Density variables

Exercise 1:
Consider the random variable X whose distribution function is:

$$F_X(t) = \begin{cases} 0 & \text{if } t \leq 2 \\ -0.5.t^2 + 3.t - 4 & \text{if } 2 \leq t \leq 3 \\ 0.5.t^2 - 3.t + 5 & \text{if } 3 \leq t \leq 4 \\ 1 & \text{if } t \geq 4 \end{cases}$$

1. Draw the graphe. IS X a dsicrete or a density random variable?
2. Compute $P(X = 2.2)$, $P(1 \leq X \leq 2.5)$, $P(3.2 \leq X \leq 4.2)$, $P(2.5 \leq X \leq 3.5)$.

Exercise 2:
In hours, the duration time D for an electrical component is a random variable whose density function is given by:

$$f_D(t) = \begin{cases} \frac{c}{t^2} & \text{if } t > 200 \\ 0 & \text{otherwise} \end{cases}$$

1. What should be the value of c ?
2. We control the component after a use of 300h. That is the probability that the component does not work anymore?

Exercise 3:
Let $n \in \mathbb{N}^*$.
Consider X a random variable whose density function is:

$$f_X(t) = \begin{cases} a.t^{n-1} & \text{if } 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

1. What should be the value of a ?
2. Compute the expectation and variance if they exist?
3. Determine the distribution function of X.

Exercise 4:
Consider X a random variable whose density function is:

$$f_X(t) = \begin{cases} 2.\theta.t.exp(-\theta.t^2) & \text{if } 0 < t \\ 0 & \text{otherwise} \end{cases}$$

avec $\theta > 0$.

1. Prove that f is a density function.
2. Determiner the distribution function F_X.
3. Compute the associated expectation and variance.
4. Consider Y defined by $Y = \theta.X^2$. What is the distribution of Y?