Signal Processing in Quantum Mechanics

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Presentation Overview

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Introduction

Signal Processing

Signal is a function that conveys information about the behaviour of a system.

- Common operations on signals:
  - Scaling, Addition, and Subtraction
  - Delays, Phase Changes
  - Filtering and Feedback

Introduction

Signal Processing

- **Control Blocks** often perform following operations:
  - Scaling
  - Delays
  - Filtering

Introduction

Signal Processing

- Signals often representable as sum of many sinusoids
- Signals commonly represented in **Frequency Domain**
- **Laplace Transform** commonly used for Frequency Domain form

$$\mathcal{L}\{f(t)\} \rightarrow \mathrm{F}(s)$$

Quantum Control Blocks

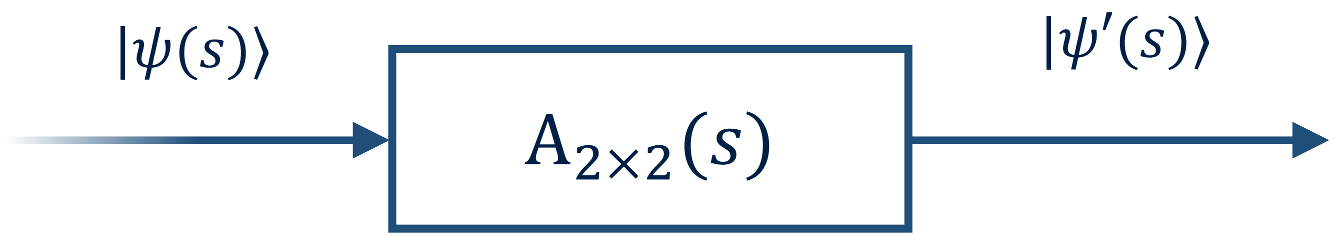
Developing Quantum Control Blocks

- We use Quantum Operators as **Quantum Control Blocks**
- **Quantum States** are input and output signals

Quantum Control Blocks

Mathematical Notation

- **Quantum Transfer Functions** represent Quantum Control Blocks
- Quantum Transfer Functions are operator matrices



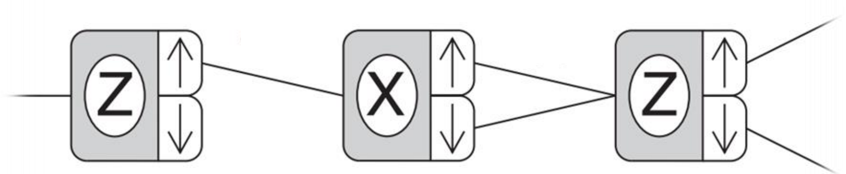
$$|\psi'(s)\rangle = \mathrm{A}(s)|\psi(s)\rangle$$

Quantum Control Blocks

Projection Blocks

- **Projection Blocks** represent act of observing the quantum state
- Observers are used to predictably alter quantum state

$$|\psi'(t)\rangle = \frac{\mathrm{P}_{\pm n}|\psi(t)\rangle}{\sqrt{\langle\psi(t)|\mathrm{P}_{\pm n}|\psi(t)\rangle}}$$



Quantum Control Blocks

Projection Blocks

- Can be used to prepare an input signal to a certain state
- *Example:* Given an arbitrary input, we want an output signal of state $|\psi'(t)\rangle = |-\rangle$

$$|\psi(t)\rangle = k_1|+\rangle + k_2|-\rangle \quad \rightarrow \quad |\psi'(t)\rangle = |-\rangle$$

$$|\psi(s)\rangle = \frac{k_1}{s}|+\rangle + \frac{k_2}{s}|-\rangle \quad \rightarrow \quad |\psi'(s)\rangle = \frac{k_2}{s}|-\rangle$$

$$|\psi'(s)\rangle = \underbrace{\boxed{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}}_{\mathbf{T}(s) = \mathbf{P}_{-z}}|\psi(s)\rangle$$

Quantum Control Blocks

Projection Blocks

- **Disadvantage:** Projection Blocks *do not* guarantee an output
- Projection Blocks behave similar to traditional filters

$$|\psi(t)\rangle = k_1|+\rangle + k_2|-\rangle \quad \rightarrow \quad |\psi'(t)\rangle = |-\rangle$$

$$|\psi(s)\rangle = \frac{k_1}{s}|+\rangle + \frac{k_2}{s}|-\rangle \quad \rightarrow \quad |\psi'(s)\rangle = \frac{k_2}{s}|-\rangle$$

$$|\psi'(s)\rangle = \underbrace{\boxed{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}}_{\mathbf{T}(\boldsymbol{s}) = \mathbf{P}_{-z}}|\psi(s)\rangle$$

Quantum Control Blocks

Projection Blocks

- **What if a projection block has no output?**
- We defined a **Zero Signal** to indicate absence of a quantum state

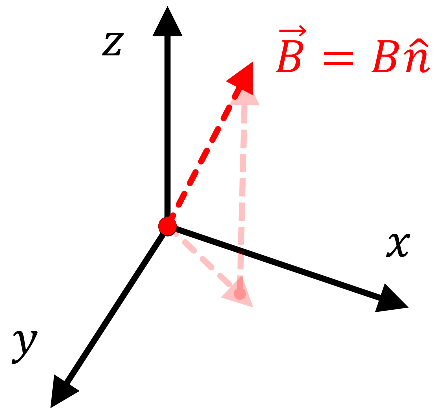
$$|\psi(s)\rangle = \frac{k_1}{s}|+\rangle \longrightarrow \boxed{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}} \longrightarrow |\psi'(s)\rangle = 0|+\rangle + 0|-\rangle$$

Quantum Control Blocks

Precession Blocks

- **Precession Blocks** represent time-evolution under influence of magnetic fields
- Magnetic fields of strength B in direction $\hat{n}$ applied for time T used to modify relative phase

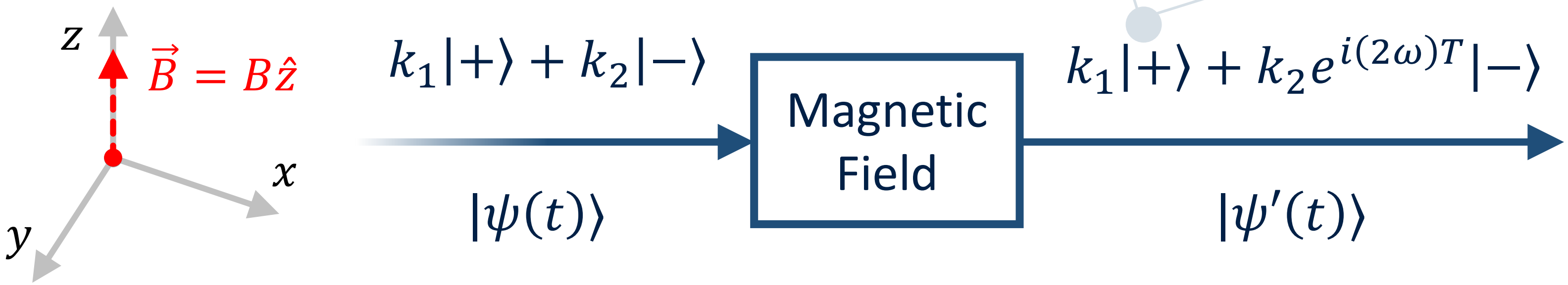
$$|\psi'(T)\rangle = k_1 e^{-i\omega T}|E_1\rangle + k_2 e^{i\omega T}|E_2\rangle$$



Quantum Control Blocks

Precession Blocks: Phase Changer

- **Phase Changer** is used to modify relative phase of an input state



$$|\psi'(s)\rangle = \overset{\mathbf{T}(\boldsymbol{s})}{\boxed{\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix}}} |\psi(s)\rangle \quad \text{when,} \quad T = \frac{\pi}{2\omega}$$

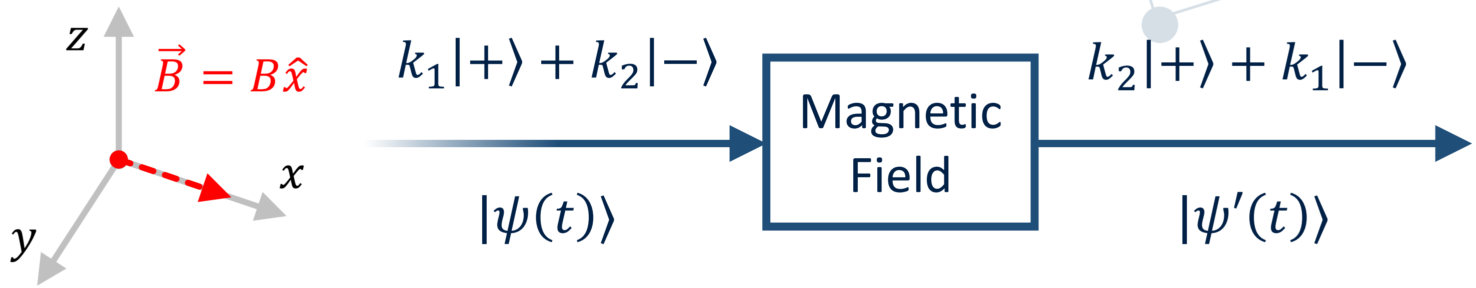
$$= k_1|+\rangle - k_2|-\rangle$$

- Since $\omega = \frac{Be^-}{m_{e^-}} \gg \pi$, the time $T \approx 0$ and delays can be ignored

Quantum Control Blocks

Precession Blocks: Amplitude Reversal

- **Amplitude Reversal** is used to implement quantum **NOT** gates



$$|\psi'(s)\rangle_x = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} |\psi(s)\rangle_x$$

$$|\psi'(s)\rangle = \underbrace{\boxed{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}}_{\mathbf{T}(\boldsymbol{s})} |\psi(s)\rangle$$

- Minuscule delays are ignored in favor of simplicity

Operations on Quantum Signals

Signal Filtering

- Filtering in traditional control systems refers to filtering frequency components

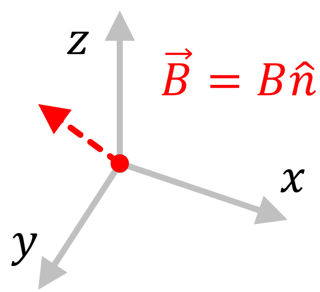
- Filtering in our context refers to removing super-position

$$|\psi(s)\rangle = \frac{k_1}{s}|+\rangle + \frac{k_2}{s}|-\rangle \longrightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \longrightarrow |\psi'(s)\rangle = \frac{k_2}{s}|-\rangle$$

Operations on Quantum Signals

Signal Phase Changes

- **Phase Changer** is used to modify relative phase of an input state



$$k_1|E_1\rangle + k_2|E_2\rangle \quad \longrightarrow \quad \boxed{\begin{pmatrix} 1 & 0 \\ 0 & e^{e^{i(2\omega)T}} \end{pmatrix}} \quad \longrightarrow \quad k_1|E_1\rangle + k_2 e^{i(2\omega)T}|E_2\rangle$$

$$|\psi(t)\rangle \qquad\qquad |\psi'(t)\rangle$$

Operations on Quantum Signals

Signal Scaling

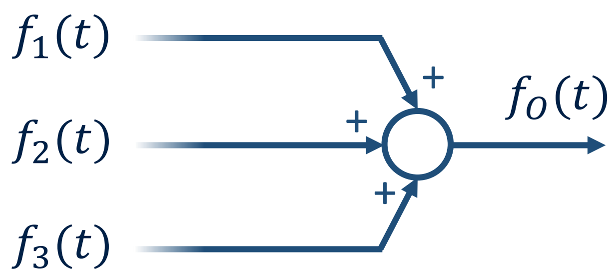
- Scaling in traditional control systems refers to changing signal magnitude

- Quantum states are *always* of a unit magnitude; scaling is not required

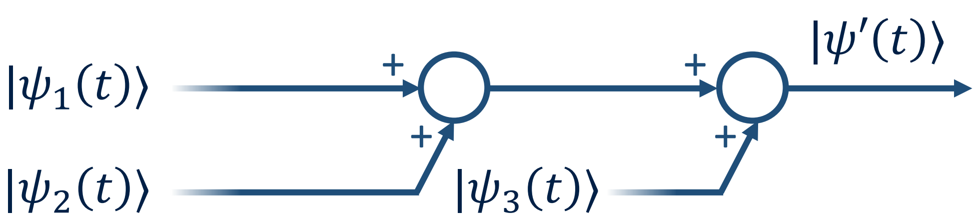
Operations on Quantum Signals

Summing Junction

- Summing Junctions in traditional control systems are used to add multiple, different signals



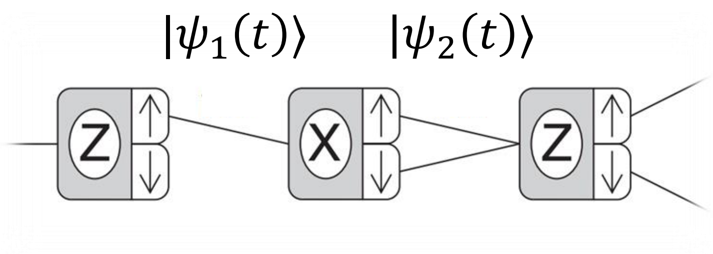
- Summing Junctions are binary operations in our context



Operations on Quantum Signals

Summing Junction: Derivation

- Concept of Quantum Summing junctions originates from the following case:



$$|\psi_2(t)\rangle = \frac{(\mathrm{P}_{+x} + \mathrm{P}_{-x})|\psi_1(t)\rangle}{\sqrt{\langle\psi_1(t)|(\mathrm{P}_{+x} + \mathrm{P}_{-x})|\psi_1(t)\rangle}}$$

$$= \frac{\boxed{\mathrm{P}_{+x}|\psi_1(t)\rangle + \mathrm{P}_{-x}|\psi_1(t)\rangle}}{\sqrt{\langle\psi_1(t)|(\mathrm{P}_{+x} + \mathrm{P}_{-x})|\psi_1(t)\rangle}}$$

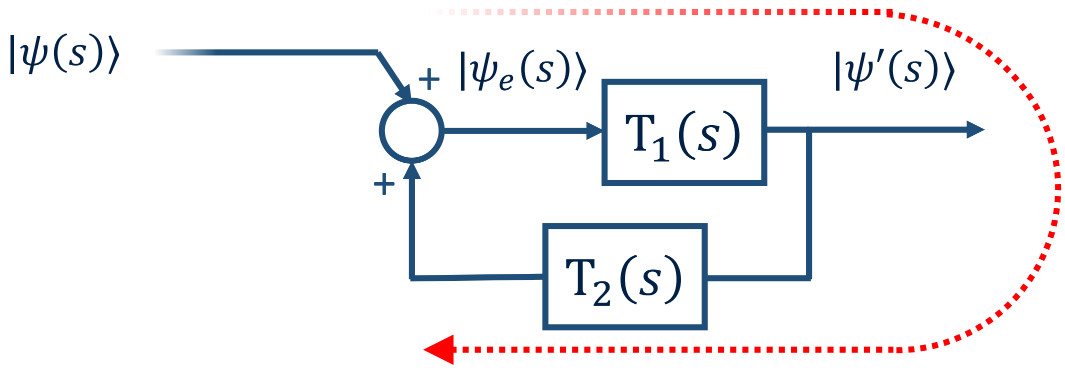
- Multiply input by appropriate operators
- *Retain* the global phase of different terms before summation

Operations on Quantum Signals

Signal Feedback, Summing Junction

- **Signal Feedback** involves using the output as a part of the input signal itself

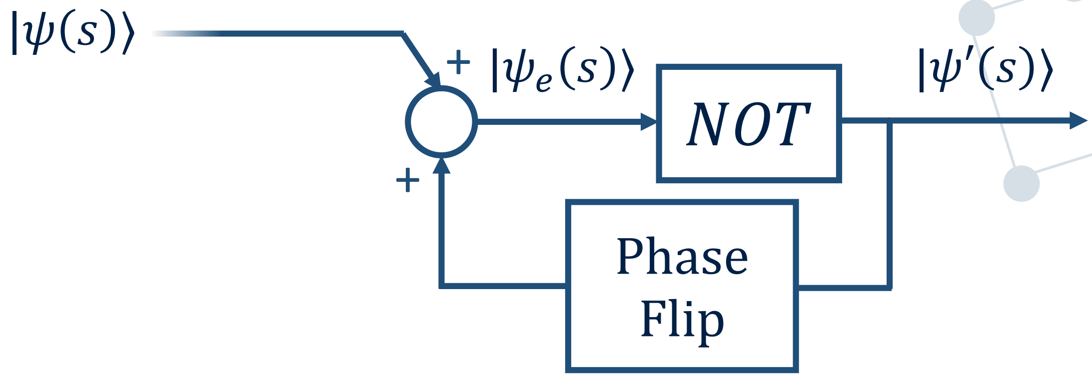
$$|\psi_{e_{(i+1)}}(t)\rangle = \mathrm{I}|\psi(s)\rangle + \mathrm{T}_2(s)\mathrm{T}_1(s)|\psi_{e_{(i)}}(t)\rangle$$

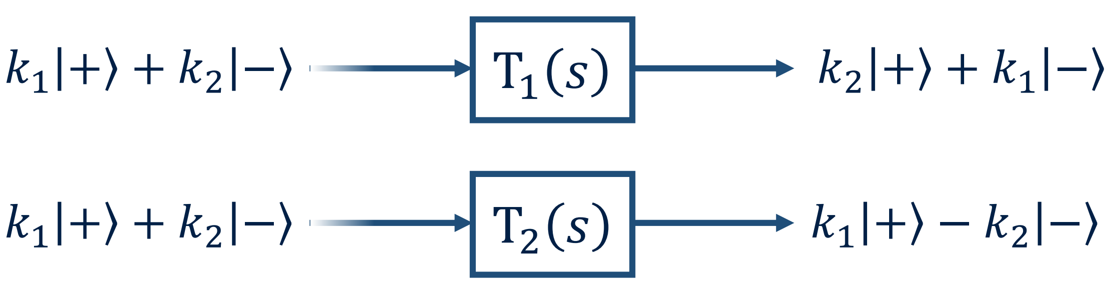


$$|\psi'(s)\rangle = (\mathrm{I} - \mathrm{T}_1(s)\mathrm{T}_2(s))^{-1}(\mathrm{T}_1(s))|\psi(s)\rangle$$

Operations on Quantum Signals

Signal Feedback: Example



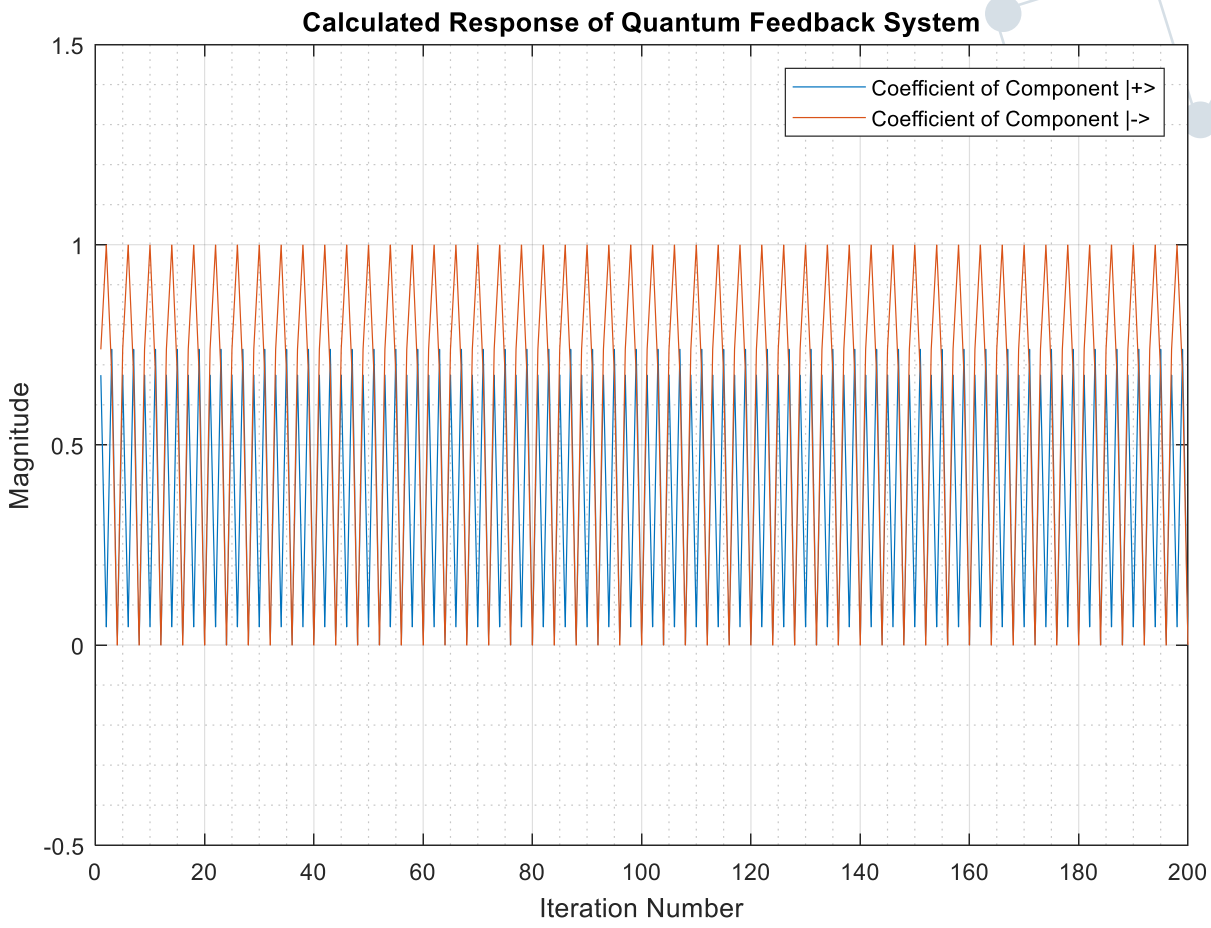


$\mathrm{T}_1(\mathrm{s})$ is a **NOT** gate
$\mathrm{T}_2(\mathrm{s})$ is a **Phase Flip**

$$\boxed{|\psi'(s)\rangle = (\mathrm{I} - \mathrm{T}_1(s)\mathrm{T}_2(s))^{-1}(\mathrm{T}_1(s))|\psi(s)\rangle}$$

Operations on Quantum Signals

Signal Feedback: Simulation of a Quantum Feedback System



Questions?

References

- [1] David H. McIntyre, “Quantum Mechanics: A Paradigms Approach”, Pearson, 2012
- [2] Lucas V. Barbosa, “Fourier Series”, Wikimedia
  `https://en.wikipedia.org/wiki/Fourier_series`