More on Learning Algorithms

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Learning by classifier combination: boosting

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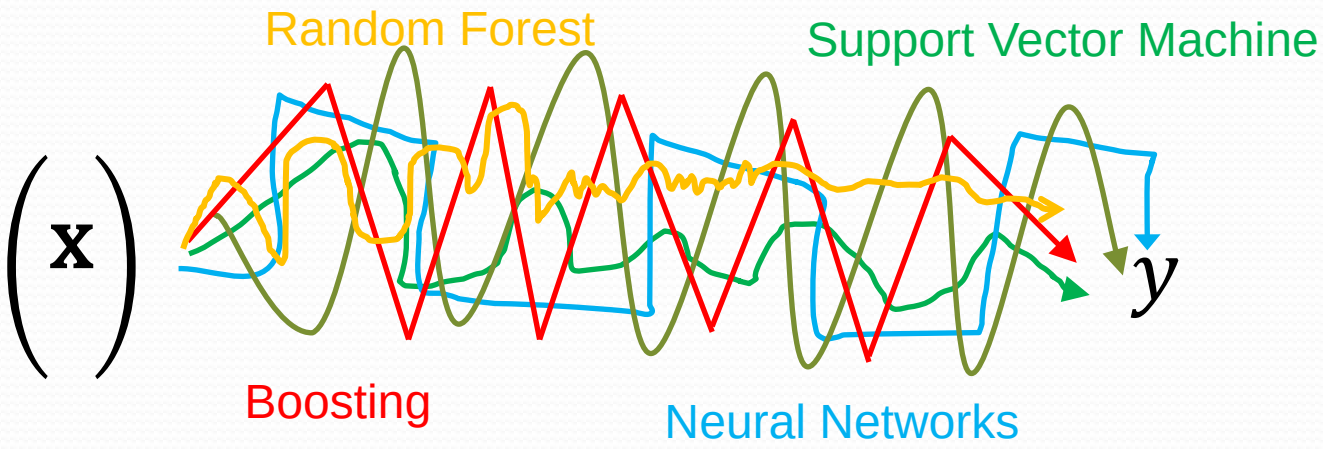
Machine Learning

$$\left(\mathbf{x}\right) \xrightarrow{f(\mathbf{X},\boldsymbol{\alpha})\ ?} y$$



Main Idea

Groups are collectively better at decision making than individuals.

Ensemble Learning

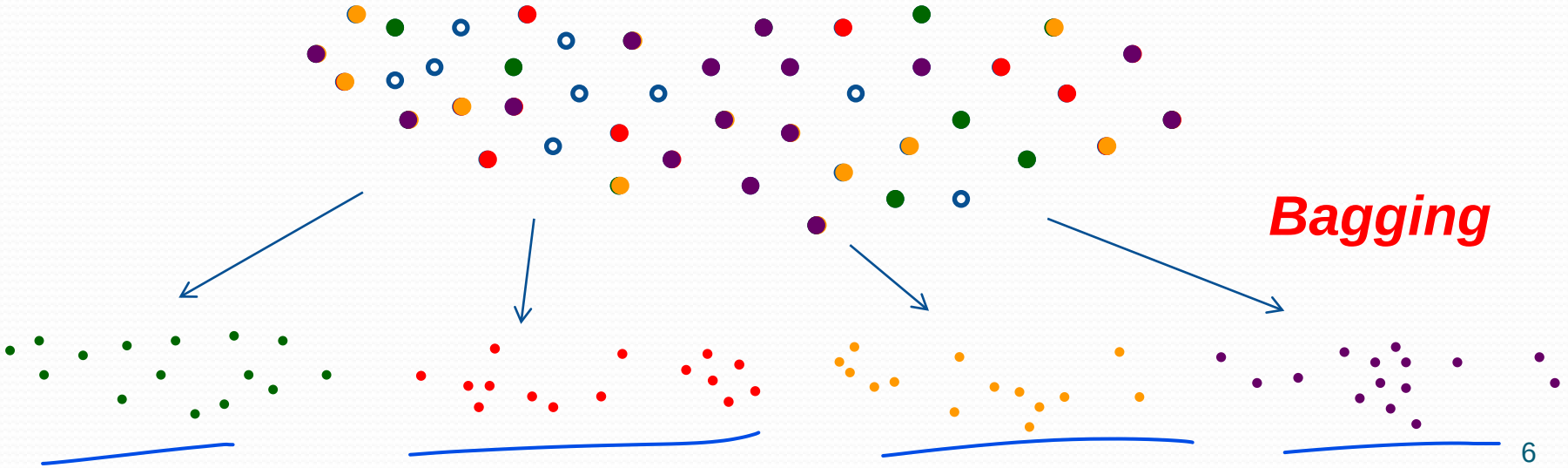
- meta-algorithm combining different learners different methods :
  - bagging :
    - weak learners learned independently usually homogeneous weak learners
    - voting (classification) or averaging (regression) bootstrapping - features sampling - Random Forests
  - boosting :
    - weak learners learned sequentially usually homogeneous weak learners
    - each new weak learners should improve the ensemble learning algorithm
  - stacking :
    - weak learners learned independently often heterogeneous week learners
    - meta-model built on top of the output of the weak learners

Outlook

- Bagging
- Boosting algorithm
  - Idea
  - A first simple example to understand boosting idea
  - Probabilistic boosting and Adaboost
  - A simple example for understanding Adaboost
- Stacking

Bagging

- bagging = combining the results of multiple models, e.g. decision trees.
  - need different models, often homogeneous
  - for regression : average the different results
  - for classication : vote for the classes
- bootstrapping : repeatedly selects a subset with replacement of the training set
  - a model is learned with each subset



Outlook

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Context: Binary Classficiation

- Separation between data corresponding to some criterions and data not corresponding.

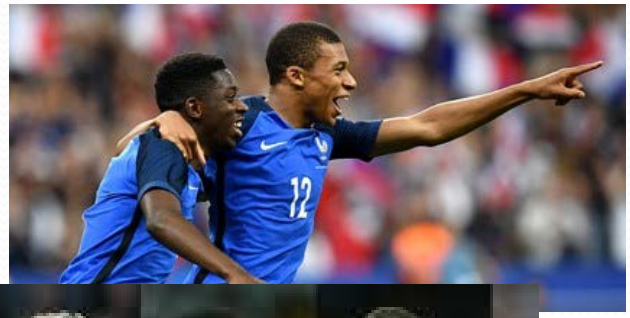
Main idea of boosting

- Build a strong learner from weak learners Example :
  - Green corresponds to fir tree.
  - Vertical shapes corresponds to fir tree.
  - Shapes with bottom larger than top corresponds to fir tree. When dominant color is red, it is not a fir tree.
  - ….
- Example of sport bet and heuristics.
  - We ask experts simple rules that are true more than 50%
  - We focus on examples for which a rule fails and we ask other rules to experts.
  - And loop the process

Ideas of boosting: Football Bets

If Lemar is not injured, French Football team wins.

If Mbappé and Dembélé are happy together, French Football team wins.



If Frenc
France

If Pogba is happy, French Football team wins.

How to win?

- Ask to professional gamblers
- Lets assume:
  - That professional gamblers can provide one single decision rule simple and relevant
  - But that face to several games, they can always provide decision rules a little bit better than random
- Can we become rich?

Idea

- Ask **heuristics** to the *expert*
- Gather a set of cases for which these **heuristics** fail (**difficult cases**)
- Ask again the *expert* to provide **heuristics for the difficult cases**
- And so one…

- **Combine** these heuristics
- *expert* stands for ***weak learner***

Boosting

- **boosting** = general method to convert several poor decision rules into one very powerful decision rule
- More precisely:
  - Let have a weak learner which can always provide a decision rule with an error $\leq \frac{1}{2} - \gamma$
  - A boosting algorithm can build (theoretically) a global decision rule with an error rate $\leq \varepsilon$

Probabilistic boosting

3 main ideas to generalize towards ***probabilistic boosting***:

1. A set of specialized experts and ask them to vote to take a decision.

2. Adaptive weighting of votes by multiplicative update.

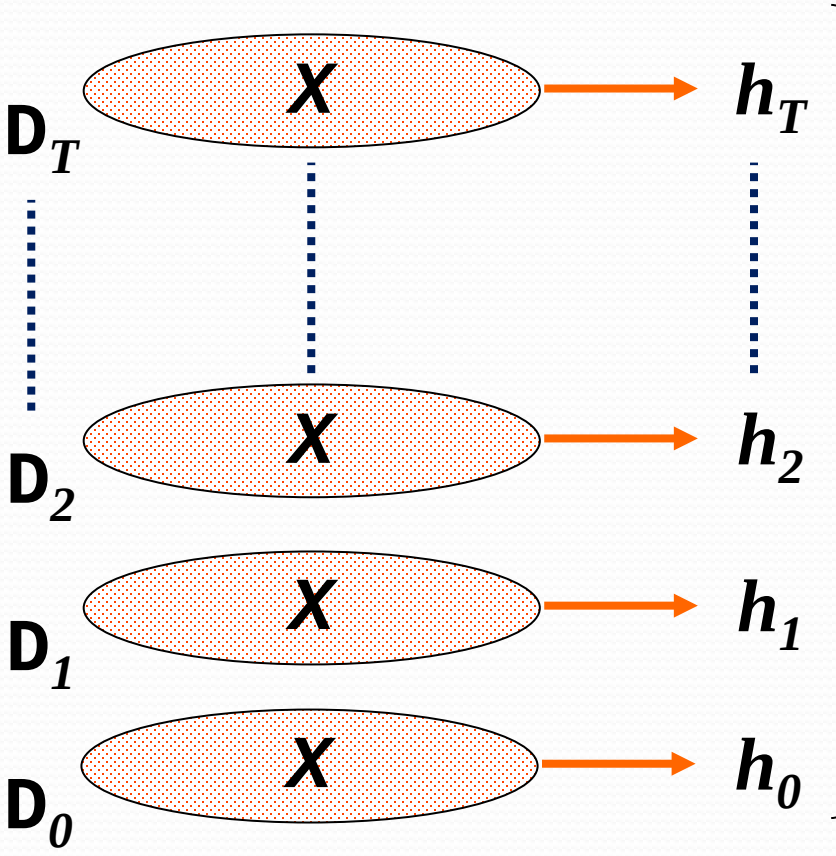
3. Modifying example distribution to train each expert, increasing the weights iteratively of examples misclassified at previous iteration.

Boosting: Basics

Parameter: weak learners ***h***
Input: A set ***S***
Initialization: All examples have same weights

For *t* = 1,*T* do
    use ***h*** with ***S*** and current distribution
    increase weight of misclassified examples at current iteration
done
Output *H* : a majority weighted vote of all the ***h***

Boosting: Basics



$$H_{final}(x) = sgn\left[\sum_{t=0}^{T} \alpha_t \cdot h_t(x)\right]$$

- How to update D_t to D_{t+1} ?
- How to compute the weight α_t ?
- What error is minimized ?

Probabilistic boosting: AdaBoost

The standard algorithm is **AdaBoost** (*Adaptive Boosting*).

The main idea is to define at each iteration $1< t < T$, a new distribution of *a priori probabilities* on training samples with respect to classification result at previous iteration.

The weight at iteration t of an example $(\mathbf{x}_i, y_i)$ of index i is denoted $D_t(i)$.

Initially, all the examples get a similar weight, at each iteration, weights of misclassified examples are increased

Thus the learner focuses on difficult examples of training set

Boosting: the algorithm

- A training set: S = $\{(x_1,y_1),\ldots,(x_m,y_m)\}$
- $y_i \in \{-1,+1\}$ label (annotation) of example $x_i \in$ S

- For $t = 0,\ldots,T$:
  - Build the distribution D_t on $\{1,\ldots,m\}$
  - Find the weak decision ("heuristic")

$$h_t : S \rightarrow \{-1,+1\}$$

with a small error ε_t on D_t :

$$\varepsilon_t = \Pr_{D_t}[h_t(x_i) \neq y_i] = \sum_{i:h_t(x_i)\neq y_i} D_t(i)$$

- Find the final decision h_{final}

The AdaBoost Algorithm

What goal the AdaBoost wants to reach?

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X$, $y_i \in \{-1, +1\}$

Initialization: $D_1(i) = \frac{1}{m}, i = 1, \ldots, m$

For $t = 1, \ldots, T$:

- Find classifier $h_t : X \to \{-1, +1\}$ which minimizes error wrt D_t ,i.e.,

$$h_t = \arg\min_{h_j} \varepsilon_j \quad \text{where } \varepsilon_j = \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

- Weight classifier: $\alpha_t = \frac{1}{2} \ln \frac{1 - \varepsilon_t}{\varepsilon_t}$

- Update distribution: $D_{t+1}(i) = \frac{D_t(i) \exp[-\alpha_t y_i h_t(x_i)]}{Z_t}$, Z_t is for normalization

Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

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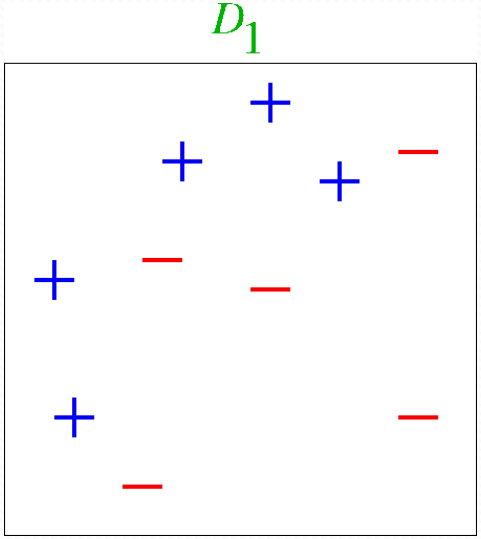
Example “Toy”



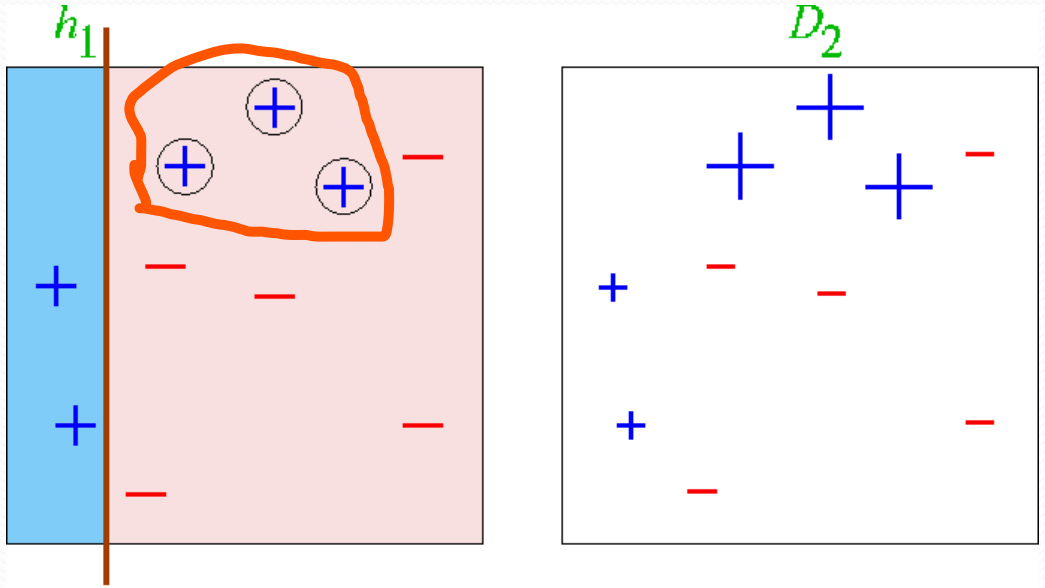
Very popular «Toy» example, 2D points in domain $\boldsymbol{D_1}$, to illustrate boosting schema.

(these points can be seen as vectors $\in \boldsymbol{R^2}$).

10data points
so d1 = 1/10 as uniform distribution and assign same weights to each data points initially.

D1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Step 1



$\varepsilon_1 = 0.30$

$\alpha_1 = 0.42$

epslion1 = 0.1*3 =0.3

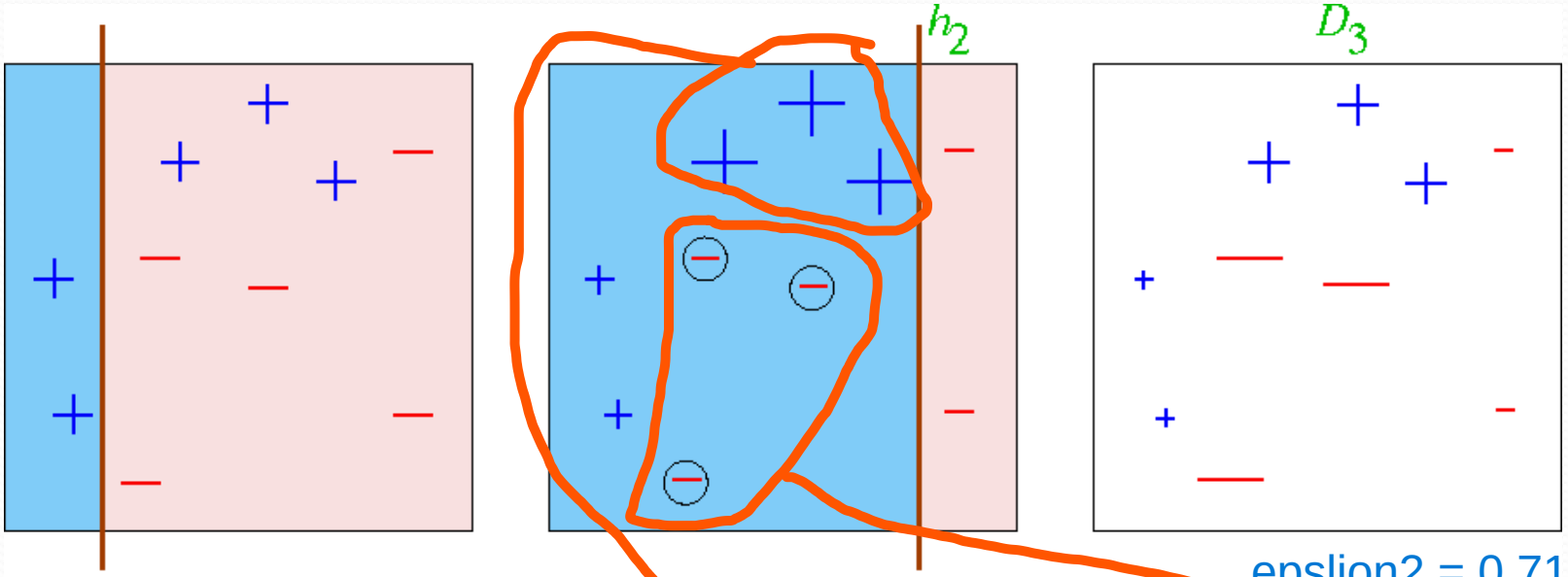
$$\alpha_1 = \frac{1}{2}\ln\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) = \frac{1}{2}\ln\left(\frac{1-0.3}{0.3}\right) = \frac{1}{2}\ln\left(\frac{0.7}{0.3}\right) = \ln\left(\sqrt{\frac{7}{3}}\right)$$

$$D'_1 = \begin{cases} 0.1e^{-\alpha_1} = 0.1\sqrt{\frac{3}{7}} = 0.065 \\ 0.1e^{\alpha_1} = 0.1\sqrt{\frac{7}{3}} = 0.152 \end{cases}$$

$$Z_1 = 3 \times 0.152 + 7 \times 0.065 = 0.911$$

$D_2=D'_1/Z_1$	0.167	0.167	0.167	0.071	0.071	0.071	0.071	0.071	0.071	0.071

Step 2



epslion2 = 0.71*3 =0.21

$\varepsilon_2 = 0.21$

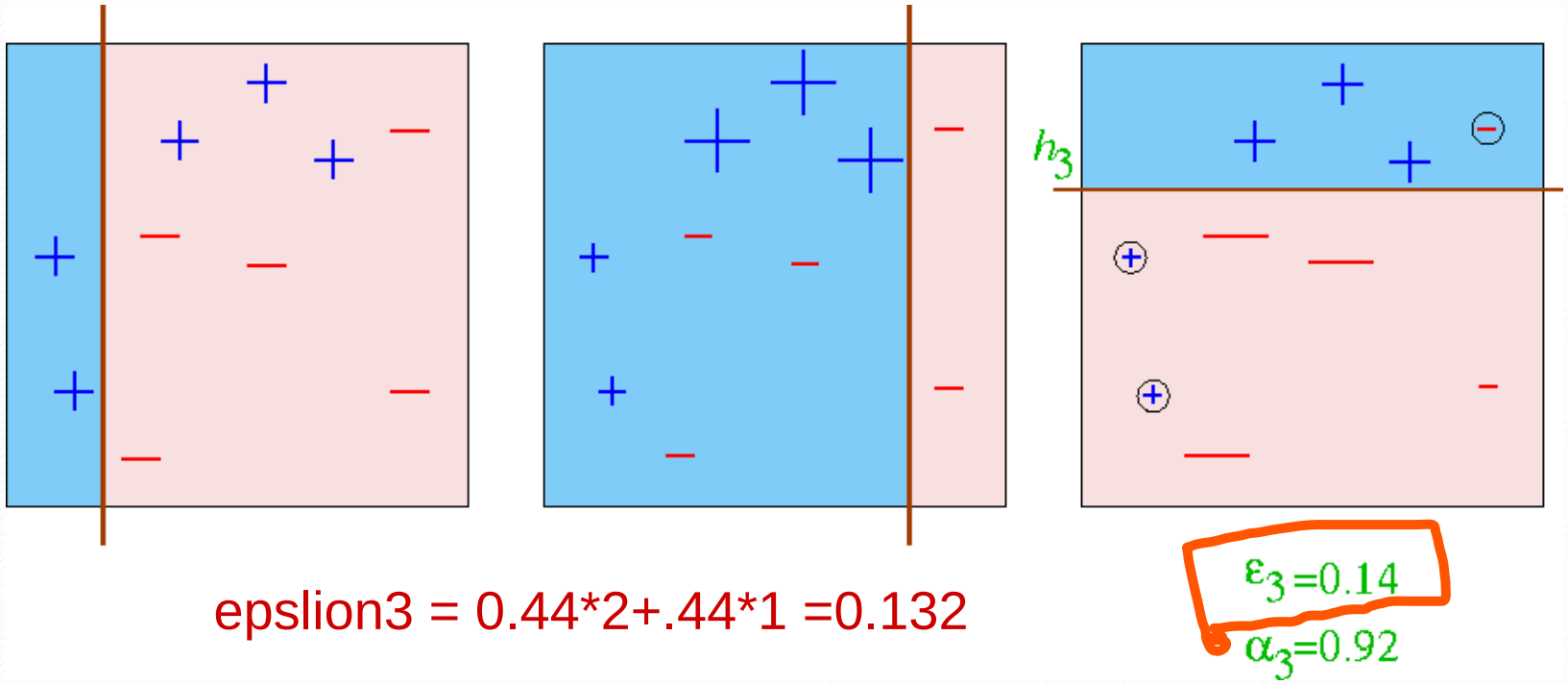
$\alpha_2 = 0.65$

$$D'_2 = \begin{cases} 0.17\, e^{-\alpha_2} = 0.0876 \\ 0.07\, e^{-\alpha_2} = 0.036 \\ 0.07\, e^{\alpha_2} = 0.1357 \end{cases}$$

$$Z_2 = 3 \times 0.0876 + 4 \times 0.036 + 3 \times 0{,}1357 = 0.814$$

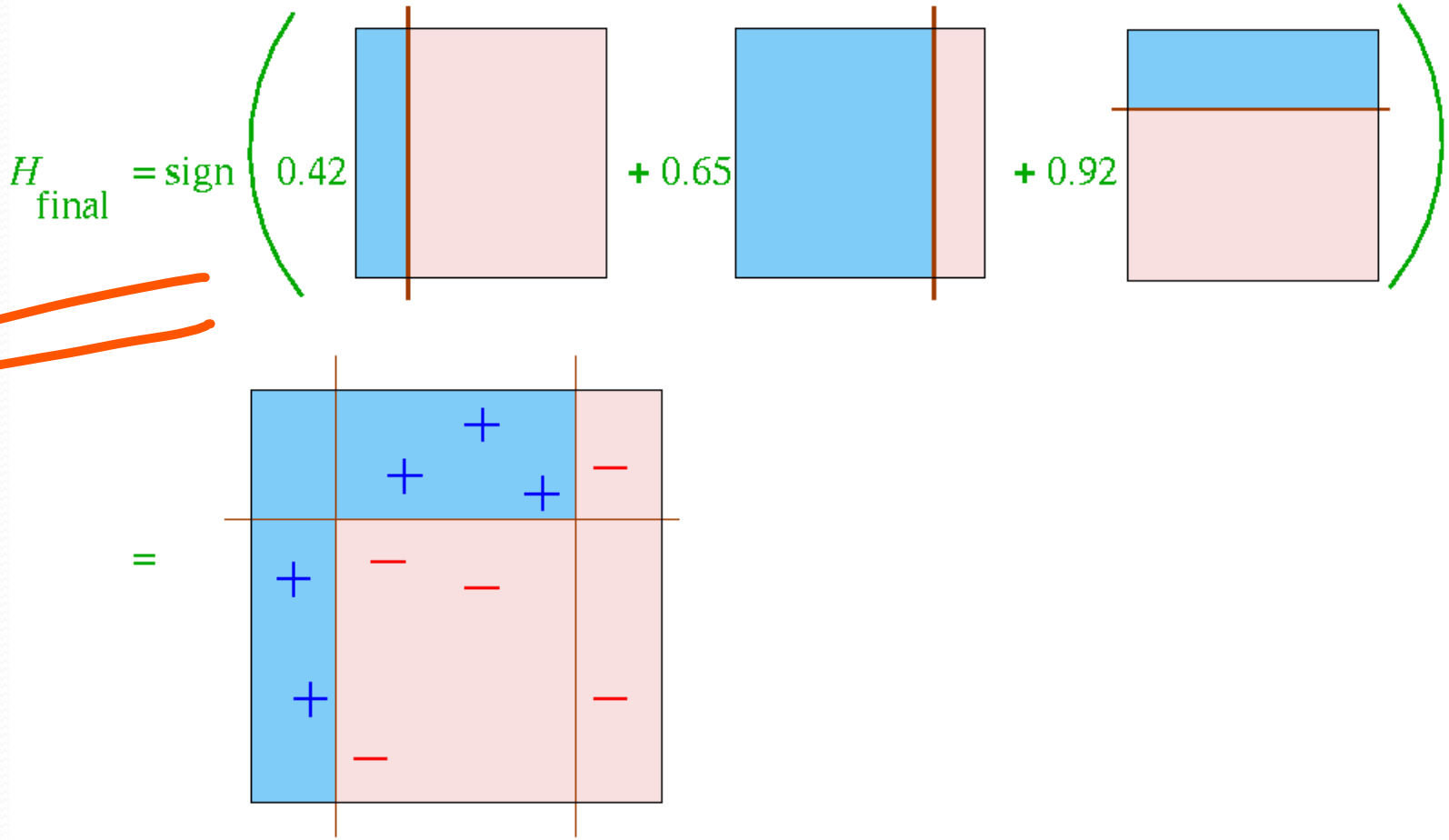
$D_3 = D'_2/Z_2$	0.107	0.107	0.107	0.044	0.044	0.044	0.044	0.166	0.166	0.166

Step 3



epslion3 = 0.44*2+.44*1 =0.132

Final decision



Boosting: global classifier

$$H_{\text{final}}(x) = \text{sgn}\left(\sum_t \alpha_t h_t(x)\right)$$

The boosting algorithm builds the finale decision as a summation on a function basis $\{\boldsymbol{h_t}\}$.

This is a classic process of machine learning algorithms (as SVM, bayesian classifier, neural networks, etc.).

AdaBoost Hinge Loss

Lets write the error ε_t of h_t as: $1/2 - \gamma_t$, where γ_t measure the improvement brought by h_t regarding the basic error of 1/2.

Freund and Schapire, have shown that the hinge loss (the amount of misclassified data over training set **S**) of finale decision **H** is bounded by:

$$\textbf{Error}_{\textbf{Hinge\_Loss}}(H_{\text{final}}(x)) \leq exp\left(-2\sum_t {\gamma_t}^2\right)$$

Hence, if each weak learner is slightly better than random ($\gamma_t > 0$) then the hinge loss decrease exponentially with t.

Proof

Let $H_{FINAL}(x) = f(x) = \sum_t \alpha_t h_t(x)$

We have: $D_{Final}(x_i, y_i) = \frac{D_t}{Z_t} \cdot \exp(-\alpha_t \cdot y_i \cdot h_t(x_i)) = \frac{D_{t-1}}{Z_{t-1}} \exp(-y_i \alpha_{t-1} \cdot h_{t-1}(x_i)) \frac{\exp(-y_i \alpha_t \cdot h_t(x_i))}{Z_t}$

$$= \frac{1}{m} \frac{\exp\left(-y_i \sum_t \alpha_t \cdot h_t(x_i)\right)}{\prod_t Z_t} = \frac{1}{m} \frac{\exp(-y_i f(x_i))}{\prod_t Z_t}$$

We can proove that the *Hinge loss* (i.e. error when counting 1 for every wrong result) is less than: $\prod_t Z_t$

$$\textbf{\textit{Error}}_{\textbf{\textit{Hinge\_Loss}}}(H(x)) = \frac{1}{m} \sum_i 1_{H(x_i) \neq y_i} = \frac{1}{m} \sum_i 1_{y_i f(x_i) < 0}$$

$$\leq \frac{1}{m} \sum_i \exp(-y_i f(x_i)) = \sum_i D_{final}(i) \prod_t Z_t = \prod_t Z_t$$

Proof

And $$Z_t = \sum_i D_t(i)\exp\left(-\alpha_t y_i f(x_i)\right) = \sum_{i:y_i \neq h_t(x_i)} D_t(i)e^{\alpha_t} + \sum_{i:y_i = h_t(x_i)} D_t(i)e^{-\alpha_t}$$

$$= \varepsilon_t e^{\alpha_t} + (1-\varepsilon_t)e^{-\alpha_t} = 2\sqrt{\varepsilon_t(1-\varepsilon_t)}$$

Then the **Hinge Loss** is bounded by:

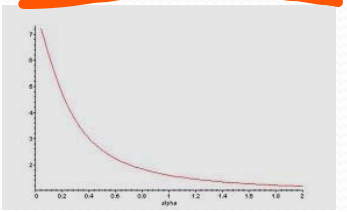
$$\prod_t Z_t = \prod_t \left[2\sqrt{\varepsilon_t(1-\varepsilon_t)}\right] = \prod_t \sqrt{1-4\gamma_t{}^2} \leq exp\left(-2\sum_t \gamma_t{}^2\right)$$

The error of learning decreases exponentially with *t*

Error of generalizaton

Error of generalization of H can be bounded by:

$$E_{Real}(H_T) = E_{HingeLoss}(H_T) + O\left(\sqrt{\frac{T.d}{m}}\right)$$



where T is the number of boosting iterations, m the number of examples, and d a dimension (VC-dimension) of H_T space (« weaks learner complexity »)

When T becomes large, the 2nd term becomes large too, then one can think that the learner would ***overfit***

⇒ the solution = over-weighting of difficult examples

Advantages of AdaBoost

- (very) fast
- simple + easy to code
- One unique parameter to set: the number of boosting iterations (T)
- Can be use with any machine learning algorithm
- No-overfitting (margin theory)
- Is adapted to multi-class problems where $y_i \in \{1,..,c\}$ and to multi-label problems
- Allow to find out outliers

Drawbacks of AdaBoost

- Performances depend on training data & weak learners
- AdaBoost can fail if
  - The weak learner is to complex
    - Low margins → overfitting
  - The weak learner is too weak
    - $\gamma_t \to 0$ too fast → underfitting
- AdaBoost seems specifically sensitive to noise

Applications

- Detection, tracking and face recognition
- Spam, Zip Code OCR
- Text classification: Schapire and Singer - Used stumps with normalized term frequency and multi-class encoding
- OCR: Schwenk and Bengio (neural networks)
- Natural language Processing: Collins; Haruno, Shirai and Ooyama
- Image retrieval: Thieu and Viola
- Medical diagnosis: Merle et al.
- Fraud Detection: Rätsch & Müller 2001
- Drug Discovery: Rätsch, Demiriz, Bennett 2002
- Elect. Power Monitoring: Onoda, Rätsch & Müller 2000

Background

- [Valiant'84]

  introduced theoretical PAC model for studying machine learning

- [Kearns&Valiant'88]

  open problem of finding a boosting algorithm

- [Schapire'89], [Freund'90]

  first polynomial-time boosting algorithms

- [Drucker, Schapire&Simard '92]

  first experiments using boosting

Backgroung (cont.)

- [Freund&Schapire '95]
  - Provided AdaBoost algorithm
  - Bigger advantage in pratice than previous boosting algorithms
- experiences using AdaBoost:

[Drucker&Cortes '95]
[Jackson&Cravon '96]
[Freund&Schapire '96]
[Quinlan '96]
[Breiman '96]
[Schapire&Singer '98]
[Maclin&Opitz '97]
[Bauer&Kohavi '97]
[Schwenk&Bengio '98]
[Dietterich'98]

- Further development on theory & algorithms:

[Schapire,Freund,Bartlett&Lee '97]
[Breiman '97]
[Grive and Schuurmans'98]
[Schapire&Singer '98]
[Mason, Bartlett&Baxter '98]
[Friedman, Hastie&Tibshirani '98]

Variations

- Solve overfitting when noisy data:
  - ***Weight Decay*** (1998): introduces slack variables regularizing the error
  - ***GentleBoost*** (1998)
  - ***LogitBoost*** (2000): additive logistic regression model (statistical point of view)
  - ***Regularized AdaBoost*** (2000): « soft » margins
  - ***BrownBoost*** (2001): removes examples many times misclassified, considered as « noisy »
  - ***WeightBoost*** (2003): weights in final decision dependent on input
- Reduce the number of features (or Weak Learner) :
  - ***FloatBoost*** (2003): removes the worst WL after each iteration
  - ***JointBoost*** (2004): multi-class, jointly train N binary classifier sharing same features
- Adaboost Multi-class
  - ***AdaBoost.M1:*** same as standard but $y \in \{1,\ldots,k\}$
  - ***AdaBoost.M2:*** introduces a confidence degree

Demonstration of Adaboost equations

The AdaBoost Algorithm

What goal the AdaBoost want

They are goal dependent.

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where

Initialization: $D_1(i) = \frac{1}{m}, i = 1, \ldots,$

For $t = 1, \ldots, T$:

- Find classifier $h_t : X \to \{-1, +1\}$ which minimizes error wrt D_t ,i.e.,

$$h_t = \arg\min_{h_j} \varepsilon_j \quad \text{where} \quad \varepsilon_j = \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

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Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

Goal

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

Minimize exponential loss

$$loss_{\exp}\left[H(x)\right] = E_{x,y}\left[e^{-yH(x)}\right]$$

Maximize the margin $yH(x)$

Goal

Minimize $loss_{\exp}\left[H(x)\right] = E_{x,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$E_{x,y}\left[e^{-yH_t(x)}\right] = E_x\left[E_y\left[e^{-yH_t(x)} \mid x\right]\right]$$

$$= E_x\left[E_y\left[e^{-y[H_{t-1}(x)+\alpha_t h_t(x)]} \mid x\right]\right]$$

$$= E_x\left[E_y\left[e^{-yH_{t-1}(x)} e^{-y\alpha_t h_t(x)} \mid x\right]\right]$$

$$= E_x\left[E_y\left[e^{-yH_{t-1}(x)}\left[e^{-\alpha_t} P(y = h_t(x)) + e^{\alpha_t} P(y \neq h_t(x))\right] \mid x\right]\right]$$

Goal

$\alpha_t = ?$

Minimize $loss_{\exp}[H(x)] = E_{x,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$E_{x,y}\left[e^{-yH_t(x)}\right] = E_x\left[E_y\left[e^{-yH_{t-1}(x)}\left[e^{-\alpha_t}P(y = h_t(x)) + e^{\alpha_t}P(y \neq h_t(x))\right] | x\right]\right]$$

Set $\frac{\partial}{\partial \alpha_t} E_{x,y}\left[e^{-yH_t(x)}\right] = 0$

$$\Rightarrow E_x\left[E_y\left[e^{-yH_{t-1}(x)}\left[\underbrace{-e^{-\alpha_t}P(y = h_t(x)) + e^{\alpha_t}P(y \neq h_t(x))}_{0}\right] | x\right]\right] = 0$$

Goal

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Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$\Rightarrow E_x\left[E_y\left[e^{-yH_{t-1}(x)}\left[\underbrace{-e^{-\alpha_t}P(y = h_t(x)) + e^{\alpha_t}P(y \neq h_t(x))}_{0}\right] | x\right]\right] = 0$$

$$\Rightarrow \alpha_t = \frac{1}{2}\ln\frac{P(y = h_t(x))}{P(y \neq h_t(x))} \quad \Rightarrow \alpha_t = \frac{1}{2}\ln\frac{1-\varepsilon_t}{\varepsilon_t}$$

$P(x_i, y_i) = D_t(i)$

$$\varepsilon_t = P(\text{error}) \approx \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

Goal

$\alpha_t = ?$

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X, y_i \in \{-1, +1\}$

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Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

Define $H_t(x) =$

Then, $H(x) = H$

$$\Rightarrow E_x \left[E_y \left[e^{-yH_{t-1}(x)} \left[\underbrace{-e^{-\alpha_t} P(y = h_t(x)) + e^{\alpha_t} P(y \neq h_t(x))}_{0} \right] \mid x \right] \right] = 0$$

$$\Rightarrow \alpha_t = \frac{1}{2} \ln \frac{P(y = h_t(x))}{P(y \neq h_t(x))} \Rightarrow \boxed{\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}}$$

$$P(x_i, y_i) = D_t(i)$$

$$\varepsilon_t = P(\text{error}) \approx \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

Goal

$D_{t+1} = ?$

Define $H_t(x) =$

Then, $H(x) = H$

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X, y_i \in \{-1, +1\}$

Initialization: $D_1(i) = \frac{1}{m}, i = 1, \ldots, m$

For $t = 1, \ldots, T$:

- Find classifier $h_t : X \to \{-1, +1\}$ which minimizes error wrt D_t ,i.e.,

$$h_t = \arg\min_{h_j} \varepsilon_j \quad \text{where } \varepsilon_j = \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

- Weight classifier: $\alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$

- Update distribution: $D_{t+1}(i) = \frac{D_t(i) \exp[-\alpha_t y_i h_t(x_i)]}{Z_t}$, Z_t is for normalization

Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

$$\Rightarrow E_x \left[E_y \left[e^{-yH_{t-1}(x)} \left[\underbrace{-e^{-\alpha_t} P(y = h_t(x)) + e^{\alpha_t} P(y \neq h_t(x))}_{0} \right] \mid x \right] \right] = 0$$

$$\Rightarrow \alpha_t = \frac{1}{2} \ln \frac{P(y = h_t(x))}{P(y \neq h_t(x))} \quad \Rightarrow \alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$$

$$P(x_i, y_i) = D_t(i)$$

$$\varepsilon_t = P(\text{error}) \approx \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

Goal

$D_{t+1} = ?$

Minimize $loss_{\exp}\left[H(x)\right] = E_{x\sim D,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$E_{x,y}\left[e^{-yH_t}\right] = E_{x,y}\left[e^{-yH_{t-1}}e^{-y\alpha_t h_t}\right] \approx E_{x,y}\left[e^{-yH_{t-1}}\left(1 - y\alpha_t h_t + \frac{1}{2}\alpha_t^2 y^2 h_t^2\right)\right]$$

$$\Rightarrow h_t = \arg\min_h E_{x,y}\left[e^{-yH_{t-1}}\left(1 - y\alpha_t h + \frac{1}{2}\alpha_t^2 y^2 h^2\right)\right] \qquad y^2h^2 = 1$$

$$\Rightarrow h_t = \arg\min_h E_{x,y}\left[e^{-yH_{t-1}}\left(1 - y\alpha_t h + \frac{1}{2}\alpha_t^2\right)\right]$$

$$\Rightarrow h_t = \arg\min_h E_x\left[E_y\left[e^{-yH_{t-1}}\left(1 - y\alpha_t h + \frac{1}{2}\alpha_t^2\right) \mid x\right]\right]$$

Goal

$D_{t+1} = ?$

Minimize $loss_{\exp}[H(x)] = E_{x\sim D,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$\Rightarrow h_t = \arg\min_h E_x\left[E_y\left[e^{-yH_{t-1}}\left(1 - y\alpha_t h + \frac{1}{2}\alpha_t^2\right)\right]|\, x\right]$$

$$\Rightarrow h_t = \arg\min_h E_x\left[E_y\left[e^{-yH_{t-1}}\left(-y\alpha_t h\right)\right]|\, x\right]$$

$$\Rightarrow h_t = \arg\max_h E_x\left[E_y\left[e^{-yH_{t-1}}\left(yh\right)\right]|\, x\right]$$

$$\Rightarrow h_t = \arg\max_h E_x\left[1\cdot h(x)e^{-H_{t-1}(x)}\cdot P(y=1|\, x) + (-1)\cdot h(x)e^{H_{t-1}(x)}\cdot P(y=-1|\, x)\right]$$

Goal

$D_{t+1} = ?$

Minimize $loss_{\exp}[H(x)] = E_{x\sim D,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$$\Rightarrow h_t = \arg\max_h E_x\left[1\cdot h(x)e^{-H_{t-1}(x)}\cdot P(y=1\mid x) + (-1)\cdot h(x)e^{H_{t-1}(x)}\cdot P(y=-1\mid x)\right]$$

$$\Rightarrow h_t = \arg\max_h E_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}\left[yh(x)\right]$$ maximized when $y = h(x) \quad \forall x$

$$\Rightarrow h_t = sign\left(E_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}\left[y\mid x\right]\right)$$

$$\Rightarrow h_t = sign\left(P_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}(y=1\mid x) - P_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}(y=-1\mid x)\right)$$

Goal

$D_{t+1} = ?$

Minimize $loss_{\exp}[H(x)] = E_{x\sim D,y}\left[e^{-yH(x)}\right]$

Final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Define $H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$ with $H_0(x) = 0$

Then, $H(x) = H_T(x)$

$\Rightarrow h_t = sign\left(P_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}(y=1|x) - P_{x,y\sim e^{-yH_{t-1}(x)}P(y|x)}(y=-1|x)\right)$

At time t $\quad x, y \sim e^{-yH_{t-1}(x)}P(y|x)$

Goal

$D_{t+1} = ?$

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X, y_i \in \{-1, +1\}$

Initialization: $D_1(i) = \frac{1}{m}, i = 1, \ldots, m$

For $t = 1, \ldots, T$:

- Find classifier $h_t : X \to \{-1, +1\}$ which minimizes error wrt D_t ,i.e.,

$$h_t = \arg\min_{h_j} \varepsilon_j \quad \text{where } \varepsilon_j = \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

- Weight classifier: $\alpha_t = \frac{1}{2} \ln \frac{1 - \varepsilon_t}{\varepsilon_t}$

- Update distribution: $D_{t+1}(i) = \frac{D_t(i) \exp[-\alpha_t y_i h_t(x_i)]}{Z_t}$, Z_t is for normalization

Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

Define $H_t(x) =$

Then, $H(x) = H$

At time t $\quad x, y \sim e^{-yH_{t-1}(x)} P(y \mid x)$

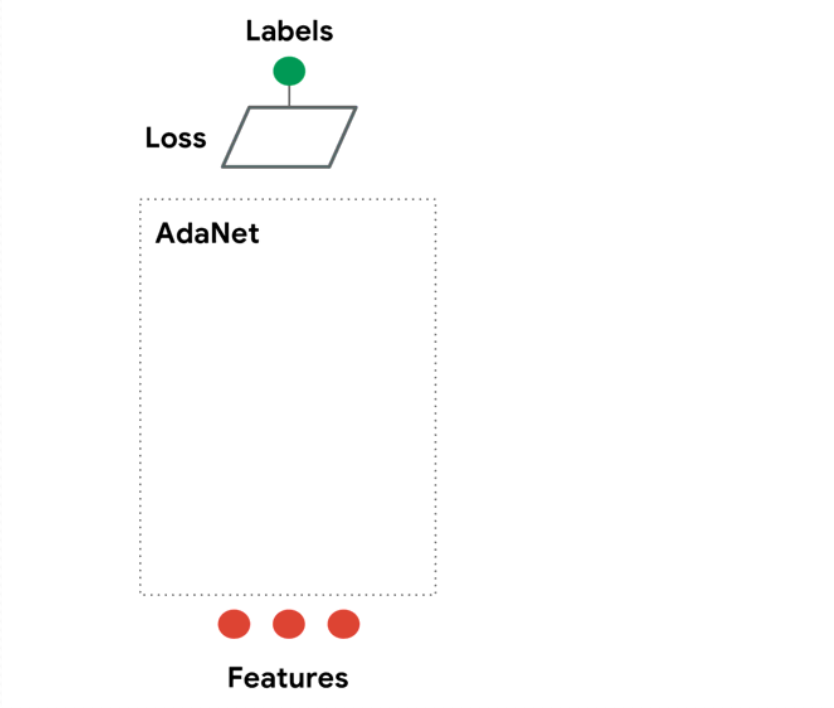
At time 1 $\quad x, y \sim P(y \mid x)$ $\qquad P(y_i \mid x_i) = 1 \Rightarrow D_1(i) = \frac{1}{Z_1} = \frac{1}{m}$

At time t+1 $\quad x, y \sim e^{-yH_t(x)} P(y \mid x) \equiv D_t e^{-\alpha_t y h_t(x)}$

$$\Rightarrow D_{t+1}(i) = \frac{D_t(i) \exp[-\alpha_t y_i h_t(x_i)]}{Z_t}, \; Z_t \text{ is for normalization}$$

Boosting and Deep Learning

Network Architecture Search (NAS)

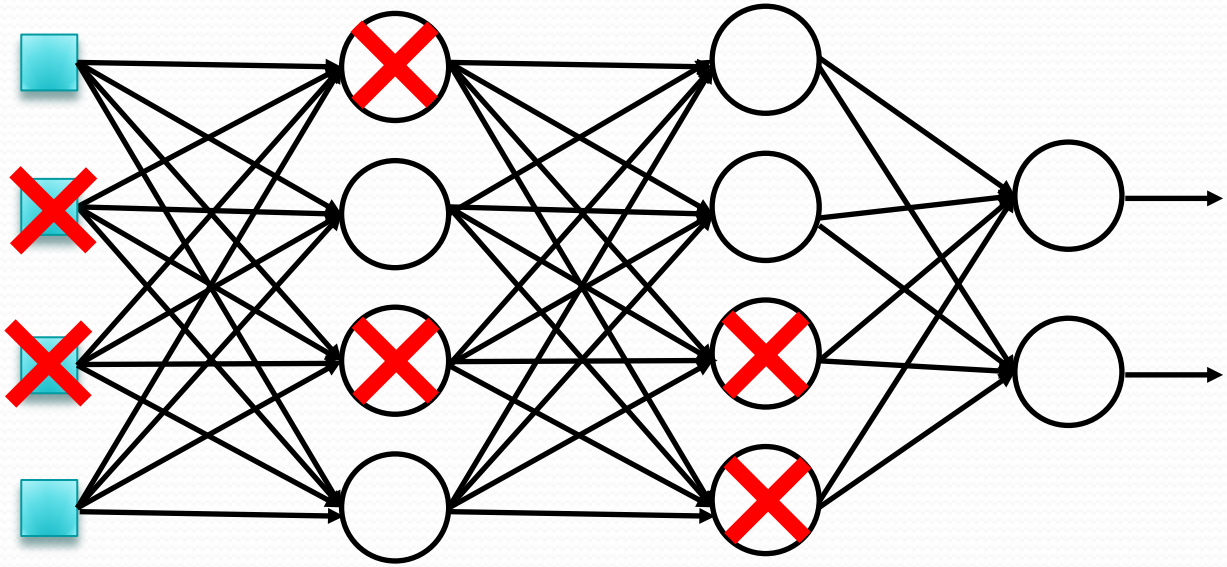


AdaNet : https://github.com/tensorflow/adanet

Ensemble and Deep Learning

- Dropout

Training:

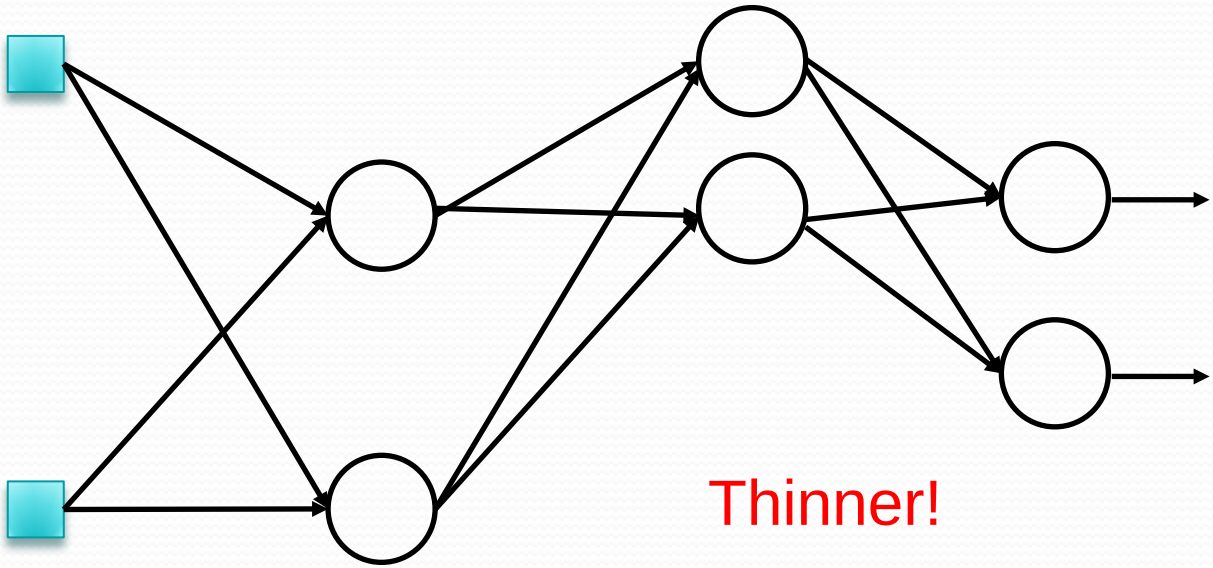


- **Each time before updating the parameters**
  - Each neuron has p% to dropout

Ensemble and Deep Learning

- Dropout

Training:



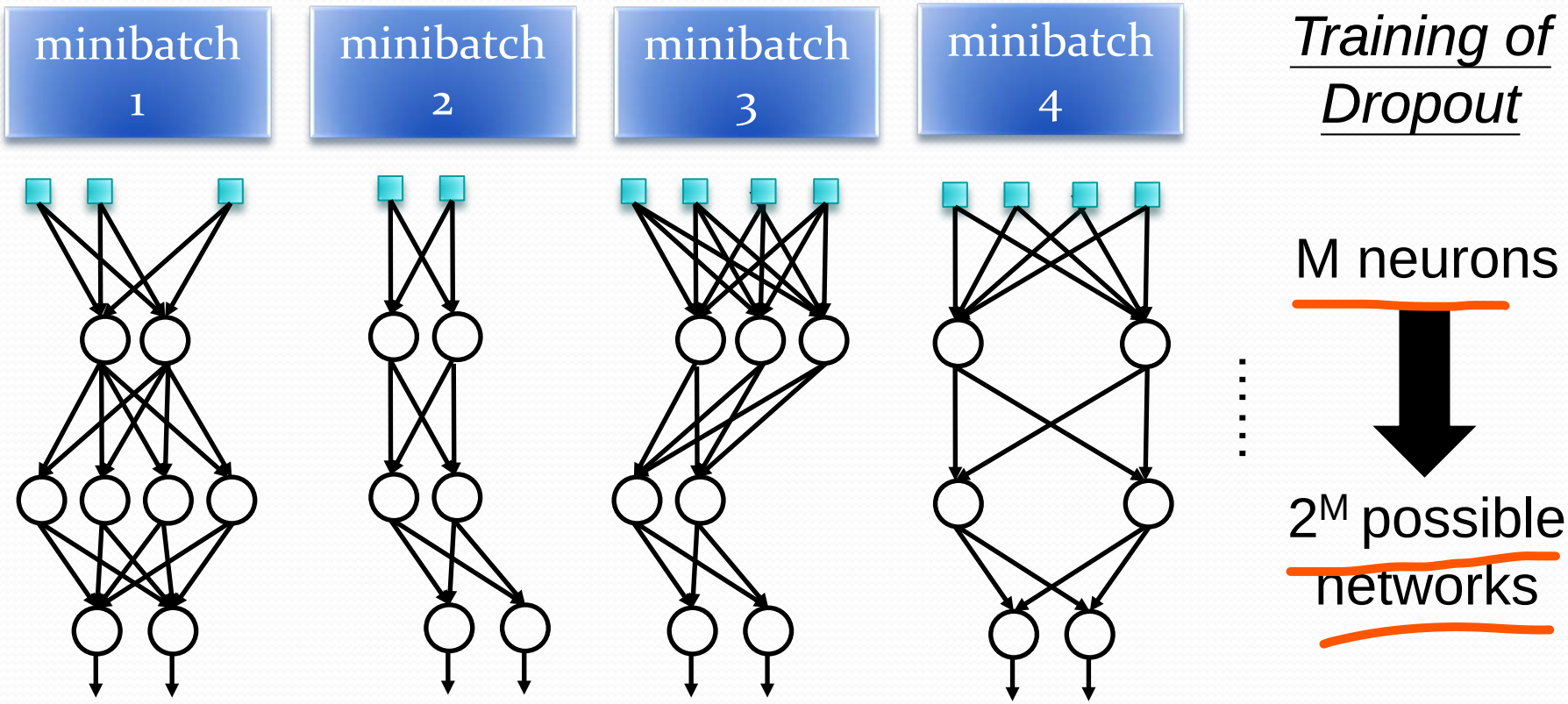
- **Each time before updating the parameters**
  - Each neuron has p% to dropout

    The structure of the network is changed.
  - Using the new network for training

For each mini-batch, we resample the dropout neurons

Ensemble and Deep Learning

- Dropout

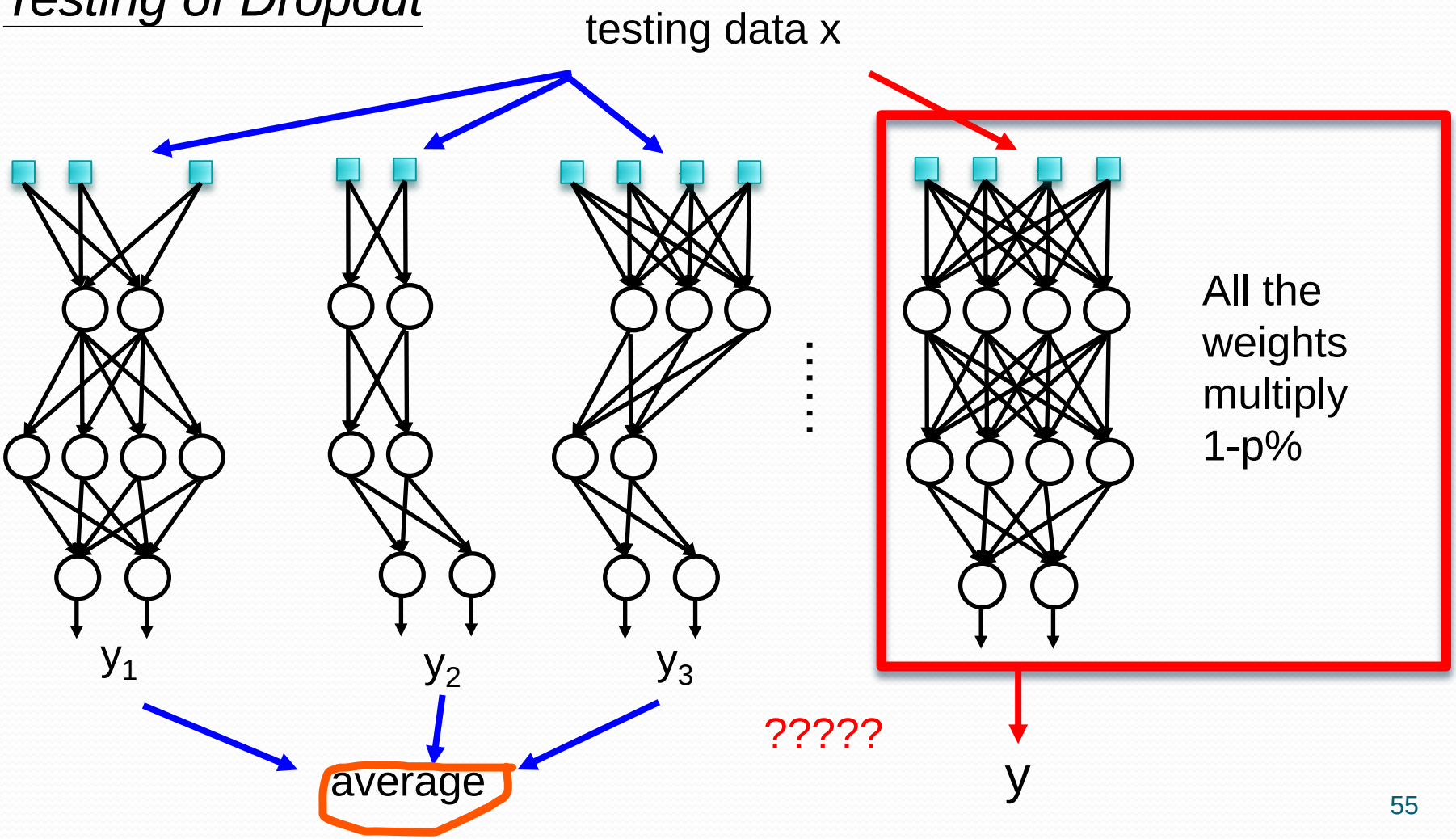


- Using one mini-batch to train one network
- Some parameters in the network are shared

Ensemble and Deep Learning

- Dropout

Testing of Dropout



Gradient Boosting

- can be used with any loss function
- weak learners are usually DT
- gradient descent
  - loss function
  - new parameterized weak learner
    - learned on residuals
- subject to overfitting
  - constraints on trees (nb, depth, nb of data per split)
  - regularisation
- implementations:
  - in scikit-learn:
    - GradientBoostingClassifier and GradientBoostingRegressor
    - HistGradientBoostingClassifier and HistGradientBoostingRegressor for large datasets
  - eXtreme Gradient Boosting (XGBoost): add the second derivate

Decission Tree

Gradient Boosting Algorithm

- Initialisation as a constant value

$$H_0(x) = h_0(x) = \arg\min_{\alpha} \sum_{i=1}^{n} \mathcal{L}(y_i, \alpha)$$

- For t ∈ {1 . . .T} :
  - Compute the pseudo-residuals for each i:

$$r_{it} = -\left[\frac{\partial \mathcal{L}(y_i, H(x_i))}{\partial H(x_i)}\right] (H(x) = H_{t-1}(x))$$

  - fit a weak learner h_t using dataset $\{(x_i , r_{it})\}$
  - compute α_t:

$$\alpha_t = \arg\min_{\alpha} \sum_{i=1}^{n} \mathcal{L}(y_i, H_{t-1}(x_i) + \alpha h_t(x_i))$$

  - update the strong classifier : $H_t(x) = H_{t-1} + \alpha_t h_t(x)$

Outlook

- Bagging
- Boosting algorithm
  - Idea
  - A first simple example to understand boosting idea
  - Probabilistic boosting and Adaboost
  - A simple example for understanding Adaboost
- Stacking

Stacking

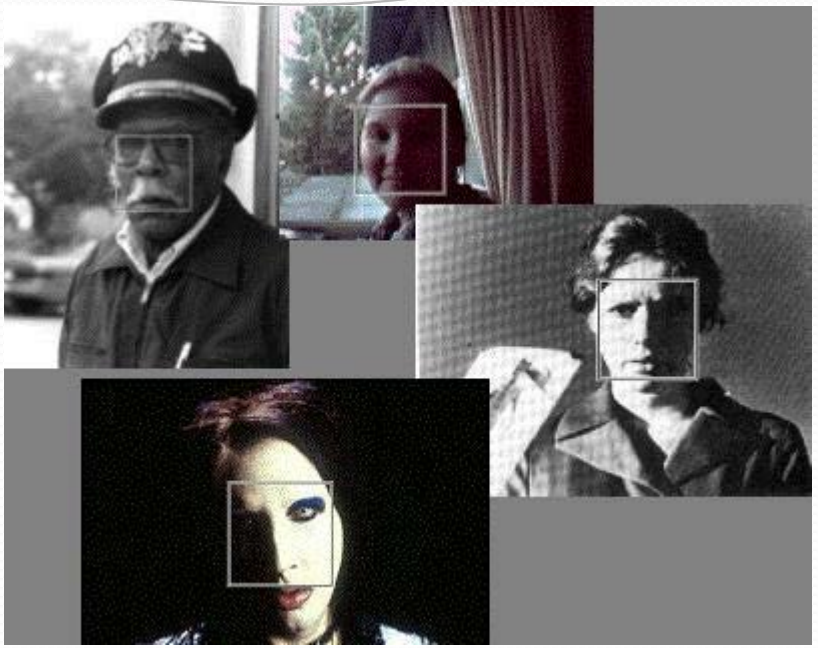
- for each sample i
  - for each learner
    - compute the prediction of the sample by the learner
  - arrange them in a vector x^i
- learn a new machine learning algorithm
  - with the dataset $\{x^i\}$
  - could be a logistic regression or any other Machine Learning algorithm
- scikit-learn implementation :
  `sklearn.ensemble.StackingClassifier` [1]

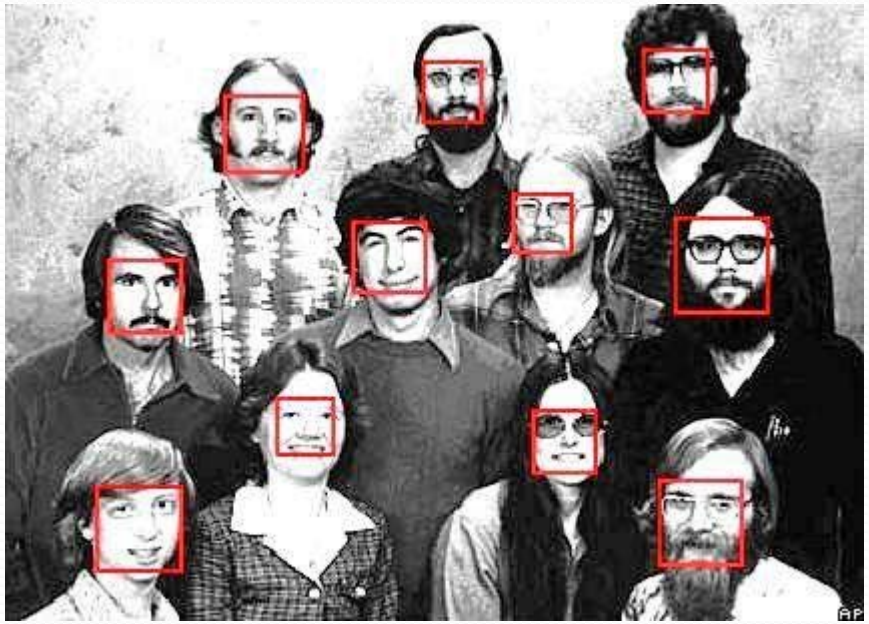
[1]. https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.StackingClassifier.html#sklearn.ensemble.StackingClassifier

LAB

Boosting example: Face detection

The Task of Face Detection



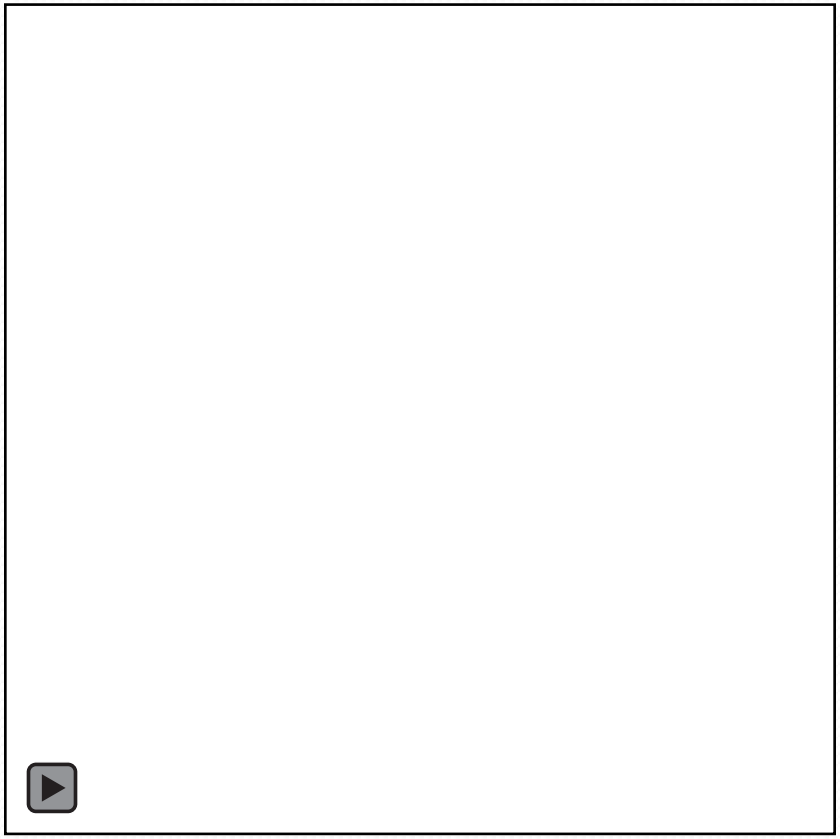


Many slides adapted from P. Viola

Basic Idea

- Slide a window across image and evaluate a face model at every location.

Face Detect Computational Complexity



Challenges

- Slide a window across image and evaluate a face model at every location.
- Sliding window detector must evaluate tens of thousands of location/scale combinations.
- Faces are rare: 0–10 per image
  - For computational efficiency, we should try to spend as little time as possible on the non-face windows
  - A megapixel image has ~10^6 pixels and a comparable number of candidate face locations
  - To avoid having a false positive in every image image, our false positive rate has to be less than 10^{-6}

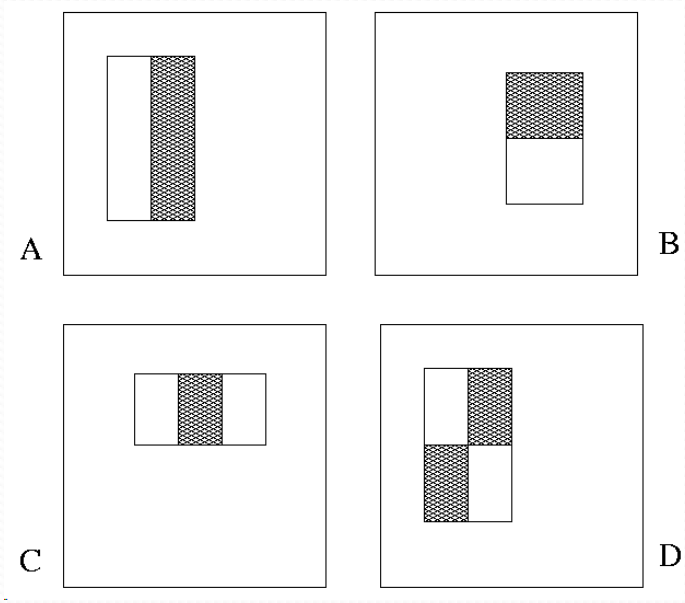
The Viola/Jones Face Detector

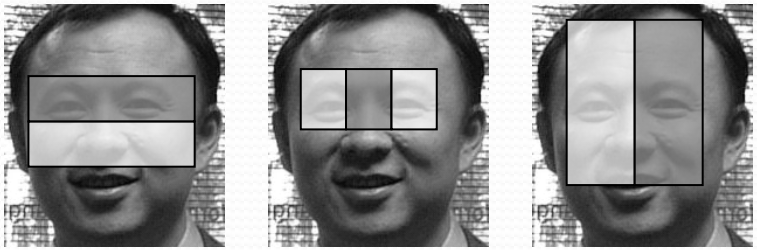
- A seminal approach to real-time object detection
- Training is slow, but detection is very fast
- Key ideas
  - *Integral images* for fast feature evaluation
  - *Boosting* for feature selection
  - *Attentional cascade* for fast rejection of non-face windows

P. Viola and M. Jones. ***Rapid object detection using a boosted cascade of simple features.*** CVPR 2001.

P. Viola and M. Jones. ***Robust real-time face detection.*** IJCV 57(2), 2004.

Image Features



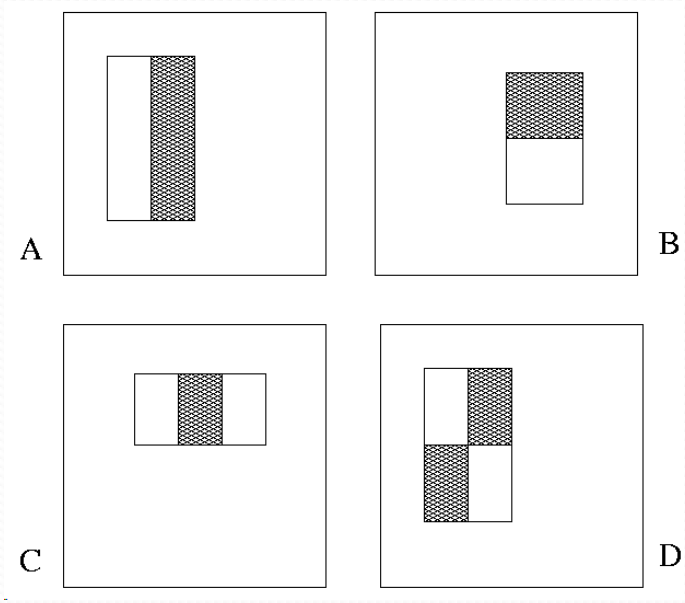


Rectangle filters

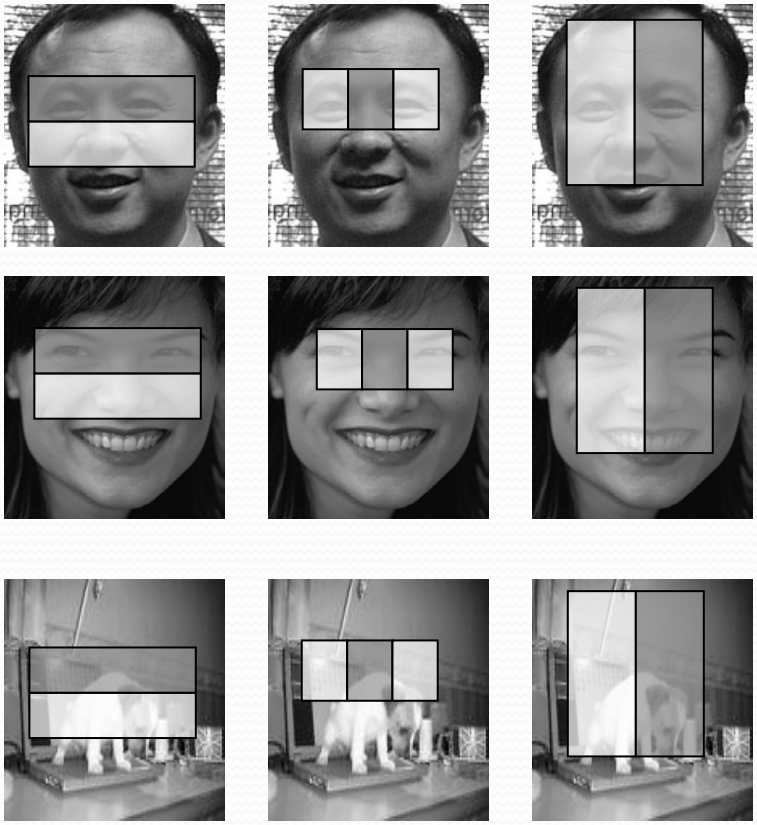
$$\text{Feature Value} = \sum(\text{Pixel in white area}) - \sum(\text{Pixel in black area})$$

Image Features

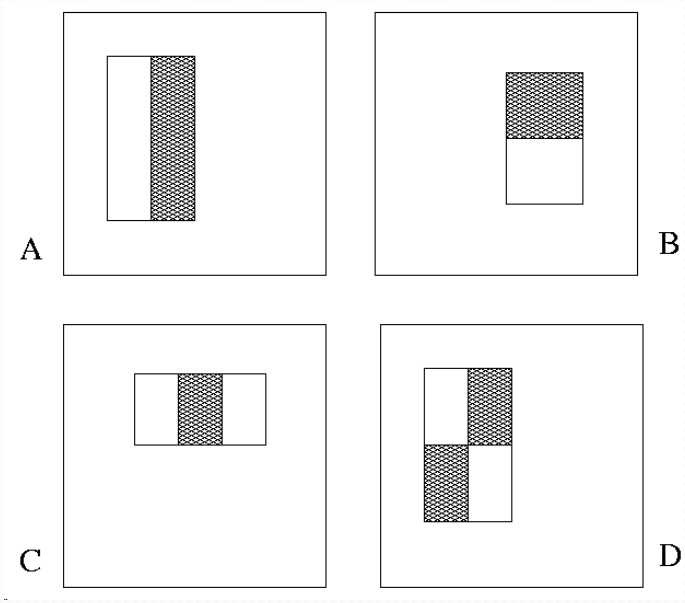
$$\text{Feature Value} = \sum(\text{Pixel in white area}) - \sum(\text{Pixel in black area})$$



Rectangle filters



Size of Feature Space



Rectangle filters

- How many number of possible rectangle features for a 24x24 detection region?

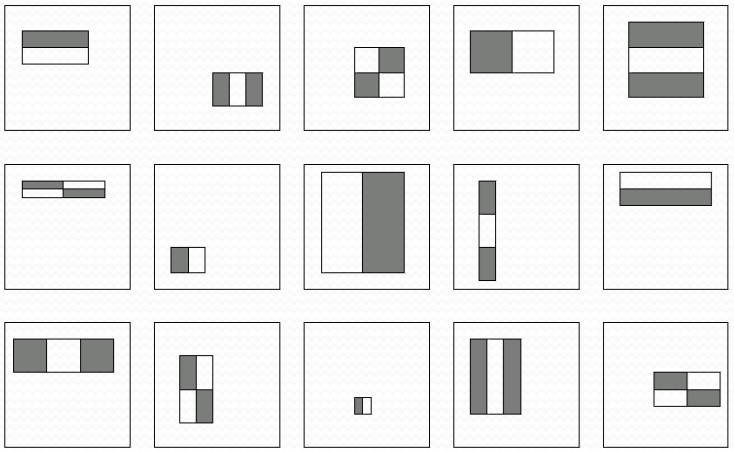
A+B $2\sum_{w=1}^{12}\sum_{h=1}^{24}(24-2w+1)(24-h+1)+$

C $2\sum_{w=1}^{8}\sum_{h=1}^{24}(24-3w+1)(24-h+1)+$

D $\sum_{w=1}^{12}\sum_{h=1}^{12}(24-2w+1)(24-2h+1)$

$\approx 160\,000$

Feature Selection



What features are good for face detection?

- How many number of possible rectangle features for a 24x24 detection region?

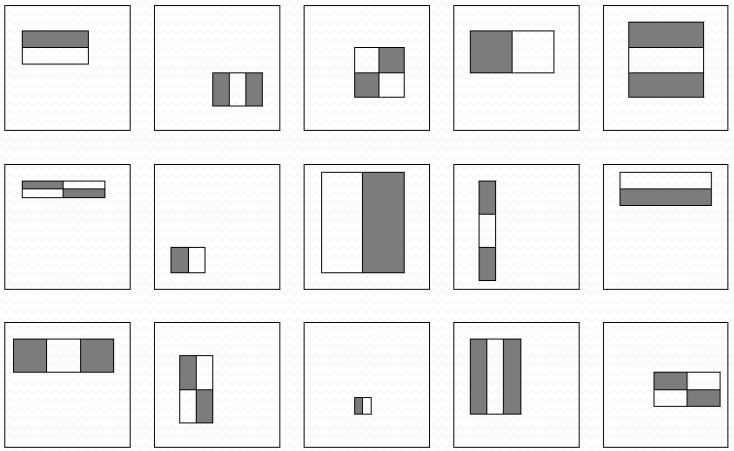
A+B $$2\sum_{w=1}^{12}\sum_{h=1}^{24}(24-2w+1)(24-h+1)+$$

C $$2\sum_{w=1}^{8}\sum_{h=1}^{24}(24-3w+1)(24-h+1)+$$

D $$\sum_{w=1}^{12}\sum_{h=1}^{12}(24-2w+1)(24-2h+1)$$

$\approx 160\,000$

Feature Selection



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C $$2\sum_{w=1}^{8}\sum_{h=1}^{24}(24-3w+1)(24-h+1)+$$

D $$\sum_{w=1}^{12}\sum_{h=1}^{12}(24-2w+1)(24-2h+1)$$

$\approx 160\,000$

- Can we create a good classifier using just a small subset of all possible features?
- How to select such a subset?

Weak Learners for Face Detection

What base learner is proper for face detection?

Given: $(x_1, y_1), \ldots, (x_m, y_m)$

Initialization: $D_1(i) = \frac{1}{m}, i = 1, \ldots, m$

For $t = 1, \ldots, T$:

- Find classifier $h_t : X \to \{-1, +1\}$ which minimizes error wrt D_t ,i.e.,

$$h_t = \arg\min_{h_j} \varepsilon_j \quad \text{where } \varepsilon_j = \sum_{i=1}^{m} D_t(i)[y_i \neq h_j(x_i)]$$

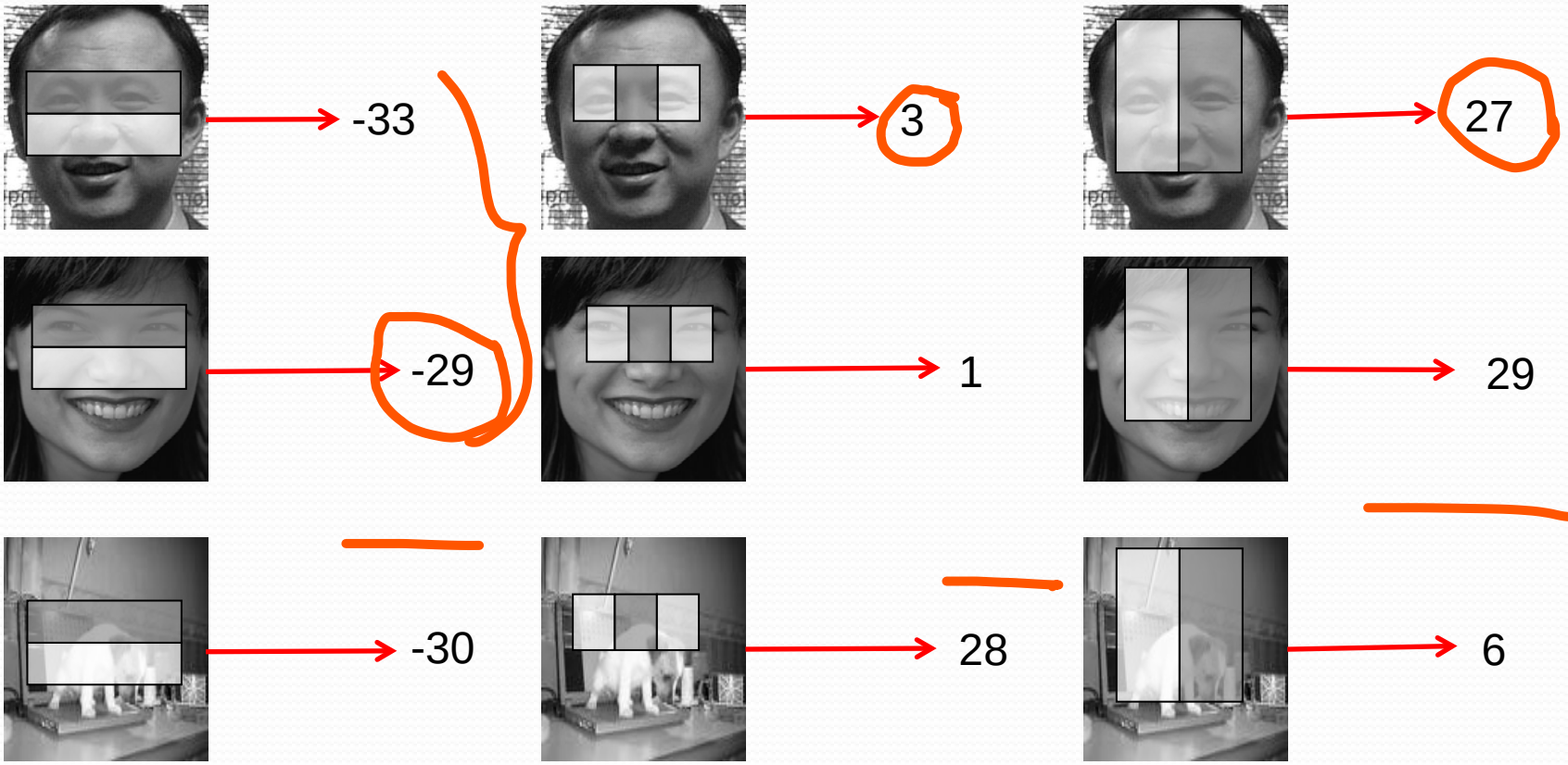
- Weight classifier: $\alpha_t = \frac{1}{2} \ln \frac{1 - \varepsilon_t}{\varepsilon_t}$

- Update distribution: $D_{t+1}(i) = \frac{D_t(i) \exp[-\alpha_t y_i h_t(x_i)]}{Z_t}$, Z_t is for normalization

Output final classifier: $sign\left(H(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \right)$

Image Features

Feature Value = $\sum$(Pixel in white area) – $\sum$(Pixel in black area)



Weak Learners for Face Detection

$$h_1(\,\boxed{}\,) = \begin{cases} 1 & \text{if } \boxed{} < -29 \\ 0 & \text{otherwise} \end{cases}$$

$$h_2(\,\boxed{}\,) = \begin{cases} 1 & \text{if } \boxed{} < 26 \\ 0 & \text{otherwise} \end{cases}$$

$$h_3(\,\boxed{}\,) = \begin{cases} 1 & \text{if } \boxed{} > 11 \\ 0 & \text{otherwise} \end{cases}$$

Weak Learners for Face Detection

window

value of rectangle feature

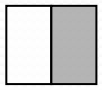
parity

threshold

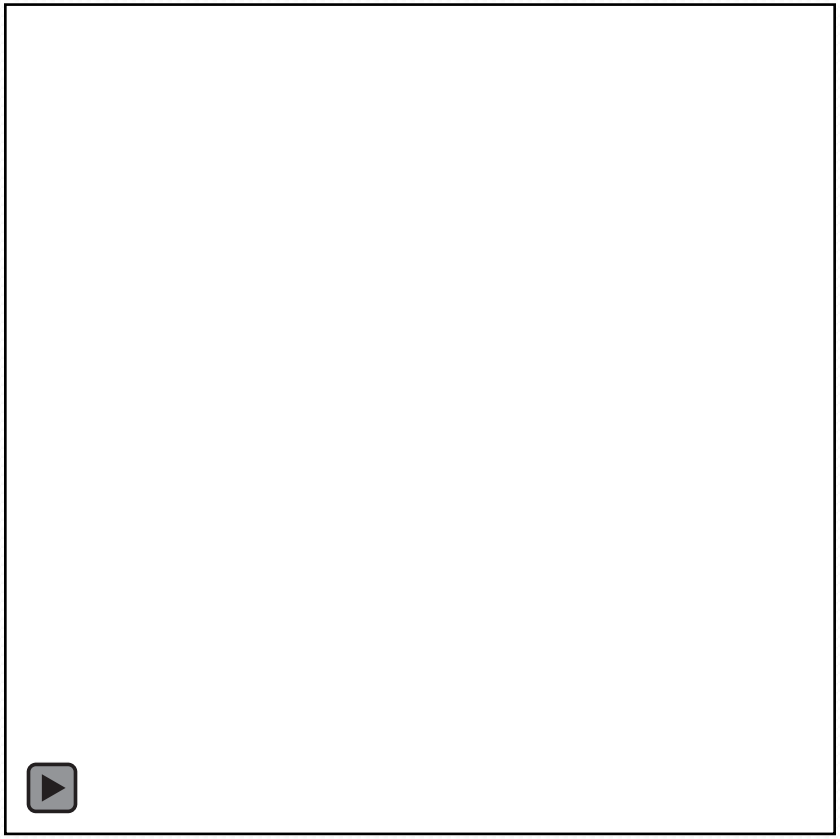
$$h_t(W) = \begin{cases} 1 & \text{if } p_t f_t(W) > p_t \theta_t \\ 0 & \text{otherwise} \end{cases}$$

$h_1($  $) = \begin{cases} 1 & \text{if } (-1)$  $> (-1)(-29) \\ 0 & \text{otherwise} \end{cases}$

$h_2($  $) = \begin{cases} 1 & \text{if } (-1)$  $> (-1)26 \\ 0 & \text{otherwise} \end{cases}$

$h_3($  $) = \begin{cases} 1 & \text{if } (+1)$  $> (+1)11 \\ 0 & \text{otherwise} \end{cases}$

Face Detect Computational Complexity

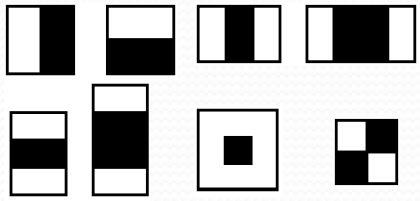


Boosting

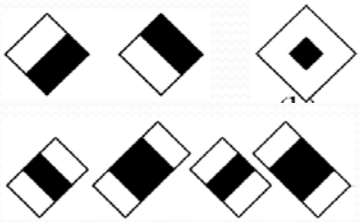
- Training set contains face and nonface examples
  - Initially, with equal weight
- For each round of boosting:
  - Evaluate each rectangle filter on each example
  - Select best threshold for each filter
  - Select best filter/threshold combination
  - Reweight examples
- Computational complexity of learning: $O(MNK)$
  - M rounds, N examples, K features

AdaBoost

- 1990 Schapire, Boosting
- 1996 Freund & Schapire, AdaBoost
  - Do not require any a priori knwoledge
- 2001 Viola & Jones
  - Face detection
  - Haar descriptors, integral image, cascades
  - Robust, fast
- 2002 Lienhart et al.
  - Extended descriptors, 45 degres rotation



Haar descriptors

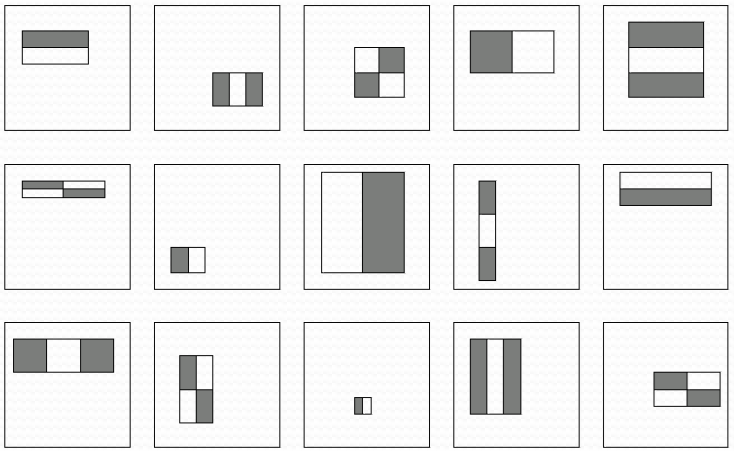


Extended descriptors

Out of scope for the LAB

- Apart from using Boosting to select the best subset of features, the two other amazing ideas from Viola & Jones are:
  - Integral Image
  - Attentional Cascade

Feature Selection



- How many number of possible rectangle features for a 24x24 detection region?

A+B $$2\sum_{w=1}^{12}\sum_{h=1}^{24}(24-2w+1)(24-h+1)+$$

C $$2\sum_{w=1}^{8}\sum_{h=1}^{24}(24-3w+1)(24-h+1)+$$

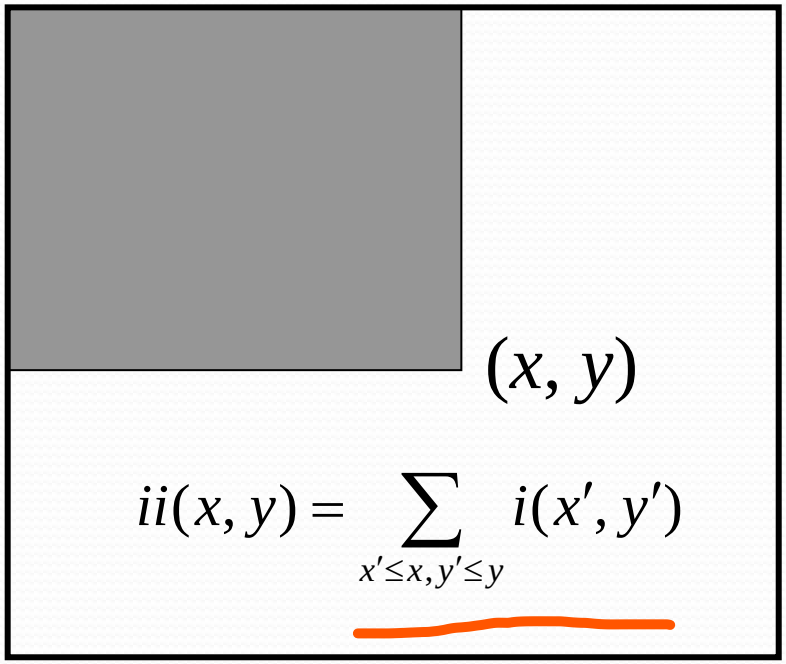
D $$\sum_{w=1}^{12}\sum_{h=1}^{12}(24-2w+1)(24-2h+1)$$

$$\approx 160\,000$$

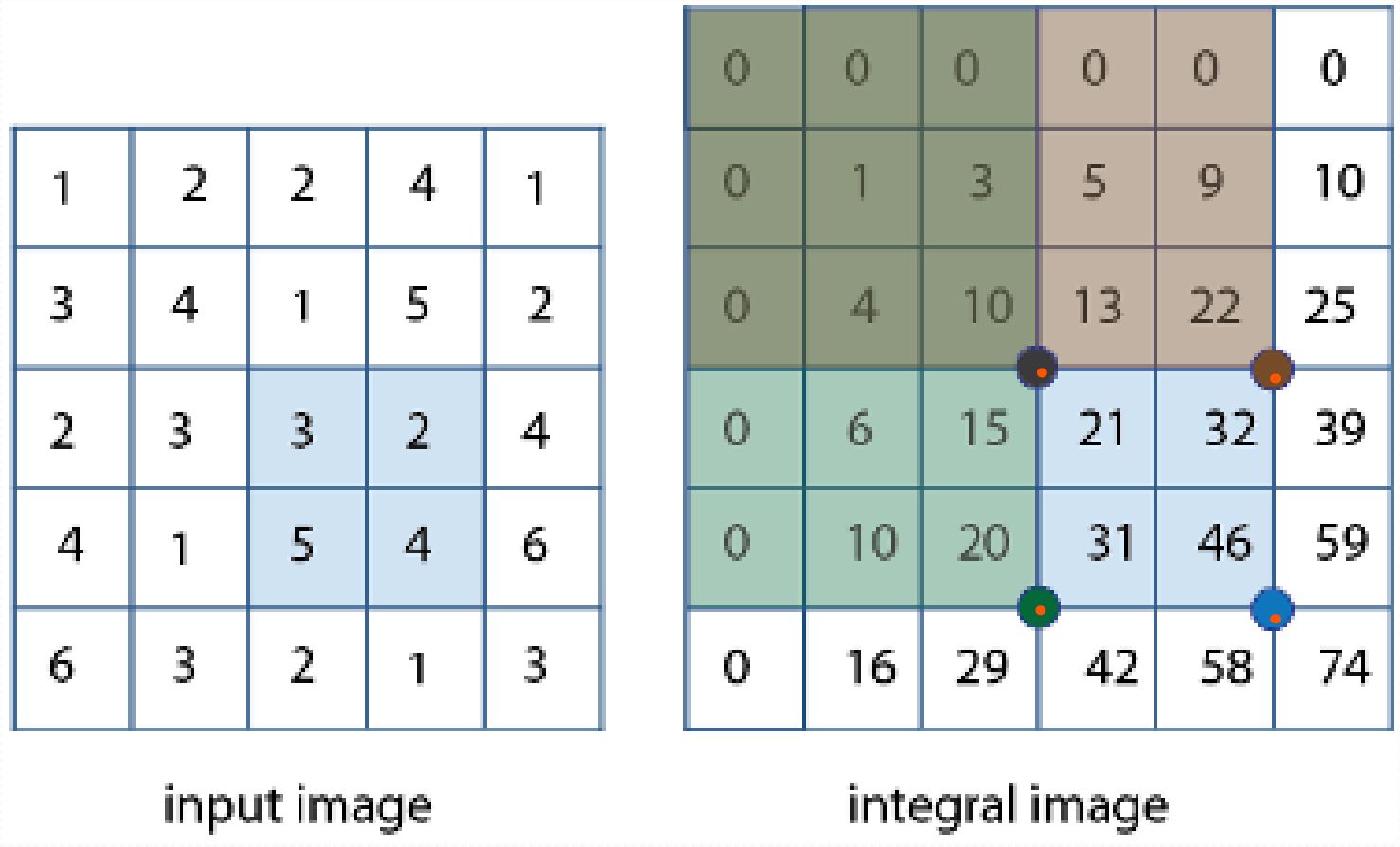
- Can we create a good classifier using just a small subset of all possible features?
- How to select such a subset?

Integral images

- The integral image computes a value at each pixel (x, y) that is the sum of the pixel values above and to the left of (x, y), inclusive.



Computing the Integral Image

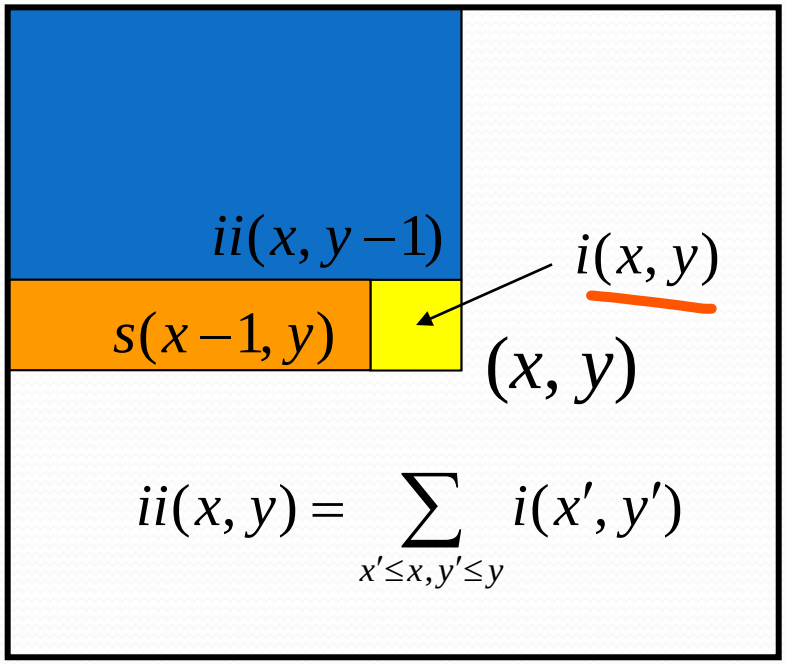


Computing the Integral Image

- The integral image computes a value at each pixel (x, y) that is the sum of the pixel values above and to the left of (x, y), inclusive.
- This can quickly be computed in one pass through the image.

$$s(x, y) = s(x-1, y) + i(x, y)$$

$$ii(x, y) = ii(x, y-1) + s(x, y)$$



Computing the Integral Image

1	2	2	4	1
3	4	1	5	2
2	3	3	2	4
4	1	5	4	6
6	3	2	1	3

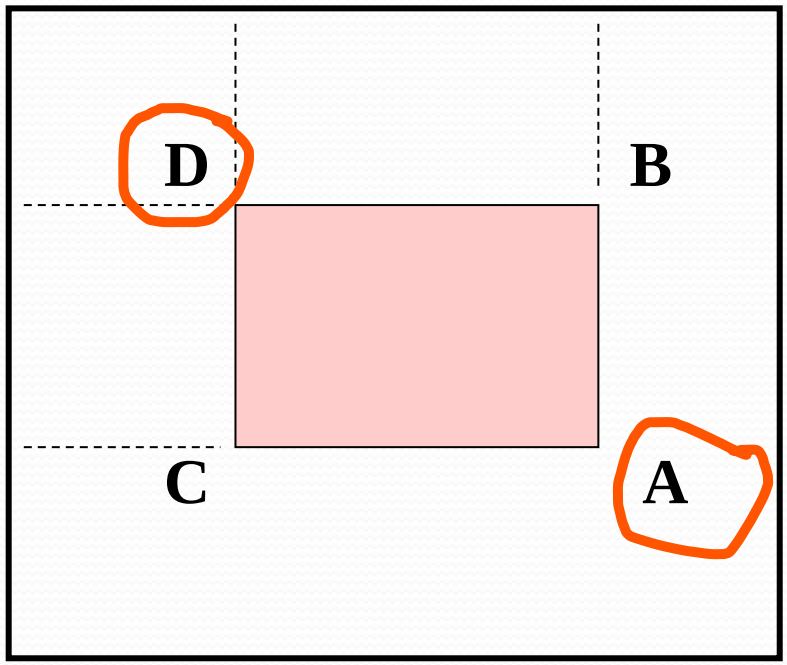
input image

0	0	0	0	0	0
0	1	3	5	9	10
0	4	10	13	22	25
0	6	15	21	32	39
0	10	20	31	46	59
0	16	29	42	58	74

integral image

Computing Sum within a Rectangle

$$sum = ii_A - ii_B - ii_C + ii_D$$



✓ Only 3 additions are required for any size of rectangle!

Computing the Integral Image

1	2	2	4	1
3	4	1	5	2
2	3	3	2	4
4	1	5	4	6
6	3	2	1	3

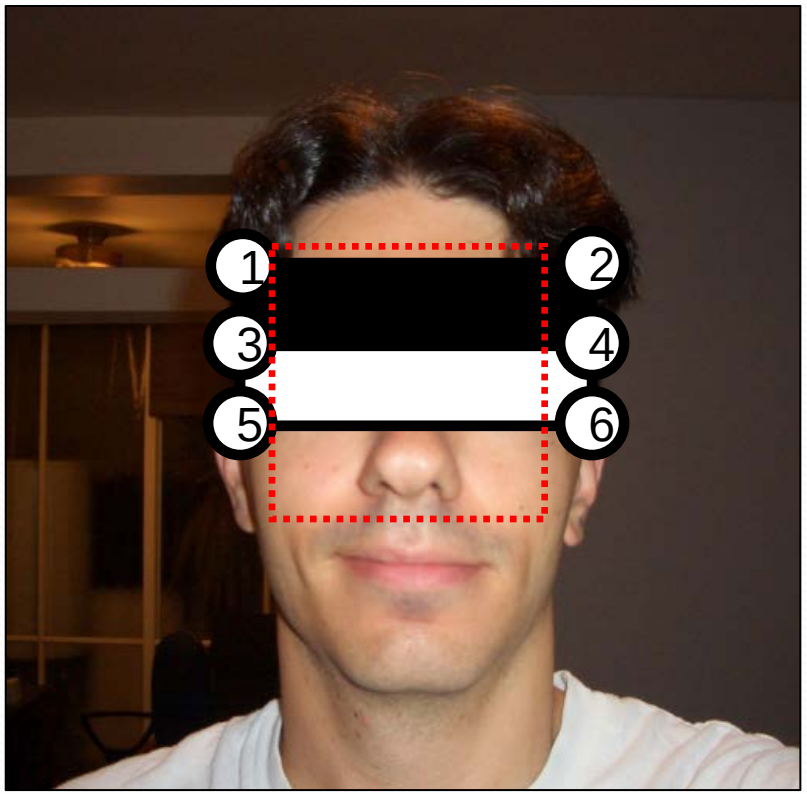
input image

0	0	0	0	0	0
0	1	3	5	9	10
0	4	10	13	22	25
0	6	15	21	32	39
0	10	20	31	46	59
0	16	29	42	58	74

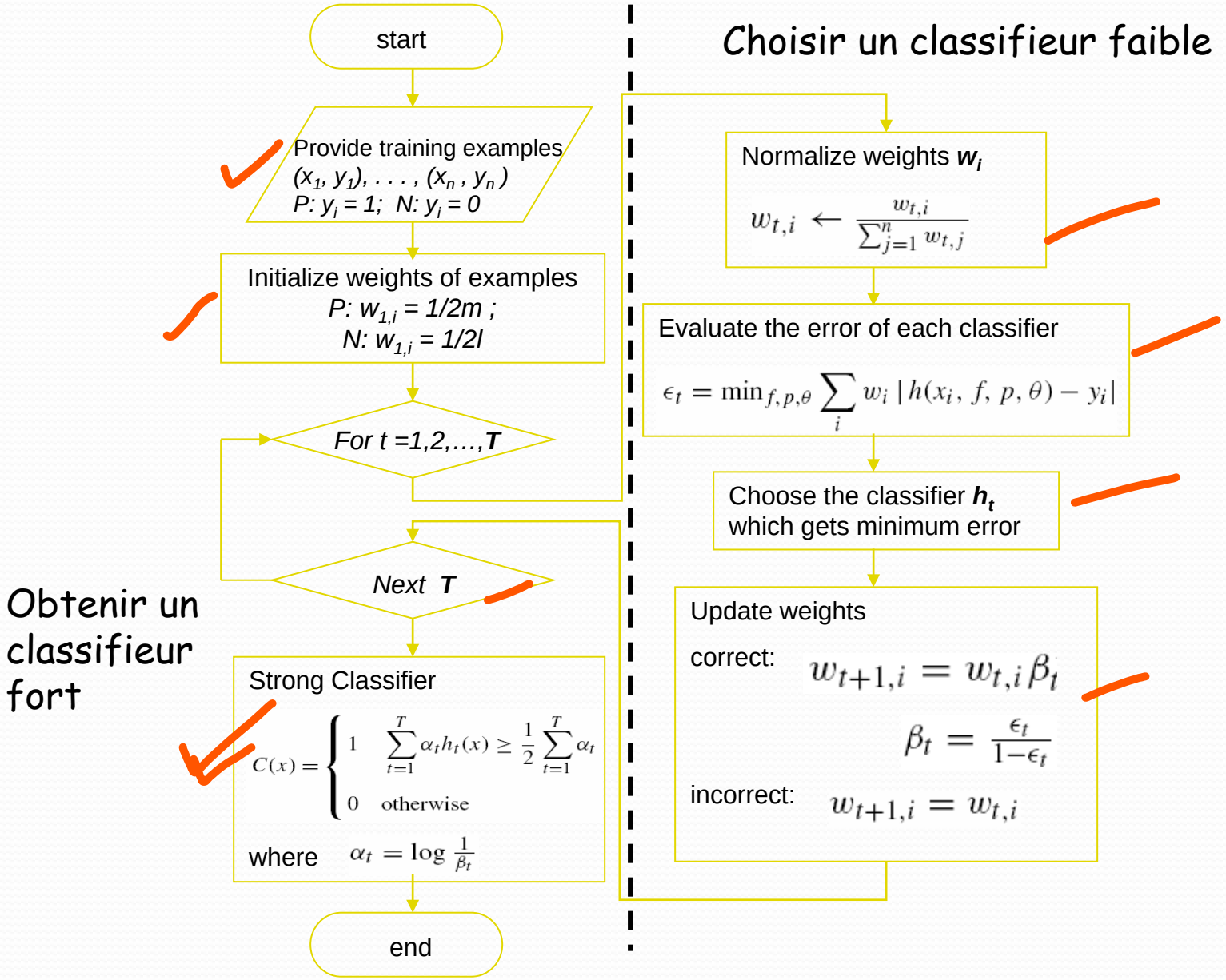
integral image

Scaling

- Integral image enables us to evaluate *all* rectangle sizes in constant time.
- Therefore, *no* image scaling is necessary.
- Scale the rectangular features instead!



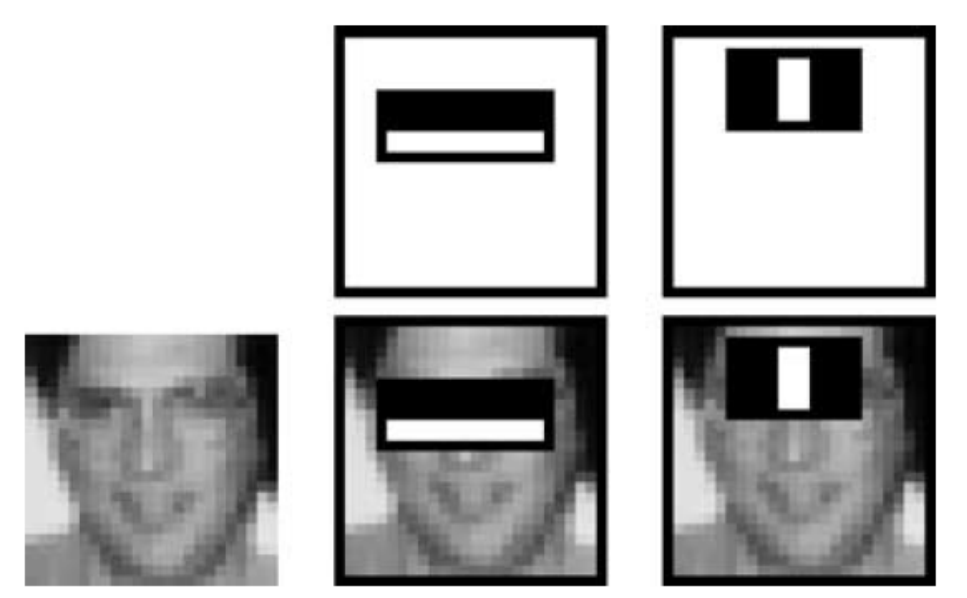
AdaBoost



Obtenir un classifieur fort

Features Selected by Boosting

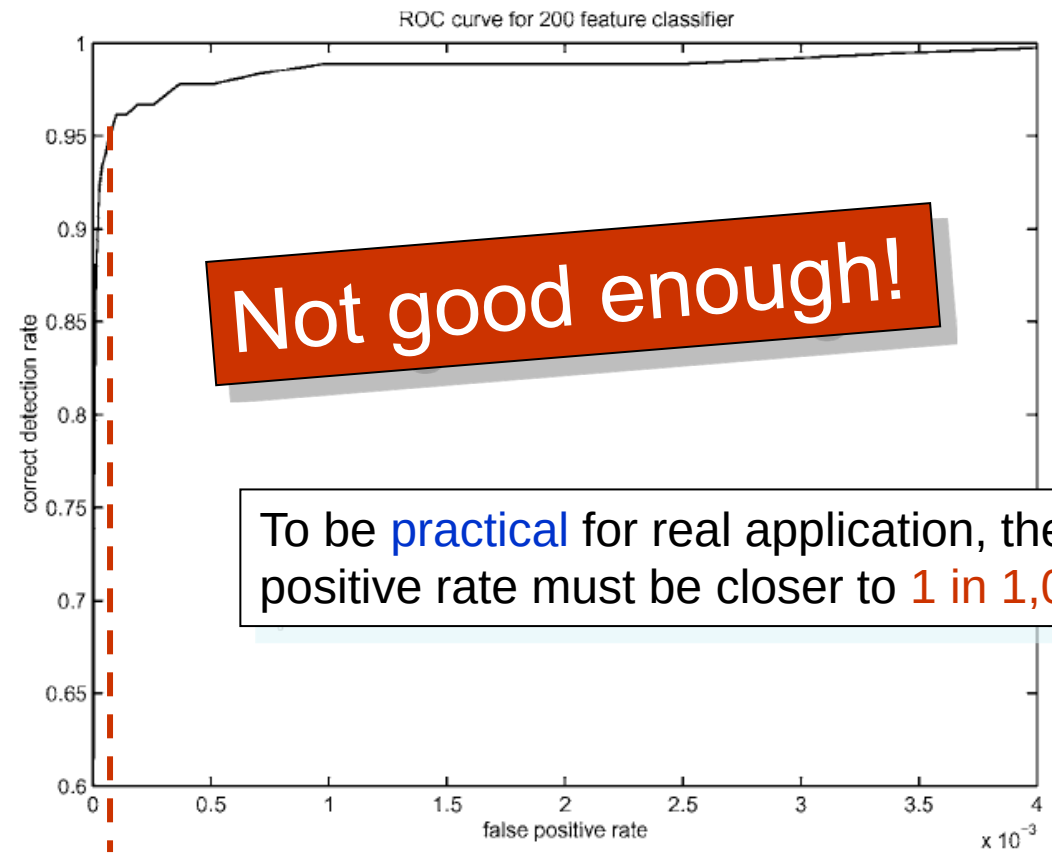
First two features selected by boosting:



This feature combination can yield 100% detection rate and 50% false positive rate

ROC Curve for 200-Feature Classifier

A 200-feature classifier can yield 95% detection rate and a false positive rate of 1 in 14084.

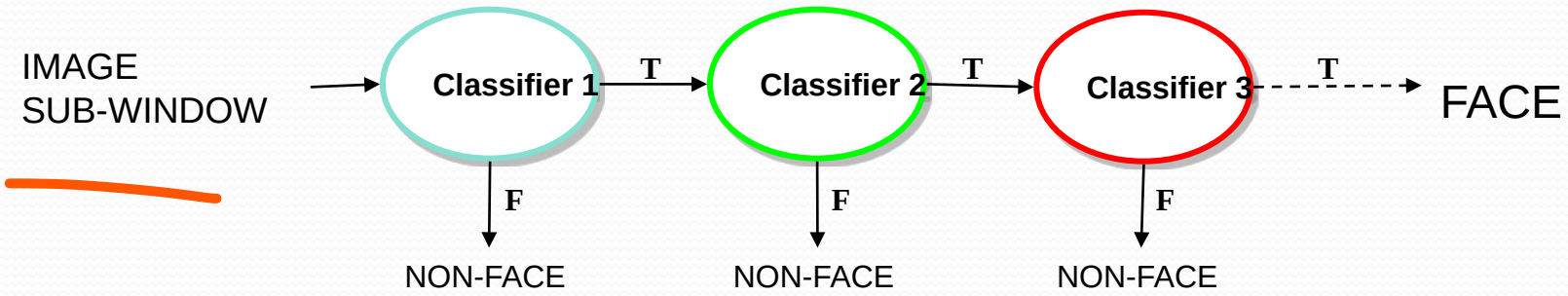


Not good enough!

To be practical for real application, the false positive rate must be closer to 1 in 1,000,000.

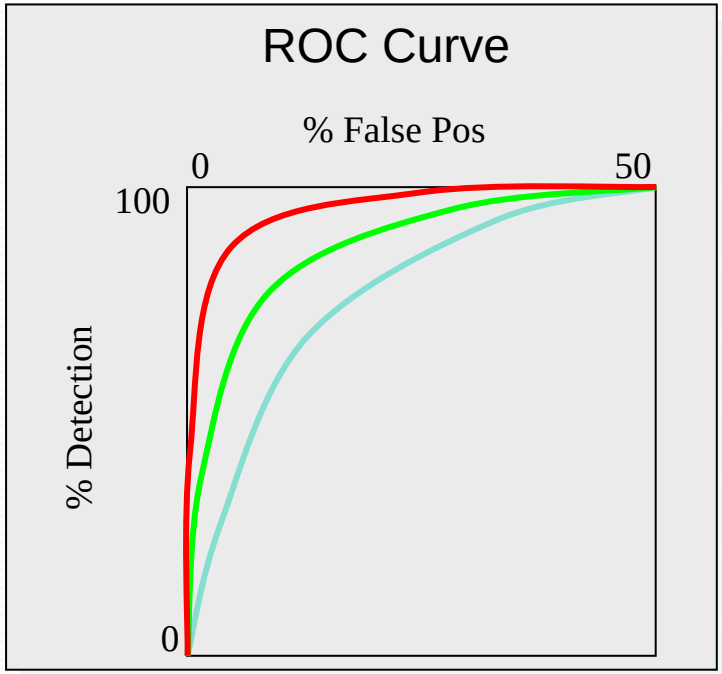
Attentional Cascade

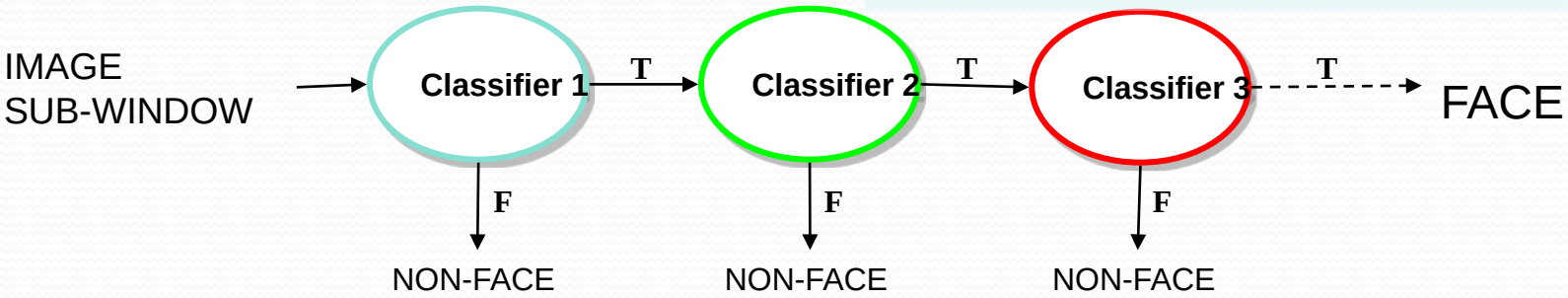
- We start with simple classifiers which reject many of the negative sub-windows while detecting almost all positive sub-windows
- Positive response from the first classifier triggers the evaluation of a second (more complex) classifier, and so on
- A negative outcome at any point leads to the immediate rejection of the sub-window



Attentional Cascade

- Chain classifiers that are progressively more complex and have lower false positive rates

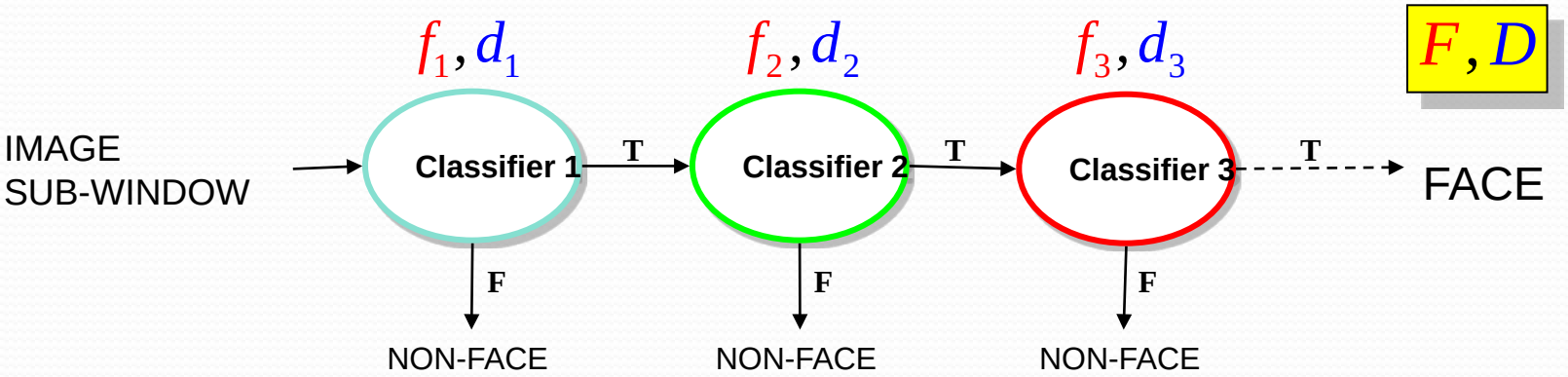




Detection Rate and False Positive Rate for Chained Classifiers

$$F = \prod_{i=1}^{K} f_i, \quad D = \prod_{i=1}^{K} d_i$$

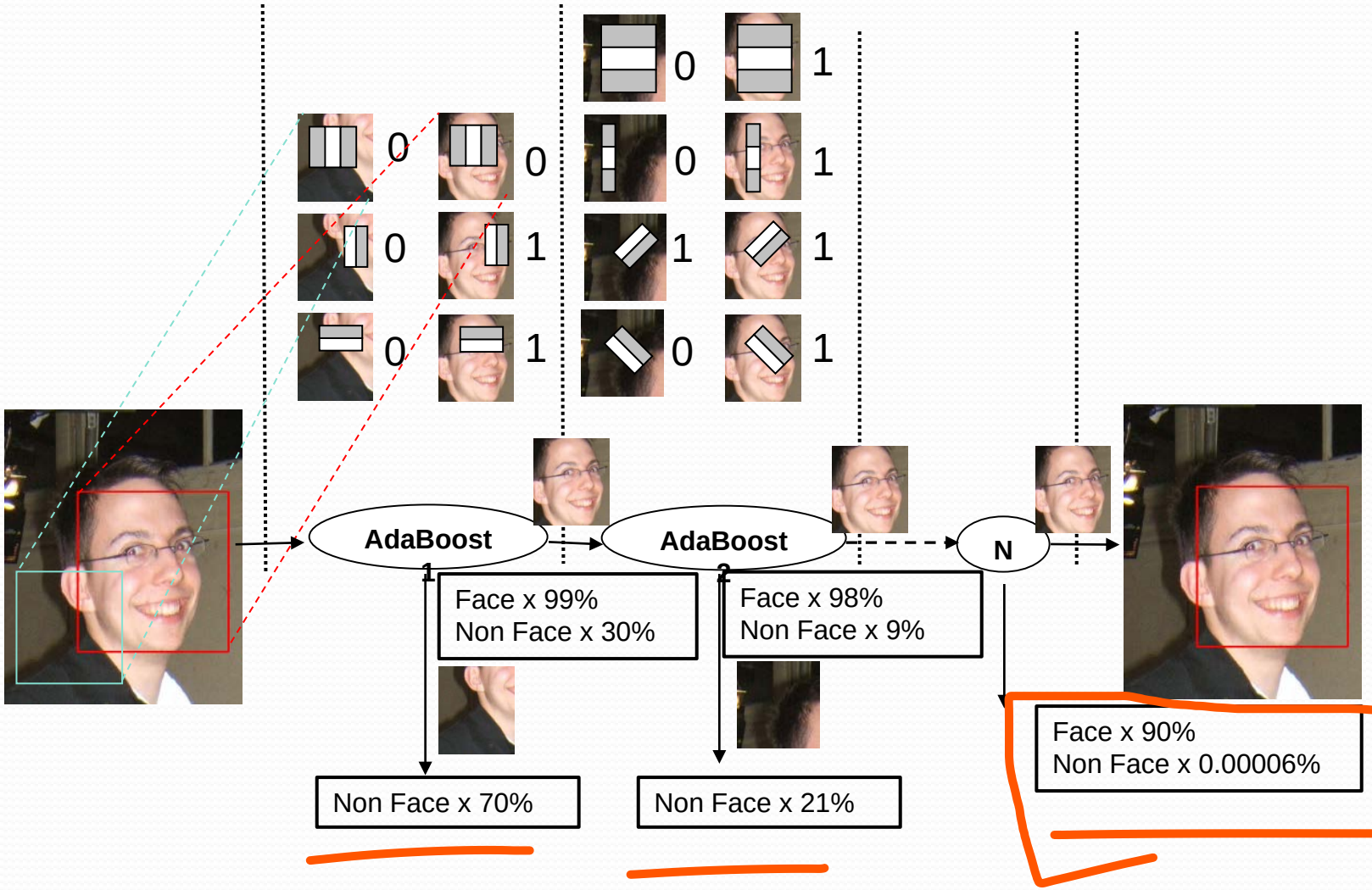
- The detection rate and the false positive rate of the cascade are found by multiplying the respective rates of the individual stages
- A detection rate of **0.9** and a false positive rate on the order of 10^{-6} can be achieved by a 10-stage cascade if each stage has a detection rate of **0.99** ($0.99^{10} \approx 0.9$) and a false positive rate of about **0.30** ($0.3^{10} \approx 6 \times 10^{-6}$)



Training the Cascade

- Set target detection and false positive rates for each stage
- Keep adding features to the current stage until its target rates have been met
  - Need to lower AdaBoost threshold to maximize detection (as opposed to minimizing total classification error)
  - Test on a *validation set*
- If the overall false positive rate is *not* low enough, then add another stage
- Use false positives from current stage as the negative training examples for the next stage

AdaBoost (6) – Cascade Principle



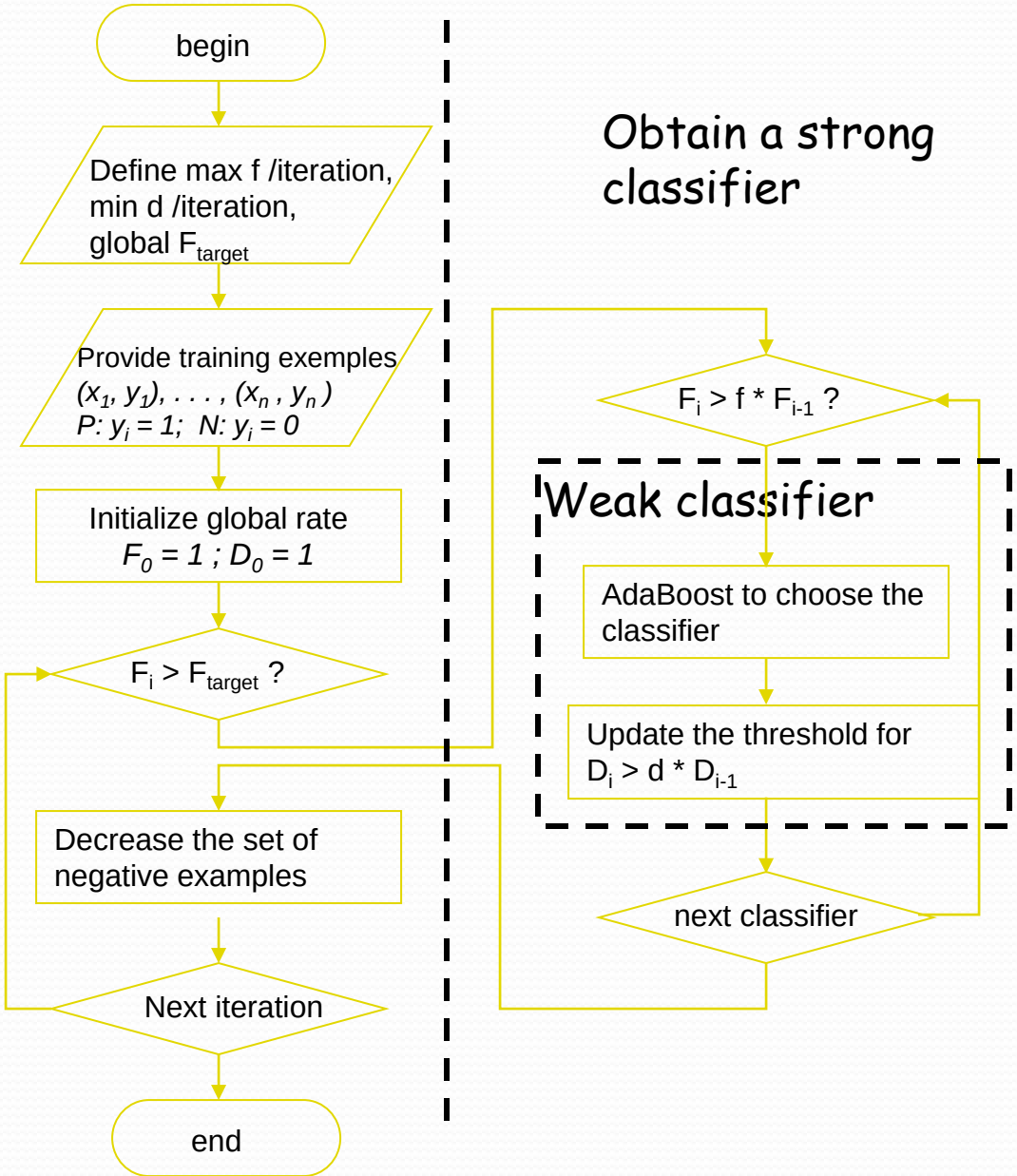
Algo

Table 2. The training algorithm for building a cascaded detector.

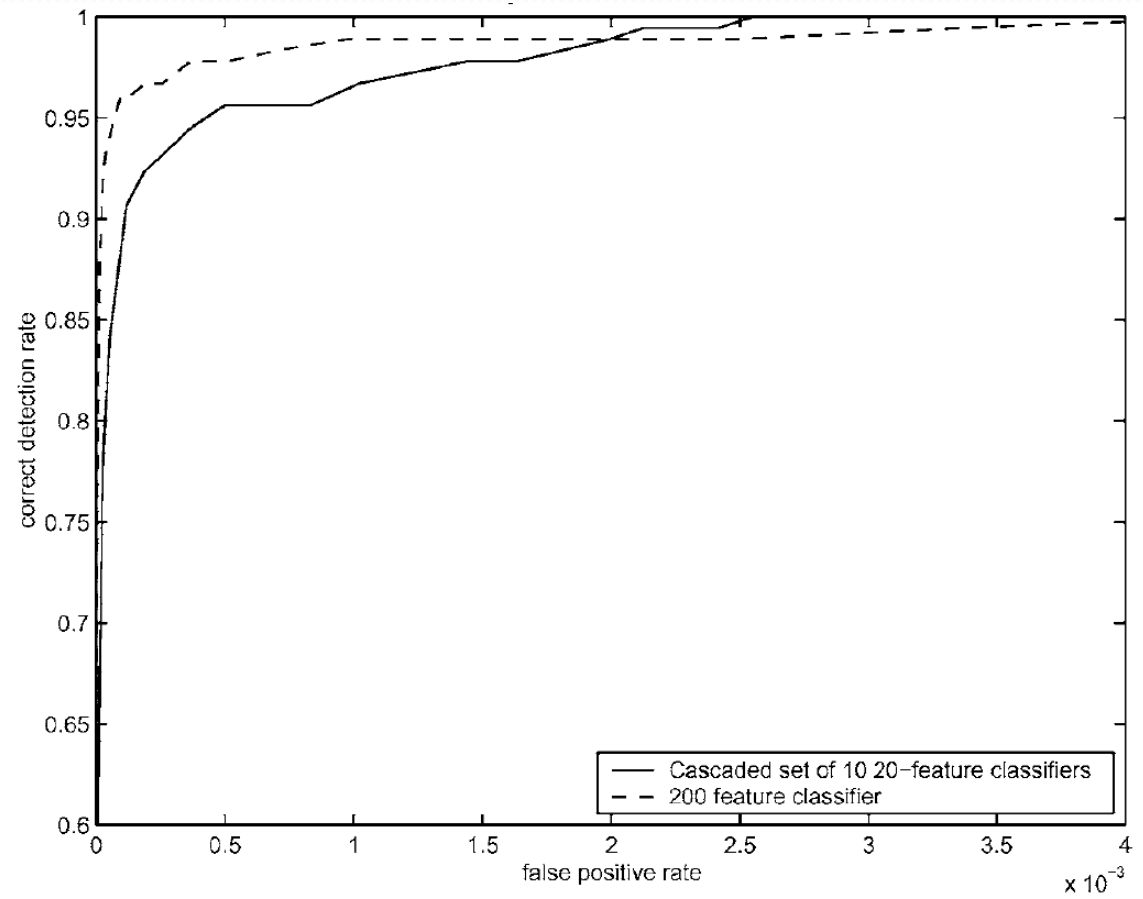
- User selects values for f, the maximum acceptable false positive rate per layer and d, the minimum acceptable detection rate per layer.
- User selects target overall false positive rate, F_{target}.
- P = set of positive examples
- N = set of negative examples
- $F_0 = 1.0$; $D_0 = 1.0$
- $i = 0$
- while $F_i > F_{target}$
  - $i \leftarrow i + 1$
  - $n_i = 0$; $F_i = F_{i-1}$
  - while $F_i > f \times F_{i-1}$
    - $n_i \leftarrow n_i + 1$
    - Use P and N to train a classifier with n_i features using AdaBoost
    - Evaluate current cascaded classifier on validation set to determine F_i and D_i.
    - Decrease threshold for the ith classifier until the current cascaded classifier has a detection rate of at least $d \times D_{i-1}$ (this also affects F_i)
  - $N \leftarrow \emptyset$
  - If $F_i > F_{target}$ then evaluate the current cascaded detector on the set of non-face images and put any false detections into the set N

Attentional Cascade

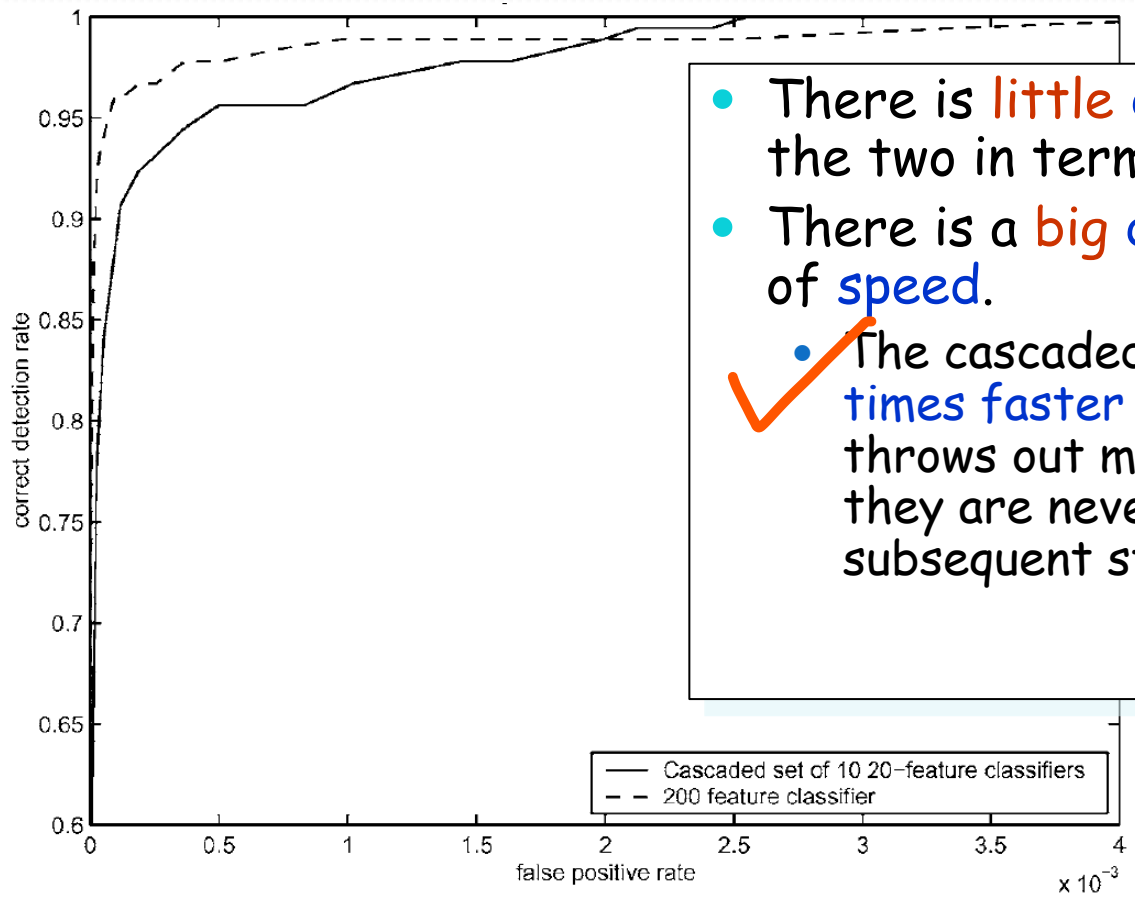
Obtain a cascade of strong classifiers



ROC Curves Cascaded Classifier to Monlithic Classifier



ROC Curves Cascaded Classifier to Monlithic Classifier



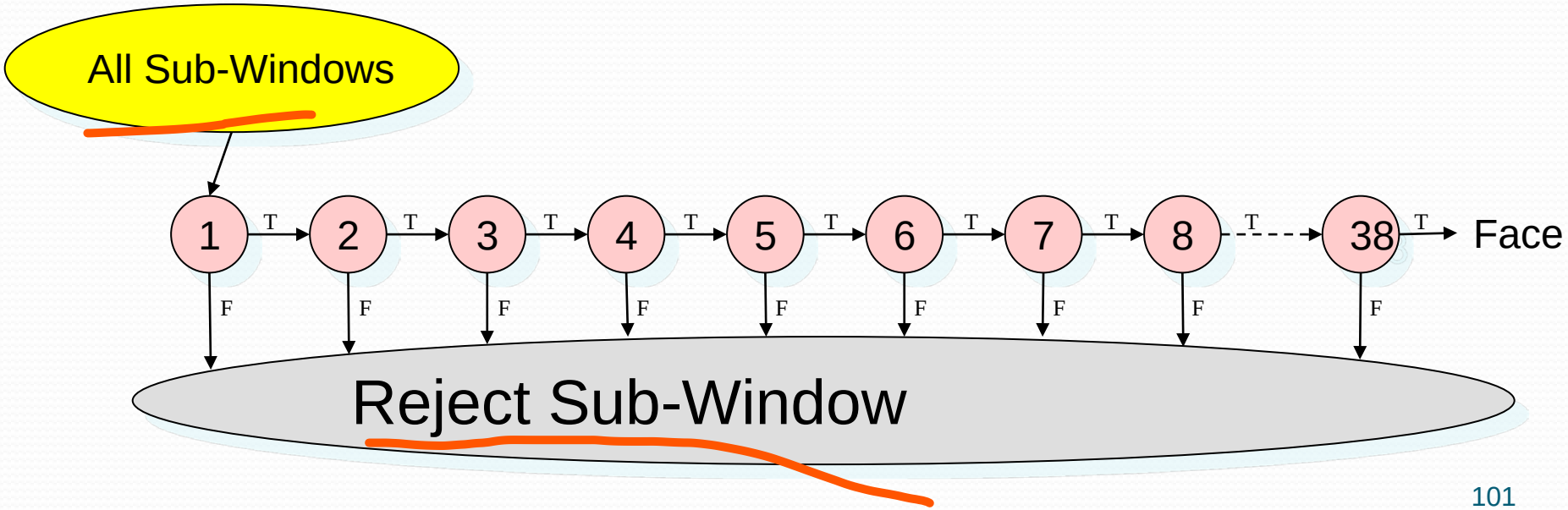
- There is little difference between the two in terms of accuracy.
- There is a big difference in terms of speed.
  - The cascaded classifier is nearly 10 times faster since its first stage throws out most non-faces so that they are never evaluated by subsequent stages.

The Implemented System

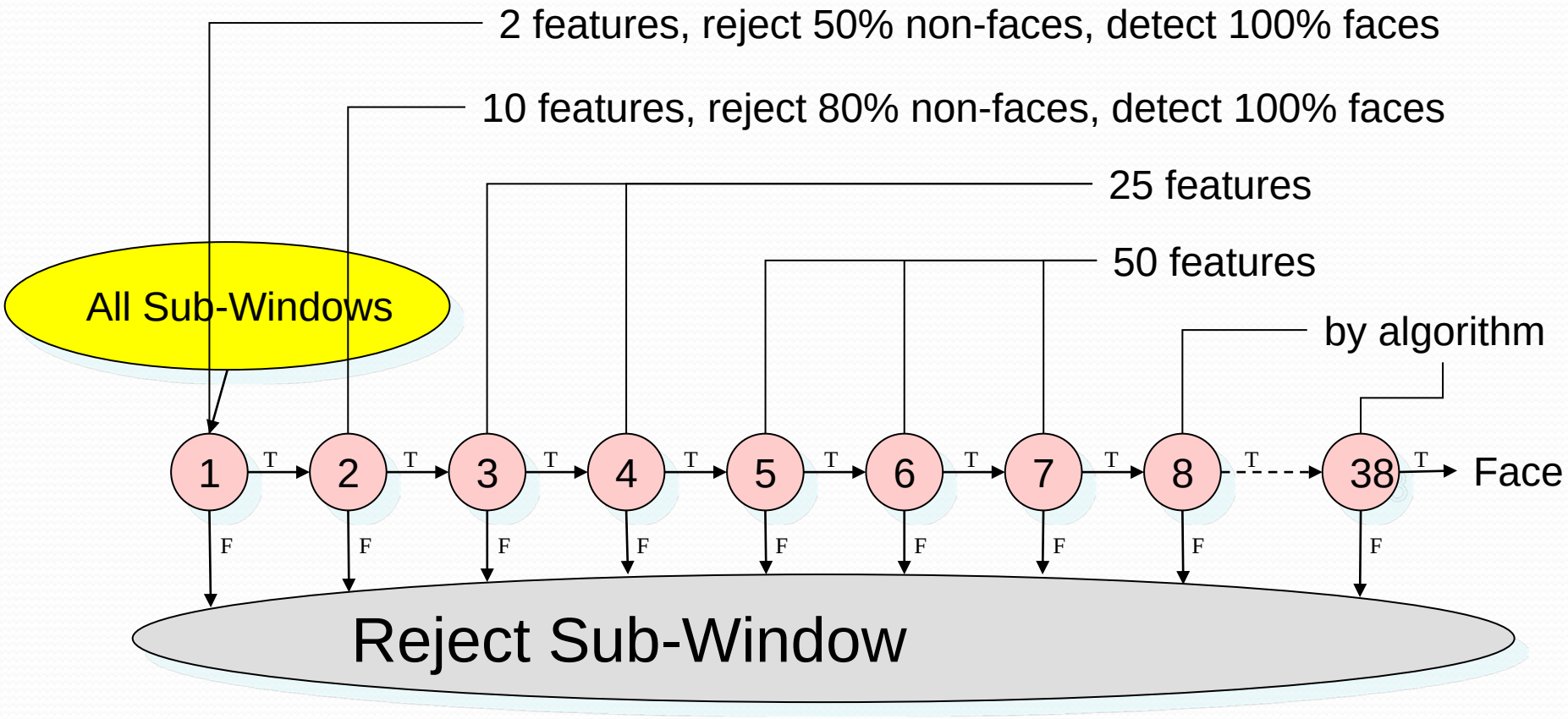
- Training Data
  - 5000 faces
    - All frontal, rescaled to 24x24 pixels
  - 300 million non-faces
    - 9500 non-face images
  - Faces are normalized
    - Scale, translation
- Many variations
  - Across individuals
  - Illumination
  - Pose

Structure of the Detector Cascade

- Combining successively more complex classifiers in cascade
  - 38 stages
  - included a total of 6060 features



Structure of the Detector Cascade



Speed of the Final Detector

- On a 700 Mhz Pentium III processor, the face detector can process a 384 ×288 pixel image in about .067 seconds"
  - 15 Hz
  - 15 times faster than previous detector of comparable accuracy (Rowley et al., 1998)

- Average of 8 features evaluated per window on test set