PROBLEM BACKGROUND

Multi-Server System Analysis

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Multi-Server System Analysis

We aim to demonstrate that for any large *t*, the number of busy servers converge to a constant:

$$\mathrm{E}[B(t)] \xrightarrow{t\to\infty} \frac{e^{-\mu/\lambda}}{1-e^{-\mu/\lambda}}$$

$B(\mathrm{t}) \overset{\Delta}{=}$ Busy servers at time *t*

$\lambda \overset{\Delta}{=}$ Request arrival rate

$\frac{1}{\mu} \overset{\Delta}{=}$ Mean service duration

Multi-Server System Analysis

Let,

$X_k \overset{\Delta}{=}$ Arrival time of request k

$S_k \overset{\Delta}{=}$ Service duration for request k

where,

$$X_k \sim f_{X_k}(x) = \delta(x - (k-1)\cdot \tfrac{1}{\lambda})$$

$$S_k \sim f_{S_k}(x) = \tfrac{1}{\lambda} e^{x/\lambda} \cdot \mathrm{u}(x)$$

$$B(t) \overset{\Delta}{=} \sum_{k=1}^{n} \mathbf{1}_{X_k \le t < X_k + S_k}$$

$$= \sum_{k=1}^{n} [u(t - X_k) - u(t - (X_k + S_k))]$$

Then if $Z_k = u(t - X_k) - u(t - (X_k + S_k))$,

$$B(t) \sim f_{B(t)}(x) = \mathop{\ast}_{k=1}^{n} f_{Z_k}(x)$$

$$\mathrm{E}[B(t)] = \sum_{k=1}^{n} \mathrm{E}[Z_k]$$

SIMULATION RESULTS

Section 1: Convergence till test T_β

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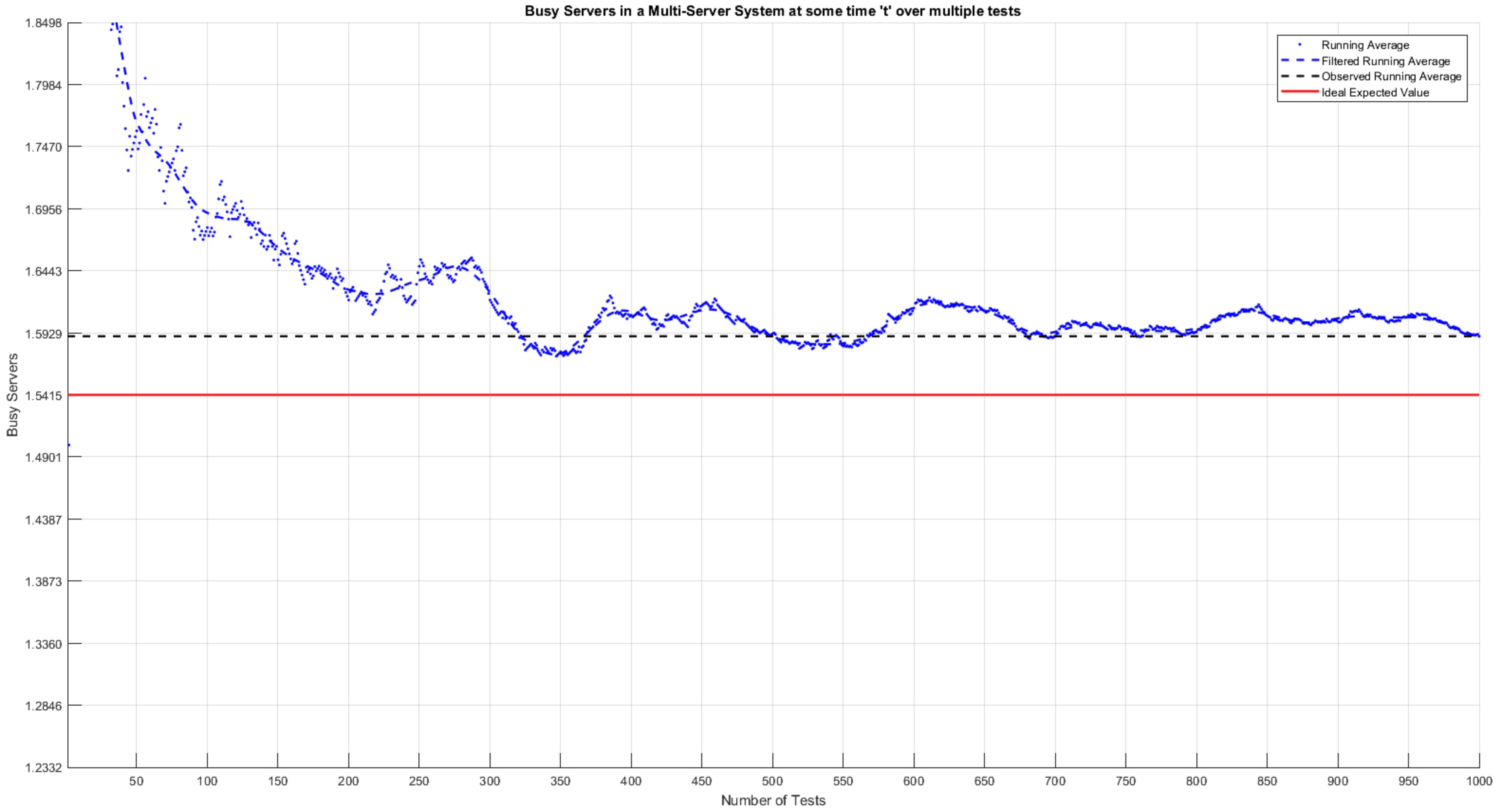
Question: "What does $B(t)$ converge to for some large t?"

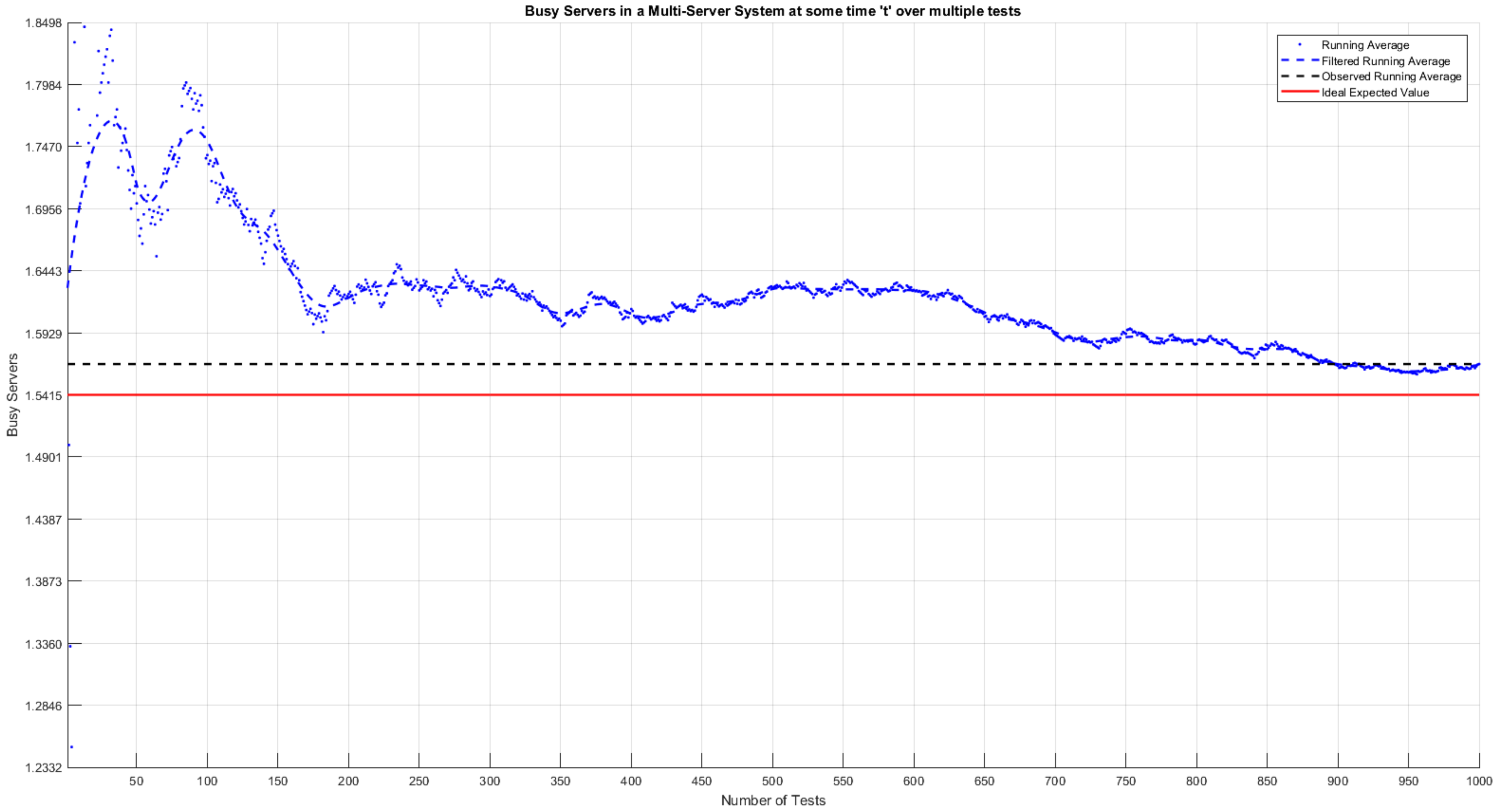
Solution:

- Conduct multiple tests.
- Sample value of $B(t)$ at some $t=k$
- Find mean of $B(k)$ over T_β tests.
- Terminate testing if $B(k) \in B(t \to \infty) \pm 0.1\%$

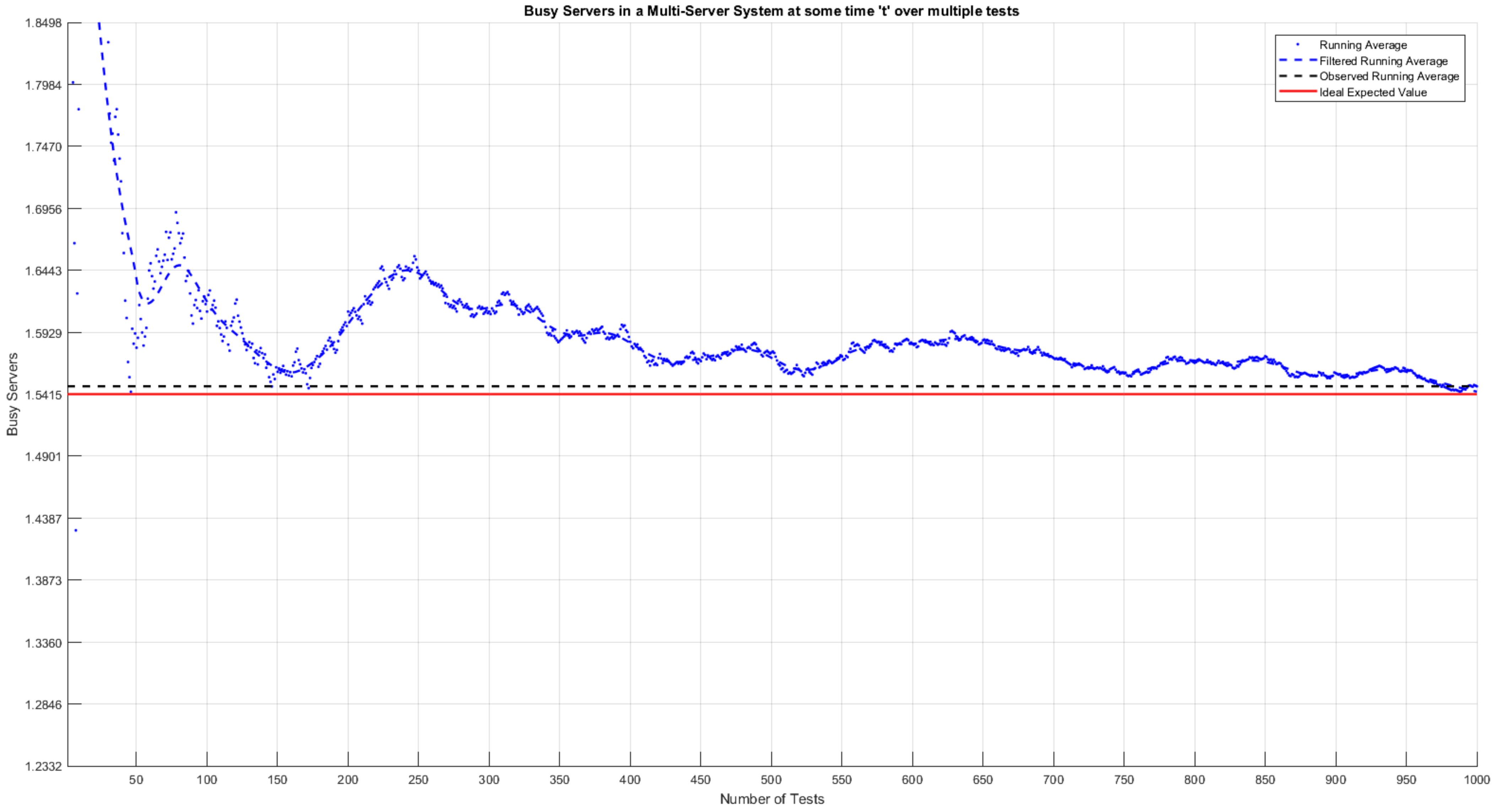
Section 1: Convergence till test T_β



Section 1: Convergence till test T_β



Section 1: Convergence till test T_β



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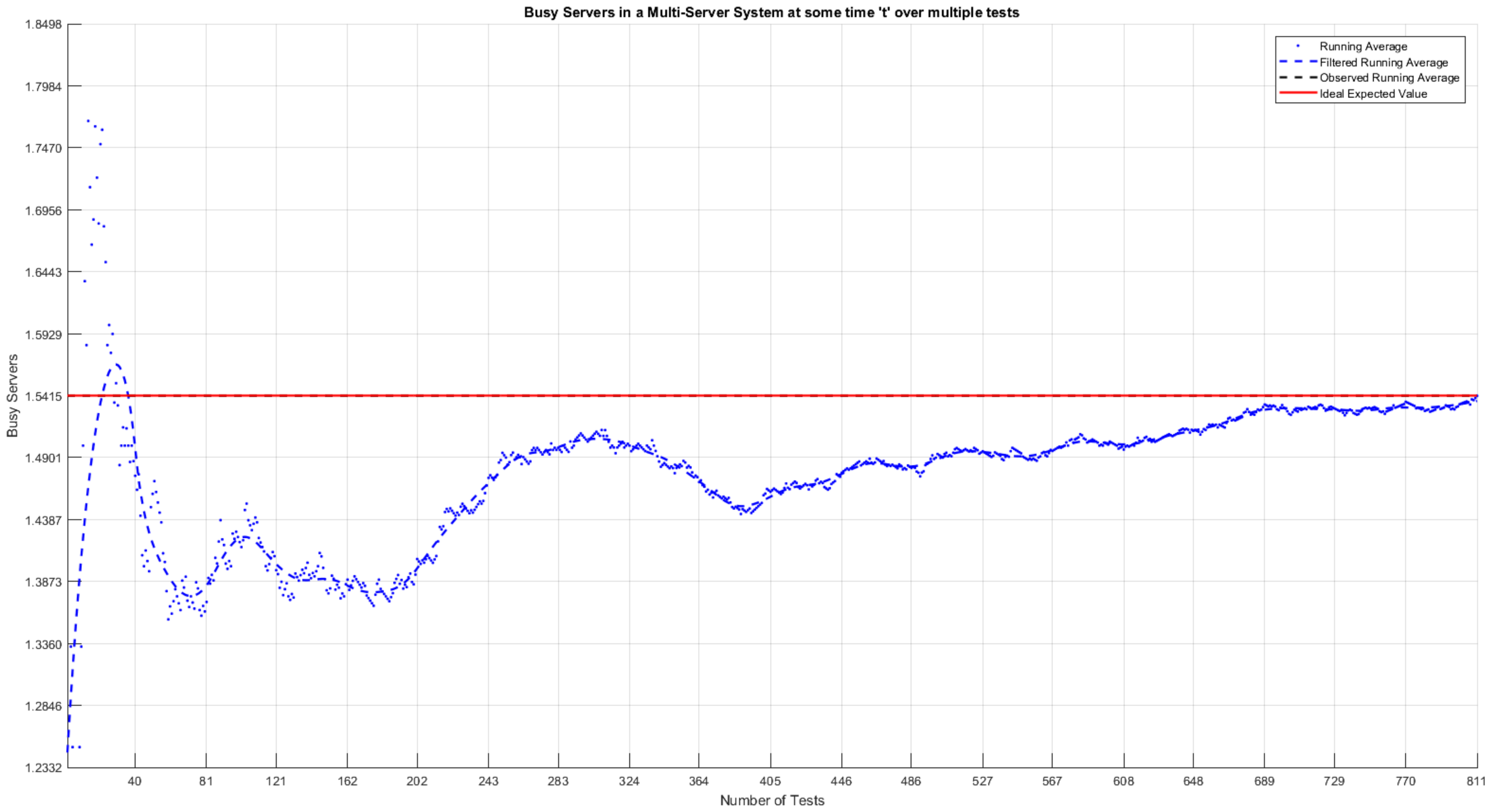

SIMULATION RESULTS

Section 2: Distribution of T_{β}

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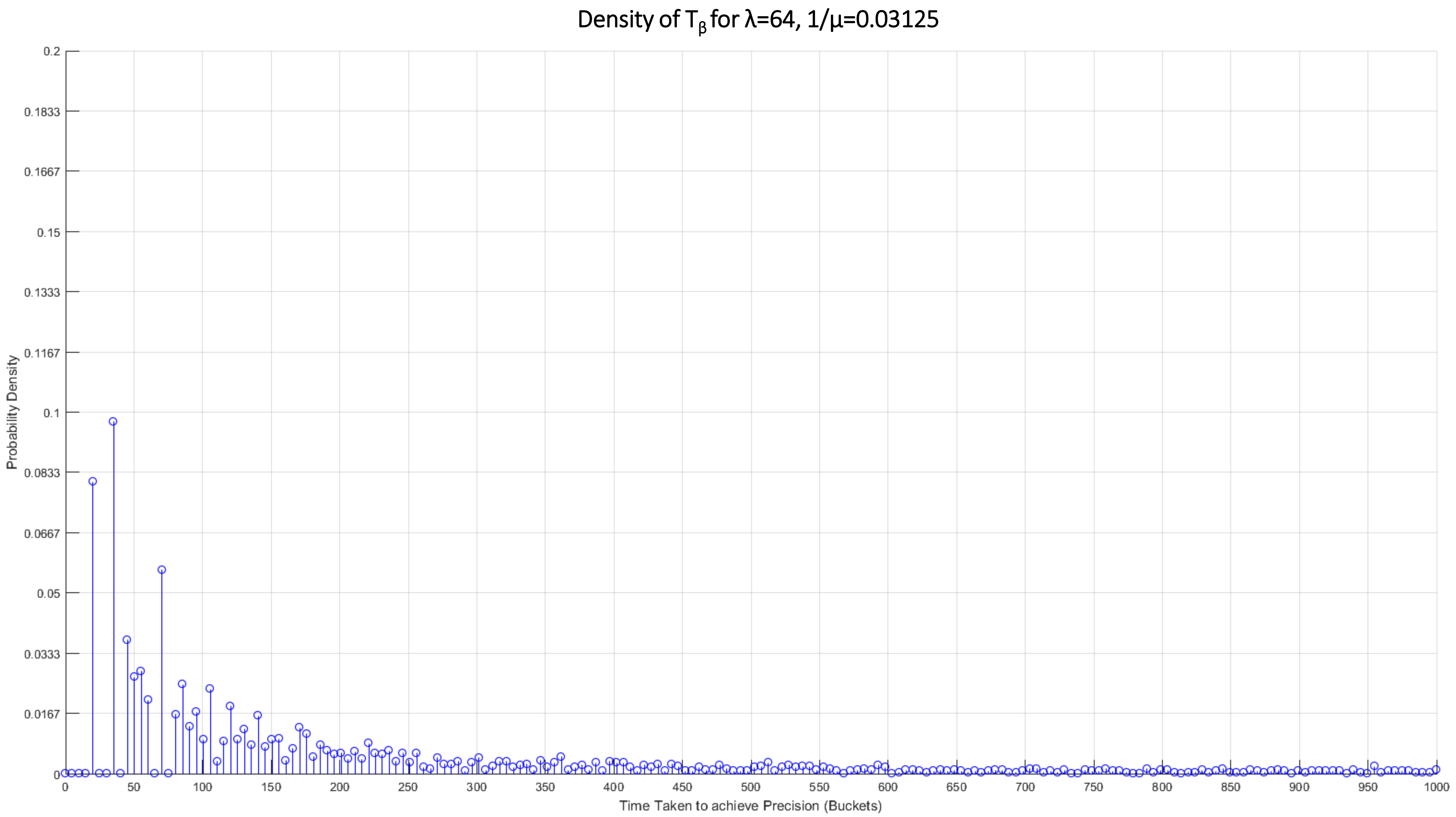


SIMULATION RESULTS

Section 3: Convergence till time t_β

Section 2: Distribution of T_β

Question: "What time guarantees that the system is stationary?"

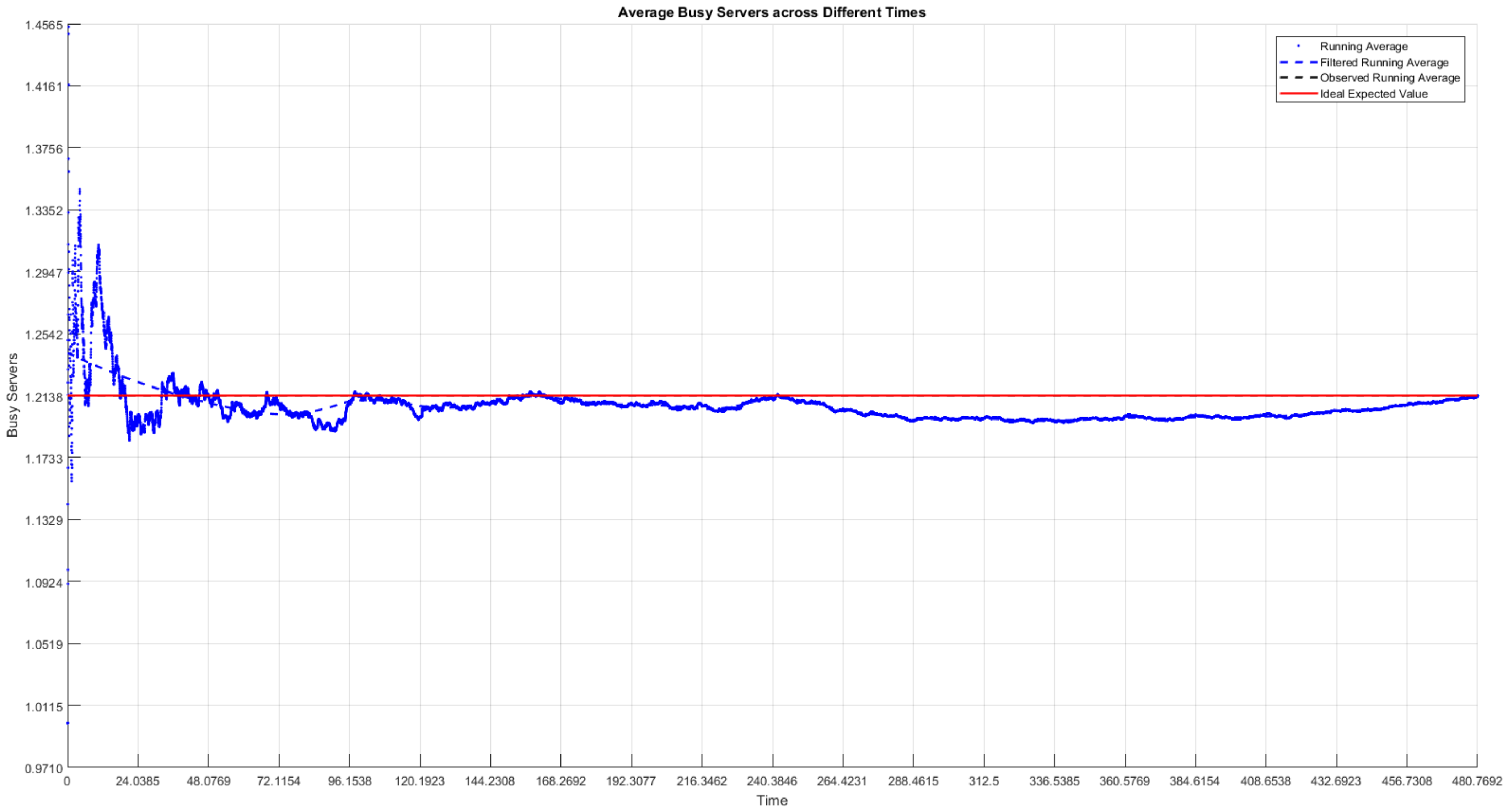
or

Question: "What does $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{B(k\lambda)}{n\lambda}$ converge to?"

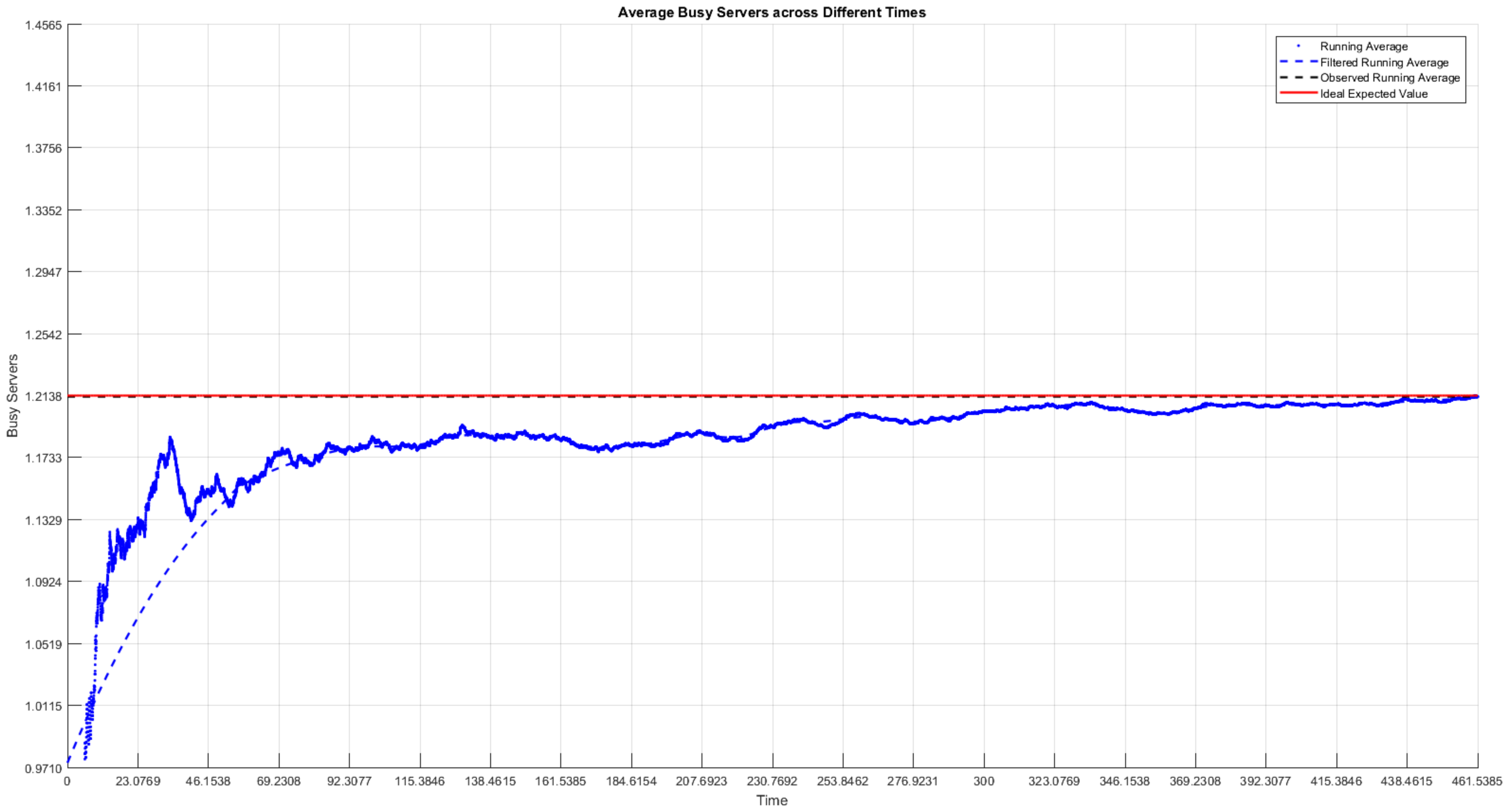
Solution:

- Conduct multiple tests.
- Sample value of $B(t)$ at some $t=k$
- Find mean of $B(k)$ over T_β tests.
- Terminate testing if $B(k) \in B(t \to \infty) \pm 0.1\%$

Section 3: Convergence till time t_β



Section 3: Convergence till time t_β



SIMULATION RESULTS

Section 4: Distribution of t_β

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