PROBLEM BACKGROUND

“Gambler’s Ruin” Analysis

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“Gambler’s Ruin” Analysis

Question: “How long does it take for the following game to end for 3, or 4 people?”

Game Description:

- Let 3 (or 4) people start with amounts \$$a$, \$$b$, \$$c$, (and \$$d$).
- Randomly choose a pair of people from $\binom{3}{2}$ (or from $\binom{4}{2}$) choice
- Randomly choose a winner, who gains \$1, and a loser, who loses \$1
- Terminate when one of the players loses all their money.

Aim:

- Find E[$T(a, b, c)$] or E[T(a, b, c, d)], where ‘T’ is the completion time

“Gambler’s Ruin” Analysis

We know that for m = 2, 3,

$$\mathrm{E}[T(k_1, k_2, \ldots, k_m)] \xrightarrow{k_1, k_2, \ldots, k_m \longrightarrow \infty} \frac{m \cdot \prod_{i=1}^{m} k_i}{\sum_{i=1}^{m} k_i}$$

where,

$$m \stackrel{\Delta}{=} \text{Number of people}$$

$$T \stackrel{\Delta}{=} \text{Termination Time}$$

But what if $\boldsymbol{m} > \mathbf{3}$?

SIMULATION RESULTS

Section 1: Testing with 3 players

“Gambler’s Ruin” Analysis

We perform a simulation with:

[*a*, *b*, *c*, *d*] = [$20, $20, $20]

Thus, with $\mathrm{E}[T] = \frac{3abc}{a+b+c} = 400$, we performed around $\mathbf{2 \times 10^5}$ gambles.

SIMULATION RESULTS

Section 2: Testing with 4 players

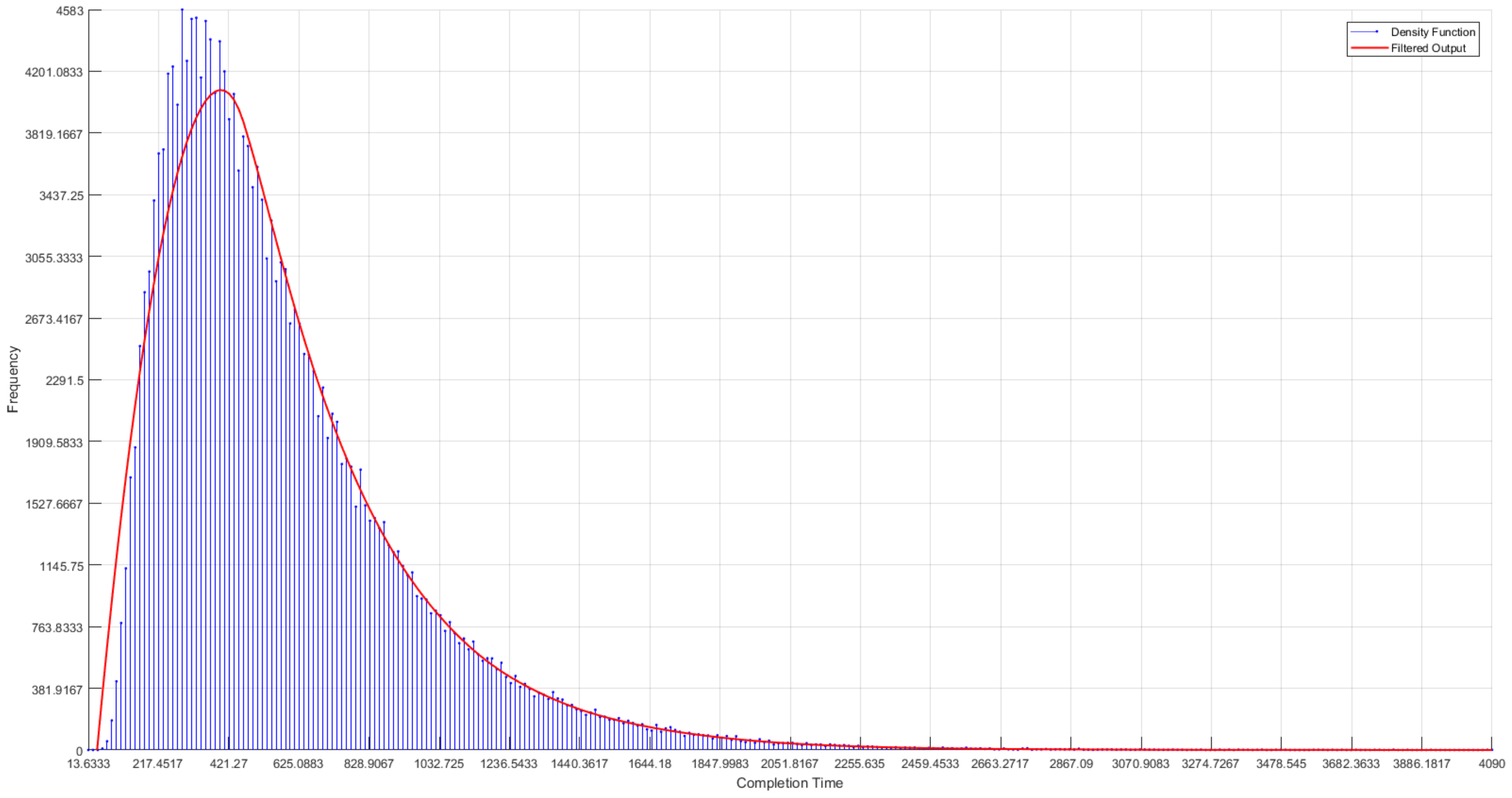
We perform a simulation with:

[a, b, c, d] = [\$20, \$20, \$20, k] where, $k \in \{\$20, \$30, \$40, \$50\}$

Thus, with E[T] ~ 700, we performed around $\mathbf{1.4 \times 10^8}$ gambles.

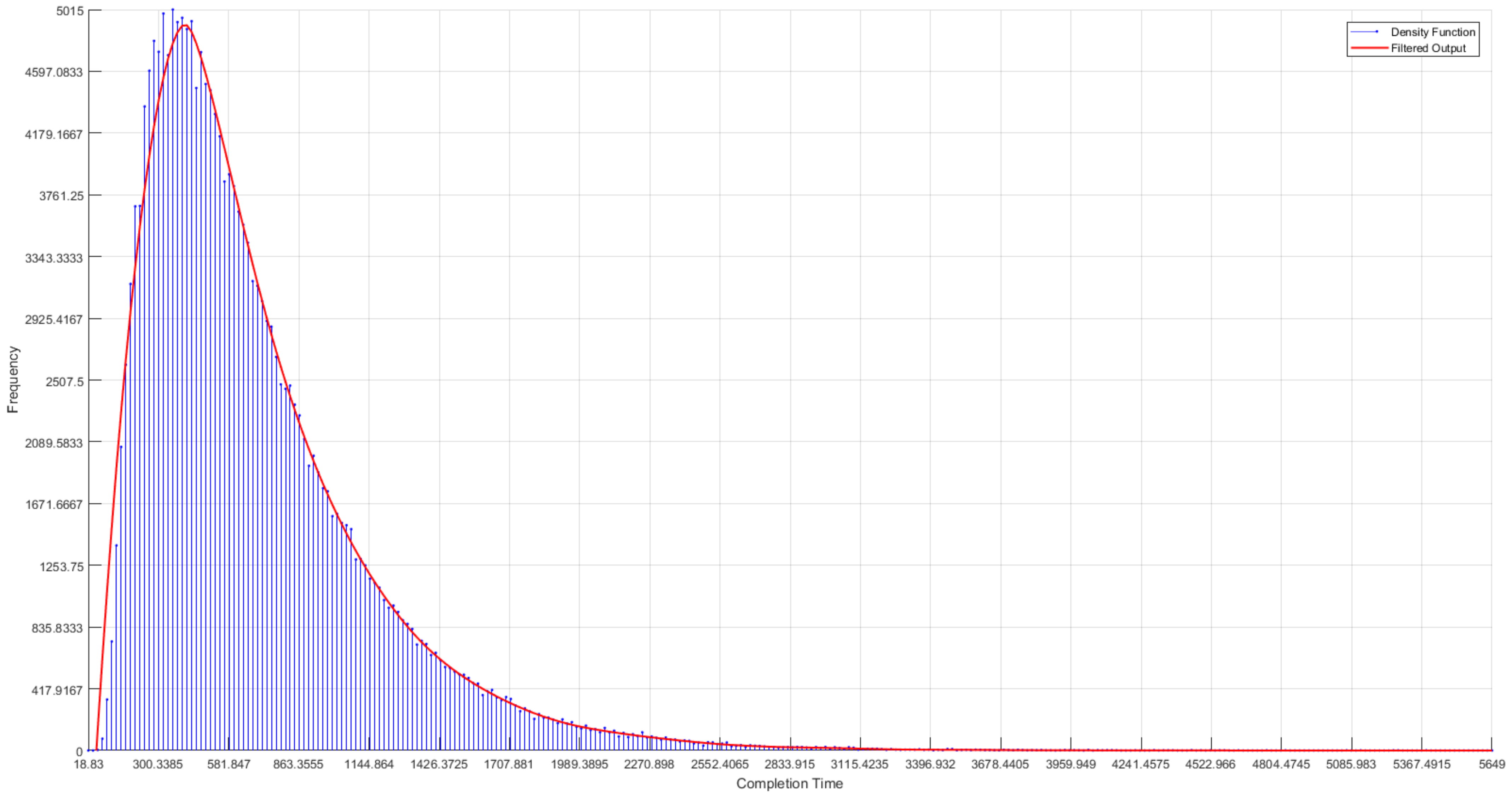
Section 2: Testing with 4 players

Empirical PDF of Completion Time @ *d*=20



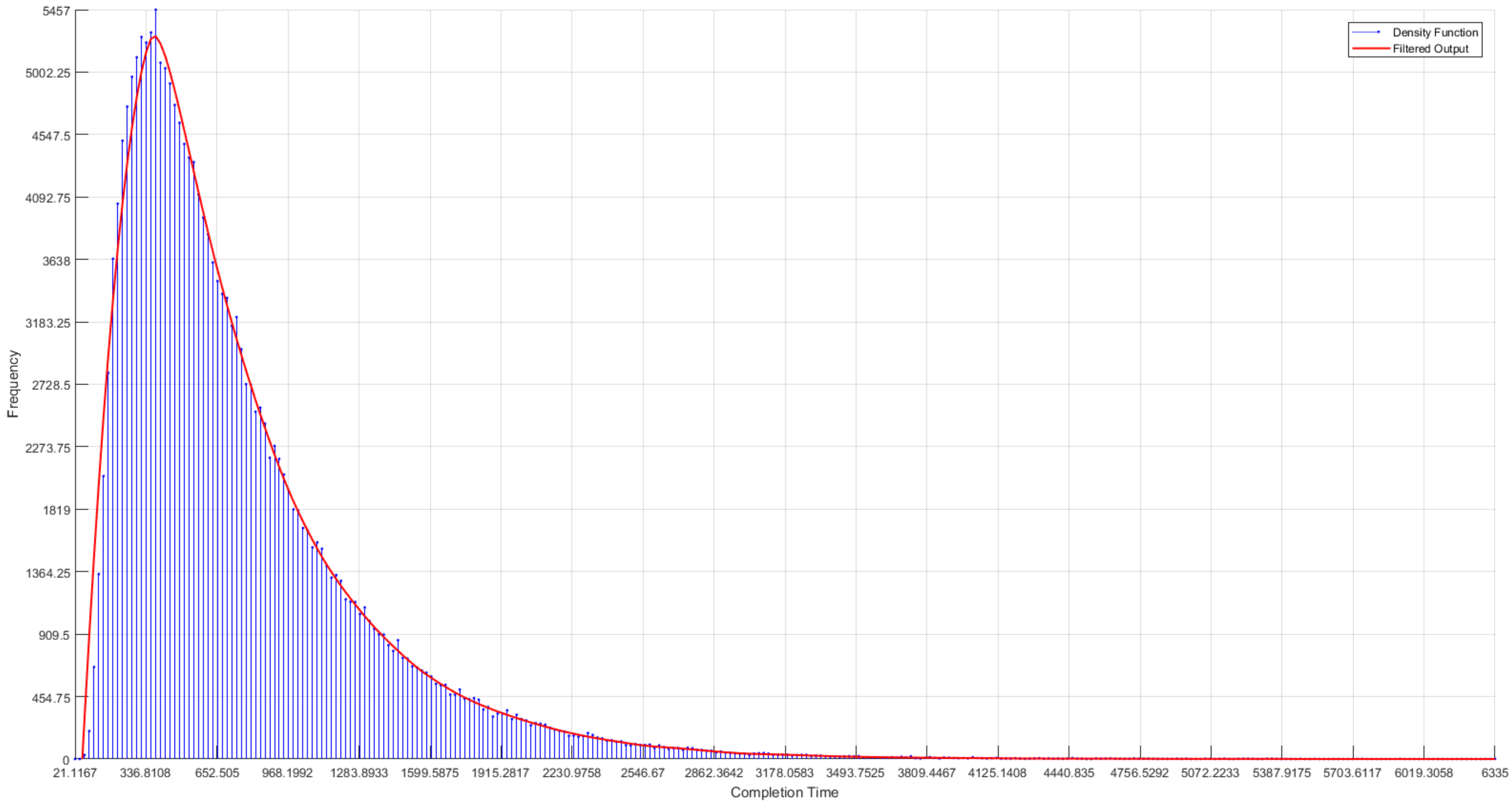
Section 2: Testing with 4 players

Empirical PDF of Completion Time @ *d*=30



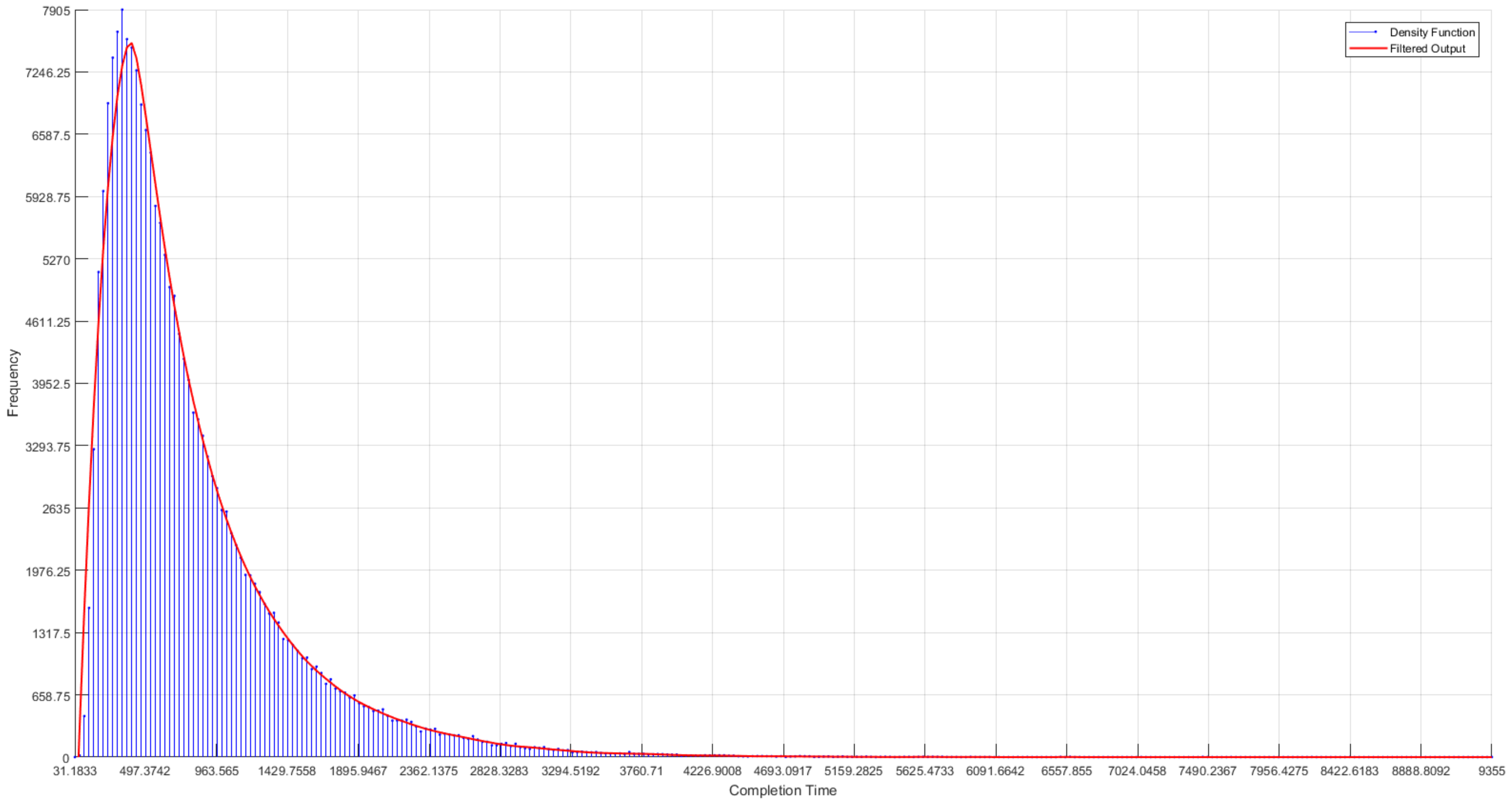
Section 2: Testing with 4 players

Empirical PDF of Completion Time @ *d*=40



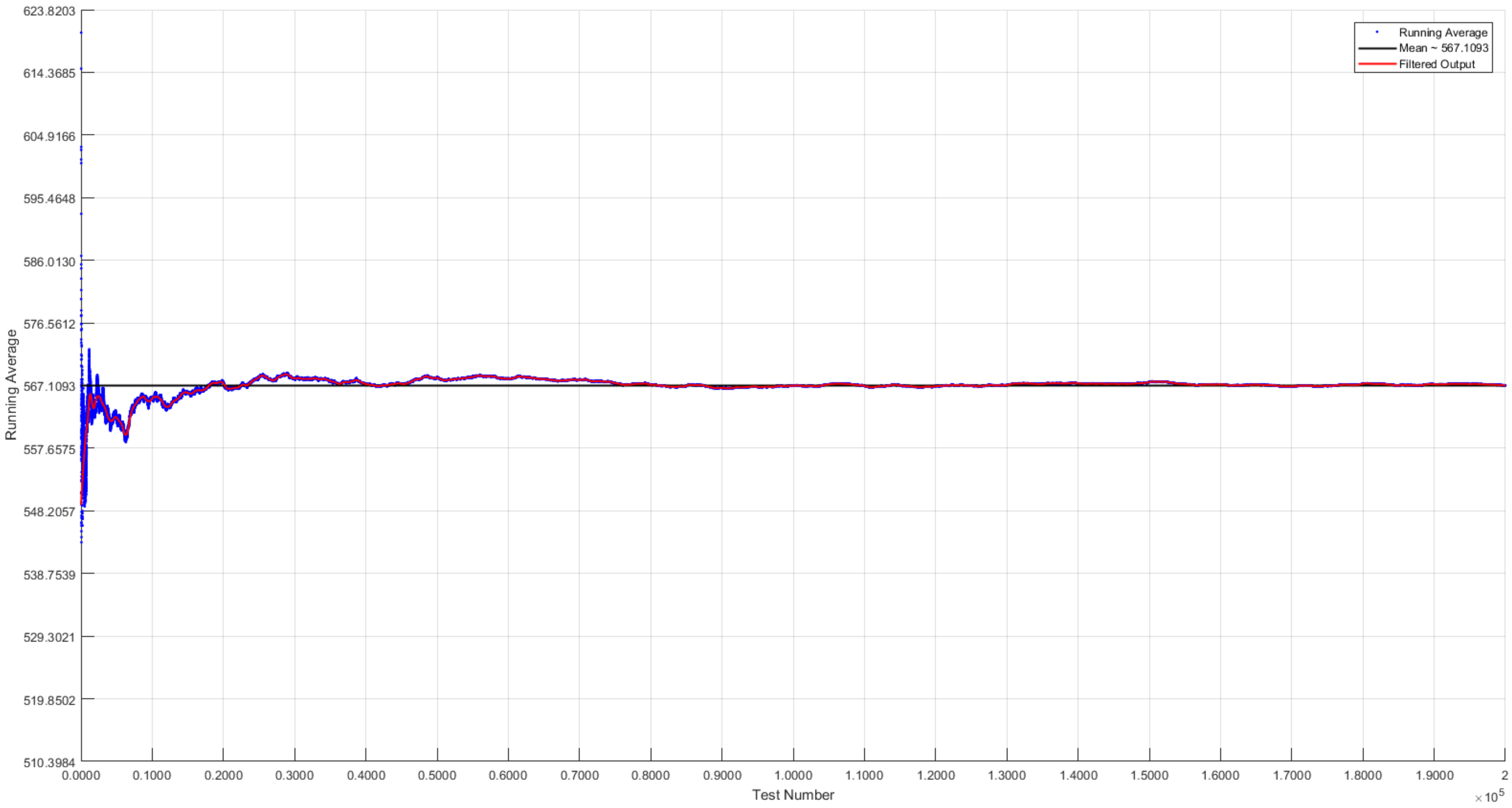
Section 2: Testing with 4 players

Empirical PDF of Completion Time @ *d*=50

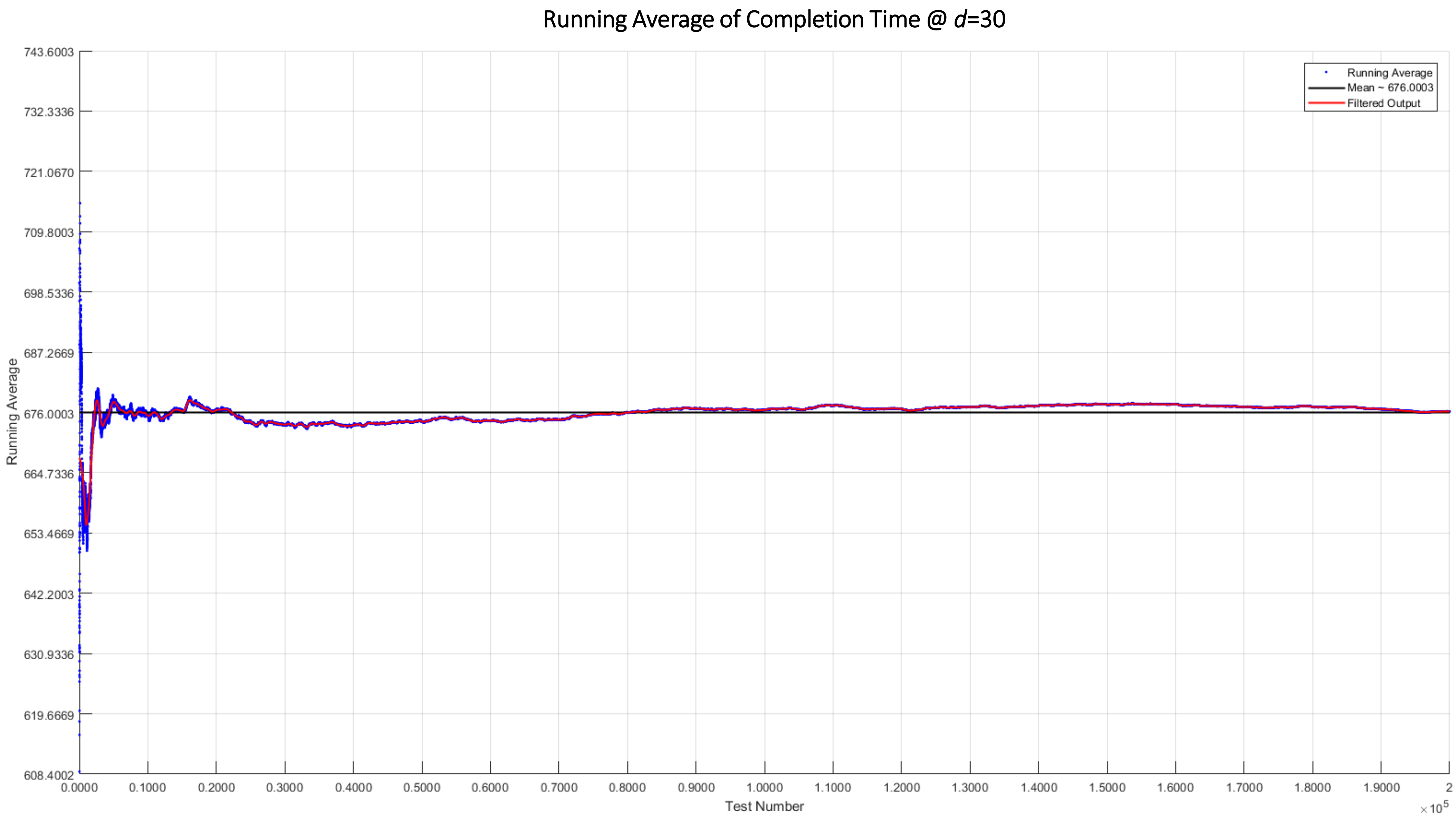


Section 2: Testing with 4 players

Running Average of Completion Time @ *d*=20

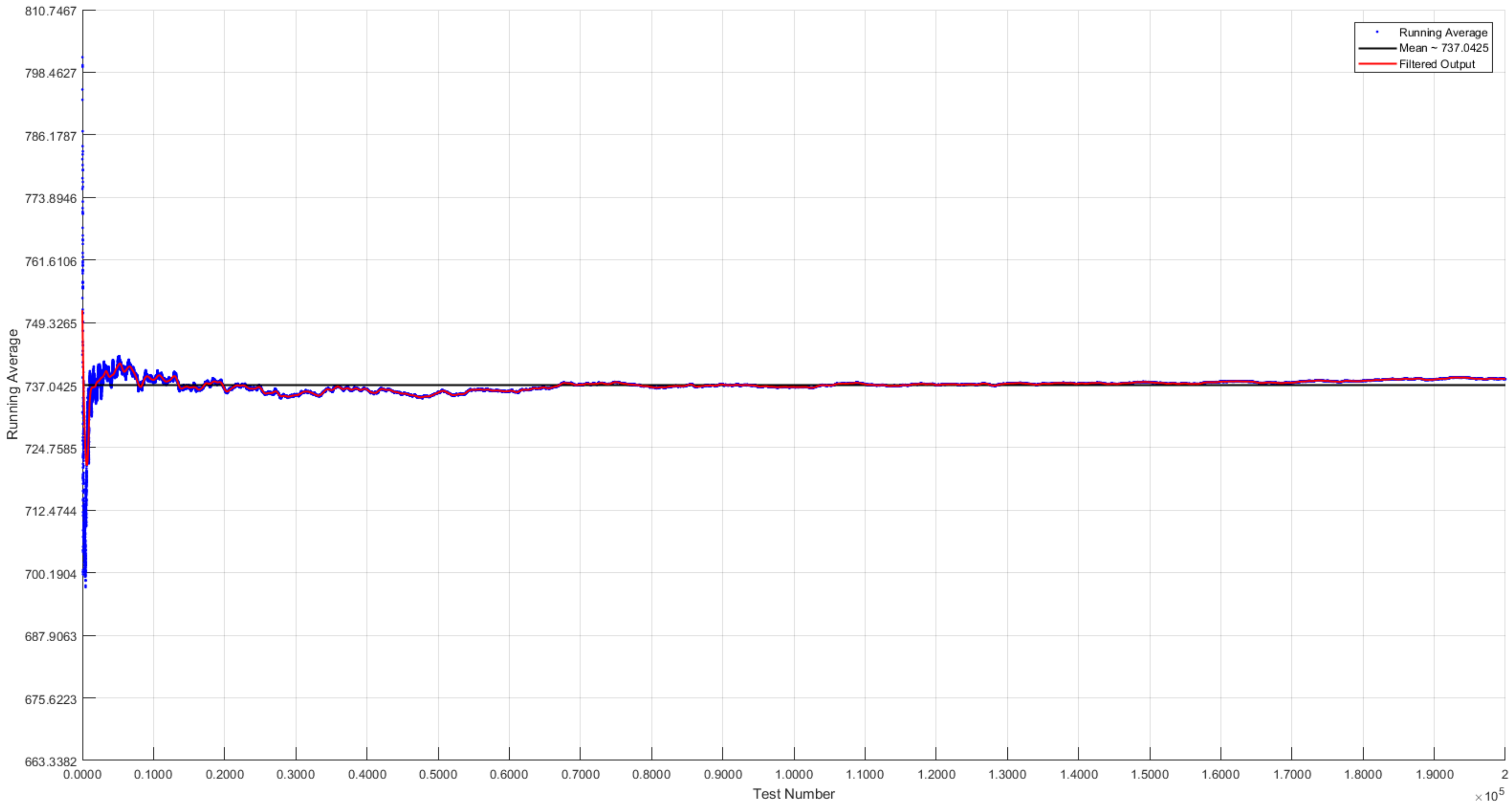


Section 2: Testing with 4 players



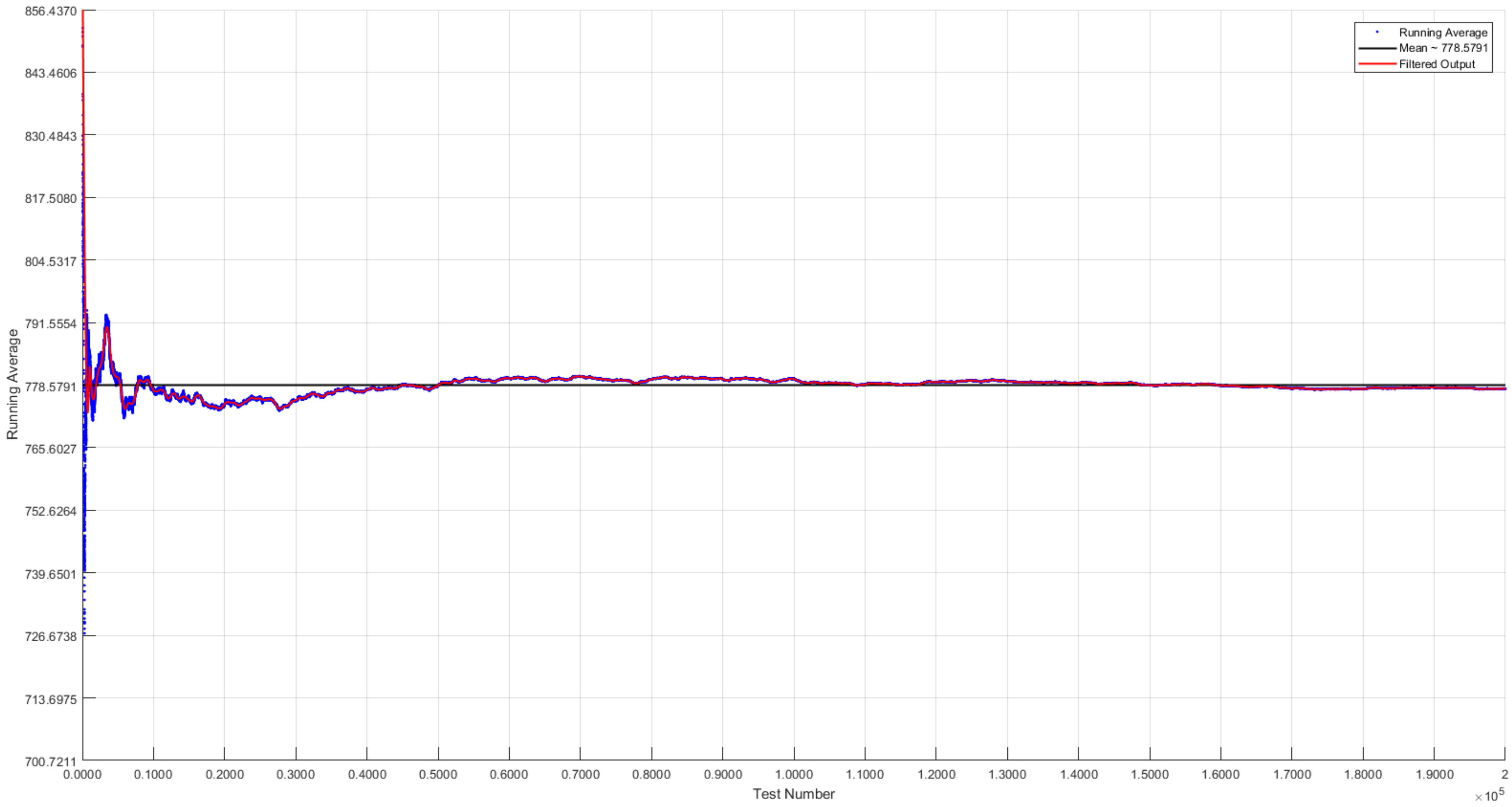
Section 2: Testing with 4 players

Running Average of Completion Time @ *d*=40



Section 2: Testing with 4 players

Running Average of Completion Time @ *d*=50



SIMULATION RESULTS

Section 4: Analysis with 4 players

Section 4: Analysis with 4 players

Question: "Why is $E[T_4] > E[T_3]$?"

Possible Reason:

- In the 4 player case, there are $\binom{4}{2} = 6$ pairs, of which $\binom{3}{1} = 3$ include the 4th player.
- On average, the 4th player contributes and takes away ~$0 from each player as $k_1, k_2, k_3, k_4 \to \infty$.
- Thus, case with 4th player is *very* similar to the case with 3rd player, given $k_1, k_2, k_3, k_4 \to \infty$.
- We can consider any gamble involving the 4th player as a *wasted turn*.
- Since ~3/6 random pairs include the 4th player, we can say that $\mathbf{E[T_4] \sim 2 \times E[T_3]}$ when $k_1, k_2, k_3, k_4 \to \infty$.