

PROBLEM BACKGROUND

“Gambler’s Ruin” Analysis

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- **Problem Background:** Gambler's Ruin
 - **Simulation:** Testing with 2 players
 - **Simulation:** Testing with 3 players
 - **Simulation:** Testing with 4 players
 - **Simulation:** Analysis with 4 players
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“Gambler’s Ruin” Analysis



Question: “How long does it take for the following game to end for 2, 3, or 4 people?”

Game Description:

- Let 3 (or 4) people start with amounts $\$a$, $\$b$, $\$c$, (and $\$d$).
- Randomly choose a pair of people from $\binom{3}{2}$ (or from $\binom{4}{2}$) choice
- Randomly choose a winner, who gains \$1, and a loser, who loses \$1
- Terminate when one of the players loses all their money.

Aim:

- Find $E[T(a, b, c)]$ or $E[T(a, b, c, d)]$, where ‘ T ’ is the completion time

“Gambler’s Ruin” Analysis



We know that for $m = 2, 3$

$$E[T(k_1, k_2)] \xrightarrow{k_1, k_2 \rightarrow \infty} k_1 \times k_2$$

$$E[T(k_1, k_2, k_3)] \xrightarrow{k_1, k_2, k_3 \rightarrow \infty} \frac{3 \times k_1 k_2 k_3}{k_1 + k_2 + k_3}$$

where,

$\overset{\Delta}{m}$ = Number of people

$\overset{\Delta}{T}$ = Termination Time

But what if $m > 3$?



SIMULATION RESULTS

Section 1: Testing with 2 players

Section 1: Testing with 2 players

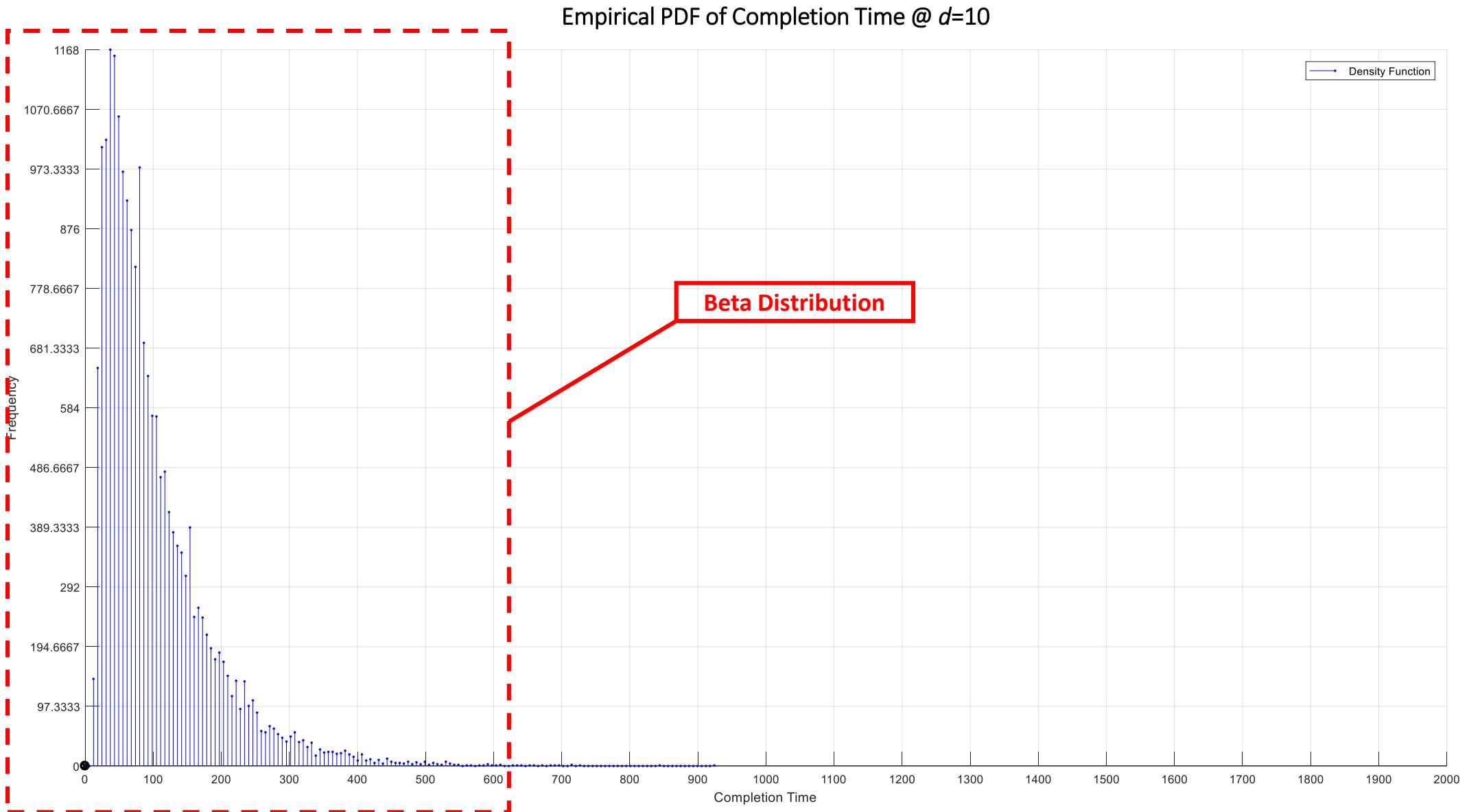


We perform a simulation with:

$$[a, b] = [\$10, \$10]$$

Thus, with $E[T] = ab = 100$, we performed around 2×10^6 gambles.

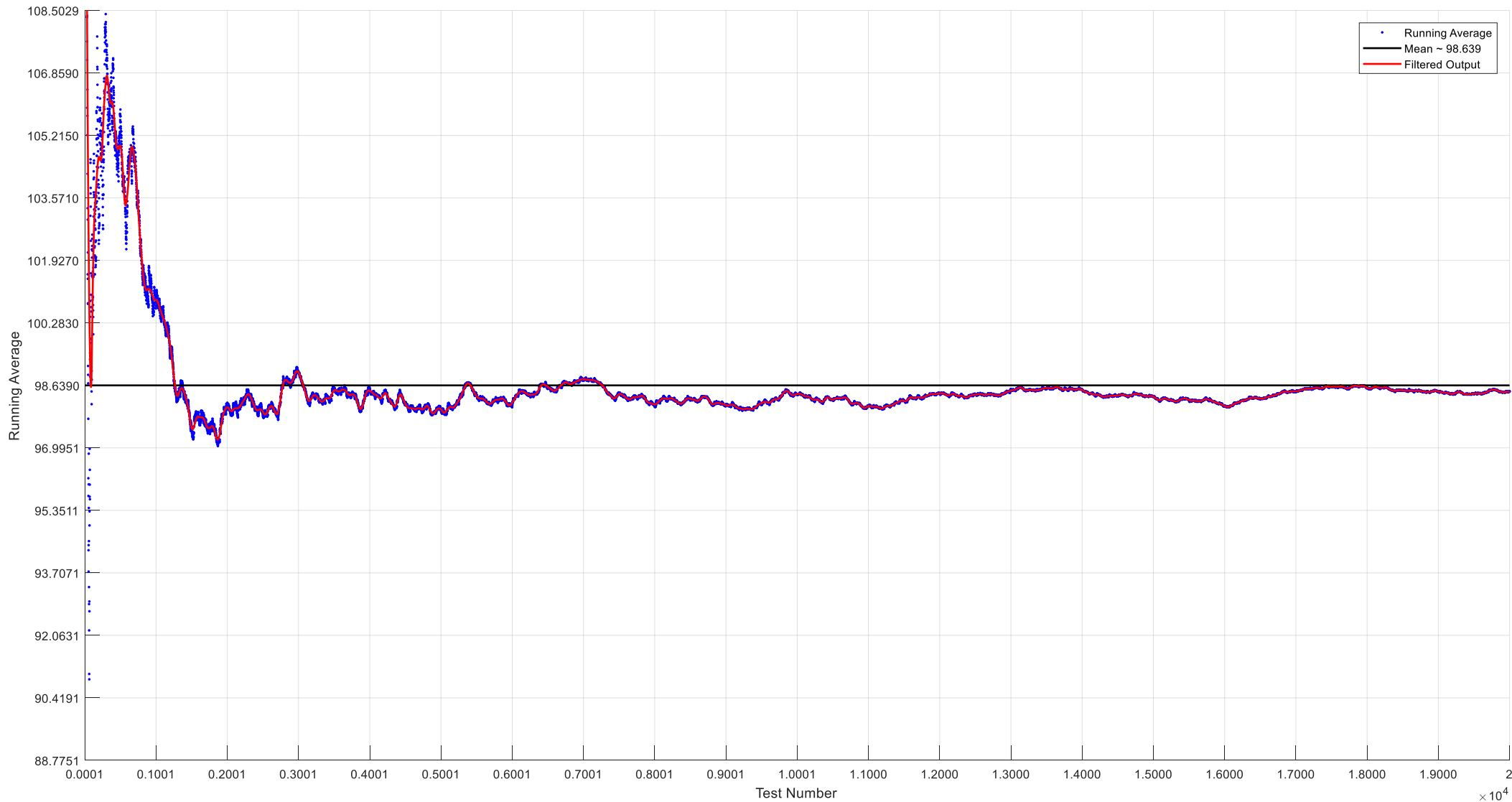
Section 1: Testing with 2 players



Section 1: Testing with 2 players



Running Average of Completion Time @ $d=10$





SIMULATION RESULTS

Section 2: Testing with 3 players

Section 2: Testing with 3 players

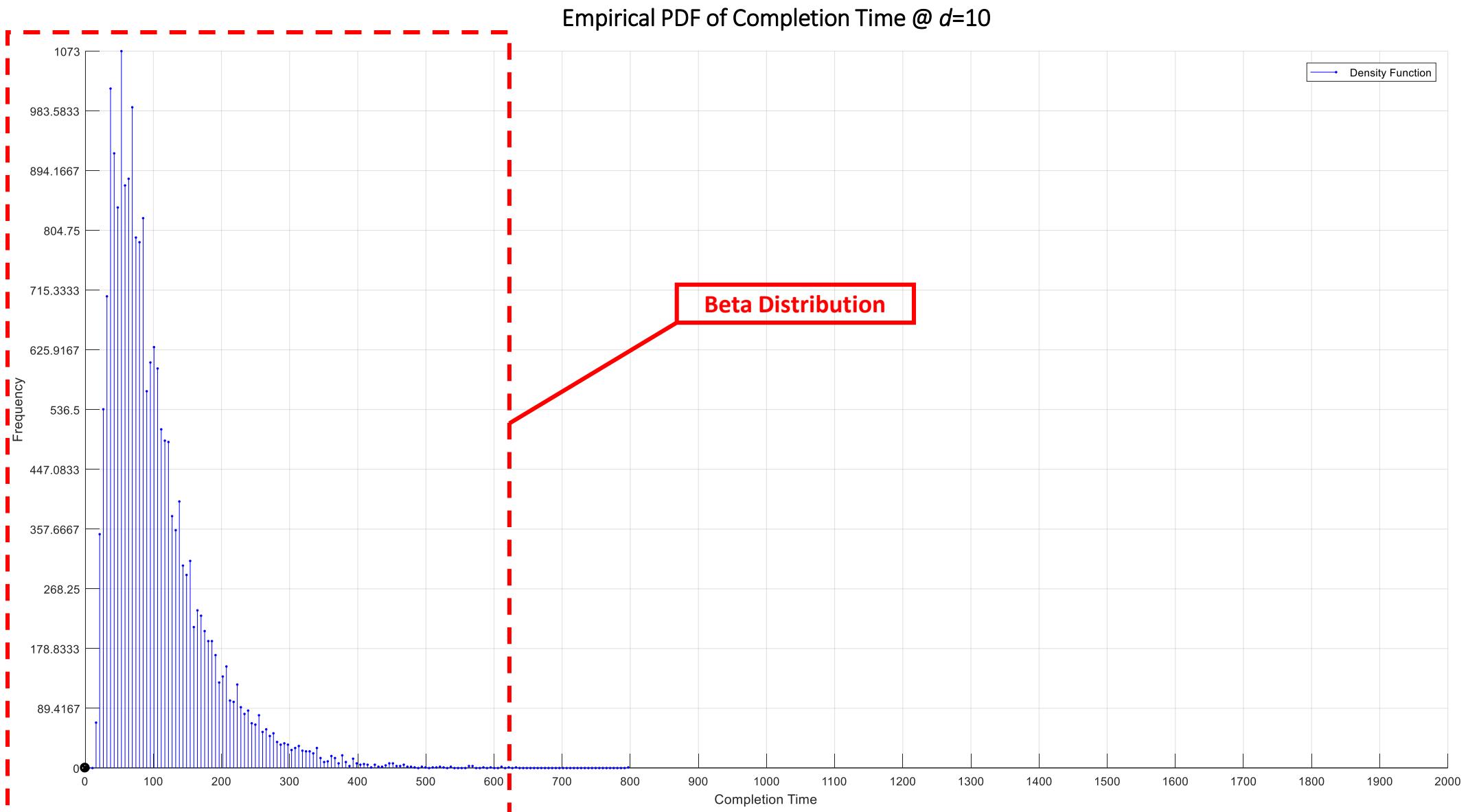


We perform a simulation with:

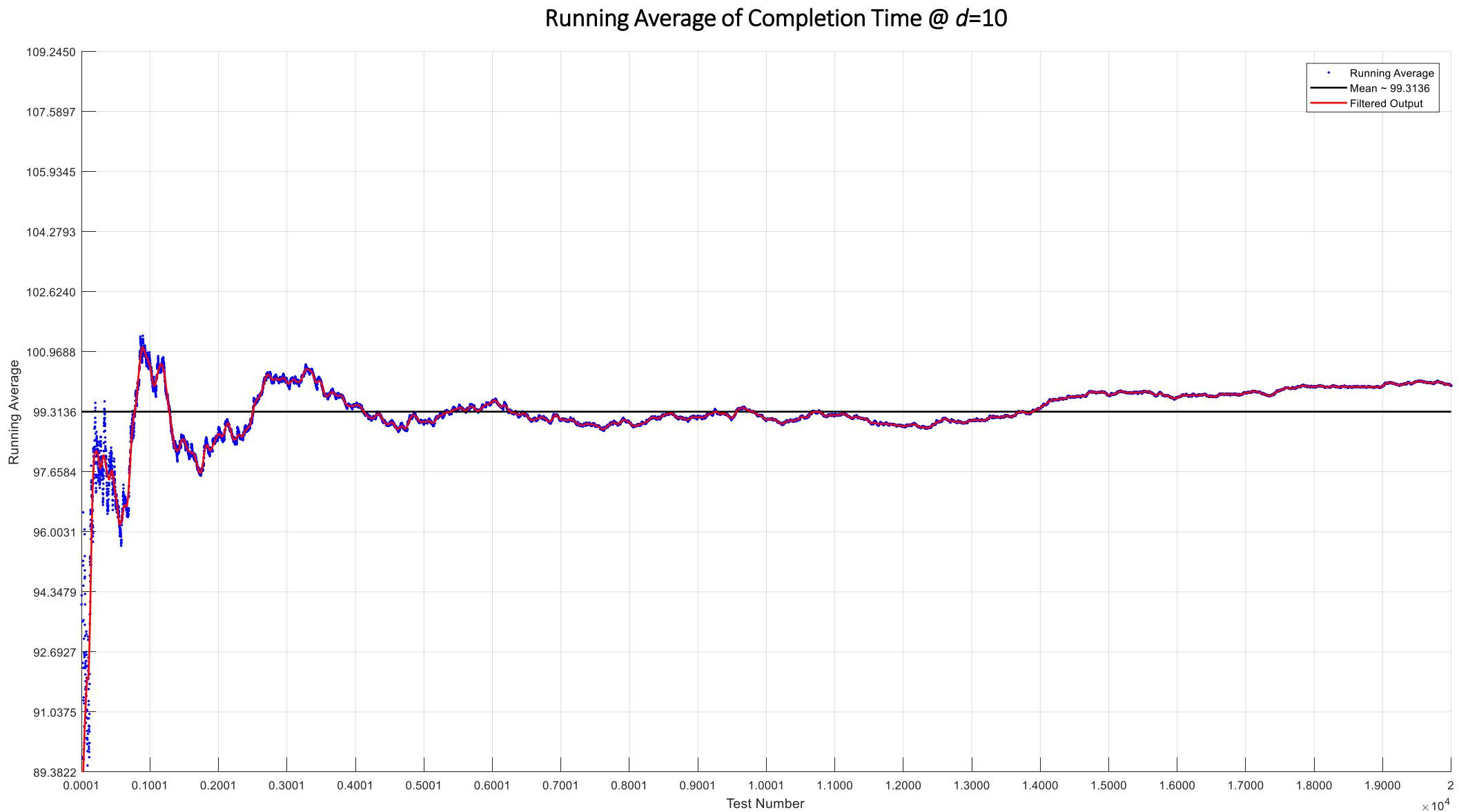
$$[a, b, c] = [\$10, \$10, \$10]$$

Thus, with $E[T] = \frac{3abc}{a+b+c} = 100$, we performed around 2×10^6 gambles.

Section 2: Testing with 2 players



Section 2: Testing with 2 players





SIMULATION RESULTS

Section 3: Testing with 4 players

Section 3: Testing with 4 players



We perform a simulation with:

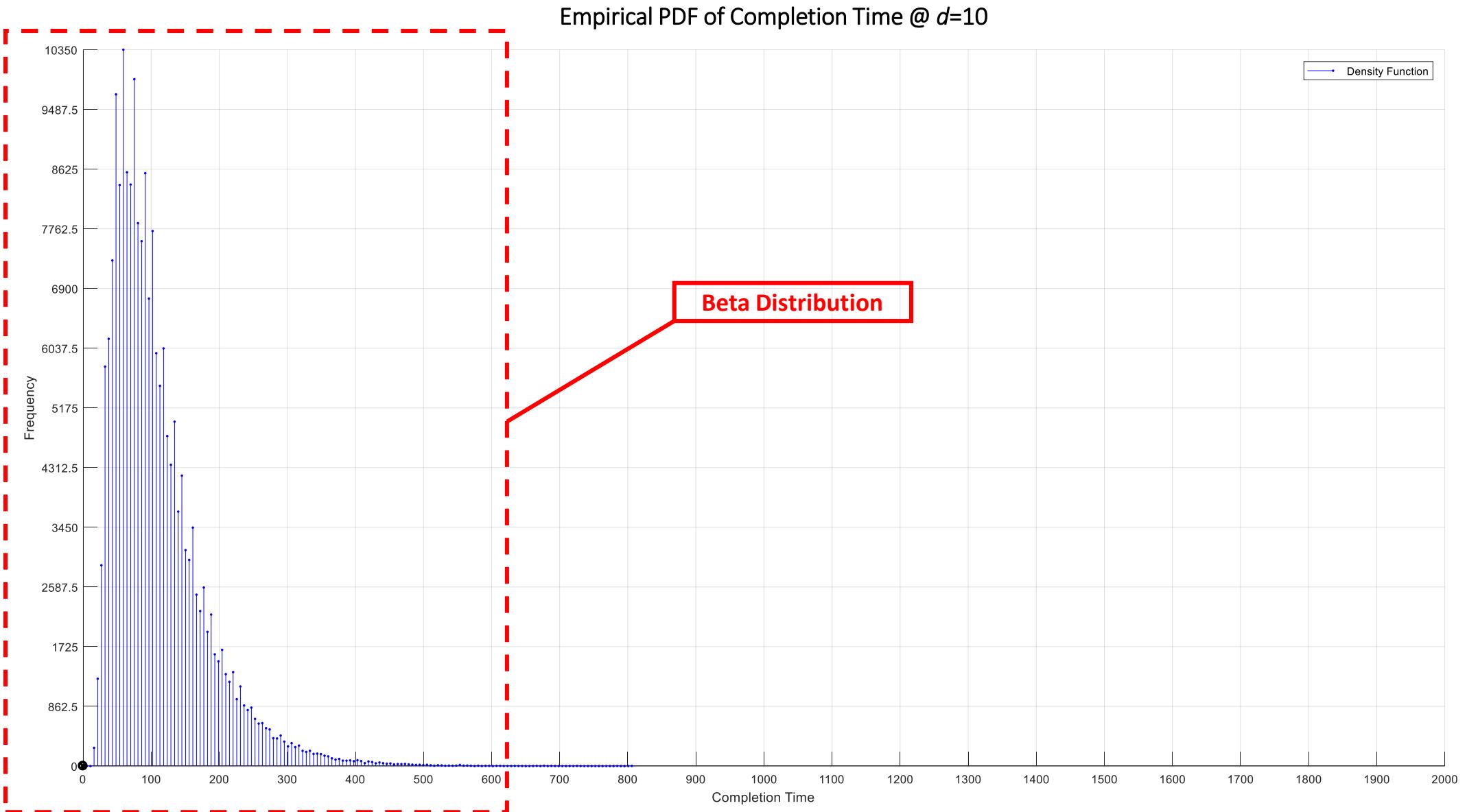
$$[a, b, c, d] = [\$10, \$10, \$10, k]$$

where,

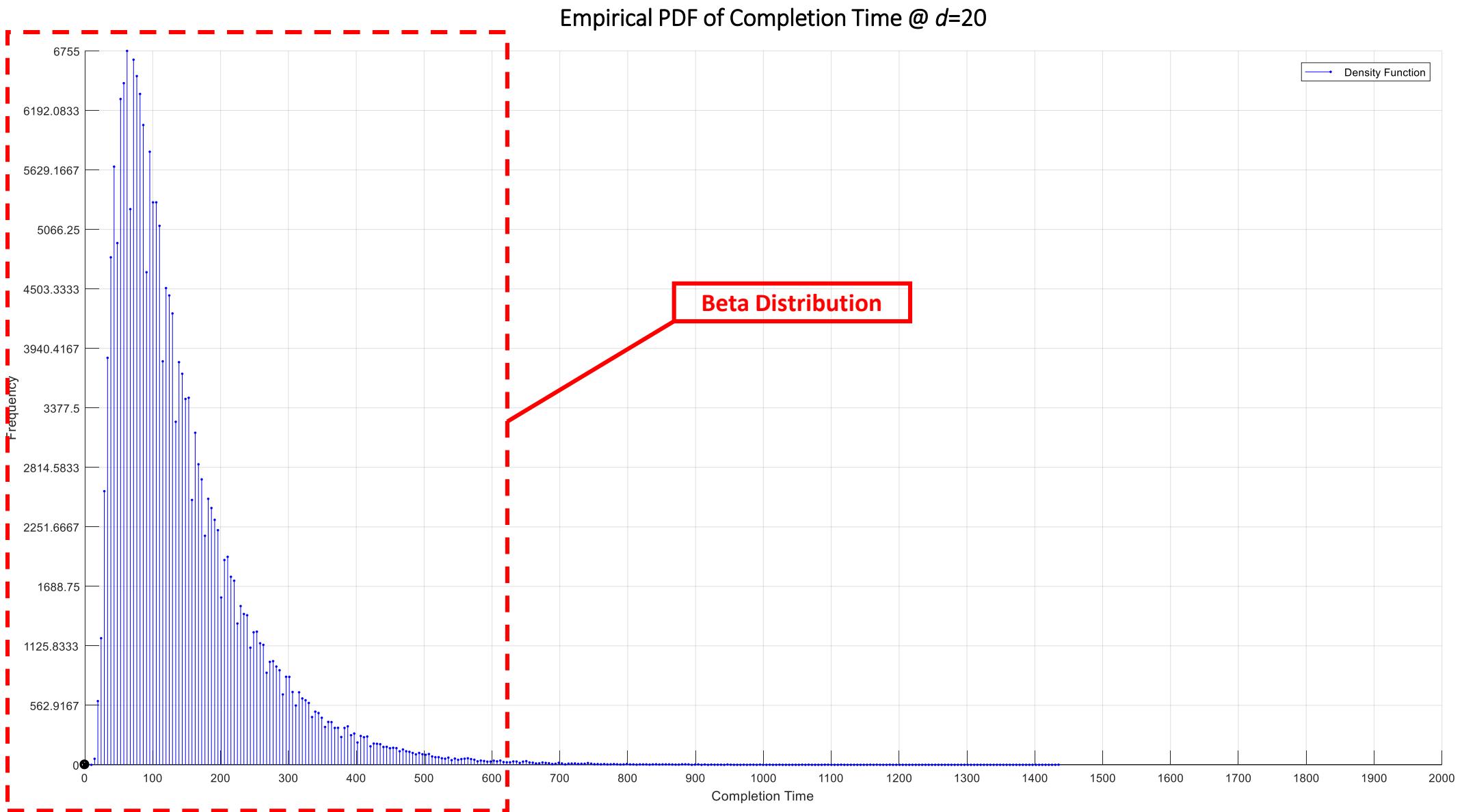
$$k \in \{\$10, \$20, \$40, \$80, \$160\}$$

Thus, with $E[T] \sim 160$, we performed around 3.8×10^8 gambles.

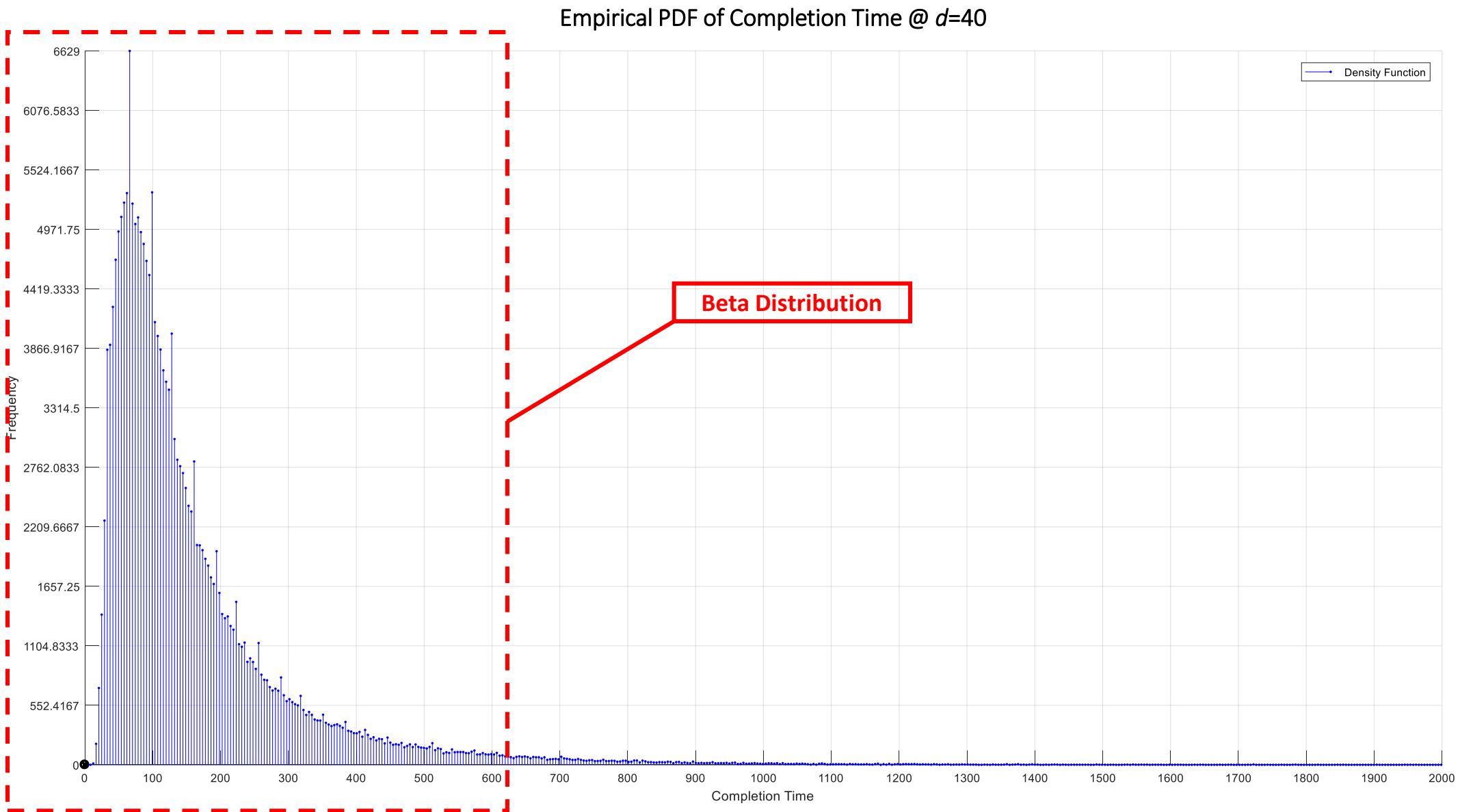
Section 3: Testing with 4 players



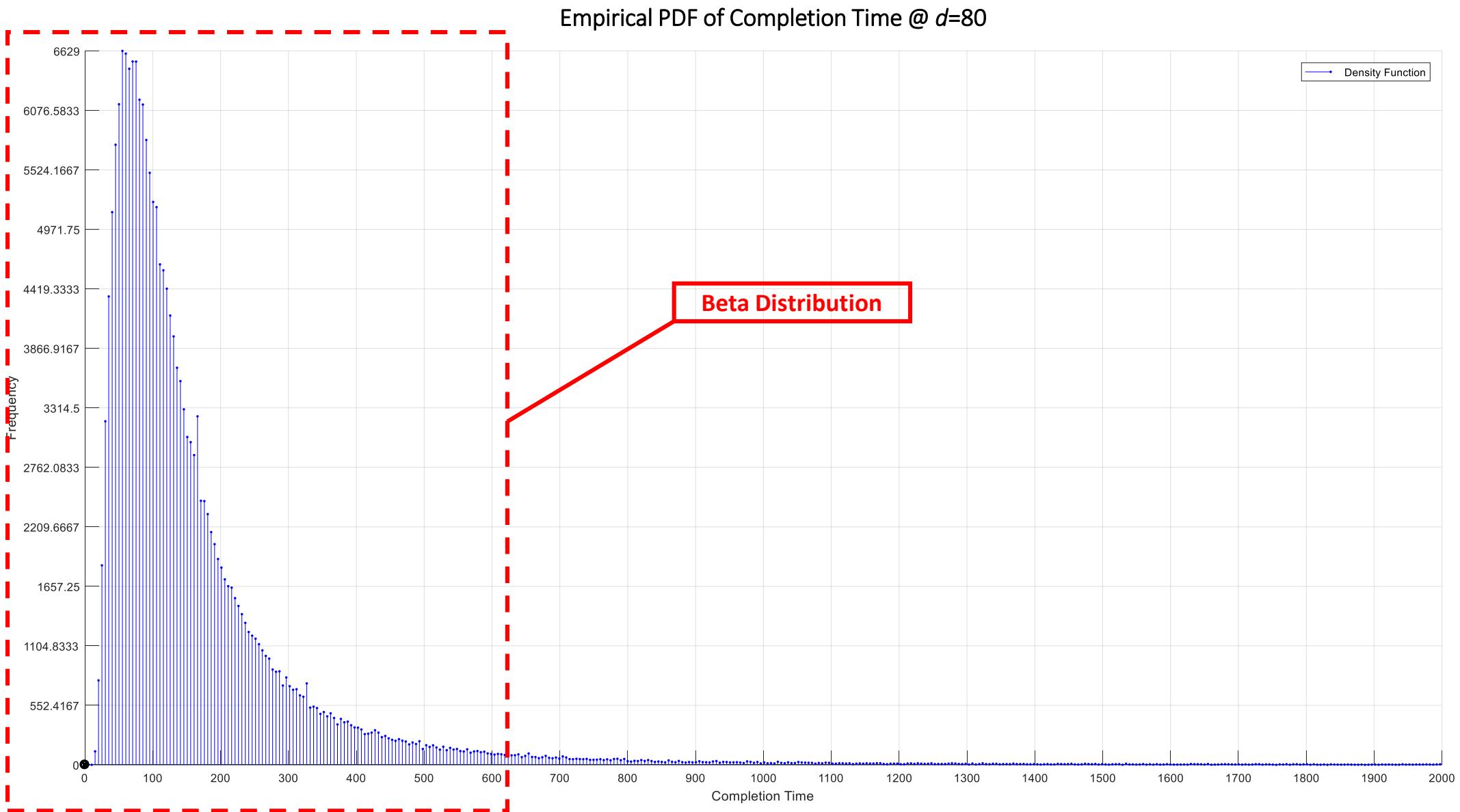
Section 3: Testing with 4 players



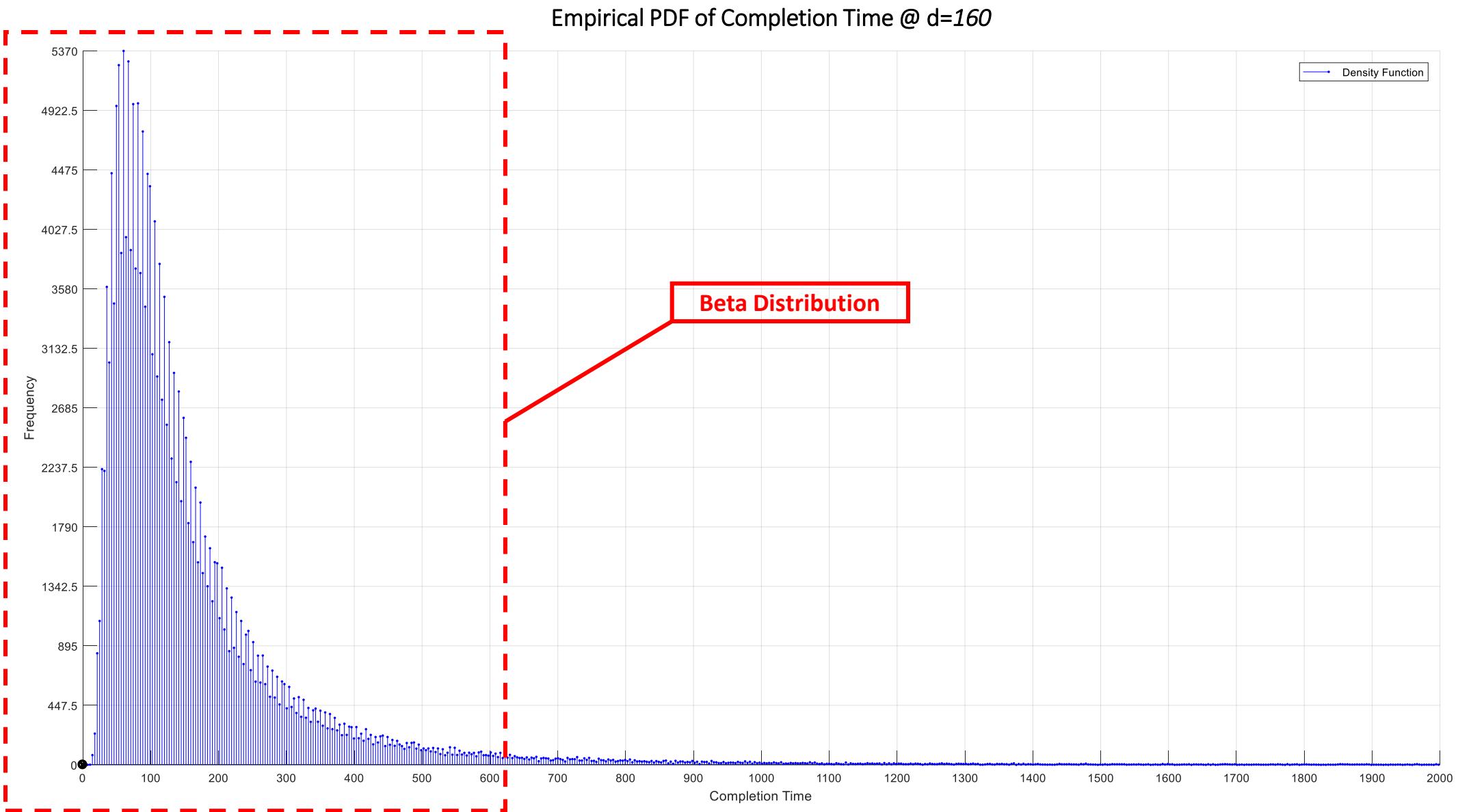
Section 3: Testing with 4 players



Section 3: Testing with 4 players



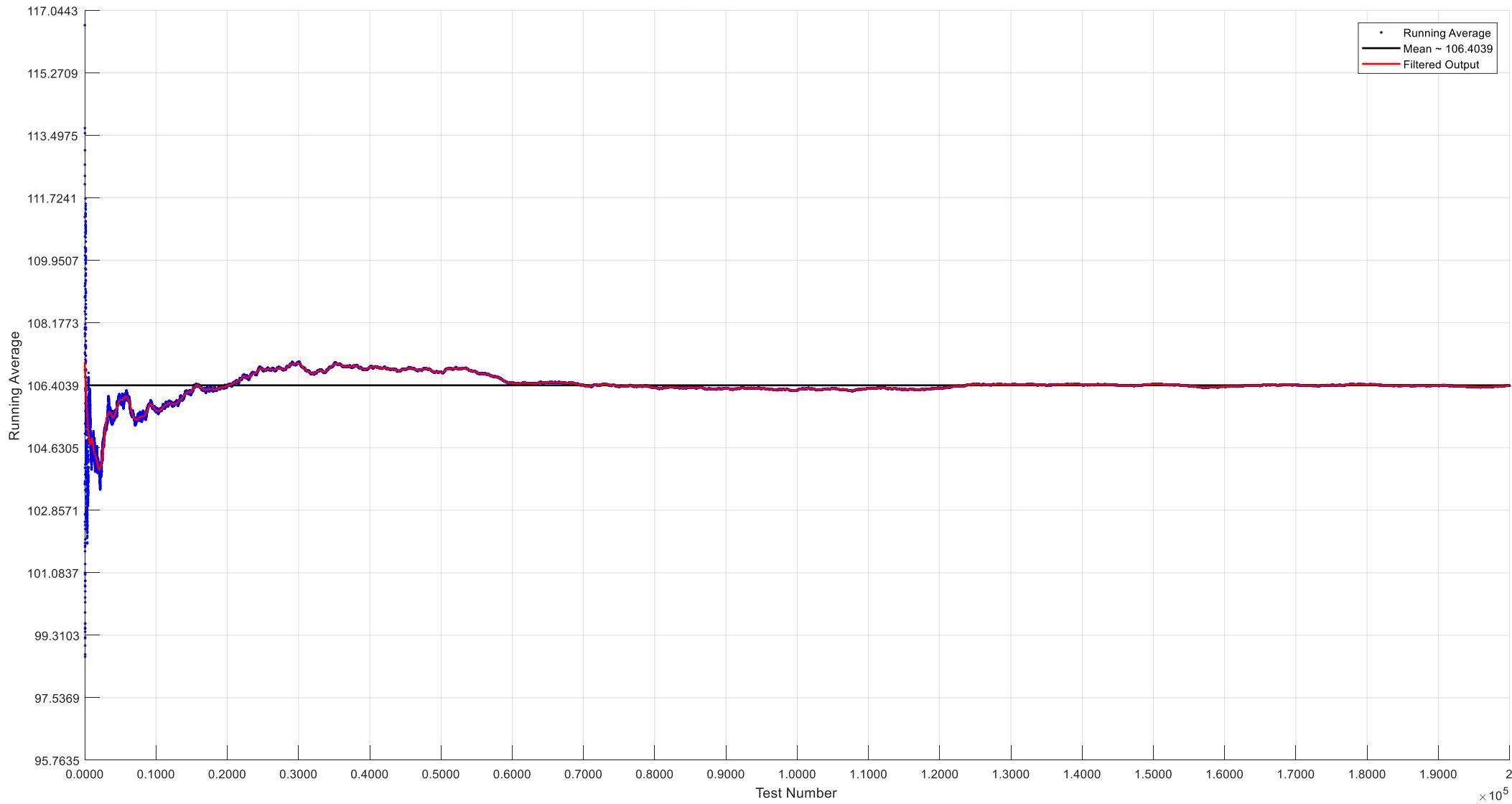
Section 3: Testing with 4 players



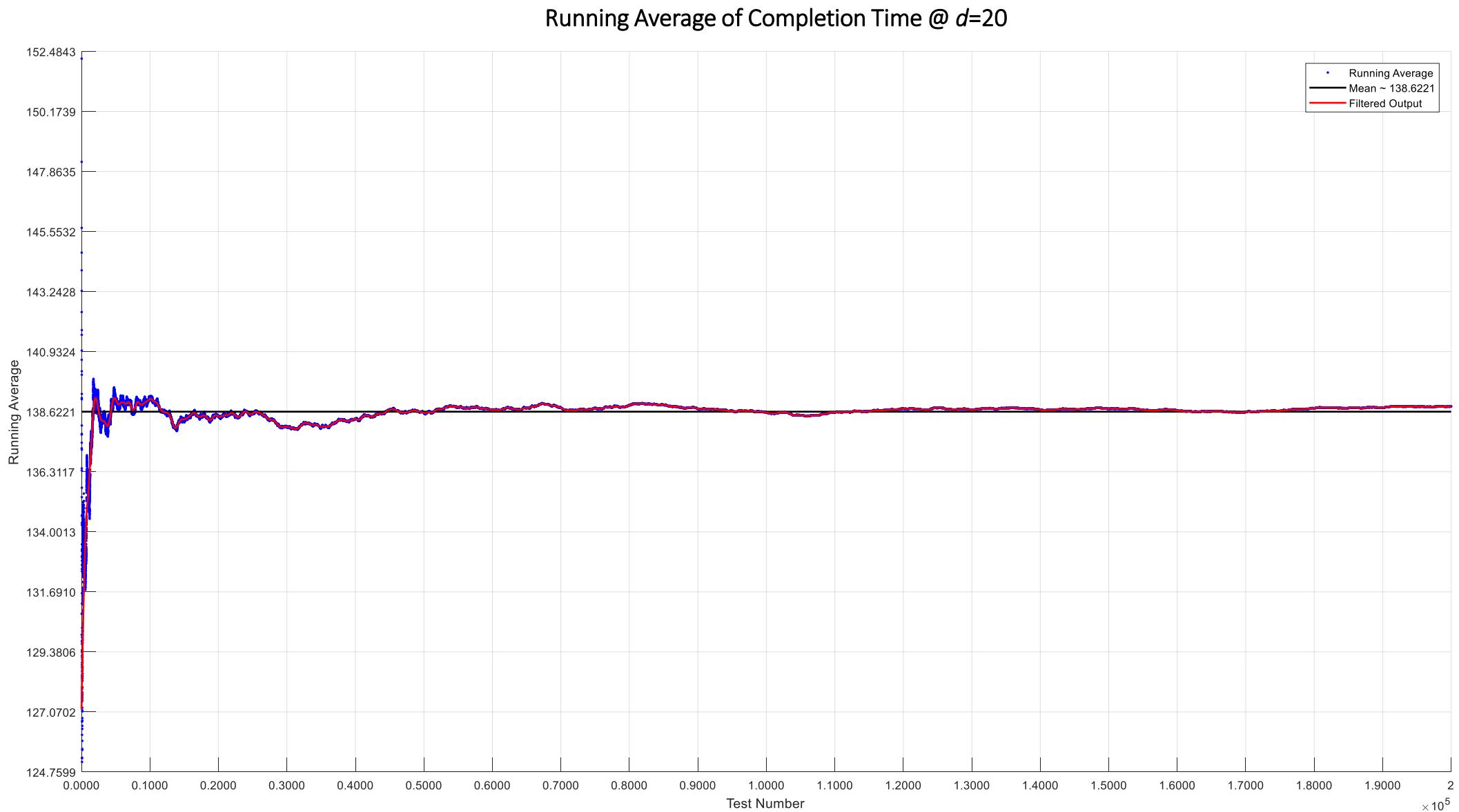
Section 3: Testing with 4 players



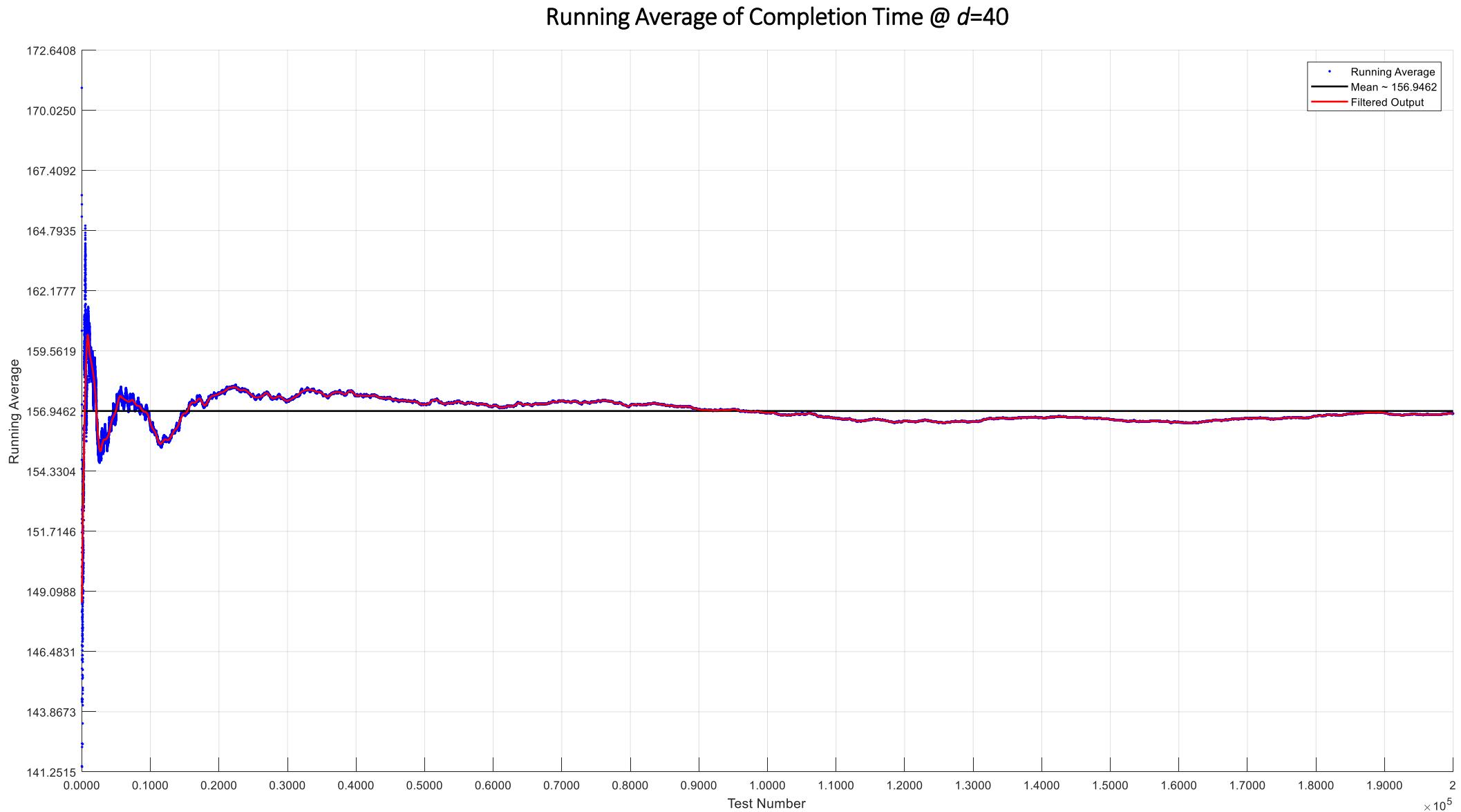
Running Average of Completion Time @ $d=10$



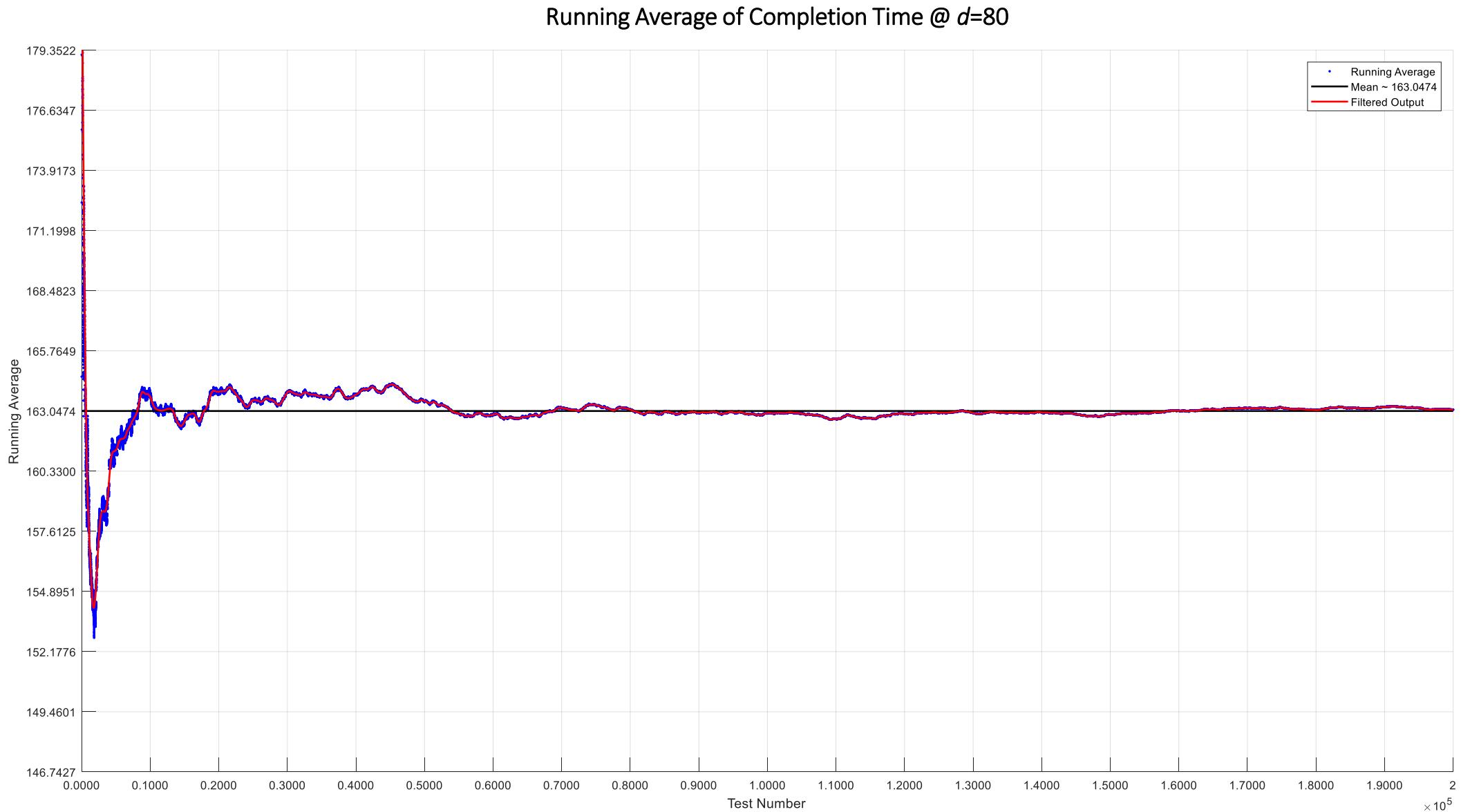
Section 3: Testing with 4 players



Section 3: Testing with 4 players



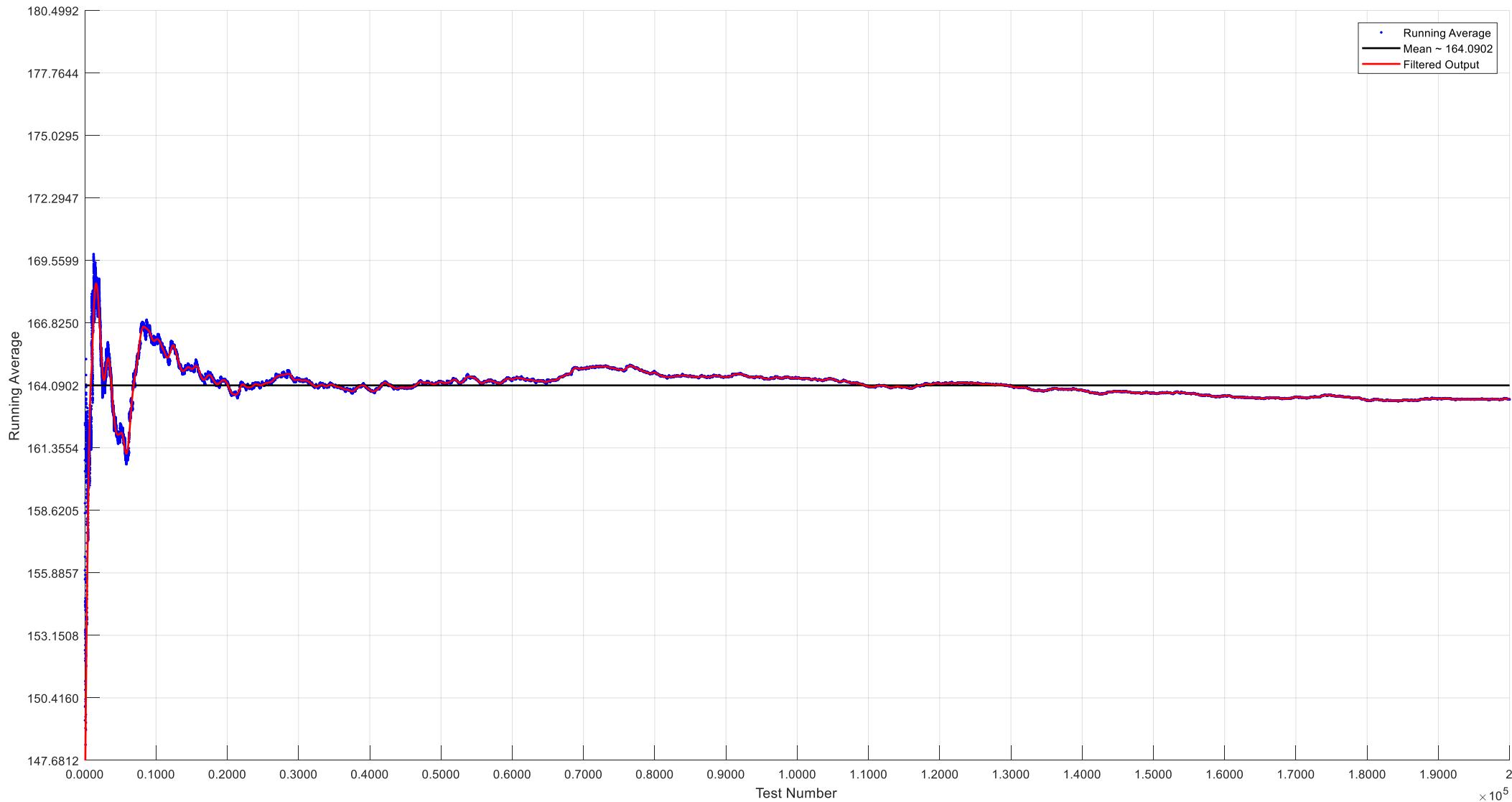
Section 3: Testing with 4 players



Section 3: Testing with 4 players



Running Average of Completion Time @ $d=160$





SIMULATION RESULTS

Section 4: Analysis with 4 players

Section 4: Analysis with 4 players



Question: “Why is $E[T_4] > E[T_3]$? ”

Possible Reason:

- In the 4 player case, there are $\binom{4}{2} = 6$ pairs, of which $\binom{3}{1} = 3$ include the 4th player.
- On average, the 4th player contributes and takes away $\sim \$0$ from each player as $k_1, k_2, k_3, k_4 \rightarrow \infty$.
- Thus, case with 4th player is *very similar* to the case with 3rd player, given $k_1, k_2, k_3, k_4 \rightarrow \infty$.
- We can consider any gamble involving the 4th player as a *wasted turn*.
- Since $\sim 3/6$ random pairs include the 4th player, we can say that $E[T_4] > E[T_3]$ when $k_1, k_2, k_3, k_4 \rightarrow \infty$.