

PROBLEM BACKGROUND

Queue Simulation

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- **Problem Background:** Queue Simulation
 - **Simulation:** Variable n_D
 - **Simulation:** Variable n_S
 - **Simulation:** Variable n_D, n_S
 - **Simulation:** Service Duration Distribution Function
 - **Simulation:** Wait Duration Distribution Function
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Queue Simulation



We aim to perform the simulation
under this constraint:

$$\frac{\lambda_D}{\lambda_S} = \frac{1}{2}$$

$S \stackrel{\Delta}{=} \text{Sum of Exponential Random Variables for Service Time}$

$D \stackrel{\Delta}{=} \text{Sum of Exponential Random Variables for Inter-arrival Delay}$

Queue Simulation



Let,

$$S \stackrel{\Delta}{=} \sum_{k=1}^{n_S} S_k, \quad D \stackrel{\Delta}{=} \sum_{k=1}^{n_D} D_k$$

with,

$$S_k \sim f_{S_k}(x) = \lambda_{S_k} e^{-\lambda_{S_k} x} \cdot u(x)$$

$$D_k \sim f_{D_k}(x) = \lambda_{D_k} e^{-\lambda_{D_k} x} \cdot u(x)$$

where,

$$\lambda_{S_k} = \frac{\lambda_S}{n_S}, \quad \lambda_{D_k} = \frac{\lambda_D}{n_D}$$



$$S \sim * f_{S_k}(x) = \frac{\lambda_{S_k}^{n_S}}{(n_S - 1)!} \cdot x^{(n_S - 1)} e^{-\lambda_{S_k} x} \cdot u(x)$$

$$D \sim * f_{D_k}(x) = \frac{\lambda_{D_k}^{n_D}}{(n_D - 1)!} \cdot x^{(n_D - 1)} e^{-\lambda_{D_k} x} \cdot u(x)$$



SIMULATION RESULTS

Section 1: Variable n_D

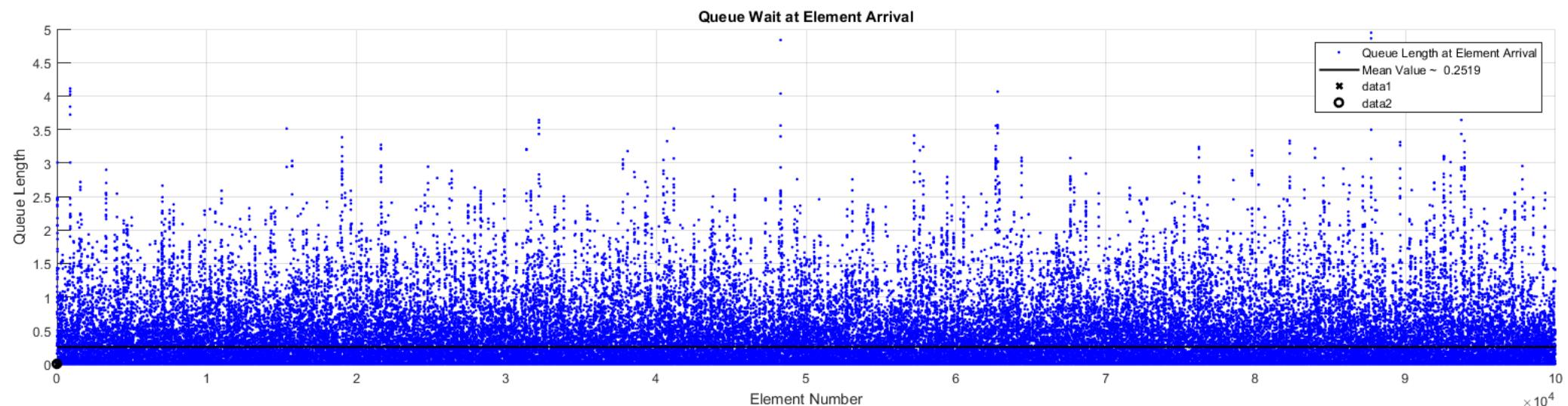
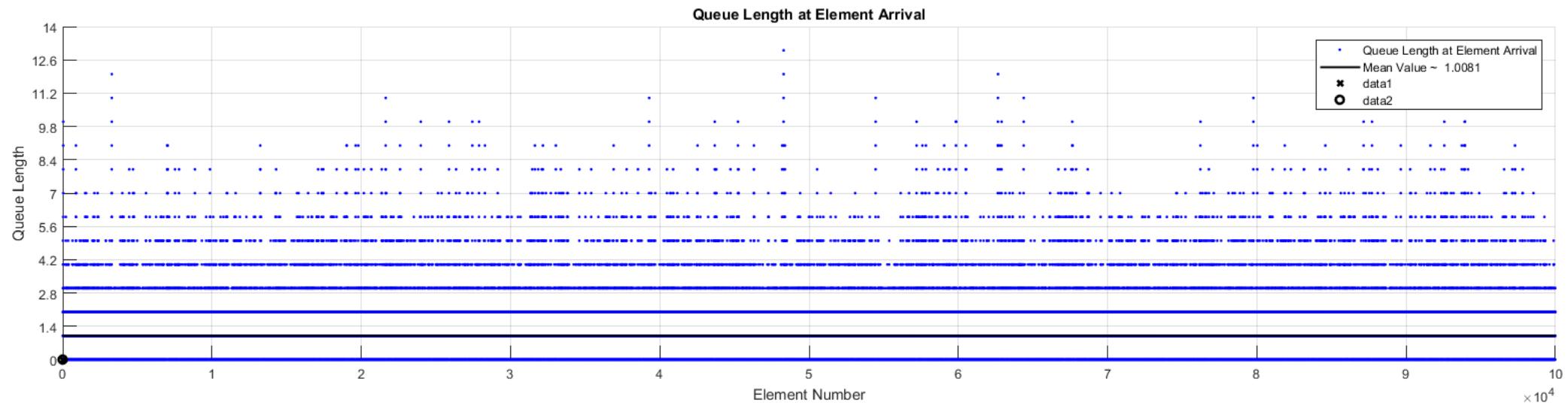
Section 1: Variable n_D



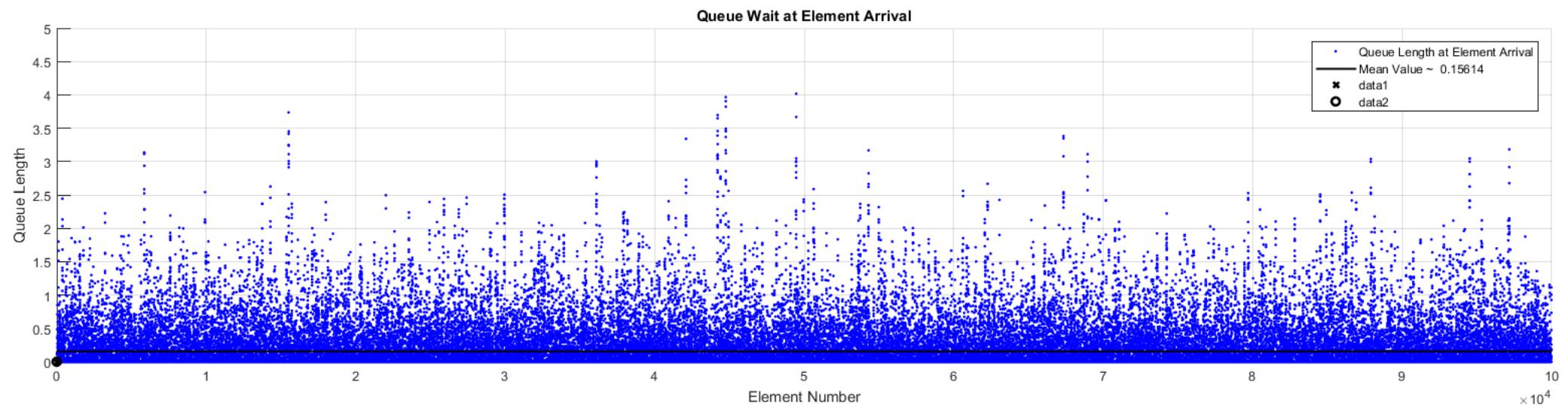
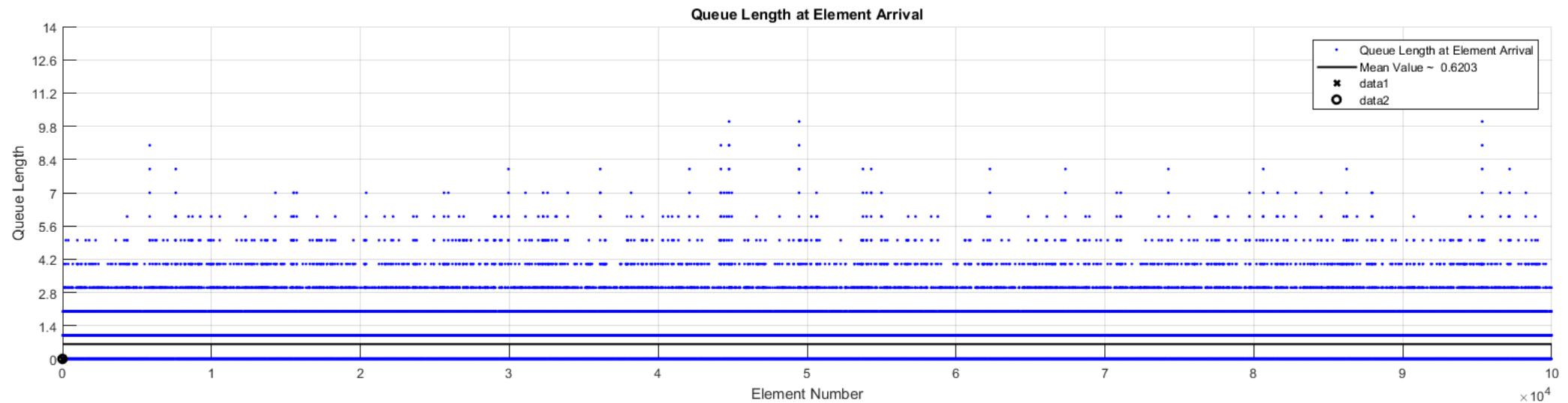
We iterate through a range of n values with $\lambda_D^{-1} = 0.5$, $\lambda_S^{-1} = 0.25$:

n_D	Number of Points	Mean Queue Length	Mean Queue Wait Time
2^0	100,000	1.0081	0.2519
2^1	100,000	0.6203	0.15614
2^2	100,000	0.43436	0.10816
2^3	100,000	0.34816	0.088109
2^4	100,000	0.30009	0.074664
2^5	100,000	0.27533	0.068502
2^6	100,000	0.27046	0.067925
2^7	100,000	0.25605	0.063816
2^8	100,000	0.25729	0.063594
2^9	100,000	0.2577	0.064077

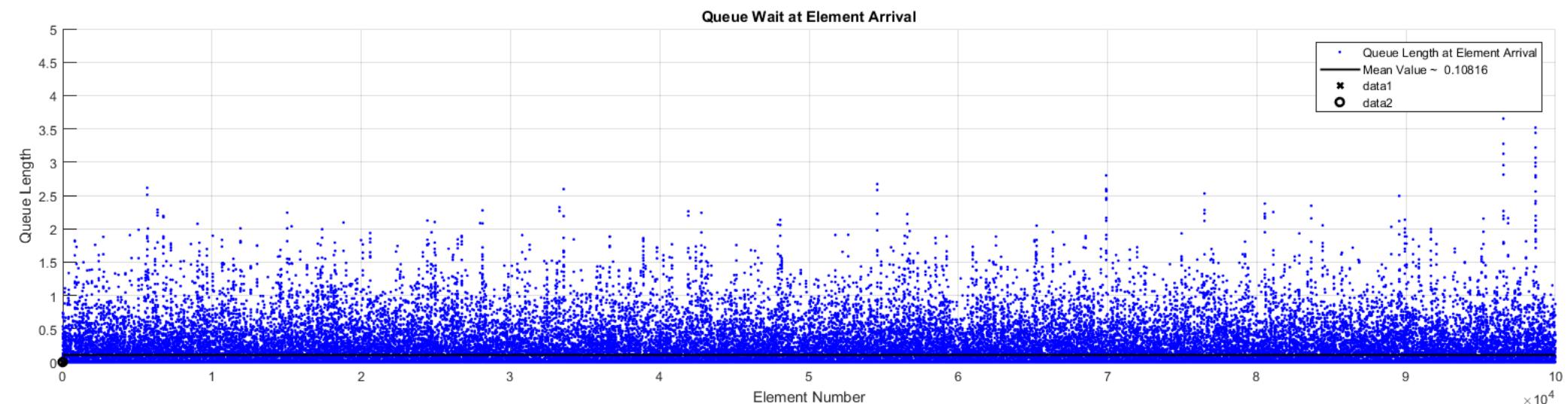
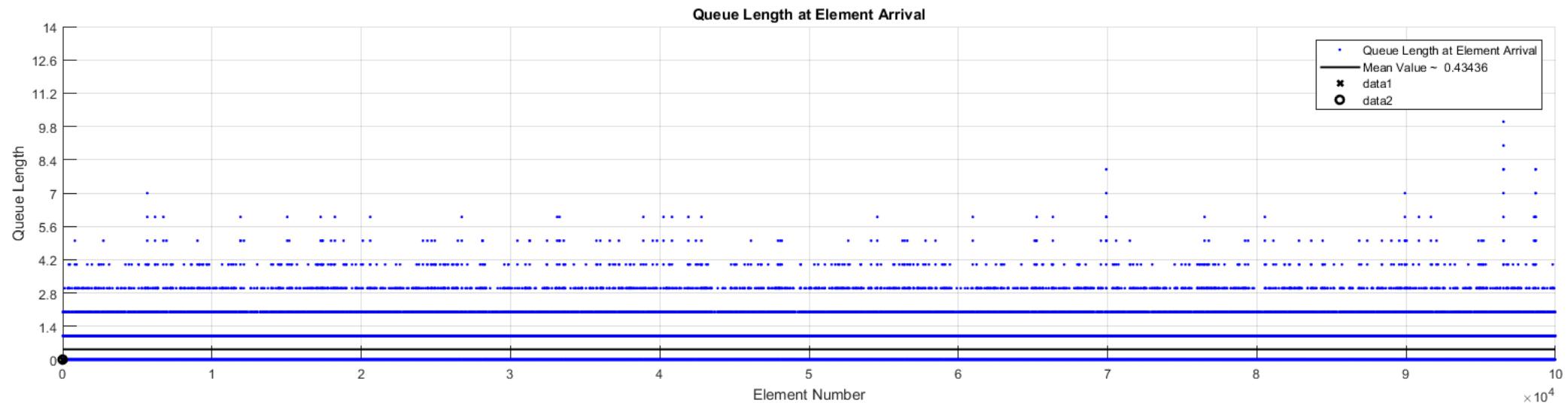
Section 1: Variable n_D



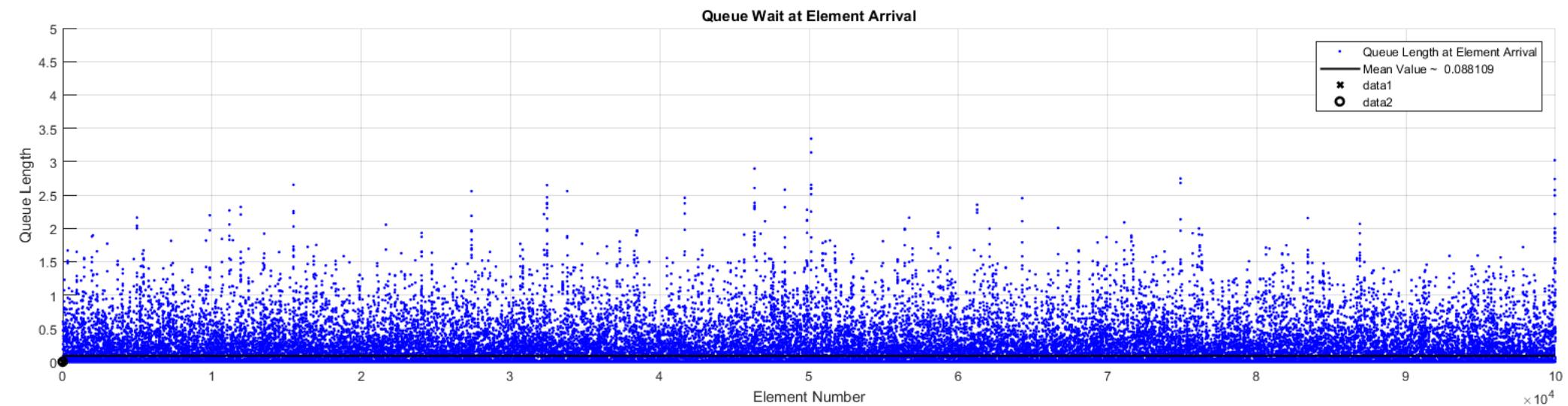
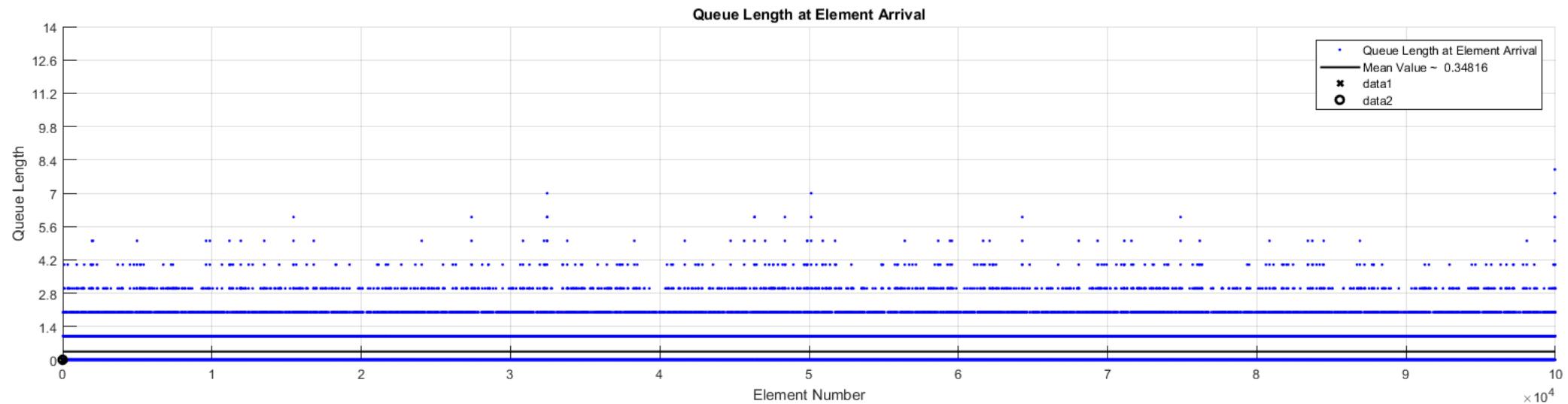
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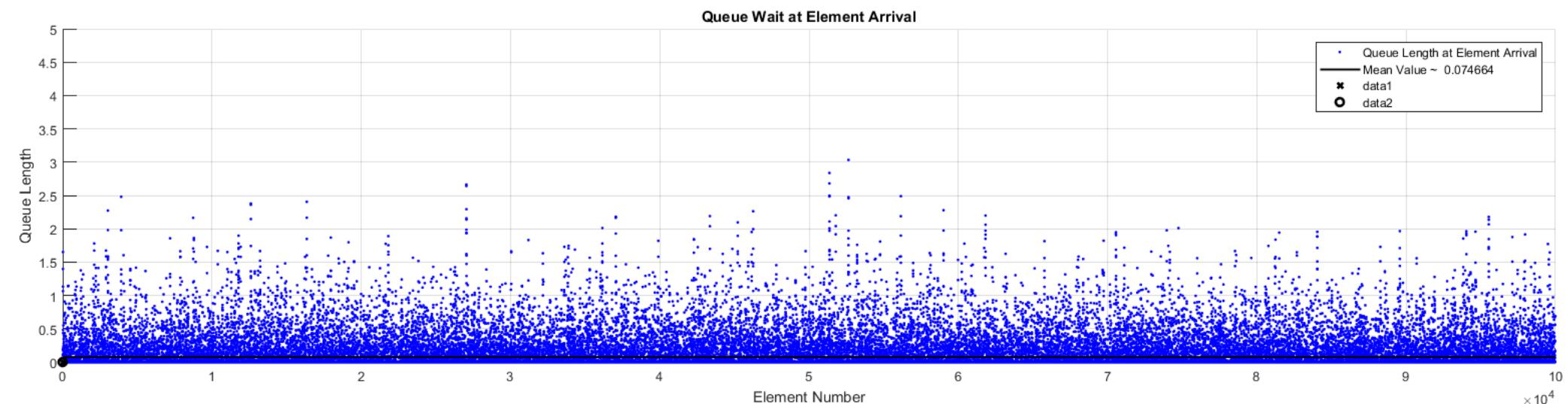
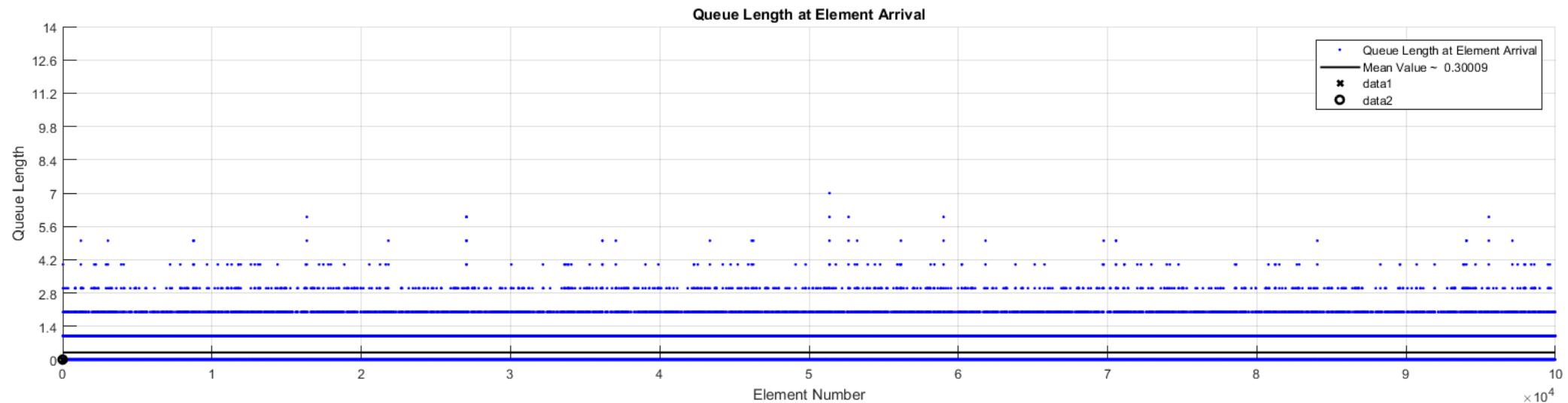
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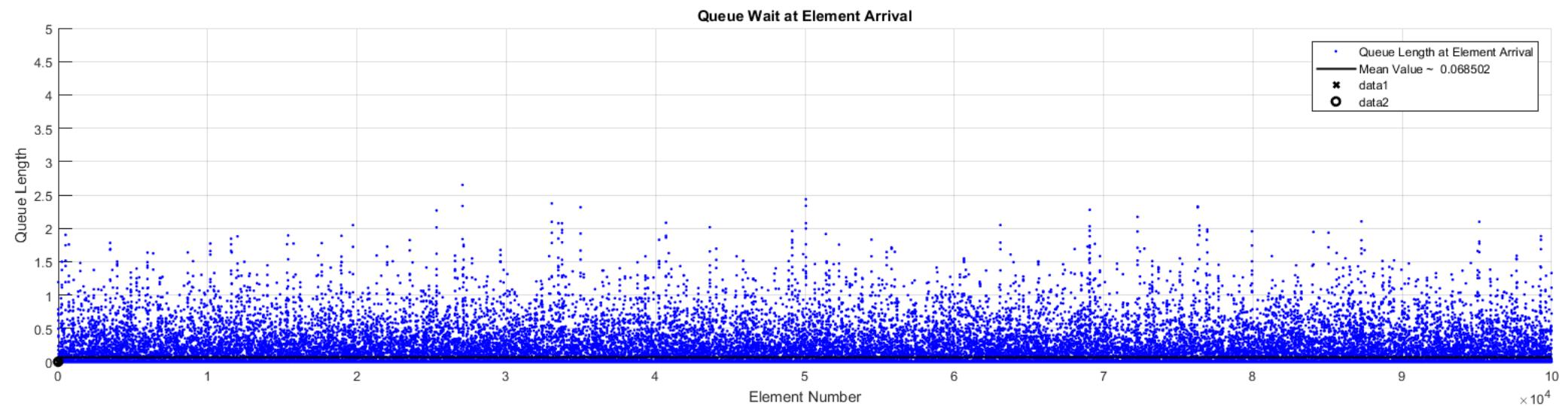
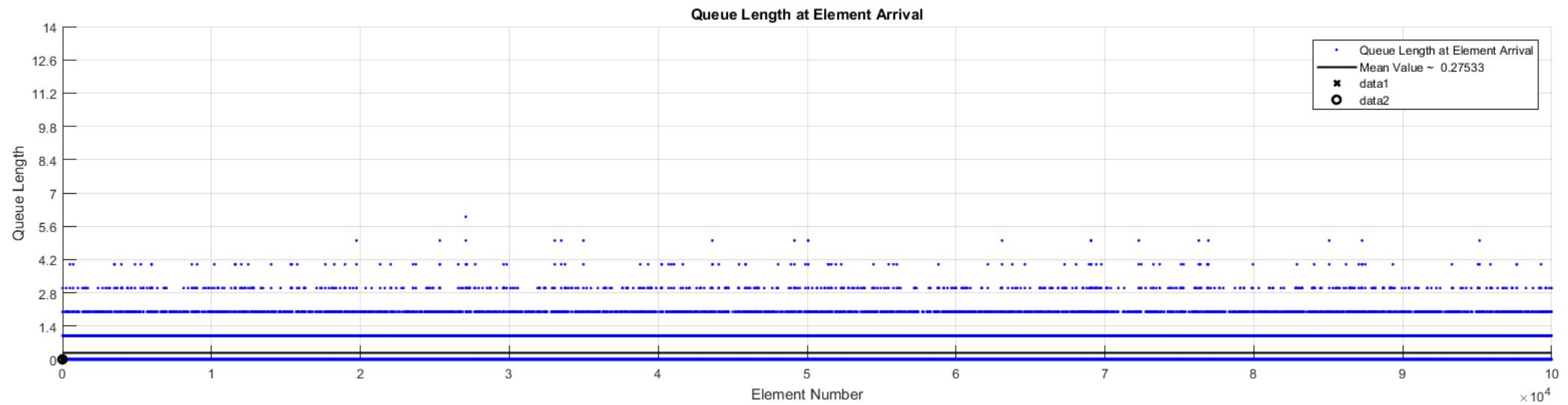
Section 1: Variable n_D



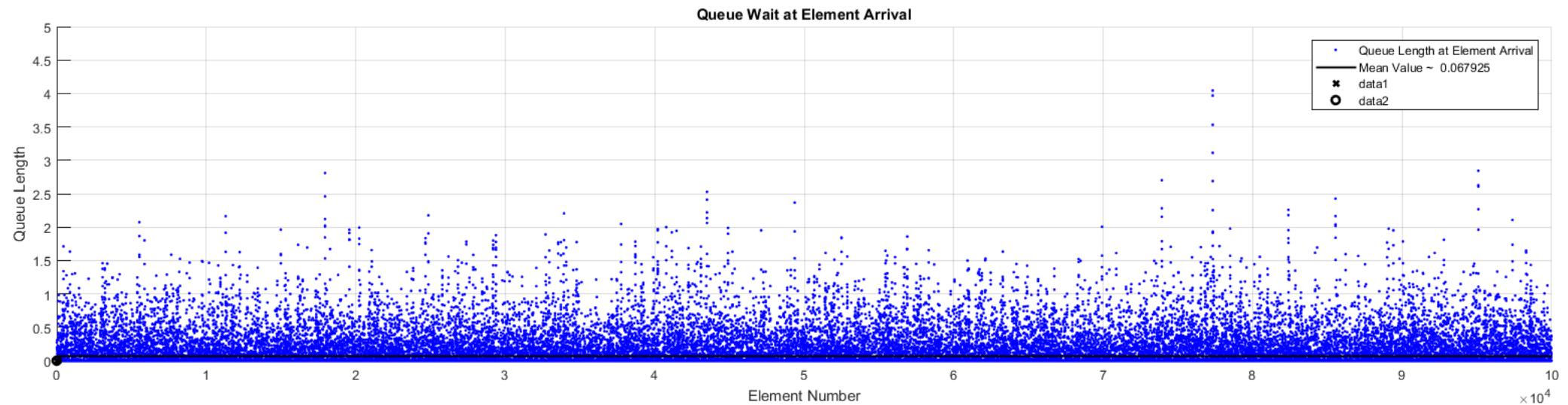
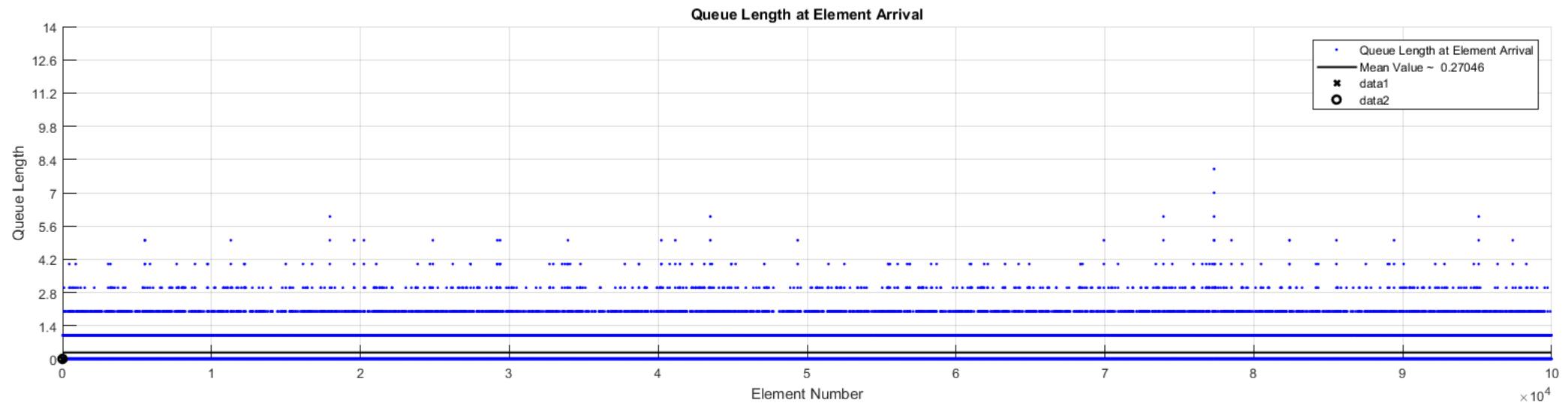
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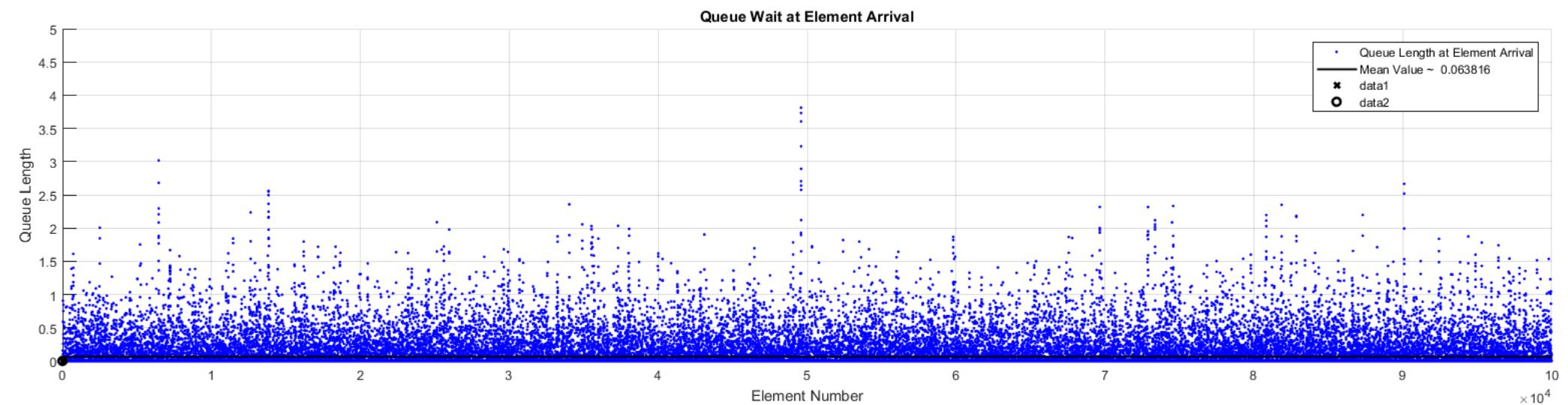
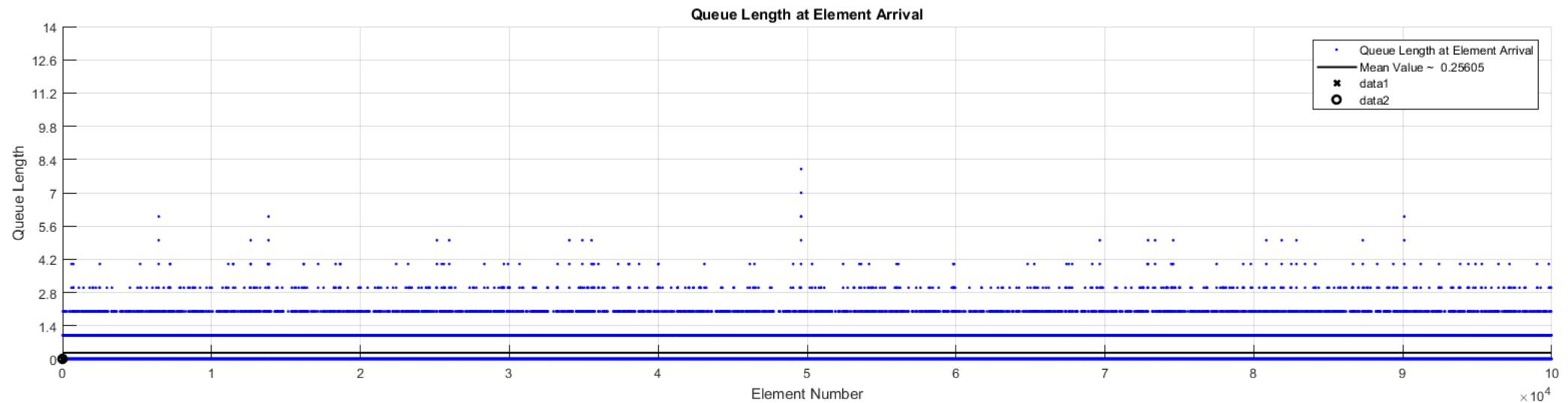
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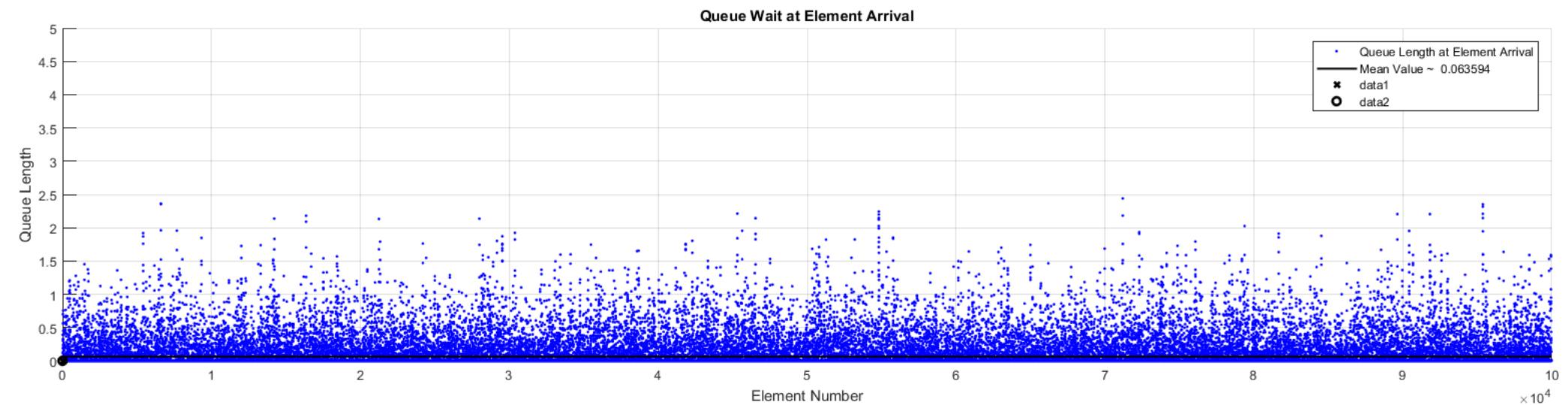
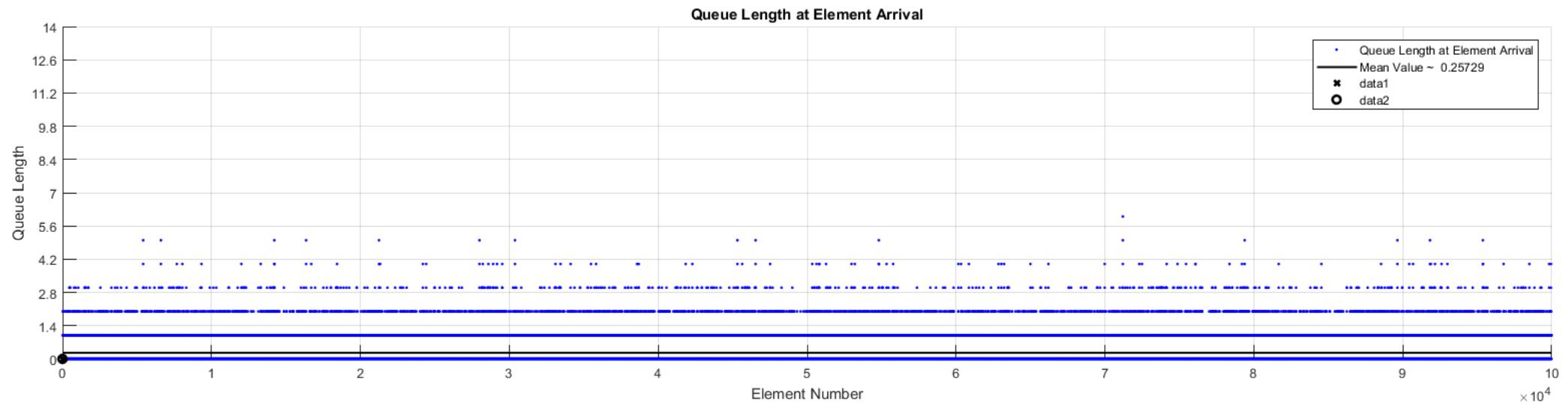
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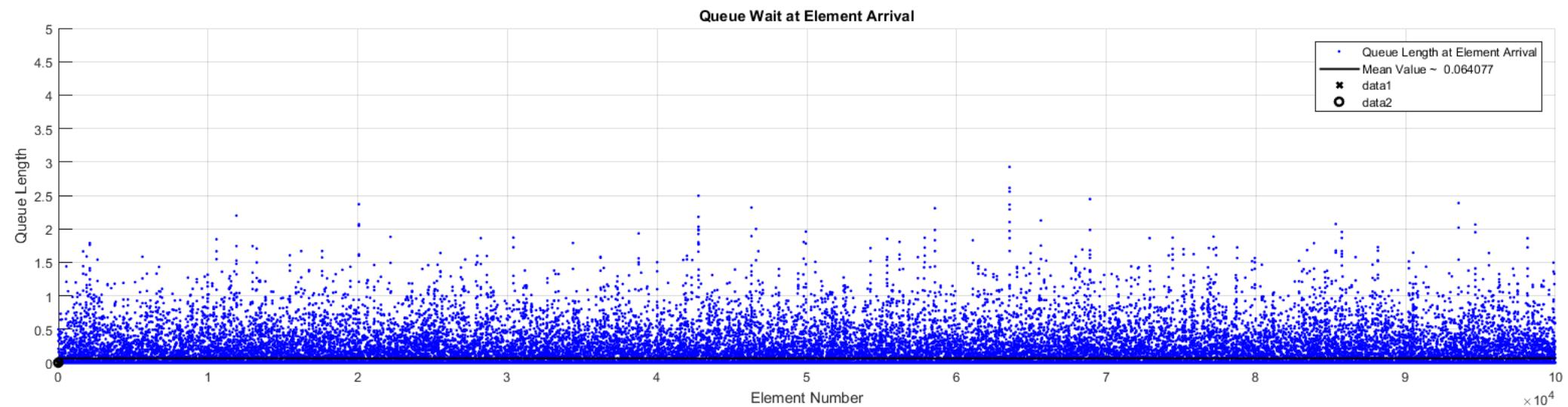
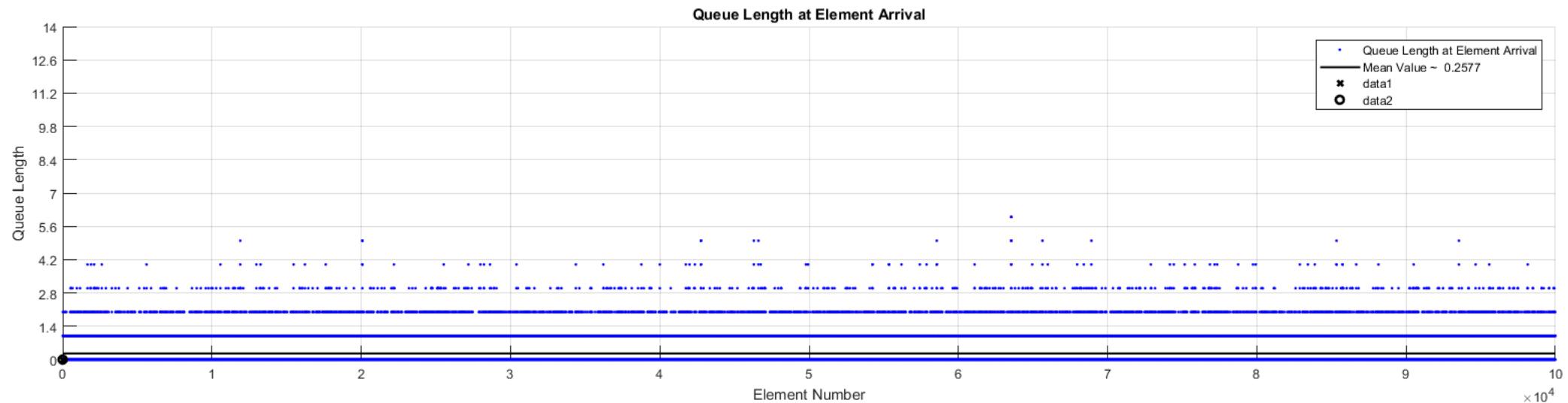
Section 1: Variable n_D



Section 1: Variable n_D



Section 1: Variable n_D





SIMULATION RESULTS

Section 2: Variable n_s

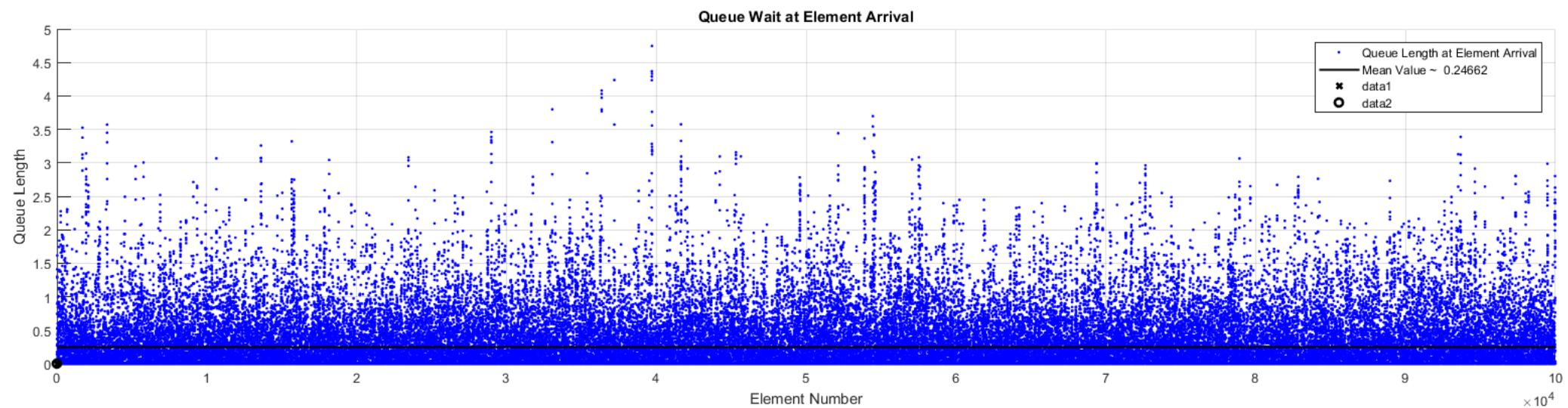
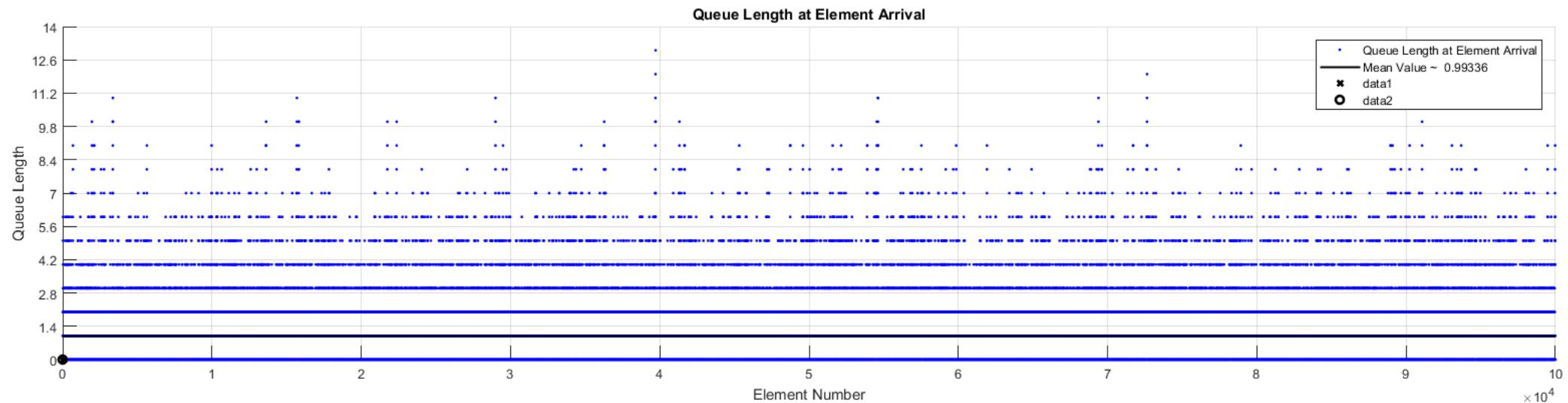
Section 2: Variable n_S



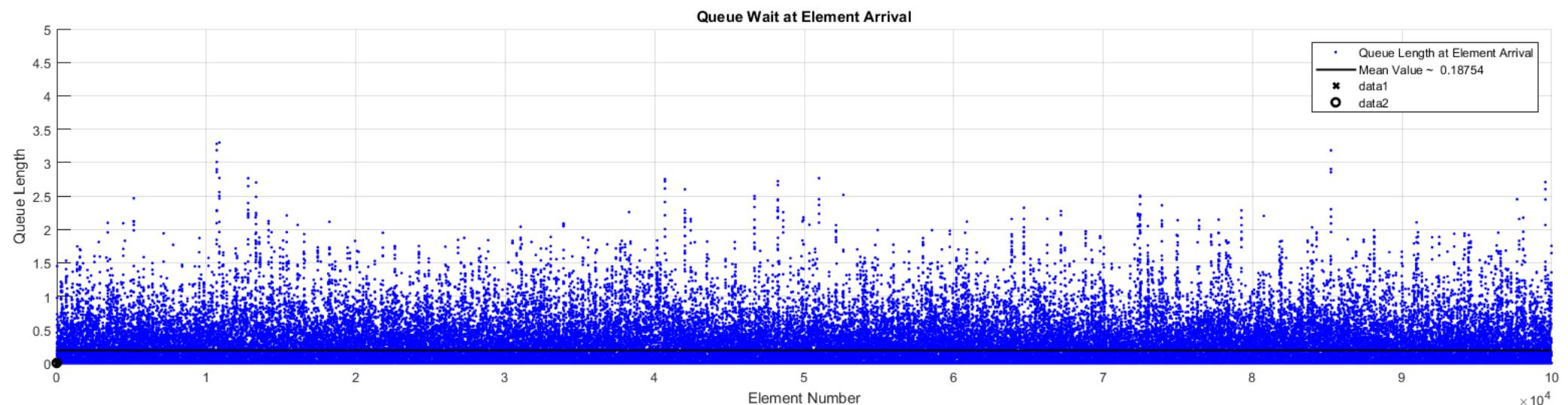
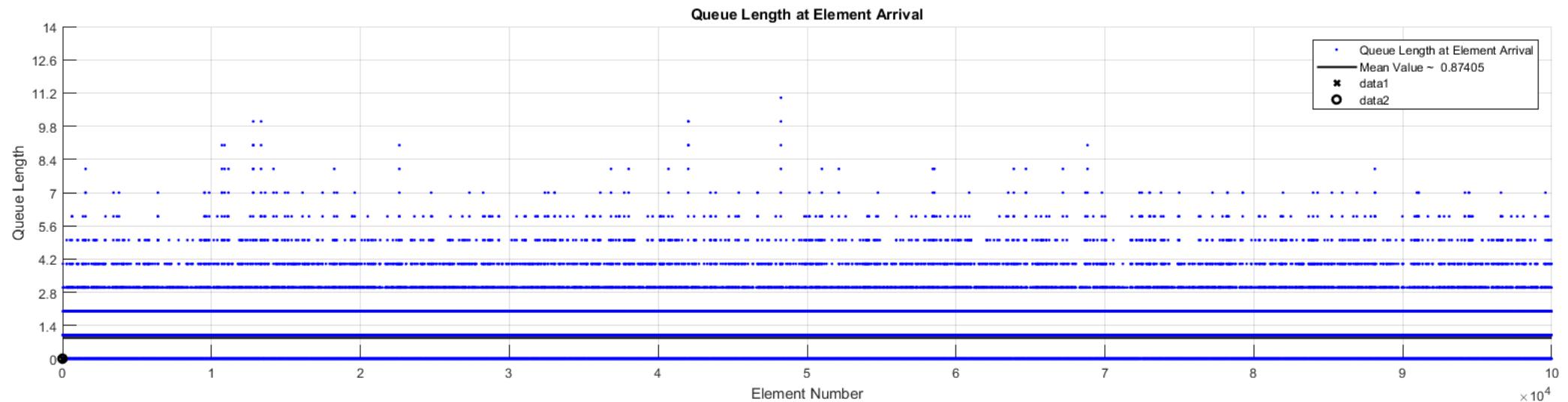
We iterate through a range of n values with $\lambda_D^{-1} = 0.5$, $\lambda_S^{-1} = 0.25$:

n_S	Number of Points	Mean Queue Length	Mean Queue Wait Time
2^0	100,000	0.99336	0.24662
2^1	100,000	0.87405	0.18754
2^2	100,000	0.81063	0.15581
2^3	100,000	0.77732	0.13941
2^4	100,000	0.76748	0.13342
2^5	100,000	0.74584	0.12602
2^6	100,000	0.74792	0.12591
2^7	100,000	0.75703	0.12699
2^8	100,000	0.75266	0.12607
2^9	100,000	0.74236	0.12377

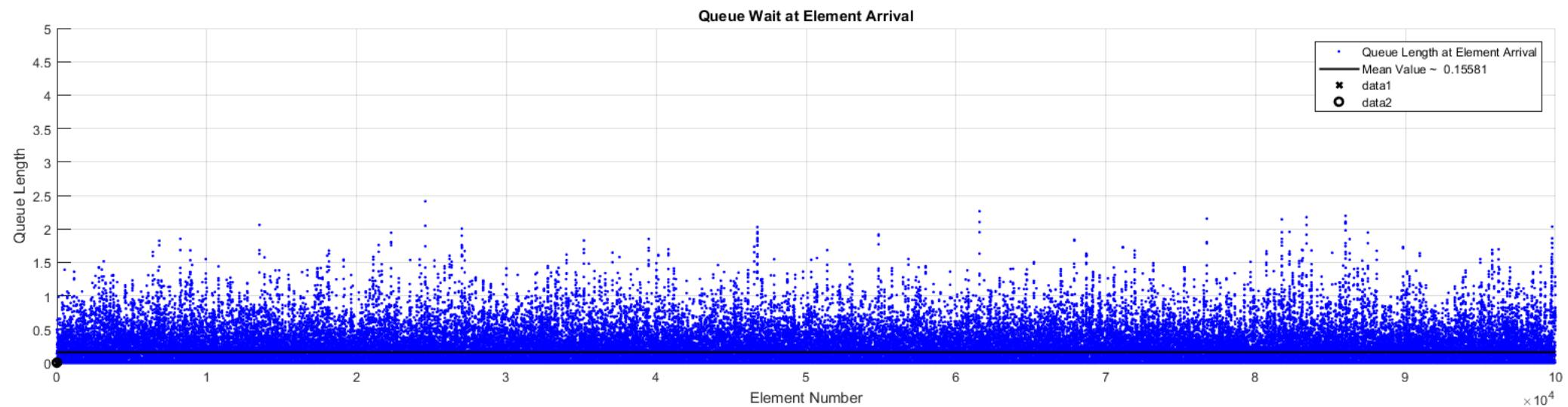
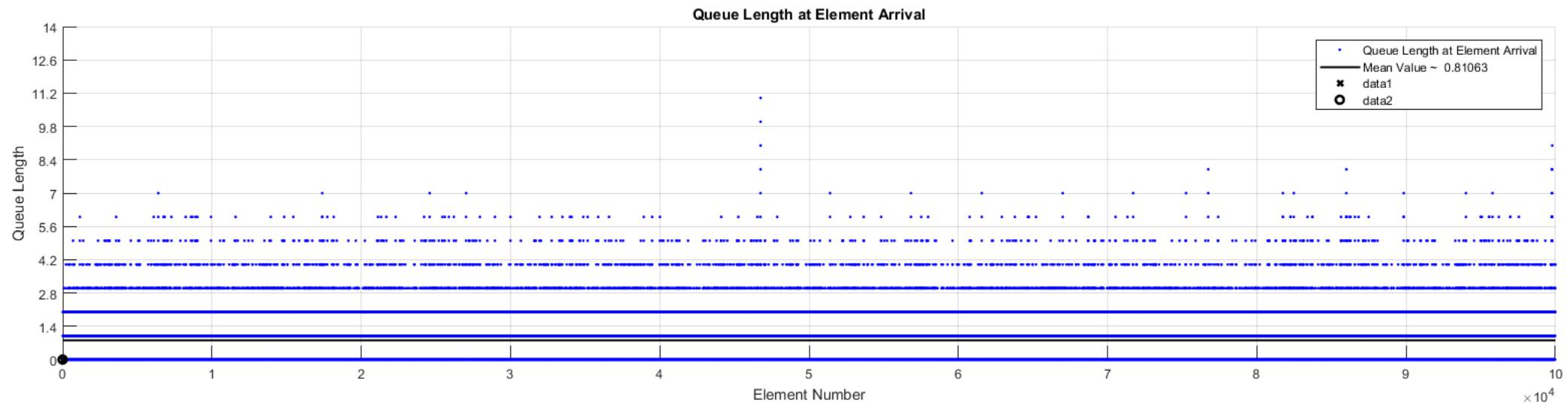
Section 2: Variable n_S



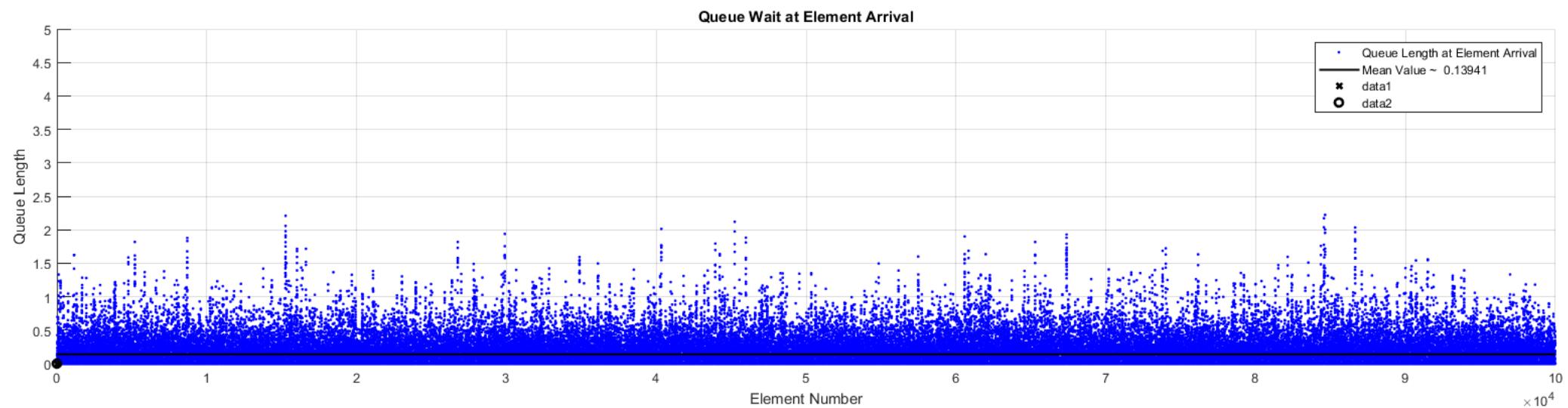
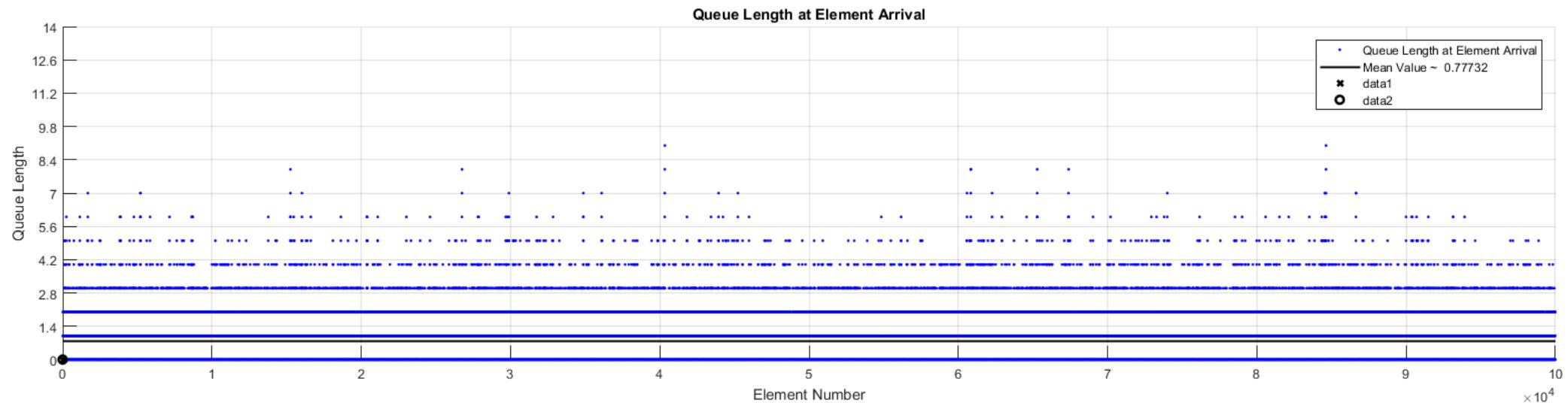
Section 2: Variable n_S



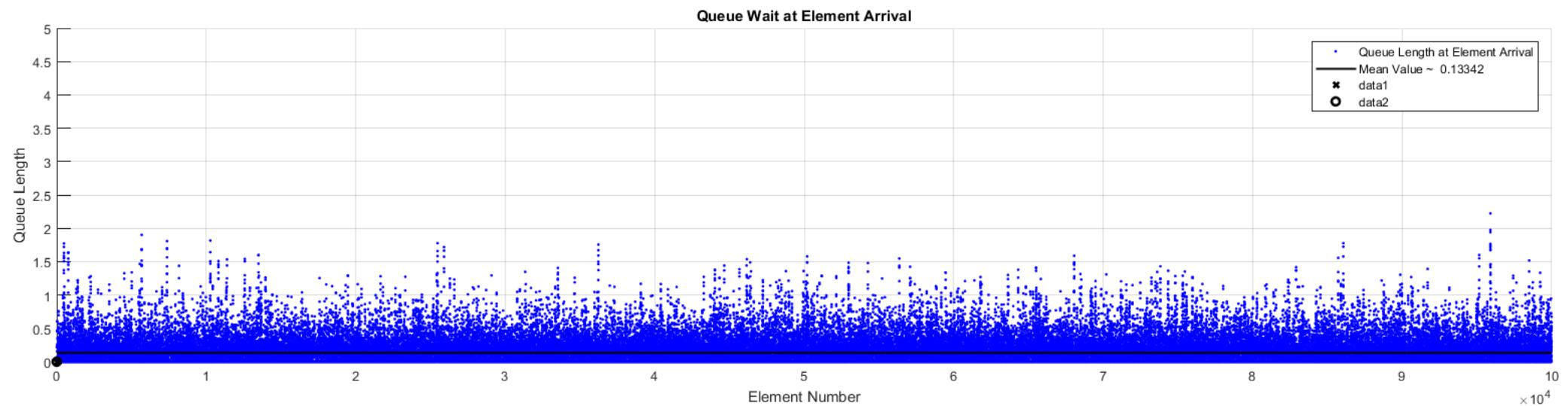
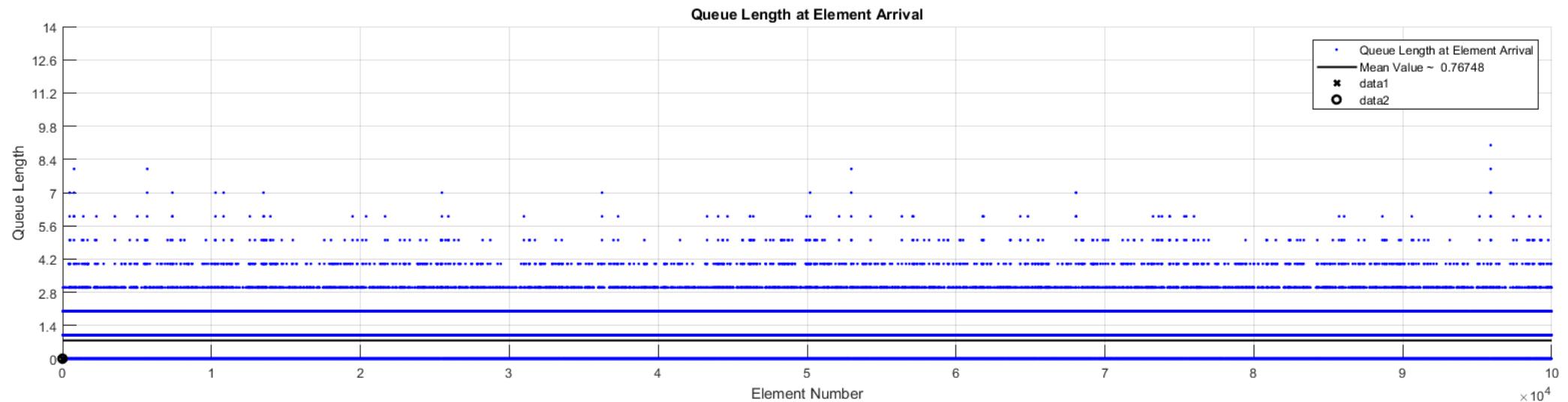
Section 2: Variable n_S



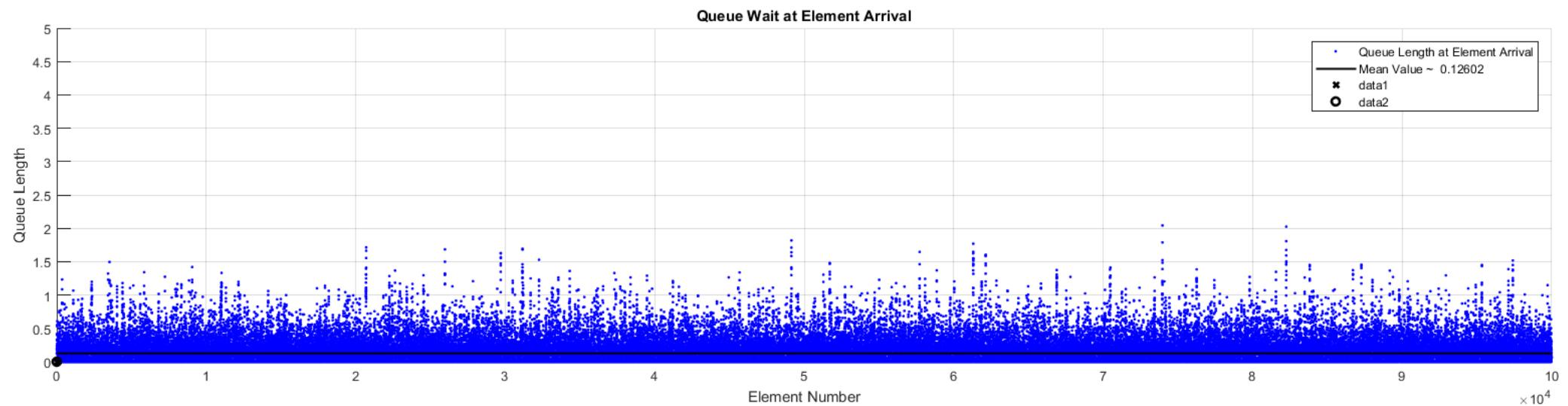
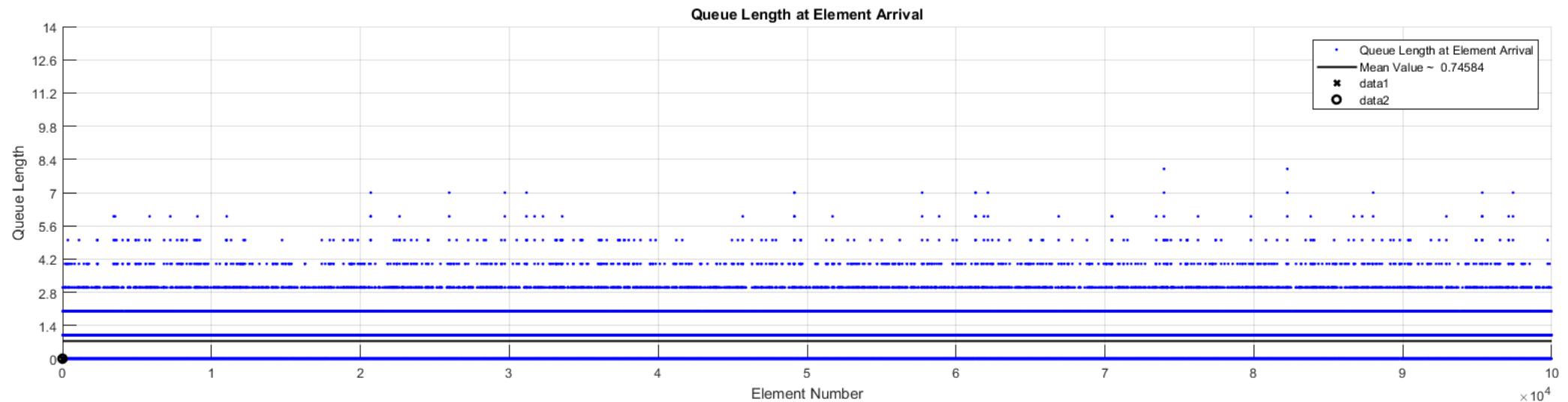
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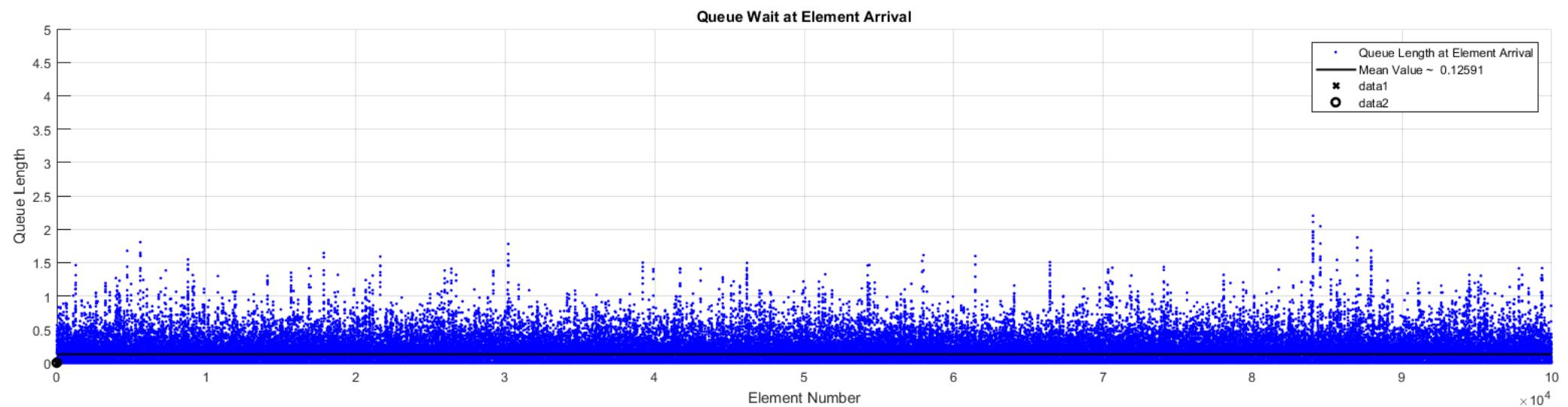
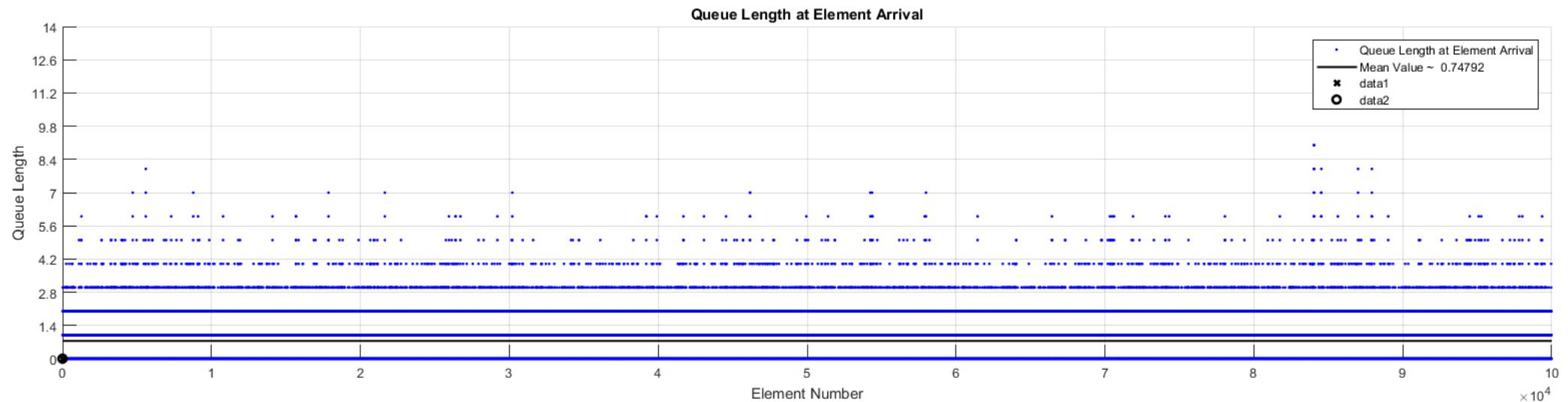
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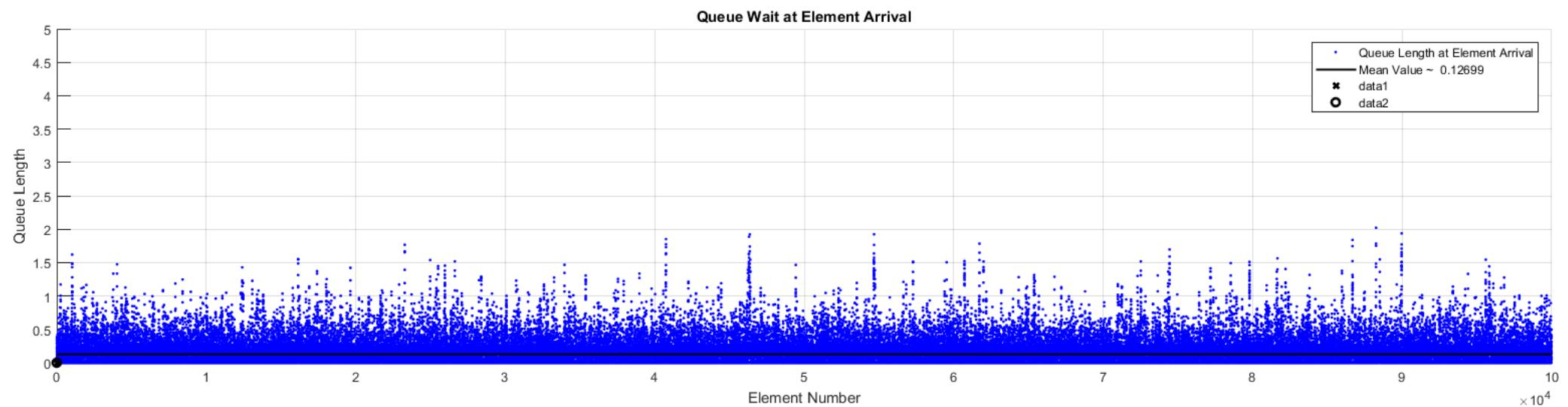
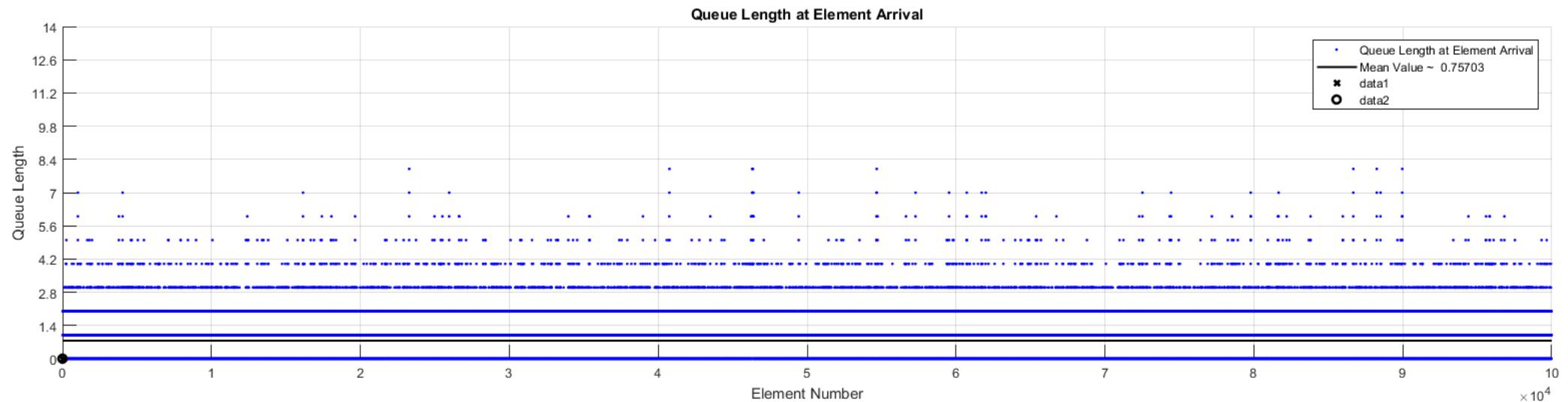
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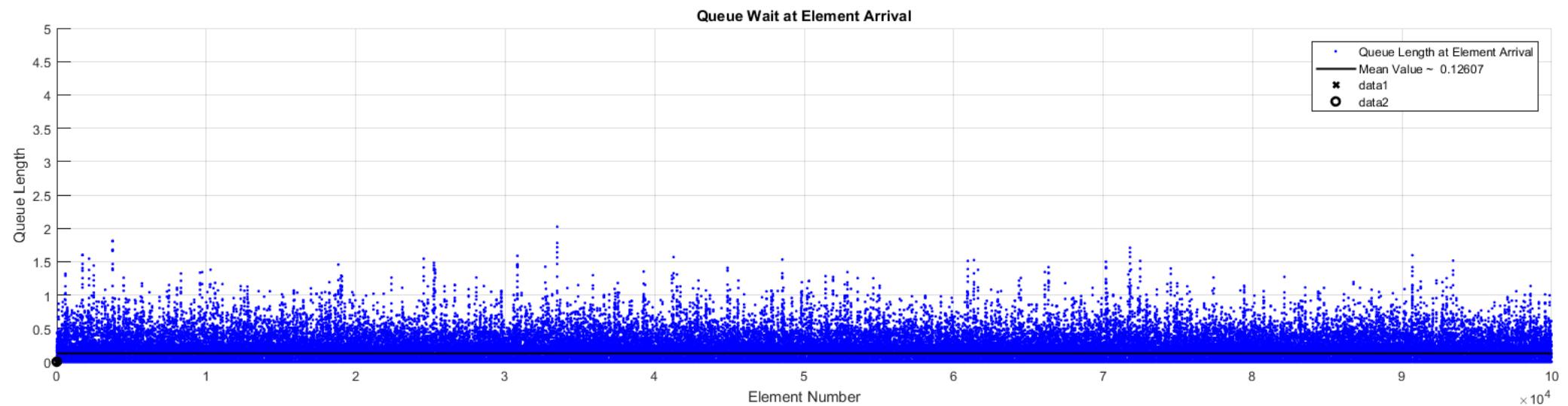
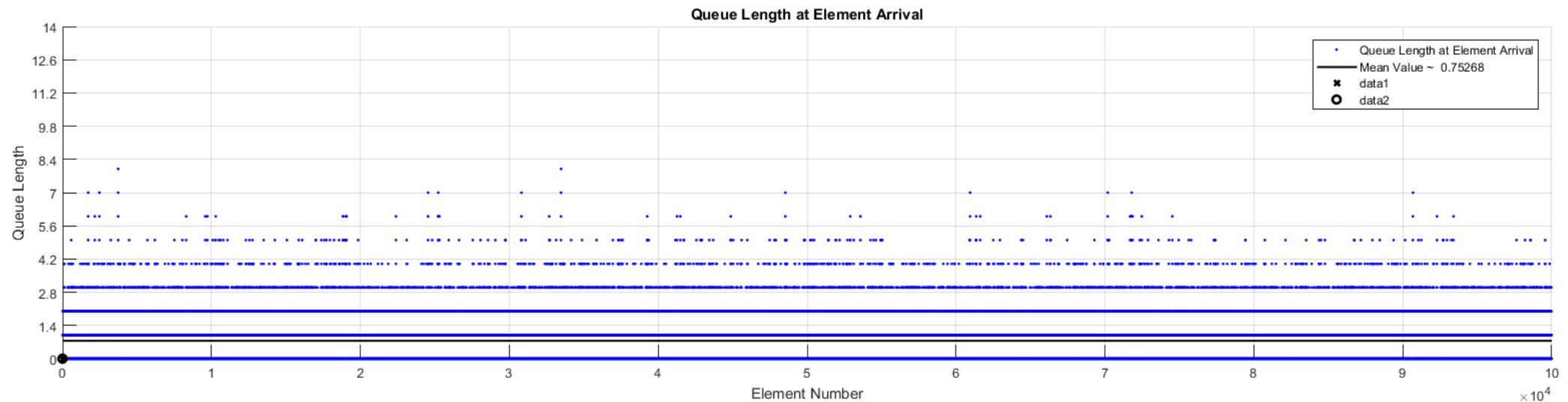
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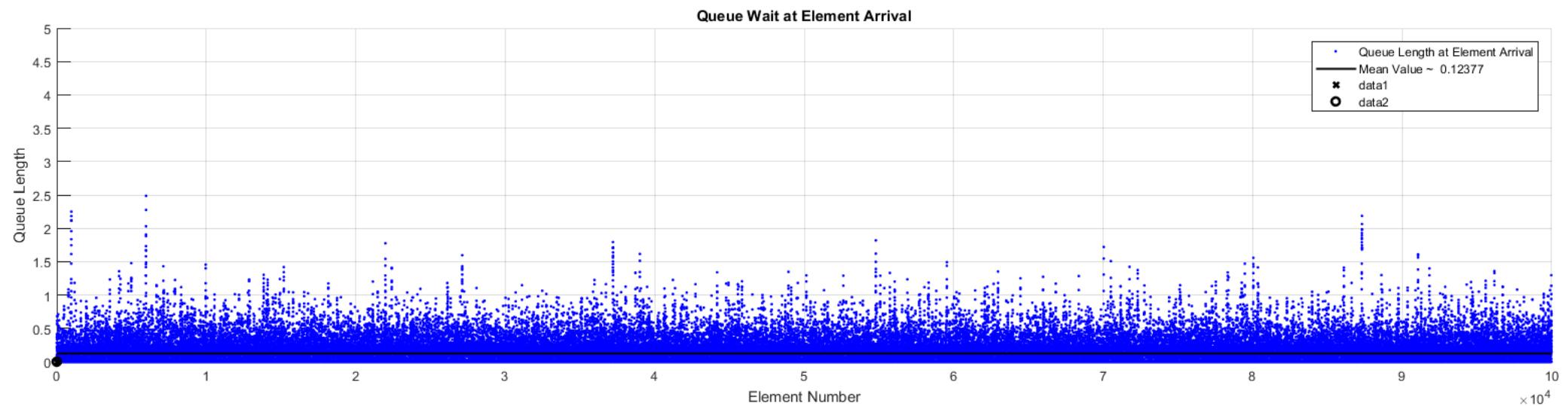
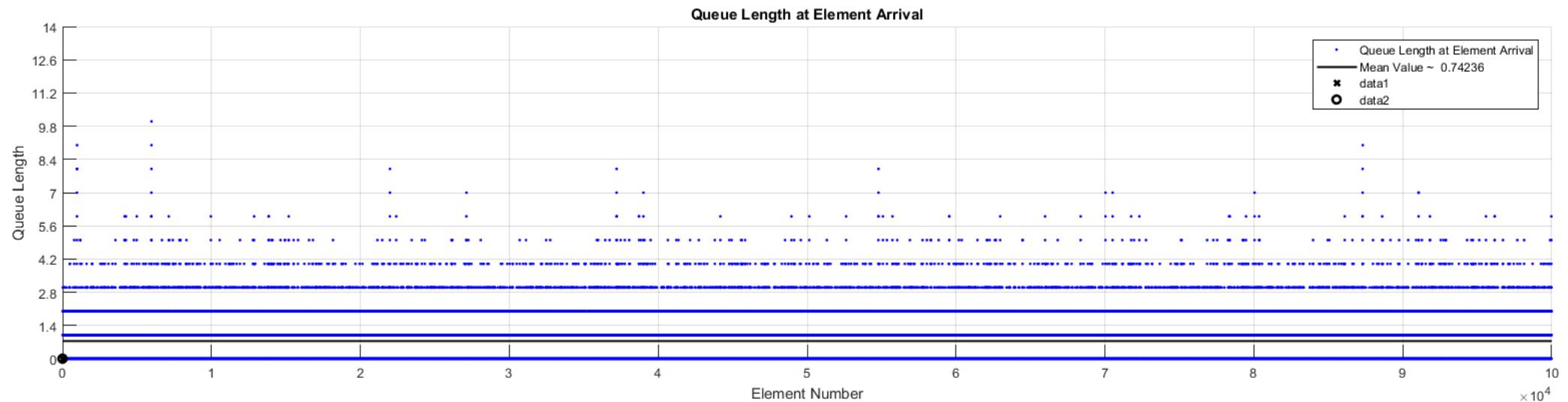
Section 2: Variable n_S



Section 2: Variable n_S



Section 2: Variable n_S





SIMULATION RESULTS

Section 3: Variable n_D , n_S

Section 3: Variable n_D , n_S

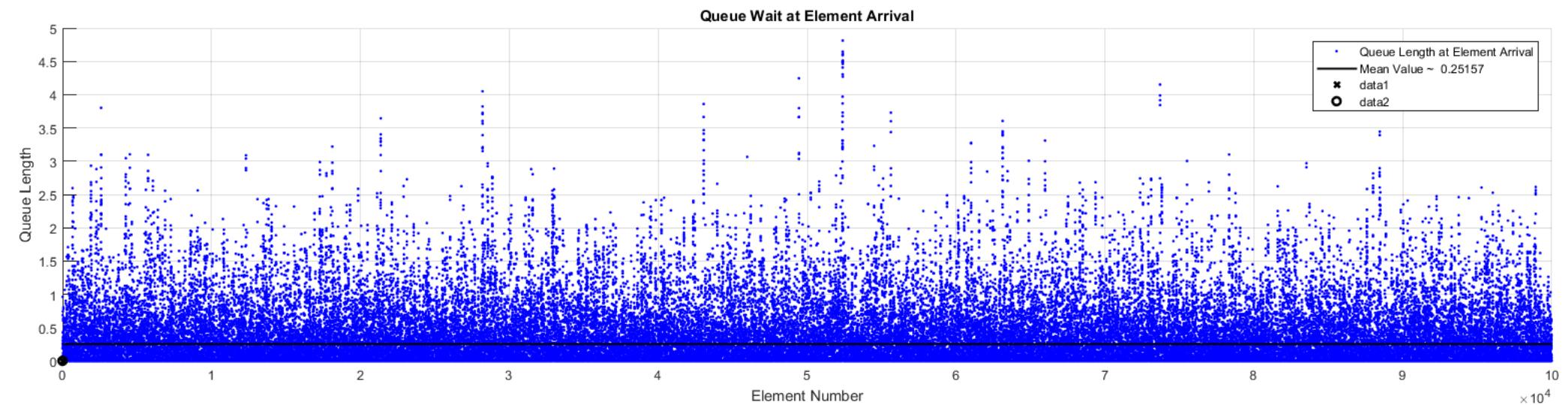
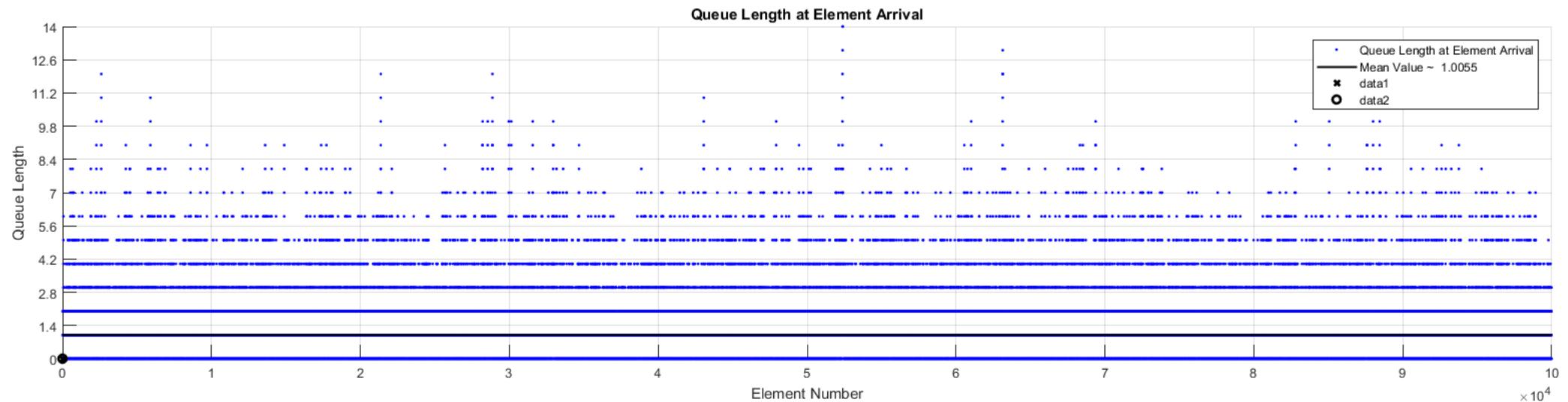


We iterate through a range of n values with $\lambda_D^{-1} = 0.5$, $\lambda_S^{-1} = 0.25$:

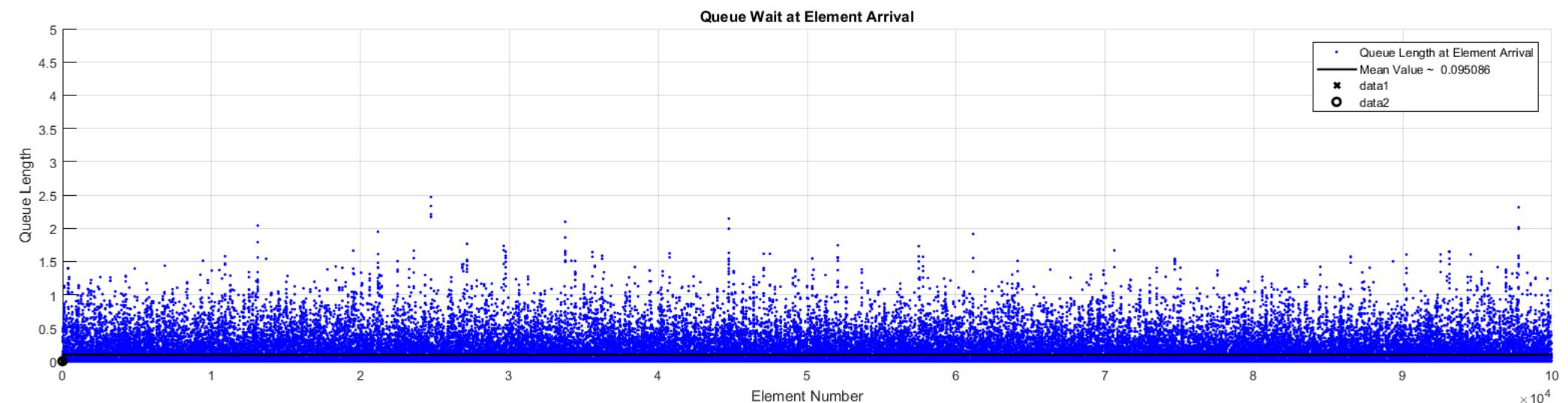
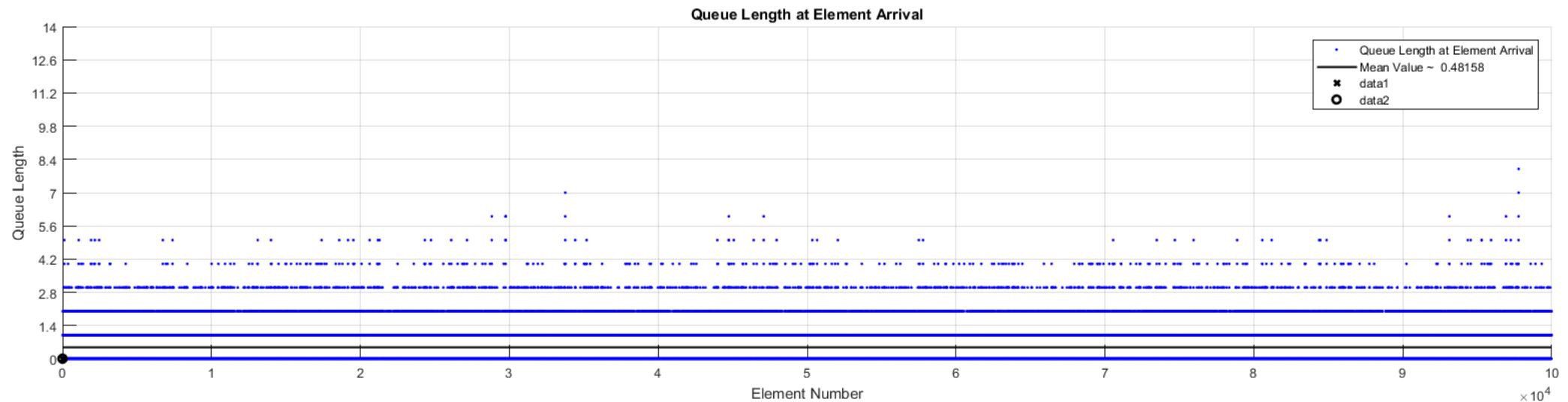
n_D, n_S	Number of Points	Mean Queue Length	Mean Queue Wait Time
2^0	100,000	1.0055	0.25157
2^1	100,000	0.48158	0.95086
2^2	100,000	0.24013	0.032588
2^3	100,000	0.10152	0.0084182
2^4	100,000	0.02834	0.0012837
2^5	100,000	0.00333	8.9225e-05
2^6	100,000	5e-05	7.9388e-07
2^7	100,000	0	0
2^8	100,000	0	0
2^9	100,000	0	0



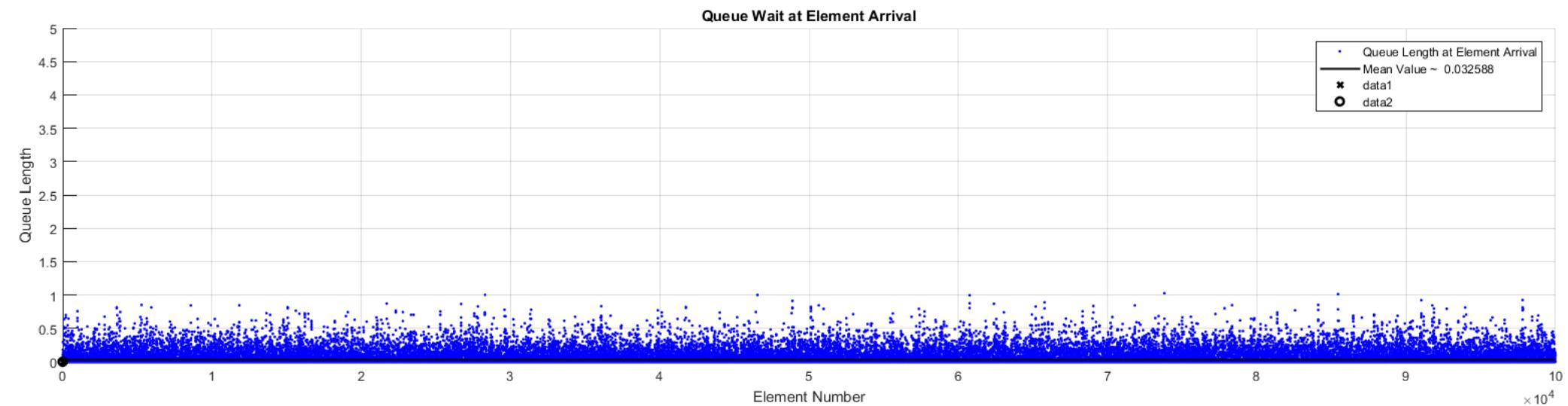
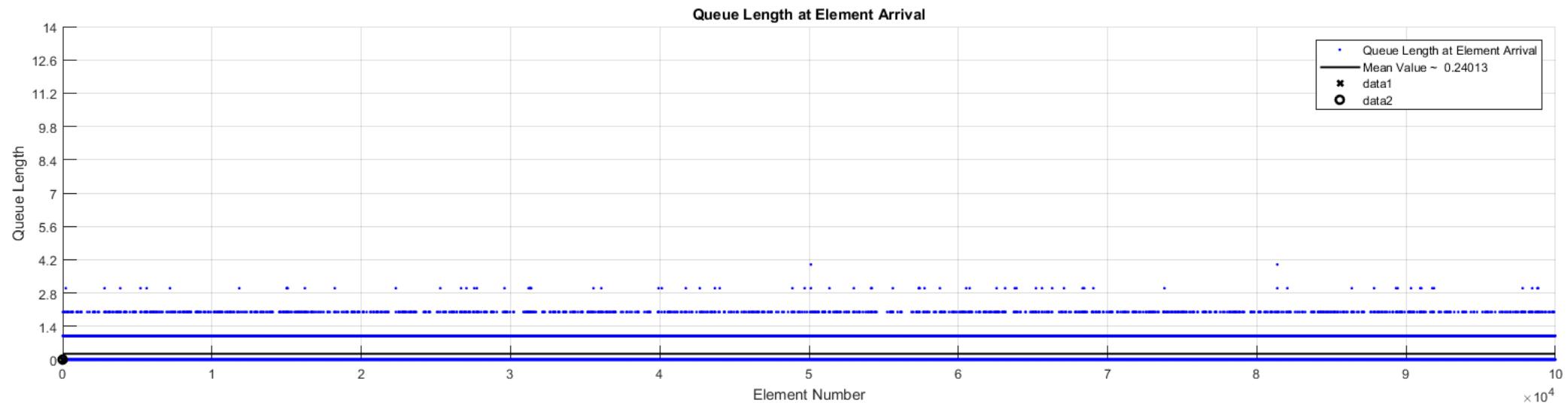
Section 3: Variable n_D , n_S



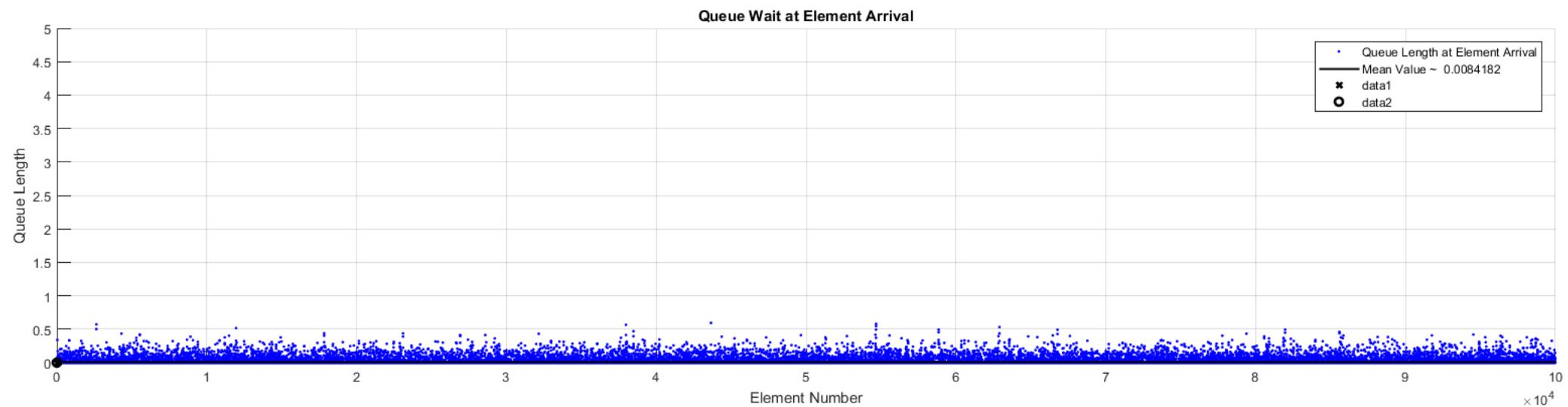
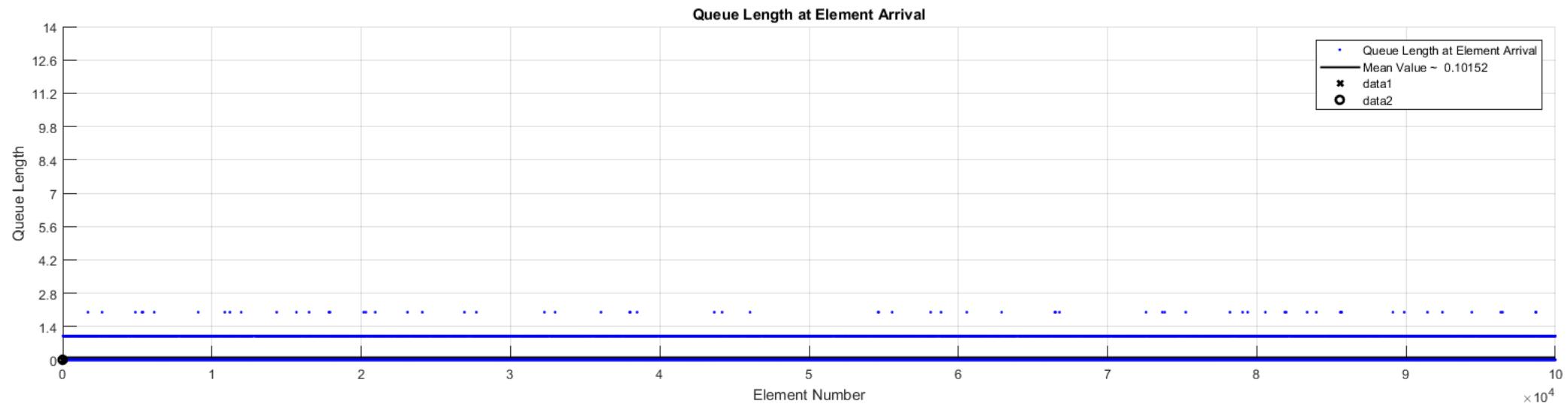
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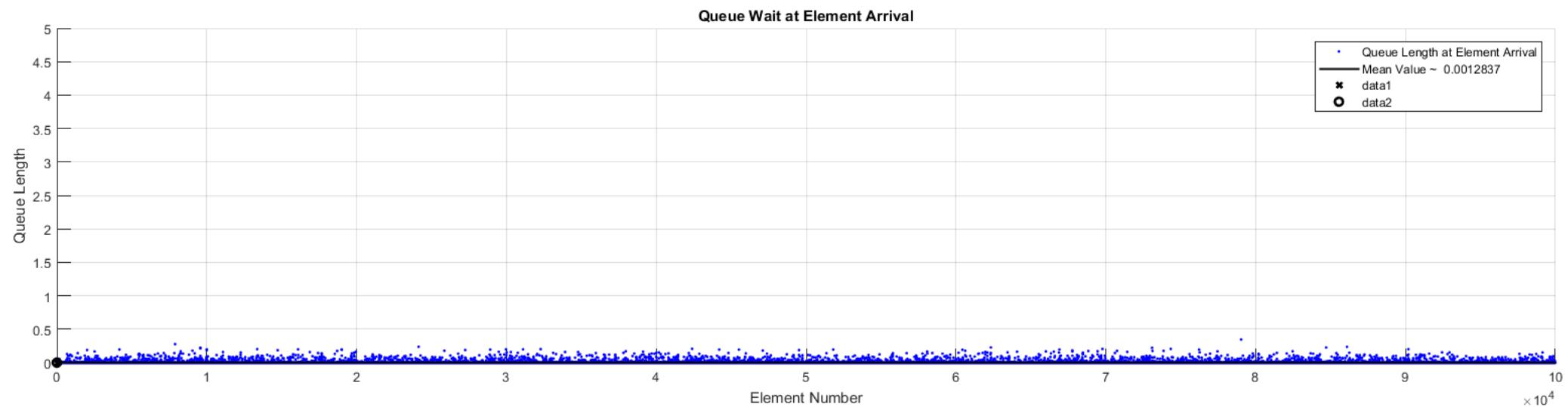
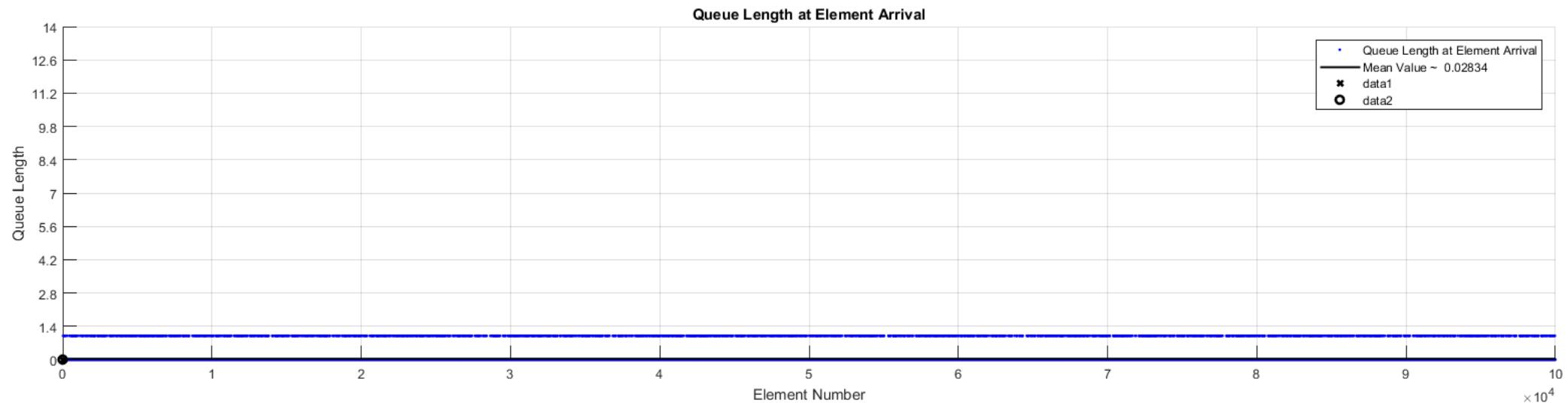
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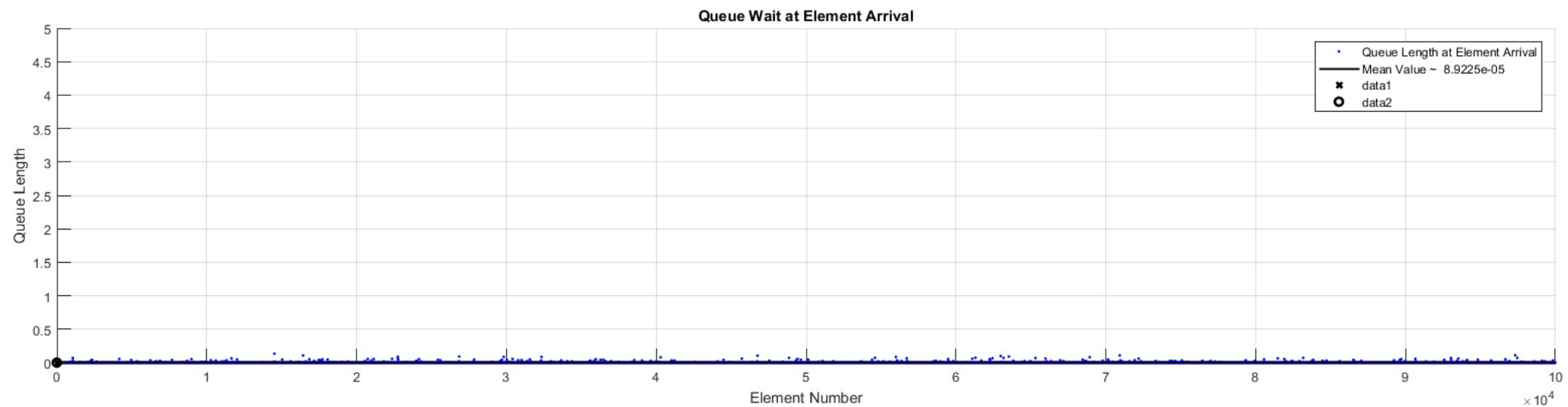
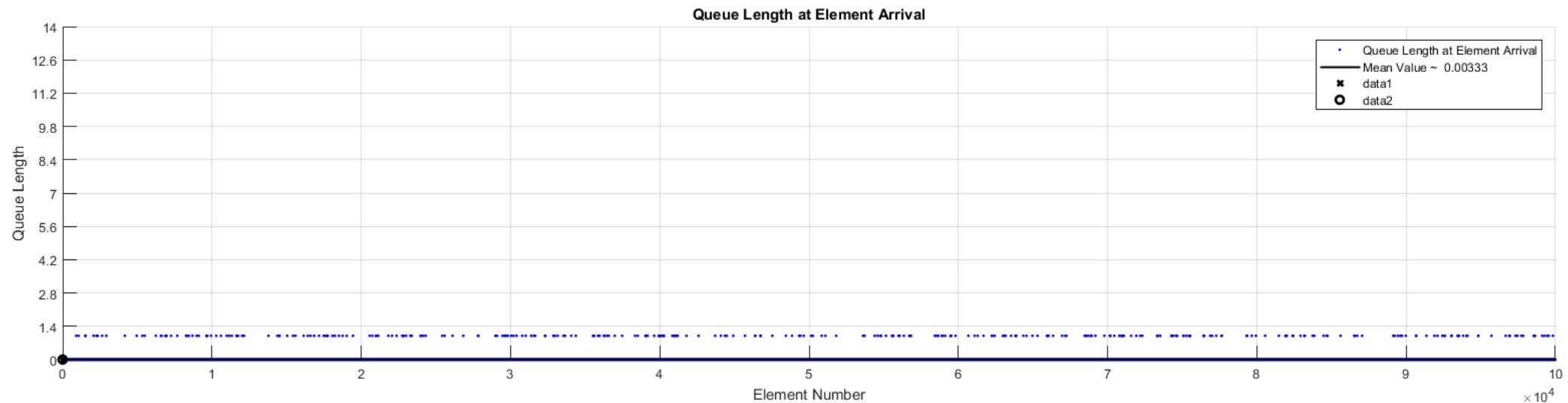
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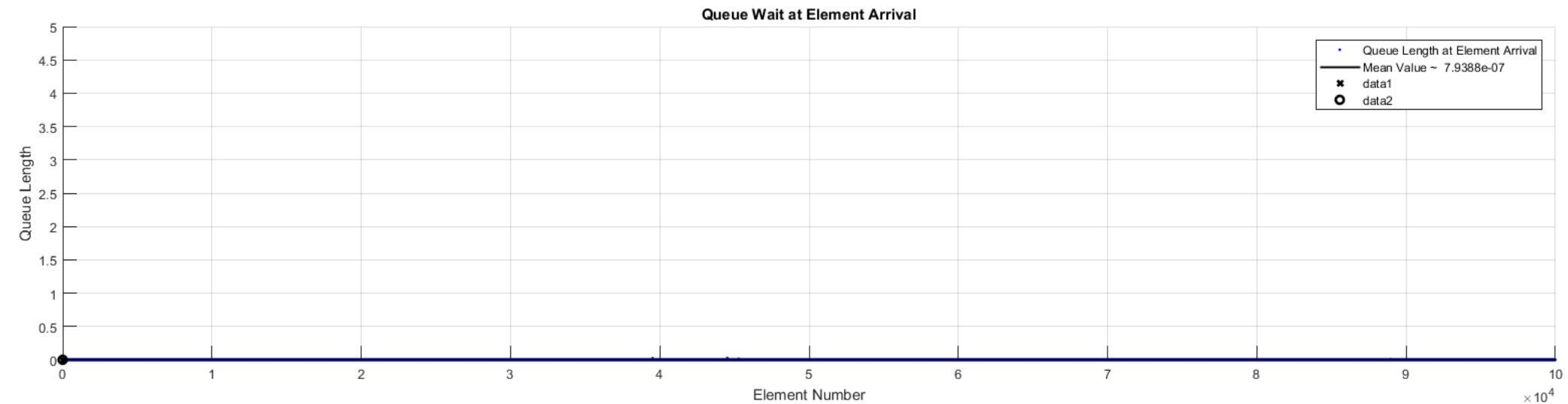
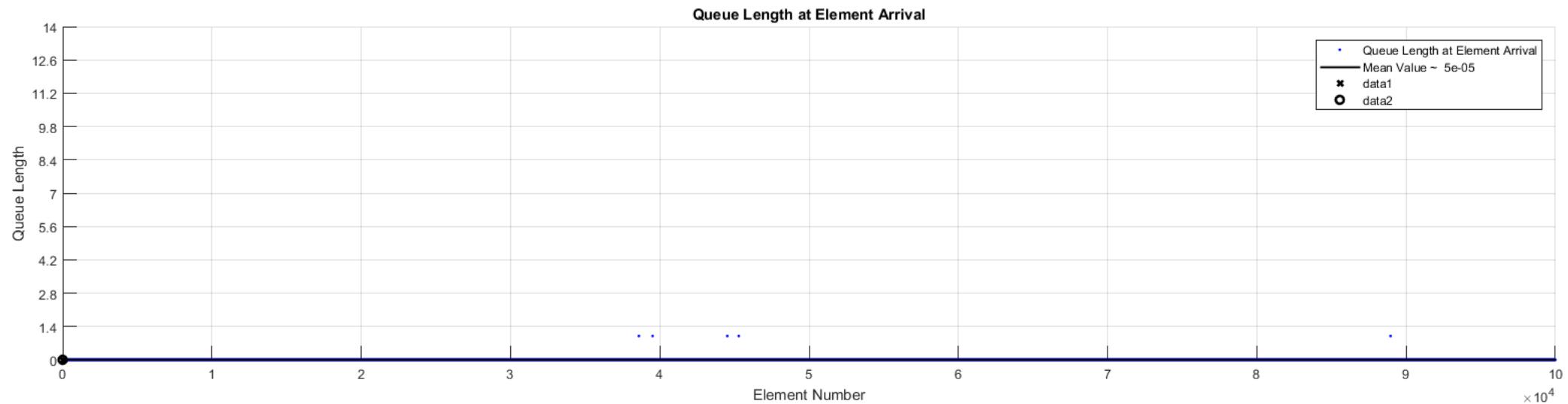
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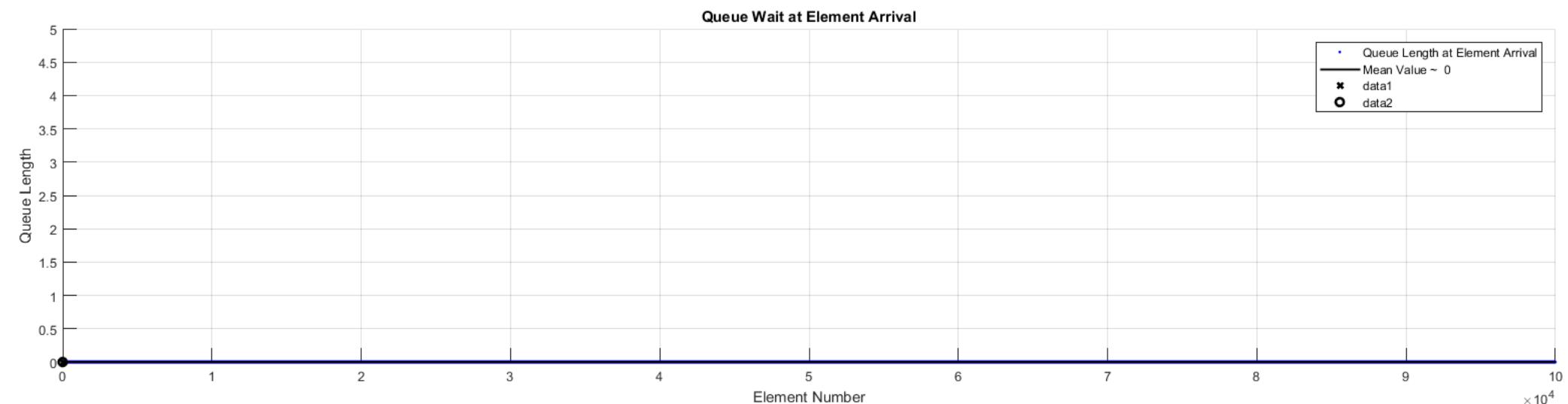
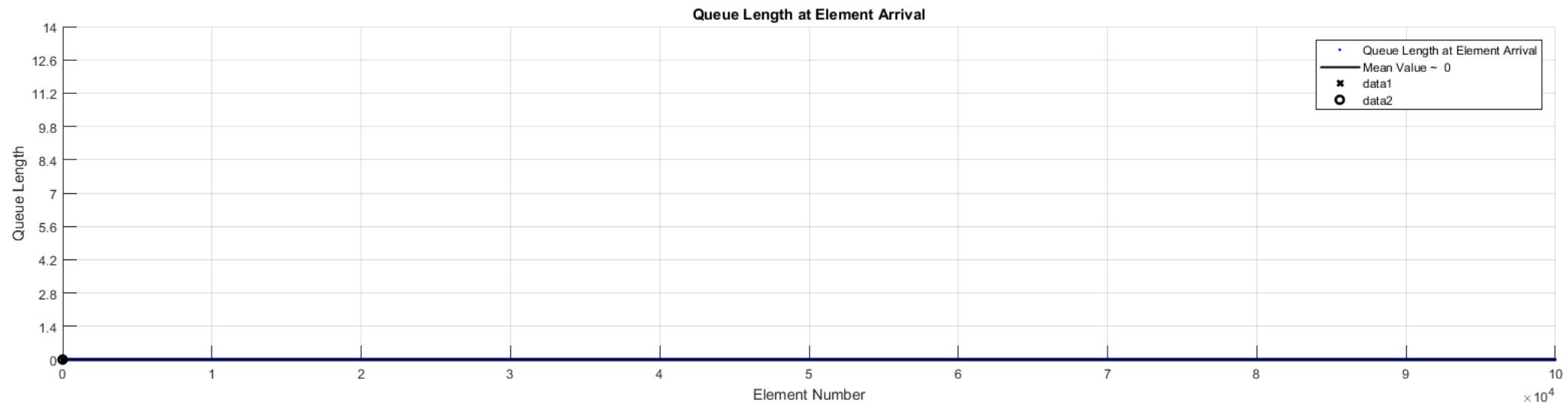
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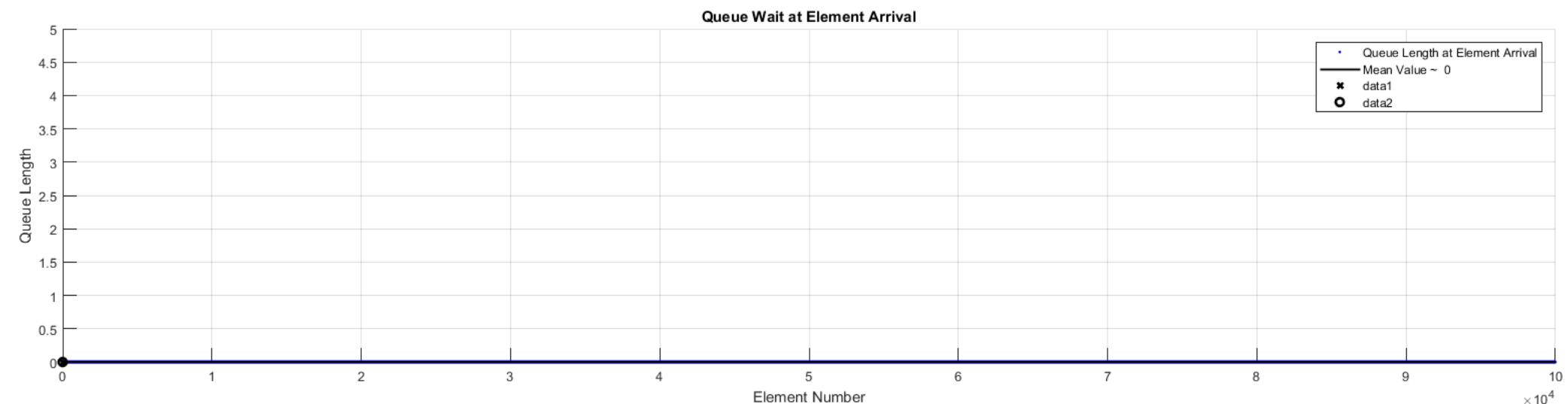
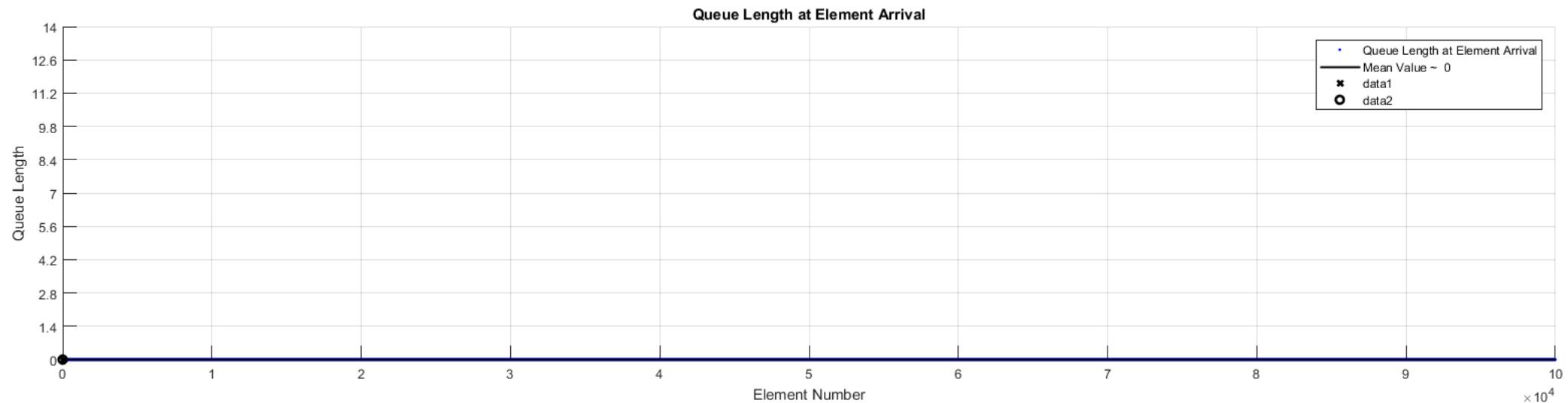
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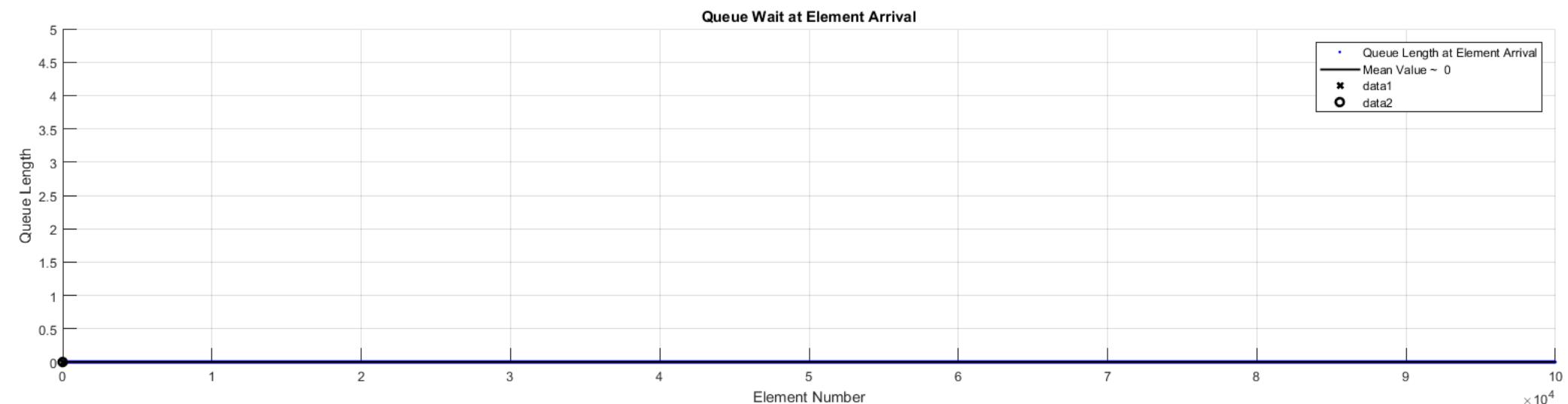
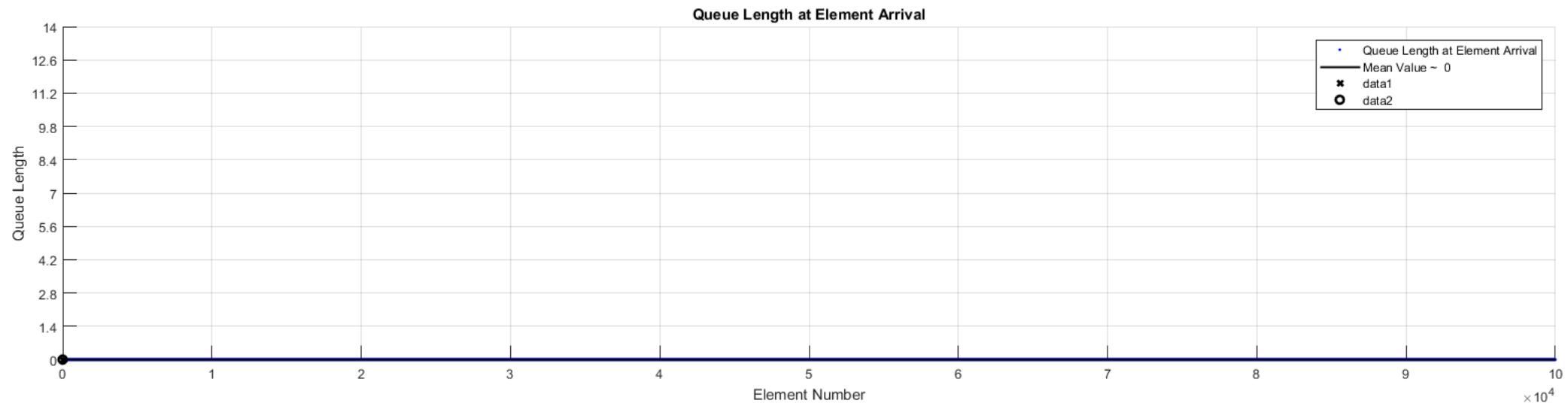
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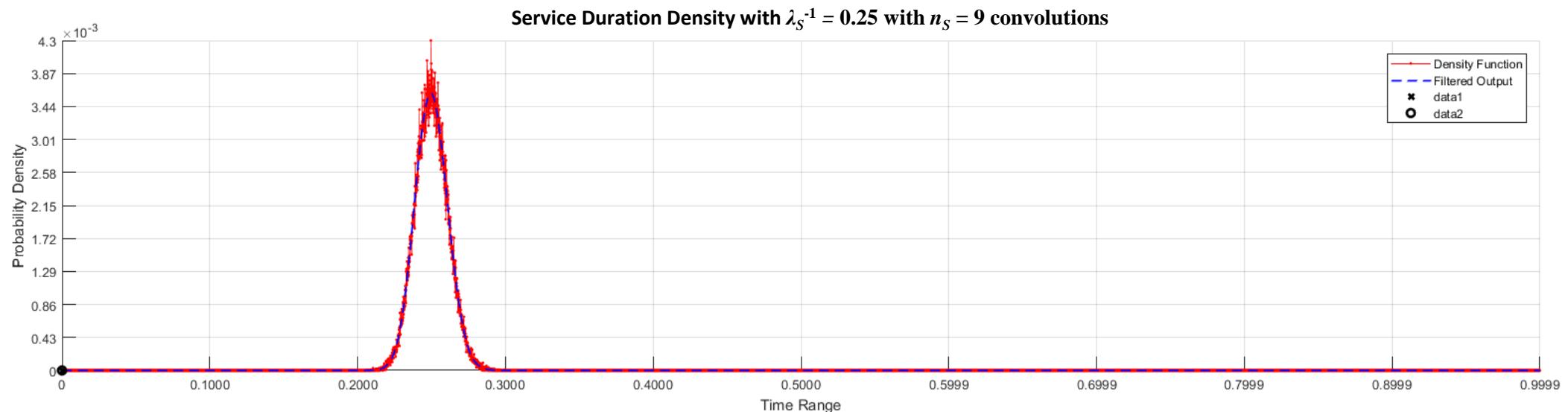
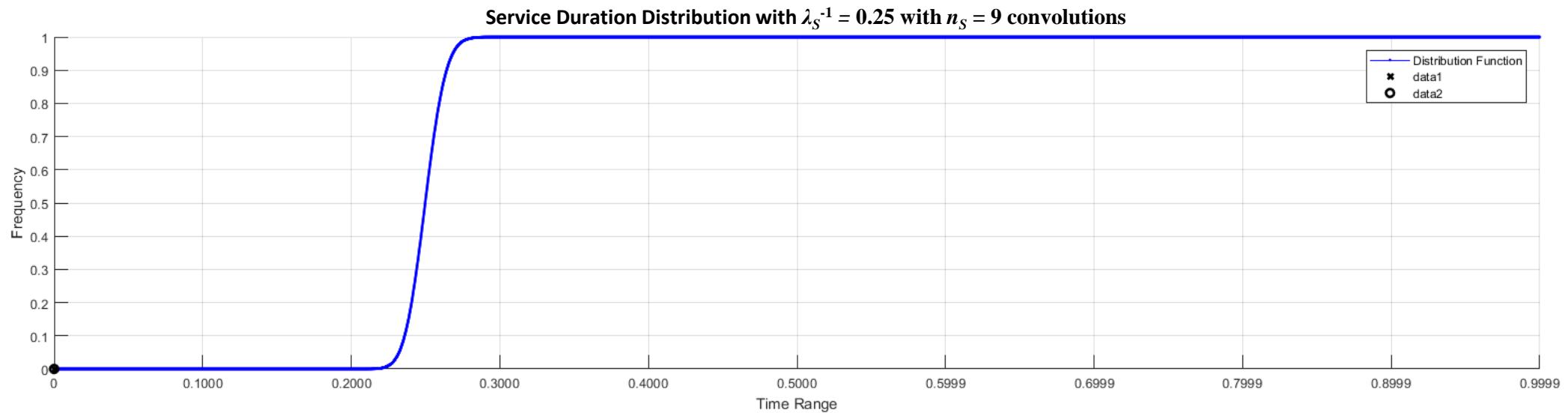




SIMULATION RESULTS

Section 4: Distribution and Density Functions

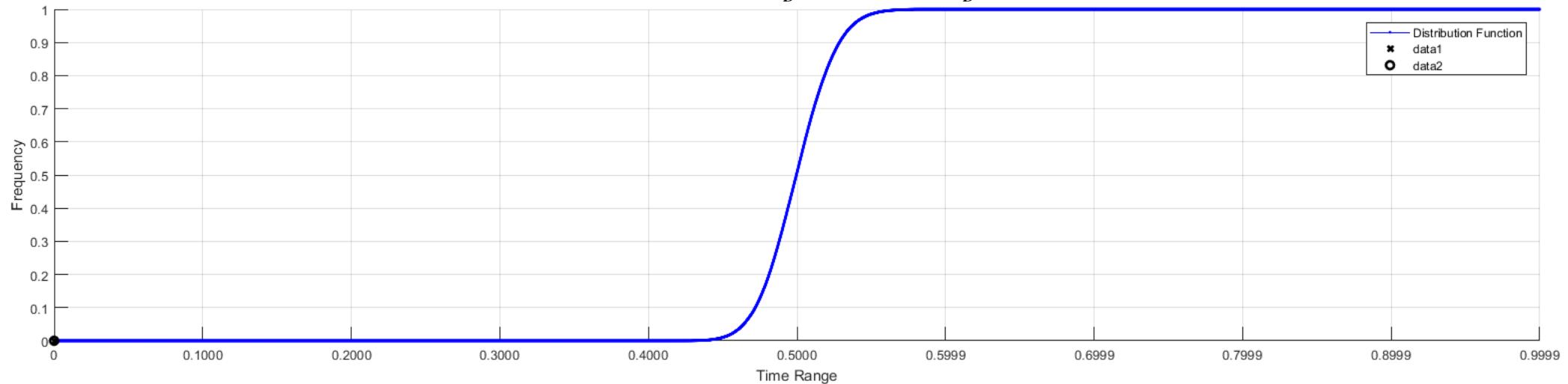
Section 4: Distribution and Density Functions



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Wait Duration Distribution with $\lambda_D^{-1} = 0.5$ with $n_D = 9$ convolutions



Wait Duration Density with $\lambda_D^{-1} = 0.5$ with $n_D = 9$ convolutions

