

CS 361  
Homework 7  
FA 2025

Possible Points: 60

Pumping Lemma for Context Free Languages + Prove Language IS Context Free

Name:

1. (30 points) Use the pumping lemma for context free languages to prove that the following language is NOT context free:

$$L_1 = \{a^r b^s c^t \mid r > s, s > t, t > 0\}$$

Show all steps of the proof as demonstrated in class, and show *all* cases (and all subcases, subsubcases, etc.).

2. (30 points) Two of the languages below are context free. One of them is not context free. Prove one of the languages IS context free by drawing the state machine for a PDA corresponding to it. Prove one of the languages is context free by giving a CFG for it. For the language that is NOT context free, show the first three steps of the proof (1. Assume CF, 2. Let  $p$  be the pumping length, 3. choose a string...), list *all* cases for the decompositions, and do the decomposition and proof for **two** of the subcases that overlap two characters (e.g. “ $v$  is in the  $a$ ’s,  $y$  is in the  $a$ ’s and  $b$ ’s”)

a.  $L_{2a} = \{a^r b^s c^t \mid r > s, s > t, t > 0, r < 4\}$

b.  $L_{2b} = \{a^r b^s c^t d^q \mid r = t; s < 2q; q, r, s, t > 0\}$

c.  $L_{2c} = \{a^r b^s c^t d^q \mid r = q, s < t; r, q, s, t > 0\}$