Discrete random variables

Exercise 1:
Consider the random variable whose distribution is given by:

k	-2	0	3
$P(X = k)$	0.1	a	0.55

1. Give $X(\Omega)$
2. What should be the value of a?

Exercise 2:
Consider a deck with32 cards.
We pick up at random in the same time 4 cards.
We define the variable X by:

$X = 10$ if the 4 cards have the same value

$X = 5$ if 3 cards only are a figure

$X = 0$ if 2 cards are figure with the same color aand the two others are not a figure

$x = -5$ otherwise

1. Determine Ω the universe of the discribed experiment
2. Determine the distribution of X.

Exercise 3:
Consider a six side die.
Cnsider X the variable equal to the number of the die that appears when we roll it.
We assume that the probability of each number is proportional to the number.

1. Determine the distribution of X and compute its expectation.
2. Consider $Y = \frac{1}{X}$. Determiner the distribution of Y and compute its expectation.

Exercise 4:
We roll two six-side dice, a blue one and a green one.
The random variable X is equal to the sum of the two dice.

1. Determine the distribution of X.

2. Consider $Y = 14 - X$. Determine the distribution of Y. Do you notice something? Do we have $X = Y$?

Exercise 5:
Consider X a radom variable whose values are $\{0; 1; 2; \ldots; n\}$ with :

$$\forall i \in \{0; 1; 2; \ldots; n\}, \ P(X = i) = \frac{1}{n+1}$$

1. Prove that we define a distribution.

2. Compute the expectation of X.

3. We assule that te expectation of X is 6. What is the value for n in this case?

Exercise 6:
We roll 3 six-side dice with one blue, one green and one red.
We put a bet of 1 euro and then we win X euros according to the following rules:

$X = 36$ if we obtain six for the 3 dice

$X = 7$ if only 2 sixt

$X = 1$ if only 1 six

$X = 0$ otherwise.

1. Determine the distribution of X.

2. Is it fair?

3. Consider $Y = 3.X + 20$. What is the value for the expectation of Y?

4. Considere $Z = X^2$. What is the value for the expectation of Z?

Exercise 7:
Consider a random variable X whose distribution is:

k	-2	2	3	5
$P(X = k)$	0.2	0.05	a	0.4

1. What should be the value of a ?

2. Compute the expectation and the variance for X.

3. Determine the distribution function for X.

4. Consider $Y = -2.X + 5$. Determiner the expectation for Y, its variance, and at the end its distributioni.

5. Do the same for $Z = X^2$.

Exercise 8:
Consider the random variable X whose distribution function is:

$$F_X(t) = \begin{cases} 0 & \text{si } t < 0 \\ \frac{1}{4} & \text{if } 0 \le t < 1 \\ \frac{2}{3} & \text{if } 1 \le t < 2 \\ \frac{11}{12} & \text{if } 2 \le t < 4 \\ 1 & \text{if } t \ge 4 \end{cases}$$

1. Determine the distribution of X
2. Compute its expectation and its variance.