Homework #2

February 4, 2025

Problem I

Consider the following optimisation problem

$$\min_{x \in \mathbb{R}^n} \frac{1}{2}\|Ax - y\|_2^2 + \lambda \mathcal{R}_{\alpha,\beta}(x) + \mathtt{i}_{\geq 0}(x)$$

where

$$\mathcal{R}_{\alpha,\beta}(x) = \|x\|_1 + \langle \alpha, x \rangle + \beta, \qquad \alpha \in \mathbb{R}^n, \beta > 0$$

and A is a convolution operator: $Ax = h \star x$.

1. Derive the proximal operator of the regularisation term $\mathcal{R}_{\alpha,\beta}$.
2. Give the pseudo-code for one iteration of FISTA.
3. Analyse the role of the parameter β in the reconstruction quality of the solution.