PROBLEM BACKGROUND

“Gambler’s Ruin” Analysis

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"Gambler's Ruin" Analysis

Question: "How long does it take for the following game to end for 2, 3, or 4 people?"

Game Description:

- Let 3 (or 4) people start with amounts \$$a$, \$$b$, \$$c$, (and \$$d$).
- Randomly choose a pair of people from $\binom{3}{2}$ (or from $\binom{4}{2}$) choice
- Randomly choose a winner, who gains \$1, and a loser, who loses \$1
- Terminate when one of the players loses all their money.

Aim:

- Find E[$T(a, b, c)$] or E[T(a, b, c, d)], where 'T' is the completion time

“Gambler’s Ruin” Analysis

We know that for m = 2, 3

$$\mathrm{E}[T(k_1,k_2)] \xrightarrow{k_1,k_2 \longrightarrow \infty} k_1 \times k_2$$

$$\mathrm{E}[T(k_1,k_2,k_3)] \xrightarrow{k_1,k_2,k_3 \longrightarrow \infty} \frac{3 \times k_1 k_2 k_3}{k_1 + k_2 + k_3}$$

where,

$$m \overset{\Delta}{=} \text{Number of people}$$

$$T \overset{\Delta}{=} \text{Termination Time}$$

But what if ***m* > 3**?

SIMULATION RESULTS

Section 1: Testing with 2 players

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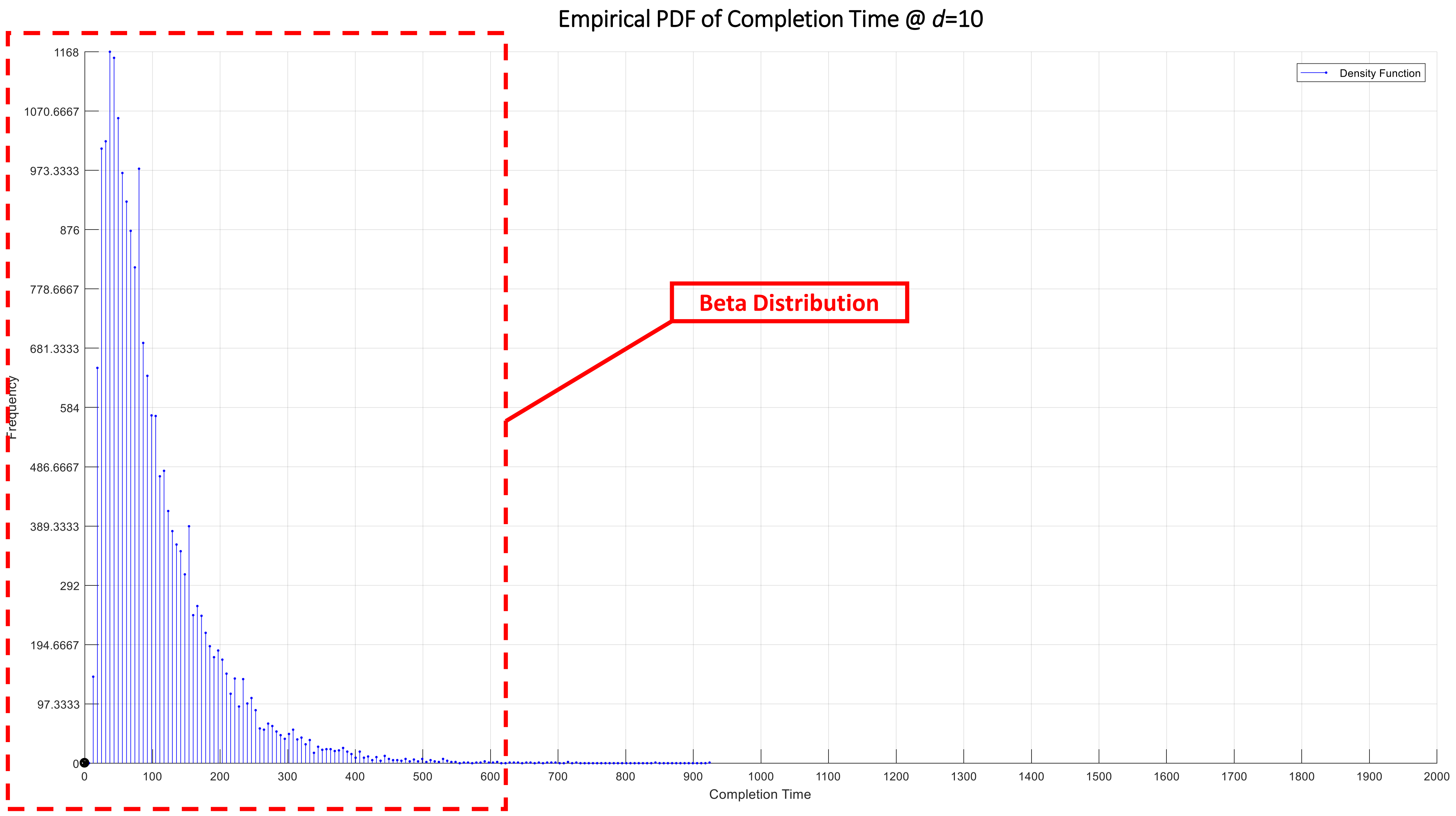
We perform a simulation with:

$$\boldsymbol{[a, b] = [\$10, \$10]}$$

Thus, with $E[T] = ab = 100$, we performed around $\mathbf{2 \times 10^6}$ gambles.

Section 1: Testing with 2 players



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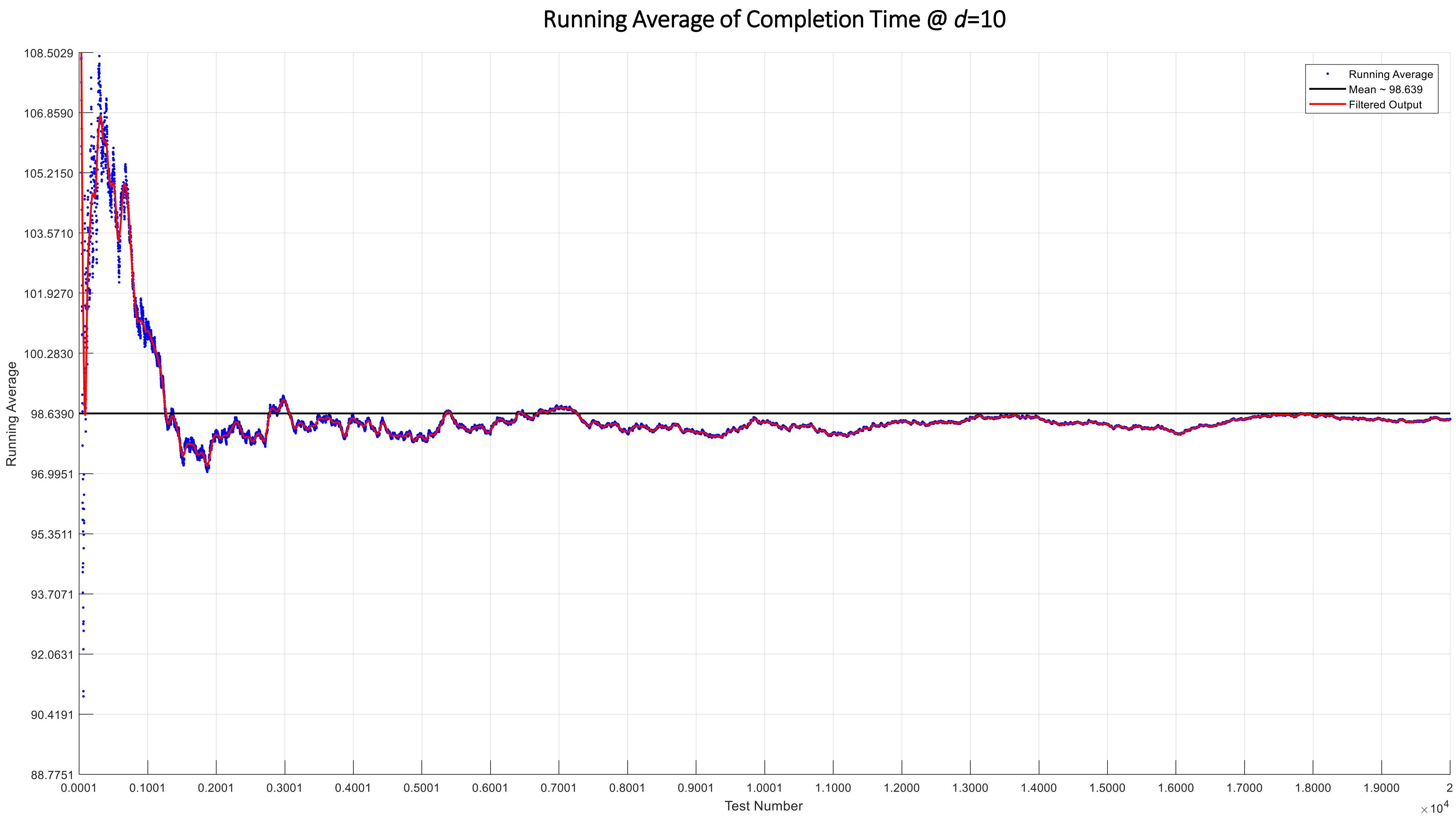


SIMULATION RESULTS

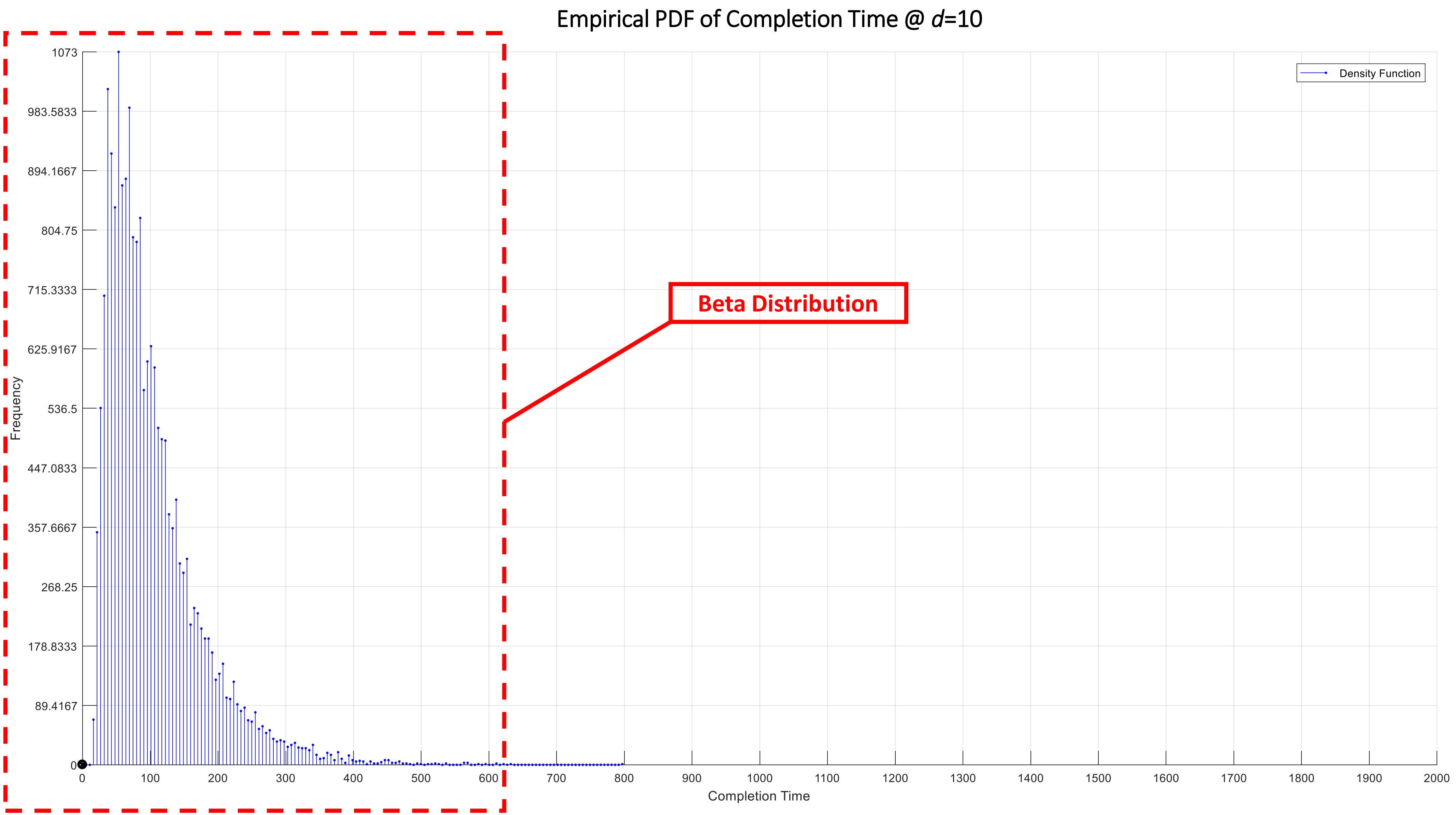
Section 2: Testing with 3 players

We perform a simulation with:

[a, b, c] = [\$10, \$10, \$10]

Thus, with $\mathrm{E}[T] = \frac{3abc}{a+b+c} = 100$, we performed around $\mathbf{2 \times 10^6}$ gambles.

Section 2: Testing with 2 players



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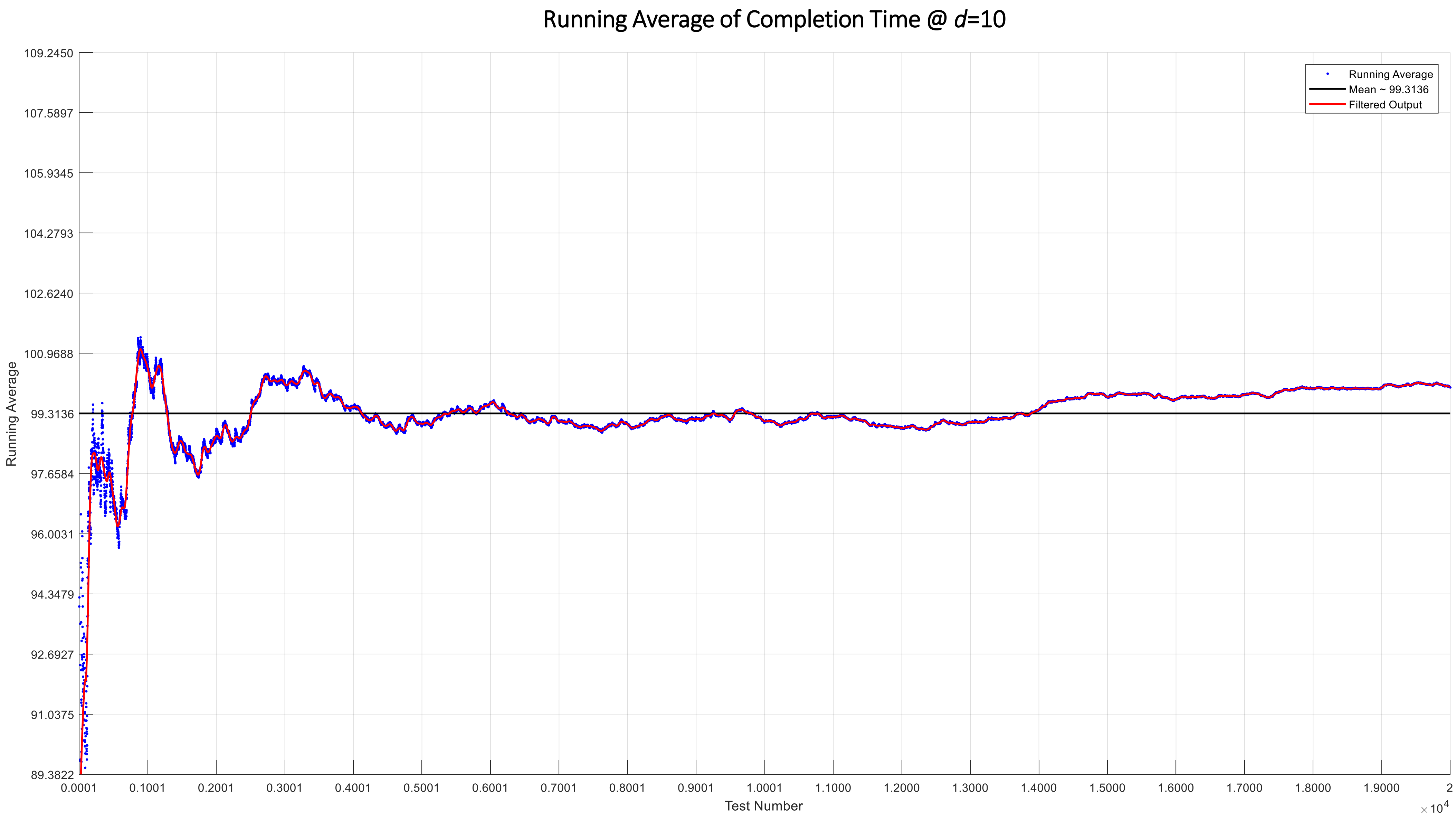


SIMULATION RESULTS

Section 3: Testing with 4 players

We perform a simulation with:

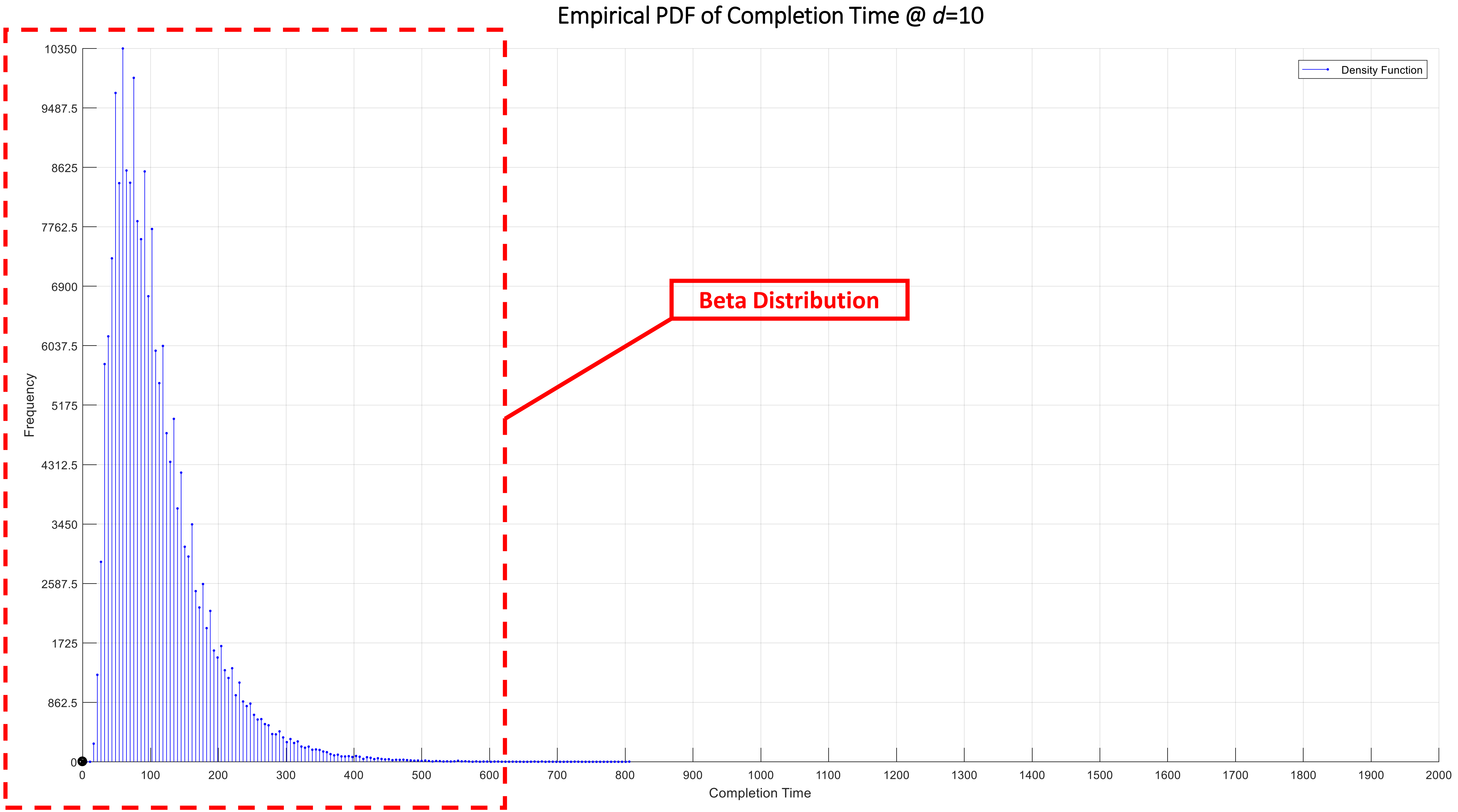
[*a*, *b*, *c*, *d*] = [$10, $10, $10, *k*]

where,

$$k \in \{\$10, \$20, \$40, \$80, \$160\}$$

Thus, with E[T] ~ 160, we performed around $\mathbf{3.8 \times 10^8}$ gambles.

Section 3: Testing with 4 players



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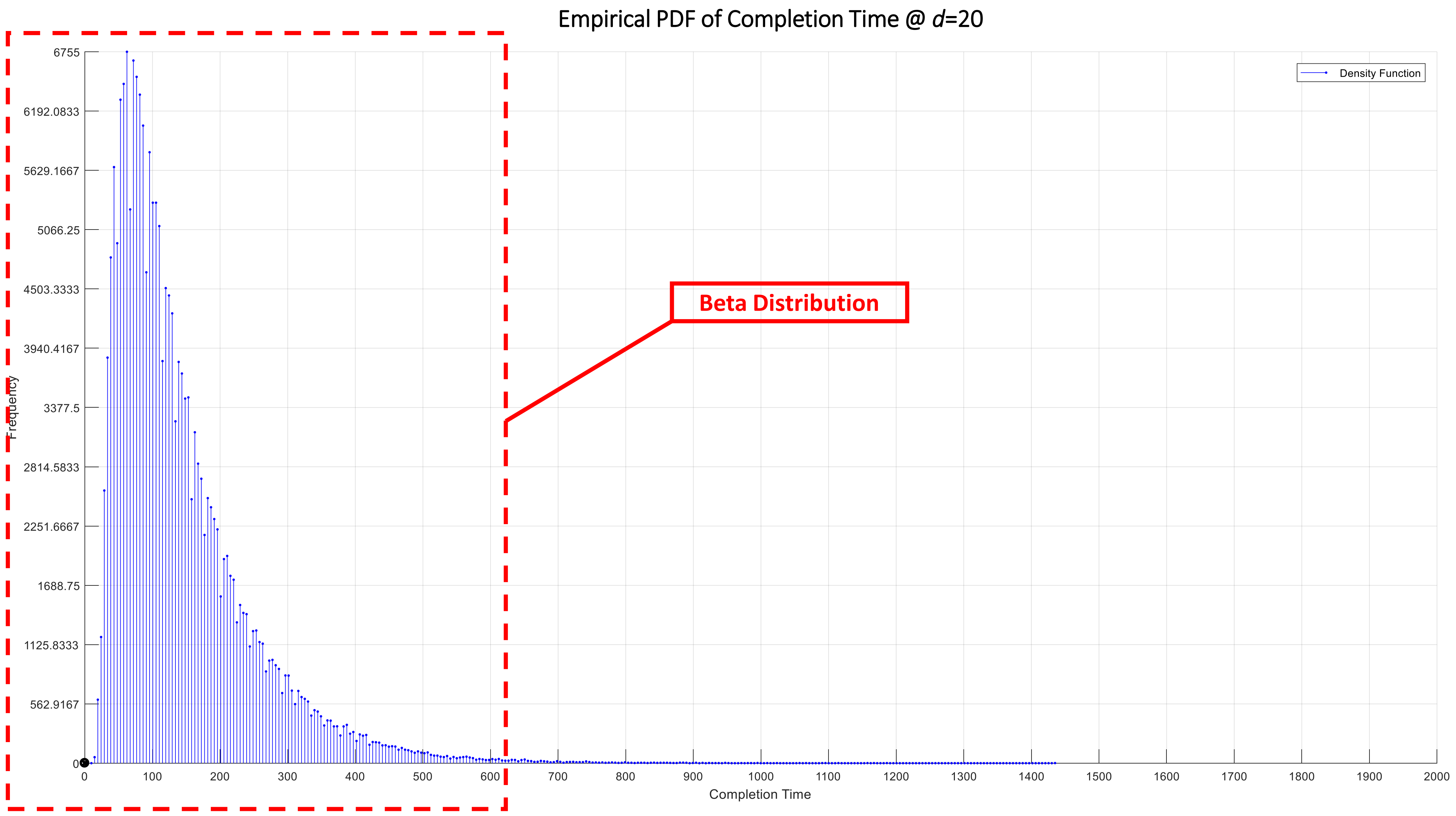

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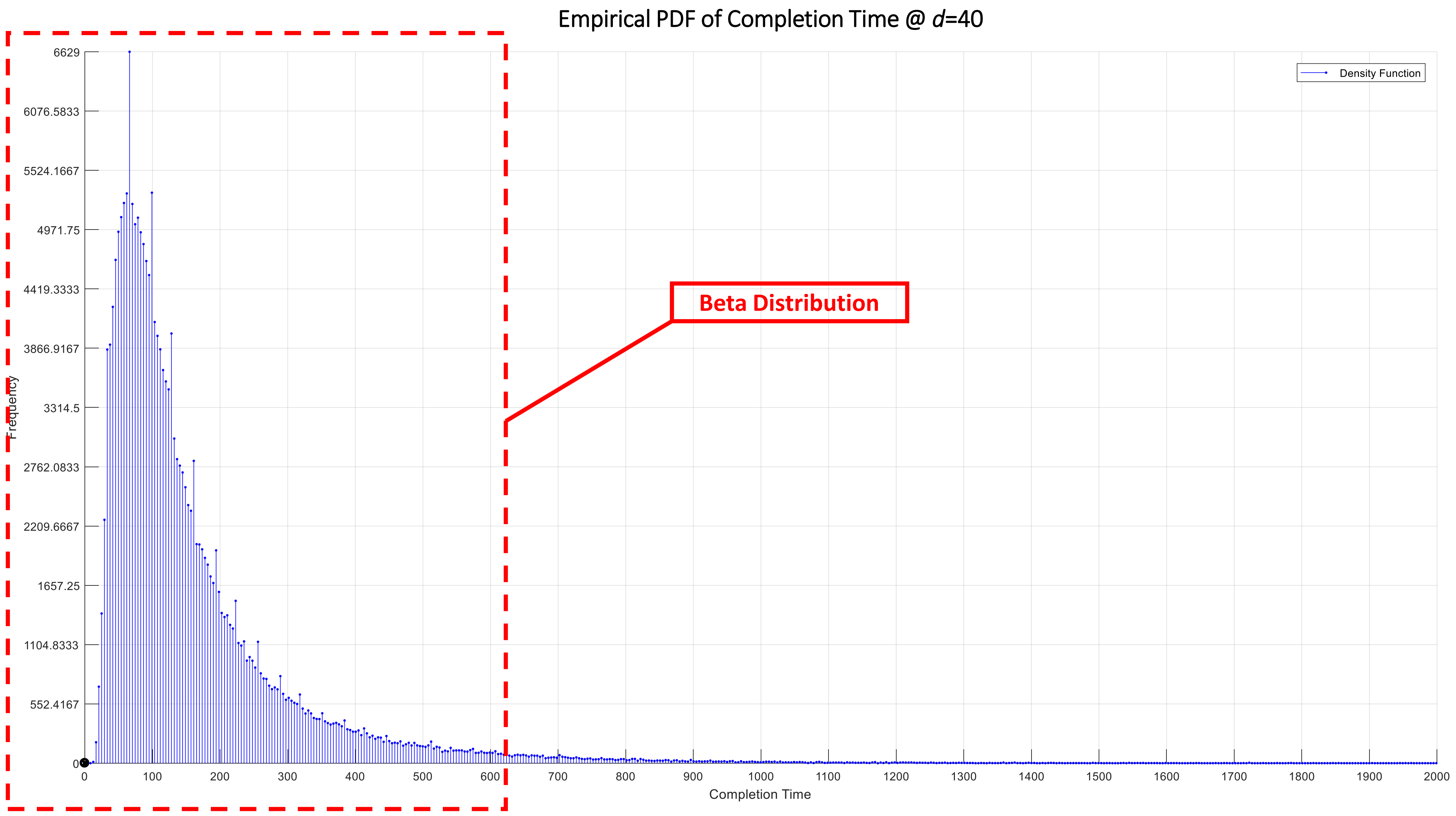

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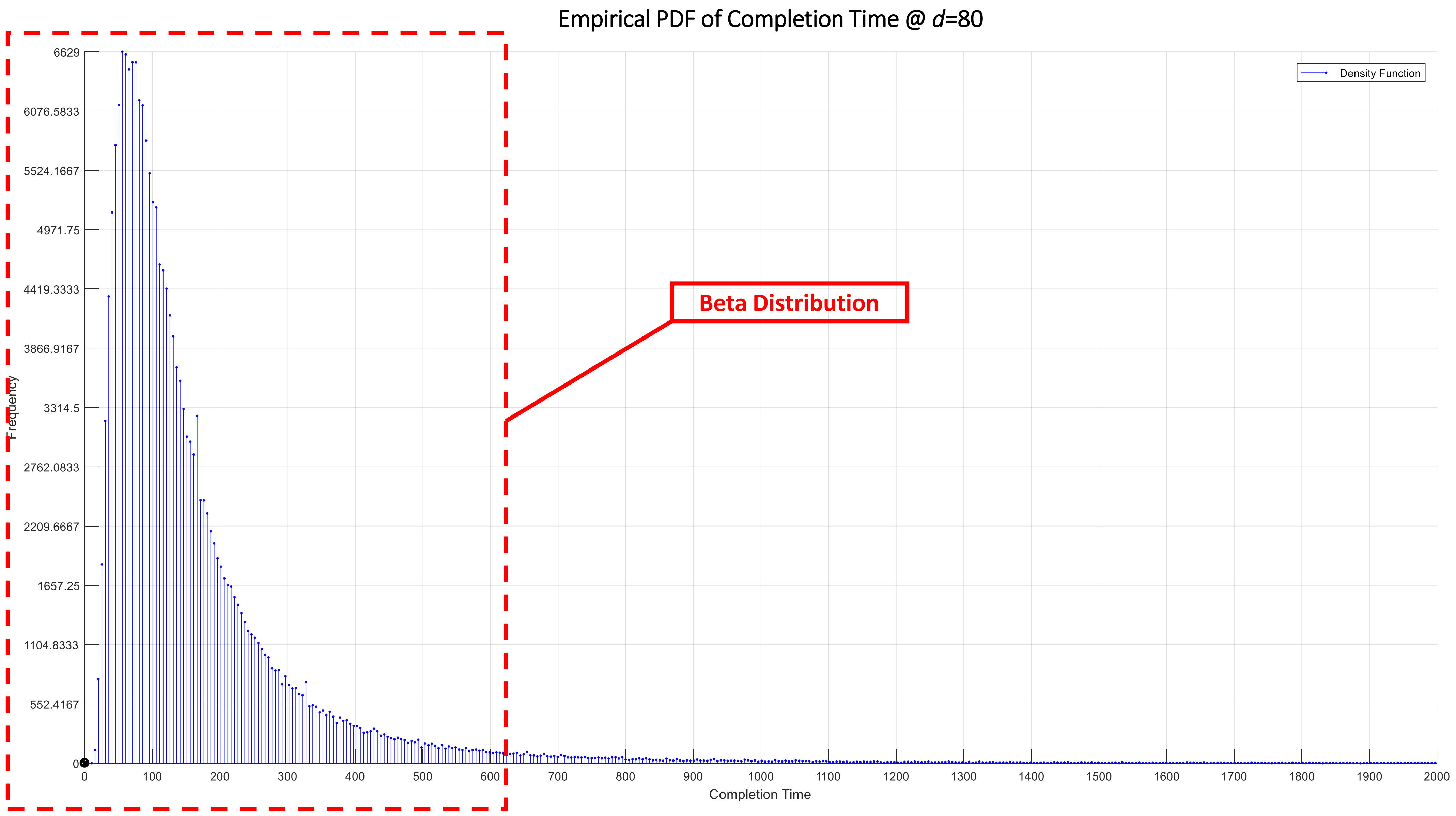

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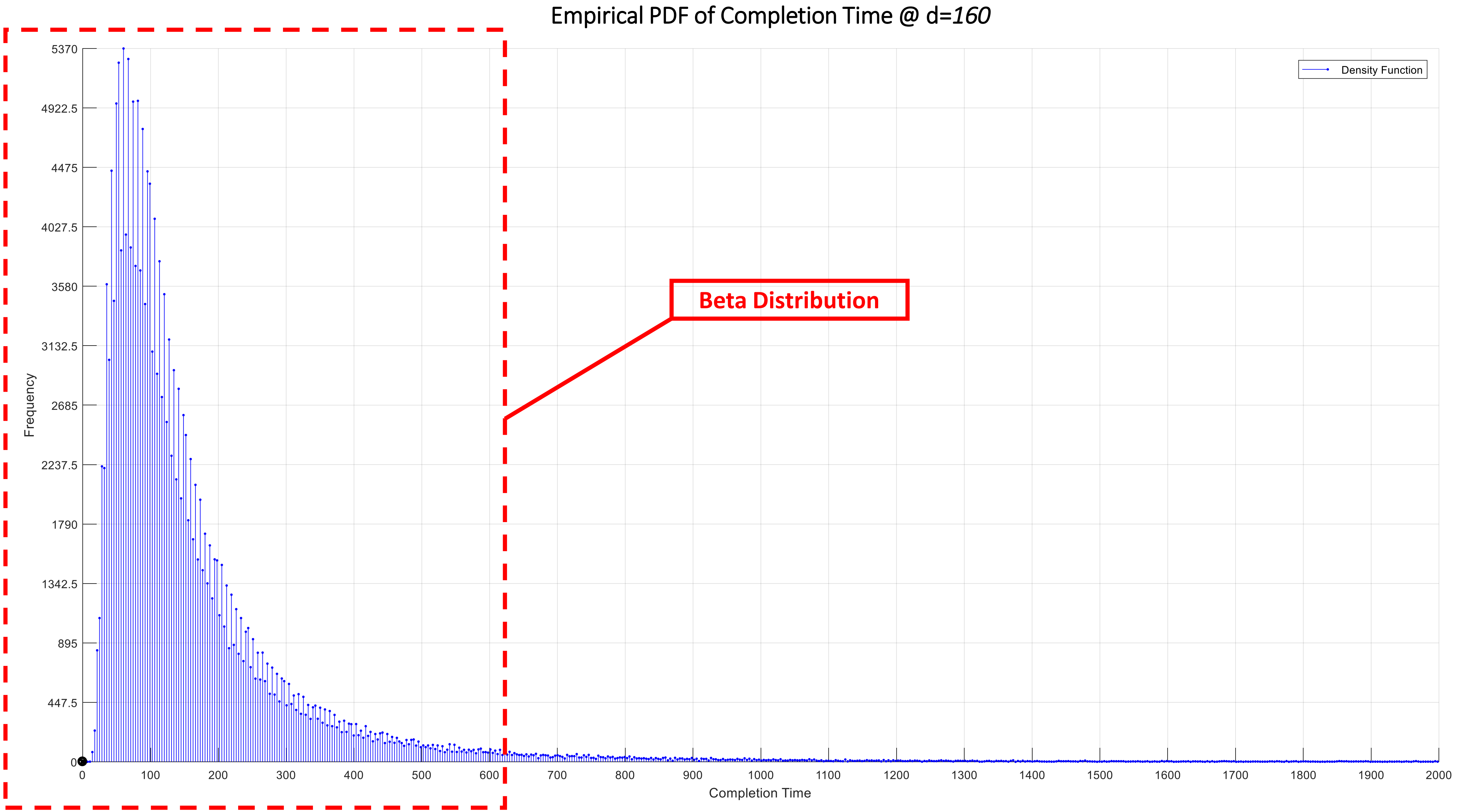

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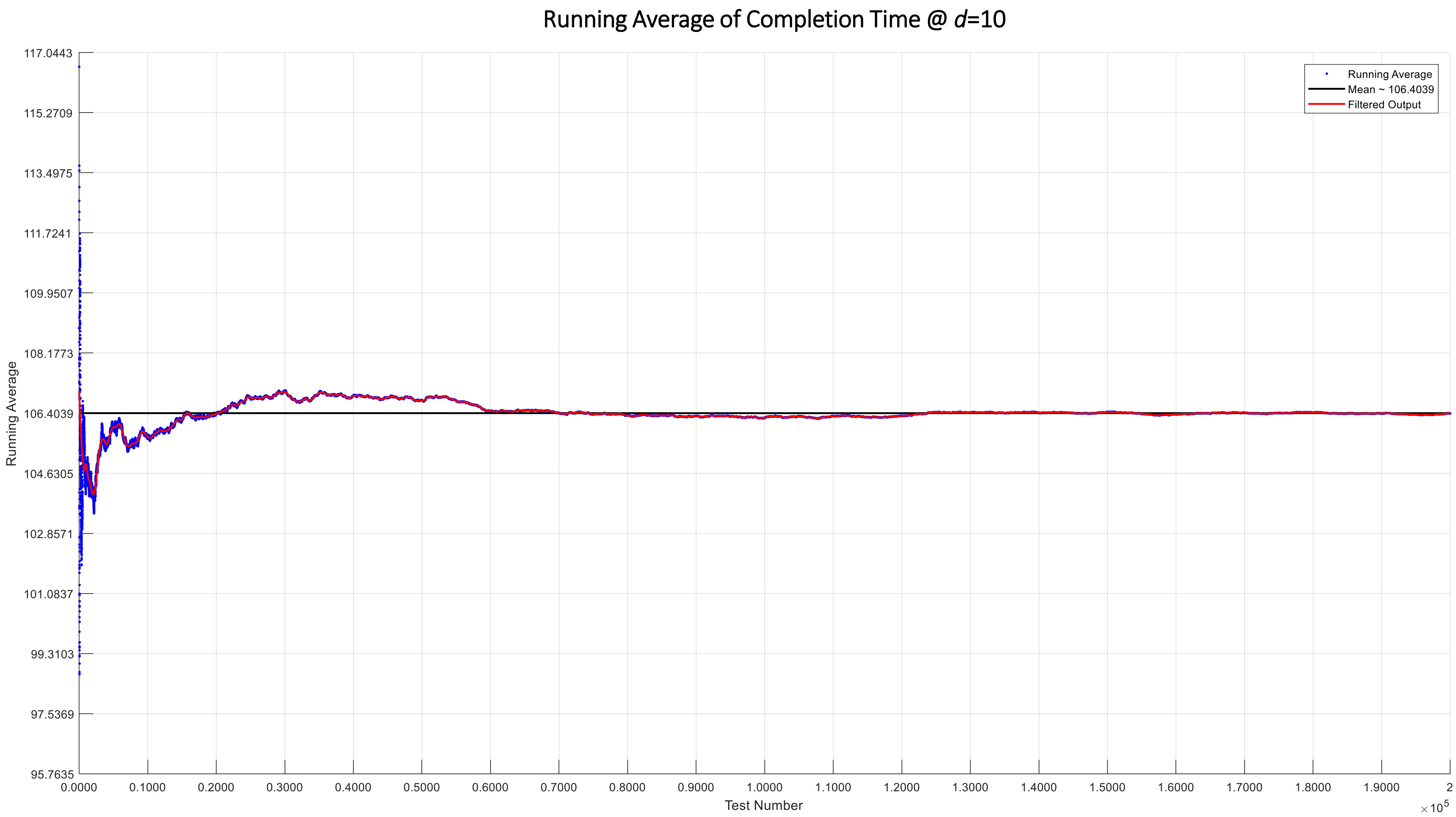

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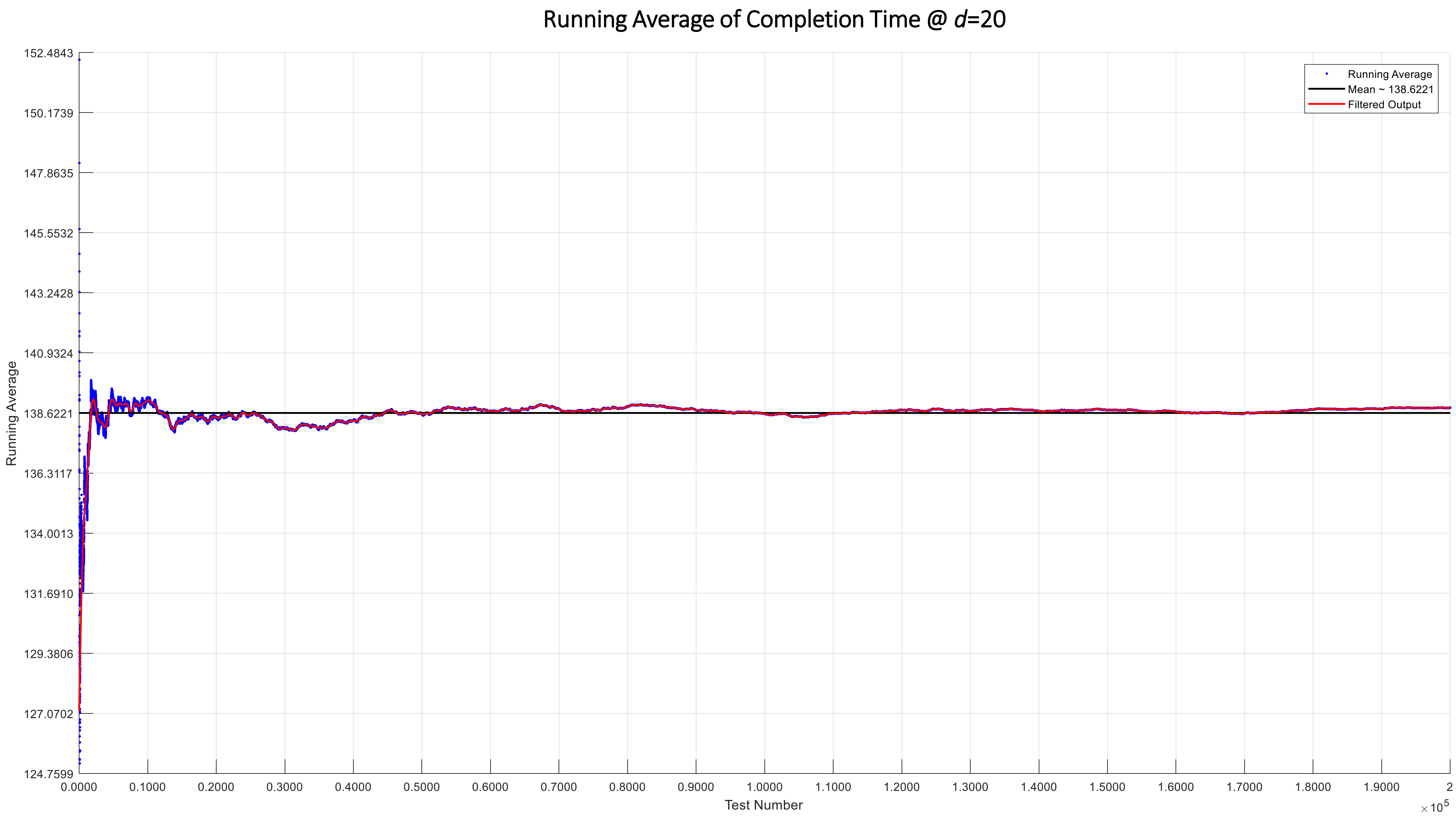

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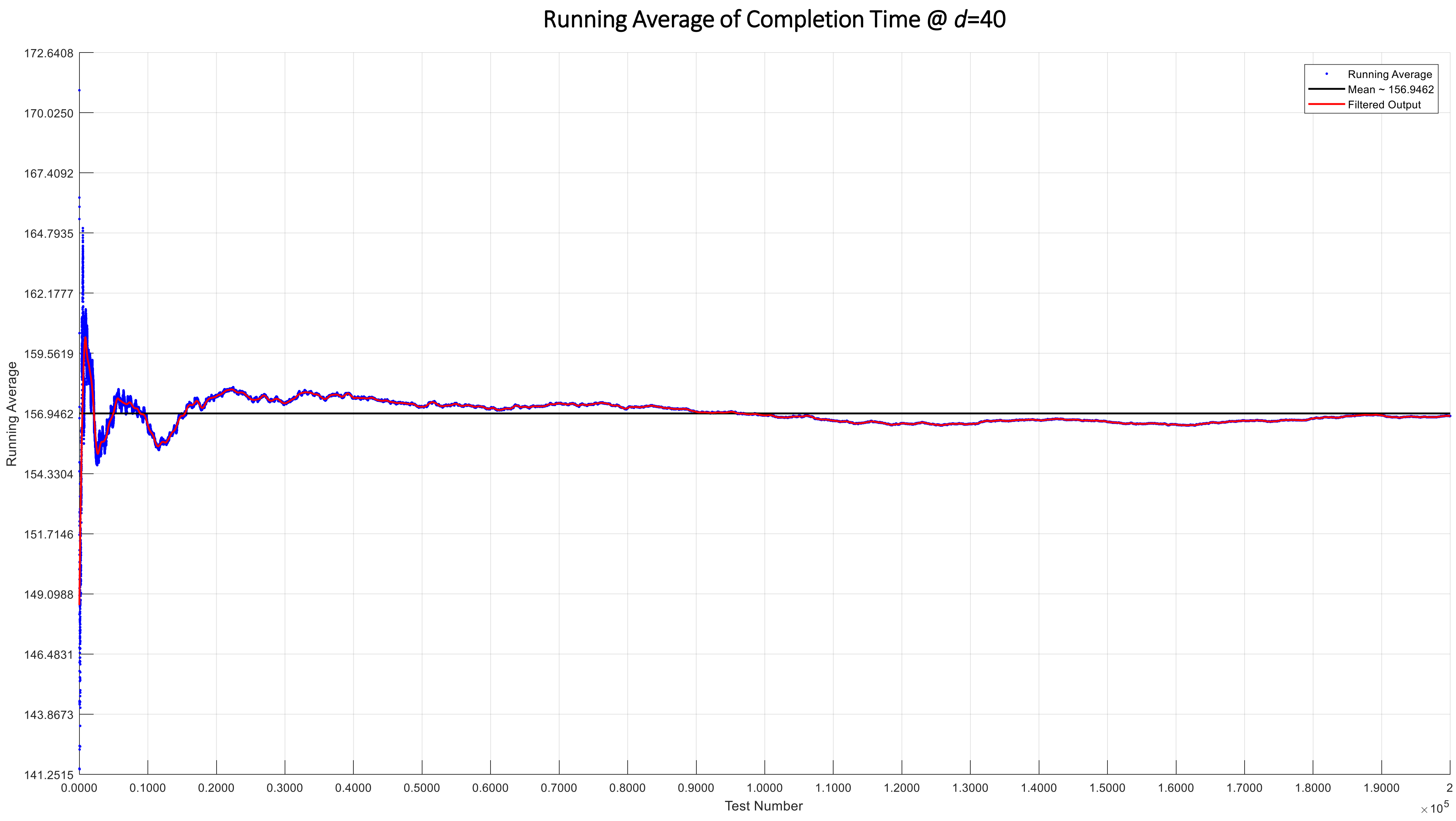

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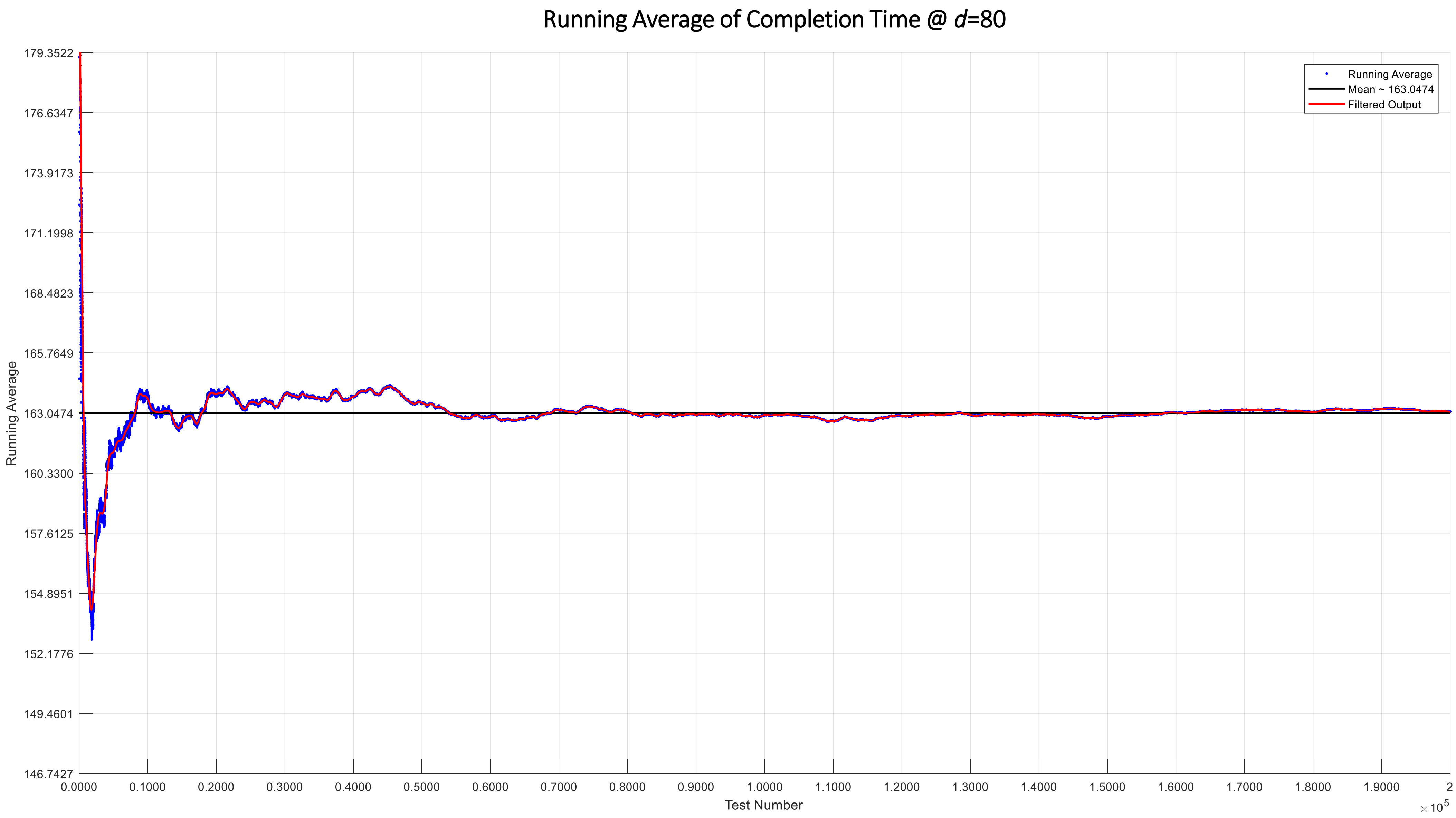

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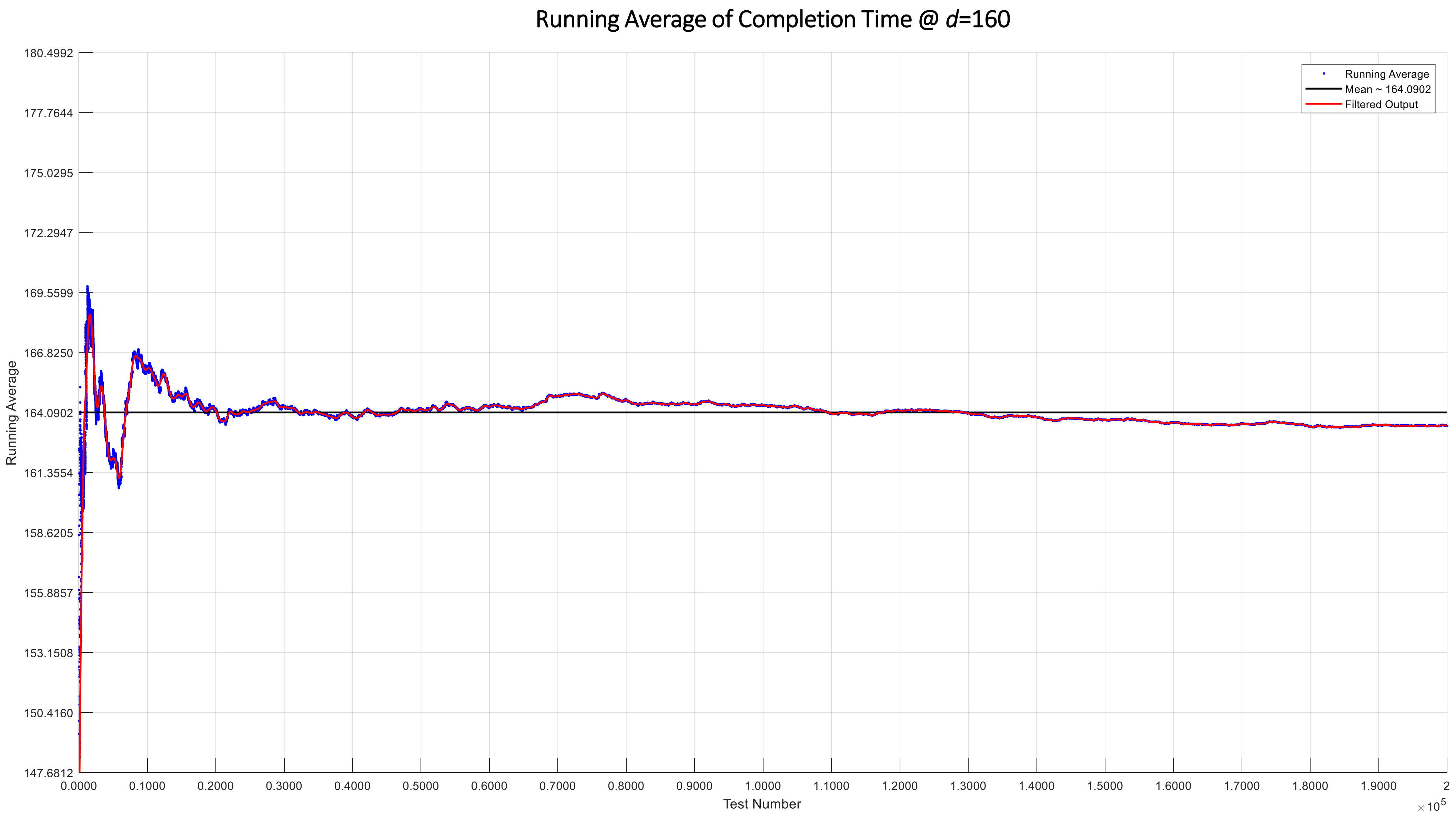

SIMULATION RESULTS

Section 4: Analysis with 4 players

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Question: "Why is $E[T_4] > E[T_3]$?"

Possible Reason:

- In the 4 player case, there are $\binom{4}{2} = 6$ pairs, of which $\binom{3}{1} = 3$ include the 4th player.
- On average, the 4th player contributes and takes away ~$0 from each player as $k_1, k_2, k_3, k_4 \rightarrow \infty$.
- Thus, case with 4th player is *very* similar to the case with 3rd player, given $k_1, k_2, k_3, k_4 \rightarrow \infty$.
- We can consider any gamble involving the 4th player as a *wasted turn*.
- Since ~3/6 random pairs include the 4th player, we can say that $\mathbf{E[T_4] > E[T_3]}$ when $k_1, k_2, k_3, k_4 \rightarrow \infty$.