Reader Report: Pearl's *Causal Inference in Statistics*

Chapter 3: The Effects of Interventions

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1. Notes

Do-Operator & Graph Surgery. The chapter distinguishes *seeing* from *doing*: $P(Y \mid X = x)$ conditions on observation, while $P(Y \mid do(X = x))$ represents intervention. Intervention is formalized as **graph surgery**—deleting all arrows into X, yielding $G_{\overline{X}}$. The post-intervention distribution follows the truncated factorization: $P(Y \mid do(X = x)) = \sum_z \prod_{i:V_i \neq X} P(V_i \mid pa_i)\big|_{X=x}$.

Back-Door Criterion. A set Z satisfies the back-door criterion relative to (X, Y) if: (i) no node in Z is a descendant of X, and (ii) Z blocks every back-door path from X to Y. When satisfied, the **adjustment formula** applies: $P(Y \mid do(X = x)) = \sum_z P(Y \mid X = x, Z = z) \cdot P(Z = z)$. This simulates randomization by averaging over confounder strata.

Front-Door Criterion. When confounders are unobserved, identification may proceed through mediators. A set M satisfies the front-door criterion if it intercepts all directed $X \to Y$ paths, has no unblocked back-door from X, and all $M \to Y$ back-doors are blocked by X: $P(Y \mid do(X = x)) = \sum_m P(M = m \mid X = x) \sum_{x'} P(Y \mid M = m, X = x')P(X = x')$.

IPW Connection. The adjustment formula rewrites as inverse probability weighting: $E[Y \mid do(X = x)] = E\left[Y \cdot \mathbb{1}(X = x)/P(X = x \mid Z)\right]$, linking to propensity score methods.

2. Questions

1. **Soft interventions:** How does graph surgery extend to stochastic policies that don't fully sever incoming edges?
2. **Adjustment set selection:** With multiple valid back-door sets, what are variance-efficiency tradeoffs? Can over-adjustment introduce bias?
3. **Front-door rarity:** Why is the front-door criterion rarely applicable? What makes complete mediating pathways uncommon?
4. **Positivity:** Adjustment requires $P(X \mid Z) > 0$ for all strata. How do violations manifest and what remedies exist?
5. **Sequential treatments:** How do these results generalize to time-varying interventions and the g-formula?

3. Summary

Chapter 3 operationalizes Pearl's framework: *How do we compute causal effects from observational data?* The **do-operator** formalizes intervention as graph surgery, separating $P(Y \mid X)$ (association) from $P(Y \mid do(X))$ (causation). The **back-door criterion** enables identification when confounders are observed; the **front-door criterion** provides an alternative through mediators when confounders are hidden.

The core insight: causal effects are *identifiable* when the graph has appropriate structure—the graph dictates both *whether* identification is possible and *how* to achieve it. This chapter grounds randomized experiments, observational study design, propensity scores, and mediation analysis—transitioning from "what is a causal model?" to "what can we compute with it?"