Chapter 2 : Density random variable

Sommaire

Density function

Définition

Consider $f : \mathbb{R} \to \mathbb{R}$ a function.
f is a density function if:

Density function

Définition

Consider $f : \mathbb{R} \to \mathbb{R}$ a function.
f is a density function if:

- $\forall x \in \mathbb{R},\ f(x) \geq 0$

Density function

Définition

Consider $f : \mathbb{R} \to \mathbb{R}$ a function.
f is a density function if:

- $\forall x \in \mathbb{R},\ f(x) \geq 0$
- $\int_{\mathbb{R}} f(x).dx = 1$

Density function

Définition

Consider $f : \mathbb{R} \to \mathbb{R}$ a function.
f is a density function if:

- $\forall x \in \mathbb{R},\ \ f(x) \geq 0$
- $\int_{\mathbb{R}} f(x).dx = 1$

Remark

Since f is a positive function, $\int_{\mathbb{R}} f(x).dx$ is the area between the graph of f and the x-axis.

Density variable

Définition

Consider X a random variable.
X is a density variable, called also continuous variable, if

Density variable

Définition

Consider X a random variable.
X is a density variable, called also continuous variable, if

- $X(\Omega)$ is a part of $\mathbb{R}$

Density variable

Définition

Consider X a random variable.
X is a density variable, called also continuous variable, if

- $X(\Omega)$ is a part of $\mathbb{R}$
- the distribution of X is given by the density function f such that

$$\forall I \subset \mathbb{R},\ P(X \in I) = \int_I f(x).dx$$

Density variable

Définition

Consider X a random variable.
X is a density variable, called also continuous variable, if

- $X(\Omega)$ is a part of $\mathbb{R}$
- the distribution of X is given by the density function f such that

$$\forall I \subset \mathbb{R},\ P(X \in I) = \int_I f(x).dx$$

Remark

$X(\Omega)$ eis the support of f i.e.

$$X(\Omega) = \{x \in \mathbb{R} \text{ such that } f(x) \neq 0\}$$

Density variable

Proposition

Consider X a density random variable. Then

$$\forall a \in \mathbb{R},\ \ P(X = a) = 0$$

Density variable

Proposition

Consider X a density random variable. Then

$$\forall a \in \mathbb{R},\ \ P(X = a) = 0$$

Proposition

Consider X a density random variable. Then

$$\forall a, b \in \mathbb{R},\ \ P(X \in [a, b]) = P(X \in]a, b]) = P(X \in]a, b[) = P(X \in [a, b[)$$

Example

Example

Consider X a density random variable whose density function f is:

$$f(x) = \begin{cases} a & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

What should be the value of a?

Example

Answer

- To have a density function f, we need f positive with and integral equal to 1.

Example

Answer

- To have a density function f, we need f positive with and integral equal to 1.
- the positivity of f induces $a \geq 0$

Example

Answer

- To have a density function f, we need f positive with and integral equal to 1.
- the positivity of f induces $a \geq 0$
- $\int_{\mathbb{R}} f(x).dx = \int_{-2}^{-1} a.dx + \int_0^3 a.dx = 4a$

Example

Answer

- To have a density function f, we need f positive with and integral equal to 1.
- the positivity of f induces $a \geq 0$
- $\int_{\mathbb{R}} f(x).dx = \int_{-2}^{-1} a.dx + \int_{0}^{3} a.dx = 4a$
- the integral equal to 1, induces $a = \frac{1}{4} \geq 0$

Example

Answer

- To have a density function f, we need f positive with and integral equal to 1.
- the positivity of f induces $a \geq 0$
- $\int_{\mathbb{R}} f(x).dx = \int_{-2}^{-1} a.dx + \int_{0}^{3} a.dx = 4a$
- the integral equal to 1, induces $a = \frac{1}{4} \geq 0$
- Thus f is a density function if and only if $a = \frac{1}{4}$

Example

Example

Consider X a density random variable whose density function f is:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

What is the value for $P(X \in [-1.5; 2])$?

Example

Answer

- $P(X \in [-1.5; 2]) = \int_{-1.5}^{2} f(x).dx$

Example

Answer

- $P(X \in [-1.5; 2]) = \int_{-1.5}^{2} f(x).dx$
- $P(X \in [-1.5; 2]) = \int_{-1.5}^{-1} \frac{1}{4}.dx + \int_{0}^{2} \frac{1}{4}.dx$

Example

Answer

- $P(X \in [-1.5; 2]) = \int_{-1.5}^{2} f(x).dx$
- $P(X \in [-1.5; 2]) = \int_{-1.5}^{-1} \frac{1}{4}.dx + \int_{0}^{2} \frac{1}{4}.dx$
- $P(X \in [-1.5; 2]) = \frac{1}{4}.(0.5 + 2) = \frac{2.5}{4}$

Sommaire

Expectation

Definition

Consider X a density random variable with density function f.
If $\int_{\mathbb{R}} |x|.f(x).dx < +\infty$, f ist integrable.
Then X qhas an expectation denoted $\mathbb{E}[X]$ given by:

$$\mathbb{E}[X] = \int_{\mathbb{R}} x.f(x).dx$$

Expectation

Definition

Consider X a density random variable with density function f.
If $\int_{\mathbb{R}} |x|.f(x).dx < +\infty$, f ist integrable.
Then X qhas an expectation denoted $\mathbb{E}[X]$ given by:

$$\mathbb{E}[X] = \int_{\mathbb{R}} x.f(x).dx$$

Remark

If $X(\Omega)$ is a bounded part of $\mathbb{R}$ and f a continuous function, then the expectation of X exists.

Expectation

Properties

Consider X and Y two integrable ransdom variables. Let λ a real.

Expectation

Properties

Consider X and Y two integrable ransdom variables. Let λ a real.

- $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$

Expectation

Properties

Consider X and Y two integrable ransdom variables. Let λ a real.

- $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$
- $\mathbb{E}[X+\lambda] = \mathbb{E}[X] + \lambda$

Expectation

Properties

Consider X and Y two integrable ransdom variables. Let λ a real.

- $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$
- $\mathbb{E}[X+\lambda] = \mathbb{E}[X] + \lambda$
- $\mathbb{E}[\lambda.X] = \lambda.\mathbb{E}[X]$

Expectation

Properties

Consider X and Y two integrable ransdom variables. Let λ a real.

- $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$
- $\mathbb{E}[X+\lambda] = \mathbb{E}[X] + \lambda$
- $\mathbb{E}[\lambda.X] = \lambda.\mathbb{E}[X]$
- $\mathbb{E}[X]$ is a non random real

Example

Example

Consider X a density variable whose density function f is :

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

What is the value off $\mathbb{E}[X]$?

Example

Answer

- $X(\Omega) = [-2, -1] \cup [0, 3]$ and f continuous on its support, so we know that the expectation of X exists

Example

Answer

- $X(\Omega) = [-2, -1] \cup [0, 3]$ and f continuous on its support, so we know that the expectation of X exists
- $\mathbb{E}[X] = \int_{\mathbb{R}} x.f(x).dx = \int_{-2}^{-1} \frac{x}{4}.dx + \int_{0}^{3} \frac{x}{4}.dx$

Example

Answer

- $X(\Omega) = [-2, -1] \cup [0, 3]$ and f continuous on its support, so we know that the expectation of X exists
- $\mathbb{E}[X] = \int_{\mathbb{R}} x.f(x).dx = \int_{-2}^{-1} \frac{x}{4}.dx + \int_{0}^{3} \frac{x}{4}.dx$
- $\mathbb{E}[X] = [\frac{x^2}{8}]_{-2}^{-1} + [\frac{x^2}{8}]_{0}^{3}$

Example

Answer

- $X(\Omega) = [-2, -1] \cup [0, 3]$ and f continuous on its support, so we know that the expectation of X exists
- $\mathbb{E}[X] = \int_{\mathbb{R}} x.f(x).dx = \int_{-2}^{-1} \frac{x}{4}.dx + \int_{0}^{3} \frac{x}{4}.dx$
- $\mathbb{E}[X] = [\frac{x^2}{8}]_{-2}^{-1} + [\frac{x^2}{8}]_{0}^{3}$
- $\mathbb{E}[X] = \frac{1}{8} - \frac{4}{8} + \frac{9}{8} - \frac{0}{8} = \frac{6}{8}$

Transfert formula

Definition

Consider X an integrable variable with density function f.
Let h a real function and $Y = h(X)$.
Then

$$\mathbb{E}[Y] = \int_{\mathbb{R}} h(x).f(x).dx$$

Variance

Definition

Consider X an integrable variable with density function f.
If $\int_{\mathbb{R}} x^2.f(x).dx < +\infty$, f is a squared integrable density function.
Then X has a variance denoted $\mathbb{V}[X]$ given by:

$$\mathbb{V}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2] = \int_{\mathbb{R}} (x - \mathbb{E}[X])^2.f(x).dx$$

Variance

Properties

Consider X and Y two squared integrable variables. Let λ a reall.

Variance

Properties

Consider X and Y two squared integrable variables. Let λ a reall.

- $\mathbb{V}[X+\lambda] = \mathbb{V}[X]$

Variance

Properties

Consider X and Y two squared integrable variables. Let λ a reall.

- $\mathbb{V}[X+\lambda] = \mathbb{V}[X]$
- $\mathbb{V}[\lambda.X] = \lambda^2.\mathbb{V}[X]$

Variance

Properties

Consider X and Y two squared integrable variables. Let λ a reall.

- $\mathbb{V}[X+\lambda] = \mathbb{V}[X]$
- $\mathbb{V}[\lambda.X] = \lambda^2.\mathbb{V}[X]$
- $\mathbb{V}[X]$ is a non random positive real

Variance

Properties

Consider X and Y two squared integrable variables. Let λ a reall.

- $\mathbb{V}[X+\lambda] = \mathbb{V}[X]$
- $\mathbb{V}[\lambda.X] = \lambda^2.\mathbb{V}[X]$
- $\mathbb{V}[X]$ is a non random positive real
- $\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.
- We have to compute $\mathbb{E}[X^2]$

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.
- We have to compute $\mathbb{E}[X^2]$
- $\mathbb{E}[X^2] = \int_{\mathbb{R}} x^2.f(x).dx = \int_{-2}^{-1} \frac{x^2}{4}.dx + \int_0^3 \frac{x^2}{4}.dx$

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.
- We have to compute $\mathbb{E}[X^2]$
- $\mathbb{E}[X^2] = \int_{\mathbb{R}} x^2.f(x).dx = \int_{-2}^{-1} \frac{x^2}{4}.dx + \int_0^3 \frac{x^2}{4}.dx$
- $\mathbb{E}[X^2] = [\frac{x^3}{12}]_{-2}^{-1} + [\frac{x^3}{12}]_0^3$

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.
- We have to compute $\mathbb{E}[X^2]$
- $\mathbb{E}[X^2] = \int_{\mathbb{R}} x^2.f(x).dx = \int_{-2}^{-1} \frac{x^2}{4}.dx + \int_0^3 \frac{x^2}{4}.dx$
- $\mathbb{E}[X^2] = [\frac{x^3}{12}]_{-2}^{-1} + [\frac{x^3}{12}]_0^3$
- $\mathbb{E}[X] = \frac{-1}{12} - \frac{-8}{12} + \frac{27}{12} - \frac{0}{12} = \frac{20}{12}$

Example

Consider X a random variable whose density function f is defined by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Que vaut $\mathbb{V}[X]$?

- We see that $\mathbb{E}[X] = \frac{6}{8}$.
- We have to compute $\mathbb{E}[X^2]$
- $\mathbb{E}[X^2] = \int_{\mathbb{R}} x^2.f(x).dx = \int_{-2}^{-1} \frac{x^2}{4}.dx + \int_0^3 \frac{x^2}{4}.dx$
- $\mathbb{E}[X^2] = [\frac{x^3}{12}]_{-2}^{-1} + [\frac{x^3}{12}]_0^3$
- $\mathbb{E}[X] = \frac{-1}{12} - \frac{-8}{12} + \frac{27}{12} - \frac{0}{12} = \frac{20}{12}$
- $\mathbb{V}[X] = \frac{20}{12} - \frac{6}{8}^2 = \frac{212}{192}$

Moments

As for discrete case, we can generalize those two notions:

Moments

As for discrete case, we can generalize those two notions:

Definition

Consider X a random variable with a density function f andt $k \in \mathbb{N}^*$. We define:

Moments

As for discrete case, we can generalize those two notions:

Definition

Consider X a random variable with a density function f andt $k \in \mathbb{N}^*$. We define:

- the moment of order k for X is $\mathbb{E}[X^k] = \int_{\mathbb{R}} x^k.f(x).dx$

Moments

As for discrete case, we can generalize those two notions:

Definition

Consider X a random variable with a density function f andt $k \in \mathbb{N}^*$. We define:

- the moment of order k for X is $\mathbb{E}[X^k] = \int_{\mathbb{R}} x^k.f(x).dx$
- the centered moment of order k for X is $\mathbb{E}[(X - \mathbb{E}[X])^k] = \int_{\mathbb{R}}(x - \mathbb{E}[X])^k.f(x).dx$

Sommaire

Distribution function

As for discrete case, we have :

Definition

Consider X a random variable with a density function f.
The distribution function for X, denotede F_X is defined by:

$$\forall t \in \mathbb{R}, \ F_X(t) = P(X \leq t)$$

Distribution function

As for discrete case, we have :

Definition

Consider X a random variable with a density function f.
The distribution function for X, denotede F_X is defined by:

$$\forall t \in \mathbb{R},\ F_X(t) = P(X \leq t)$$

Remark

Since for all $a \in \mathbb{R}$, we have $P(X = a) = 0$, we can notice that :

$$F_X(t) = P(X < t)$$

Thus for continuous case, the two definitions are equivalent.

Distribution function

Propriétés

Consider X a random variable with a density function f.
The distribution function F_X is

Distribution function

Propriétés

Consider X a random variable with a density function f.
The distribution function F_X is

- an increasing function

Distribution function

Propriétés

Consider X a random variable with a density function f.
The distribution function F_X is

- an increasing function
- $\underset{+\infty}{lim} F_X = 1$ and $\underset{-\infty}{lim} F_X = 0$

Distribution function

Propriétés

Consider X a random variable with a density function f.
The distribution function F_X is

- an increasing function
- $\underset{+\infty}{lim} F_X = 1$ and $\underset{-\infty}{lim} F_X = 0$
- $\forall t \in \mathbb{R}\ F_X(t) \in [0; 1]$

Distribution function

Propriétés

Consider X a random variable with a density function f.
The distribution function F_X is

- an increasing function
- $\underset{+\infty}{lim} F_X = 1$ and $\underset{-\infty}{lim} F_X = 0$
- $\forall t \in \mathbb{R}\ F_X(t) \in [0; 1]$
- is a continuous function

Example

Consider X a random variable with a density function f given by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2, -1] \cup [0, 3] \\ 0 & \text{otherwise} \end{cases}$$

What is F_X?

Example

Consider X a random variable with a density function f given by:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

What is F_X?

$$F_X(t) = \begin{cases} 0 & \text{if } t \leq -2 \\ \frac{1}{4}(t+2) & \text{if } -2 < t \leq -1 \\ \frac{1}{4} & \text{if } -1 < t \leq 0 \\ \frac{1}{4}(t+1) & \text{if } 0 < t \leq 3 \\ 1 & \text{if } 3 < t \end{cases}$$

Example

Explanations :

Example

Explanations :

- if $t \leq -2$, $F_X(t) = \int_{-\infty}^{t} f(x).dx = \int_{-\infty}^{-t} 0.dx = 0$

Example

Explanations :

- if $t \leq -2$, $F_X(t) = \int_{-\infty}^{t} f(x).dx = \int_{-\infty}^{-t} 0.dx = 0$
- if $-2 < t \leq -1$, $F_X(t) = \int_{-2}^{t} f(x).dx = \int_{-2}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+2)$

Example

<u>Explanations :</u>

- if $t \leq -2,\ F_X(t) = \int_{-\infty}^{t} f(x).dx = \int_{-\infty}^{-t} 0.dx = 0$
- if $-2 < t \leq -1,\ F_X(t) = \int_{-2}^{t} f(x).dx = \int_{-2}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+2)$
- if $-1 < t \leq 0,\ F_X(t) = \int_{-2}^{-1} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx = \frac{1}{4}$

Example

Explanations :

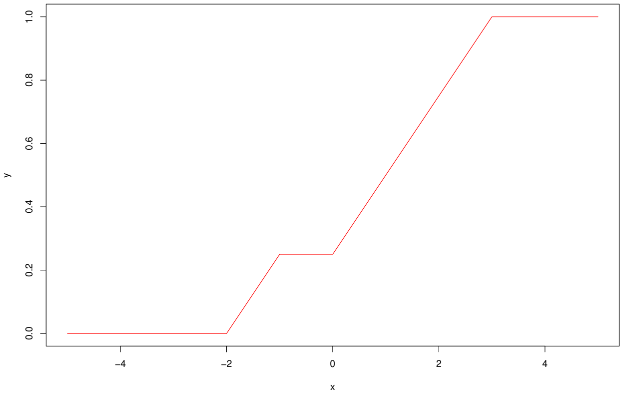
- if $t \leq -2$, $F_X(t) = \int_{-\infty}^{t} f(x).dx = \int_{-\infty}^{-t} 0.dx = 0$
- if $-2 < t \leq -1$, $F_X(t) = \int_{-2}^{t} f(x).dx = \int_{-2}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+2)$
- if $-1 < t \leq 0$, $F_X(t) = \int_{-2}^{-1} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx = \frac{1}{4}$
- if $0 < t \leq 3$,
  $F_X(t) = \int_{-2}^{1} f(x).dx + \int_{0}^{t} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx + \int_{0}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+1)$

Example

Explanations :

- if $t \leq -2$, $F_X(t) = \int_{-\infty}^{t} f(x).dx = \int_{-\infty}^{-t} 0.dx = 0$
- if $-2 < t \leq -1$, $F_X(t) = \int_{-2}^{t} f(x).dx = \int_{-2}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+2)$
- if $-1 < t \leq 0$, $F_X(t) = \int_{-2}^{-1} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx = \frac{1}{4}$
- if $0 < t \leq 3$,
  $F_X(t) = \int_{-2}^{1} f(x).dx + \int_{0}^{t} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx + \int_{0}^{t} \frac{1}{4}.dx = \frac{1}{4}(t+1)$
- if $3 < t$, $F_X(t) = \int_{-2}^{1} f(x).dx + \int_{0}^{3} f(x).dx = \int_{-2}^{-1} \frac{1}{4}.dx + \int_{0}^{3} \frac{1}{4}.dx = 1$

Example



Properties

Properties

Consider X a random variable with a density function f.
We have :

Properties

Properties

Consider X a random variable with a density function f.
We have :

- $\forall a, b \in \mathbb{R},\ a < b,\ P(X \in [a, b]) = F_X(b) - F_X(a)$

Properties

Properties

Consider X a random variable with a density function f.
We have :

- $\forall a, b \in \mathbb{R}, \ a < b, \ P(X \in [a, b]) = F_X(b) - F_X(a)$
- $\forall t \in \mathbb{R}$, with t a point where the distribution function is differantiable

$$F'_X(t) = f(t)$$

Use of the distribution function

Consider X a random variable with a density function f.
Let h a function, we definet

$$Y = h(X)$$

Use of the distribution function

Consider X a random variable with a density function f.
Let h a function, we definet

$$Y = h(X)$$

What is the distribution of Y?

Use of the distribution function

Consider X a random variable with a density function f.
Let h a function, we definet

$$Y = h(X)$$

What is the distribution of Y?
here is th emethod to answer :

Use of the distribution function

Consider X a random variable with a density function f.
Let h a function, we definet

$$Y = h(X)$$

What is the distribution of Y?
here is th emethod to answer :

- We determine the distribution of Y thanks to the one of X

Use of the distribution function

Consider X a random variable with a density function f.
Let h a function, we definet

$$Y = h(X)$$

What is the distribution of Y?
here is th emethod to answer :

- We determine the distribution of Y thanks to the one of X
- We differentiate we distribution function obtained to get the density function of Y.

Example

Consider X a random variable with a density function f is given by

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2, -1] \cup [0, 3] \\ 0 & \text{otherwise} \end{cases}$$

Consider $Y = X^2$. What is the distribution of Y?

Example

Consider X a random variable with a density function f is given by

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } x \in [-2,-1] \cup [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Consider $Y = X^2$. What is the distribution of Y?
The distribution of Y is given by the density function g whose expression is

$$g(t) = \begin{cases} \frac{1}{8.\sqrt{t}} & \text{if } t \in]0;1[\\ \frac{1}{4.\sqrt{t}} & \text{if } t \in]1;4[\\ \frac{1}{8.\sqrt{t}} & \text{if } t \in]4;9[\\ 0 & \text{otherwise} \end{cases}$$

Example

Explanations:

Example

Explanations:

- We determine the distribution function of Y

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$
- If $t \geq 0$, then
  $F_Y(t) = P(Y \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$
- If $t \geq 0$, then
  $F_Y(t) = P(Y \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$
- If $t \in [0;1]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4} = \frac{1}{4}\sqrt{t}$
- If $t \in [1;4]$,
  $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4}(-\sqrt{t}+2) = \frac{1}{4}(2.\sqrt{t}-1)$

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$
- If $t \geq 0$, then
  $F_Y(t) = P(Y \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$
- If $t \in [0;1]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4} = \frac{1}{4}\sqrt{t}$
- If $t \in [1;4]$,
  $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4}(-\sqrt{t}+2) = \frac{1}{4}(2.\sqrt{t}-1)$
- If $t \in [4;9]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - 0 = \frac{1}{4}(\sqrt{t}+1)$

Example

<u>Explanations:</u>

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$
- If $t \geq 0$, then
  $F_Y(t) = P(Y \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$
- If $t \in [0;1]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4} = \frac{1}{4}\sqrt{t}$
- If $t \in [1;4]$,
  $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4}(-\sqrt{t}+2) = \frac{1}{4}(2.\sqrt{t}-1)$
- If $t \in [4;9]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - 0 = \frac{1}{4}(\sqrt{t}+1)$
- If $t > 9$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = 1 - 0 = 1$

Example

Explanations:

- We determine the distribution function of Y
- By definition, $F_Y(t) = P(Y \leq t) = P(X^2 \leq t)$
- If $t < 0$, $\{X^2 \leq t\} = \emptyset$ and $F_Y(t) = 0$
- If $t \geq 0$, then
  $F_Y(t) = P(Y \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$
- If $t \in [0;1]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4} = \frac{1}{4}\sqrt{t}$
- If $t \in [1;4]$,
  $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - \frac{1}{4}(-\sqrt{t}+2) = \frac{1}{4}(2.\sqrt{t}-1)$
- If $t \in [4;9]$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = \frac{1}{4}(\sqrt{t}+1) - 0 = \frac{1}{4}(\sqrt{t}+1)$
- If $t > 9$, $F_Y(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t}) = 1 - 0 = 1$
- We just need now to differentiate the g function