

# ASSIGNMENT 1 REPORT

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## QUESTION 1:

1.1)

$$\text{PDF} = (1 / (2 * b)) * \exp(-\text{abs}(x - \mu) / b)$$

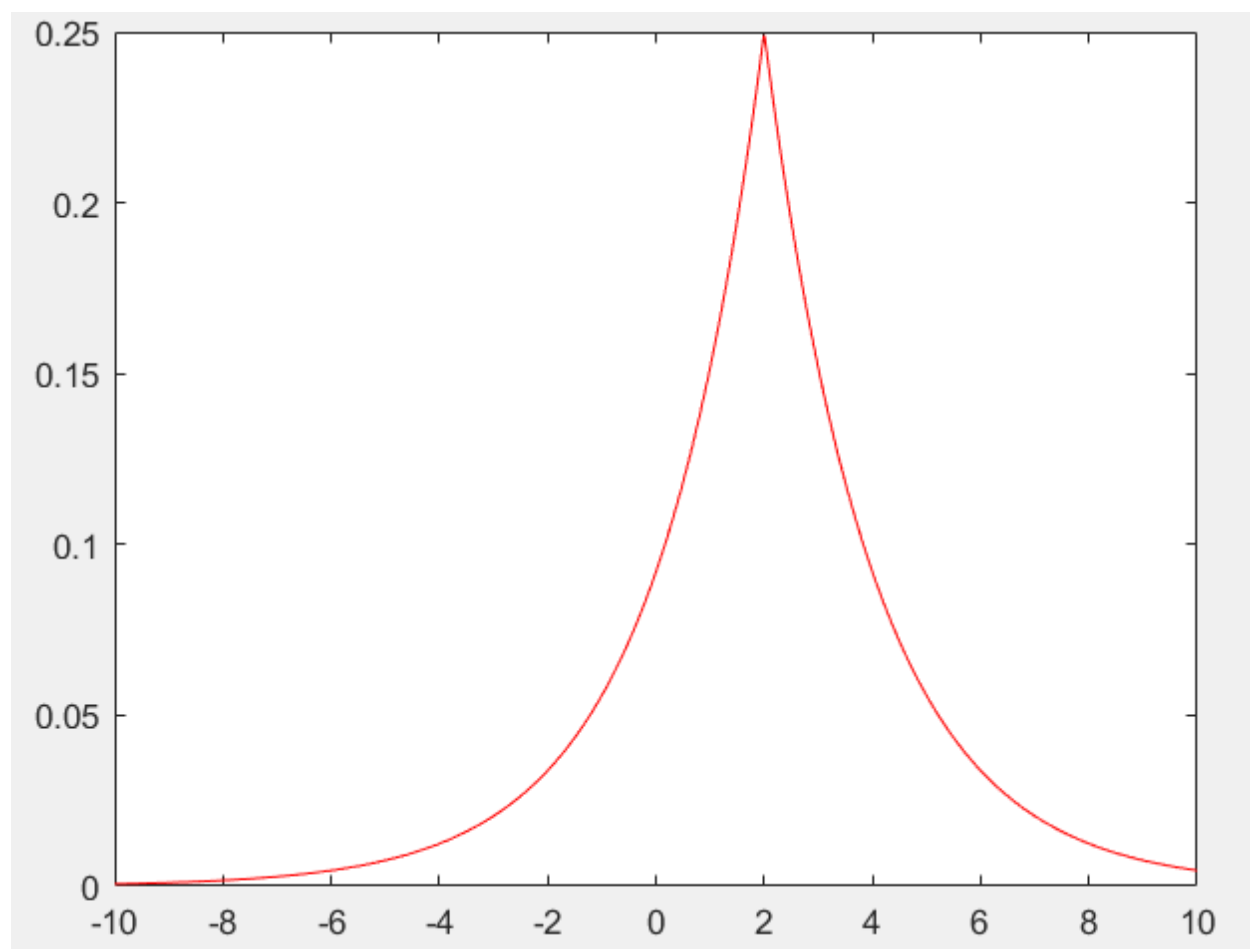
With  $\mu=2$  and  $b=2$

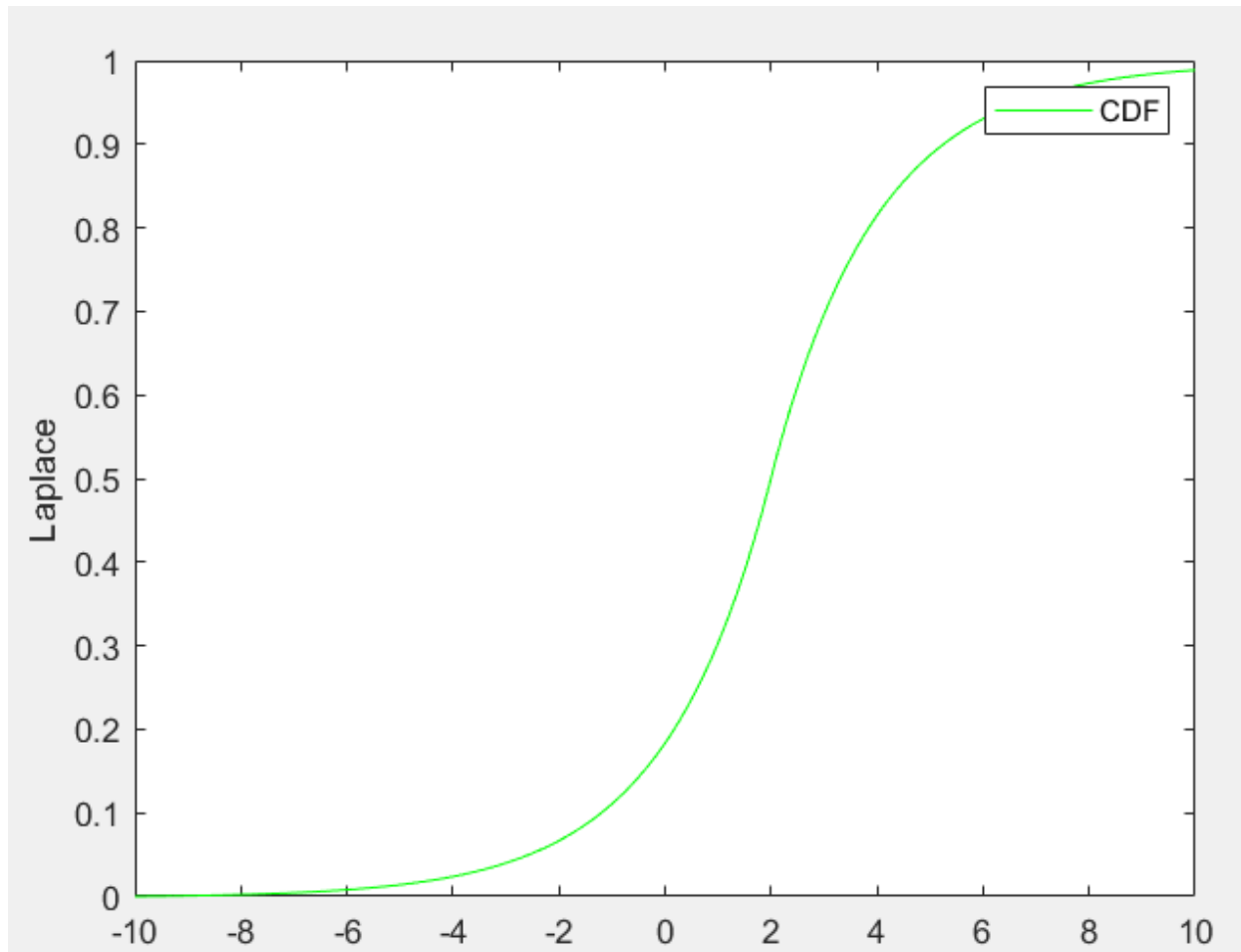
**Variance = 8(theoretically)**

By taking 10000 points and between -50 and 50 we were able to obtain a value of 7.9992.

P.S. I didn't calculate mean separately as I have used  $\mu$  from the PDF equation.

Which is `var(a)` in my code.





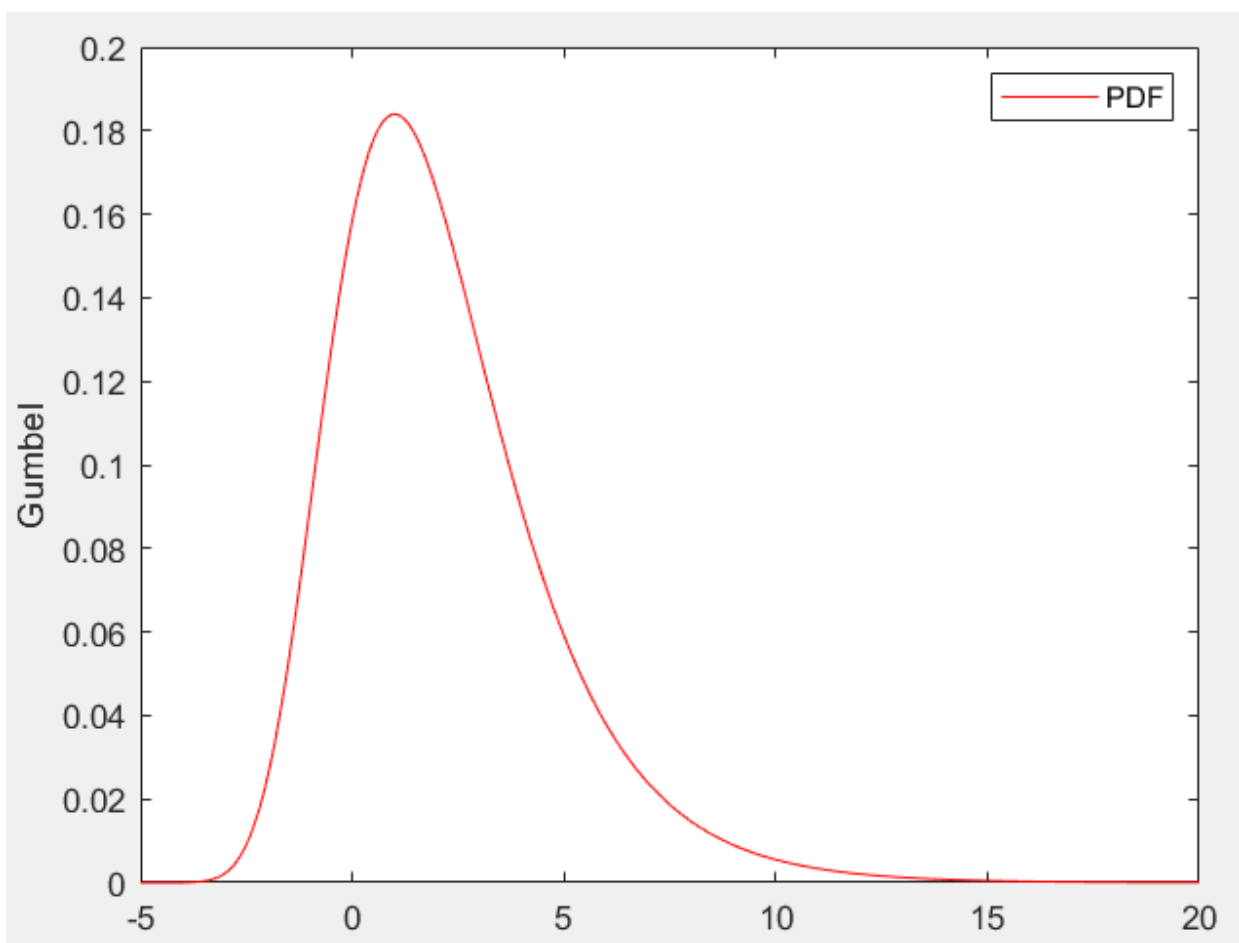
1.2)

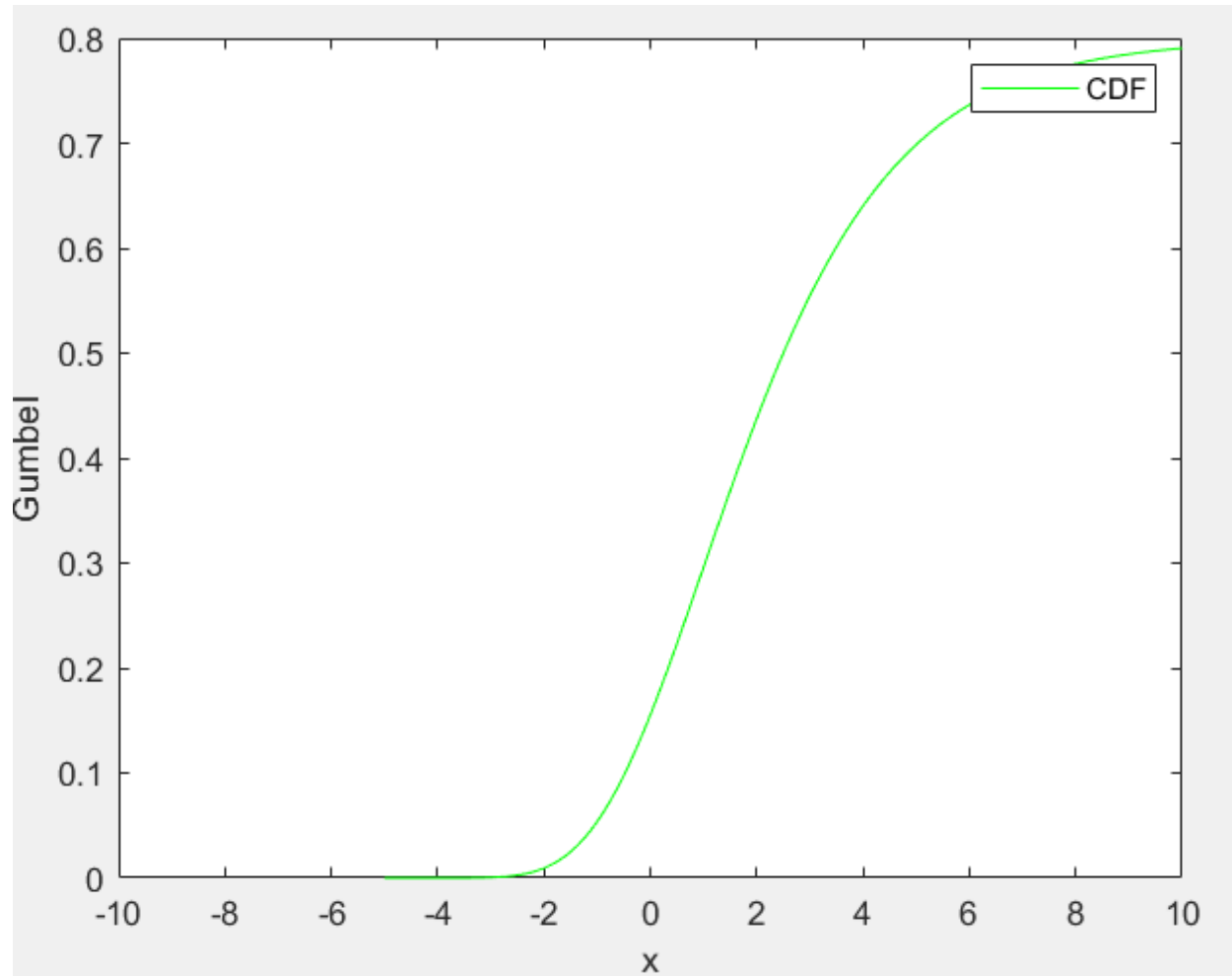
$$\text{PDF} = (1/b) * \exp(-((x-a)/b) + \exp(-(x-a)/b))$$

With  $a=1$  and  $b=2$

**Variance = 6.5797(theoretically)**

By taking 10000 points and between -50 and 50 we were able to obtain a value of 6.5791.



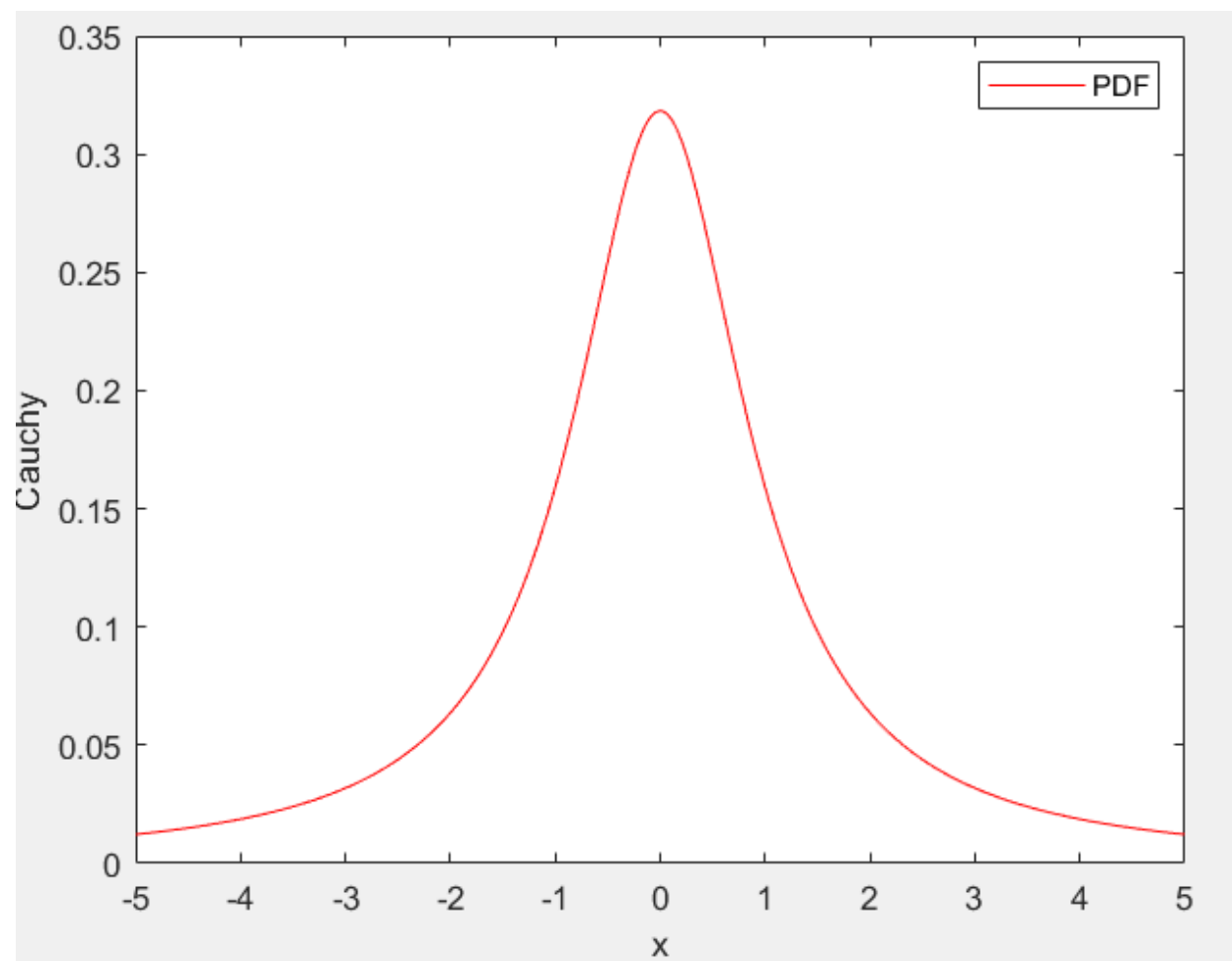


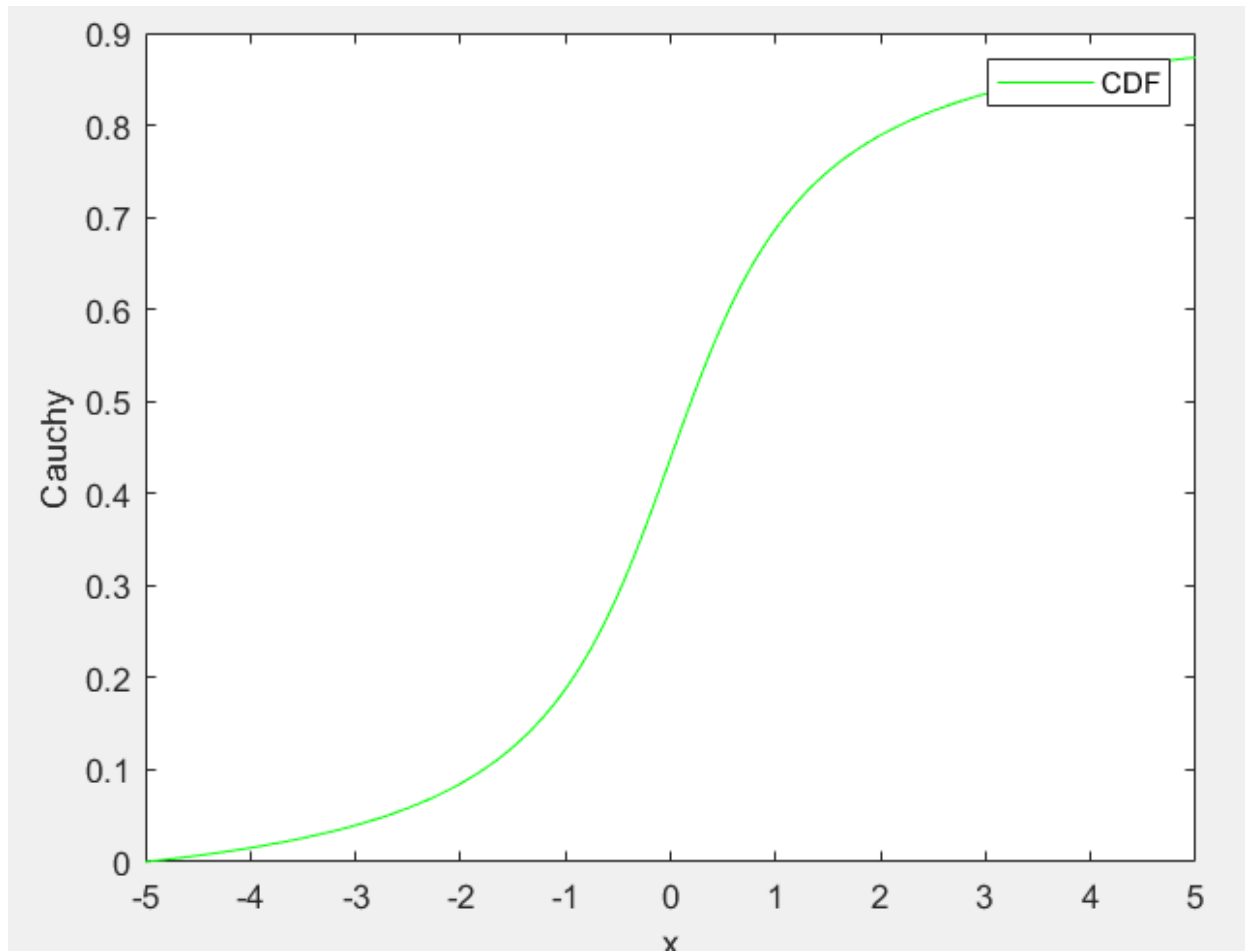
1.3)

$$\text{PDF} = (1/\pi) * (1 / (1+x^2))$$

With  $x_0=0$  and  $\gamma=1$

Integral does not converge variance is likely undefined.





Images sent are taken over smaller range of numbers for high resolution , the ranges taken in the code are for the purpose of calculating variance .

## QUESTION 2:

### Part A

Generate two poisson distribution data one with  $\lambda=3$  and other with  $\lambda=4$  . $Z=X+Y$  add all the sample instances from data 1 and data 3 and calculate the PMF empirically. Now use the theoretical PMF of  $Z$  (we know it also follows poisson distribution with  $\lambda=7$ ) Code contains two arrays  $p\_exp$  and  $p\_the$  both PMFs are very close.

The difference between them is in the order of  $10^{-4}$  or  $10^{-5}$  which is done by displaying comp in the code.

Difference from k=1 to k=25:

```
1.0e-03 *
0.0488
0.0669
0.2967
0.0908
0.2787
0.2288
0.0828
0.0096
0.1107
0.0177
0.0972
0.0412
0.1116
0.0768
0.0044
```

## Part B

Create a sample size of  $10^5$  elements which satisfy poisson distribution. Now pass each element of the sample through binornd() function to generate a binomial distribution of the poisson sample.

This PMF calculated is p\_exp .

Theoretically this can be calculated by using poisson distribution with  $\lambda=3.2$

Instead of  $\lambda=4$ .

Experimental and theoretical values match to a great extent

Except for K=1 and K=2 the order of difference is less than  $10^{-4}$  while 1 and 2

are in the order of  $10^{-3}$ .

Difference from k=1 to k=25:

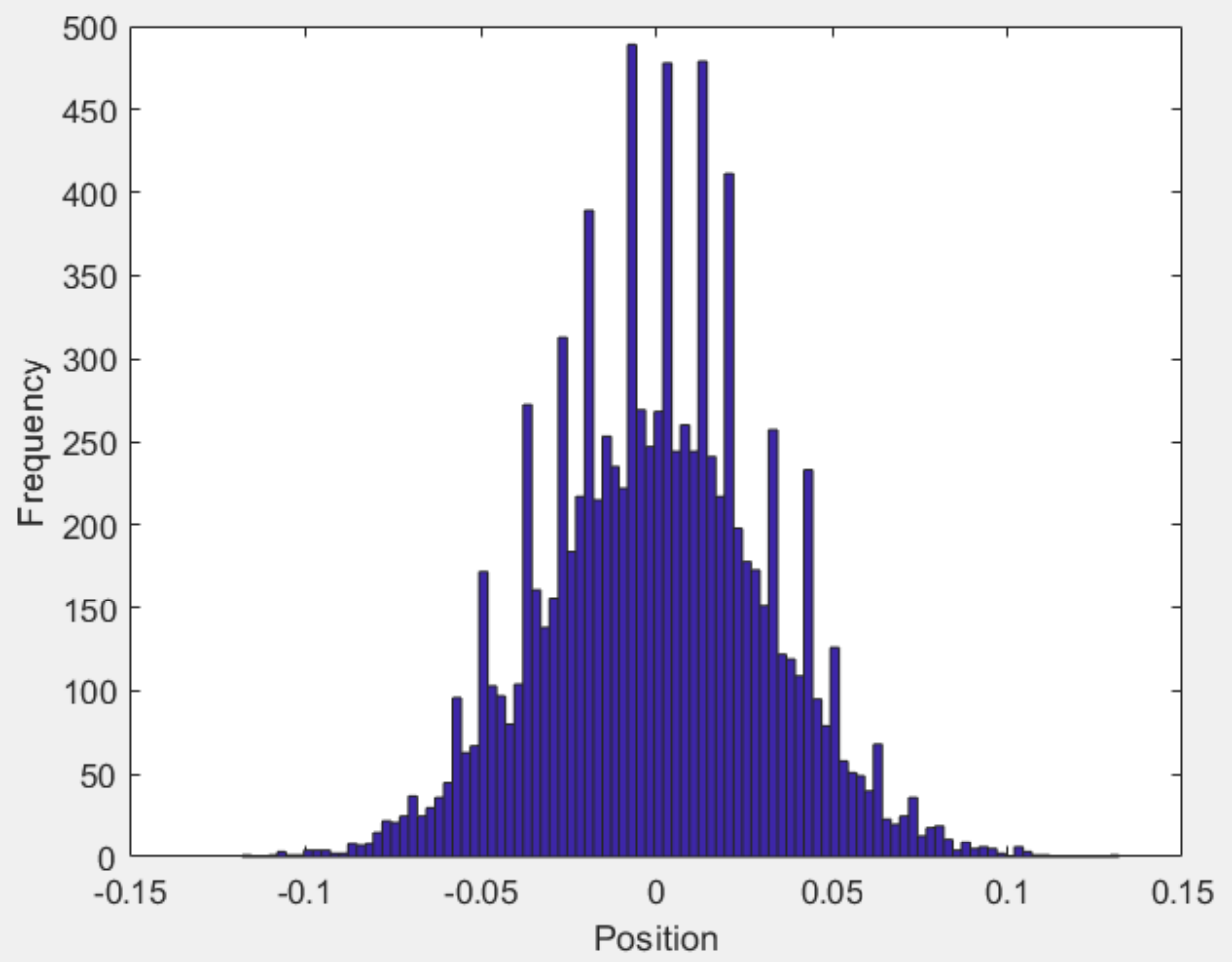
```
0.0016
0.0012
0.0009
0.0016
0.0004
0.0002
```

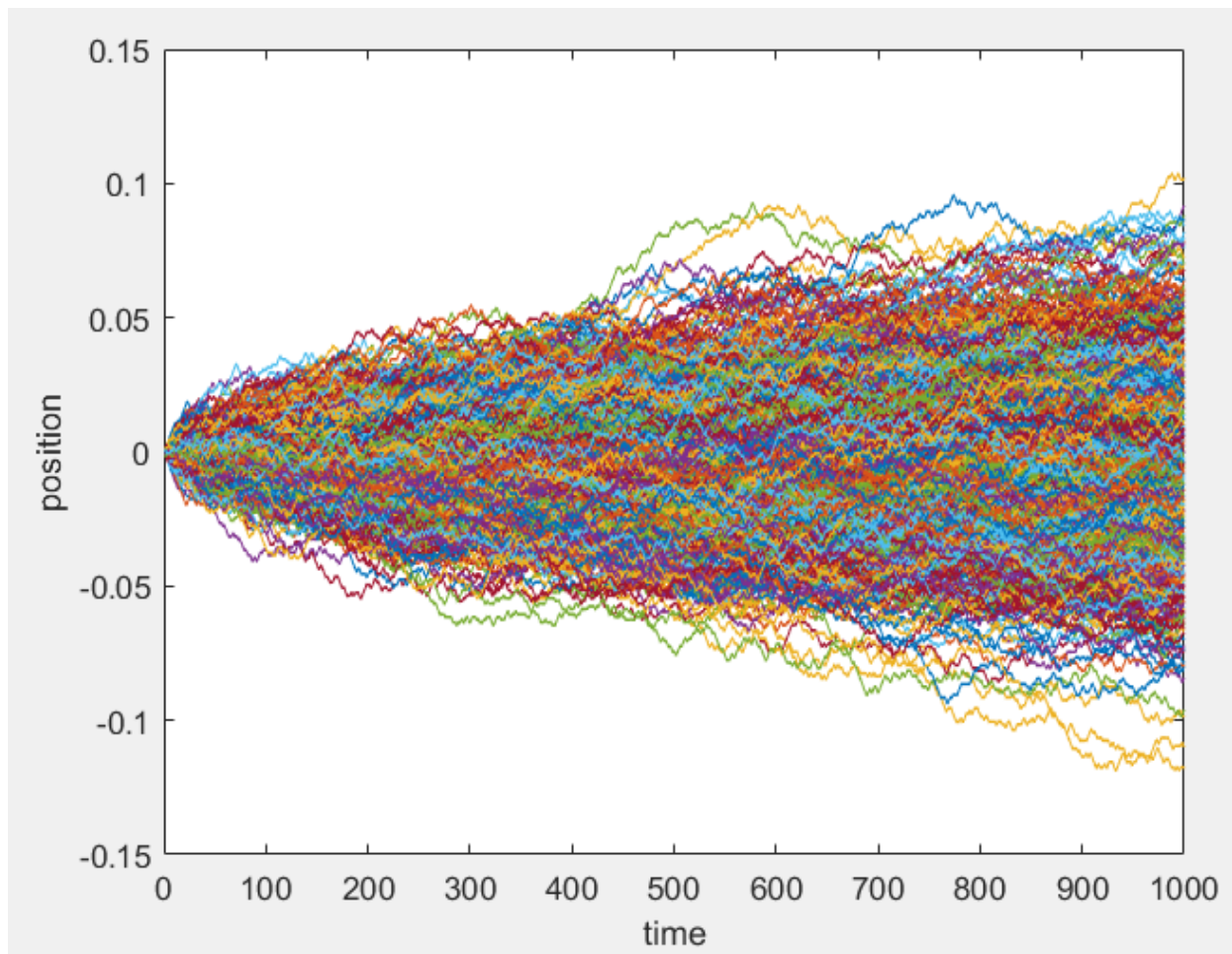


0.0001  
0.0009  
0.0001  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000  
0.0000

## QUESTION 3:

### Part A





### Part B

We dont know how to write this in LaTeX and its not so clear in Google Docs . So we have submitted our written copy for this variance question .

Law Of Large Numbers:-

\* The random variable  $\hat{M} = \frac{X_1 + X_2 + X_3 + \dots + X_N}{N}$  converges to the true mean,

$M := E[X]$  as  $N \rightarrow \infty$ . This can be written as:

For all  $\epsilon > 0$ , as  $N \rightarrow \infty$ ,  ~~$P(|\hat{M} - M| \geq \epsilon) \rightarrow 0$~~

$$P(|\hat{M} - M| \geq \epsilon) \rightarrow 0.$$

\* Proof for this would be as shown:

We use the Chebyshev's inequality to prove this statement. It says "If  $X$  is a random variable with PDF  $P(\cdot)$ , finite expectation  $E[X]$  and finite variance  $\text{Var}(X)$ , then,

$$P(|X - E[X]| \geq a) \leq \frac{\text{Var}(X)}{a^2}.$$

Using this inequality,

$$P(|\hat{M} - M| \geq \epsilon) \leq \frac{\text{Var}(X)}{\epsilon^2}.$$

$$P(|\hat{M} - M| \geq \epsilon) = \frac{\text{Var}(X)}{N\epsilon^2}.$$

$$P(|\hat{M} - M| \geq \epsilon) = 0.$$

(Since  $N \rightarrow \infty$ )

\* Hence, we have proved above that for all  $\epsilon > 0$ , as  $N \rightarrow \infty$ ,  $P(|\hat{M} - M| \geq \epsilon) \rightarrow 0$ .

\* The expected value of the random variable  $\hat{V}$  tends to true variance  $\hat{V} = \text{Var}(X)$ ,  $N \rightarrow \infty$  where  $\hat{V}$  is defined as:

$$\hat{V} = \sum_{i=1}^N \frac{(X_i - \hat{M})^2}{N}.$$

\* Proof for this would be as shown:

$$\hat{M} = \frac{(X_1 + X_2 + X_3 + \dots + X_N)}{N}$$

$$\hat{V} = \frac{\sum_{i=1}^N (X_i - \hat{M})^2}{N}, \quad \hat{V} = \frac{\sum_{i=1}^N (X_i^2 - 2X_i\hat{M} + \hat{M}^2)}{N} \quad \text{--- (1)}$$

$$\hat{V} = \frac{\sum_{i=1}^N X_i^2}{N} - \hat{M}^2 \quad \text{--- (2)}$$

We already know that  $\text{Var}(X) = E[X^2] - \hat{M}^2$  --- (3)

Applying Law of Large Numbers on the term  $\sum_{i=1}^N \frac{X_i^2}{N}$  in above eq'n (2), take  $X^2$  as a random variable and now applying Law of Large Numbers, we get that average of squares of  $X$  is equal to  $E[X^2]$ .

Now, substituting the value of  $E[X^2]$  found in eq(3) in eq(2), we get

$$\hat{V} = (\text{Var}(X) + \hat{M}^2) - \hat{M}^2$$

$$\Rightarrow \boxed{\hat{V} = \text{Var}(X)}$$

Hence, proved.



## Part C

1) Variance and Mean of final location of walkers found empirically.  
After generating the data let's calculate variance and mean.

```
Variance = 0.0010;  
Mean     = 1.386e-4;  
  
>> assignment3  
0.0010  
  
1.3860e-04
```

2) Theoretically it's a limiting case of binomial distribution.

With variance of walking right =  $n \cdot p \cdot q$

Mean of walking right =  $n \cdot p$

Location (z) when walked x steps right is

$$z = \Delta z (2x - n)$$

Therefore,  $\text{Var}(z) = 4 \cdot (\Delta z)^2 \cdot \text{Var}(x)$

$$= 4 \cdot 10^{-6} \cdot 10^3 \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \quad (p=q=\frac{1}{2}) \text{ As}$$

rand generator is uniform distribution)

$$= 0.001$$

$$\text{Mean}(z) = \Delta z (2 \cdot \text{Mean}(x) - n)$$

$$= \Delta z \cdot n (2 \cdot p - 1)$$

$$= 0$$

3)

Deviation of variance from theoretical value is 0.0000  
which is not exactly zero but in the order of  $10^{-5}$

While deviation of mean from theoretical is about  
 $1.386 \cdot 10^{-4}$

## QUESTION 4:

## 1 ) Generating Draws from $P_x(.)$ :

⇒ q4.m is code which generates independent draws from  $P_x(.)$  .

Command to run the program is “ q4 “ .

Here  $P_x(.)$  is defined as follows :

$$\begin{aligned} P_x(x) &= 0 && \text{for } |x| > 1 . \\ &= |x| && \text{for } x \text{ in } [-1,1] . \end{aligned}$$

$$\begin{aligned} F(x) &= 0 && \text{for } x < -1 . \\ &= (1 - x^2) / 2 && \text{for } x \text{ in } [-1,0) \\ &= (1 + x^2) / 2 && \text{for } x \text{ in } [0,1) \\ &= 1 && \text{for } x > 1 . \end{aligned}$$

⇒ Actually **rand()** generates a random number between 0 and 1 with uniform probability . Now if we generate two random numbers say a and b and if we select maximum of these two, let's say 'a' . Now 'b' to be less than 'a' , it should be in b/w 0 and 'a' which means that for generating 'b' the probability is proportional to 'a' .

⇒ I have developed this by defining a function for generating a random number b/w -1 and 1 , for this I have used **sign(randn)** generates +1 or -1 .

⇒ After generating 100000 draws from the generator we wrote , we stored the generated values in an array, which I used to plot the histogram . Then I calculated the CDF and plotted that in a graph .

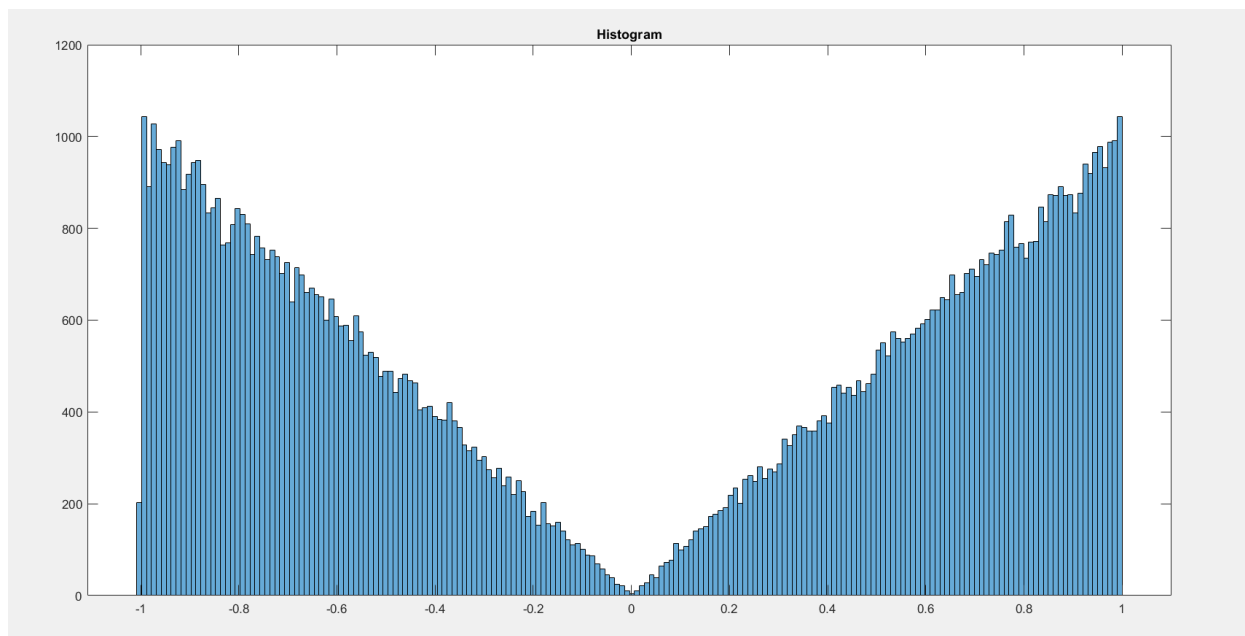
## 2 ) Generating Draws from $Y_N(.)$ :

⇒ For  $Y_N(.)$  I used the generator which I developed above N times that was looped N times  $X_1, X_2, X_3, X_4, \dots, X_N$  and then

taking their average .

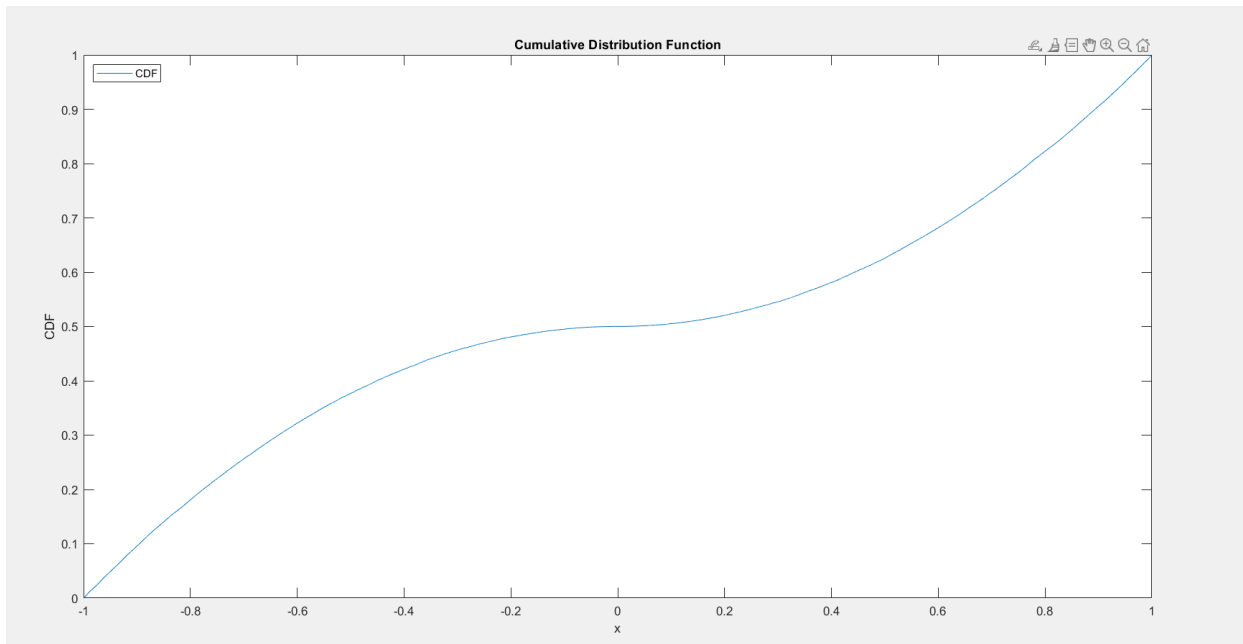
- ⇒ We can see the average of  $N$   $X_i$  's would result in a  $P_Y(.)$  slowly tend to a random variable which has Gauss distribution as  $N$  becomes larger .
- ⇒ Similarly, CDF would also resemble that of the Gaussian CDF as  $N$  becomes larger and larger .
- ⇒ We have plotted the histograms and CDFs for  $N = 1, 2, 4, 8 \dots 64$ .
- ⇒ I have submitted 9 files( all histograms and CDFs ) .This would contain a histogram, a plot containing CDF for  $P_X(.)$  and histograms for  $Y_N(.)$  (  $N = 2, 4 \dots 64$  ) and a plot containing CDF for  $Y_N(.)$  (  $N = 1, 2, 4 \dots 64$  ) .

### Histogram for $P_X(.)$

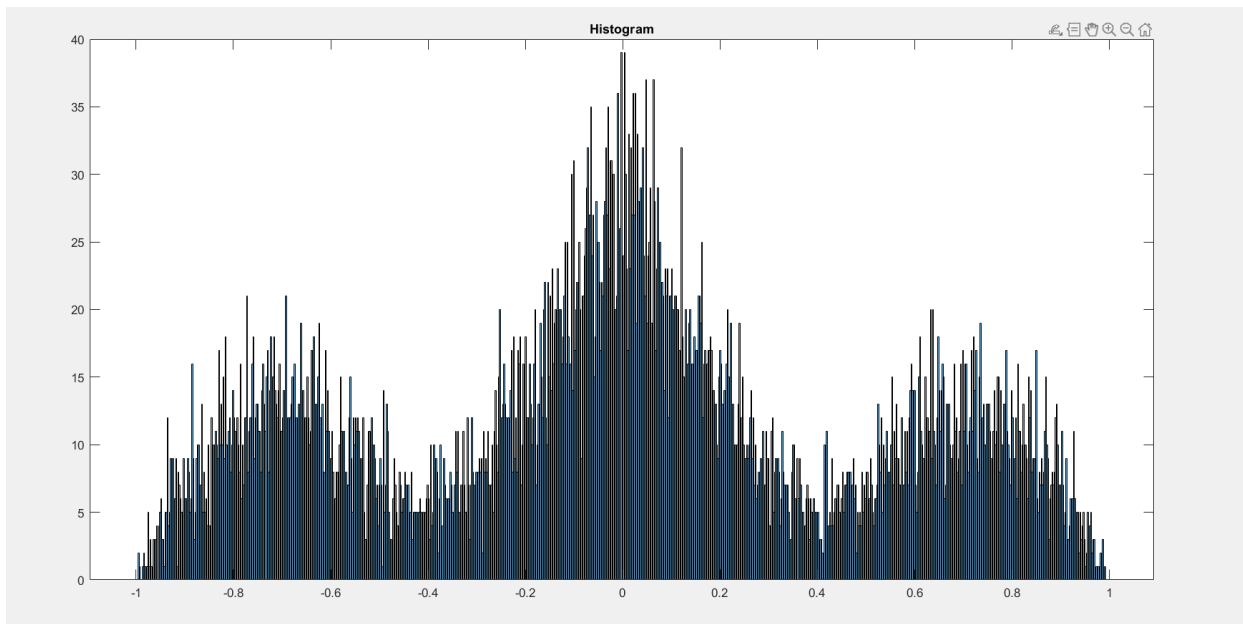




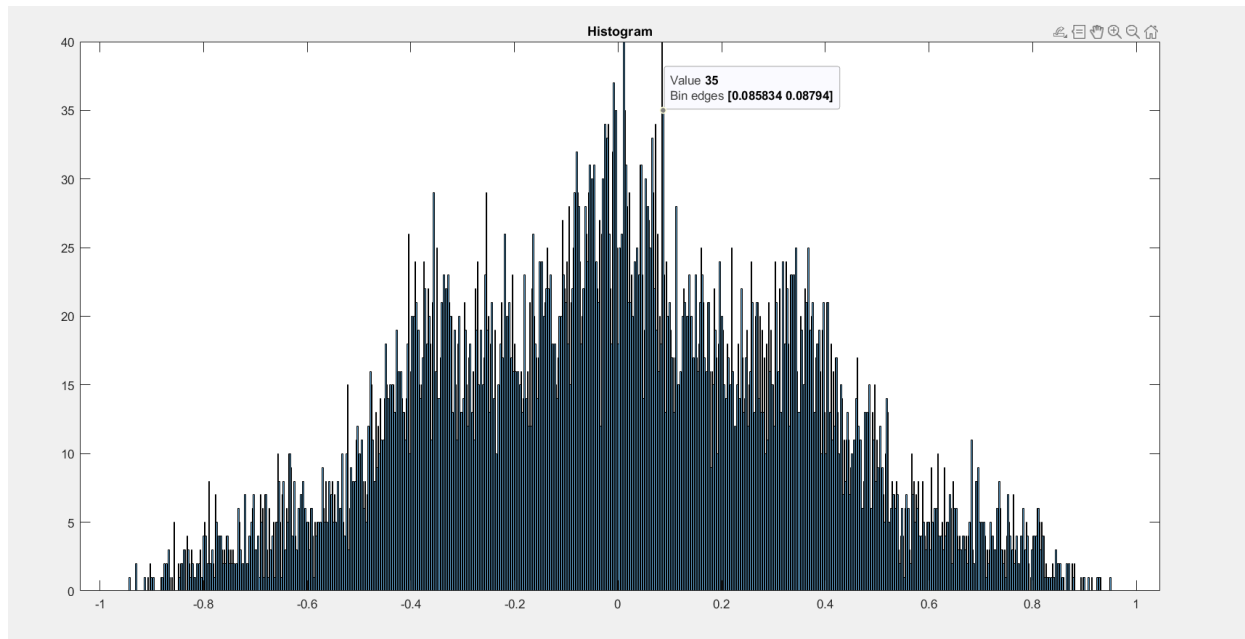
## CDF for $P_x (.)$



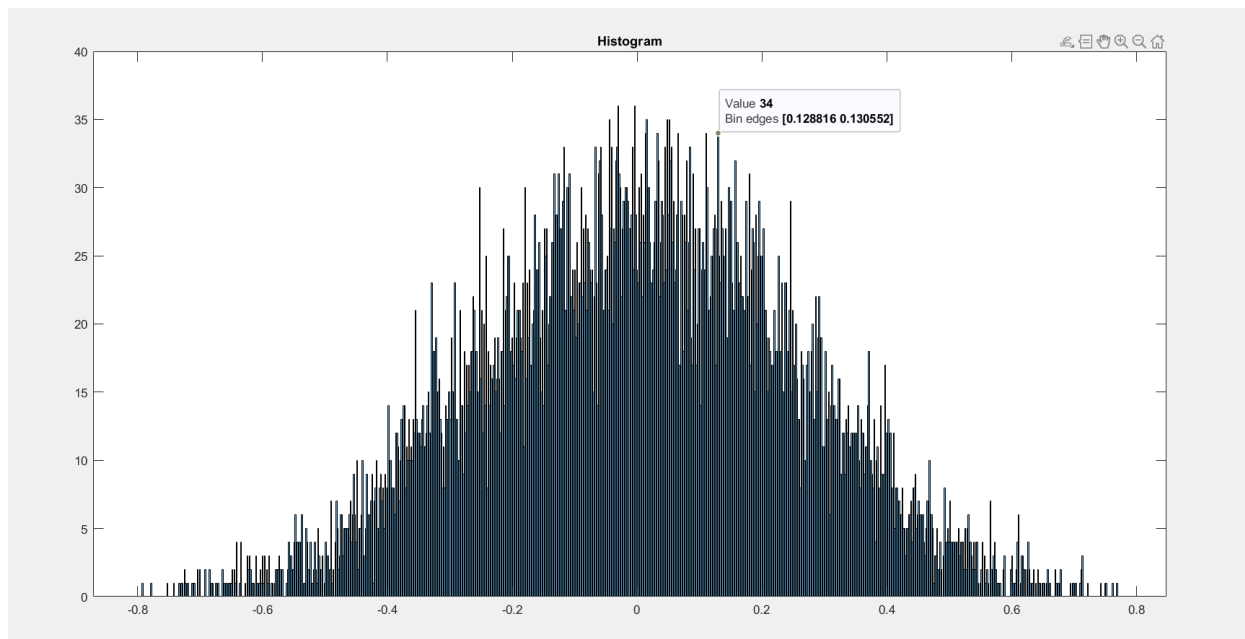
## Histogram for N=2



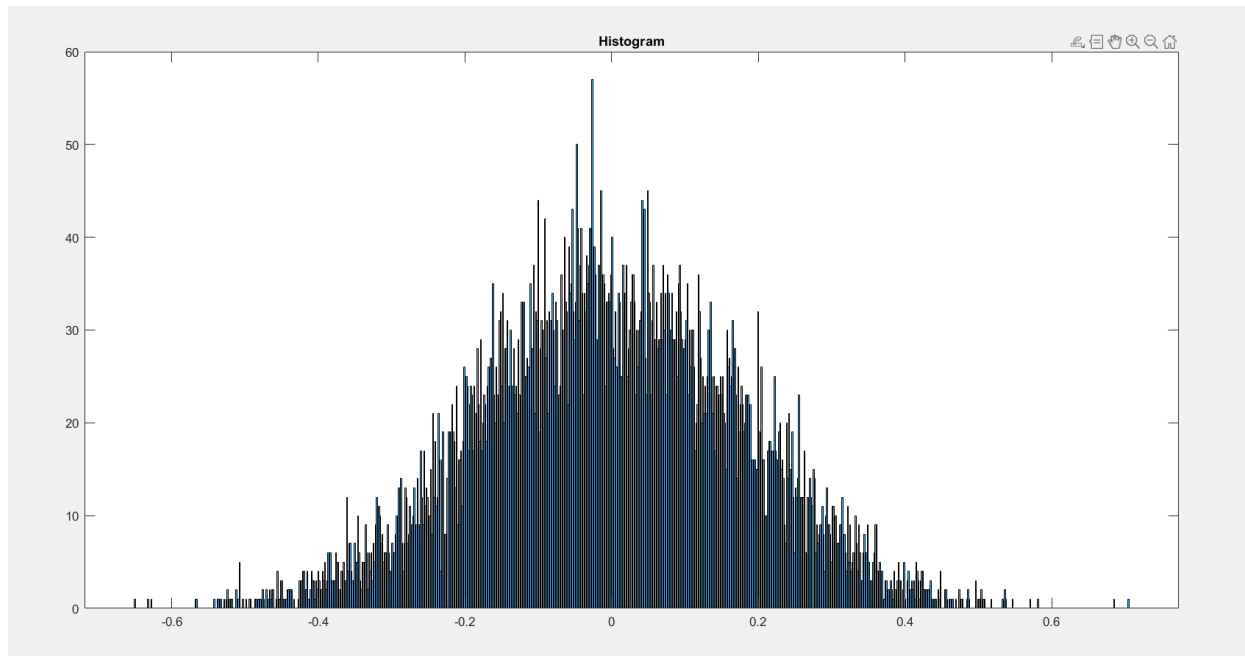
## Histogram for N=4



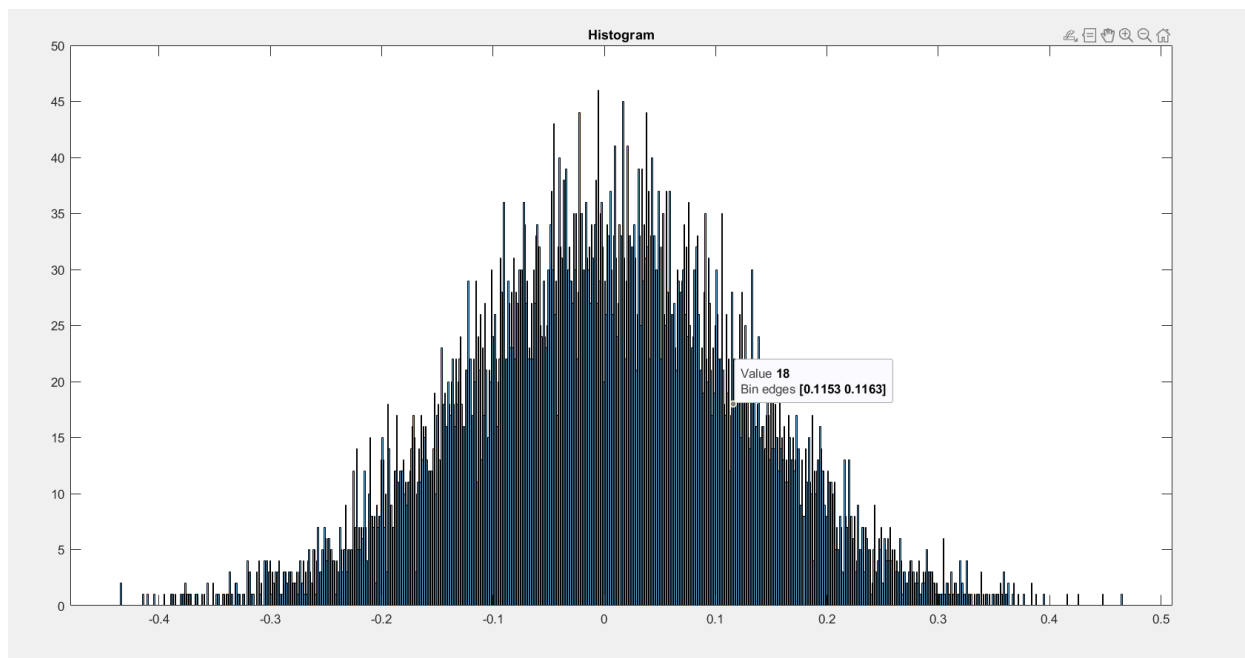
## Histogram for N=8



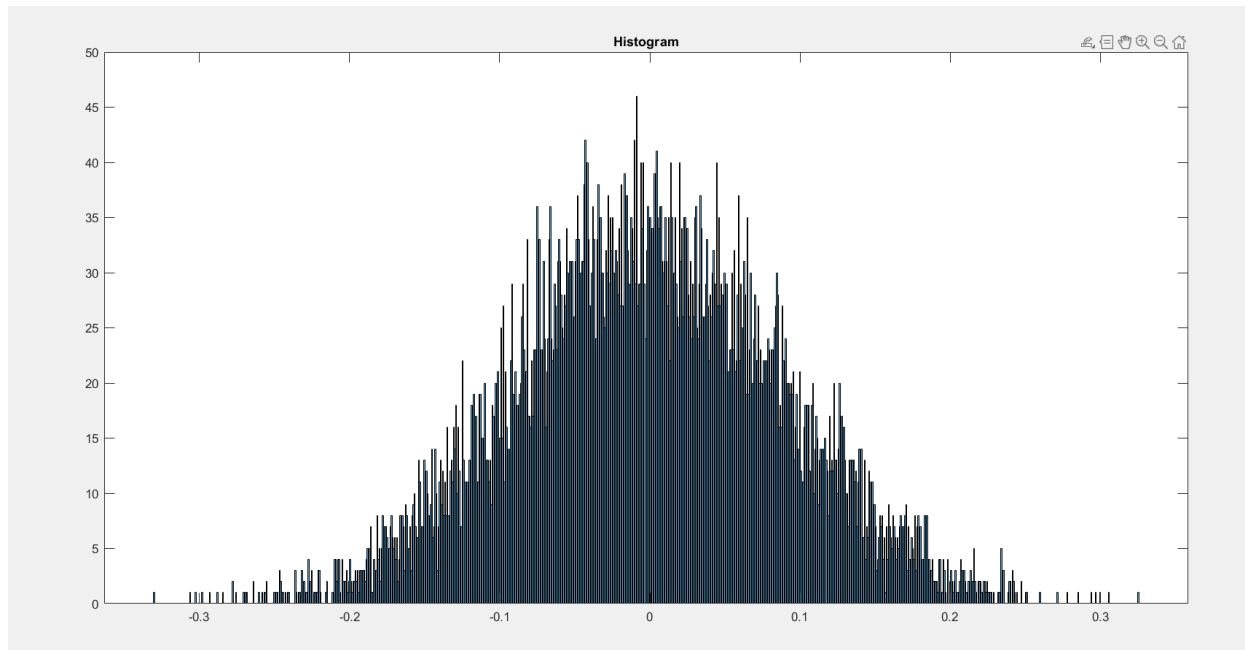
## Histogram for N=16



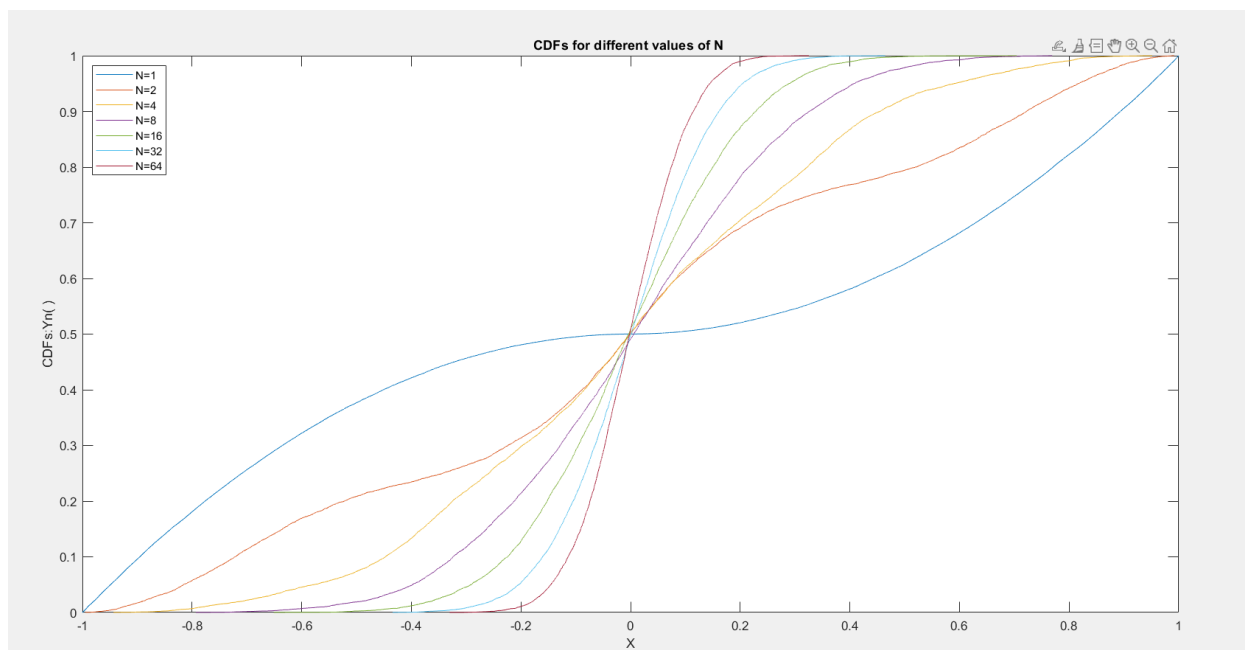
## Histogram for N=32



## Histogram for N=64



## CDF for all N=1,2,4....64



**QUESTION 5:**

This question is an experimental evidence of LAW OF LARGE NUMBERS  
we can see that error i.e, abs(avg\_mean-true mean) decreases as size increases.

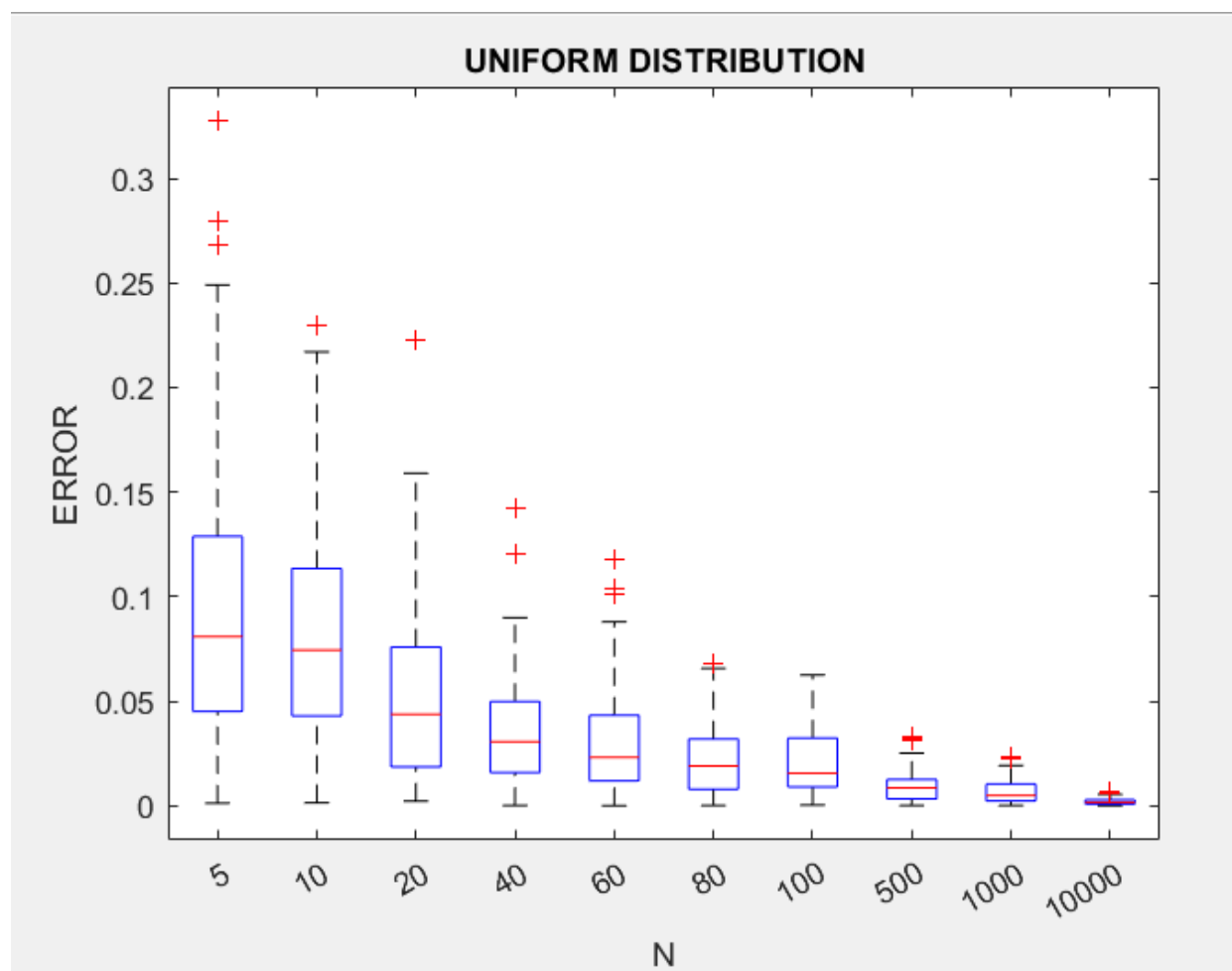
And between normal distribution and gauss distribution error is significantly low for the same (smaller N )in case of normal distribution.

DATA AND BOXPLOTS:

N= 5	10	20	40	60	80	100	500		
0.1786	0.0399	0.2231	0.0119	0.0025	0.0351	0.0373	0.0094	0.0038	0.0004
0.0691	0.0529	0.0439	0.0701	0.0205	0.0273	0.0570	0.0009	0.0176	0.0029
0.1742	0.0619	0.0773	0.0054	0.0072	0.0026	0.0044	0.0109	0.0105	0.0013
0.1462	0.0518	0.0641	0.0107	0.0192	0.0060	0.0080	0.0009	0.0013	0.0007
0.1307	0.1122	0.0746	0.0113	0.0111	0.0030	0.0134	0.0136	0.0023	0.0047
0.0440	0.0797	0.0386	0.0523	0.0116	0.0645	0.0427	0.0172	0.0011	0.0003
0.2684	0.0854	0.0757	0.0070	0.0125	0.0346	0.0119	0.0006	0.0052	0.0010
0.0640	0.1799	0.0067	0.0363	0.0362	0.0425	0.0133	0.0185	0.0041	0.0000
0.0136	0.0431	0.0104	0.0184	0.0041	0.0372	0.0093	0.0034	0.0037	0.0001
0.1091	0.2067	0.0288	0.0168	0.0192	0.0177	0.0010	0.0027	0.0229	0.0013
0.1215	0.2170	0.0322	0.0758	0.0039	0.0183	0.0026	0.0125	0.0031	0.0027
0.0215	0.1458	0.0030	0.0135	0.0313	0.0209	0.0088	0.0329	0.0047	0.0011
0.1205	0.1211	0.0732	0.0202	0.0376	0.0138	0.0341	0.0042	0.0062	0.0025
0.0896	0.0398	0.1486	0.0277	0.0498	0.0253	0.0006	0.0074	0.0014	0.0003
0.1164	0.1749	0.0575	0.0095	0.0133	0.0117	0.0223	0.0020	0.0109	0.0047
0.1226	0.0991	0.0534	0.0177	0.0038	0.0236	0.0465	0.0198	0.0005	0.0005
0.1469	0.0352	0.0868	0.0148	0.0314	0.0080	0.0164	0.0108	0.0056	0.0017
0.0490	0.0649	0.0494	0.0263	0.0052	0.0272	0.0298	0.0036	0.0039	0.0015
0.0252	0.1315	0.0361	0.0170	0.0150	0.0339	0.0387	0.0119	0.0094	0.0037
0.1966	0.0984	0.0307	0.0743	0.0043	0.0003	0.0126	0.0081	0.0127	0.0008
0.1076	0.0551	0.0850	0.0517	0.0677	0.0197	0.0212	0.0175	0.0086	0.0026
0.0913	0.1084	0.0666	0.0527	0.0412	0.0260	0.0127	0.0126	0.0123	0.0017
0.1342	0.1114	0.0088	0.1422	0.0275	0.0264	0.0254	0.0312	0.0043	0.0037
0.0762	0.0512	0.0381	0.0594	0.0149	0.0230	0.0529	0.0038	0.0039	0.0022
0.0331	0.1788	0.0774	0.0218	0.0240	0.0108	0.0048	0.0004	0.0022	0.0006
0.0174	0.1144	0.0769	0.0222	0.0273	0.0055	0.0043	0.0108	0.0231	0.0027
0.1133	0.0415	0.1004	0.0251	0.0879	0.0363	0.0109	0.0049	0.0050	0.0043
0.0440	0.0109	0.0608	0.0454	0.0148	0.0079	0.0100	0.0006	0.0154	0.0011
0.0482	0.0593	0.0752	0.0014	0.0226	0.0009	0.0159	0.0247	0.0014	0.0020
0.3274	0.0413	0.0617	0.0309	0.0000	0.0013	0.0064	0.0047	0.0001	0.0014
0.0611	0.0278	0.1349	0.0336	0.0602	0.0041	0.0145	0.0090	0.0056	0.0002

0.0585	0.0652	0.0811	0.0480	0.0163	0.0163	0.0118	0.0002	0.0052	0.0017
0.1176	0.0762	0.0150	0.0446	0.0300	0.0153	0.0146	0.0091	0.0034	0.0024
0.0330	0.1817	0.0342	0.0453	0.0018	0.0362	0.0094	0.0318	0.0011	0.0013
0.2065	0.0730	0.0320	0.0001	0.0880	0.0096	0.0625	0.0006	0.0001	0.0011
0.0297	0.0742	0.0131	0.0621	0.0565	0.0057	0.0269	0.0252	0.0030	0.0038
0.0072	0.0736	0.0650	0.0257	0.1175	0.0105	0.0210	0.0203	0.0128	0.0001
0.0922	0.1566	0.0381	0.0072	0.0440	0.0080	0.0097	0.0025	0.0108	0.0012
0.1149	0.1696	0.0259	0.0696	0.0283	0.0543	0.0220	0.0032	0.0046	0.0023
0.0876	0.0690	0.1240	0.0751	0.0004	0.0183	0.0143	0.0183	0.0024	0.0012
0.0436	0.0021	0.0024	0.0439	0.0154	0.0170	0.0471	0.0080	0.0157	0.0015
0.2149	0.0020	0.0466	0.0492	0.0728	0.0215	0.0456	0.0032	0.0180	0.0028
0.0891	0.0379	0.0138	0.0057	0.0015	0.0001	0.0214	0.0088	0.0002	0.0002
0.2490	0.0234	0.0894	0.0345	0.0125	0.0453	0.0075	0.0076	0.0161	0.0047
0.0509	0.0196	0.0207	0.0181	0.0061	0.0230	0.0143	0.0023	0.0113	0.0006
0.0607	0.0307	0.0266	0.0627	0.0561	0.0372	0.0020	0.0026	0.0039	0.0062
0.0214	0.0675	0.0255	0.0102	0.0000	0.0569	0.0131	0.0126	0.0072	0.0015
0.1670	0.0685	0.0049	0.0468	0.0430	0.0233	0.0483	0.0006	0.0007	0.0005
0.0786	0.0467	0.0677	0.0230	0.0238	0.0657	0.0174	0.0107	0.0146	0.0011
0.0013	0.1236	0.0427	0.0552	0.0018	0.0459	0.0042	0.0004	0.0018	0.0061
0.0503	0.0559	0.0034	0.0869	0.1036	0.0263	0.0158	0.0229	0.0171	0.0051
0.1108	0.0751	0.0296	0.0439	0.0334	0.0101	0.0398	0.0209	0.0101	0.0034
0.0462	0.1334	0.0506	0.0816	0.0123	0.0385	0.0072	0.0003	0.0022	0.0009
0.0731	0.1025	0.0674	0.0155	0.0703	0.0565	0.0061	0.0051	0.0004	0.0028
0.0498	0.0552	0.0306	0.0222	0.0440	0.0027	0.0066	0.0075	0.0028	0.0034
0.1051	0.1426	0.0663	0.0417	0.0177	0.0562	0.0017	0.0086	0.0090	0.0005
0.0319	0.1008	0.0111	0.0168	0.0866	0.0261	0.0028	0.0131	0.0033	0.0034
0.1938	0.1124	0.1328	0.0372	0.0243	0.0298	0.0119	0.0093	0.0016	0.0002
0.1344	0.0794	0.0551	0.0058	0.0246	0.0423	0.0341	0.0158	0.0017	0.0007
0.0662	0.0195	0.1580	0.0303	0.0162	0.0017	0.0143	0.0113	0.0092	0.0030
0.2246	0.0878	0.0109	0.0210	0.0438	0.0317	0.0182	0.0071	0.0049	0.0072
0.0209	0.0212	0.0207	0.0104	0.0186	0.0094	0.0184	0.0021	0.0110	0.0019

0.1144	0.1368	0.1350	0.0900	0.0184	0.0109	0.0301	0.0064	0.0052	0.0052
0.0481	0.1495	0.0515	0.0014	0.0319	0.0010	0.0016	0.0100	0.0104	0.0007
0.0256	0.1094	0.0372	0.0257	0.0168	0.0414	0.0037	0.0004	0.0038	0.0011
0.1219	0.0719	0.1049	0.0092	0.0487	0.0230	0.0153	0.0032	0.0192	0.0002
0.1988	0.1070	0.0647	0.0457	0.0500	0.0049	0.0096	0.0040	0.0017	0.0021
0.0863	0.0813	0.1590	0.0247	0.0436	0.0321	0.0282	0.0012	0.0002	0.0046
0.1628	0.0458	0.0871	0.0667	0.0614	0.0680	0.0467	0.0108	0.0047	0.0024
0.0812	0.0508	0.0128	0.0180	0.0052	0.0109	0.0403	0.0185	0.0185	0.0041
0.0714	0.0015	0.1314	0.0508	0.0530	0.0248	0.0181	0.0206	0.0050	0.0052
0.0329	0.2016	0.0034	0.0748	0.1008	0.0026	0.0609	0.0098	0.0101	0.0028
0.0646	0.0429	0.0113	0.0501	0.0266	0.0522	0.0174	0.0101	0.0014	0.0014
0.0756	0.0241	0.0188	0.1201	0.0305	0.0288	0.0340	0.0076	0.0022	0.0001
0.1192	0.0852	0.0671	0.0155	0.0013	0.0078	0.0388	0.0044	0.0013	0.0043
0.0502	0.0194	0.0184	0.0140	0.0169	0.0076	0.0056	0.0123	0.0081	0.0009
0.0497	0.1268	0.0073	0.0676	0.0083	0.0199	0.0125	0.0206	0.0014	0.0008
0.1918	0.0747	0.0430	0.0356	0.0026	0.0470	0.0262	0.0074	0.0051	0.0055
0.2799	0.1052	0.0195	0.0574	0.0335	0.0140	0.0447	0.0148	0.0041	0.0022
0.0435	0.2301	0.0078	0.0132	0.0047	0.0487	0.0329	0.0107	0.0107	0.0032
0.1479	0.0877	0.0088	0.0161	0.0008	0.0233	0.0384	0.0112	0.0031	0.0005
0.1263	0.0353	0.0436	0.0400	0.0139	0.0057	0.0105	0.0061	0.0233	0.0009
0.0083	0.1025	0.0760	0.0169	0.0176	0.0094	0.0605	0.0152	0.0029	0.0014
0.1574	0.0246	0.0604	0.0442	0.0341	0.0117	0.0160	0.0052	0.0061	0.0028
0.0162	0.0876	0.1284	0.0497	0.0259	0.0182	0.0082	0.0054	0.0155	0.0013
0.1596	0.0479	0.0415	0.0217	0.0160	0.0475	0.0140	0.0055	0.0119	0.0028
0.0200	0.0545	0.0957	0.0387	0.0449	0.0052	0.0595	0.0123	0.0040	0.0007
0.0329	0.0222	0.0028	0.0355	0.0129	0.0190	0.0457	0.0101	0.0045	0.0016
0.0097	0.1411	0.1230	0.0684	0.0053	0.0304	0.0231	0.0245	0.0012	0.0017
0.0470	0.0833	0.0974	0.0204	0.0032	0.0054	0.0464	0.0231	0.0071	0.0008
0.0231	0.0548	0.1358	0.0361	0.0186	0.0181	0.0087	0.0038	0.0050	0.0042
0.1270	0.1270	0.0474	0.0361	0.0283	0.0002	0.0052	0.0007	0.0061	0.0052
0.0396	0.1380	0.0022	0.0063	0.0562	0.0207	0.0022	0.0216	0.0040	0.0013
0.1796	0.1383	0.0409	0.0382	0.0010	0.0191	0.0225	0.0032	0.0017	0.0024
0.0753	0.0081	0.0189	0.0405	0.0384	0.0302	0.0004	0.0023	0.0156	0.0020
0.1233	0.0104	0.0128	0.0624	0.0567	0.0072	0.0319	0.0084	0.0075	0.0020
0.0755	0.0259	0.0168	0.0178	0.0222	0.0032	0.0194	0.0067	0.0101	0.0002
0.0806	0.0841	0.0065	0.0064	0.0288	0.0150	0.0142	0.0093	0.0048	0.0020
0.0922	0.0481	0.0089	0.0036	0.0276	0.0074	0.0167	0.0101	0.0156	0.0030





0.2587	—								
0.9898	0.1714	0.1087	0.0378	0.0438	0.0962	0.0791	0.0256	0.0200	0.0005
0.6124	0.1629	0.2511	0.1029	0.0474	0.1528	0.0433	0.0107	0.0045	0.0005
0.7974	0.2557	0.2667	0.0699	0.0269	0.1090	0.0063	0.0360	0.0044	0.0126
0.4601	0.2802	0.1341	0.1351	0.0636	0.1308	0.0349	0.0223	0.0011	0.0133
0.1929	0.0534	0.3628	0.0110	0.1328	0.1317	0.0609	0.0299	0.0019	0.0145
1.0163	0.2049	0.0183	0.0269	0.1366	0.0148	0.1114	0.0390	0.0214	0.0085
0.1335	0.2476	0.0132	0.0336	0.0921	0.0203	0.0041	0.0474	0.0087	0.0062
0.1153	0.1703	0.2183	0.0676	0.1363	0.1417	0.0996	0.0548	0.0406	0.0035
0.5271	0.3897	0.1504	0.1956	0.1379	0.0730	0.0992	0.0024	0.0010	0.0024
0.6242	0.3488	0.0615	0.2362	0.1810	0.0963	0.0858	0.1066	0.0167	0.0050
0.4052	0.5526	0.1515	0.1806	0.1550	0.2034	0.1423	0.0268	0.0032	0.0095
0.4282	0.3917	0.2380	0.0946	0.1979	0.0262	0.0094	0.0943	0.0150	0.0111
0.1718	0.1284	0.4290	0.1195	0.0448	0.0172	0.1306	0.0020	0.0717	0.0022
0.0128	0.6709	0.5419	0.0443	0.0175	0.1157	0.1205	0.0481	0.0077	0.0150
0.5420	0.4585	0.5552	0.0583	0.0559	0.0177	0.0140	0.0579	0.0358	0.0036
0.5384	0.1718	0.0728	0.1494	0.0333	0.1530	0.0282	0.0028	0.0491	0.0027
0.0676	0.1658	0.3581	0.3313	0.1142	0.0166	0.0273	0.0163	0.0442	0.0054
0.3196	0.5054	0.0768	0.2229	0.2756	0.0614	0.1435	0.0316	0.0581	0.0116
0.8124	0.3015	0.3518	0.3561	0.0322	0.0251	0.1189	0.0337	0.0275	0.0087
0.0377	0.1830	0.3262	0.4563	0.0713	0.0926	0.0574	0.0270	0.0031	0.0054
0.4294	0.0183	0.3430	0.0062	0.0648	0.0506	0.0229	0.0773	0.0230	0.0223
0.3469	0.2220	0.0365	0.0562	0.0353	0.0812	0.0521	0.0290	0.0248	0.0004
0.3611	0.1713	0.2783	0.3262	0.1962	0.0689	0.0806	0.0377	0.0554	0.0029
0.7171	0.7033	0.1511	0.0254	0.2112	0.1700	0.0802	0.0660	0.0609	0.0083
0.1604	0.5250	0.3286	0.0307	0.0755	0.0899	0.0920	0.0272	0.0345	0.0033
0.3929	0.0320	0.3017	0.0347	0.0397	0.0471	0.0638	0.0293	0.0424	0.0069
0.0243	0.3375	0.2914	0.2178	0.0739	0.1274	0.0827	0.0678	0.0246	0.0181
0.6213	0.1087	0.4368	0.0561	0.1352	0.0638	0.0317	0.0298	0.0151	0.0059
0.7415	0.1049	0.1381	0.0182	0.0825	0.1058	0.0648	0.0282	0.0191	0.0052
0.1372	0.2763	0.0272	0.1536	0.0248	0.0169	0.0595	0.0077	0.0003	0.0036
0.0620	0.2055	0.0531	0.1342	0.0025	0.0917	0.1503	0.0351	0.0121	0.0028
0.3839	0.2228	0.0168	0.2208	0.0436	0.0030	0.1343	0.1201	0.0157	0.0070
	0.4301	0.1394	0.0133	0.2783	0.0416	0.0817	0.0181	0.0405	0.0028

0.2400	0.0774	0.0984	0.0186	0.1636	0.1028	0.0491	0.1179	0.1054	0.0012
0.8772	0.2138	0.2277	0.0441	0.1362	0.0432	0.1120	0.0401	0.0532	0.0024
0.1782	0.3209	0.1951	0.0038	0.0159	0.0317	0.0912	0.0346	0.0149	0.0055
0.3075	0.6752	0.3164	0.0347	0.0015	0.0343	0.0889	0.0030	0.0198	0.0034
0.3549	0.0491	0.1048	0.1742	0.0399	0.0273	0.0314	0.0480	0.0519	0.0024
0.1283	0.3129	0.1993	0.0673	0.1324	0.0855	0.0396	0.0320	0.0471	0.0024
0.6471	0.0319	0.0070	0.1406	0.1097	0.0470	0.1206	0.0135	0.0022	0.0112
0.1090	0.1771	0.1795	0.0892	0.1860	0.0876	0.0001	0.0571	0.0604	0.0035
0.0946	0.1715	0.0323	0.1970	0.2554	0.2144	0.2188	0.0218	0.0033	0.0100
0.1843	0.3181	0.0784	0.0606	0.1264	0.1116	0.0775	0.0110	0.0000	0.0098
0.0719	0.0598	0.1414	0.1563	0.1003	0.1733	0.0211	0.0046	0.0099	0.0018
0.0076	0.1163	0.0232	0.1147	0.1443	0.1430	0.0543	0.0166	0.0231	0.0217
0.0284	0.3626	0.2555	0.1360	0.0569	0.1333	0.0985	0.0310	0.0477	0.0059
0.6111	0.0997	0.2452	0.2604	0.2096	0.0536	0.0684	0.0402	0.0249	0.0070
0.5104	0.5659	0.1130	0.1307	0.1119	0.1374	0.0516	0.0904	0.0586	0.0135
0.0742	0.0172	0.0535	0.2179	0.0201	0.1958	0.0485	0.0653	0.0569	0.0007
0.0188	0.2021	0.1699	0.0910	0.2651	0.0926	0.0385	0.0016	0.0031	0.0047
0.0958	0.1842	0.3168	0.2505	0.1193	0.2162	0.0149	0.0040	0.0040	0.0039
0.4192	0.1808	0.2690	0.1358	0.0531	0.0736	0.0266	0.0512	0.0187	0.0016
0.2219	0.1004	0.2203	0.1846	0.1697	0.1065	0.0072	0.0083	0.0083	0.0033
0.3386	0.8430	0.0659	0.3391	0.0602	0.0105	0.1278	0.1305	0.0021	0.0028
0.4044	0.3686	0.0668	0.0799	0.2600	0.0522	0.1114	0.0147	0.0338	0.0088
0.2792	0.4244	0.2155	0.0838	0.0050	0.1083	0.0294	0.0380	0.0340	0.0019
0.8072	0.2091	0.3733	0.2933	0.1824	0.0325	0.0012	0.0056	0.0001	0.0091
0.3044	0.1691	0.0060	0.1470	0.1065	0.0269	0.0563	0.0329	0.0187	0.0085
0.0180	0.0393	0.0160	0.0166	0.1259	0.0467	0.0495	0.0050	0.0269	0.0065
1.2975	0.2914	0.0806	0.1732	0.1515	0.0101	0.0725	0.0044	0.0127	0.0168
0.2155	0.3360	0.3278	0.1859	0.0700	0.0933	0.1409	0.0165	0.0346	0.0011
0.2442	0.0650	0.1894	0.0580	0.1894	0.0586	0.0094	0.0211	0.0316	0.0149
0.0994	0.3383	0.0721	0.0457	0.0038	0.1112	0.0863	0.0234	0.0041	0.0008
0.1367	0.3967	0.4161	0.0928	0.1199	0.0168	0.1475	0.0533	0.0433	0.0016
0.2678	0.4086	0.1029	0.0255	0.1323	0.1397	0.0908	0.0349	0.0338	0.0024
0.3387	0.3309	0.0352	0.0241	0.0427	0.0953	0.1016	0.0016	0.0155	0.0093

0.7073	0.0906	0.1742	0.1188	0.2350	0.0492	0.0927	0.0103	0.0028	0.0183
0.1400	0.3878	0.1428	0.0092	0.2850	0.0602	0.1361	0.0522	0.0173	0.0003
0.2028	0.0983	0.0459	0.0425	0.0538	0.0403	0.0515	0.0872	0.0296	0.0104
0.4346	0.6298	0.0261	0.0067	0.1009	0.0164	0.0198	0.0212	0.0079	0.0130
0.0946	0.2165	0.0240	0.3061	0.0951	0.1091	0.0999	0.0554	0.0044	0.0131
0.1458	0.1722	0.2643	0.1484	0.0433	0.1401	0.1798	0.0158	0.0057	0.0010
0.1620	0.4409	0.1228	0.0805	0.0139	0.0752	0.0600	0.0129	0.0136	0.0077
0.2706	0.0853	0.2041	0.1202	0.1007	0.0268	0.0690	0.0260	0.0465	0.0068
0.3715	0.2303	0.0078	0.1530	0.0568	0.1560	0.0888	0.0241	0.0088	0.0043
0.3285	0.3907	0.0189	0.2058	0.0870	0.0877	0.0833	0.0073	0.0060	0.0030
0.4921	0.9619	0.1394	0.0191	0.1257	0.0973	0.2726	0.0347	0.0688	0.0234
0.6100	0.4756	0.2916	0.1388	0.0323	0.0601	0.0452	0.0572	0.0089	0.0076
0.6748	0.4732	0.0346	0.1393	0.1264	0.0739	0.0459	0.0334	0.0857	0.0143
0.1440	0.1792	0.1650	0.1453	0.0065	0.0993	0.0491	0.0054	0.0506	0.0011
0.4373	0.3909	0.2079	0.0372	0.0458	0.2334	0.1253	0.0401	0.0052	0.0090
0.5371	0.1438	0.3036	0.2021	0.0492	0.0197	0.1440	0.0247	0.0806	0.0041
0.1363	0.2612	0.0239	0.1786	0.1000	0.0793	0.0623	0.0211	0.0373	0.0100
0.2840	0.3653	0.2532	0.0894	0.0728	0.0299	0.0529	0.0329	0.0214	0.0010
0.0161	0.1601	0.0462	0.0702	0.0166	0.1538	0.0555	0.0479	0.0200	0.0096
0.6633	0.0091	0.4865	0.0936	0.0899	0.0050	0.0234	0.0323	0.0098	0.0065
0.1451	0.0216	0.3011	0.0551	0.0701	0.0308	0.0519	0.0382	0.0548	0.0014
0.2446	0.0134	0.2505	0.0264	0.0988	0.0355	0.0641	0.0658	0.0354	0.0007
0.0096	0.3275	0.0055	0.0675	0.1646	0.1106	0.0365	0.0074	0.0341	0.0021
0.6568	0.0846	0.0991	0.0234	0.0632	0.1655	0.1464	0.0459	0.0011	0.0128
0.5019	0.2086	0.0892	0.0549	0.0618	0.0756	0.1290	0.0052	0.0100	0.0103
0.3027	0.2746	0.1580	0.1796	0.2395	0.1157	0.0242	0.0048	0.0688	0.0013
0.2467	0.3907	0.1191	0.1546	0.0117	0.0980	0.1337	0.0253	0.0256	0.0017
0.0654	0.2243	0.0711	0.2339	0.0905	0.0343	0.1021	0.0270	0.0353	0.0134
0.3866	0.4173	0.0158	0.1015	0.1827	0.1668	0.0365	0.0184	0.0132	0.0048
0.2151	0.1049	0.0894	0.1554	0.1095	0.0224	0.1026	0.0720	0.0368	0.0037
0.2760	0.7373	0.1513	0.0920	0.1003	0.1431	0.0256	0.0960	0.0002	0.0064
0.1122	0.5860	0.1366	0.2506	0.0990	0.1322	0.0809	0.0124	0.0159	0.0047
0.4162	0.0781	0.2672	0.0629	0.1049	0.0763	0.0129	0.0373	0.0334	0.0039

