times 1}{\psi_k} \quad \text{and}$$

$$t = (t_1, \dots, t_{n_k})^T$$