Chapter 1 : Discrete random variables

Sommaire

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- Is the game fair for the player depending on the values of x, G_1, G_2 and G_3?

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Remark :

$X(\Omega)$: this is the set of all attainable values for the random variable X .

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Remark :

For a given random experiment, it is possible to define several random variables. For example, on the previous example, we can also consider:

$$\begin{cases} Y : \Omega \to & \mathbb{R} \\ (a,b) \to & a + b \end{cases}$$

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Remark

$\{X = G_1\}$ is an event since this is a part of Ω since:

$$\{X = G_1\} = \{x \in \Omega,\ X(\omega) = G_1\}$$

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the distribution of X can be summarized by:

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Remark

Another method to compute $P(X = G_3)$ is

$$P(X = G_3) = 1 - P(X = G_1) - P(X = G_2)$$

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In gambling, a common question os whether the game is fair, meaning if the players face the same risk of losing.

To answer the last question, we need to compute the expectation.

Expectation

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When the expectation exists, we denote it $\mathbb{E}[X]$, and its formula is:

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Remark

What are the conditions for the existence of the expectation?
The expectation of X exists if X is a integrable variable, which means :

$$\mathbb{E}[|X|] = \sum_{k \in X(\Omega)} |k|.P(X = k) < +\infty$$

Xintegrable can be denoted $X \in L^1$.

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- If $X \leq Y$, then $\mathbb{E}[X] \leq \mathbb{E}[Y]$
- $\mathbb{E}[X]$ is a real, not a random quantity.

Expectation

come back onto the previous example:

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- since $X(\Omega)$ is a finite set, we are sure about the existence of the expectation
- So we have:

$$\mathbb{E}[X] = G_1.\frac{1}{36} + G_2.\frac{10}{36} + G_3.\frac{25}{36}$$

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- let take $x = 2,\ G_1 = 10,\ G_2 = 5$ et $G_3 = 0$, then $\mathbb{E}[X] = \frac{60}{36} < 2$.
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- let take $x = 2,\ G_1 = 30,\ G_2 = 5$ et $G_3 = 0$, then $\mathbb{E}[X] = \frac{80}{36} > 2$.
  The expectation being positive, thus the game is favorable to the player.

Transfert formula

Proposition

Let X an integrable discrete random variable.
Let h a real function.
Consider $Y = h(X)$.
We assume that h has properties to ensure the existence of the expectation for Y.
Then :

$$\mathbb{E}[Y] = \sum_{k \in X(\Omega)} h(k).P(X = k)$$

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Remark

The interest of this formula is that it is possible to compute the expectation for $h(X)$ without having to dtermine the distribution of $h(X)$.

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What is the expectation for Y?
By applying the transfert formula with $h(x) = x^2$, we get:

$$\mathbb{E}[Y] = \sum_{k \in X(\Omega)} k^2.P(X=k) = (-2)^2.\frac{1}{2} + (0)^2.\frac{1}{4} + (1)^2.\frac{1}{4} = \frac{9}{4}$$

Variance

Definition

Consider a discrete random variable X
When the variance exists, we denote it $\mathbb{V}[X]$, and its formula is :

$$\mathbb{V}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2]$$

with :

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Remark

What are the conditions for the existence of the variance?
The variance of X exists if X is a squared integrable variable, i.e.:

$$\mathbb{E}[X^2] = \sum_{k \in X(\Omega)} k^2 . P(X = k) < +\infty$$

X squared integrable variable is written $X \in L^2$.

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$$L^2 \subset L^1$$

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The previous formula for the variance is the definition, in practice we use more:

$$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

with $\mathbb{E}[X^2] = \sum_{k \in X(\Omega)} k^2.P(X = k)$ (transfert formula)

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By using the practical formula for the variance :

$$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{9}{4} - ((-2).\frac{1}{2} + 0 * \frac{1}{4} + 1.\frac{1}{4})^2 = \frac{9}{4} - \frac{9}{16} = \frac{27}{16}$$

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- The centered moment of order 1 for X has no sense since is it equal to 0
- Let X an integrable random variable.
  Then $Y = X - \mathbb{E}[X]$ is a centered variable, which means that its expectation is equal to 0.

Sommaire

Distribution function

Definition

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The distribution function for X, denoted F_X, is the function given by:

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Remark

Be careful, there exists another definition which is the english one:

$$\forall t \in \mathbb{R},\ F_X(t) = P(X < t)$$

Those txo definitions do not provide the same results for discrete random variables.

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Example

Consider the variable X whose distribution is :

k	-2	0	1
$P(X=k)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

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the distribution function for X is given by

$$F_X(t) = \begin{cases} 0 & \text{if } t < -2 \\ \frac{1}{2} & \text{if } -2 \leq t < 0 \\ \frac{3}{4} & \text{if } 0 \leq t < 1 \\ 1 & \text{if } 1 < t \end{cases}$$

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- If $0 \leq t < 1$, then $\{X \leq t\} = \{-2; 0\}$ and $F_X(t) = P(X = -2) + P(X = 0) = \frac{3}{4}$
- If $1 \leq t$, then $\{X \leq t\} = \{-2; 0; 1\}$ and $F_X(t) = P(X = -2) + P(X = 0) + P(X = 1) = 1$

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The distribution and the distribution function provide the same information. Thanks to the distribution, we can determine the distribution function, but he inverse is true also.

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