

Mathematics

Grade 9

Government of Nepal

**Ministry of Education, Science and Technology
Curriculum Development Centre
Sanothimi, Bhaktapur**

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Send your comments and suggestions to:
Curriculum Development Centre
Phone: 01-6630588
Email: cde@ntc.net.np
Website: www.moecdce.gov.np

Preface

School education is the foundation for preparing the citizen who are loyal to the nation and nationality, committed to the norms and values of federal democratic republic, self-reliant and respecting the social and cultural diversity. It is also remarkable for developing a good moral character with the practical know-how of the use of ICT along with the application of scientific concept and positive thinking. It is also expected to prepare the citizens who are moral and ethical, disciplined, social and human value sensitive with the consciousness about the environmental conversation and sustainable development. Moreover, it should be helpful for developing the skills for solving the real life problems. This textbook 'Mathematics, Grade 9' is fully aligned with the intent carried out by the National Curriculum Framework for School Education, 2076 and is developed fully in accordance with the new Secondary Level Mathematics Curriculum (Grade 9-10), 2078.

This textbook, initially written by Mr. Ganesh Sapkota, Mr. Narahari Acharya, Mr. Sakti Prasad Acharya, Mr. Jagannath Adhikari. It has been translated by a team of experts Mr. Loknath Bhattarai, Dr. Eakraj Pandit, Ms. Nirmala Gautam, Ms. Pramila Bakhati, Mr. Ram Chandra Dhakal and Mr. Jagannath Adhikari. The contribution made by Director General Baikuntha Prasad Aryal, Prof. Dr. Ramjee Prasad Pandit, Mr. Keshab Raj Phulara, Mr. Ram Hada and Ms. Nirmala Gautam is remarkable in bringing the book in this form. The language of this book was edited by Mr. Nabin Kumar Khadka. The layout was designed by Mr. Nawaraj Puri. The Curriculum Development Centre extends sincere gratitude to all of them.

The textbook is a primary resource for classroom teaching. Considerable efforts have been made to make the book helpful in achieving the expected competencies of the curriculum. Curriculum Development Centre always welcomes constructive feedback for further betterment of its publications.

2079 B.S.

Curriculum Development Centre
Sanothimi, Bhaktapur

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1.0 Review

This kind of information was obtained while discussing with 4 students of grade 9 about the crops planted in their fields:

- (a) Crops planted in Anjal's field = Paddy, Wheat, Millet, Mustard, Peas, Mustardy
- (b) Crops planted in Bimala's field = Wheat, Buckwheat, Mustard, Tomato, Brinjal, Barley, Potato, Beans
- (c) Crops planted in Pemba's field = Buckwheat, Barley, Potato, Beans, Maize
- (d) Crops planted in Rambilas's field = Wheat, Mustard, Paddy, Maize, Banana, Sugarcane, Cabbage

Find the answers of the following questions based on the above information:

- (i) Write each of the above information by listing method.
- (ii) What types of sets are 'Crops planted in Anjal's field' and 'Crops planted in Pemba's field'?
- (iii) What types of sets are 'Crops planted in Bimala's field' and 'Crops planted in Rambilas's field'?
- (iv) What types of sets are 'Crops planted in Bimala's field' and 'Crops planted in Enjal's field'?
- (v) Construct the following subsets from crops planted in Rambilas's field. What types of sets are formed? Write by listing method:
 - Set of grains
 - Set of fruits
 - Set of vegetables

Discuss the above questions in group and present the conclusion in the class.

1.1 Operation of sets

Activity 1

Work in pairs. Collect different colours of flowers by each pairs and construct sets from those flowers based on their colours. For example:

Set of colours of flowers collected by Supriya (S) = {red, yellow, blue, pink }

Set of colours of flowers collected by Anjila (A) = {yellow, white, blue, purple, orange, red}

Construct the following sets based on the above sets:

- Set of colours of flowers collected by Supriya or Anjila or both of them,
- Set of flowers of common colours collected by Supriya and Anjila,
- Set of colours of flowers collected by Anjila only,
- Set of colours of flowers except the colours of flowers collected by Supriya.

What types of sets are constructed in this way?

1.1.1 Union of sets

In the above sets,

Set of colours of flowers collected by Supriya(S)

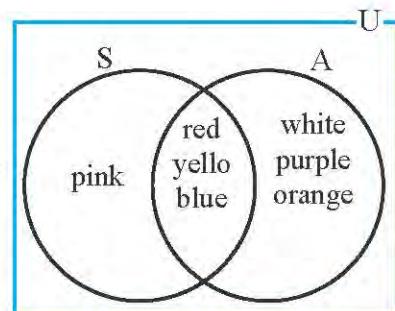
$$= \{ \text{red, yellow, blue, pink} \}$$

Set of colours of flowers collected by Anjila(A)

$$= \{ \text{yellow, white, blue, purple, orange, red} \}$$

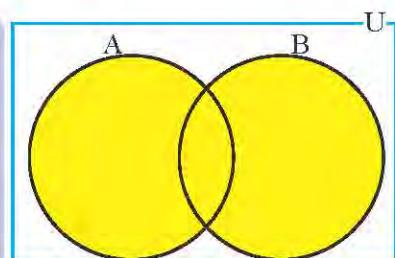
Set of colours of flowers collected by both

$$= \{ \text{red, yellow, blue, pink, white, purple, orange} \}$$

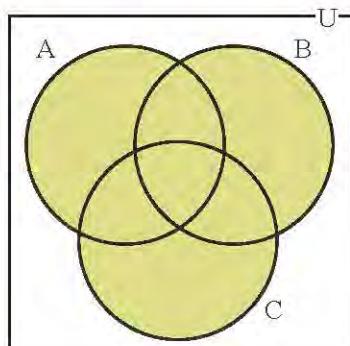


In this way, the set of colours of flowers collected by both of them is called the union of sets of colours of flowers collected by them separately.

If set A and B are the subsets of an universal set U, then the union of set A and B is denoted by $A \cup B$. ($A \cup B$) consists of all the elements of the set A or the set B. According to the set builder method, it is written as $A \cup B = \{x: x \in A \text{ or } x \in B\}$. In the given Venn diagram, the shaded portion represents the set $(A \cup B)$.



Similarly $(A \cup B \cup C)$ is written as $A \cup B \cup C = \{x: x \in A \text{ or } x \in B \text{ or } x \in C\}$. In the given Venn diagram the shaded portion represents the set $(A \cup B \cup C)$. While writing the elements in the union of sets we must write all the elements without repeating the common elements of those sets.



Example 1

If $P = \{\text{multiples of } 3 \text{ less than } 20\}$ and $Q = \{\text{multiples of } 2 \text{ less than } 20\}$, find $P \cup Q$ and present it in Venn diagram.

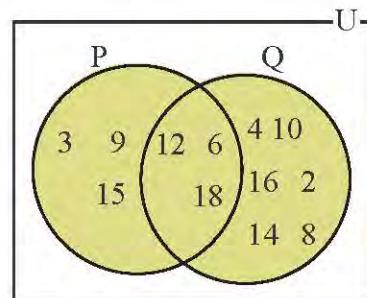
Solution

Here, $P = \{\text{multiples of } 3 \text{ less than } 20\}$

$$= \{3, 6, 9, 12, 15, 18\}$$

$Q = \{\text{multiples of } 2 \text{ less than } 20\}$

$$= \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$



Now, $P \cup Q$

$$= \{3, 6, 9, 12, 15, 18\} \cup \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

$$= \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$$

In the Venn diagram, shaded region represents $P \cup Q$.

Example 2

If $A = \{\text{wheat, buckwheat, mustard, tomato, brinjal, chilly, barley, potato, beans}\}$ and $B = \{\text{buckwheat, barley, potato, beans}\}$ find, $A \cup B$ and also present in Venn diagram.

Solution

Here, $A = \{\text{wheat, buckwheat, mustard, tomato, brinjal, chilly, barley, potato, beans}\}$

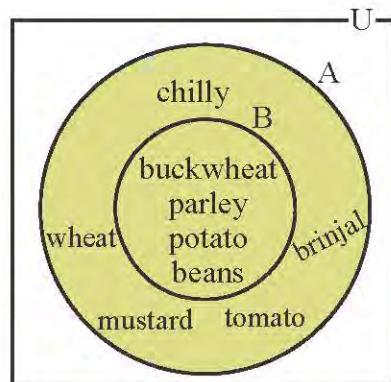
$$B = \{\text{buckwheat, barley, potato, beans}\}$$

Now, $A \cup B$

$$= \{\text{wheat, buckwheat, mustard, tomato, brinjal, chilly, barley, potato, beans}\} \\ \cup \{\text{buckwheat, barley, potato, beans}\}$$

$$= \{\text{wheat, buckwheat, mustard, tomato, brinjal, chilly, barley, potato, beans}\} = A$$

In the adjoining Venn diagram, the shaded region represents the set $A \cup B$



Activity 2

If two sets are disjoint, then in that condition, what will be their union? Discuss with each other and present by drawing Venn diagram.

Note: In two sets, if one is a subset of another then their union is the bigger one. If two sets are disjoint, then their union is formed by all the elements of both sets.

Example 3

If $P = \{\text{Even number less than } 10\}$ and $Q = \{\text{Odd number less than } 10\}$, find $P \cup Q$ and present it in Venn diagram.

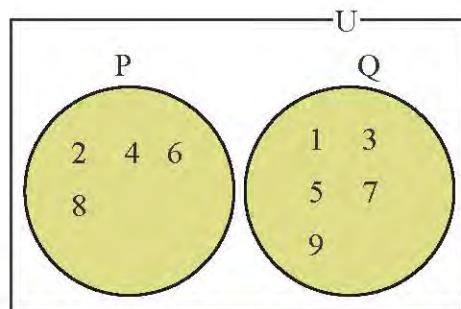
Solution

$$\begin{aligned} \text{Here, } P &= \{\text{even number less than } 10\} \\ &= \{2, 4, 6, 8\} \end{aligned}$$

$$\begin{aligned} Q &= \{\text{odd number less than } 10\} \\ &= \{1, 3, 5, 7, 9\} \end{aligned}$$

$$\begin{aligned} P \cup Q &= \{2, 4, 6, 8\} \cup \{1, 3, 5, 7, 9\} \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \end{aligned}$$

In the Venn diagram, shaded region represents $P \cup Q$.



Example 4

If $A = \{a, b, c, d, e\}$, $B = \{a, e, i, o, u\}$, $C = \{d, e, f, g\}$ find $A \cup B \cup C$. Also present it in Venn diagram.

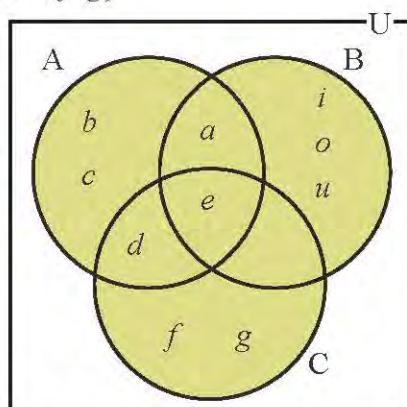
Solution

$$\text{Here, } A = \{a, b, c, d, e\}, B = \{a, e, i, o, u\}, C = \{d, e, f, g\}$$

Now, $(A \cup B \cup C)$

$$\begin{aligned} &= \{a, b, c, d, e\} \cup \{a, e, i, o, u\} \cup \{d, e, f, g\} \\ \therefore (A \cup B \cup C) &= \{a, b, c, d, e, f, g, i, o, u\} \end{aligned}$$

In the adjoining Venn diagram, shaded region represents $(A \cup B \cup C)$.



1.1.2 Intersection of sets

Among the students of grade 9 who like Science and Mathematics are as follows:

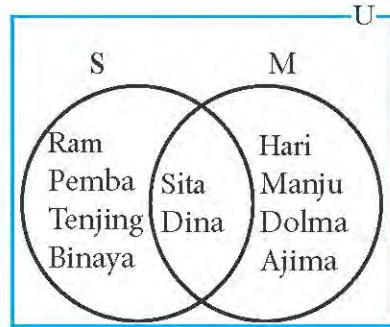
Set of students who like Science (S) = {Ram, Sita, Pemba, Tenjing, Dina, Binaya}

Set of students who like Mathematics (M) = {Hari, Sita, Manju, Dolma, Dina, Ajima}

Presenting the above sets in Venn diagram,

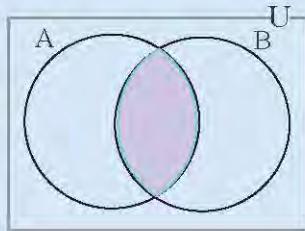
In the adjoining Venn diagram, the set of students who like both Science and Mathematics is

{Sita, Dina}.



It is the intersection of the set of students who like Science and the set of students who like Mathematics.

If A and B be are two non-empty sets, then the set containing the common elements of the set A and the set B is called the intersection of A and B. It is denoted by $A \cap B$. According to the set builder method, $A \cap B$ is written as $A \cap B = \{x : x \in A \text{ and } x \in B\}$. In the adjoining Venn diagram, the shaded region represents the intersection of the set A and B.



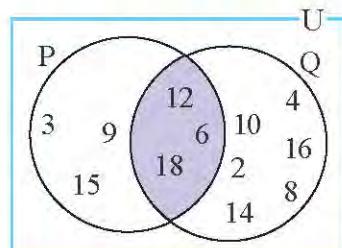
Example 5

If $P = \{\text{multiples of 3 less than 20}\}$ and $Q = \{\text{multiples of 2 less than 20}\}$, find $P \cap Q$ and present in Venn diagram.

Solution,

$$\begin{aligned} \text{Here, } P &= \{\text{multiples of 3 less than 20}\} \\ &= \{3, 6, 9, 12, 15, 18\} \end{aligned}$$

$$\begin{aligned} Q &= \{\text{multiples of 2 less than 20}\} \\ &= \{2, 4, 6, 8, 10, 12, 14, 16, 18\} \end{aligned}$$



Now $P \cap Q$

$$\begin{aligned} &= \{3, 6, 9, 12, 15, 18\} \cap \{2, 4, 6, 8, 10, 12, 14, 16, 18\} \\ &= \{6, 12, 18\} \end{aligned}$$

In the adjoining Venn diagram, the shaded region represents $P \cap Q$.

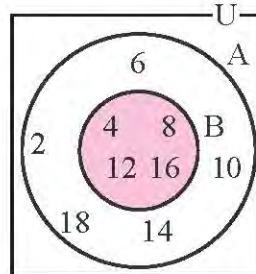
Example 6

If $A = \{\text{multiples of } 2 \text{ less than } 20\}$ and $B = \{\text{multiples of } 4 \text{ less than } 20\}$, find $A \cap B$ and present it in Venn diagram.

Solution,

$$\begin{aligned} \text{Here, } A &= \{\text{multiples of } 2 \text{ less than } 20\} \\ &= \{2, 4, 6, 8, 10, 12, 14, 16, 18\} \\ B &= \{\text{multiples of } 4 \text{ less than } 20\} = \{4, 8, 12, 16\} \end{aligned}$$

$$\begin{aligned} \text{Now } A \cap B &= \{2, 4, 6, 8, 10, 12, 14, 16, 18\} \cap \{4, 8, 12, 16\} \\ &= \{4, 8, 12, 16\} \end{aligned}$$



In the adjoining Venn diagram, shaded region represents $A \cap B$.

In two sets, if one is subset of another, then their intersection is the subset itself.

Activity 3

If two sets are disjoint what will be their intersection in that condition? Discuss in group and present in the class.

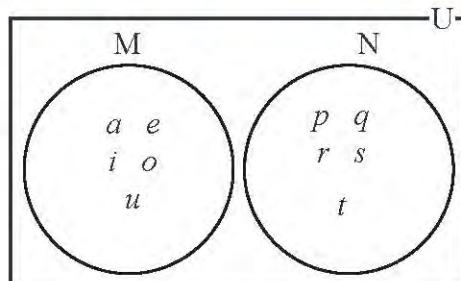
If two sets are disjoint then their intersection will be the empty set.

Example 7

If $M = \{a, e, i, o, u\}$ and $N = \{p, q, r, s, t\}$, find $M \cap N$ and present it in Venn diagram.

Solution,

$$\begin{aligned} \text{Here, } M &= \{a, e, i, o, u\}, N = \{p, q, r, s, t\} \\ \text{Now, } M \cap N &= \{a, e, i, o, u\} \cap \{p, q, r, s, t\} \\ \therefore M \cap N &= \{ \} \text{ or } \emptyset \end{aligned}$$



Shaded region is not in Venn diagram since there is no common element.

Example 8

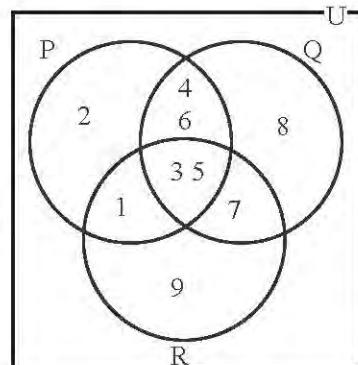
If $P = \{1, 2, 3, 4, 5, 6\}$, $Q = \{3, 4, 5, 6, 7, 8\}$ and $R = \{1, 3, 5, 7, 9\}$, find $P \cap Q \cap R$ and show it in a Venn diagram.

Solution,

Here, $P = \{1, 2, 3, 4, 5, 6\}$
 $Q = \{3, 4, 5, 6, 7, 8\}$
 $R = \{1, 3, 5, 7, 9\}$

Now $P \cap Q \cap R$

$$\begin{aligned} &= \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8\} \cap \\ &\quad \{1, 3, 5, 7, 9\} \\ &= \{3, 5\} \\ \therefore P \cap Q \cap R &= \{3, 5\} \end{aligned}$$

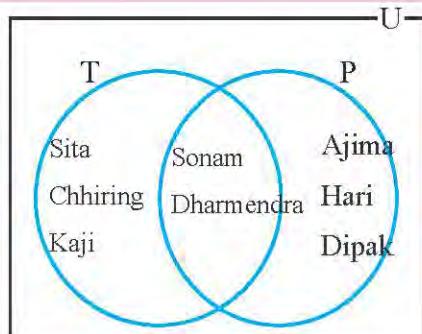


1.1.3 Difference of sets

Activity 4

Observe the given Venn diagram. Set of students who like to watch TV (T) = {Sita, Chhiring, Kaji, Sonam, Dharmendra}, Set of students who like to read newspaper (P) = {Ajima, Hari, Dipak, Sonam, Dharmendra}

Write the set of students who like to watch TV only and who like to read newspaper only.



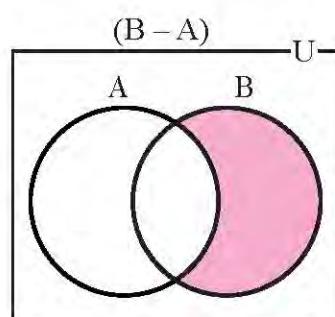
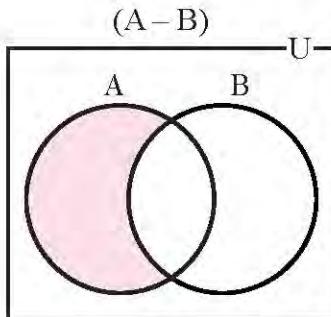
Here who like to read newspaper only do not like to watch TV. Thus it is called the difference of ‘set of students who like to read newspaper’ and ‘set of students who like to watch TV’.

Similarly, who like to watch TV only do not like to read newspaper. Thus, it is called the difference of the ‘set of students who like to watch TV’ and the ‘set of students who like to read newspaper’.

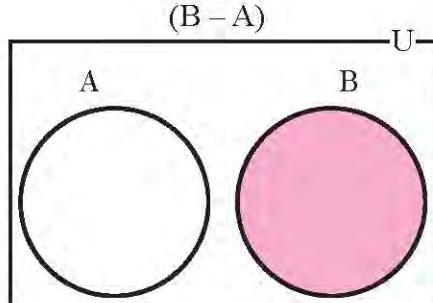
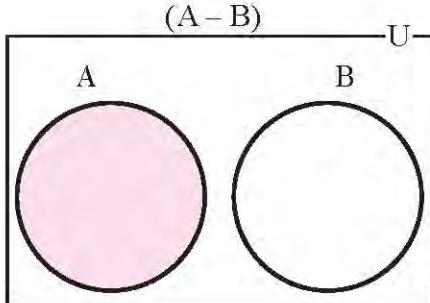
Set A and set B are the subsets of an universal set U. The set of elements which are in A but not in B is called the difference of A and B. It is written as $A - B$ and read as A difference B. Similarly, the set of elements which are in B only is called the difference of B and A. it is written as $B - A$ and read as B difference A. According to the set builder method it is written as $A - B = \{x : x \in A \text{ and } x \notin B\}$ and $B - A = \{x : x \in B \text{ and } x \notin A\}$

A – B and B – A in Venn diagram

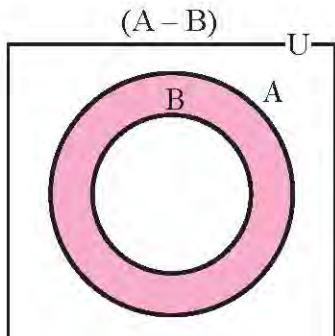
(a) If A and B are overlapping sets,



(b) If A and B are disjoint sets,



(c) If set B is the proper subset of set A,



- If A and B are equal sets, then $A - B = B - A = \emptyset$
- Difference of set A and empty set $A - \emptyset = A$
- Difference of empty set and set A, $\emptyset - A = \emptyset$
- If set A is the proper subset of set B, then $A - B = \emptyset$

Note: Union of $(A - B)$ and $(B - A)$ is called the symmetric difference of A and B. It is denoted by $A \Delta B$ and written as $A \Delta B = (A - B) \cup (B - A)$.

Example 9

From the given Venn diagram, write the following sets by listing method:

Ma) A

b) B

c) $A - B$

d) $B - A$

Solution,

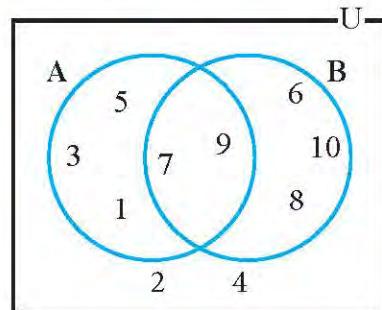
Here,

a) $A = \{1, 3, 5, 7, 9\}$

b) $B = \{6, 7, 8, 9, 10\}$

c) $A - B = \{1, 3, 5\}$

d) $B - A = \{6, 8, 10\}$



Example 10

If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8, 9, 10\}$ find $A - B$ and $B - A$. Also present them in separate Venn diagram by shading.

Solution,

Here, $A - B$

$$= \{1, 2, 3, 4, 5\} - \{4, 5, 6, 7, 8, 9, 10\}$$

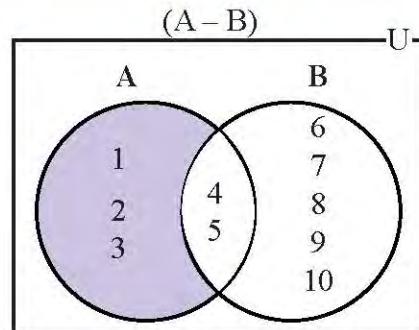
$$= \{1, 2, 3\}$$

and

$B - A$

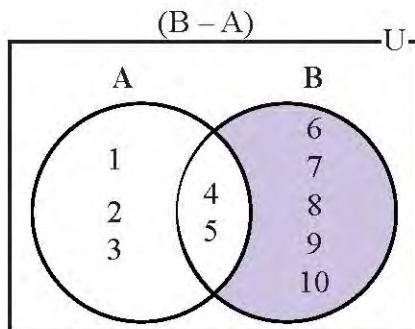
$$= \{4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5\}$$

$$= \{6, 7, 8, 9, 10\}$$



In the adjoining Venn diagram, $A - B$ and

$B - A$ are shown by shading.



Example 11

If $U = \{x: x \leq 30, x \text{ is a natural number}\}$,

$A = \{x: x \text{ is a natural number greater than } 15 \text{ and less than } 30\}$ and

$B = \{x: x \text{ is a natural number from } 1 \text{ to } 15\}$, find $A - B$ and $B - A$.

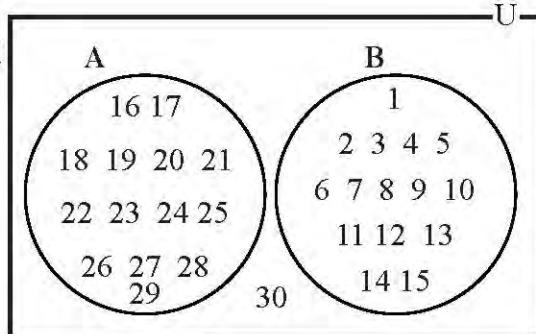
Solution,

Here, $U = \{1, 2, 3, 4, \dots, 28, 29, 30\}$

$A = \{16, 17, 18, \dots, 27, 28, 29\}$ and

$B = \{1, 2, 3, 4, \dots, 13, 14, 15\}$

Hence,



$$\begin{aligned}A - B &= \{16, 17, 18, \dots, 27, 28, 29\} - \{1, 2, 3, 4, \dots, 13, 14, 15\} \\&= \{16, 17, 18, \dots, 27, 28, 29\} = A \text{ and}\end{aligned}$$

$$\begin{aligned}B - A &= \{1, 2, 3, 4, \dots, 13, 14, 15\} - \{16, 17, 18, \dots, 27, 28, 29\} \\&= \{1, 2, 3, 4, \dots, 13, 14, 15\} = B\end{aligned}$$

If A and B are disjoint sets then, $A - B = A$ and $B - A = B$.

1.1.4 Complement of a set

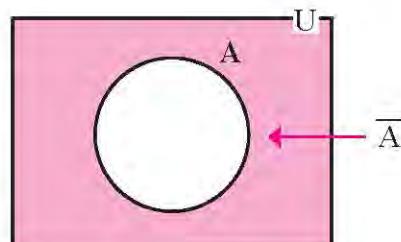
Let, A be a proper subset of an universal set U . Then the set of all elements of the set U , which do not belong to the set A is called the complement of the set A . It is denoted by A' or \bar{A} or A^c .

In other words, the difference between an universal set U and its subset A is called the complement of the set A . Symbolically it is written as $\bar{A} = U - A$.

It is written in set builder form as:

$$\bar{A} = \{x: x \in U \text{ but } x \notin A\}.$$

\bar{A} can be shown by shading in Venn diagram as in the adjoining figure.



The union of A and \overline{A} is U . Thus, A and \overline{A} are said to be compliment to each other. Also, $A \cup \overline{A} = U$ and $\overline{\overline{A}} = A$.

What will be $A \cap \overline{A}$? Discuss in group and write.

Example 12

If, $U = \{1, 2, 3, \dots, 18, 19, 20\}$, $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ and $B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, then list the elements of the following sets:

- (a) \overline{A} (b) \overline{B} (c) $\overline{A} \cup \overline{B}$ (d) $\overline{A} \cap \overline{B}$

Solution,

Here, $U = \{1, 2, 3, \dots, 18, 19, 20\}$

$$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

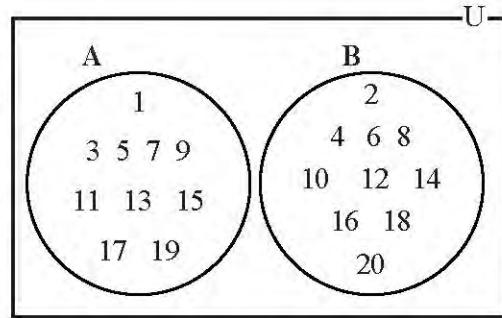
$$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$\begin{aligned} \text{(a)} \quad \overline{A} &= U - A \\ &= \{1, 2, 3, \dots, 18, 19, 20\} - \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \\ &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \overline{B} &= U - B \\ &= \{1, 2, 3, \dots, 18, 19, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \\ &= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \overline{A} \cup \overline{B} &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cup \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \\ &= \{1, 2, 3, 4, \dots, 18, 19, 20\} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \overline{A} \cap \overline{B} &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cap \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \\ &= \{ \} \end{aligned}$$



Example 13

If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 3, 5, 7, 9\}$, write the elements of the following sets by listing method:

(a) \bar{A}

(b) $A \cup \bar{A}$

(c) $A \cap \bar{A}$

(d) $\bar{\bar{A}}$

Solution,

Here, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and

$$A = \{1, 3, 5, 7, 9\}$$

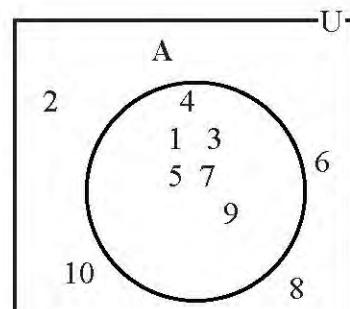
a) $\bar{A} = U - A$

$$\begin{aligned} &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

b) $A \cup \bar{A} = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U$

c) $A \cap \bar{A} = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\} = \emptyset$

d) $\bar{\bar{A}} = U - \bar{A}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\}$
 $= \{1, 3, 5, 7, 9\} = A$



Example 14

If, $U = \{a, b, c, d, e, f, g, h, i, j, k\}$, $A = \{a, b, c, d, i, j\}$, $B = \{c, d, e, f, g, h, i\}$ and $C = \{d, e, f, g, h, i, j, k\}$, list out the elements of the following sets:

(a) $\overline{(A \cup B) \cap C}$

(b) $\bar{A} \cup \bar{B}$

(c) $A \cap \bar{B}$

(d) $(A \cap \bar{C}) \cup \bar{A}$

(e) $(\bar{A} \cap C) \cup \bar{B}$

Solution,

(a) $\overline{(A \cup B) \cap C}$

$$\begin{aligned} (A \cup B) &= \{a, b, c, d, e, f, g, h, i, j\} \cup \{c, d, e, f, g, h, i\} \\ &= \{a, b, c, d, e, f, g, h, i, j\} \end{aligned}$$

$$\begin{aligned} (A \cup B) \cap C &= \{a, b, c, d, e, f, g, h, i, j\} \cap \{d, e, f, g, h, i, j, k\} \\ &= \{d, e, f, g, h, i, j\} \end{aligned}$$

Again, $\overline{(A \cup B) \cap C}$

$$\begin{aligned}
 &= U - \{(A \cup B) \cap C\} \\
 &= \{a, b, c, d, e, f, g, h, i, j, k\} - \{d, e, f, g, h, i, j\} \\
 &= \{a, b, c, k\}
 \end{aligned}$$

(b) $\bar{A} \cup \bar{B}$

$$\begin{aligned}
 \bar{A} &= U - A \\
 &= \{a, b, c, d, e, f, g, h, i, j, k\} - \{a, b, c, d, i, j\} \\
 &= \{e, f, g, h, k\}
 \end{aligned}$$

$$\begin{aligned}
 \bar{B} &= U - B \\
 &= \{a, b, c, d, e, f, g, h, i, j, k\} - \{c, d, e, f, g, h, i\} \\
 &= \{a, b, j, k\}
 \end{aligned}$$

Again, $\bar{A} \cup \bar{B}$

$$\begin{aligned}
 &= \{e, f, g, h, k\} \cup \{a, b, j, k\} \\
 &= \{a, b, e, f, g, h, j, k\}
 \end{aligned}$$

(c) $A \cap \bar{B}$

$$\begin{aligned}
 &= \{a, b, c, d, i, j\} \cap \{a, b, j, k\} \\
 &= \{a, b, j\}
 \end{aligned}$$

(d) $\bar{C} = U - C$

$$\begin{aligned}
 &= \{a, b, c, d, e, f, g, h, i, j, k\} - \{d, e, f, g, h, i, j, k\} \\
 &= \{a, b, c\}
 \end{aligned}$$

Now, $A \cap \bar{C}$

$$\begin{aligned}
 &= \{a, b, c, d, i, j\} \cap \{a, b, c\} \\
 &= \{a, b, c\}
 \end{aligned}$$

Again, $(A \cap \bar{C}) \cup \bar{A}$

$$\begin{aligned}
 &= \{a, b, c\} \cup \{e, f, g, h, k\} \\
 &= \{a, b, c, e, f, g, h, k\}
 \end{aligned}$$

(e) $(\bar{A} \cap C) \cup \bar{B}$

$$\begin{aligned}
 &= \{e, f, g, h, k\} \cap \{d, e, f, g, h, i, j, k\} \\
 &= \{e, f, g, h, k\}
 \end{aligned}$$

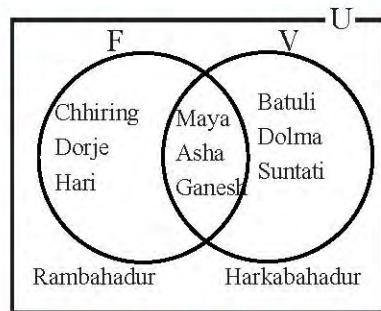
$(\bar{A} \cap C) \cup \bar{B}$

$$\begin{aligned}
 &= \{e, f, g, h, k\} \cup \{a, b, j, k\} \\
 &= \{a, b, e, f, g, h, j, k\}
 \end{aligned}$$

Exercise 1

- 1.** In the given Venn diagram, F represents the set of students who like football and V represents the set of students who like volleyball. Write the elements of the following sets by listing method:

- (a) F (b) V (c) $F \cup V$
 (d) $F \cap V$ (e) U



- 2.** If $U = \{a, b, c, d, e, f, g, h, i, j, k\}$, $A = \{a, c, e, f, g, i, k\}$, $B = \{b, d, i, j, k, h\}$ find the following sets and also present in a separate Venn diagram:

- (a) $(A \cap B)$ (b) $(B \cup A)$ (c) $A - B$ (d) $B - A$

- 3.** If, $U = \{x: x \text{ is a whole number from 1 to } 30\}$, $A = \{x: x \text{ is a multiple of 3 from 1 to } 30\}$, $B = \{x: x \text{ is a multiple of 4 from 1 to } 30\}$ and $C = \{x: x \text{ is a multiple of 5 from 1 to } 30\}$, write the following sets by listing method and present them in a Venn diagram:

- (a) $(A - B)$ (b) $(B - A)$ (c) $(A - C)$ (d) $(B - C)$
 (e) $(A \cup B)$ (f) $A \cup B \cup C$ (g) $A \cap B \cap C$ (h) $(\overline{B \cup C})$

- 4.** If $U = \{a, b, c, d, e, f, g, h, i, j, k\}$, $A = \{a, c, d, f\}$, $B = \{g, h, i\}$ then,

- (a) Construct the following sets:

- (i) \overline{A} (ii) \overline{B} (iii) $\overline{A} \cup \overline{B}$
 (iv) $\overline{A} \cap \overline{B}$ (v) $\overline{A \cup B}$ (vi) $(\overline{A} \cap \overline{B})$

- (b) Which of the sets in question A are equal? Find.

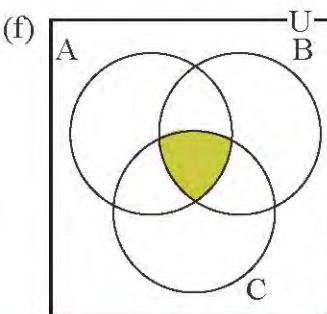
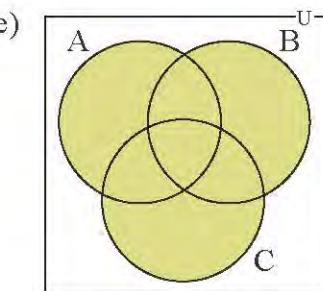
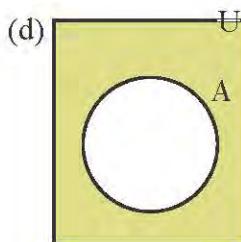
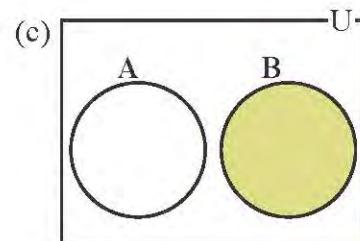
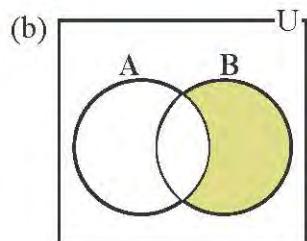
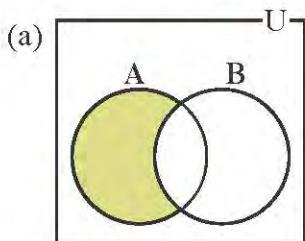
- 5.** If $U = \{\text{set of actual numbers from 1 to } 12\}$, $E = \{\text{set of even numbers from 1 to } 12\}$, $O = \{\text{set of odd numbers from 1 to } 12\}$ and $P = \{\text{set of prime numbers from 1 to } 12\}$, find the following sets and also present them in Venn diagram:

- (a) \overline{E} (b) \overline{O} (c) \overline{P} (d) $(\overline{E} \cup \overline{P})$
 (e) $\overline{P \cap Q}$ (f) $\overline{P - O}$ (g) P (h) $(\overline{E} \cup \overline{O} \cup \overline{P})$
 (i) $(\overline{E} \cap \overline{O} \cap \overline{P})$ (j) $(\overline{E} \cup \overline{P}) - (P \cap O)$ (k) $\overline{P} \cup (E \cap O)$

- 6.** If $U = \{m, n, o, p, q, r, s, t, u, v\}$, $A = \{q, r, s, t, u, v\}$, $B = \{n, o, p, q, r\}$ and $C = \{m, u, s, t, q, r\}$ find the sets of the following relations. Also present them by drawing separate Venn diagram:

- (a) $(A \cap B)$ (b) $(A \cup B) \cap C$ (c) $(A \cup B \cup C)$ (d) $(A \cap B \cap C)$
 (e) $(A - B)$ (f) $(\overline{A \cup B \cup C})$ (g) $\overline{A \cap B}$ (h) \overline{A} (i) $(A \cap C) \cup B$

7. From the following Venn diagram, write the sets represented by the shaded region in set notation:



8. If P and Q are the intersecting subsets of an universal set U, show the following sets by drawing Venn diagram:

(a) $P - Q$ (b) $Q - P$ (c) $(P - Q) \cup P$ (d) $P \cap (Q - P)$

9. Write an universal set U and two subsets X and Y. After that, write the elements of following sets by listing method:

(a) $(\bar{X} \cup \bar{Y})$ (b) $(X \cap \bar{Y})$ (c) $\bar{\bar{X}}$ (d) $\bar{X} \cap \bar{Y}$

10. Construct the following sets from the students of your class:

- (a) set of all students
(b) set of girls
(c) set of boys

Which of the above set is universal set and which sets are the subsets of the universal set? Write with appropriate symbol. After that find the compliment of all those sets.

Answers

1. (a) {Chhiring, Dorje, Hari, Maya, Aasha, Ganesh}
(b) {Batuli, Dolma, Suntali, Maya, Aasha, Ganesh}
(c) {Chhiring, Dorje, Hari, Batuli, Dolma, Suntali, Maya, Aasha, Ganesh}
(d) {Maya, Aasha, Ganesh}
(e) {Chhiring, Dorje, Hari, Batuli, Dolma, Suntali, Maya, Aasha, Ganesh, Rajbahadur, Harkabahadur}
2. (a) { i, k } (b) U (c) { a, c, e, f, g } (d) { b, d, j, h }
(e) \varnothing and show the Venn diagrams to your teacher.
3. (a) {3, 6, 9, 15, 18, 21, 27, 30} (b) {4, 8, 16, 20, 28}
(c) {3, 6, 9, 12, 18, 21, 24, 27} (d) {4, 8, 12, 16, 24, 28}
(e) {3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21, 24, 27, 28, 30}, and show Venn diagrams to your teacher.
(f) {3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30}
(g) { }
(h) {1, 2, 3, 4, 6, 7, 9, 11, 13, 14, 17, 18, 19, 21, 22, 23, 26, 27, 29}
4. (a) i) { b, e, g, h, i, j, k } (ii) { a, b, c, d, e, f, j, k }
iii) { $a, b, c, d, e, f, g, h, i, j, k$ } (iv) { b, e, j, k }
(v) { b, e, j, k } (vi) U
(b) Show to your teacher.
5. (a) {1, 3, 5, 7, 9, 11} (b) {2, 4, 6, 8, 10, 12}
(c) {1, 4, 6, 8, 9, 10, 12} (d) {1, 9} (e) U (f) Q
(g) {2, 3, 5, 7, 11} (h) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
(i) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} (j) {1, 9} (k) {1, 4, 6, 8, 9, 10}
6. (a) { q, r } (b) { u, s, t, r } (c) { $m, n, o, p, q, r, s, t, u, v$ }
(d) { q, r } (e) { s, t, u, v } (f) { } (g) { m, n, o, p, s, t, u, v }
(h) { m, n, o, p } (i) { q, r, s, t, u, n, o, p }
7. (a) $(A - B)$ (b) $(B - A)$ (c) B or $B - A$
(d) \overline{A} if $U = A$ 8 - 10. Show to your teacher.

1.2 Cardinality of sets

The total number of elements of a set is called the cardinality of that set. For example, in a set $A = \{m, a, t, h\}$, the total number of element is 4. So, the cardinality of the set A is 4. Symbolically it is written as $n(A) = 4$.

Cardinality of an empty set is always zero. For example, $B = \{\text{set of students of grade 9 having age less than 5 years}\}$. In the set B, there is no element or the cardinality is zero. So it is written as $n(B) = 0$.

Activity 1

(a) If,

$$U = \{\text{set of SAARC countries}\}$$

$$A = \{\text{Nepal, Bharat, Pakistan, Afghanistan}\}$$

$$B = \{\text{Bhutan, Bangladesh, Shreelanka, Maldives}\}$$

what will be the value of $n(A \cup B)$

Here, there is no common element in set A and set

B. So, in these disjoint sets $n(A \cup B) = 8$.

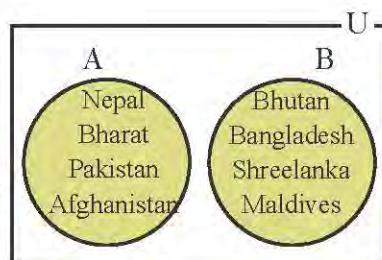
$$\text{Again, } n(A) = 4, n(B) = 4, n(A \cup B) = n(A) + n(B) = 4 + 4 = 8.$$

(b) If, $A = \{a, b, c, d, e\}$, $B = \{d, e, f, g\}$

$$(A \cup B) = \{a, b, c, d, e, f, g\} \text{ and } (A \cap B) = \{d, e\}$$

$$\text{So, } n(A \cup B) = 7 \text{ and } n(A \cap B) = 2$$

\therefore If two sets A and B are intersecting sets, then
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.



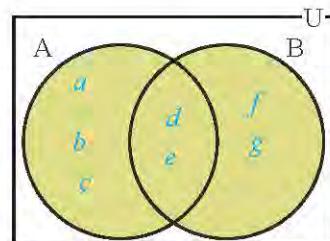
Activity 2

Observe the given Venn diagram. Find the cardinality of the following sets and present in class room.

- a) $n(A)$ b) $n(B)$ c) $n(C)$
d) $n(A \cup B \cup C)$ e) $n(A \cap B \cap C)$

Here, from the Venn diagram,

- $A = \{a, b, c, d, e\}$ $\therefore n(A) = 5$
 $B = \{a, e, i, o, u\}$ $\therefore n(B) = 5$
 $C = \{d, e, f, i\}$ $\therefore n(C) = 4$



$$\begin{array}{lll} (A \cup B \cup C) & = \{a, b, c, d, e, f, i, o, u\} & \therefore n(A \cup B \cup C) = 9 \\ (\overline{A \cup B \cup C}) & = \{g, h\} & \therefore n(\overline{A \cup B \cup C}) = 2 \end{array}$$

Activity 3

If M and N are any two overlapping sets, then the number of elements of the set M only is denoted by $n_o(M)$ and N only is denoted by $n_o(N)$.

From the given Venn diagram, $n_o(M) = 3$

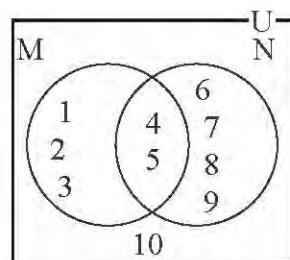
Thus, $n_o(M) = n_o(M - N)$ It can also be written as

$$n_o(M) = n(M) - n(M \cap N)$$

$$\text{Similarly, } n_o(N) = 4$$

$$\text{Similarly } n_o(N) = n(N - M) = n(N) - n(M \cap N)$$

$$\text{Here, } n(M) = 5, n(N) = 6, n_o(M) = 3, n_o(N) = 4, \\ n(M \cup N) = 9 / n(M \cap N) = 2 \text{ x'G5 .}$$



If a set is proper subset of another set, or if $A \subset B$, then $n(A \cup B) = n(B)$ and $n(A \cap B) = n(A)$.

Example 1

If, $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, prove the following:

$$(a) n(A \cup B) = n(B) \quad (b) n(A \cap B) = n(A)$$

Solution,

Here, $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$

$$\begin{aligned} (a) \quad n(A \cup B) &= \{1, 2, 3\} \cup \{1, 2, 3, 4, 5\} \\ &= \{1, 2, 3, 4, 5\} \\ \therefore n(A \cup B) &= 5 \\ \therefore n(A \cup B) &= n(B) \end{aligned}$$

$$\begin{aligned} (b) \quad n(A \cap B) &= \{1, 2, 3\} \cap \{1, 2, 3, 4, 5\} \\ &= \{1, 2, 3\} \\ \therefore n(A \cap B) &= 3 \\ n(A) &= \{1, 2, 3\} \\ \therefore n(A \cap B) &= n(A) \end{aligned}$$

Method of finding cardinality of set:

Observe the following Venn diagram and answer the questions given below:

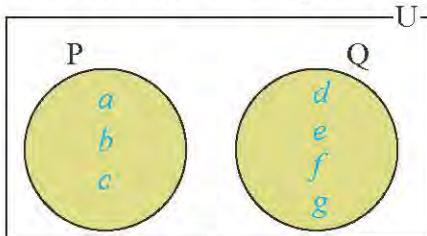


Fig. 1

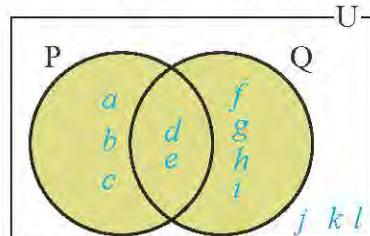


Fig. 2

- a) $n(U) = ?$ b) $n(P) = ?$ c) $n(Q) = ?$ d) $n(P \cup Q) = ?$
 e) $n(P \cap Q) = ?$ f) $n(\overline{P \cup Q}) = ?$

Here in the fig.1, P and Q are disjoint sets.

$$U = \{a, b, c, d, e, f, g\}$$

$$n(U) = 7$$

$$P = \{a, b, c\}$$

$$n(P) = 3$$

$$Q = \{d, e, f, g\}$$

$$n(Q) = 4$$

$$(P \cup Q) = \{a, b, c, d, e, f, g\}$$

$$n(P \cup Q) = 7$$

$$(P \cap Q) = \{\}$$

$$n(P \cap Q) = 0$$

$$\overline{(P \cup Q)} = \{\}$$

$$n(\overline{P \cup Q}) = 0$$

$$\therefore n(U) = n(P \cup Q)$$

Here, in the fig.2 P and Q are overlapping sets.

$$U = \{a, b, c, d, e, f, g, h, i, j, k, l\}$$

$$n(U) = 12$$

$$P = \{a, b, c, d, e\}$$

$$n(P) = 5$$

$$Q = \{d, e, f, g, h, i\}$$

$$n(Q) = 6$$

$$(P \cup Q) = \{a, b, c, d, e, f, g, h, i\}$$

$$n(P \cup Q) = 9$$

$$(P \cap Q) = \{d, e\}$$

$$n(P \cap Q) = 2$$

$$\overline{(P \cup Q)} = \{j, k, l\}$$

$$n(\overline{P \cup Q}) = 3$$

$$\therefore n(U) = n(P \cup Q) + n(\overline{P \cup Q})$$

Cardinality of sets can be written in the following formulae:

- (a) If A and B are any two disjoint sets, then, $n(A \cup B) = n(A) + n(B)$
- (b) If A and B are any two overlapping sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ Or, $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$
- (c) If U contains only the elements of set A and B, then $n(U) = n(A \cup B)$.
- (d) If U contains some more elements other than the elements of set A and set B, then $n(\overline{A \cup B}) = n(U) - n(A \cup B)$.
Similarly, $n(U) = n(A \cup B) + n(\overline{A \cup B})$.
- (e) In overlapping sets A and B, $n_o(A) = n(A) - n(A \cap B)$ and $n_o(B) = n(B) - n(A \cap B)$

Example 2

Observe the adjoining Venn diagram and answer the questions given below:

- (a) $n(A)$
- (b) $n(B)$
- (c) $n(C)$
- (d) $n(A \cap B)$
- (e) $n(A \cap B \cap C)$
- (f) $n(A \cup B \cup C)$
- (g) $n(\overline{A \cup B \cup C})$
- (h) $n_o(A)$
- (i) $n_o(A \cap B)$
- (j) $n(A - B)$

Solution,

From the given Venn diagram,

$$(a) A = \{a, b, c, d, e\}$$

$$\therefore n(A) = 5$$

$$(b) B = \{c, d, g, h, j, k\}$$

$$\therefore n(B) = 6$$

$$(c) C = \{e, f, g, h, i\}$$

$$\therefore n(C) = 6$$

$$(d) A \cap B = \{c, d\}$$

$$\therefore n(A \cap B) = 2$$

$$(e) A \cap B \cap C = \{d\}$$

$$\therefore n(A \cap B \cap C) = 1$$

$$(f) (A \cup B \cup C) = \{a, b, c, d, e, f, g, h, i, j, k\}$$

$$\therefore n(A \cup B \cup C) = 11$$

$$(g) (\overline{A \cup B \cup C}) = \{l, m\}$$

$$\therefore n(\overline{A \cup B \cup C}) = 2$$

$$(h) A = \{a, b\}$$

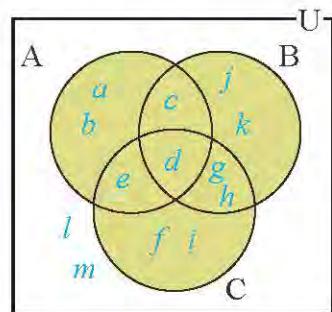
$$\therefore n_o(A) = 2$$

$$(i) A \cap B = \{c\}$$

$$\therefore n_o(A \cap B) = 1$$

$$(j) A - B = \{a, b, e\}$$

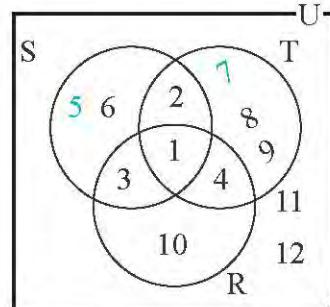
$$\therefore n(A - B) = 3$$



Exercise 1.2

1.(a) Observe the given Venn diagram and find the cardinality of the following sets:

- | | | |
|--------------------------|-------------------------------------|--------------------------|
| (a) $n(S)$ | (b) $n(T)$ | (c) $n(U)$ |
| (d) $n(S \cap T)$ | (e) $n(R \cup T)$ | (f) $n(S \cap R \cap T)$ |
| (g) $n(S \cup R \cup T)$ | (h) $n(\overline{S \cup R \cup T})$ | |
| (i) $n_o(S)$ | (j) $n_o(S \cap R)$ | (k) $n(\overline{R})$ |



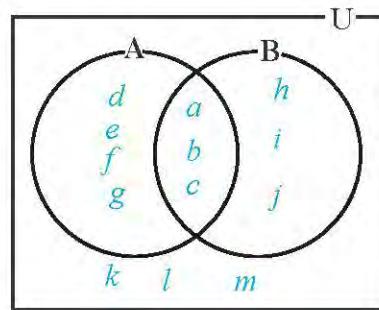
- (b) If, A and B are any two disjoint sets, $n(A) = 30$, $n(B) = 35$, find the value of $n(A \cup B)$.

2.(a) From the given Venn diagram, find the cardinality of the following sets:

- | | | |
|-------------------|--------------|-------------------|
| (a) $n(A)$ | (b) $n(B)$ | (c) $n(A \cup B)$ |
| (d) $n(A \cap B)$ | (e) $n_o(A)$ | (f) $n_o(B)$ |

- (b) If, $U = \{a, b, c, d, e, f, g\}$, $A = \{c, d, e, f\}$, $B = \{a, b, e, f\}$ and $C = \{d, e, f, g\}$, find the cardinality of the following sets:

- | | |
|---------------------------------|------------------------------------|
| (a) $n(A - B)$ | (b) $n(B - C)$ |
| (c) $n(\overline{A - C})$ | (d) $n(\overline{A})$ |
| (e) $n(A \cup B)$ | (f) $n\{(A \cup B) - (A \cap B)\}$ |
| (g) $n\{(A - B) \cup (B - A)\}$ | |



3.(a) If, $U = \{\text{set of natural numbers less than } 20\}$, $A = \{\text{set of even numbers less than } 20\}$, $B = \{\text{set of prime numbers less than } 20\}$ and $C = \{\text{set of square numbers less than } 20\}$, find the cardinality of the following sets:

- | | | | |
|-----------------------|-------------------|------------------------------|----------------|
| (a) $n(U)$ | (b) $n(C)$ | (c) $n(A \cap B)$ | (d) $n(B - C)$ |
| (e) $n(\overline{A})$ | (f) $n(A \cup C)$ | (g) $n(\overline{A \cap B})$ | |

- (b) If, $U = \{x: x \text{ is a natural number less than } 20\}$

$A = \{y: y \text{ is a prime number}\}$, $B = \{z: z \text{ is a factor of } 18\}$ and

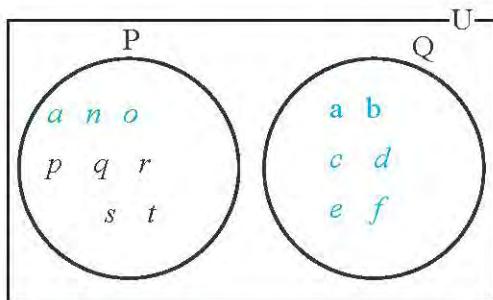
$C = \{p: p \text{ is a multiple of } 3 \text{ less than } 20\}$. Present the following sets by drawing separate Venn diagrams and find their cardinality:

- | | | |
|--------------------------|-------------------|--------------------------|
| (a) $n(A \cup B)$ | (b) $n(B \cup C)$ | (c) $n(A \cup B \cup C)$ |
| (d) $n(A \cap B \cap C)$ | (e) $n_o(A)$ | (f) $n_o(A - B)$ |

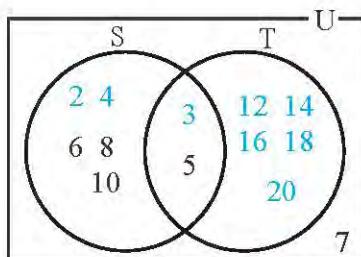
(g) $n(\overline{A \cap B})$

(h) $n(\overline{C})$

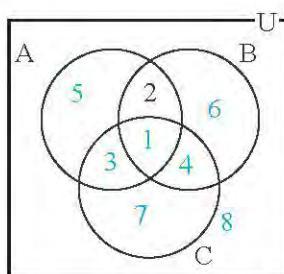
4. From the given Venn diagram, find the cardinality of the following sets:
- (a) $n(P \cup Q)$ (b) $n(P \cap Q)$ (c) $n(\overline{P \cup Q})$ (d) $n(\overline{P} \cap \overline{Q})$



5. Fund the cardinality of the following sets using the given Venn diagram.
- (a) $n(S)$ (b) $n(T)$ (c) $n(S \cap T)$ (d) $n(S \cup T)$ (e) $n_o(T)$
(f) $n_o(S)$ (g) $n(\overline{T})$ (h) $n(\overline{S \cup T})$ (i) $n(\overline{S \cap T})$ (j) $n(U)$

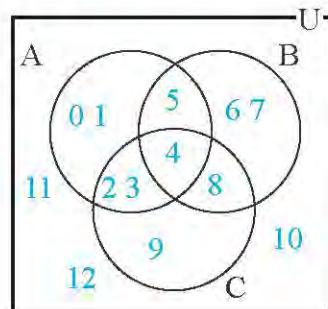


6. Observe the given Venn diagram and find the cardinality of the following sets:
- (a) $n(A)$ (b) $n(B)$ (c) $n(\overline{C})$ (d) $n(A \cup B)$
(e) $n(A \cup B \cup C)$ (f) $n(A \cap B \cap C)$ (g) $n(\overline{A \cup B \cup C})$ (h) $n_o(A)$
(i) $n_o(C)$ (j) $n(U)$



7. Verify the following relation from the adjoining Venn diagram:

- $n(\overline{A}) = n(U - A)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n_o(A) = n(A - B)$
- $n(A \cup B) = n_o(A) + n(B)$
- $n(A \cup C) = n_o(A) + n(C)$
- $n(B \cup C) = n_o(B) + n(C)$
- $n(\overline{(A \cup B \cup C)}) = U - (A \cup B \cup C)$



Project Work

- Ask at least 20 people which of the fruit either apples or orange they like. Denote the set of people who like apple by A and the set of people who like orange by O. According to their responses, write the set of people who like apple, who like orange, who like both fruits, who like apple only, who like orange only, who don't like apple and orange by listing method. From the obtained information construct the following sets. Also present them in a separate Venn diagram:
 - A
 - O
 - $A \cup O$
 - \overline{A}
 - $A - O$
 - $O - A$
 - $(O - A) \cup (A - O)$
- Ask all the students in your class which of football, cricket and basketball they like to play. Denote the set of students who like to play football by F, cricket by C and basketball by B. According to their responses, find the set of students who like to play football, who like to play cricket, who like to play basketball, who like to play any two sports, who like to play all three sports, who like to play only one sport and who do not like to play any of them by listing method. Find the cardinality of the following sets and also present them by drawing separate Venn diagram:
 - $n(F)$
 - $n(C)$
 - $n(B)$
 - $n(F \cap B)$
 - $n(B \cap C)$
 - $n(F \cap C \cap B)$
 - $n(F \cup C \cup B)$
 - $n(\overline{F \cup B})$
 - $n(\overline{B \cup C})$
 - $n(\overline{C})$
 - $n_o(F)$
 - $n_o(F \cap C)$
 - $n(F - B)$

Answers

1. (a) (i) 5 (ii) 6 (iii) 12 (iv) 2 (v) 8 (vi) 1
 (vii) 10 (ix) 2 (x) 2 (xi) 1 (xii) 8 (xiii) 65
2. (a) (i) 6 (ii) 5 (iii) 9 (iv) 2 (v) 4 (vi) 3
 (b) (i) 2 (ii) 2 (iii) 1 (iv) 3 (v) 1 (vi) 4 (vii) 4
3. (a) (i) 19 (ii) 4 (iii) 1 (iv) 8 (v) 10 (vi) 12 (vii) 18
 (b) (i) 12 (ii) 8 (iii) 14 (iv) 1 (v) 6 (vi) 6 (vii) 12 (viii) 8
4. (a) 14 (b) { } or \emptyset (c) { } or \emptyset (d) 14
5. (a) 7 (b) 7 (c) 2 (d) 12 (e) 12 (f) 5
 (g) 5 (h) 6 (i) 1 (j) 11 (k) 13
6. (a) 4 (b) 4 (c) 4 (d) 6 (e) 7 (f) 1
 (g) 1 (h) 1 (i) 1 (j) 8
7. Show to your teacher.

2.0 Review

Divide the students into appropriate number of groups and discuss the conditions given below.

- (a) Bishal received his monthly salary after deducting 1%.
- (b) When Kamala bought a mobile, she paid by adding 13 % in the price of mobile.
- (c) A headteacher received Rs. 510 out of Rs. 600 as an allowance per meeting of a program.
- (d) Shakti paid Rs. 300 to renew the bluebook of his motorcycle, registered at transportation Management Service Office, Bagmati province for the fiscal year 2078/2079.

In the above cases, the amounts added and deducted are the tax amount. Different tax rates different headings are mentioned in the law unde.

2.1 Tax

Activity 1

Discuss among your friends the following questions based on the tax provisions included in the budget statement presented at the beginning of each fiscal year in our country.

- (a) How does the government manage the administrative expenses and the development expenses of the state?
- (b) How does the government manage the salary allowance given to the civil servants?
- (c) You may have heard that the budget for the construction and maintenance of drinking water tanks, upgrading and development works such as skilled-based program etc. is being disbursed. Where do such expenses come from?

Tax is a compulsory payment that a person, firm or company has to pay to the government according to law. Tax is a main source of income of the state. The government conducts regular contingency and development Activity of the country according to the income received through this tax. So we all have to pay taxes. Paying tax is fulfilling the duty of a good citizen. Among the taxes that are in vogue in Nepal are vehicle tax, the customs tax, house-rent tax, income tax, value added tax, property tax, social security tax etc. Tax is expressed in percentage (%). For example, students going to study aboard has to pay 1 % tax on exchange money when they take money exchange facility.

2.1.1 Income tax

Activity 2

Discuss in a group with your peers regarding the income tax ceiling and questions given below. Then, present the findings in the class:

*Income tax rate applicable
to natural persons for the fiscal year 2078/2079*

Tax Rate only for Employment Income			
For Single Person		For Couple	
Title	Tax Rate	Title	Tax Rate
Income up to 4 Lakh rupees	1%	Income up to 4 Lakh and 50 Thousand rupees	1%
Income above Rs.400000 up to Rs.500000	10%	Income above Rs.450000 up to Rs.550000	10%
Income above Rs.500000 up to Rs.700000	20%	Income above Rs.550000 up to Rs.750000	20%
Income above Rs.700000 up to Rs.2000000	30%	Income above Rs.750000 up to Rs.2000000	30%
Income above Rs.2000000	36%	Income above Rs.2000000	36%

Tax Rate only for Business Income as an Individual Firm			
For Single Person		For Couple	
Title	Tax Rate	Title	Tax Rate
Income up to 4 Lakh rupees	Tax Free	Income up to 4 Lakh and 50 Thousand rupees	Tax Free
Income above Rs.400000 up to Rs.500000	10%	Income above Rs.450000 up to Rs.550000	10%
Income above Rs.500000 up to Rs.700000	20%	Income above Rs.550000 up to Rs. 750000	20%
Income above Rs.700000 up to Rs.2000000	30%	Income above Rs.750000 up to Rs.2000000	30%
Income above Rs.2000000	36%	Income above Rs.2000000	36%

- (a) What is income tax?

- (b) Why does the government collect certain percent tax on our income amount by making laws?
- (c) Which agency manages the income tax in Nepal?
- (d) From where it can be studied easily about the income tax rate and ceiling fixed by Inland Revenue Department? What types of rules are mentioned?

The tax on the income of an individual or the group of people (industry, company etc.) is called income tax. Mainly, the tax on the income, salary and profit is the income tax. Income tax is one of the sources of income of the government. Income tax is calculated in percentage. There are different types of individual tax rate based on profession, occupation and marital status. According to the Income Tax Act 2058, four headings of income are (a) Employment (b) Business (c) Investment and (d) Contingent benefits. The responsibility of managing tax in Nepal in Nepal is given to Inland Revenue Department. The Inland Revenue Department prepares the income tax calculation procedure every year, details of which can be seen on the website <http://www.ird.gov.np>.

Income Tax Exemptions		
	Title	Ceiling
(a)	On the amount deposited at employees provident fund	On the less amount in one third of taxable income and Rs. 300000
(b)	On the amount deposited at citizen investment trust	
(c)	On the premium expenses paid as life insurance	Up to Rs. 25000 (with couple)
(d)	On the expenses on religious work and on donated amount	On the less amount in 5% of adjusted taxable income and Rs. 1,00,000.
(e)	On the obtained amount as remote allowance (based on remote area)	(a) Rs. 50,000 (b) Rs. 4,00,000 (c) Rs. 20,000 (d) Rs. 10,000
(f)	On the amount on 75% of foreign allowance	For staff working in Nepal's diplomatic mission aboard
(g)	On the expenses of medical treatment	25% of 15% of total expenses
(h)	Discount facility available to persons with disabilities	An additional 50 % of specified ceiling
(i)	On the premium paid for health insurance	Up to Rs. 20,000 yearly

(j)	On the amount contributed for social security fund	Up to Rs. 5,00,000 yearly
(k)	In case of woman who has only employment income	10% discount on income tax
(l)	In all allowance given as social security	
(m)	In the amount received through dowry, scholarship in-law's will	
(n)	In premium amount paid for insurance of private building owned by self	Up to Rs. 5000 yearly
(o)	In income as pension	An additional of 25% in specified ceiling
(p)	In case of taxpayer of firm registered individually	1 % tax is not imposed for pension income of a natural person who has contributed to employment fund and contribution based social security fund.

Example 1

The monthly salary of a married teacher is Rs. 37,990. How much income tax should be paid annually, when calculating the income of 13 months including festival expenses, received by the teacher according to the income tax rate of the fiscal year 2078/ 2079?

Solution,

Here, monthly salary = Rs. 37,990

$$\begin{aligned} \text{Yearly income} &= 13 \times 37,990 \\ &= \text{Rs. } 49,3870 \end{aligned}$$

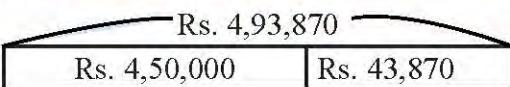
Now writing the taxable income Rs. 4,93,870 according to the income tax ceiling,

$$\text{Rs. } 4,93,870 = \text{Rs. } 4,50,000 + \text{Rs. } 43,870$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 1\% & & 10\% \end{array}$$

$$\text{Annual income tax} = 1\% \text{ of Rs. } 4,50,000 + 10\% \text{ of Rs. } 43,870$$

$$\begin{aligned} &= \frac{1}{100} \times 4,50,000 + \frac{10}{100} \times 43,870 \\ &= 4,500 + 4,387 \\ &= \text{Rs. } 8,887 \end{aligned}$$



1%

10%

Example 2

The monthly salary of an unmarried woman staff working in a bank is Rs. 30,000. 1% social security tax is levied on income up to Rs. 4,00,000 and 10% on income above Rs. 4,00,000 per annum. If she receives a salary equal to 15 months in a year, how much tax should be paid annually?

Solution,

Here, monthly income of staff = Rs. 30,000

$$\text{Annual income} = 15 \times \text{Rs. } 30,000$$

$$= \text{Rs. } 4,50,000$$

Now writing the taxable income Rs. 4,50,000 according to the income tax ceiling,

$$\text{Rs. } 4,50,000 = \text{Rs. } 4,00,000 + \text{Rs. } 50,000$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 1\% & & 10\% \end{array}$$

∴ Annual income tax = 1% of Rs. 4,00,000 + 10% of Rs. 50,000

$$\begin{aligned} &= \frac{1}{100} \times 4,00,000 + \frac{10}{100} \times 50,000 \\ &= 4,000 + 5,000 \\ &= \text{Rs. } 9,000 \end{aligned}$$

Being a woman will get 10% discount in income tax,

$$\text{Discount amount} = 10\% \text{ of Rs. } 9,000$$

$$= \text{Rs. } 900$$

$$\begin{aligned} \therefore \text{Annual tax payable the staff} &= \text{Rs. } 9,000 - \text{Rs. } 900 \\ &= \text{Rs. } 8,100 \end{aligned}$$

Example 3

A married male employee in Nepal earning Rs. 40,500 monthly including Dearness Allowance of Rs. 2,000 pays Rs. 23,500 yearly as a premium for life insurance. He receives the salary of 13 months in a year including Festival Allowance of 1 month. If he deposits 10% of his income excluding dearness allowance and festival allowance to Employees Provident Fund, then how much income tax should he pay on the remaining income? Find out.

Solution,

Here monthly salary = Rs. 40,500 – Rs. 2,000 = Rs. 38,500

Annual salary = Rs. 38,500 × 12 = Rs. 4,62,000

Dearness Allowance = 2,000 × 12 = Rs. 24,000

Festival allowance = Rs. 38,500

Employees Provident Fund = $4,62,000 \times \frac{10}{100}$ = Rs. 46,200

Determinable Income = Rs. 4,62,000 + Rs. 24,000 + Rs. 38,500 + Rs. 46,200
= Rs. 5,70,700

Deducting,

(i) Amount deposited at Employees Provident Fund

$$\text{Rs. } 46,200 + \text{Rs. } 46,200 = \text{Rs. } 92,400$$

(ii) Premium for life insurance = Rs. 23,500

$$\text{Total} = \text{Rs. } 1,15,900$$

$$\begin{aligned}\text{Now one third of determinable income} &= \text{Rs. } 5,70,700 \times \frac{1}{3} \\ &= \text{Rs. } 1,90,233.33\end{aligned}$$

Being the total amount deposited at Employees Provident Fund and premium of insurance is less than one third of determinable income,

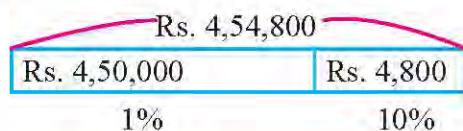
Tax free income = Rs. 1,15,900

Taxable income = 5,70,700 – 1,15,900 = Rs. 4,54,800

Being a married male staff,

Payable income tax = 1% of Rs. 4,50,000 + 10% of Rs. 4,800

$$\begin{aligned}&= \text{Rs. } 4,50,000 \times \frac{1}{100} + 4,800 \times \frac{10}{100} \\ &= 4,500 + 480 \\ &= \text{Rs. } 4,980\end{aligned}$$



∴ The employee has to pay income tax of Rs. 4,980 annually.

Example 4

If a businessman is exempted from income tax up to Rs. 4,50,000 out of his annual income, the income tax is levied at the rate of 10% from Rs. 4,50,001 to Rs. 5,50,000 and at the rate of 20% from Rs. 5,50,001 to Rs. 7,00,000, how much income tax should the businessman earning Rs. 6,75,000 annually have to pay?

Solution,

Here, expressing the total yearly income of Rs. 6,75,000 according to the given income tax ceiling,

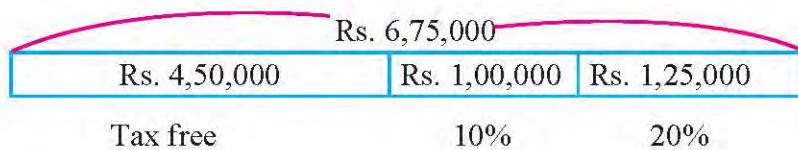
$$\text{Rs. } 6,75,000 = \text{Rs. } 4,50,000 + \text{Rs. } 1,00,000 + \text{Rs. } 1,25,000$$



$$\therefore \text{Total yearly income tax} = 10\% \text{ of Rs. } 1,00,000 + 20\% \text{ of Rs. } 1,25,000$$

$$\begin{aligned}&= \text{Rs. } 1,00,000 \times \frac{10}{100} + 1,25,000 \times \frac{20}{100} \\&= \text{Rs. } 10,000 + 25,000 \\&= \text{Rs. } 35,000\end{aligned}$$

From Model drawing method,



$$\text{Total yearly income tax} = 10\% \text{ of Rs. } 1,00,000 + 20\% \text{ of Rs. } 1,25,000$$

$$\begin{aligned}&= 1,00,000 \times \frac{10}{100} + 1,25,000 \times \frac{20}{100} \\&= 10,000 + 25,000 \\&= \text{Rs. } 35,000\end{aligned}$$

\therefore That businessman has to pay yearly income tax of Rs. 35,000.

Example 5

Dhanbahadur has deposited Rs. 35,000 in a fixed saving account of a co-operative at the rate of 10% per annum simple interest for 4 years. If 5 % income tax is levied on interest earning from that saving, how much interest does he receive after deducting the tax? Find out.

Solution,

Here give,

$$\text{Principal (P)} = \text{Rs. } 35,000$$

$$\text{Time (T)} = 4 \text{ years}$$

$$\text{Rate of simple interest (R)} = 10\%$$

$$\text{Tax rate} = 5\%$$

$$\text{Simple Interest} = ?$$

$$\text{Interest after deducting tax} = ?$$

$$\text{Tax on income as interest} = ?$$

Using formula,

$$\begin{aligned} I &= \frac{P \times T \times R}{100} \\ &= \frac{35,000 \times 4 \times 10}{100} \\ &= \text{Rs. } 14,000 \end{aligned}$$

$$\therefore \text{Simple Interest} = \text{Rs. } 14,000,$$

Here, the interest so obtained is calculated as Dhanbahadur's income from investment.

Again Income tax = 5 % of Rs. 14,000

$$\begin{aligned} &= \text{Rs. } 14,000 \times \frac{5}{100} \\ &= \text{Rs. } 700 \end{aligned}$$

$$\therefore \text{Tax on income as interest} = \text{Rs. } 700$$

$$\begin{aligned} \therefore \text{Interest after deducting tax} &= \text{Rs. } 14,000 - \text{Rs. } 700 \\ &= \text{Rs. } 13,300 \end{aligned}$$

That co-operative deducts 5% tax from obtained interest and provides only Rs. 13,300 to the saver Dhanbahadur.

Exercise 2.1

1. In Nepal, an officer level married civil servant earning Rs. 38,000 monthly pays Rs. 23,500 yearly as a premium of life insurance. If his / her income is calculated as a salary equal to 13 months in a year including festival allowance, how much income tax do they have to pay yearly. Find out. (Calculate according to the income tax ceiling mentioned on the previous page.)
2. An employee working in an organization has to pay income tax at the rate of 1% up to Rs. 4,50,000 out of his income, above Rs. 4,50,000 up to Rs. 5,50,000 at the rate of 10 %, above Rs. 5,50,000 up to Rs. 7,50,000 at the rate of 20 % and above Rs. 7,50,000 up to Rs. 20,00,000 at the rate of 30%, how much income tax should be paid to the government by the employee earning Rs. 65,000 per month?
3. **Study the following table showing the income tax threshold for entrepreneurs doing business as a sole proprietorship:**

Yearly Income (Rs.)	Rate of Tax
1 - 4,50,000	Tax free
4,50,001 - 5,50,000	10%
5,50,001 - 7,50,000	20%
7,50,001 - 20,00,000	30%
Above 20,00,000	36%

According to the income threshold, how much tax should be paid by an entrepreneur having the following yearly income?

- (a) Yearly income = Rs. 6,30,000
- (b) Yearly income = Rs. 9,25,000
- (c) Yearly income = Rs. 17,88,000
- (d) Yearly income = Rs. 22,25,000
4. What is the total interest of Rs. 10 lakh deposited in a bank when calculated at the rate of 8.5% simple interest in 4 years? If 5% income tax is levied on that interest, then what is the net simple interest? Find out. (Thus, income tax paid on interest refers to the tax paid on investment income.)

Project Work

- (a) Collect the actual details of the salary of the level secondary teachers working in your school, including including description of employees provident fund, citizen investment fund and insurance, calculate how much tax each teacher has to pay in annual income.
- (b) If the student's family members have jobs, calculate the income from them and get the actual details and find out the amount of tax to be paid annually.

Answers

1. Rs. 6,550 2. Rs. 63,500 3. (a) Rs. 26,000 3. (b) Rs. 1,02,500
3. (c) Rs. 3,61,400 3. (d) Rs. 5,06,000 4. Rs. 3,40,000 and Rs. 3,23,000

2.1.2 Value added tax

Activity 1

Study the following bill and discuss the questions given below:

Invoice No. 883

ABC Electronics

Kathmandu

TAX INVOICE

Date: 2077-11-15

VAT No

M/s

Address:

Buyers VAT No.: Mode of Payment: Cash/Cheque/Others

Customer's Sign

For ABC Electronics

- (a) What is the sales rate of refrigerator in the given bill?
- (b) Did the refrigerator buyer get a discount or not?
- (c) It is seen that the buyer has paid more than the selling rate, why did this happen?
- (d) According to the bill, Rs. 3,623.90 more amount seems to have been paid in the sale rate. Why is the extra amount paid?
- (e) Do we have to pay extra amount for the selling price when buying any goods?
- (f) What price of refrigerator does Rs. 31,500 paid by the buyer refer to here?

Value Added Tax (VAT)

Value added tax is a kind of indirect tax levied on goods and services. Value added tax is the tax levied on the increased price at various levels/ stages from production to distribution of goods or services. At present, the rate of value added tax in Nepal is 13 %.

From Mathematical Technical Dictionary,

Value Added Tax: A value added tax (VAT) is a tax levied on the price that increases at each level by deducting a discount when selling goods or services. This tax should be paid by the consumer at the end but it does not allow the price of the commodity to increase unnecessarily.

A company that manufactures television set fixed the price of a television set at Rs. 10,000 by adding production cost and profit. The following process is completed when the television produced by the company reaches to the consumer through the dealer, wholesaler and retailer.

When the manufacturing company sells to the dealer	When the dealer sells to the wholesaler
<p>The price of a television set = Rs. 10,000 Rate of value added tax = 13% Selling Price, $= \text{Rs. } 10,000 + 13\% \text{ of } \text{Rs. } 10,000$ $= 10,000 + 10,000 \times \frac{13}{100}$ $= 10,000 + 1,300$ $= \text{Rs. } 11,300$</p>	<p>Cost Price = Selling Price of manufacturing company = Rs. 11,300 Price of television including extra expenses and profit of Rs. 1200 = Rs. 12,500 Rate of value added tax = 13% Selling Price, $= \text{Rs. } 12,500 + 13\% \text{ of } \text{Rs. } 12,500$ $= \text{Rs. } 12,500 + \text{Rs. } 1,625$ $= \text{Rs. } 14,125$</p>

When the wholesaler sells to the retailor	When the retailer sells to the consumer
<p>Cost Price = Selling Price of dealer = Rs. 14,125</p> <p>Price of television including extra expenses and profit of Rs. 875 = Rs. 15,000</p> <p>Rate of value added tax = 13%</p> <p>Selling Price</p> $\begin{aligned} &= \text{Rs. } 15,000 + 13\% \text{ of Rs. } 15,000 \\ &= 15,000 + 1,950 \\ &= \text{Rs. } 16,950 \end{aligned}$	<p>Cost Price = Selling Price of wholesaler = Rs. 6,950</p> <p>Price of television including extra expenses and profit of Rs.,050 = Rs. 8,000</p> <p>Rate of value added tax = 13%</p> <p>Selling Price</p> $\begin{aligned} &= \text{Rs. } 18,000 + 13\% \text{ of Rs. } 18,000 \\ &= \text{Rs. } 18,000 + \frac{13}{100} \times \text{Rs. } 18,000 \\ &= \text{Rs. } 18,000 + \text{Rs. } 2,340 \\ &= \text{Rs. } 20,340 \end{aligned}$

Here, consumer has to pay Rs. 2,340 value added tax. When the amount is deposited in the government treasury,

$$\text{Manufacturer} = \text{Rs. } 1,300$$

$$\text{Dealer} = (1,625 - 1,300) = \text{Rs. } 325$$

$$\text{Wholesaler} = (1,950 - 1,625) = \text{Rs. } 325$$

$$\text{Retailer} = (2,340 - 1,950) = \text{Rs. } 390$$

$$\text{Total} = 1,300 + 325 + 325 + 390 = \text{Rs. } 2,340$$

Now, based on the increased value of each level,

$$\text{Manufacturer} = 13\% \text{ of Rs. } 10,000$$

$$= \frac{13}{100} \times \text{Rs. } 10,000 = \text{Rs. } 1,300$$

$$\text{Dealer} = 13\% \text{ of Rs. } (12,500 - 10,000)$$

$$= \frac{13}{100} \times \text{Rs. } 2500$$

$$= \text{Rs. } 325$$

$$\text{Wholesaler} = 13\% \text{ of Rs. } (15,000 - 12,500)$$

$$= \frac{13}{100} \times \text{Rs. } 2500$$

$$= \text{Rs. } 325$$

$$\text{Retailer} = 13\% \text{ of Rs. } (18,000 - 15,000)$$

$$= \frac{13}{100} \times \text{Rs. } 3,000$$

$$= \text{Rs. } 390$$

$$\text{Total value added tax} = \text{Rs. } 1,300 + \text{Rs. } 325 + \text{Rs. } 325 + \text{Rs. } 390 = \text{Rs. } 2,340$$

Example 1

Find the value added tax (VAT) in the given cases:

- (a) Price without value added tax = Rs. 700, rate of value added tax = 13%
- (b) Marked price without value added tax(MP) = Rs. 10,000, Discount = 15% and rate of value added tax = 13%

Solution,

- (a) Here, Price without value added tax = Rs. 7,000
Rate of value added tax = 13%
Value added tax amount = ?

Now, value added tax amount = 13% of Rs. 7000

$$\begin{aligned} &= \frac{13}{100} \times \text{Rs. } 7,000 \\ &= \text{Rs. } 910 \end{aligned}$$

∴ Value added tax amount = Rs. 910

- (b) Marked price without value added tax(MP) = Rs. 10,000

Discount = 15% and

Rate of value added tax = 13%

Value added tax amount = ?

Now, finding discount amount,

$$\begin{aligned} \text{Discount amount} &= 15\% \text{ of Rs. } 10,000 \\ &= \frac{15}{100} \times \text{Rs. } 10,000 \\ &= \text{Rs. } 1,500 \end{aligned}$$

Price after deducting the discount = Rs. 10,000 – Rs. 1500

$$= \text{Rs. } 8,500$$

Now, value added tax amount = 13% of Rs. 8,500

$$\begin{aligned} &= \frac{13}{100} \times \text{Rs. } 8,500 \\ &= \text{Rs. } 1,105 \end{aligned}$$

∴ Value added tax amount = Rs. 1,105

Example 2

A mobile set was marked at Rs. 15,000. What is the value of the mobile set after allowing a discount of 15% and adding 13% value added tax? Calculate.

Solution,

Here, the marked price of mobile set without VAT (MP) = Rs. 15,000

$$\text{Discount} = 15\%$$

$$\text{Rate of VAT} = 13\%$$

$$\text{Price with VAT} = ?$$

Now discount amount = 15% of Rs. 15,000

$$\begin{aligned}&= \frac{15}{100} \times \text{Rs. } 15,000 \\&= \text{Rs. } 2,250\end{aligned}$$

$$\begin{aligned}\text{Again, value added taxable amount} &= \text{Rs. } (15,000 - 2,250) \\&= \text{Rs. } 12,750\end{aligned}$$

$$\begin{aligned}\text{Value added tax amount} &= 13\% \text{ of Rs. } 12,750 \\&= 12,750 \times \frac{13}{100} \\&= \text{Rs. } 1,657.50\end{aligned}$$

$$\begin{aligned}\therefore \text{Price with value added tax} &= 12,750 + 1,657.50 \\&= \text{Rs. } 14,407.50\end{aligned}$$

Alternative Method

Solution,

Price of mobile without value added tax (MP) = Rs. 15000

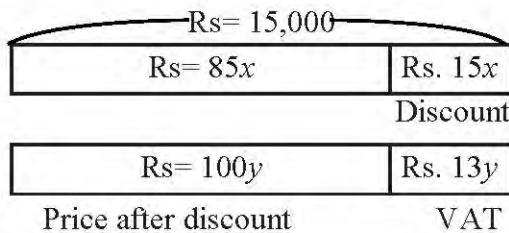
After giving 15% discount,

$$\begin{aligned}\text{Price after discount} &= 85\% \text{ of Rs. } 15,000 \\&= \frac{85}{100} \times \text{Rs. } 15,000 \\&= \text{Rs. } 12,750\end{aligned}$$

After adding 13% VAT,

$$\begin{aligned}\text{Price with VAT} &= 113\% \text{ of Rs. } 12,750 \\&= \frac{113}{100} \times \text{Rs. } 12,750 \\&= \text{Rs. } 14,407.50\end{aligned}$$

From model drawing method,



$$\text{Now, } 100x = \text{Rs. } 15,000$$

$$x = \frac{15,000}{100} = \text{Rs. } 150$$

$$85x = \text{Rs. } 150 \times 85 = 12,750$$

$$\text{Again, } 100y = 85x$$

$$=?= 12,750$$

$$y = \frac{12,750}{100}$$

$$113y = \frac{12,750}{100} \times 113 \\ = \text{Rs. } 14,407.50$$

∴ Price with value added tax = Rs. 14,407.50

Example 3

If a buyer has to pay Rs. 57,630 for an article after deducting 15 % discount on the price excluding value added tax and adding 13% value added tax, find the marked price of the article.

Solution,

Let the marked price excluding value added tax (MP) = Rs. x

Given discount = 15%

Rate of value added tax = 13%

Price with value added tax = Rs. 57,630

According to the question, Discount amount = 15% of x

$$= \frac{15}{100} \times x \\ = \frac{15x}{100}$$

$$= \text{Rs. } \frac{3x}{20}$$

Value added taxable amount

$$\begin{aligned} &= \text{Rs. } \left(x - \frac{3x}{20} \right) \\ &= \text{Rs. } \frac{17x}{20} \end{aligned}$$

Value added tax amount = 13% or $\text{Rs. } \frac{17x}{20}$

$$\begin{aligned} &= \frac{17x}{20} \times \frac{13}{100} \\ &= \text{Rs. } \frac{221x}{2000} \end{aligned}$$

Price with value added tax

$$\begin{aligned} &= \text{Rs. } \left(\frac{17x}{20} + \frac{221x}{2000} \right) \\ &= \text{Rs. } \left(\frac{1700x + 221x}{2000} \right) \\ &= \text{Rs. } \frac{1921x}{2000} \end{aligned}$$

According to the question,

$$\frac{1921x}{2000} = 57630$$

$$\text{or, } 1921x = 2000 \times 57630$$

$$\text{or, } x = \left(\frac{2,000 \times 57,630}{1921} \right)$$

$$\text{or, } x = 60,000$$

$$\therefore \text{Marked Price (MP)} = \text{Rs. } 60,000$$

Alternative Method

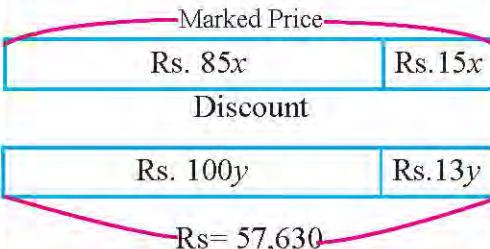
Let the marked price excluding value added tax (MP) = $\text{Rs. } x$

Given discount = 15%

Rate of value added tax = 13%

Price with value added tax = $\text{Rs. } 57,630$

From model drawing method,



$$\text{Now, } 113y = \text{Rs. } 57,630$$

$$y = \frac{\text{Rs. } 57630}{113}$$

$$100y = \text{Rs. } 51,000$$

$$\text{Again, } 85x = \text{Rs. } 51,000$$

$$x = \frac{51,000}{85}$$

$$100x = \text{Rs. } 60,000$$

$$\therefore \text{Marked Price (MP)} = \text{Rs. } 60,000$$

$$\begin{aligned}\text{Price with value added tax} &= (100 - 15)\% \text{ of Rs. } x \times (100 + 13)\% \\ &= \frac{85}{100} \times x \times \frac{113}{100}\end{aligned}$$

According to the question,

$$\text{Rs. } 57,630 = x \times \frac{85}{100} \times \frac{113}{100}$$

$$\text{or, } x = \frac{57630 \times 100 \times 100}{85 \times 113}$$

$$\text{or, } x = 60,000$$

\therefore Marked Price (MP) = Rs. 60,000

Example 4

A shopkeeper has bought a bicycle for Rs. 5,800 excluding value added tax. He has fixed the marked price by increasing 40 % on cost price. Find out how much the consumer has to pay when allowing 13 % value added tax with a 10% discount on the marked price.

Solution,

Here the cost price of bicycle excluding value added tax (CP) = Rs. 5,800

Discount = 10%

Rate of value added tax = 13%

Price with value added tax = ?

Profit % or Loss % = ?

From the question,

$$\begin{aligned}\text{Marked price of bicycle (MP)} &= (100 + 40)\% \text{ of Rs. } 5,800 \\ &= \frac{140}{100} \times \text{Rs. } 5,800 \\ &= \text{Rs. } 8,120\end{aligned}$$

Discount amount = 10 % of marked price

$$\begin{aligned}&= \frac{10}{100} \times \text{Rs. } 8120 \\ &= \text{Rs. } 812\end{aligned}$$

$$\begin{aligned}\therefore \text{Taxable income for value added tax} &= \text{Rs. } 8,120 - \text{Rs. } 812 [\text{MP} - \text{discount amount}] \\ &= \text{Rs. } 7,308\end{aligned}$$

Now value added tax amount = 13% of Rs. 7,308

$$= \frac{13}{100} \times \text{Rs. } 7,308 \\ = \text{Rs. } 950.04$$

$$\therefore \text{Price of bicycle with value added tax} = \text{Rs. } 7,308 + \text{Rs. } 950.04 \\ = \text{Rs. } 8,258.04$$

Thus, the consumer has to pay Rs. 8,258.04.

Example 5

If a shopkeeper sells a watch purchased at Rs. 2000 excluding value added tax at a profit of 25%, find out how much the consumer will have to pay on the bill after 13% value added tax.

Solution,

Here cost price of the watch (CP) = Rs. 2,000

Profit = 25%

Rate of value added tax = 13%

Amount to be paid by the consumer (price with VAT) =?

Profit = 25% of Rs. 2000

$$= \frac{25}{100} \times 2,000 \\ = \text{Rs. } 500$$

\therefore Taxable amount for value added tax = cost price + profit

$$= \text{Rs.}(2,000 + 500) \\ = \text{Rs. } 2500$$

Value added tax amount = 13% of Rs. 2500

$$= \frac{13}{100} \times \text{Rs. } 2,500 \\ = \text{Rs. } 325$$

$$\therefore \text{Amount to be paid by the consumer} = \text{Rs. } 2,500 + \text{Rs. } 325 \\ = \text{Rs. } 2,825$$

Example 6

The sports goods dealer has bought a table tennis board from a importer for Rs. 25,000 excluding value added tax. He sells that table tennis board to retailer for Rs.30,000 excluding value added tax. That retailer sells it to Bikki for Rs. 37,000 excluding value added tax. Find the answers of the following questions based on the 13% value added tax rate at each level:

- (a) What is the cost price of the dealer?
- (b) What is the cost price of the retailer?
- (c) How much will Bikki pay to buy it?
- (d) How much of the value added tax from the sale of table tennis board is deposited in the government treasury?
- (e) How much should the dealer and retailer deposit value added tax in the the government treasury?

Solution,

Sale price excluding value added tax fixed by the importer = Rs.25,000

Rate of value added tax = 13%

(a) Cost price of the dealer = $25,000 + 13\% \text{ of } 25,000$
 $= 25,000 + \frac{13}{100} \times 25,000$
 $= 25,000 + 3,250$
 $= \text{Rs. } 28,250$

Here, selling price of dealer adding profit and other expenses excluding value added tax = Rs. 30,000

(b) Cost price of retailer = $30,000 + 13\% \text{ of } 30,000$
 $= 30,000 + \frac{13}{100} \times 30,000$
 $= 30,000 + 3,900$
 $= \text{Rs. } 33,900$

Here selling price of retailer adding profit and other expenses excluding value added tax = Rs. 33,900

(c) Now, the cost price of Bikki = $37,000 + 13\% \text{ of } 37,000$
 $= \text{Rs. } 37,000 + \frac{13}{100} \times 37,000$
 $= \text{Rs. } 37,000 + 4,810$
 $= \text{Rs. } 41,810$

- (d) The value added tax from the sale of table tennis board deposited in the government treasury = Rs. 4,810
- (e) Calculating the value added tax deposited by the dealer in the government treasury,

First Method	Second Method
The value added tax he collected = Rs. 3,900	Amount to be deposited in government treasury
The value added tax he paid = Rs. 3,250	$13\% \text{ of } (\text{Rs. } 30,000 - \text{Rs. } 25,000)$
Amount to be deposited in government treasury $= \text{Rs. } 3,900 - \text{Rs. } 3,250$ $= \text{Rs. } 650$	$= \frac{13}{100} \times 5,000$ $= \text{Rs. } 650$

Calculating the value added tax deposited by retailer in the government treasury,

First Method	Second Method
The value added tax he collected = Rs. 4,810	Amount to be deposited in government treasury =
The value added tax he paid = Rs. 3,900	$13\% \text{ of } (\text{Rs. } 37,000 - \text{Rs. } 30,000)$
Amount to be deposited in govern- ment treasury $= \text{Rs. } 4,810 - \text{Rs. } 3,900$ $= \text{Rs. } 910$	$= \frac{13}{100} \times 7,000$ $= \text{Rs. } 910$

Exercise 2.2

- 1. Find the value added tax amount based on the given table:**

S.N.	Price excluding value added tax	Rate of value added tax	Value added tax amount
(a)	Rs. 300	13%	?
(b)	Rs. 750	13%	?
(c)	Rs. 6,000	13%	?
(d)	Rs. 3,75,000	13%	?
(e)	Rs. 20,27,000	13%	?

- 2. Calculate the price excluding value added tax based on the given table:**

S.N.	Price excluding value added tax	Value added tax amount	Rate of value added tax
(a)	?	? = 3,616	13%
(b)	?	? = 30,510	13%
(c)	?	? = 3,390	13%
(d)	?	? = 57,630	13%
(e)	?	? = 1,19,328	13%

- 3. Calculate the price that the customer has to pay for the given goods:**

- (a) **Marked price without VAT = Rs. 35,000**
Discount = 10%
VAT = 13%



- (b) **Marked price without VAT = Rs. 6,500**
Discount = 7.5%
VAT = 13%



- (c) **Marked price without VAT = Rs. 25,700**
Discount = 15%
VAT = 13%



- (b) **Marked price without VAT = Rs. 1,450**
Discount = 22.75%
VAT = 13%



- 4. Calculate the actual price of the goods based on the information given in the following table:**

S.N.	Marked price excluding VAT	Rate of Discount	Rate of VAT	Price with VAT
(a)	Rs. 2,000	8%	13%	?
(b)	Rs. 7,000	15%	13%	?
(c)	Rs. 27,000	20%	13%	?
(d)	Rs. 20,525.30	10%	13%	?
(e)	Rs. 1,81,500	7.5%	13%	?

- 5.** The marked price of a LED television set excluding value added tax is Rs. 37,500. If it is sold after 11% discount and 13% VAT is levied on it, how much will the consumer pay? Find out by calculation.

- 6. Find the marked price and discount amount based on the given table:**

S.N.	Marked price excluding VAT	Rate of Discount	Discount Amout	Rate of VAT	Price with VAT
(a)	?	20%	?	13%	Rs. 4,520
(b)	?	10%	?	13%	Rs. 15,225
(c)	?	15%	?	13%	Rs. 57,630
(d)	?	25%	?	13%	Rs. 2,151.52
(e)	?	15%	?	13%	Rs. 2,40125

- 7.** If the price of an electric kettle after allowing 5 % discount on the marked price excluding value added tax and adding 13 % VAT is Rs. 1,575, What will be the marked price of that kettle? Find the taxable price for valued added tax.

- 8. A shopkeeper bought an article for Rs. 27,500 excluding value added tax and marked its price Rs. 35,000. When selling the article after allowing 10.5% discount and levying 13% VAT,**

- (a) What is the price including value added tax?
- (b) What is the percentage of profit or loss from this transaction?
- (c) If he could sell at the marked price, what percentage of profit would he get?

9. A shopkeeper bought a watch for Rs. 4,000 excluding value added tax and labelled its price 25% above the cost price. After allowing 12 % discount and levying 13 % VAT,
- How much does the consumer have to pay for value added tax?
 - If it is sold at a loss of 5%, what will be its price with value added tax?
10. A wholesaler sold a washing machine to a retailer at Rs. 67,000 including 13% value added tax. If the retailer delivered the machine to the consumer's house with transportation charge Rs. 3,000, local tax Rs. 550 and a profit of Rs. 5,000, how much would the consumer pay for value added tax at the current rate? Find out.
11. A wholesaler of watch bought a watch from a dealer at Rs. 12,000 excluding value added tax and sold it to a retailer at Rs. 16,950 including value added tax. If the rate of value added tax is 13 % at each level,
- How much did the retailer pay for the watch except the value added tax?
 - How much did the retailer pay for value added tax?
 - How much profit did the dealer make?

Project Work

Be divided your friends into appropriate number of groups. Study the budget statement for the current fiscal year. Collect the following information from that budget statement and prepare a report in a group:

- Size of the budget
- Different types of taxes imposed in the budget
- Rate of value added tax
- Goods or services exempt from tax
- Goods subject to value added tax
- Other income tax

Answers

1. (a) Rs. 39 (b) Rs. 97.50 (c) Rs. 780 (d) Rs. 48,750
(e) Rs. 2,63,510

2. (a) Rs. 3,200 (b) Rs. 27,000 (c) Rs. 3,000 (d) Rs. 51,000
(e) Rs. 1,05,600

3. (a) Rs. 35,595 (b) Rs. 6,794.13 (c) Rs. 24,684.85 (d) Rs. 1,265.74

4. (a) Rs. 20,79.20 (b) Rs. 67,23.50 (c) Rs. 24,408 (d) Rs. 20,874.23
(e) Rs. 189712.88

5. Rs. 37,713.75

6. (a) Rs. 60,000, Rs. 9000 (b) Rs. 5,000, Rs. 1000
(c) Rs. 2,50,000, Rs. 37500 (d) Rs.14,970.50, Rs. 1497.05
(e) Rs. 2,538.67, Rs. 34.67

7. Rs. 1,393.81 and Rs.1,467.16

8. (a) Rs. 35,397.25 (b) 13.91% (c) 27.27%

9. (a) Rs. 572 (b) Rs. 4,294

10. Rs. 10,953.80

11.(a) Rs. 15,000 (b) Rs. 1,950 (c) Rs. 3,000

3.1 Commission

Activity 1

Form three groups from your classmates. Act as a land owner (name: Dhaniram), the leader of the first group, as a buyer (name: Aadinath), the leader of the second group and agent as a mediator (name: Sabina), the leader of the third group. A person acting as an intermediary between a seller and a buyer of goods and services is called an agent.

In this way, the amount that the agent receives at the rate of certain percentage of sale price for performing the role of mediator is called the commission. Agent can receive such amount from buyer or seller or both. The sale price is taken as the basis when taking and giving commission. Commission is expressed in percentage. Commissions are calculated in land transactions, on financial transaction in banks and financial institution, in insurance company, buying and selling goods etc.

Example 1

If an agent receives 5% commission by selling a land for Rs. 25,00,000, then

- How much amount does the agent receive as the commission?
- How much money is in the hand of the land owner after commission? Find out.

Solution,

Here selling price of land = Rs. 25,00,000

Rate of commission = 5%

- Amount of commission =?
- The amount received by the land owner =?

(a) We know that,

Amount of commission = 5 % of Rs. 25,00,000

$$\begin{aligned}
 &= \frac{5}{100} \times \text{Rs. } 25,00,000 \\
 &= \text{Rs. } 1,25,000
 \end{aligned}$$

∴ Amount of commission = Rs. 1,25,000

(b) The amount received by the land owner

$$\begin{aligned} &= \text{price after commission} = \text{selling price} - \text{commission amount} \\ &= 25,00,000 - 1,25,000 \end{aligned}$$

∴ The amount received by the land owner = Rs. 23,75,000

Example 2

The rate of commission paid by a company to agent is given below:

- 0.5% in the sale up to Rs. 15,00,000
- 1 % in the sale above Rs. 15,00,000 up to Rs. 25,00,000
- 1.5% in the sale above Rs. 25,00,000 up to Rs. 40,00,000
- 2 % in the sale above Rs. 40,00,000

How much total commission does an agent receive in a sale of Rs. 60 lakhs, based on the above rate of commission?

Solution,

Here, writing the selling price Rs. 60,00,000 breaking down according to the ceiling of commission rate,

$$\text{Rs. } 60,00,000 = \text{Rs. } 15,00,000 + \text{Rs. } 10,00,000 + \text{Rs. } 15,00,000 + \text{Rs. } 20,00,000$$

$$\begin{array}{cccc} & \downarrow & \downarrow & \downarrow \\ & 0.5\% & 1\% & 1.5\% \\ & & & 2\% \end{array}$$

$$\begin{aligned} \text{Total Commission Amount} &= 0.5\% \text{ of Rs. } 15,00,000 + 1\% \text{ of Rs. } 10,00,000 \\ &\quad + 1.5\% \text{ of Rs. } 15,00,000 + 2\% \text{ of Rs. } 20,00,000 \end{aligned}$$

$$\begin{aligned} &= \text{Rs.} \left\{ \frac{0.5}{100} \times 15,00,000 \right\} + \left\{ \frac{1}{100} \times 10,00,000 \right\} \\ &\quad + \left\{ \frac{1.5}{100} \times 15,00,000 \right\} + \left\{ \frac{2}{100} \times 20,00,000 \right\} \\ &= 7,500 + 10,000 + 22,500 + 40,000 \end{aligned}$$

$$\therefore \text{Total Commission Amount} = \text{Rs. } 80,000$$

Example 3

Monthly salary of an employee working in a construction material shop is Rs. 12,000. He also gets commission for selling goods. He has sold construction materials worth Rs. 4,75,000 in a month. If he earns the total amount of Rs. 19,125 in that month, what is the rate of the commission?

Solution,

Selling Amount = Rs. 4,75,000

Monthly salary of the employee = Rs. 12,000

Employee's monthly income including commission = Rs. 19,125

$$\therefore \text{Commission Amount} = \text{Rs. } 19,125 - \text{Rs. } 12,000 \\ = \text{Rs. } 7,125$$

$$\text{Now, rate of commission} = \frac{\text{Commission Amount}}{\text{Selling Amount}} \times 100\% \\ = \frac{7,125}{4,75,000} \times 100\% \\ = 1.5\%$$

Example 4

Monthly salary of a staff working in a bookstore is Rs. 16,000. If he sells books worth more than Rs. 5,00,000 a month, he is paid a commission of 2 %. If the total sales of the shop for the month of Baisakh is Rs. 7,25,000, what is the total income of the staff in that month? Find it.

Solution,

Here, the monthly salary of the staff = Rs. 16,000

Total selling amount of the month of Baisakh = Rs. 7,25,000

Rate of commission = 2%

Monthly income of the staff = ?

Writing the total selling amount according to the ceiling of commission,

Rs. 7,25,000 = Rs. 5,00,000 + Rs. 2,25,000

Rate of commission = 2%

Now, the amount of commission = 2% of Rs. 250000

$$= \frac{2}{100} \times \text{Rs. } 2,50,000 \\ = \text{Rs. } 5,000$$

Thus, the total income of that staff of the month of Baisakh = Rs. $(16,000 + 5000)$
= Rs. 21,000

Exercise 3.1

1. Find the amount of commission based on the given table:

S.N.	Particulars (Items)	Total Selling Price	Rate of Commission
(a)	Land	Rs. 1 crore 50 lakh	5%
(b)	House	Rs. 2 crore	3%
(c)	Photocopy machine	Rs. 4 lakhs	7.5%
(d)	Share transaction	Rs. 5 crore 25 lakh	2%

2. If an agent receives 5% commission by selling a house for Rs. 1,40,00,000 then,
- How much amount does the agent receive as the commission?
 - How much amount does the showroom have to pay to the employees for commission? Calculate.
3. A motorcycle showroom has 10 employees who get Rs. 20,000 as salary per month. They get 0.25% commission from the total monthly sale. If the total transaction in Asoj is Rs. 1,35,00,000,
- How much amount does the showroom provide to the employees as the commission? calculate.
 - What percentage is more in the monthly income of an employee than the monthly salary?
4. A company producing plastic goods gives commission to its seller at the rate of 1% in a selling amount up to Rs. 6,00,000, 1.5% above Rs. 6,00,000 up to Rs. 10,00,000 and 2% in a selling amount above Rs. 10,00,000, then find the amount of commission received by the seller in the following selling amount:
- Rs. 4,45,600
 - Rs. 7,25,000
 - Rs. 15,75,000
5. The monthly salary of a staff working in a cosmetic shop is Rs. 17,000. He gets a certain percentage of commission on the total sales. In the month of Manshir he earns Rs. 30,000 including commission on the total selling price of Rs. 10,00,000 in that month, how much amount of commission does he get from the shop?

Project Work

Visit a company or a shop nearby. Ask staff or operator about the following, prepare a report and present in your class:

- (a) Sales provision
- (b) Plan for additional sales facilities for staffs.

Answers

- | | | | |
|---------------------|------------------------|---------------------|----------------------|
| 1. (a) Rs. 7,50,000 | 1. (b) Rs. 6,00,000 | 1. (c) Rs. 30,000 | 1. (d) Rs. 10,50,000 |
| 2. (a) Rs. 7,00,000 | 2. (b) Rs. 1,33,00,000 | 3. (a) Rs. 3,37,500 | 3. (b) 168.75% |
| 4. (a) Rs. 4,456 | 4. (b) Rs. 7,875 | 4. (c) Rs. 23500 | 5. 1.3% |

3.2 Bonus

Activity 1

The edited excerpt of the news item entitled Oil Corporation distributing 6 crore bonus to employees' published in the Naya Patrika Daily of 2077 Poush 24 is as follows:

Nepal Oil Corporation is going to distribute bonus to the employees. The corporation has decided to distribute bonus to the employees in view of the legal provision of distributing bonus in the year of profit. Yet the corporation has to get the permission of the government to give bonus to the employees.

Oil Corporation is going to distribute around 6 crores to the employees from the amount allocated for the bonus of two fiscal years 2075/076 and 2076/077. According to the official president of trade union of the corporation, Gopal Rai, it has been decided to distribute bonus equal to two months and 15 days to the assistant level employees and two months' salary to the officer level or higher level employees. "We believe that performance will be more effective if bonuses can be distributed to the employees." He said ' We are hopeful that Ministry of Finance will approve it this time."

The Bonus Act provides for a bonus of up to 1 % of the net profit to be distributed to the employees of public enterprises. The law also stipulates that a maximum of 3 months' salary is equal to the bonus received by an employee in a fiscal year. The corporation has to distribute the bonus amount to the employees and deposit the remaining 80% to the government fund and 20 % in the welfare fund of the corporation.

Study the news except given above and answer the given questions.

- (a) Why would a company provide bonus?
- (b) On what basis is the bonus distributed?
- (c) Does a company distribute bonus if it cannot each profit?

In addition to the regular salary that an employee receives from his or her work, a certain percentage of that profit is paid annually to the employee as a lump sum incentive if the employee is able to earn a profit from the organization in which he or she works is called the bonus. In case of Nepal, if Nepal Electricity Authority, Nepal Telecom, Nepal Oil Corporation, Nepal Water Supply Corporation and other private institutions including several banks make a profit, the employees working in such institutions will get a lump sum bonus annually as per the rules and regulations.

Example 1

An industry has made a profit of Rs. 40,00,000 in a year. It has decided to distribute 60 % of profit to 80 employees working in that industry, find out how much amount each employee will receive as a bonus that year.

Solution,

Total profit = Rs. 40,00,000

Bonus amount to be distributed = 60% of Rs. 40,00,000

$$\begin{aligned} &= \frac{60}{100} \times 40,00,000 \\ &= \text{Rs. } 24,00,000 \end{aligned}$$

Total number of employees = 80

$$\begin{aligned} \text{Bonus amount received by an employee} &= \text{Rs. } \frac{24,00,000}{80} \\ &= \text{Rs. } 30,000 \end{aligned}$$

∴ Each employee receives Rs. 30,000 as a bonus in that year.

Example 2

A hotel has made a profit of Rs. 50,00,000 in a year. A certain percentage of profit was distributed to 55 employees working in that hotel equally at the rate of Rs. 50,000, what percentage of profit did the hotel distribute? Calculate.

Solution

Total yearly profit of the hotel = Rs. 50,00,000

Bonus distributed to each employee = Rs. 50,000

Total number of employee = 55

Percentage of bonus distributed = ?

Total amount distributed for bonus = $55 \times \text{Rs. } 50,000$

$$= \text{Rs. } 27,50,000$$

$$\begin{aligned} \therefore \text{Percentage of bonus distributed} &= \frac{274,500,000}{50,00,000} \times 100\% \\ &= 55\% \end{aligned}$$

3.3 Dividend

Activity 2

The edited excerpt of the news item entitled " Telecom Passed Dividend" published in Gorkhapatra Online on 14th Chaitra, 2070 BS is as follows:

Nepal Telecom Company has passed a proposal to distribute cash dividend to the shareholders at the rate of Rs. 40 per share from the profits of the fiscal year 2076/77.

Study the above news excerpt.

Answer the question given below.

- (a) What is share?
- (b) Have you or a member of your family bought shares in a company?
- (c) What is dividend? On what basis does the company distribute dividends?
- (d) Companies distribute a portion of their profit to shareholders each year. Have you or your family received such amount?

If a certain percentage of the profit earned by the company on the basis of investing in shares in a company or profit oriented organization is given to the shareholder, then that amount is called dividend. When distributing dividends, a certain percentage of the net profit of the corporation is divided by the number of shares. There are two types of such dividends:

- (a) Cash dividend
- (b) Share dividend

Example 3

If a microfinance company has 6,00,000 unit of shares at the rate of Rs.100 and that company has made a profit of Rs.40,00,000 in a year. If the company decides to distribute 20 % of the profits as the cash dividend to its shareholders. Find how much cash dividend does the person who owns 60 shares of that company in the year?

Solution,

Here, total profit = Rs. 40,00,000

Cash dividend to be distributed = 20% of Rs. 40,00,000

$$\begin{aligned} &= \frac{20}{100} \times 40,00,000 \\ &= \text{Rs. } 8,00,000 \end{aligned}$$

Again, total number of shares = 6,00,000

$$\begin{aligned}\text{Cash dividend for one share} &= \frac{8,00,000}{6,00,000} \\ &= \text{Rs. } 1.33\end{aligned}$$

The dividend received from the company by the person having 60 shares = $60 \times \text{Rs. } 1.33$
= Rs. 80

Exercise 3.2

1. Calculate the bonus based on the following table:

Company	Profit	Rate of Bonus
(a)	Rs. 60 lakh	25%
(b)	Rs. 2 crore 50 lakh	67.5%
(c)	Rs. 80 lakh	48%
(d)	Rs. 9 crore	55%

2. A textile industry has made a profit of Rs. 5 lakhs and sixty thousand in the year 2076 BS. It has decided to distribute 60 % of the profits as a bonus to 105 employees working in the industry, how much amount each will an employee receive? Find out.
3. An insurance company has decided to distribute two third of the total profit equally to its 150 staffs as a bonus. If each staff receives Rs. 40,000, what is the total profit of the company in that year?
4. **Calculate the total dividend and per share dividend from the given table:**

Company	Total number of shares	Rate of Dividend	Net profit
(a)	4,50,000	40%	Rs. 50,00,000
(b)	1,15,62,486	55.5%	Rs. 12,00,00,000
(c)	3,28,50,000	23.5%	Rs. 5,20,60,000
(d)	21,63,25,030	60%	Rs. 2,75,67,36,894

5. Suryalaxmi cooperative has 13,63,497 units of shares of Rs. 100 each. Mamata has 350 units of share of that cooperative. If that cooperative earns Rs. 1 crore and 20 lakh in one year and decides to distribute cash dividend to its shareholder equal to 45 % of the profit, how much cash dividend does Mamata receive? Find out.

6. An insurance company having 22,39,440 units of shares of Rs.100 each earns profit Rs. 1,52,05,675 in a year. If the board of directors of that company has decided to distribute cash dividend to its shareholder equals to 65 % of total profit, how much cash dividend does a shareholder holding 280 units of shares receive? Calculate.

Project Work

Divide your classmates into 5 groups. Visit 5 different public or private financial company/organization. Collect the information of 5 companies about how they distribute bonus to their employees and 5 companies how they distribute dividend to their shareholders. Prepare report in a group and present in the classroom.

Answers

1. (a) Rs. 1,5,00,000 (b) Rs. 1,68,75,000
(c) Rs. 38,40,000 (d) Rs. 4,95,00,000

2. Rs. 3,200

3. Rs. 90,00,000

4. (a) Rs. 20,00,000 and Rs. 4.44 (b) Rs. 6,66,00,000 and Rs. 5.76
(c) Rs. 1,22,34,100 and Rs. 0.37 (d) Rs. 1,65,40,42,136 and Rs. 7.65

5. Rs. 1,386.14

6. Rs. 1,235.77

4.0 Review

Discuss and resolve the following questions:

- If the cost of 25 copies is Rs. 500, what is the cost of one copy?
- If the cost of 1 dozen copies is Rs. 600, what is the value of 7 copies?
- How much is it if you add 10 % amount to your Rs. 750?
- What is the amount of 2 % discount on a calculator with a price of Rs 500? What is the price after discount?

We have already studied the calculation of unitary method, discount amount and value added tax amount in the previous class. The use of them makes it easy to calculate from the basic mathematical operations while paying the electricity bill, water and telephone bill consumed in our daily life.

Household expenses like electricity tariff, water tariff, telephone tariff, taxi fare etc. can also be paid online using apps in mobile or computer set. Nowadays, due to information and technology, the compulsion to physically attend the office for hours to pay various tariffs and fees is coming to an end.

4.1 Household expenses for use of electricity**Activity 1**

The Nepal Electricity Authority has fixed the tariff rates for electricity services provided according to the capacity of electricity meter.

According to the electricity consumer tariff rate fixed from the 140 th meeting of the Electricity Regulatory Body in 2078/7/8,

Kilowatt-Hour unit	5 Ampere		15 Ampere		30 Ampere		60 Ampere	
	Service Charge (Rs.)							
0 – 20	30	0	50	4.00	75	5	125	6
21 – 30	50	6.50	75	6.50	100	6.50	125	6.50
31 – 50	50	8.00	75	8.00	100	8.00	125	8.00
51 – 100	75	9.50	100	9.50	125	9.50	150	9.50
101 – 250	100	9.50	125	9.50	150	9.50	200	9.50
Above 250	150	11.00	175	11.00	200	11.00	250	11.00

Note: In case of 5 Ampere customer, if they consume more than 20 Kilowatt hour unit per month, energy charges up to 1 -20 Kilo watt hour unit will be charged at the rate of Rs. 3 per unit.

Study the above rate of tariff and the given bill, then discuss the questions given below with your friends:

Note: Kilo Watt is written as KW in short form.

- (a) What month is the given bill for?
- (b) According to the bill, how much capacity of meter is fixed?
- (c) What capacity of meter is fixed in your house?
- (d) How many units of electricity can be seen consumed in this bill this month?
- (e) According to this bill, what is the minimum charge to be paid?
- (f) How do you calculate the minimum charge of the meter fixed in your house?
- (g) How much is the energy charge on the bill?
- (h) What is meant by minimum charge in the above tariff rate table? What is meant of energy charge?
- (i) What are the details mentioned in the bill lading from the automatic machine currently used in your meter?
- (j) According to this bill, on which date was the meter reading done?



According to the electricity consumer tariff rate fixed from the 140th meeting of the Electricity Regulatory Body in 2078/7/8, the charge for single phase electricity consumption is billed as follows:

Method of Billing (for 5Ampere)			
S.N.	Consumed unit block	Rate per unit (Rs.)	Way of Billing
1.	0-20	0.00	Minimum charge is Rs. 30 and no energy charge
2.	From 21 to 30	6.50	Minimum charge: Rs. 50 Energy charge: up to 20 units at the rate of Rs.3 per unit, 21 to 30 units at the rate of Rs.6.50 per unit has to be paid.
3.	From 31 to 50	8.00	Minimum charge: Rs. 50 and Energy charge: up to 20 units at the rate of Rs.3 per unit, <ul style="list-style-type: none">• 21to 30 units per unit Rs.6.50• 31 to 50 units at the rate of Rs. 8.00 has to be paid
4.	From 51 to 100	9.50	Minimum charge: Rs. 50 and Energy charge: up to 20 units at the rate of Rs.3 per unit, <ul style="list-style-type: none">• 21to 30 units per unit Rs.6.50• 31 to 50 units at the rate of Rs. 8.00• 51 to 100 units at the rate of Rs. 9.50 per unit has to be paid.
5.	From 101 to 250	9.50	Minimum charge: Rs. 50 and Energy charge: up to 20 units at the rate of Rs.3 per unit, <ul style="list-style-type: none">• 21to 30 units per unit Rs.6.50• 31 to 50 units at the rate of Rs. 8.00• 51 to 250 units at the rate of 9.50 per unit has to be paid.

6.	Above 250 units	11.00	Minimum charge: Rs. 50 and Energy charge: up to 20 units at the rate of Rs.3 per unit, <ul style="list-style-type: none">● 21to 30 units per unit Rs.6.50● 31 to 50 units at the rate of Rs. 8.00● 51 to 250 units at the rate of 9.50● Above 250 units at the rate of Rs. 11 per unit has to be paid.
7.	Above 400 units	12.00	Minimum charge: Rs. 50 and Energy charge: up to 20 units at the rate of Rs.3 per unit, <ul style="list-style-type: none">● 21to 30 units per unit Rs.6.50● 31 to 50 units at the rate of Rs. 8.00● 51 to 250 units at the rate of 9.50● 251 to 400 units at the rate of Rs. 11 per unit● Above 400 units at the rate of Rs. 12.00 per unit has to be paid.

Note: Similarly, the billing of 14, 30 and 60 Ampere are done.

The following provision of discount and penalties are provisioned for the customer in the rules while paying the tariff.

1. 2% discount is given if the bill is paid within 7 days from the date of meter reading.
2. Amount is taken according to the bill if it is paid from 8th day to 15th day from the meter reading date.
3. 5 % extra charge (fine or penalty) is added to the amount of bill if the bill is paid from 16th day to 30th day from the meter reading date.
4. 10 % extra charge (fine or penalty) is added to the amount of bill if the bill is paid from 31st day to 40th day from the meter reading date.
5. 25 % extra charge (fine or penalty) is added to the amount of bill if the bill is paid from 41st day to 60th day from the meter reading date.
6. If the electricity tariff is not paid by the 60th day of the meter reading date, the power will be cut off at any time without notice. The connection and the remaining amount will be recovered from the concerned customer like the government balance and the reconnection fee will be Rs.500.

The tariff rate of Nepal Electricity Authority varies according to the energy capacity of the meter connected to our house. When installing a meter with more capacity, we have to pay more than the minimum tariff. Similarly, the more electricity consumed, the more tariffs have to be paid. When the tariff is calculated, the amount received by the service provider is understood as the amount charged for consuming electricity. Currently, the Nepal Electricity Authority (NEA) has given high priority to the use of information technology (IT) for meter reading and payment of electricity tariff to the customer.

If the electricity tariff is paid within one week from the date of meter reading, exemption will be given, after one week if it is paid within 15 days, there is no discount or it is according to the bill amount, fine to pay if the tariff is paid after 15 days and if the tariff is not paid within 60 days, the Electricity Authority can cut the power line at any time and charges re connection fee are mentioned in the regulation.

Example 1

A meter of 5 Ampere capacity is fixed in Rambaran's house. Find out the electricity bill for the month of Push, 2078 BS by calculating the answers of the following questions based on the given two details:

PRESENT RDG = 42973

PREVIOUS RDG = 42828

RDG here in the bill means reading.

- How many units of electricity has been consumed in Rambaran's house in the month of Push?
- How much is the total tariff?
- If the tariff is paid on the 5th day from the date of meter reading, how much was the discount?
- How much money is required to pay the tariff on the 13th day from the date of meter reading?
- If the tariff was paid on the 22nd day of the meter reading, how much would Rambaran have been fined?
- If he had paid the tariff on the 39th day from the date of meter reading, how much fine would he have to pay?
- If the tariff is to be paid on the 50th day from the date of meter reading, now how much is required to be paid to Nepal Electricity Authority with additional fee? Calculate.

Solution,

Here, Present reading = 42973

Previous reading = 42828

- (a) Consumed unit in the month of Push in Rambaran's house = $42973 - 42828$
= 145 units

- (b) Let's study the rate of tariff given in the previous page,
Being consumed unit is 145 units, and the capacity of meter is 5 ampere, 145 units lies in the group 101 to 150. According to that group 145 units can be broken as follows:

$$145 \text{ units} = 20 \text{ units} + 10 \text{ units} + 20 \text{ units} + 95 \text{ units}$$



$$\text{Minimum charge} = \text{Rs.}100$$

$$\begin{aligned}\text{Total energy charge} &= 20 \times 3 + 10 \times 6.50 + 20 \times 8.00 + 95 \times 9.50 \\ &= 60 + 65 + 160 + 902.50 \\ &= \text{Rs. } 1187.50\end{aligned}$$

$$\text{Minimum charge of the group (101-150)} = \text{Rs. } 100$$

$$\begin{aligned}\text{Total tariff} &= \text{Minimum charge} + \text{Total energy charge} \\ &= \text{Rs. } 100 + \text{Rs. } 1187.50 \\ &= \text{Rs. } 1287.50\end{aligned}$$

- (c) 2 % discount is given if the bill is paid within 7 days from the date of meter reading.

Here, if Rambaran pays the tariff on the 5th day,

$$\begin{aligned}\text{Discount amount in tariff} &= 2\% \text{ of } \text{Rs. } 1287.50 \\ &= \frac{2}{100} \times \text{Rs. } 1,287.50 \\ &= \text{Rs. } 25.75\end{aligned}$$

- (d) As a rule, the amount is paid according to the bill if it is paid on 8th day to 15th day from the date of meter reading

Here, if Rambaran pays the tariff on the 13th day, he has to pay the bill amount of Rs.1287.50

- (e) As a rule, 5% extra charge has to pay if the bill is paid on 16th day to 30th day from the date of meter reading.

Here, if Rambaran pays the bill on 22th day, he has to pay 5% extra charge.

So, the extra charge amount to be paid = 5 % of Rs.1287.50

$$= \frac{5}{100} \times \text{Rs. } 1,287.50 \\ = \text{Rs. } 64.38$$

- (f) As a rule, 10 % extra charge has to pay if the bill is paid on 31st day to 40th day from the date of meter reading.

Here, Rambaran pays the bill on 39th day,

Extra charge to be paid (fine amount) = 10 % of Rs.1287.50

$$= \frac{10}{100} \times \text{Rs. } 1,287.50 \\ = \text{Rs. } 128.75$$

- (g) As a rule, 25% extra charge has to pay if the bill is paid on 41st day to 60th day from the date of meter reading.

Here, Rambaran pays the bill on the 50th day,

Fine amount = 25 % of Rs.1287.50

$$= \frac{25}{100} \times \text{Rs. } 1,287.50 \\ = \text{Rs. } 321.88$$

∴ Total amount of tariff with fine amount = Rs. 1,287.50 + Rs. 321.88

$$= \text{Rs. } 1609.38$$

Exercise 4.1

1. A meter of capacity 5 ampere capacity is installed in Dambar Kumari Tamang's house. The meter reading of her house from the month Kartik to Fagun of the year 2078 BS is given in the following table. Study the table and answer the questions given below:

Month	Kartik	Manshir	Push	Magh	Fagun
Meter Reading Unit	3528	3593	3700	3904	3980

- (a) What is the total unit of electricity consumed on the month of Manshir?
- (b) In which months did Dambar Kumari's house consume the most and the least amount of electricity?
2. According to the current tariff rate, calculate the tariff to be paid to Nepal Electricity Authority in the following cases:
- (a) Present reading: 2575, Previous reading: 2472,
Capacity of meter: 5 Ampere
The bill is paid on the 4th day of meter reading.
- (b) Present reading: 3036, Previous reading: 2831,
Capacity of meter: 15 Ampere
The bill is paid on the 41st day of meter reading.
- (c) Present reading: 3603, Previous reading: 3294,
Capacity of meter: 30 Ampere
The bill is paid on the 39th day of meter reading.
- (d) Present reading: 4311, Previous reading: 3944,
Capacity of meter: 60 Ampere
The bill is paid on the 14th day of meter reading.
- (e) Present reading: 5555, Previous reading: 5107,
Capacity of meter: 30 Ampere
The bill is paid on the 17th day of meter reading.
- (f) Present reading: 6452, Previous reading: 6443,
Capacity of meter: 5 Ampere
The bill is paid on the 57th day of meter reading.

Project Work

Collect the electricity bills of 6 months of your house or school. Show the teacher individually the process of calculation tariff with bills obtained from automatic machine.

Answers

- | | | |
|-------------------|--|------------------|
| 1. (a) 65 units | (b) Maximum: Magh and Minimum: Manshir | |
| 2. (a) Rs. 870.73 | (b) Rs. 2,378.13 | (c) Rs. 3,381.40 |
| (d) Rs. 3,782 | (e) Rs. 4,833.15 | (f) Rs. 37.50 |

4.2 Household expenses for use of water

Activity 1

Study the given bill of water consumed through a pipe of size $\frac{1}{2}$ inch and the rate of tariff table, then discuss on the questions given below, where 1000 liter = 1 unit.

- What are the details mentioned on the bill?
- How much is the bill amount?
- What is the discount amount on the bill?
- How much amount is paid for penalty on the bill? Why?
- How much is the arrears amount on the bill?
- How much is the sewerage charge?
- According to this bill, how many units of water have been consumed in the calculated month?

Kathmandu Upatyaka Khanepani Limited

Branch: Tripureshwor
PAN No.: 600041601



PAYMENT DUPLICATE RECEIPT

Receipt No.	:	1115117730000577
Date	:	2077-11-17
Customer ID	:	1115004313
Connection No	:	20198
Area No.	:	14C-17-55
Name	:	Jana bikas pra.bi.
Payment Mode	:	Cash
Cheque No	:	
Arrears	:	1083.00
Water Charges	:	100.00
Sewerage Charges	:	50
Meter Rent Charges	:	0.00
Miscellaneous	:	0.00
Penalty	:	200.50
Rebate	:	13.95
Previous Balance	:	0.00
Bill Amount	:	1238.00
Total To Be Paid	:	1424.55
Total Amt Paid Rs.	:	<u>1425.00</u>
Payment Period	:	2077 Shrawan - 2077 Falgun
Advance	:	0.45
Advance Rebute	:	0.01
Balance (After Paid)	:	0.46
Received By	:	(rbhattarai - 3)
----- Computer Generated Receipt -----		

Kathmandu Upatyaka Khanepani Limited

Head Office, Tripureshwor

Tariff rate table applicable from Shrawan 1, 2070

Size of pipe (in inch)	Minimum consumption (in litre)	Minimum unit	In the meter connected taps	
			Minimum charge (in Rs.)	Based on the volume when more water is used (per 1000 litre) Rs.
½"	10000	10	100	32
¾"	27000	27	1,910	71
1"	56000	56	3,960	71
1.5"	155000	155	10,950	71
2"	320000	320	22,600	71
3"	881000	881	62,240	71
4"	1810000	1810	1,27,865	71

Note: 50% tariff is fixed for the sewage service being taken by the company at the tariff.

The above bill is calculated at the following tariff rate. According to the rules, a minimum of 10000 litre or 10 units of water (1 unit = 1000 litre) is consumed in a ½ inch pipe.

The exemption and penalty for payment of drinking water tariff will be as presented in the table below:

Payment (by the date the bill is distributed)	Discount / Fine
Within the first and second month	3 % discount
Within the third month	As per bill
Within the fourth month	10% Fine
Within the fifth month	20% Fine
After the fifth month	50% Fine

Example 1

A $\frac{1}{2}$ inch-sized pipe was found in the house of Sadikshya and the meter reading of water was 3754 during the month of Chaitra. If the meter reading in the month of Fagun was 3727, what is the tariff of Chaitra including 50% sewerage service? What will be the total tariff to be paid if the bill is paid within the second month?

Solution,

Here, Present reading = 3,754

Previous reading = 3,727

$$\begin{aligned}\text{Consumed water} &= 3754 - 3727 \\ &= 27 \text{ units}\end{aligned}$$

Charge of minimum 10 units (10000 litre) = Rs.100

Here, 27 unit = 10 units + 17 unit

  Minimum charge Rs.100 at the rate of Rs.32 per unit

$$\begin{aligned}\therefore \text{Total tariff} &= 100 + 17 \times 32 \\ &= 100 + 544 \\ &= \text{Rs.644}\end{aligned}$$

Sewerage Charge = 50 % of Rs.644

$$\begin{aligned}&= \frac{50}{100} \times 644 \\ &= \text{Rs. 322}\end{aligned}$$

$$\text{Total amount of bill} = \text{Rs.644} + \text{Rs.322} = \text{Rs.966}$$

Thus, the total amount to be paid by Sadikshya in the month of Chaitra for consuming water = Rs.966

As per rule, if Sadikshya pays the tariff within the second month from the date of bill distributing, she will get a discount of 3 %,

$$\begin{aligned}\text{Discount amount} &= 3 \% \text{ of Rs.966} \\ &= \frac{3}{100} \times \text{Rs. 966} \\ &= \text{Rs. 28.98}\end{aligned}$$

$$\begin{aligned}\therefore \text{The amount to be paid by Sadikshya within the second month} &= \text{Rs. 966} - \text{Rs. 28.98} \\ &= \text{Rs. 937.02}\end{aligned}$$

Example 2

123 units of water was consumed in a month at Central colony from a 1 inch sized water supply pipe. If the bill is paid within the fourth month from the bill distribution date, how much total fine amount has to be paid?

Solution,

Total consumption of water = 123 unit

Tariff for consumption of minimum 56 units (56000 liter) of water = Rs. 3,960

$$\begin{aligned}\text{Here, } 123 \text{ unit} &= 56 \text{ unit} + (123 - 56) \text{ unit} \\ &= 56 \text{ units} + 67 \text{ units}\end{aligned}$$



Minimum charge Rs.3960 at the rate of Rs.71 per unit

$$\begin{aligned}\therefore \text{Total tariff} &= \text{Rs.}3960 + 67 \times \text{Rs.}71 \\ &= \text{Rs.}8717\end{aligned}$$

$$\begin{aligned}\text{50 \% tariff for sewerage service} &= 50 \% \text{ of Rs.}8,717 \\ &= \frac{50}{100} \times \text{Rs.}8717 \\ &= \text{Rs.} \frac{8717}{2} \\ &= \text{Rs.}4358.50\end{aligned}$$

$$\begin{aligned}\therefore \text{Total amount of bill} &= \text{Rs.}8,717 + \text{Rs.}4,358.50 \\ &= \text{Rs.}13,075.50\end{aligned}$$

If the bill is paid within the fourth month of distribution, an additional 10 % charge or fine or penalty has to be paid.

So, fine amount = 10 % of Rs. 13,075.50

$$\begin{aligned}&= \frac{10}{100} \times 13,075.50 \\ &= \text{Rs.}1,307.55\end{aligned}$$

Exercise 4.2

1. There is $\frac{1}{2}$ inch drinking water supply pipe is connected in Kopila's house. The current and previous reading in the month of Shrawan are 4225 and 4197 respectively. Solve the following questions on the basis of tariff table given on the previous page:
 - (a) How much does Kopila have to pay for 50% of that month's sewage service?
 - (b) What is the total bill amount?
 - (c) If the bill is paid within the first and second month of distribution, how much tariff will Kopila have to pay?
 - (d) If the bill is paid within the third month of distribution, how much tariff will she have to pay?
2. 423 unit of water was consumed in a month at a hotel from a 1.5" sized water supply pipe. Solve the following questions on the basis of tariff table given on the previous page:
 - (a) What will be the total tariff including sewerage in that month?
 - (b) If the bill is paid within five months of the distribution of the bill, how much penalty has to be paid?

Answers

1. (a) Rs. 338 (b) Rs. 1,014 (c) Rs. 983.58 (d) Rs. 1,014
2. (a) Rs. 44,967 (b) Rs. 53,960.40

4.3 Household expenses for use of telephone

Activity 1

Study the given bill and answer the following questions:

	नेपाल टेलिकम Nepal Telecom (Nepal Doorsanchar Company Limited)	TAX INVOICE	TPIN/PAN : 300044614
Transaction No.	541427661	Invoice No.	R21062141402621
Customer Company's Name	Date (B/S A.D.)
Hesco Account No.	6001190372	Customer's TPIN/PIN
Subscription Number	015531957	Subscription Type	PSTN
Payment Type	PAYMENT/ADVANCE PAYMENT	Payment Mode	Cash
Bal of	Baisakh Jatra of 2078 Chaitra of 2077		
Charge Items	Charge(NPR)	TSC(NPR)	VAT(NPR)
Bill Amount	464.01	52.52	59.34
Fine	7.21	0.00	0.00
Maintenance	0.00	0.00	0.00
Rebate	0.0	0.00	0.00
Adjustment	0.00	0.00	0.00
Advance	1.92	0.00	0.00
OT	0.00	0.00	0.00
Sum Amount	413.14	52.52	59.34
Invoice Amount	525.00		
IPTV Cash Received	0.00		
Total Receivable Amount	525.00		
Tender Amount Rs	525.00	Return Amount Rs	0.00
			Total Received Rs 525.00
Received Amount (In words)	Five Hundred And Twenty Five Rupees Only		
Counter Thecho	Received By	No. Of Copy	1 [Original]
Note: In case of Deposit and Advance Payment, this invoice is considered as Receipt as well IPTV Tax invoice is provided by NT Partner's Billing System			

- (a) What details are mentioned in the bill?
- (b) What is the amount of value added tax in the bill?
- (c) What is the total tariff?

The above bill is related to local call tariff. On the basis of rule of local call, tariff of minimum 175 calls is Rs.200, After that, tariff is calculated by adding Re. 1 per call extra amount. Costumer has to pay 10 % service charge (for service provider company) of the tariff of total calls. 13 % value added tax amount of total tariff with service charge should be deposited to Nepal Government as a revenue.

So, the total tariff (TC) = minimum tariff + extra tariff

Service charge (TSC) = 13 % of total tariff

Value added tax (VAT) amount = 13 % of (TC + TSC)

Total tariff = TC + TSC + VAT amount

Tariff Rate of GSM Prepaid of Nepal Telecom

S.N.	Types of Service	Charge
1.	On-Net call	Rs. 1.50 per minute
	Off-Net call	Rs. 2 per minute
2.	SMS service	Rs. 1 per SMS (in network of telecom)
		Rs. 1.25 per SMS (in other networks)
3.		Rs. 5 per SMS (for international SMS)
	Friends and family service (FNF service)	Rs. 0.70 per minute for maximum 5 persons
4.	Video call charge	Rs. 2 per minute

Tariff Rate of GSM Postpaid of Nepal Telecom

S.N.	S.N.	Peak hour 6:00 AM - 18:00 PM	Off peak hour 10:00 PM - 6:00 AM	Monthly rental charge
1.	On-net call	Re. 1 per minute	Re. 0.55 per minute	Rs. 300
	Off-net call	Re. 1.5 per minute	Re. 1.5 per minute	
2.	Short message service (SMS)	Re. 1 per SMS (on - net sms)		
		Rs. 5 per international SMS		
3.	FNF Service	Rs.0.55 per minute for maximum 6 person		
4.	Video call charge	Rs. 2 per minute		

Note:

1. Tax is not included in the above tariff rate.
2. 13 % telecom service charge and 13 % value added tax are charged on the tariff. (Applicable from Shrawan-1, 2075)

3. 2 % ownership tax (OT) is charged on the total tariff including 13% telecom service charge and 13 % value added tax.
4. Detail information is in the website www.ntc.net.np of Nepal Telecom.

General Notice:

Please pay your bills on time to avoid call restriction by Credit Control System.

To know due amount and available credit limit, please type CB for GSM/ CDMA Postpaid and send sms to 1415, CB*PSTN Number and send sms to 1545.

13% TSC & 13% VAT will be applied as per Government rule from 1st Shrawan 2075.

GSM/CDMA Outgoing calls will be blocked (1-Way bar) after finishing remaining credit limit. Outgoing and incoming both calls will be blocked (2-Way bar) on crossing 90 days of 1-way bar.

Both Outgoing and Incoming calls will be blocked (2-Way bar)) automatically in PSTN after finishing remaining credit limit.

For PSTN Numbers distributed from Soft Switches like NGN, IMS MSAN, MSAG: Meter reading value remains same in each month bill statement; local calls are charged based on call data record (CDR), not on meter reading.

To download monthly Bill statement, please visit: <http://gsmbl.ntc.net.np/>
For any complaints, Please contact nearest customer care center or dial 191.1498.

2% Ownership Tax (OT) will be applied on every GSM and CDMA Postpaid mobile recharge as per Government rule from 1st Shrawan 2077.

Example 1

If there is 267 call in a month from PSTN telephone line,

- Find the total charge (TC) of that month.
- Calculate the service charge (TSC) and value added tax (VAT).
- What is the total telephone tariff of that month?

Solution,

Here, the total telephone call of one month is 267.

And minimum charge of 175 call = Rs.200

$$\text{Now, } 267 \text{ call} = 175 \text{ call} + (267 - 175) \text{ call}$$

$$= 175 \text{ call} + 92 \text{ call}$$

Minimum charge Rs. 200 at the rate of Re.1 per call

$$\begin{aligned}\therefore \text{Total tariff (TC)} &= \text{Rs. } 200 + 92 \times \text{Re. } 1 = \text{Rs. } 200 + \text{Rs. } 92 \\ &= \text{Rs. } 200 + \text{Rs. } 92 \\ &= \text{Rs. } 292\end{aligned}$$

Service charge (TSC) = 13% of Rs. 292

$$\begin{aligned}&= \frac{13}{100} \times \text{Rs. } 292 \\ &= \text{Rs. } 37.96\end{aligned}$$

$$\begin{aligned}\text{Total tariff including (C + TSC) including service charge (TSC)} &= \text{Rs. } 292 + \text{Rs. } 37.96 \\ &= \text{Rs. } 329.96\end{aligned}$$

Now, value added tax amount = 13 % of Rs.329.96

$$\begin{aligned}&= \frac{13}{100} \times \text{Rs. } 329.96 \\ &= \text{Rs. } 42.89\end{aligned}$$

$$\begin{aligned}(c) \text{Total tariff} &= C + TSC + \text{VAT amount} \\ &= 292 + 37.96 + 42.89 \\ &= \text{Rs. } 372.85\end{aligned}$$

\therefore The total telephone tariff of that month = Rs. 372.85

Alternative Method,

Total call = 267 call

Extra call = $267 - 175 = 92$ call

$$\begin{aligned}\text{Total tariff (TC)} &= \text{minimum charge} + \text{extra charge} \\ &= \text{Rs. } 200 + \text{Rs. } 92 \times 1 \\ &= \text{Rs. } 292\end{aligned}$$

$$\begin{aligned}\text{Tariff including TSC and VAT} &= 113\% \times 113\% \times \text{TC} \\ &= \frac{113}{100} \times \frac{113}{100} \times 292 \\ &= \text{Rs. } 372.85\end{aligned}$$

Example 2

Under GSM prepaid mobile service of Nepal Telecom, when recharging from a recharge card of Rs.100, how much rupees of talk time (voice call) can be used by the concerned costumer? 13 % telecom service charge (TSC), 13 % value added tax and 2% ownership tax (OT) are included in that Rs.100. If the rate of voice call charge within NT Network is Rs.1.50 per minute, how many minutes can be talked? Find out.

Solution,

Telecom service charge (TSC) = 13%

Value added tax (VAT) = 13%

Ownership tax (OT) = 2%

Recharge amount = Rs.100

Let, the price before adding OT = Rs. x

$\therefore x + 2\% \text{ of } x = \text{Rs. } 100$

$$\text{or, } x + \frac{2}{100} \times \text{Rs. } x = 100$$

$$\text{or, } \frac{102x}{100} = 100$$

$$\therefore x = 98.04$$

Again, let the price before applying 13 % VAT = Rs. y

$\therefore y + 13\% \text{ of } y = 98.04$

$$\text{or, } y + y \times \frac{13}{100} = 98.04$$

$$\text{or, } \frac{113y}{100} = 98.04$$

$$\therefore y = 86.76$$

Similarly,

Let the price before adding TSC = Rs. z

$$\therefore z + 13\% \text{ of } z = \text{Rs. } 86.76$$

$$\text{or, } z + z \times \frac{13}{100} = 86.76$$

$$\text{or, } \frac{113z}{100} = 86.76$$

$$\therefore z = 76.78$$

∴ While recharging of Rs.100, talk time equal to Rs.76.78 can be used.

There is a charge of Rs.1.50 per minute for making phone calls and talking within the network of Nepal Telecom.

$$\therefore \text{Time to talk for Rs.} 76.78 = \frac{76.78}{1.50} \text{ minute}$$

∴ By using the recharge card of Rs.100, we can talk for 51.10 minutes in phone call within the network of Nepal Telecom.

Exercise 4.3

- 1.** The current reading telephone calls of Ramlal's house in the month of Baishakh is 4444 and previous reading is 3992, calculate:

 - How many total calls have been made?
 - What is the total tariff (TC) of the calls if Rs.200 is charged for first 175 calls and after that Re. 1 per call?
 - Calculate service charge and value added tax amount.
 - Calculate the total tariff after adding value added tax.

2. Calculate the total tariff including 13% service charge (TSC) and 13% value added tax for the following telephone calls: (Where Rs.200 is charged for first 175 calls and after that Re. 1 per call is added)

 - 550 calls
 - 695 calls
 - 793 calls

3. Minimum tariff for the first 175 calls is Rs.200 and after that Re. 1 is charged for each extra call. How many telephone calls can be made from the following amount excluding service charge (TSC) and value added tax?

 - Rs. 275
 - Rs. 695
 - Rs. 890

4. Minimum tariff for the first 175 calls is Rs.200 and after that Re. 1 is charged for each extra call. If the total tariff including 13 % service charge and 13 % value added tax is Rs.696.08, how many telephone calls had been made?
5. Minimum service charge (Rental charge) of GSM postpaid mobile service of Nepal Telecom is Rs.300. After applying 13 % service charge on this amount, 13 % value added tax is added. If after that again 2 % ownership tax is added on the total tariff, how much minimum monthly tariff has to be paid by a postpaid sim user?

Answers

- | | | |
|------------------|-------------------|--|
| 1.(a) 452 calls | 1.(b) Rs. 477 | 1.(c) Rs. 47.70, Rs. 68.211.(d) Rs. 592.91 |
| 2.(a) Rs. 714.73 | 2.(b) $Rs=894.96$ | 2.(c) $Rs=1016.77$ |
| 3.(a) 250 calls | 3.(b) 670 calls | 3.(c) 865 calls |
| 4. 535 calls | 5. Rs. 390.73 | |

4.4 Calculation of amount of taxi meter

Activity 1

Discuss the following questions:

- (a) Have you ever used taxi to go from one place to another?
- (b) Have you seen noticeboard about taxi fare somewhere?
- (c) How much taxi meter is maintained by the office under the Nepal Bureau of Standards of Metrology (NBSM) at present? How and from where can we get the information about it?

Taxies used in urban areas may or may not be connected with meter. The fares of metered taxies are uniform and the chances of fraud are also very low. Complaints can be lodged with the nearest Traffic Police Office along with the taxi number in case of any inconvenience related to taxi fare and fraud.

The rates of fare mentioned here may change over a period of time.

The taxi fare rate posted on the website of the Nepal Bureau of Standards of Metrology are mentioned in the following table:

From 6.00 am to 9.00 pm	From 9 pm to 6 am
Initial rate: Rs.14	Initial rate: Rs.21
Per 200 meter: calculated at the rate of Rs.7.80 per km adding Rs.39, fare of initial 1km: Rs.53	Per 200 meter: calculated at the rate of Rs.11.70 per km adding Rs.58.50, fare of initial 1km: Rs.79.50
Additional fare rate per kilometer: Rs.39	Additional fare rate per kilometer: Rs.58.50
Waiting charge: Rs.7.80 per 2 minute	Waiting charge: Rs.11.70 per 2 minute

Example 1

Bimakumari took a taxi to travel 6 km from Balkhu to Putalisadak in Kathmandu. Initially the taxi meter charged a minimum of Rs.14 and then Rs. 7.80 per 200 meters. During the journey, if 10 minutes waiting charge at the rate of Rs.7.80 per 2 minute was also charged, how much taxi fare did she pay? Find it.

Solution,

Minimum (initial) charge = Rs. 14

The distance travelled by the taxi = 6 km

$$= 6 \times 1000 \text{ m}$$

$$= 6000 \text{ m}$$

Here, the fare of 200 m = Rs. 7.80

$$\text{Fare of 1 m} = \frac{7.80}{200}$$

$$\text{Fare of } 6000 \text{ m} = \frac{7.80}{200} \times 6000 = \text{Rs. 234}$$

$$\begin{aligned}\text{The amount for waiting charge} &= \frac{7.80}{2} \times 10 \\ &= \text{Rs. 39}\end{aligned}$$

$$\begin{aligned}\text{The total fare paid by Bimakumari} &= 14 + 234 + 39 \\ &= \text{Rs. 287}\end{aligned}$$

Alternative Method

Minimum (initial) charge = Rs. 14

The distance travelled by the taxi = 6 km

Fare per km = Rs. 39

Waiting time = 10 minutes

The amount for waiting charge per 2 minutes = Rs. 7.80

The amount for waiting charge for 10 minutes = $\frac{\text{Rs. 7.80}}{2} \times 10 = \text{Rs. 39}$

Total fare = Rs. 14 + $39 \times 6 + 39$

= Rs. 14 + 234 + 39

= Rs. 287

Exercise 4.4

1. Shreenath took a taxi to travel 9 km from Baneshwar to Bhaktapur. Initially the taxi meter charged a minimum of Rs. 14 and then Rs. 7.80 per 200 meters. During the journey, if 6 minutes waiting charge at the rate of Rs. 7.80 per 2 minute was also charged, how much taxi fare did he pay? Find it.
2. Shristi took a taxi to travel 18 km from Jagati, Bhaktapur to Sanepa, Lalitpur. At first, the taxi meter charged a minimum of Rs. 14 and then Rs. 7.80 per 200 meters. During the journey, if 20 minutes waiting charge at the rate of Rs. 7.80 per 2 minute was also charged, how much taxi fare did he pay? Find it.
3. Calculate the taxi fare in the following cases based on the rate mentioned in this lesson:

Distance (in km)	Time	Waiting time (in minute)
(a) 6	8 am	15
(b) 15	3:30 pm	10
(c) 7.5	11 pm	-
(d) 8.75	4 am	7

4. Samyog pays Rs.228.50 for taking a taxi with 10 minutes of waiting charge. Taxi meter shows the fare at first, a minimum of Rs.14 and then Rs.7.80 per 200 meters. If the waiting charge is at the rate of Rs.7.80 per 2 minutes, what distance does he travel?
5. Phulmaya payed Rs.1027.20 for taking a taxi including 12 minutes of waiting charge. She had to pay Rs 14 at first, and then Rs.11.70 per 200 meters. If the waiting charge was at the rate of Rs.11.70 per 2 minutes, what distance did she travel?

Project Work

Take a recharge card of Rs.100 of prepaid mobile. Calculate service charge, amount of value added tax mentioned in that card and present in your classroom.

Answers

- | | | | |
|-------------------|-------------------|-------------------|---------------|
| 1. Rs. 388.40 | 2. Rs.794 | 3. (a) Rs. 306.50 | 3.(b) Rs. 638 |
| 3. (c) Rs. 459.75 | 3. (d) Rs. 573.83 | 4. 4.5 km | 5. 16 km |

Miscellaneous Exercise

1. A person earning Rs. 7 lakhs per annum has to pay income tax depending on the number of dependent families as 1 % up to Rs. 4,50,000 and 10 % on the remaining income. How much income tax does he or she have to pay?
2. A person who earns Rs. 4,60,000 per annum, has to pay Rs. 5,600 income tax. If the rate of tax is 1% up to Rs. 4,50,000, then what is its rate for remaining amount? Find it.
3. If an organization has to pay the income tax at the rate of 2.5 % on its income up to Rs. 10 lakhs, 4 % on the income above Rs. 10 lakhs and up to Rs. 20 lakhs, 5. 25 % on the income above Rs. 20 lakhs and up to 40 lakhs and 12.75 % on the income above 40 lakhs, calculate the tax amount for the following yearly income:
 - (a) Rs. 7,60,000 (b) Rs. 15,70,500 (c) Rs. 23,75,600 (d) Rs. 55,75,000
4. What is the simple interest of Rs. 50,000 in 1 year at the rate of 2 paisa per Rs. 3 per month? If 5 % tax is levied on that interest, what is the net amount of simple interest?
5. **A shopkeeper bought a bicycle for Rs. 7,520 and sold it at a marked price of Rs. 10,000. If the rate of value added tax is 13 %,**
 - (a) How much does the costumer have to pay?
 - (b) What is the percentage of profit made by the shopkeeper in this transaction?
 - (c) If it had sold at a discount of 30 % on the marked price, what would be its value including value added tax?
6. A laptop was bought at Rs. 1,50,000 and sold at a profit of 10 %, what would be its value with 13 % value added tax?
7. If an item is sold at Rs. 9040 with 20 % discount and 13 % value added tax, how much discount amount is given? Find.
8. A dealer has bought a computer from an importer at Rs. 2,600 including value added tax. The dealer adds the transportation charge and profit then sells it to a retailer at Rs. 25,990 including value added tax. If the rate of value added tax is 13%, calculate the following:
 - (a) At what price did the dealer buy from the importer excluding value added tax?
 - (b) What was the profit of the retailer?
 - (c) How much amount of value added tax was deposited to government account?
 - (d) How much amount of value added tax should be deposited by the retailer to the governments account?

9. The monthly salary of a sales manager is Rs. 22,000 and 0.75% commission of total sales is added to his salary. If the sales of a month is Rs. 6,24,000, what will be his total income? How much percentage of income is added to his monthly salary?
10. A house owner gives 3 % commission of rent amount to an agent for renting his house. If the total monthly rent of the house is Rs. 75,000, how much amount does the agent receive in a year?
11. Rohan receives Rs. 1 crore, 28 lakhs and 25 thousand by selling his house after giving 5% commission to the agent, what was the selling price of his house? Find it.
12. 25 % of the profit of a company is Rs. 1,50,000. The company has decided to distribute a bonus of 65 % of the total profits equally to its 15 employees as an incentive, how much bonus does each employee receive?
13. If a bank has decided to distribute 55 % bonus of its profits in a year to its 275 employees at the rate of Rs. 65000 each, what was the total profit of the bank?
14. Out of 3,50,000 shares of a cement manufacturing company worth Rs. 100, Hari bahadur has 250 shares of IPO. If the company decides to pay 40 % cash dividend in case of annual profit of Rs. 9,00,00,000, how much dividend will the shareholder Hari bahadur get in total?
15. A meter of capacity 5 ampere capacity is connected in a house. The meter reading of that house from Baishakh to Ashoj are given in the following table:

Month	Baishakh	Jestha	Ashar	Shrawan	Bhadra	Ashoj
Meter Reading	1577	1622	1647	1662	1689	1731

From the above table, if the meter was read on the last day of each month, calculate the electricity tariff based on the current rate:

- (a) How many units of electricity have been consumed from the month of Jestha to Ashoj?
- (b) Which month's bill has the highest electricity tariff and how much?
- (c) Which month's bill has the lowest electricity tariff and how much?
- (d) How much is to be paid if the tariff for the month of Jestha is paid on 13th of Ashar?

Answers

1. Rs. 42,000
2. 10%
3. (a) Rs. 19,000
3. (b) Rs. 47,820
3. (c) Rs. 84,719
3. (d) Rs. 3,70,812.50
4. Rs. 4,000 and Rs. 3,800
5. (a) Rs. 1,13,00
5. (b) 32.98%
5. (c) Rs. 7,910
5. (d) Loss 6.91%
6. Rs. 1,86,450
7. Rs. 2,000
8. (a) Rs. 20,000
- (b) Rs. 2,000
- (c) Rs. 3,250
- (d) Rs. 260
9. Rs. 2,66,680 and 0.21%
10. Rs. 27000
11. Rs. 1,21,83,750
12. Rs. 26,000
13. Rs. 3,25,00,000
14. Rs. 25,714.29
15. (a) Jestha 45 units, Ashar 25 units, Shrawan 15 units, Bhadra 27 units, Ashoj 42 units
(b) Jestha, Rs. 332.50
(c) Shrawan, Rs. 75
(d) Rs. 332.50 as per bill
(e) Rs. 152.25
(f) Rs. 300.86
16. Rs. 6,783.15
17. (a) Rs. 383.07
(b) Rs. 446.92
(c) Rs. 630.79
18. (a) 575 calls
(b) 675 calls
(c) 775 calls
(d) 875 calls
19. (a) Rs. 170
(b) Rs. 365
(c) Rs. 521
(d) Rs. 599

5.0 Review

Be divided into appropriate number of groups. Draw a triangle in each group, measure the sides. What type of triangle is formed? Discuss and present in the class.

- If all three sides of triangle are equal, then it is called an equilateral triangle.
 - If any two sides of triangle are equal, then it is called an isosceles triangle.
 - If the measures of all three sides are different, then it is called scalene triangle.

5.1 Area of scalene triangle

Activity 1

Study the given triangle and discuss:

Let, ABC be a triangle, in which, side BC = a unit, CA = b unit and AB = c unit. Perpendicular AD is drawn from the vertex A to the base BC, i.e., $AD \perp BC$. In the base side BC, if we suppose DC by x unit then BD will be $(a - x)$ unit. Let the height of the triangle ABC be $AD = h$ unit.

Now, the perimeter of the triangle ABC (P) = $a + b + c$ and its semi-perimeter (s) becomes $(s) = \frac{P}{2} = \frac{a + b + c}{2}$

Here, in the right angled triangle ADB,

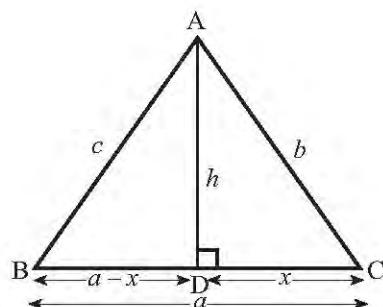
$$AD^2 + BD^2 = AB^2$$

$$\text{or, } h^2 + (a - x)^2 = c^2$$

Again in the right angled triangle ADC

$$AD^2 + DC^2 = AC^2$$

$$\text{or, } h^2 + x^2 = b^2$$



From equation (i) and (ii)

$$c^2 - (a-x)^2 = b^2 - x^2$$

$$\text{or, } c^2 = b^2 - x^2 + (a - x)^2$$

$$\text{or, } c^2 = b^2 - x^2 + a^2 - 2ax + x^2$$

$$\text{or, } c^2 = b^2 + a^2 - 2ax$$

$$\text{or, } 2ax = b^2 + a^2 - c^2$$

$$\text{or, } x = \frac{b^2 + a^2 - c^2}{2a} \quad \dots \dots \dots \text{(iii)}$$

Substituting the value of x in equation (ii),

$$h^2 = b^2 - \left(\frac{b^2 + a^2 - c^2}{2a} \right)^2$$

$$\text{or, } h^2 = b^2 - \frac{(b^2 + a^2 - c^2)^2}{4a^2}$$

$$\text{or, } h^2 = \frac{4a^2b^2 - (a^2 + b^2 - c^2)^2}{4a^2}$$

$$\text{or, } h^2 = \frac{(2ab)^2 - (a^2 + b^2 - c^2)^2}{4a^2}$$

$$\text{or, } h^2 = \frac{(2ab + a^2 + b^2 - c^2)(2ab - a^2 - b^2 + c^2)}{4a^2}$$

$$\text{or, } h^2 = \frac{[(a+b)^2 - c^2][c^2 - (a-b)^2]}{4a^2}$$

$$\text{or, } h^2 = \frac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{4a^2} \quad \dots \dots \text{ (iv)}$$

From above, $s = \frac{a+b+c}{2}$

$$\text{or, } a + b + c = 2s \quad \dots\dots\dots \text{(v)}$$

$$\text{or, } a + b = 2s - c$$

Subtracting c from both sides

$$\text{or, } a + b - c = 2s - c - c$$

$$\text{or, } a + b - c = 2s - 2c = 2(s - c)$$

$$\therefore a + b - c = 2(s - c)$$

Similarly, $a + c - b = 2s - 2b = 2(s - b)$

$$b + c - a = 2s - 2a = 2(s - a) \quad \dots \dots \dots \text{(viii)}$$

from equations (iv), (v), (vi), (vii) and (viii),

$$h^2 = \frac{2s \times 2(s-c) \times 2(s-b) \times 2(s-a)}{4a^2}$$

or, $h^2 = \frac{16s(s-a)(s-b)(s-c)}{4a^2}$

or, $h = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$

we know that,

$$\text{Area of triangle ABC} = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times a \times h$$

$$= \frac{1}{2} \times a \times \frac{2\sqrt{(s-a)(s-b)(s-c)}}{a}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$\therefore \text{Area of triangle ABC} = \sqrt{s(s-a)(s-b)(s-c)} \text{ square units}$$

Formula to find the area of scalene triangle,

Area of scalene triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi perimeter of the triangle. It is called Heron's formula

Example 1

If the length of edges of a triangular field are 5m, 12m and 13m, find its area:

Solution,

Here,

Let $a = 5$ m, $b = 12$ m and $c = 13$ m

$$\text{Semi-perimeter } (s) = \frac{a+b+c}{2} = \frac{5+12+13}{2} \text{ m} = 15 \text{ m}$$

According to the formula,

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)((s-b)(s-c))} \\ &= \sqrt{15(15-5)(15-12)(15-13)} \text{ m}^2 \\ &= \sqrt{15 \times 10 \times 3 \times 2} \text{ m}^2 \\ &= \sqrt{900} \text{ m}^2 \\ &= 30 \text{ m}^2\end{aligned}$$

Example 2

Find the area of the given quadrilateral:

Solution,

Here,

there are two scalene triangles in the given quadrilateral.

In, $\triangle ABC$, $AB = c = 16 \text{ cm}$, $BC = a = 12 \text{ cm}$, $AC = b = 20 \text{ cm}$

$$\text{Semi-perimeter } (s) = \frac{a + b + c}{2} = \frac{12 + 20 + 16}{2} = 24 \text{ cm}$$

According to the formula,

$$\begin{aligned}\text{Area of } \triangle ABC (A_1) &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{24(24-12)((24-20)(24-16)} \\ &= \sqrt{24 \times 12 \times 4 \times 8} \\ &= \sqrt{9216} \\ &= 96 \text{ cm}^2\end{aligned}$$

Again in $\triangle ACD$,

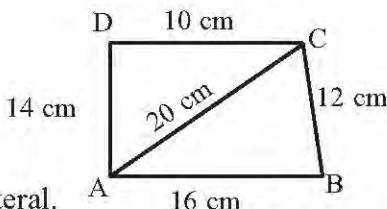
Let $AD = a = 14 \text{ cm}$, $CD = b = 10 \text{ cm}$ and $AC = c = 20 \text{ cm}$

$$\text{Semi-perimeter } (s) = \frac{a + b + c}{2} = \frac{14 + 10 + 20}{2} = \frac{44}{2} = 22 \text{ cm}$$

$$\begin{aligned}\text{Area of } \triangle ACD (A_2) &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{22(22-14)((22-10)(22-20)} \\ &= \sqrt{22 \times 8 \times 12 \times 2} \\ &= \sqrt{4224} \\ &= 8\sqrt{66} \text{ cm}^2\end{aligned}$$

\therefore Area of the quadrilateral ABCD = $A_1 + A_2$

$$\begin{aligned}&= (96 + 8\sqrt{66}) \text{ cm}^2 \\ &= 160.99 \text{ cm}^2\end{aligned}$$



Example 3

If the edges of a triangular kitchen garden are in the ratio 3: 4: 5 and area 216 square meter, what will be the perimeter of that kitchen garden? Find out.

Solution,

Here let the length of edges of the kitchen garden be

$$a = 3x \text{ m } b = 4x \text{ m } \text{ and } c = 5x \text{ m}$$

$$\text{Now, Semi-perimeter (s)} = \frac{a + b + c}{2} = \frac{3x + 4x + 5x}{2} = 6x \text{ m}$$

$$\text{Area of kitchen garden (A)} = 216 \text{ m}^2$$

According to the formula,

$$\text{Area of the kitchen garden (A)} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$216 = \sqrt{6x(6x - 3x)(6x - 4x)(6x - 5x)}$$

$$\text{or, } 216 = \sqrt{6x \times 3x \times 2x \times x}$$

$$\text{or, } 216 = \sqrt{36x^4}$$

$$\text{or, } 216 = 6x^2$$

$$\text{or, } \frac{216}{6} = x^2$$

$$\therefore x = 6$$

Thus the length of the edges of the kitchen garden,

$$a = 3x = 3 \times 6 = 18 \text{ m}$$

$$b = 4x = 4 \times 6 = 24 \text{ m}$$

$$c = 5x = 5 \times 6 = 30 \text{ m}$$

Again, perimeter of kitchen garden (P) = $a + b + c$

$$= (18 + 24 + 30) \text{ m}$$

$$= 72 \text{ m}$$

Example 4

The area and perimeter of a triangular park having length 26 m are 336m² and 84 m respectively. Find the length of remaining edges of that park.

Solution,

Here, area of the triangular park (A) = 336 m^2

$$\text{Perimeter (P)} = 84 \text{ m}$$

Length of one edge (a) = 26 m

Remaining two edges (b)=? and c ?=?

$$\text{Now, semi-perimeter } (S) = \frac{P}{2} = \frac{84}{2} = 42 \text{ m}$$

We know that,

$$P = a + b + c$$

$$84 = 26 + b + c$$

$$\text{or, } b + c = (84 - 26)$$

$$\text{or, } b + c = 58$$

$$b = (58 - c) \dots \dots \dots \quad (i)$$

$$\text{Area of park (A)} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$336 = \sqrt{42(42 - 26)(42 - 58 + c)(42 - c)} \quad [\because \text{from equations (i)}]$$

$$\text{or, } 336 = \sqrt{42 \times 16(c - 16)(42 - c)}$$

$$\text{or, } 336 = \sqrt{672(42x - c^2 - 672 + 16c)}$$

Squaring on both sides,

$$\text{or, } (336)^2 = 672(-c^2 + 58c - 672)$$

$$\text{or, } \frac{112896}{672} = -c^2 + 58c - 672$$

$$\text{or, } 168 + c^2 - 58c + 672 = 0$$

$$\text{or, } c^2 - 58c + 840 = 0$$

$$\text{or, } c^2 - (30 + 28)c + 840 = 0$$

$$\text{or } c^2 - 30c + 28c + 840 = 0$$

$$\text{or, } c(c - 30) - 28(c - 30) = 0$$

$$\text{or } (c - 30)(c - 28) = 0$$

or, $c - 30 = 0 \Rightarrow c = 30$ m

or, $c - 28 = 0 \Rightarrow c = 28$ m

(a) when substituting $c = 30$ m in equation (i)

$$b = (58 \text{ m} - 30 \text{ m}) = 28 \text{ m}$$

(b) When substituting $c = 28$ m in equation (i)

$$b = 58 \text{ m} - 28 \text{ m} = 30 \text{ m}$$

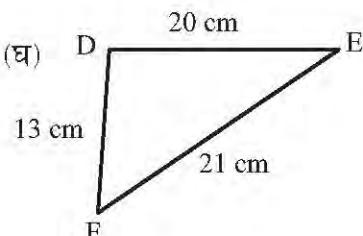
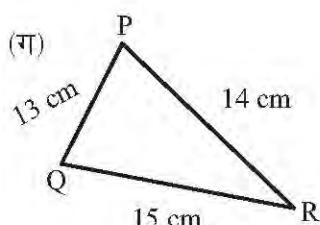
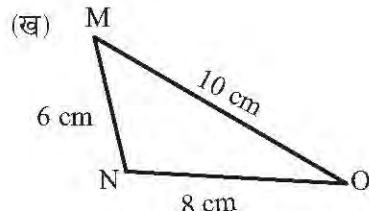
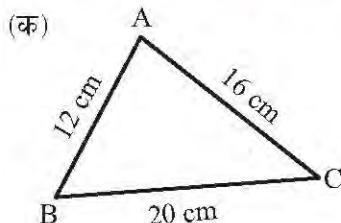
\therefore Length of remaining edges are 28 m and 30 m.

Exercise 5.1

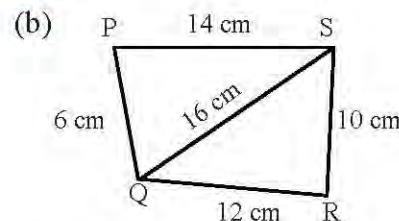
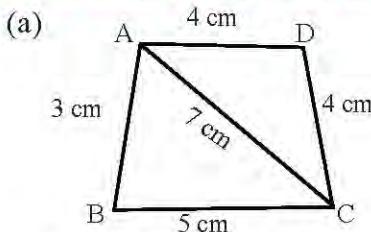
1. Answer the given questions:

- What is meant by scalene triangle?
- If the length of three sides of a triangle are p cm, q cm and r cm respectively, what is the perimeter of the triangle?
- Write the formula to find the area of scalene triangle (Heron's formula).

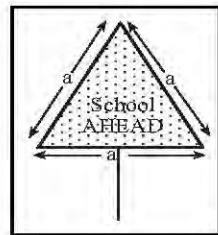
2. Find the area of the given scalene triangles.



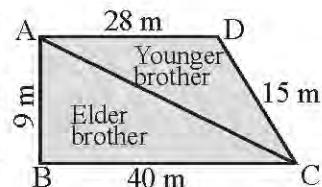
3. Find the area of the given quadrilateral:



4. The edges of a triangular field are in the ratio of 3: 5: 7 and its perimeter is 300m. What is the area of that field? Find out.
5. The edges of a triangular field are in the ratio of 12: 17: 25 and its perimeter is 540 ft. What is the area of that field? Find out.
6. The edges of a triangular garden are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$ and its perimeter is 62m. What is the area of that field? Find out.
7. Area of a triangular field having perimeter 20 m and measure of one side 9m is $6\sqrt{5}$ m², find the measure of remaining edges of that field.
8. The given traffic symbol board is of equilateral triangular shape having length of sides 'a' cm. If the perimeter of the board is 360 cm, find its area using Heron's formula.



9. The given figure is the scale drawing of a quadrilateral land. The land will have to be divided for two brothers as shown in the figure. Compare whose land is more by area. Also, find out the area of that land before dividing it.



10. There is a garden ABCD made in quadrilateral shape, in which $\angle C = 90^\circ$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. Find the area occupied by the garden.
11. Area of triangle and parallelogram having same base are equal. Length of sides of triangle are 26cm, 28cm and 30 cm. If the length of base of that parallelogram is 28 cm then what is height? Find out.

Project Work

- Make a triangular shape by cutting a cardboard or a strong cartoon box. Measure the length of its sides using a ruler and note down on your copy. Then, find the area of that triangle by using Heron's formula and present in the class.
- Divide the sheet of your copy into two triangles and find the area of the surface of the sheet by finding the area of triangles. Measure the length and breadth of that sheet and find the area using the formula for the area of a rectangle. Compare the area of that sheet obtained from both methods and present in the class.
- Prepare sketches by taking actual measurements of the edges of the land allotted for making house around your home or school. Also take actual measurements of the edges needed to make triangular pieces and find the area of that piece of land (ghaderi).

Answers

- Show to the teacher.
- (a) 96 cm^2 (b) 24 cm (c) 84 cm (d) 126 cm^2
- (a) 13.27 cm^2 (b) 101.48 cm^2
- $2,598.07 \text{ m}^2$ 5. $9,000 \text{ ft}^2$ 6. 80.49 m^2
- $4 \text{ m} / 7 \text{ m}$ 8. 56118.45 cm^2
- $180 \text{ m}^2, 126 \text{ m}^2, 54 \text{ m}^2$ extra, 306 m^2 10. 65.49 m^2
11. 12 cm

5.2 Area of four walls, floor and ceiling

Activity 1



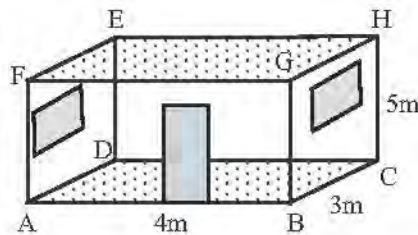
Be divided into appropriate groups. Study the given figures and present the answers of the following questions in the class:

- How many plane surfaces are there?
- Observe and discuss in groups how to find the surface area of a plane surface can be found and which surface area is equal between those areas.

Activity 2

Measure the length, breadth and height of any one of the rooms in your house. Prepare a drawing of the room with that measurement. Present in the class, then discuss in group and find the area of four walls, floor and ceiling.

Among them, the model of a room presented by one member of any group is presented here in the figure.



$$\text{Length of room (AB)} = l = 4 \text{ m}$$

$$\text{Breadth of room (BC)} = b = 3 \text{ m}$$

$$\text{Height of the room (HC)} = h = 5 \text{ m}.$$

Area of the floor (ABCD) of that room (A_1)

$$= AB \times BC = l \times b = 4 \times 3 = 12 \text{ m}^2$$

Since the area of floor is equal to area of the ceiling,

$$\text{Area of the ceiling} = l \times b = 12 \text{ m}^2$$

Among four walls area of two walls along to its length

$$(A_2) = 2 (AB \times HC) = 2 (l \times h) = 2 (4 \times 5) = 40 \text{ m}^2$$

Area of two walls along to its breadth (A_3) = $2(BC \times HC) = 2(b \times h) = 2(3 \times 5) = 30 \text{ m}^2$
So, the area of 4 walls, floor and ceiling = $12 + 40 + 30 + 12 = 94 \text{ m}^2$
The walls on the opposite side are equal. The area of floor and ceiling is also equal.

There are four walls, one floor and one ceiling. The walls on opposite sides are equal. The area of floor and ceiling in also equal.

Area of four walls = sum of four surfaces of rectangular walls

= Area of ABGF + Area of ABGF + Area of CDEH + Area of BCHG + Area of ADEF

$$\begin{aligned}&= l \times h + l \times h + b \times h + b \times h \\&= 2lh + 2bh \\&= 2h(l + b)\end{aligned}$$

Area of floor and ceiling = Area of ABCD + Area of EFGH

$$= l \times b + l \times b = 2lb$$

Area of four walls, floor and ceiling = $2h(l + b) + 2lb$

$$\begin{aligned}&= 2(hl + bh + lb) \\&= 2(lb + bh + hl)\end{aligned}$$

Example 1

If the length, breadth and height of a room are 6m, 4m and 3m respectively, then,

- What is the area of the floor?
- Find the area of four walls.
- Find the area of four walls, floor and ceiling.

Solution

Here in the rectangular room,

$$\text{Length } (l) = 6 \text{ m}$$

$$\text{Breadth } (b) = 4 \text{ m}$$

$$\text{Height } (h) = 3 \text{ m}$$

According to the formula,

(a) Area of the floor (A_1) = $l \times b$
= $6 \times 4 \text{ m}^2$
= 24 m^2

(b) Area of four walls (A_2) = $2h(l + b)$
= $2 \times 3(6 + 4) \text{ m}^2$
= $(6 \times 10) \text{ m}^2 = 60 \text{ m}^2$

(c) let the area of ceiling = (A_3)

We know that,

$$\begin{aligned}\text{Area of floor} &= \text{area of ceiling } (A_3) = 24 \text{ m}^2 \\ \therefore \text{Area of four walls, floor and ceiling} &= A_1 + A_2 + A_3 \\ &= (24 + 60 + 24) \text{ m}^2 \\ &= 108 \text{ m}^2\end{aligned}$$

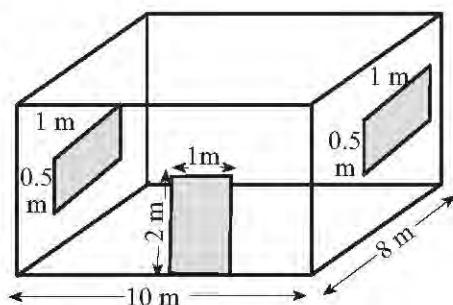
Alternative Method,

$$\begin{aligned}\text{Area of four walls, floor and ceiling} &= 2(lb + bh + lh) \\ &= 2(6 \times 4 + 4 \times 3 + 6 \times 3) \\ &= 2(24 + 12 + 18) \\ &= 2 \times 54 \\ &= 108 \text{ m}^2\end{aligned}$$

Activity 3

The length, breadth and height of a room are 10m, 8m and 3m respectively. There is a door of height 2 m and breadth 1 m and there are two windows of same shape and size with length 1 m and breadth 0.5 m.

How the area of walls without door and windows can be found, discuss in a group and present in the class.



Here, area of four walls with one door and two windows (A_1) = $2h(l + b)$
= $2 \times 3(10 + 8)$
= 6×18
= 108 m^2

Area of the door (A_2) = $2 \text{ m} \times 1 \text{ m}$
= 2 m^2

Area of two windows having same measurement (A_3) = $2(1 \text{ m} \times 0.5 \text{ m})$
= $2 \times 0.5 \text{ m}^2$
= 1 m^2

Now, the area of four walls without door and windows = $A_1 - A_2 - A_3$
= $(108 - 2 - 1) \text{ m}^2$
= 105 m^2

To find the area of four walls without door and window

- (a) Find the area of four walls (A_1)
- (b) Find the total area of windows, denote it by (A_2)
- (c) Find the total area of doors, denote it by (A_3)
- (d) Subtract the area of doors and windows from area of four walls
or

The area of four walls without doors and windows (A) = $A_1 - A_2 - A_3$

Example 2

The length, breadth and height of a room are 15 ft, 10 ft and 9 ft respectively. There are two square shaped windows having length of edges 3 ft and one door of size 6 ft. \times 3 ft. Find the area of four walls excluding windows and door.

Solution

Here,

In the rectangular room,

$$\text{Length } (l) = 15 \text{ ft}$$

$$\text{Breadth } (b) = 10 \text{ ft}$$

$$\text{Height } (h) = 9 \text{ ft}$$

Area of four walls excluding door and windows =?

We know that,

$$\begin{aligned}\text{Area of four walls } (A_1) &= 2h(l + b) \\ &= 2 \times 9 (15 + 10) \\ &= 18 \times 25 \\ &= 450 \text{ ft}^2\end{aligned}$$

$$\begin{aligned}\text{Area of two square windows } (A_2) &= 2 (l_1^2) \\ &= 2(3^2) \\ &= 2 \times 9 \\ &= 18 \text{ ft}^2\end{aligned}$$

$$\text{Area of a door } (A_3) = 6 \text{ ft} \times 3 \text{ ft} = 18 \text{ ft}^2$$

Again area of four walls excluding door and windows

$$\begin{aligned}&= \text{Area of four walls } (A_1) - \text{Area windows } (A_2) - \text{Area of door } (A_3) \\ &= (450 - 18 - 18) \text{ ft}^2 \\ &= 414 \text{ ft}^2\end{aligned}$$

Activity 4

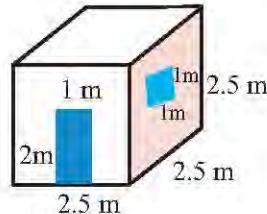
One student has brought a model drawing of a square room made for security guard of the school. Based on that model, discuss the following questions and present in the class:

- What is the relationship between the area of the plane surfaces of the room?
- Is there any difference between the area of the floor and the area of ceiling, or not?
- What is the area of walls, floor and ceiling?
- What is the area of four walls excluding door and windows?

Here,

- Length, breadth and height of the room are equal, so, the areas of all plane surfaces are equal.
- In this room, length = breadth = height
 $l = b = h = 2.5 \text{ m}$

$$\text{Area of the floor of the room} = l \times b = 2.5 \text{ m} \times 2.5 \text{ m} = 6.25 \text{ m}^2$$



Area of the ceiling of the room = $l \times b = 2.5 \text{ m} \times 2.5 \text{ m} = 6.25 \text{ m}^2$

∴ The area of the floor and ceiling of the room are equal.

(c) Area of four walls of the room including door and windows = $2h(l + b)$

$$\begin{aligned}&= 2 \times 2.5 (2.5 + 2.5) \text{ m}^2 \\&= 5 \times 5 \text{ m}^2 \\&= 25 \text{ m}^2\end{aligned}$$

Now, the area of four walls, floor and ceiling

$$\begin{aligned}&= \text{Area of the floor} + \text{Area of four walls} + \text{Area of ceiling} = (6.25 + 25 + 6.25) \text{ m}^2 \\&= 37.50 \text{ m}^2\end{aligned}$$

(d) Area of windows and door = $(1 \text{ m} \times 1 \text{ m} + 2 \text{ m} \times 1 \text{ m}) = 3 \text{ m}^2$

Now, area of four walls, floor and ceiling excluding door and windows = Area of four walls including door and windows - Area of door and windows

$$\begin{aligned}&= 37.50 \text{ m}^2 - 3 \text{ m}^2 \\&= 34.50 \text{ m}^2\end{aligned}$$

In the above presentation, area of all 6 plane surfaces are equal. Area of floor and area of each wall area also equal. Thus, it is known as cubical room.

In a room having equal length, breadth and height

- i. Area of floor = l^2
- ii. Area of four walls = $4l^2$
- iii. Area of ceiling = l^2
- iv. Area of four walls, floor and ceiling
 $= l^2 + 4l^2 + l^2 = 6l^2$
- v. If area of window is (a_1), area of door (a_2) then area of four walls, floor and ceiling excluding windows and doors = $6l^2 - a_1 - a_2$

Example 3

If the length, breadth and height of a room are 9 ft, 9 ft and 9 ft respectively, then

- Find the area of the floor.
- Find the area of four walls.
- What is the area of four walls, floor and ceiling? Find out.

Solution,

In the given cubical room, length (l) = breadth (b) = height (h) = 9 ft

Using formula,

- Area of floor (A_1) = $l^2 = (9 \text{ ft})^2 = 81 \text{ ft}^2$
- Area of four walls (A_2) = $4 l^2 = 4 \times 9 \text{ ft}^2 = 4 \times 81 \text{ ft}^2 = 324 \text{ ft}^2$
- We know that,

$$\text{Area of ceiling } (A_3) = \text{Area of floor } (A_1) = 81 \text{ ft}^2$$

$$\begin{aligned}\text{Area of four walls, floor and ceiling} &= A_1 + A_2 + A_3 \\ &= (324 + 81 + 81) \\ &= 486 \text{ ft}^2\end{aligned}$$

Alternative Method

$$\begin{aligned}\text{Area of four walls, floor and ceiling } (A_3) &= 6 l^2 \text{ ft}^2 \\ &= 6 \times (9 \text{ ft})^2 \\ &= 6 \times 81 \text{ ft}^2 \\ &= 486 \text{ ft}^2\end{aligned}$$

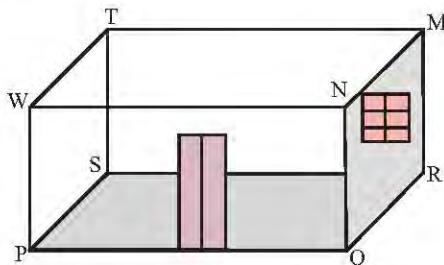
Exercise 5.2

1. Answer the following questions:

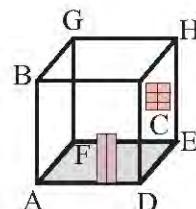
- What is the relationship between the area of opposite walls of a rectangular room?
- What is the relationship between the four walls of a cubical room and its relation with the area of floor?
- Are the area of floor and ceiling of a rectangular room equal?
- Which of the plane surfaces of a rectangular room are equal?

2. Write the name of surfaces which represents the area of floor, area of four walls and area of ceiling from the following rectangular and cubical model figures of rooms.

(a)



(b)



- If the length, breadth and height of a rectangular room are 8m, 5m and 2.5 m respectively, then find (a) the area of four walls, and (b) the area of floor and ceiling.
- In a rectangular room of length 8.5m, breadth 6m and height 2.4m, calculate the following:
 - What is the area of floor? Find it.
 - What is the area of ceiling? Find it.
 - What is the area of two walls along its length? Find it.
 - What is the area of two walls along its breadth? Find it.
 - Find the area of four walls.
 - What is the area of four walls, floor and ceiling? Find it.
- The area of four walls of a meeting hall is 135 m^2 . If the length and breadth of the hall are 15 m and 12 m respectively, what is its height? Find it.
- The area of four walls of a room is 432 ft^2 . If the breadth and height of the room are 14 ft and 9 ft respectively, what is its length? Find it.

7. If the perimeter of Ishan's bedroom is 60m and the area of four walls 144 m^2 , find the height of the room.
8. **A large hall has been constructed at Sonam's school to conduct various program. Inner length, breadth and height of the hall are 30m, 25m and 4m respectively. If there are 5 windows of size $2.5\text{m} \times 1.5\text{m}$ and 2 doors of size $5\text{m} \times 3\text{m}$, then**
- Find the area of the floor and ceiling.
 - What is the area of four walls excluding doors and windows? Find it.
 - If there are 3 patterns on the ceiling per square meter, how many patterns are made in total?
 - If there are three national flags per 15 square meters on the wall, find out the minimum number of flags that can be made on the wall.
9. **The length, breadth and height of a classroom are 8m, 6m and 3m respectively. If there are 2 doors of size $2\text{m} \times 1.5\text{m}$ and 2 windows of size $1.5\text{m} \times 1\text{m}$, then**
- What is the area of four walls excluding doors and windows? Find it.
 - If the students have pasted one sheet of paper with one picture per square meter on four walls excluding doors and windows, find out how many sheets of paper are pasted on that wall.
10. **Sijan's father has rented a dark room with a single door of $2\text{m} \times 1\text{m}$ for photo shoot. If the room is of cubical shape of side length 2.7m, then**
- What is the area of four walls excluding door? Find it.
 - If one photo frame is hung per 4 m^2 area on the four walls excluding doors and windows, find how many photo frames can be hung on that wall?

Project Work

Take a measurement of length, breadth and height of your bed room. Also take the measurement of the length and breadth of doors and windows of that room. Find the areas given in the question below and present them in the class:

- (a) Separate area of window and door
- (b) Area of four walls including doors and windows
- (c) Area of four walls excluding windows and doors
- (d) Area of four walls, floor and ceiling including doors and windows
- (e) Area of four walls, floor and ceiling excluding doors and windows
- (f) If the size of the length and breadth of your bedroom is increased by 25%, how much more will the area of four walls, floor and ceiling of that room be than the previous room? Compare.

Answers

- | | | | | |
|-----|-------------------------|--------------------------|-------------------------|--------|
| 3. | (a) 65 m^2 | (b) 80 m^2 | | |
| 4. | (a) 51 m^2 | (b) 51 m^2 | (c) 40.8 m^2 | |
| | (d) 28.8 m^2 | (e) 69.6 m^2 | (f) 171.6 m^2 | |
| 5. | 2.5 m | 6. 10 ft | 7. 2.4 m | |
| 8. | (a) 1500 m^2 | (b) 391.25 m^2 | (c) 2250 | (d) 88 |
| 9. | (a) 75 m^2 | (b) 75 | | |
| 10. | (a) 27.16 m^2 | (b) 6 | | |

5.3 Problems related to cost estimation

Activity 1

Measure your classroom length, and width in pairs and find the area to lay the carpet in your classroom. Go to a nearby carpet shop. Find out how much it costs for laying 1 square meter of carpet and find out the cost of carpet laying in your classroom and present it to the class.

For example, the work of a couple is as follows:

Length of the classroom (l) = 6 m

Breadth of the classroom (b) = 4 m

$$\text{So, the area of room (A)} = l \times b$$

$$= 6 \times 4 \text{ m}^2$$

$$= 24 \text{ m}^2$$

We know that, area of the room = area of the carpet

So, the area of the carpet = 24 m²

When asked in the shop, the rate of laying carpets in a room with ordinary carpet with wages is Rs 500 per square meter. So, the cost for laying carpet in the classroom

$$= 500 \times 24 = \text{Rs. } 12,000.$$

1. The floor area of the room = the area of the carpet.
2. The total cost for laying carpet is obtained by multiplying the cost of one square meter of carpet and the area of the carpet.

So, the total cost (T) = area of the carpet (A) \times cost of one unit of carpet (R)

Therefore, $T = A \times R$

Likewise, $R = \frac{T}{A}$ and $A = \frac{T}{R} = A \times R$

Example 1

A room is 7m long, 6m wide and 3m high. If the price of carpet per square meter is Rs 300. Find out how much it will cost to lay carpet on the floor of the room.

Solution

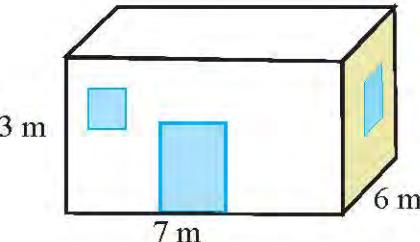
Here, length of the room (l) = 7 m

Breadth of the room (b) = 6 m

We know that,

Area of the floor of the room = $l \times b$

$$\begin{aligned} &= 7 \times 6 \text{ m}^2 \\ &= 42 \text{ m}^2 \end{aligned}$$



Now, the cost per square meter of carpet is Rs 300

So, total cost of 42 m^2 carpet = Rs. $42 \times 300 = \text{Rs. } 12,600$

Activity 2

Measure the length, width, and height of some of the rooms in the school by forming the appropriate number of groups. The four walls and ceiling of the room have to be painted (except doors and window). Now, find the answer to the following questions discussing in the group:

- How can the area of four walls be found?
- How can the area of four walls beside windows and doors be found?
- How can the area of ceilings be found?
- How can the total cost of painting be found?

Example 2

The length, breadth, and height of one room of Ram's house are 5m, 4m, and 3m respectively. Find out how much it costs to paint inside of all walls(except windows, doors, and ceilings) at a rate of Rs 5 per square meter.

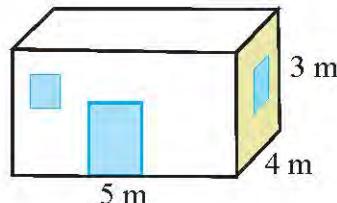
Solution

Here,

Length of the room (l) = 5 m

Breadth of the room (b) = 4 m

Height of the room (h) = 3 m



Per square meter for colouring (R) = Rs. 50

Total Cost (T) =?

By formula,

$$\begin{aligned}\text{Area of four walls } (A) &= 2h(l+b) = 2 \times 3(5+4) \text{ m}^2 = 6 \times 9 \text{ m}^2 \\ &= 54 \text{ m}^2\end{aligned}$$

$$\text{Total Cost } (T) = A \times R = 54 \times 50 = \text{Rs. 2700}$$

So, Ram needs Rs. 2,700 to colour on the four walls.

Example 3

The length, width and height of a given room are 15ft, 10ft and 8ft respectively. The room has two square windows with 3ft edges and two doors of 6ft \times 2ft. Find the total cost of painting four walls and ceilings at a rate of Rs. 175.

Solution

Here,

$$\text{Length of the room } (l) = 15 \text{ ft}$$

$$\text{Breadth of the room } (b) = 10 \text{ ft}$$

$$\text{Height of the room } (h) = 8 \text{ ft}$$

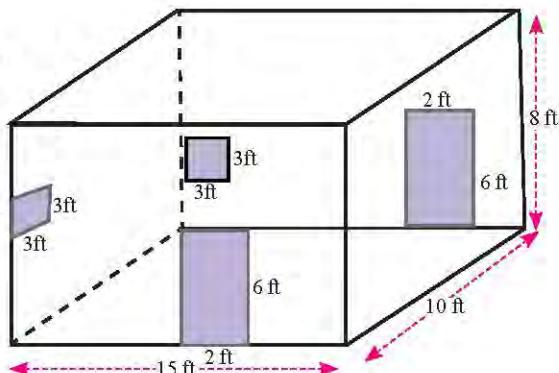
Length of square window (l_1) = 3ft, height of square door (h_1) = 6 ft and width of the door (b_1) = 2ft

We know that,

$$\begin{aligned}\text{Area of the wall and ceiling of the room } (A_1) &= 2h(l+b) + lb \\ &= [2 \times 8(15+10) + 15 \times 10] \text{ ft}^2 \\ &= (16 \times 25 + 150) \text{ ft}^2 \\ &= (400 + 150) \text{ ft}^2 \\ &= 550 \text{ ft}^2\end{aligned}$$

$$\text{Area of two square windows } (A_2) = 2 \times (l_1)^2$$

$$= 2 \times 3^2 \text{ ft}^2$$



$$= 18 \text{ ft}^2$$

$$\begin{aligned}\text{Area of two doors } (A_3) &= 2(h_1 \times b_1) \\ &= 2 \times 6 \times 2 \text{ ft}^2 \\ &= 24 \text{ ft}^2\end{aligned}$$

The area of four walls except window and doors

$$\begin{aligned}(A) &= A_1 - A_2 - A_3 \\ &= (550 - 18 - 24) \text{ ft}^2 \\ &= 508 \text{ ft}^2\end{aligned}$$

Per square colouring cost (R) = Rs. 175

$$\begin{aligned}\text{Total cost } (T) &= A \times R \\ &= 508 \times \text{Rs. } 175 \\ &= \text{Rs. } 88,900\end{aligned}$$

So, the total cost for colouring is Rs. 88,900.

Example 4

Laying carpets in a square meeting hall costs Rs. 10,800 at the rate of Rs. 75 per square meter. Similarly, except for the windows and doors, the cost for plastering four walls of the hall at the rate of Rs 25 per square meter costs Rs 6,000. The total area of four windows and two doors in that hall is 24m^2 , find the height of the hall.

Solution

Here,

Per square meter cost for laying carpet in the room (R_1) = Rs. 75

The total cost for laying carpet (T_1) = Rs. 10,800

Per square meter cost to plaster four walls (R_2) = Rs. 25

Total cost for plastering the four walls (T_2) = Rs. 6,000

Length of the square room (l) = ?

Height of the square room (h) = ?

Area of the floor of the square room (A_1) = $\frac{T_1}{R_1}$

$$\text{or, } l^2 = \frac{\text{Rs. } 10,800}{\text{Rs. } 75} \text{ m}^2$$

$$[\because A = l^2]$$

$$\text{or, } l^2 = 144 \text{ m}^2$$

$$\therefore l = 12 \text{ m}$$

Again, the area of the four walls except for the window and doors (A_2)

$$= \frac{T_2}{R_2}$$

$$= \frac{6,000}{25} [\because A = 4hl]$$

$$= 240 \text{ m}^2$$

Area of the four walls with windows doors = $240 + 24 = 264 \text{ m}^2$

$$\text{or, } 4lh = 264$$

$$\text{or, } 4 \times 12 \times h = 264$$

$$\text{or, } h = \frac{264}{4 \times 12}$$

$$\text{or, } h = 5.5 \text{ m}$$

So, height of the square room is 5.5m.

Example 5

It costs Rs. 10,240 for laying carpet in a room which has a length twice its width and width twice its height at the rate of Rs. 80 per square meter. If the total area of windows and doors in the room is 5 m^2 , find how much it costs to paint on four walls and ceiling at the rate of 90 per square meter.

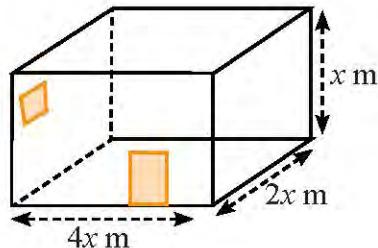
Solution,

Here let the height of the room (h) = x m

Width of the room (b) = $2x$ m

And length of the room (l) = $2b$

$$= 2 \times 2x \text{ m} = 4x \text{ m}$$



The per square meter cost for laying carpet in the room (R_1) = Rs. 80

The total cost for laying carpet in the room (T_1) = Rs. 10,240

The per square meter cost for painting (R_2) = Rs. 90

Total cost for painting the ceiling and four walls (T_2) = ?

Total cost for laying carpet (T_1)

Now, the area of the floor of the room (A_1) = $\frac{\text{Per square meter cost for laying carpet } (R_1)}{\text{Total cost for laying carpet } (T_1)}$

$$\text{or, } l \times b = \frac{10,240}{80}$$

$$[\because A = l \times b]$$

$$\text{or, } 4x \times 2x = 128$$

$$\text{or, } 8x^2 = 128$$

$$\text{or, } x^2 = \frac{128}{8}$$

$$\text{or, } x^2 = 16 \quad \text{or, } x = 4$$

\therefore Height of the room (h) = 4 m

Width of the room (b) = $2x$ m = 2×4 m = 8 m

Length of the room (l) = $4x$ m = 4×4 m = 16 m

$$\begin{aligned} \text{Again, area of four walls and ceiling } (A_2) &= 2h(l + b) + lb \\ &= 2 \times 4(16 + 8) + 16 \times 8 \\ &= 8 \times 24 + 128 \\ &= 320 \text{ m}^2 \end{aligned}$$

Now, area of the ceiling and four walls except windows and doors (A_3) = $320 - 5 = 315 \text{ m}^2$

Total cost for painting the ceiling and four walls (T_2) = $A_3 \times R_2$

$$= \text{Rs. } 90 \times 315 = \text{Rs. } 28,350$$

\therefore The total cost for painting the ceiling and four walls is Rs. 28,350.

Exercise 5.3

- Find the total cost for laying a carpet costing Rs. 130 per square ft. on the floor of the room having length 12 ft and breadth 11 ft.
- Interior length, width, and height of a room are 15 ft, 12 ft, and 8 ft respectively. The room has two windows of $6\text{ft} \times 4\text{ft}$ 6 inches and a door of $3\text{ft} \times 6\text{ft}$ 6 inches. Except the window, door, and ceiling of the room, find out how much money is needed to paint the four walls at the rate of Rs 125 per square ft.
- If the length of the square room is 15 ft and the height is 8 ft., find out how much it will cost to plaster the floor, four walls, and ceiling of the room at the rate of 120 per square ft.
- The length, width, and height of an assembly hall are 32m, 32m, and 4m respectively. The building has 6 windows of $2.5\text{m} \times 1.8\text{m}$ and 2 doors of $4\text{m} \times 3\text{m}$, then:
 - Find the total area of the floor and ceiling of the building.

- (b) Find the total area of windows and doors.
 - (c) Find the areas of four walls except for windows and doors.
 - (d) If one chair occupies 2 square meter area of the floor, how many chairs of the same size can fit in that building?
 - (e) Find the total cost of plastering all four walls at the rate of Rs 350 per square meter.
5. The floor area of the largest room in Simran's house was found 500 ft^2 . There are three windows of size $6\text{ft} \times 5\text{ft}$ and one door of size $3\text{ft} \times 6.5\text{ft}$. if the length of the room is 25 ft and height is 9 ft, find the total cost for coloring its four walls (except windows and door) at the rate of 300 per square foot.
6. The height of a squared room is 8 ft. The total cost for laying carpet on that room at the rate of 200 per square ft is 39,200. Find the cost for coloring on the four walls of the room at the rate of 350 per square ft.
7. A school pays Rs 7,056 for laying carpet at the rate of 144 per square meter on a square room for teachers and staff. Except for the windows and doors of the same room, when the paper is pasted on four walls at the rate of 400 per square meter a total of Rs 30,000 is needed. If the total area of windows and doors is 9 m^2 , find the height of the room.
8. In Nima's house, the length of room is double of it's breadth and height of a room is 2.8 meters. Apart from the window, door, and ceiling of the room, Rs 54,000 is needed to draw the picture on its four walls at the rate of Rs 900 per square meter. If the area of windows and doors is 7.2 square meters, find out the total cost of laying tiles on the floor of the room at the rate of Rs 200 per square meter.
9. The length of a school meeting hall is twice the width and the width is twice the height. The school pays Rs 43,200 for coloring its four walls including windows and doors at the rate of 225 per square meter. Find the total cost for laying tiles on the floor of the room at the rate of 250 per square meter.
10. **The size of a park decorated by a municipality is $50\text{m} \times 40\text{m}$. If the square slab of *dubo* sized 20 cm is land,**
- (a) Find, how many slabs are needed?
 - (b) Find the total cost for laying slabs if the cost for laying per slab is Rs. 225.
 - (c) After a few years, considering the park to be small, the municipality decided to increase the length and width by 15 m and 10 m respectively. Find out how many slabs need to be added to cover the square of the same length in the increased area and how much it costs at the same rate.

Project Work

1. Sit in an appropriate group, Measure the length, width, and height of the classroom of the school you are studying in or the room you live in, as well as the length and height of windows and doors, and present the solution to the given questions in the class.
 - (a) Find the area of the floor and ceilings.
 - (b) Find the areas of the four walls.
 - (c) Find the areas of the four walls except for windows and doors.
 - (d) Find out much it will cost to plaster four walls, ceiling, and floor according to the current rate.
 - (e) Find out much it costs to paint four walls, and the ceiling according to the current rate in your village and town.
2. You need to paint the walls of your school's ninth-grade classroom. The school asks you to prepare an estimate of the cost of painting the classroom. Find out the cost of painting the classroom and present it to the class.

Answers

- | | | |
|----------------------------|----------------------|-------------------------------|
| 1. Rs. 17160 | 2. Rs. 67500 | 3. Rs. 1,11,600 |
| 4. (a) $1,200 \text{ m}^2$ | (b) 51 m^2 | (c) 365 m^2 (d) 320 |
| (e) Rs. 1,27,750 | 5. Rs. 2,10,150 | 6. Rs. 1,12,000 |
| 7. 3 m | 8. Rs. 6,400 | 9. Rs. 32,000 |
| 10. (a) 50,000 | (b) Rs. 1,12,50,000 | (c) 3,750, Rs. 8,43,750 |

5.4 Area of triangular and quadrilateral shaped land

Activity 1

In order to prepare *kittanapi* in a systematic manner in Nepal, the Department of Surveying and Mapping was established in 2014 BS. Even now, under the department of surveying, *kittanapi*, *Geojedic napi*, and *Toponapi* has been developed in a detailed and systematic manner, and various surveys required in the country have been conducted. Study the following situations and discuss how to do a land survey in the area where you live is conducted.

1. Durga kumari's permanent home is in the hilly district. She has a 3 *Ropani* farm, 1 *Ropani* of *pakha bari*, and a house constructed on the land of 8 *aana* 3 *paisa* area. She at so has a triangular plot of an area 4 *aana* on the main roadside.
2. Ravilal's permanent home is in tarai. He is a landlord there. He has three *Biggha* of the farm, 1 *biggha* 3 *kattha* bari, and a house constructed on 1 *kattha* 2 *dhur* of land. Also, he has 4 *kattha* areas of the quadrangular plot on the main roadside.
3. Bhim Bahadur is an Amin. He calculates the area of the land according to the criteria issued by the Department of Survey. Here is the formula for finding the area of a triangle by converting the quadrilateral land into two triangles and find the area of both triangles and the area of the quadrilateral land from its sum.



There are land and ground surfaces in a variety of shapes. In this lesson, we study about finding the area of triangular and quadrilateral lands. When trading land, it is mandatory to measure its area. Square feet or square meters is the universal unit of land measurement. But the method and unit of measuring land area may vary from place to place. Units used for measuring land in use in different parts of Nepal are:

1. In hilly region, Ropani, Aana, Paisa, Daam are used to measure lands in hilly region.
2. In the terai region, Bigha, Kattha, Dhur etc are used to measure lands the terai region.

Activity 2

Based on the given table, it is possible to understand how much or how the area of land or plot is measured by converting it to universally accepted units (sq. feet or sq. meter). Discuss what units are being used to measure the area of land in the area where you live and present the findings in the classroom.

Units used in Terai region		sq. meter	sq. feet	Conversion into the units used in hilly region	Relationship between meter, feet and inch
1 Bigha	20 Kattha	6772.63	72900	13.31 Ropani	1 m = 3.2808 ft
1 Kattha	20 Dhur	338.63	3645	10.65 Aana	1 m = 39.37 inches
1 Dhur		16.93	182.25	2.13 Paisa	
Units used in hilly region				Conversion into the units used in terai region	
1 Ropani	16 Aana	508.72	5476	1.5023 Kattha	1 foot = 0.3048 m
1 Aana	4 Paisa	31.79	342.25	1.8777 Dhur	1 foot = 12 inches
1 Paisa	4 Daam	7.95	85.56		
1 Daam		1.99	21.39		

Conversion of units used in measuring the land

Example 1

Somnath had recently bought 8 *katthas* and 3 *dhurs* of land in Nawalparasi. If the area of land is only in square meter units, how many square meters area should be kept?

Solution

$$\text{Area of land} = 8 \text{ Kattha } 3 \text{ Dhur}$$

$$= 338.63 \times 8 \text{ m}^2 + 16.93 \times 3 \text{ m}^2$$

[\because since 1 *Kattha* = 338.63 m^2 and 1 *Dhur* = 16.93 m^2]

$$= 2709.04 \text{ m}^2 + 50.79 \text{ m}^2$$

$$= 2759.83 \text{ m}^2$$

The 8 *Kattha* 3 *Dhur* land purchased by Somnath has an area of 2759.83 *square meters*.

Example 2

Convert the following area of land into 'Aana'.

- a) 2 Ropani 3 Aana b) 3 Kattha 9 Dhur

Solution,

$$(a) 2 \text{ Ropani } 3 \text{ Aana} = 16 \times 2 \text{ Aana} + 3 \text{ Aana}$$

$$= 32 \text{ Aana} + 3 \text{ Aana}$$

$$= 35 \text{ Aana}$$

$$(b) 3 \text{ Kattha } 9 \text{ Dhur} = 338.63 \times 3 \text{ sq. meter} + 16.93 \times 9 \text{ sq. meter}$$

[\because 1 *Kattha* = 338.63 *sq. meter* and 1 *Dhur* = 16.93 *sq. meter*]

$$= 1015.89 \text{ sq. meter} + 152.37 \text{ sq. meter}$$

$$= 1168.26 \text{ sq. meter}$$

$$= \frac{1168.26}{31.79} \text{ Aana} \quad [\because 1 \text{ Aana} = 31.79 \text{ m}^2]$$

$$= 36.75 \text{ Aana}$$

Example 3

The area of a land is 1560 ft^2 . Convert the area of the land as follows:

- In Ropani, Aana, Paisa and Daam according to the units circulated in the himalayan and mountain region.
- In Bigha, Kattha and Dhur according to the units circulated in the terai region.

Solution,

- (a) Area of the given land = 1560 ft^2

$$\begin{aligned}&= \frac{1560}{5476} \text{ Ropani} [\because 1 \text{ Ropani} = 5476 \text{ ft}^2] \\&= 0.28489 \text{ Ropani} \\&= 0.28489 \times 16 \text{ Aana} [\because 1 \text{ Ropani} = 16 \text{ Aana}] \\&= 4.55824 \text{ Aana} \\&= (4 + 0.55824) \text{ Aana} \\&= 4 \text{ Aana and } 0.55824 \times 4 \text{ Paisa} [\because 1 \text{ Aana} = 4 \text{ Paisa}] \\&= 4 \text{ Aana and } 2.23296 \text{ Paisa} \\&= 4 \text{ Aana and } (2 + 0.23296) \text{ Paisa} \\&= 4 \text{ Aana } 2 \text{ Paisa and } (0.23296 \times 4) \text{ Daam} [\because 1 \text{ Paisa} = 4 \text{ Daam}] \\&= 4 \text{ Aana } 2 \text{ Paisa } 0.93 \text{ Daam}\end{aligned}$$

It is written as *Ropani - Aana - Paisa - Daam 0 - 4 - 2 - 0.93 (Ropani)*.

Alternative Method

Area of land = 1560 sq. feet

$$\begin{aligned}&= \frac{1560}{5476} \text{ Ropani} [\because 1 \text{ Ropani} = 5476 \text{ sq. feet}] \\&= 0.28489 \text{ Ropani}\end{aligned}$$

Now, convert 0.28489 Ropani into '*Aana*' as,

$$\begin{aligned}0.28489 \text{ Ropani} &= 0.28489 \times 16 \text{ Aana} [\because 1 \text{ Ropani} = 16 \text{ Aana}] \\&= 4.55824 \text{ Aana} \\&= (4 + 0.55824) \text{ Aana}\end{aligned}$$

Now, convert 0.55824 *Aana* into '*Paisa*' as,

$$\begin{aligned}&= 0.55824 \times 4 \text{ Paisa} [\because 1 \text{ Aana} = 4 \text{ Paisa}] \\&= 2.23296 \text{ Paisa} \\&= (2 + 0.23296) \text{ Paisa}\end{aligned}$$

Now, convert 0.23296 *Paisa* into '*Daam*' as,

$$\begin{aligned}&= 0.23296 \times 4 \text{ Daam} [\because 1 \text{ Paisa} = 4 \text{ Daam}] \\&= 0.93 \text{ Daam}\end{aligned}$$

So, area of given land = 1560 sq. feet = 0 *Ropani*, 4 *Aana*, 2 *Paisa* 0.93 *Daam*.

It is written as 0 - 4 - 2 - 0.93 (*Ropani*).

(b) Area of given land = 1560 sq. feet

$$\begin{aligned}&= \frac{1560}{72900} \text{ Bigha} [\because 1 \text{ Bigha} = 72900 \text{ sq. feet}] \\&= 0.021399 \text{ Bigha}\end{aligned}$$

Now, convert 0.021399 *Bigha* info *Kattha* as,

$$\begin{aligned}0.021399 \text{ Bigha} &= 0.021399 \times 20 \text{ Kattha} [\because 1 \text{ Bigha} = 20 \text{ Kattha}] \\&= 0.42798 \text{ Kattha} \\&= (0 + 0.42798) \text{ Kattha}\end{aligned}$$

Now, convert 0.42798 *Kattha* info *Dhur* as,

$$\begin{aligned}&= 0.42798 \times 20 \text{ Dhur} [\because 1 \text{ Kattha} = 20 \text{ Dhur}] \\&= 8.5596 \text{ Dhur} \\&= 8.56 \text{ Dhur}\end{aligned}$$

So, the area of land = 1560 sq. feet = 0 *Bigha*, 0 *Kattha*, 8.56 *Dhur* = 8.56 *Dhur*.

It is written in the form of *Bigha* – *Kattha* – *Dhur* – 0 as 0-0-8.56-0 *Bigha*.

Example 4

Plot No.	विवरण (घर, आवादी, इत्यादि)	जग्गाध नीको हकहिस्सा	मोहीको नाम थर	किसिम वा वर्ग	Area / sq. meter	Remarks	प्रमाणित गर्नेको दस्तखत
244	आवादी	एकलौटी		धनहर/ खेत	0-1-15-0 (Bigha) (0-20-20-4)		
0	रैकर निजी			दोयम	592.52		
247	आवादी	एकलौटी		धनहर/ खेत	0-1-15-0 (Bigha) (0-20-20-4)		
0	रैकर निजी			दोयम	592.52		
			Total Area	1185.04 sq. meter			

In the above *Lalpurja* the area of the land on plot no. 244 is 1 *Kattha* and 15 *Dhur* or 592.52 m². Test the area using the area conversion table on the front page.

Solution,

The area of the land on plot no. 244 in given *Lalpurja* = 1 *Kattha* 15 *Dhur*

$$\begin{aligned}
 &= 20 \text{ } Dhur + 15 \text{ } Dhur \quad [\because 1 \text{ } Kattha} = 20 \text{ } Dhur] \\
 &= 35 \text{ } Dhur \\
 &= 16.93 \times 35 \text{ } m^2 \quad [\because 1 \text{ } Dhur} = 16.93 \text{ } m^2] \\
 &= 592.55 \text{ } m^2
 \end{aligned}$$

Example 5

गा.वि.सं / न.पा	वडा नं नक्सा सिट	कित्ता नं.	विवरण (धर, आवादी, इत्यादि)	ज.ध्र को हक हिस्सा	मोहिको नाम धर	किसिम वा वर्ग	शेवफल विग्राहा/वर्गमिटर	मोठ / पाना न.
??	3	2731	आवादी	सबै		खेत अवल	0-0-10-0	28
	3 (a)		रेकर निजी				169.32	10833
Total area 169.32 sq. Meter								

In the above Lalpurja the area of the land on plot no. 2731 is 0-0-10-0 (*Biggha*) or 169.32 m². Test the area using the area conversion table on the front page.

Solution,

The area of the land on plot no. 2731 is given $Lalpurja = 0 \text{ Biggha } 0 \text{ Kattha } 10 \text{ Dhur } 0 \text{ Kunua}$

$$\begin{aligned}
 &= 10 \text{ Dhur} \\
 &= 10 \times 16.93 \text{ m}^2 \quad [\because 1 \text{ Dhur} = 16.93 \text{ m}^2] \\
 &= 169.30 \text{ m}^2
 \end{aligned}$$

You can also convert from one unit to another using the area conversion option by installing NaaPI apps on mobile. Using the plotter option we can draw the picture and find the area on the basis of the length of each edge using the primary and secondary unit tool.

Area of triangular and quadrangular land

Example 6

A blueprint map of one of the places of Kathmandu is given in the picture below. Here you can see places of the land of different sizes. This map print is drawn on a scale of 1: 500. Some of these are given in the picture below:



नेपाल सरकार
भूमिसंधार तथा व्यवस्था मन्त्रालय
नापी विभाग

जिल्ला: काठमाण्डौ
सीट नं. १०३ १०२३ ०६

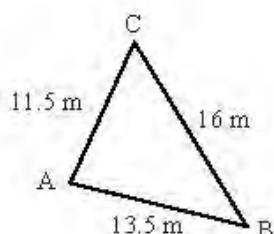
नापी कार्यालय, डिल्लीबजार

बड़ा नं. १६
स्कैल: १:५००



The shape of plot no. 559 in this blue print is triangular. The actual measurement of each sides is given in the figure.

- (a) Find the area of the given plot.
 (b) Find the area as Ropani - Aana - Paisa - Daam.



Solution,

(a) In given triangular plot,

$$\text{length of edge AB } (c) = 13.5 \text{ m}$$

$$\text{length of edge BC } (a) = 16 \text{ m}$$

$$\text{length of edge CA } (b) = 11.5 \text{ m}$$

$$\text{Perimeter } (p) = a + b + c = 13.5 + 16 + 11.5 = 41 \text{ m}$$

$$\text{Semi-perimeter } (s) = \frac{p}{2} = \frac{41}{2} = 20.5 \text{ m}$$

$$\text{Area (A)} = ?$$

We know that,

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{20.5(20.5-16)(20.5-11.5)(20.5-13.5)} \\ &= \sqrt{20.5 \times 4.5 \times 9 \times 7} \\ &= \sqrt{5811.75} \\ &= 76.23 \text{ m}^2 \end{aligned}$$

∴ Area of the plot is 76.23 sq. meter.

(b) The area of the plot is less than 'Ropani'. So convert into 'Aana' as

$$76.23 \text{ m}^2 = \frac{76.23}{31.79} \text{ Aana} \quad [:: 1 \text{ Aana} = 31.79 \text{ m}^2]$$

$$= 2.3979 \text{ Aana}$$

$$= (2 + 0.3979) \text{ Aana}$$

$$= 2 \text{ Aana and } 0.3979 \times 4 \text{ Paisa} \quad [:: 1 \text{ Aana} = 4 \text{ Paisa}]$$

$$= 2 \text{ Aana and } 1.5916 \text{ Paisa}$$

$$= 2 \text{ Aana } 1 \text{ Paisa and } 0.5916 \times 4 \text{ Daam} \quad [:: 1 \text{ Paisa} = 4 \text{ Daam}]$$

$$= 2 \text{ Aana } 1 \text{ Paisa } 2.37 \text{ Daam}$$

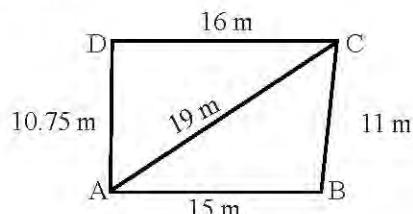
$$= 0 - 2 - 1 - 2.37 \text{ (Ropani)}$$

Example 7

The actual measurement of plot no. 278 in above example 4 is presented below:

(a) Find the area of the plot.

(b) Find the area as *Bigha* – *Kattha* – *Dhur* – 0.



Solution,

- (a) The plot is rectangular shape; which is made in the ratio of 1:500. Diagonal AC divided the rectangle into two triangle.

To find area of $\triangle ABC$,

$$\text{length of side } BC (a) = 11 \text{ m}$$

$$\text{length of side } AC (b) = 19 \text{ m}$$

$$\text{length of side } AB (c) = 15 \text{ m}$$

$$\text{Semi-perimeter of } \triangle ABC (s) = \frac{a + b + c}{2} = \frac{11 + 19 + 15}{2} = 22.5 \text{ m}$$

$$\text{Area of } \triangle ABC = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{22.5(22.5 - 11)(22.5 - 19)(22.5 - 15)}$$

$$= \sqrt{22.5 \times 11.5 \times 3.5 \times 7.5}$$

$$= \sqrt{6792.1875}$$

$$= 82.41 \text{ m}^2$$

To find area of $\triangle DAC$,

$$\text{length of side } DC (d) = 16 \text{ m}$$

$$\text{length of side } AC (b) = 19 \text{ m}$$

$$\text{length of side } DA (e) = 10.75 \text{ m}$$

$$\text{Semi-perimeter of } \triangle DAC (s') = \frac{d + b + e}{2} = \frac{16 + 19 + 10.75}{2} = 22.88 \text{ m}$$

$$\text{Area of } \triangle DAC (A) = \sqrt{s'(s' - a)(s' - b)(s' - c)}$$

$$= \sqrt{22.88(22.88 - 16)(22.88 - 19)(22.88 - 10.75)}$$

$$= \sqrt{22.88 \times 6.88 \times 3.88 \times 12.13}$$

$$= \sqrt{7408.6142}$$

$$= 86.07 \text{ m}^2$$

Now, area of rectangle ABCD = Area of $\triangle ABC$ + Area of $\triangle DAC$

$$= 82.41 \text{ m}^2 + 86.07 \text{ m}^2$$

$$= 168.48 \text{ m}^2$$

\therefore Area of the given plot is 168.48 m^2 .

- (b) Area of plot = 168.48 m^2

Here, area of the given plot 1 *Kattha* is less than 338.63 sq. meter. So, we convert into *Dhur*,

$$\text{Area of plot} = 168.48 \text{ m}^2$$

$$= \frac{168.48}{16.93} \text{ Dhur} \quad [\because 1 \text{ Dhur} = 16.93 \text{ sq. meter}] \\ = 9.95 \text{ Dhur}$$

So, area of the plot is 0-0-9.95-0 'Biggha'.

Exercise 5.4

- 1. Answer the following questions based on the area conversion table presented on the front page:**

- According to the methods of mapping lands in Nepal, what are the ways used in mountainous and hilly areas?
- What is the method of measuring land in the area where you live?
- How many square feet of area is in one *Ropani*?
- How many *aanas* are in one *Ropani* and how many square feet are in one *aana*?
- How many square meters of the area is in one *Bigha*?
- How many *Katthas* are there in one *Biggha* and how many square feet areas are in one *Kattha*?
- Which unit of an *aana* and a *kattha* has more square meter areas?
- If the area of rectangular land with a width of 37 feet is one *ropani*, what is the length of that land?
- A landlord of Terai has 10 *Bigghas* of land. According to the *Ropani* unit, how many *ropansis* of land does he have?

- 2. Convert the following unit areas of land into square feet units:**

- | | | |
|---------------------|----------------------------------|------------------------------------|
| (a) 2 <i>Kattha</i> | (b) 4 <i>aana</i> | (c) 8 <i>Dhur</i> |
| (d) 5 <i>aana</i> | (e) 6 <i>aana</i> 3 <i>Paisa</i> | (f) 4 <i>Kattha</i> 10 <i>Dhur</i> |

- 3. Convert the following unit areas of land into square meter units**

- | | | | | |
|---------------------|--------------------|--------------------|---------------------|------------------------------------|
| (a) 2 <i>ropani</i> | (b) 1 <i>Bigha</i> | (c) 16 <i>Dhur</i> | (d) 3 <i>Kattha</i> | (e) 5 <i>Kattha</i> 10 <i>Dhur</i> |
|---------------------|--------------------|--------------------|---------------------|------------------------------------|

4. Convert the following unit areas of land into *Aana* units:

- | | | |
|------------------------------------|---------------------|-----------------------------------|
| (a) 16 <i>Dhur</i> | (b) 4 <i>Kattha</i> | (c) 3 <i>Kattha</i> 2 <i>Dhur</i> |
| (d) 6 <i>Kattha</i> 10 <i>Dhur</i> | (e) 2 <i>Ropani</i> | (f) 1 <i>Bigha</i> |

5. Convert the following unit areas of land into *Dhur* units:

- | | | |
|------------------------------------|----------------------------------|------------------------------------|
| (a) 3 <i>aana</i> | (b) 6 <i>aana</i> 3 <i>paisa</i> | (c) 16 <i>Kattha</i> 5 <i>Dhur</i> |
| (d) 1 <i>Bigha</i> 4 <i>Kattha</i> | (e) 2 <i>Biggha</i> | |

6. A plot of land with one Ropani area of Aitaman is faced on 20 feet road. The length of the face on the side of the road is 148 feet. When the land has to be divided into four equal areas, what are the areas of each plot? What is the length and width of each plot?

7. Convert the areas of the following lands as follows:

- | | | | |
|---|------------------------------|-------------------------------|--|
| (a) <i>Ropani-Aana-Paisa-Daam</i> (R-A-P-D) | | | |
| (b) <i>Biggha-Kattha-Dhur</i> (B-K-D) | | | |
| (i) 3056 ft ² | (ii) 1426.85 ft ² | (iii) 5026.80 ft ² | |
| (iv) 2167.24 ft ² | (v) 80736.75 ft ² | | |
8. The area of land given in the following table is 0-5-3-0 *Biggha* or 1743.70 ft². Test using the area conversion table on the front page:

दाढ़ न. / नक्शा सीट न.	कित्ता न. / खण्डे न.	विवरण (घर, आवादी, इत्यादि)	जग्गाधनीको हक्कसिस्ता	मोहीको नाम दर	किसिम वा वर्ग	झेत्रफल/वर्गमि	कैफियत
5	166	आवादी	एकलौटी		धनहर/खेत	0-5-30-30-4	
		रैकर निजी			जम्मा	1743.70	
							जम्मा झेत्रफल 1743.70 वर्गफिट

9. The area of the land in the given *Lalpurja* is 677.26 ft². In this map, the area of the map is given in square meters only. Find the area using the area conversion table on the front page:

- (a) According to the hilly system Ropani- Aana- Paisa -Daam format
- (b) According to the terai system Biggha- Kattha- Dhur format
- (c) In square feet.

बाडीन./ नक्सा सीट न.	फिला न. / खण्डे न.	विवरण (घर, आवादी, इत्यादि)	जमगाउनीको हक्कसिस्ता	जमगाउनीको हक्कहिस्ता	मोहीको नाम वर	किसिम वा वर्ग	लेनफाल/ व.मि.	कैफियत
४	३२४	आवादी,	एकलौटी			घनहर / खेत /	०-०-०-०	
४८	३२४	रेफर निजी				दोयम	६७७ रु	
	०					जम्मा लेनफाल ६७७ रु, वर्गमिटर		

10. The figure on the right shows a map of the fragmented land as a plot. The area of each plot is given according to the Ropani-Aana-Paisa-Daam.

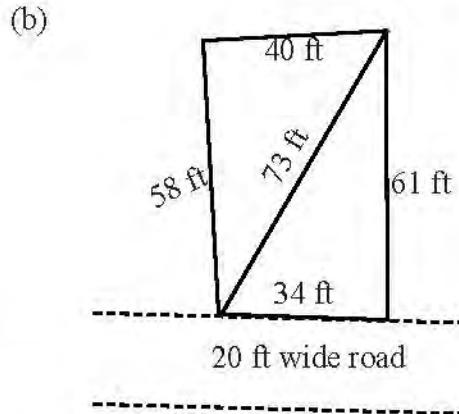
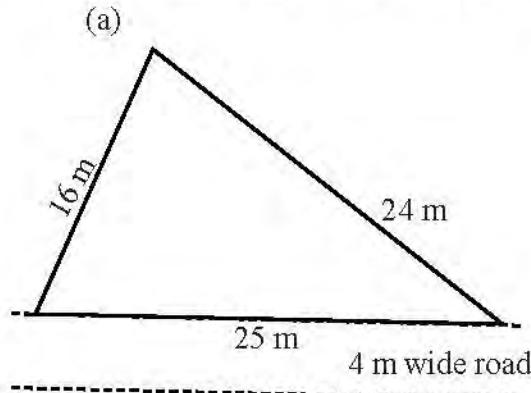
- (a) Convert the area of plots no 1, 2, 6, 8, and 9 into square meter units.
- (b) Convert the area of plots no 4, 18, 20, and 22 into square meter units.
- (c) Convert the area of plots no 9, and 22 into Biggah- Kattha- Dhur units.



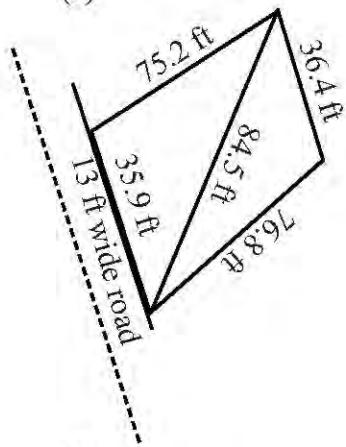
11. Find the area of the land given in the following measurements:

(a) According to Ropani- Aana- Paisa -Daam units.

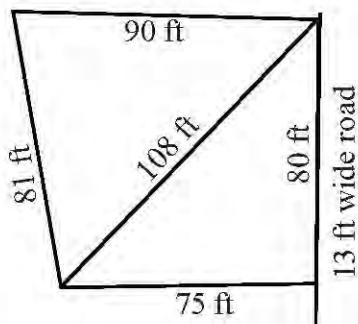
(b) According to Biggah- Kattha- Dhur units



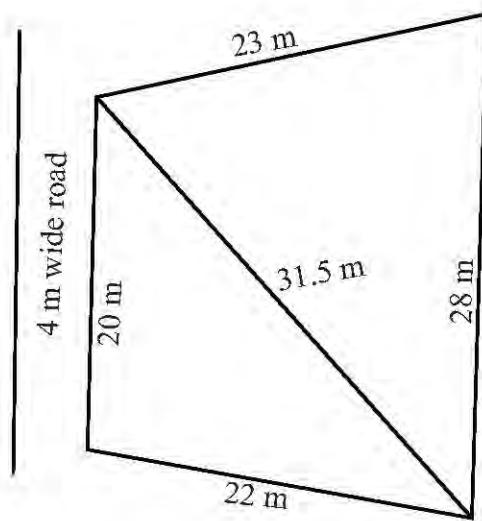
(c)



(d)



(e)



12. The perimeter of a triangular land is 160 feet. If the ratio of each edge of the land is 5:7:8, find its area into Dhur.

Project Work

1. Divide the students in the class into appropriate groups (with at least 5 people in each group). Divide the responsibilities for each person in the group (two people for measuring with a measuring tape, 1 person for observing measurement, 1 person for taking notes, one person for overseeing all the work). With the necessary materials (measuring tape, ruler, pencil, etc), two groups measure the area of certain parts of the school play ground(triangular and quadrangular part) in their own way. Similarly, other two groups find out the area of the open space of any public places (temple premises, park, play ground, open space, and open space of the school etc.) near the school. Draw an area with with a map of the work done in this way and present it in the classroom. Compare the similarities between the area of the same site discovered by each of the two groups and choose the right option with the help of the teacher.
 2. How much land does each student have in the name of his/her family? How much and how has the area been kept in the Lalpurja for its verification? Can you find out the area of land in the name of your family?

When, from whom and how was the land surveyed? Gather information and present it in the classroom with the help of your parents, community elders and related people.

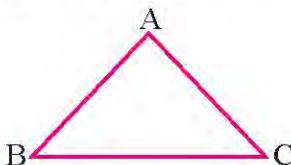
Answers

1. Show to your teacher.
 2. (a) 72.90 ft² (b) 1369 ft² (c) 1458 ft²
(d) 1711.25 ft² (e) 2310.18 ft² (f) 16402 ft²
 3. (a) 1017.44 ft² (b) 6772.63 ft² (c) 270.88 ft²
(d) 1015.89 ft² (e) 1862.45 ft²
 4. (a) 8.53 Aana (b) 42.60 Aana (c) 33.03 Aana
(d) 69.24 Aana (e) 32 Aana (f) 213.05 Aana
 5. (a) 5.63 Dhur (b) 12.67 Dhur (c) 325 Dhur
(d) 480 Dhur (e) 800 Dhur

6. 1369 ft^2 , length 37 ft, width 37 ft
7. (i)–(a) 0-8-3-2.87 (*Ropani*) (b) 0-0-16.77 (*Biggha*)
(ii)–(a) 2-12-3-2.02 (*Ropani*) (b) 0-4-4.2 (*Biggha*)
(iii)–(a) 0-14-2-3 (*Ropani*) (b) 0-1-7.58 (*Biggha*)
(iv)–(a) 4-4-0-2.6 (*Ropani*) (b) 0-6-7.99 (*Biggha*)
(v)–(a) 14-11-3-2.39 (*Ropani*) (b) 1-2-3 (*Biggha*)
8. (a) 1-5-1-0.2 (*Ropani*) (b) 0-1-20 (*Biggha*) (c) 7289.92 ft^2
9. (a) plot no. (1) 0-3-2-3 / 117.42 m^2
plot no. (2) 0-4-0-2 / 143.06 m^2
plot no. (6) 0-4-0-0 / 127.16 m^2
plot no. (8) 0-3-1-0 / 103.32 m^2
plot no. (9) 0-5-1-3 / 172.87 m^2
- (b) plot no. (4) 0-4-2-0 / 1540.12 ft^2
plot no. (18) 0-3-2-2 / 1240.65 ft^2
plot no. (20) 0-4-0-0 / 1433.17 ft^2
plot no. (22) 0-7-0-0 / 2395.75 ft^2
- (c) plot no. (9) 0-0-10.2 (*Biggha*)
plot no. (22) 0-0-13.14 (*Biggha*)
10. (a) 184.89 ft^2 (i) 0-5-3-1.04 (*Ropani*) (ii) 0-0-10.91 (*Biggha*)
(b) 2187.22 ft^2 (i) 0-6-1-2.25 (*Ropani*) (ii) 0-0-11.77 (*Biggha*)
(c) 2746.54 ft^2 (i) 0-8-0-0.3 (*Ropani*) (ii) 0-0-15.07 (*Biggha*)
(d) 6565.80 ft^2 (i) 1-3-0-2.9 (*Ropani*) (ii) 0-1-16.02 (*Biggha*)
(e) 526.15 ft^2 (i) 1-0-2-0.7 (*Ropani*) (ii) 0-1-11.07 (*Biggha*)
11. (a) $350.54, 1.92 \text{ Dhur}$

6.0 Review

Draw the following pictures and discuss the following questions in your group. Present your answers in your class:



- How many sides (edges) are there in the given figures?
- How can we calculate the perimeters of plain surfaces in each figure?
- How can we calculate the area of plain surfaces in each figure?
- If we pile up the same shape and size of cardboard papers, what type of shape will be formed?
- Do you see the pile-up of papers?

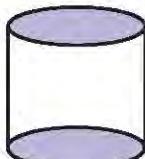
6.1 Introduction to prism

Activity 1

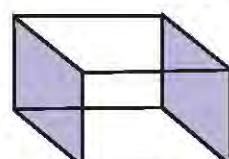
Divide the students in class into a group of 5 students in each. Take one solid object as given below in each group, observe it and discuss the following questions:



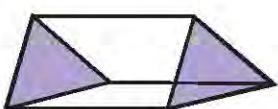
(i)



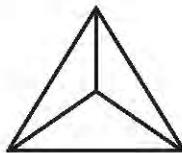
(ii)



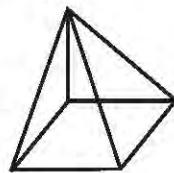
(iii)



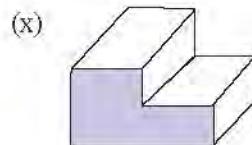
(iv)



(v)



(vi)



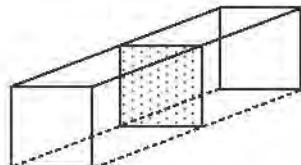
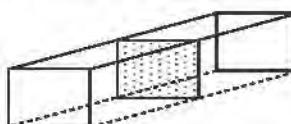
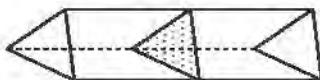
- (a) Which of the above objects have their opposite surfaces congruent and parallel?
- (b) How many straight edges consist in each object?
- (c) What are the shapes of polygons with congruent and parallel opposite sides in given objects
- (d) How many vertices are there in each object?

A prism is a three-dimensional solid object with parallel and congruent two opposite polygons.

6.2 Cross-section area of prism

Activity 2

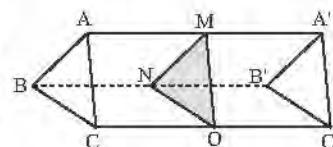
Divide the students in class into a group of 5 students in each. Take one solid object of triangular or rectangular or square shapes.



Cut the prism from its middle part as shown in the figure. Observe the surface of the object, discuss in your group and present it in your class. Eg. the group work of the third group was as follows:

The area of the triangle MNO is the cross-sectional area of the prism if we cut the solid object of the triangular base making the triangular face parallel and perpendicular to its length or height.

Therefore, ($\triangle ABC \cong \triangle MNO \cong \triangle A'B'C'$)



Characteristics of Prism

- (a) It has two parallel congruent opposite polygons, called the base of the prism. The prism of the base triangle is a triangular prism and the prism of the base rectangle is a rectangular prism.
- (b) The surface parallel with the base in a prism is called the cross-section of the prism. This cross-section is congruent with the base.
- (c) Generally there are two types of prism called oblique and right. In this class, we discussed on right prism only (all the faces except the base are perpendicular to the base).
- (d) The areas of all faces other than their bases in the prism is called the lateral surface area of the prism.
- (e) The perpendicular distance between two bases is the height or length of the prism.
- (f) The volume of the prism (V) = Area of the base (A) \times height (h)

Activity 3

Is our textbook a prism? What type of prism is this? Compare the area of the base of the first page, middle page, and last page of the book. What does each page in the book of the prism represents?

Area of the bases in a triangular prism

- (a) Area of the equilateral triangle (A) = $\frac{\sqrt{3}}{4} a^2$
- (b) Area of the isosceles triangle (A) = $\frac{b}{4} \sqrt{4a^2 - b^2}$
- (c) Area of the scalen triangle (A) = $\sqrt{s(s-a)(s-b)(s-c)}$
where, $s = \frac{a+b+c}{2}$
- (d) Area of right angled triangle (A) = $\frac{1}{2} \times p \times b$
- (e) Area of right angle isosceles triangle (A) = $\frac{1}{2} p^2$
or, $\frac{1}{2} b^2$
- (f) Area of the base in a rectangular prism (A) = $l \times b$
- (g) Area of the base in square prism (A) = l^2

6.3 Lateral surface area of prism

Activity 4

Distribute solid objects of triangular, rectangular and square base in different groups and trace the object in your copy.

The group work of group 3 is as below:

They took prism of six faces and denoted its length, breadth and height by l , b and h respectively. The faces of the prism except the bases are

ABHE, DCGF, ADFE and BCGH

All the faces are rectangular and each surface has:

$$\text{Area of the face ABHE } (A_1) = l \times h = lh$$

$$\text{Area of the face DCGF } (A_2) = l \times h = lh$$

$$\text{Area of the face ADFE } (A_3) = b \times h = bh$$

$$\text{Area of the face BCGH } (A_4) = b \times h = bh$$

Sum of the areas of all faces:

$$= A_1 + A_2 + A_3 + A_4$$

$$= lh + lh + bh + bh$$

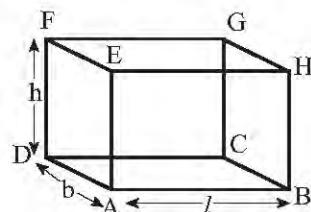
$$= 2lh + 2bh$$

$$= 2h(l + b) = 2(l + b) \times h = P \times h \quad \boxed{\text{Where, } P = \text{perimeter of the base}}$$

So, the sum of areas of all faces except its two bases is the lateral surface area of the prism.

$$\text{Lateral surface area of the prism (LSA)} = 2h(l + b)$$

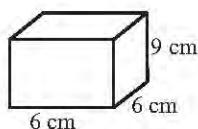
$$\text{Also, Lateral surface area of prism (LSA)} = P \times h$$



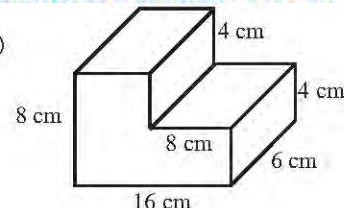
Example 1

Find the cross sectional area and lateral surface area of following prism.

(क)



(ख)



Solution,

Here,

- a) In given prism,
length (l) = 6 cm, breadth (b) = 6 cm and height (h) = 9 cm
We know that,

$$\text{Cross sectional area of prism} = \text{base area of prism}$$

$$= l \times b = 6 \times 6 = 36 \text{ cm}^2$$

So, cross sectional area of prism = 36 cm^2

Again, perimeter of the base (P) = $(6 + 6 + 6 + 6)$ cm = 24 cm

Lateral surface area of prism = $P \times h = 24 \text{ cm} \times 9 \text{ cm} = 216 \text{ cm}^2$

- b) Extend line CD upto G in the figure. Area of rectangle ABCG (A_1)

$$= AB \times BC$$

$$= 16 \times 4$$

$$= 64 \text{ cm}^2$$

Area of the rectangle DEFG (A_2)

$$= DE \times EF = 4 \times 8$$

$$= 32 \text{ cm}^2$$

Now, Cross sectional area of prism is

$$= A_1 + A_2$$

$$= 64 + 32$$

$$= 96 \text{ cm}^2$$

Again, Perimeter of the base (P)

$$= AB + BC + CD + DE + EF + FA$$

$$= 16 + 4 + 8 + 4 + 8 + 8$$

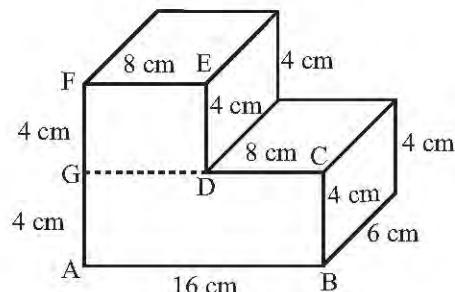
$$= 48 \text{ cm}$$

length of prism (l) = 6 cm

Lateral surface area of prism = $P \times l$

$$= 48 \times 6$$

$$= 288 \text{ cm}^2$$

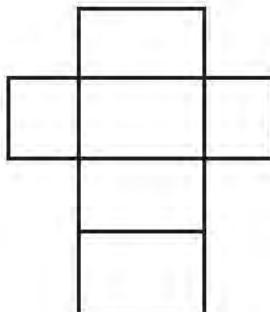
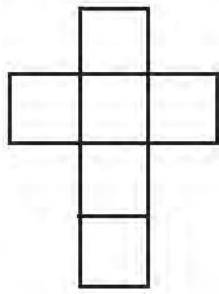
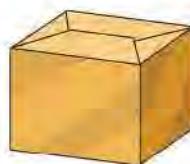


6.4 Total surface area of prism

Activity 5

Distribute some cubical hollow boxes of chalk or ink and rectangular hollow boxes of a shop or matchbox to the student's group. Open its faces slowly and trace them on chart paper. Present the answer to the following questions in your class.

- What type of shape is formed?
- How many faces are congruent and how many are not congruent in those shapes?
- If we fold the cubical and rectangular objects, can we find the original shape?
- Find the area of all faces of these cubical and rectangular objects and sum the areas. Then find the formula to find the total surface areas of cubical and rectangular objects.



The chalk box is cubical and it has six faces. So, all faces are square in shape. The area of all faces is equal. If the side of one edge is l the area will be l^2 . The area of all square surfaces = $6l^2$ sq. unit

The area of each faces in a cubical prism = l^2

The total surface area of the cubical prism (A) = $6l^2$

Likewise, there are six faces in match boxes. The box is cuboid in shape. Three faces out of six are congruent. The total surface area will be found by adding the areas of all faces.

The sum of areas of all six bases is the total surface area of rectangular based prism.

$$\text{Total surface area of rectangular base prism (TSA)} = 2(lb + bh + hl)$$

$$\text{or, TSA} = 2A + p \times h$$

where, A = Area of base

P = Perimeter of base

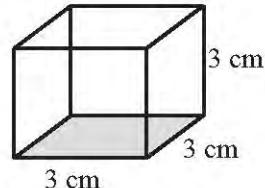
Example 2

Find total surface area of given prism:

Solution,

Here, length of cubical prism (l) = 3 cm

$$\text{Total surface area} = 6l^2 = 6 \times (3 \text{ cm})^2 = 54 \text{ cm}^2$$



Example 3

A school constructed a rectangular water tank as the figure below, for drinking water. The length of the tank is 12 ft., breadth of the tank is 10 ft and height of the tank is 6 ft. Find the total surface area of the water tank:

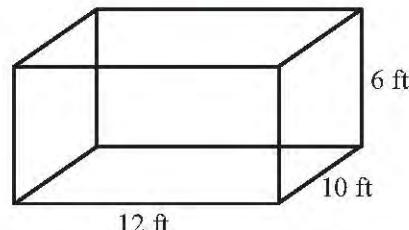
Solution,

Here,

$$\text{length of the water tank } (l) = 12 \text{ ft}$$

$$\text{breadth } (b) = 10 \text{ ft}$$

$$\text{height } (h) = 6 \text{ ft}$$



By formula,

$$\begin{aligned}\text{Total surface area of the tank (A)} &= 2(lb + bh + hl) \\&= 2(12 \times 10 + 10 \times 6 + 6 \times 12) \text{ ft}^2 \\&= 2(120 + 60 + 72) \text{ ft}^2 \\&= 2 \times 252 \text{ ft}^2 \\&= 504 \text{ ft}^2\end{aligned}$$

∴ Total surface area of the tank is 504 ft^2 .

Alternative Method

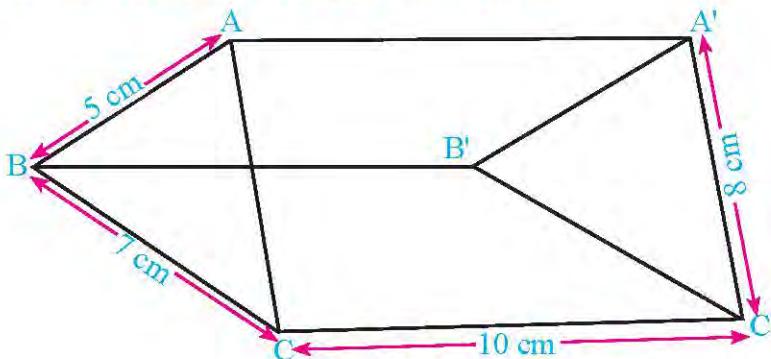
Here, area of the base (A) = $l \times b$
= 12×10
= 120 ft^2

Perimeter of the base (P) = $2(l + b)$
= $2(12 + 10)$
= 44 ft

Total surface area of the tank (TSA) = $2A + P \times h$
= $2 \times 120 + 44 \times 6$
= $240 + 264$
= 504 ft^2

Example 4

Find total surface area of the triangular base prism:



Solution,

In base of prism ΔABC ,

$$AB (c) = 5 \text{ cm}$$

$$BC (a) = 7 \text{ cm}$$

$$CA (b) = 8 \text{ cm}$$

$$\text{Now, half perimeter } (s) = \frac{a + b + c}{2} = \frac{7 + 8 + 5}{2} = 10 \text{ cm}$$

$$\begin{aligned}\text{Area of the triangular base } \Delta ABC (A) &= \sqrt{s(s - a)(s - b)(s - c)} \\ &= \sqrt{10(10 - 7)(10 - 8)(10 - 5)} \\ &= \sqrt{10 \times 3 \times 2 \times 5}\end{aligned}$$

$$= \sqrt{300}$$

$$= 17.32 \text{ cm}^2$$

Perimeter of the base (P) = $a + b + c$

$$= 7 + 8 + 5$$

$$= 20 \text{ cm}$$

length of the prism (l) = 10 cm

Now, Total surface area (TSA) = $2A + P \times l$

$$= 2 \times 17.72 + 20 \times 20$$

$$= 34.64 + 200$$

$$= 234.64 \text{ cm}^2$$

6.5 Volume of prism

Activity 5

How can we find the volume of rectangular base solid objects? On the basis of formula to find volume of rectangular base prism, ask students to find the volume of other prism:

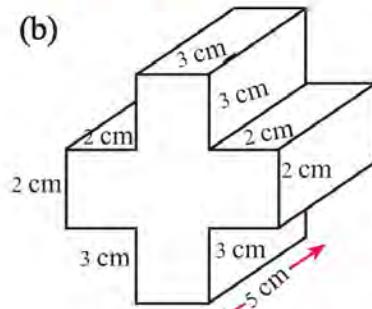
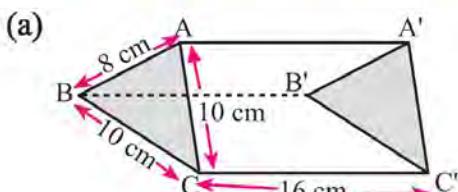
Volume of prism (V) = Area of base \times height = $A \times h$

Relationship between liter, cubic centimeter and cubic meter,

$$1000 \text{ cm}^3 = 1 \text{ l} \text{ and } 1 \text{ m}^3 = 1000 \text{ l}$$

Example 5

Find the volume and total surface area of following prism:



Solution

Here,

- (a) The given prism is isosceles triangle base prism.

where, BC = AC = (a) = 10 cm

base (AB) = (b) = 8 cm

$$\begin{aligned}\text{Area of the triangular base (A)} &= \frac{8}{4} \sqrt{4a^2 - b^2} \\ &= \frac{8}{4} \sqrt{4 \times (10)^2 - (8)^2} \\ &= \frac{8}{4} \sqrt{400 - 64} \\ &= 8\sqrt{21} = 836.66 \text{ cm}^2\end{aligned}$$

Perimeter of the base (P) = a + a + b = (10 + 10 + 8) = 28 cm

Height or length of the prism (h) = 16 cm

$$\begin{aligned}\text{Lateral surface area of the prism (LSA)} &= P \times h = 28 \times 16 \\ &= 448 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area of the prism (TSA)} &= 2 \times A + \text{LSA} \\ &= 2 \times 36.66 \\ &= 73.32 + 448 \\ &= 521.32 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume of the prism (V)} &= A \times h = 8\sqrt{21} \times 16 \text{ cm}^3 \\ &= 128\sqrt{21} \text{ cm}^3\end{aligned}$$

- (b) Join I, L, and C, F as figure.

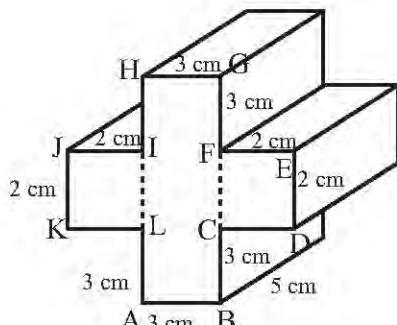
$$\begin{aligned}\text{Now, area of square IJKL (A}_1\text{)} &= (2 \text{ cm})^2 \\ &= 4 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the rectangle ABGH (A}_2\text{)} &= (3 + 2 + 3) \text{ cm} \times 3 \text{ cm} \\ &= 8 \times 3 \text{ cm}^2 = 24 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the square CDEF (A}_3\text{)} &= (2 \text{ cm})^2 \\ &= 4 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Cross sectional area of the prism (A)} &= A_1 + A_2 + A_3 \\ &= (4 + 24 + 4) \text{ cm}^2 \\ &= 32 \text{ cm}^2\end{aligned}$$

Perimeter of the base (P)



$$\begin{aligned}&= AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA \\&= (3 + 3 + 2 + 2 + 2 + 3 + 3 + 3 + 2 + 2 + 2 + 3) \text{ cm} \\&= 30 \text{ cm}\end{aligned}$$

Height (h) = 5 cm

Lateral surface area (LSA) = $P \times h = 30 \times 5 \text{ cm}^2 = 150 \text{ cm}^2$

$$\begin{aligned}\text{Total surface area (TSA)} &= 2 \times A + \text{LSA} \\&= 2 \times 32 + 150 \\&= 64 + 150 \\&= 214 \text{ cm}^2\end{aligned}$$

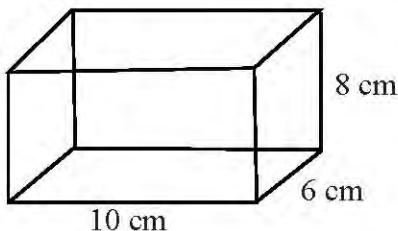
$$\begin{aligned}\text{Volume of prism (V)} &= A \times h \\&= 32 \times 5 = 160 \text{ cm}^3\end{aligned}$$

Exercise 6.1

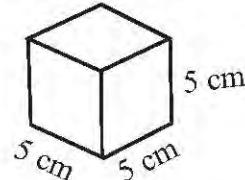
- What is the volume of prism whose cross-sectional area is 35 cm^2 and height is 10 cm?
- Find the lateral surface area of the prism of the perimeter of 36 cm and height of 8 cm.
- Find the total surface area of a prism cross-sectional area and lateral surface area are 40 cm^2 and 175 cm^2 respectively

- 4.** Find the cross-sectional area, lateral surface area, total surface area, and volume of the following prisms.

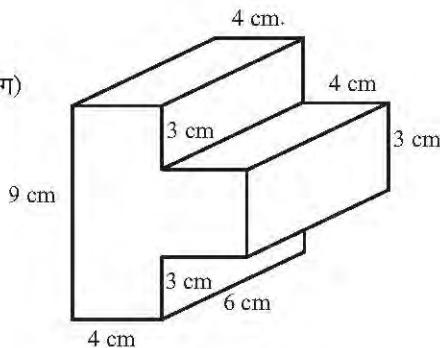
(a)



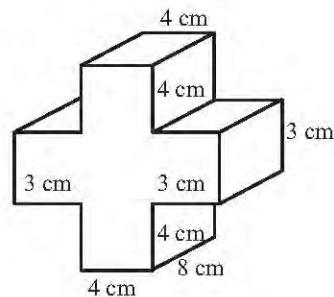
(b)



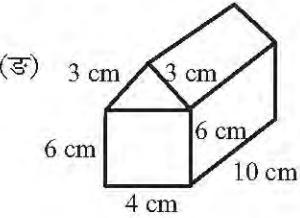
(c)



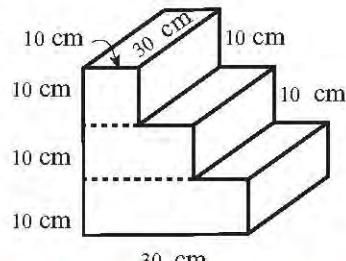
(d)



(e)



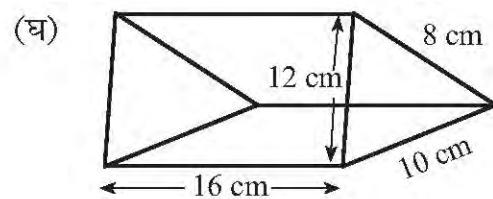
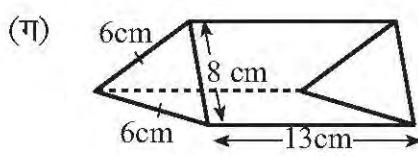
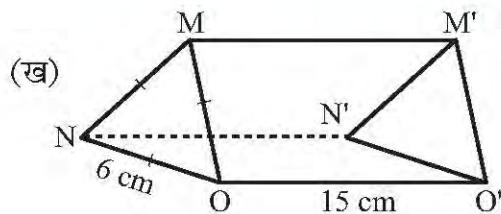
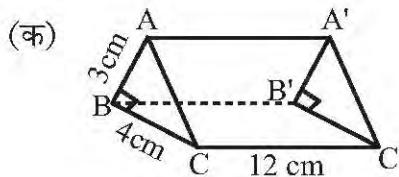
(f)



- 5.** If there is a cuboidal prism having 6 cm length, 5 cm breadth, and 8 cm height:

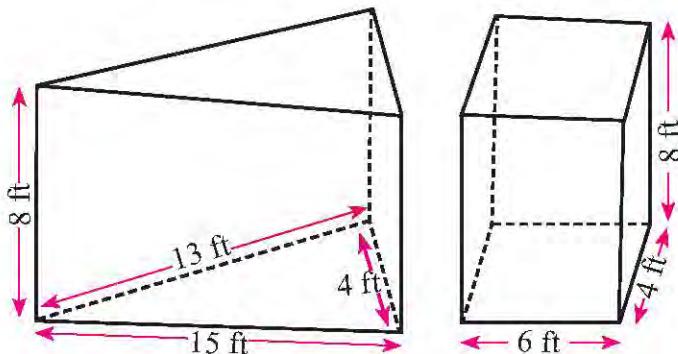
- Find the area of cross section?
 - Find the lateral surface area?
 - Find the volume?
6. A rectangular tank is 2 m long 1.5 m broad and 1 m high. Find the capacity of the tank in liters.
7. Divide a timber cube of 20 cm long into 8 equal parts. Find the length of each piece.

- 8.** Find the cross-sectional area, lateral surface area, total surface area, and volume of the following prism.



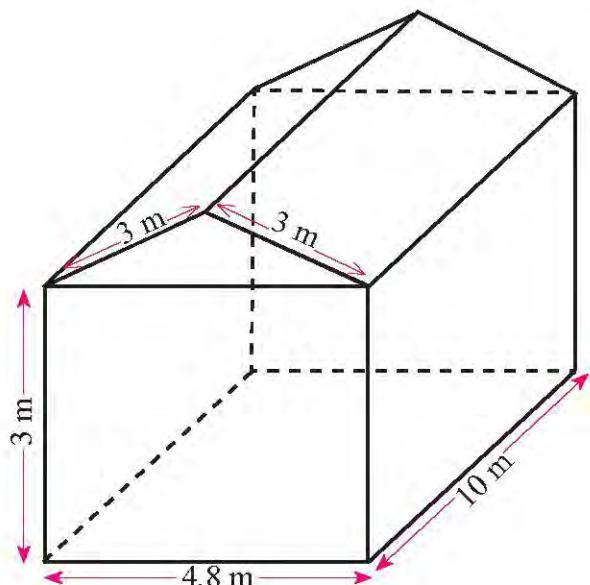
9. The volume of the right triangular prism is 864 m^3 . If the length of side to make right angle of right triangles are 8 cm and 9 cm respectively find the height of prism.
10. The height of the right triangle prism is 30 cm. If the length of the base and perpendicular of the right triangle is 4 cm and 3 cm respectively. Find the areas of rectangular faces.
11. The total surface area of the triangular prism is 660 cm^2 and the base of the prism is a right triangle. The length of the base and hypotenuse of the triangle are 12 cm and 13 cm respectively. Find the height/ length of the prism.
12. **The volume of the triangular prism is 480 cm^3 . If the length of the prism is $l \text{ cm}$ and the sides of the base are 4 cm, 13 cm and 15 cm:**
- Find the length (l) of the prism.
 - Find the areas of the rectangular faces.
 - Find the total surface area of the prism.
13. **The lateral surface area of the triangular prism is 2160 cm^2 . The perimeter of the base is 54 cm and the cross-sectional area is 126 cm^2 . Now find:**
- Height of the prism
 - Volume of the prism

14. Santaman plans to construct any one of the following metal boxes to storing rice. Find which of the following box have high capacity to store. Also find which box need less amount of metal and by how much less it is.

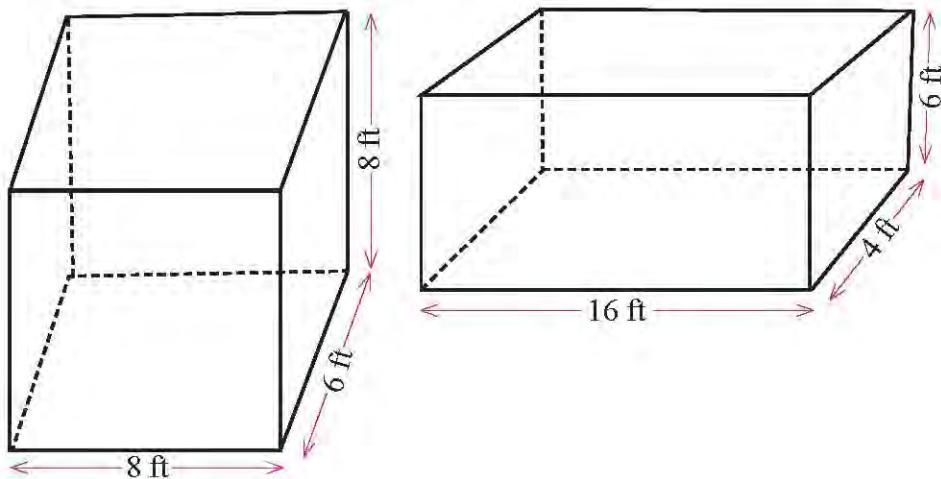


15. The following type of tent is necessary for a mathematical exhibition hall in a school. They plan for carpeting and making an entry gate using one face of breadth. Then;

- Find the clothes needed to construct such type of tent.
- Find the area of carpet for carpetting.
- If one exhibition cabin occupied 7m^2 , then how many exhibition cabins can be constructed in this hall?



16. A reserve water tank will be constructed in a village for supplying drinking water. One of the following water tank designs is to be selected. Which of the following design is appropriate for construction on the basis of the capacity of the tank and materials for construction?



Project Work

1. Collect different models of the prism from your community. Find their measurement of any two objects using the ruler and present the answer to following questions in your class:
 - (a) Find the cross-sectional area of the objects
 - (b) Find the total surface area of the objects
 - (c) Find the volume of the objects.
2. Divide the students of the class into a group of 5/5 students. Construct the triangular prism of different shapes for each group using paper, string, or wood. Find the measurement of each part of the prism find;
 - (a) cross- sectional area.
 - (b) lateral surface area.
 - (c) how many times is the lateral surface area more than its cross-sectional area (in percent)?

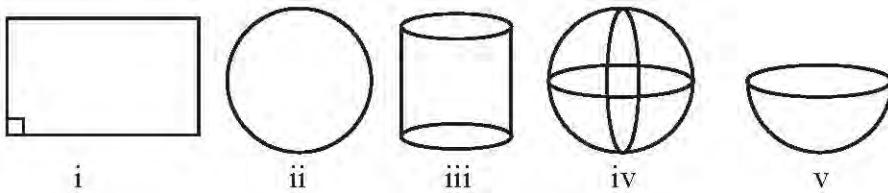
Answers

1. 350 cm^3
2. 288 cm^2
3. 255 cm^2
4. (a) $80 \text{ cm}^2, 216 \text{ cm}^2, 376 \text{ cm}^2, 480 \text{ cm}^3$
(b) $25 \text{ cm}^2, 100 \text{ cm}^2, 150 \text{ cm}^2, 125 \text{ cm}^3$
(c) $48 \text{ cm}^2, 180 \text{ cm}^2, 276 \text{ cm}^2, 288 \text{ cm}^2$
(d) $62 \text{ cm}^2, 336 \text{ cm}^2, 460 \text{ cm}^2, 496 \text{ cm}^3$
(e) $24 + 2\sqrt{5} \text{ cm}^2, 220 \text{ cm}^2, 276.94 \text{ cm}^2, 284.72 \text{ cm}^3$
(f) $600 \text{ cm}^2, 3600 \text{ cm}^2, 4800 \text{ cm}^2, 18000 \text{ cm}^3$
5. (a) 30 cm^2 (b) 276 cm^2 (c) 240 cm^3
6. 3000 l
7. 10 cm
8. (a) $6 \text{ cm}^2, 144 \text{ cm}^2, 156 \text{ cm}^2, 72 \text{ cm}^3$
(b) $9\sqrt{3} \text{ cm}, 270 \text{ cm}^2, 301.16 \text{ cm}^2, 135\sqrt{3} \text{ cm}^3$
(c) $8\sqrt{5} \text{ cm}^2, 260 \text{ cm}^2, 295.77 \text{ cm}^2, 104\sqrt{5} \text{ cm}^2$
(d) $39.69 \text{ cm}^2, 480 \text{ cm}^2, 559.38 \text{ cm}^2, 634.98 \text{ cm}^3$
9. $24 \text{ cm}, 10. 360 \text{ cm}^2$
10. 20 cm
11. 20 cm
12. (a) 20 cm (b) 640 cm^2 (c) 688 cm^2
13. (a) 40 cm (b) 5040 cm^2
- 14 -16. Show to your teacher.

7.0 Review

Make an appropriate number of groups and take one figure as given below in different groups. Observe these figures and present the conclusion in your class from group discussion on the basis of the following questions:

- How and what type of plane surface are there on the given figures? Count them and write.
- How many vertices and edges are there on given figures? Count them.
- Count how many straight edges are there on given figures?



Here,

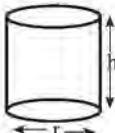
- In the first figure, there is one rectangular surface. There are four vertices and four straight edges. It is a rectangle.
- In the second figure, there is a circular surface. There are no vertices.
- In the third figure, there are two circular surfaces. There are no edges and one curved surface.
- Similarly, in the fourth figure there are no plane surfaces, edges, and vertices.
- In the fifth figure, there is one plane surface and one curved surface. There are no vertices.

7.1 Cylinder

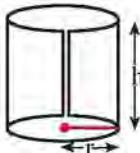
Surface area of the cylinder

Activity 1

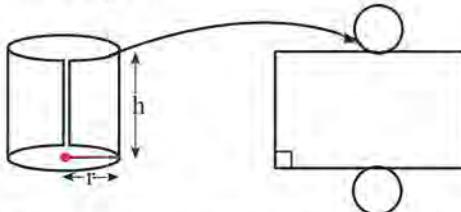
Take one paper-made cylinder having different sizes in each group.



Cut the cylinder vertically as shown in the above figure.



Make a rectangle after cutting the paper cylinder such that the circumference of the circle is converted into length and height into the breadth of the rectangle as shown in the above figure.



Now, observe the rectangle and cylinder, then discuss the following questions and present them in your class.

- What is the length of the circumference of the circle? Write.
- If the length and breadth are given, then which formula is appropriate to find the area of the rectangle?
- Which part of the cylinder is represented by the area of the rectangle?

The following conclusions can be drawn from the discussion:

The length of the circumference of the circle is the perimeter of the circle.

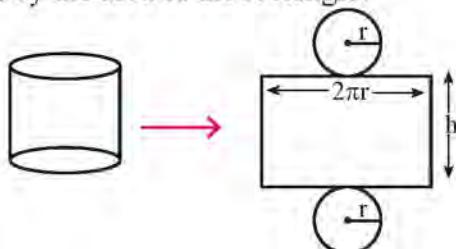
So, the circumference of the circle = length of the perimeter = $2\pi r$

Likewise, the formula to find the area of the rectangle is $l \times b$.

Here, the area of the rectangle is represented by the curved surface area of the cylinder.

The curved surface area of the cylinder = area of rectangle = $l \times b = 2\pi r \times h = 2\pi rh$ sq. unit.

Again, add the area of two circles of the cylinder on the curved surface area of the cylinder. Are the areas of two circles equal on the cylinder? Yes, they are congruent.



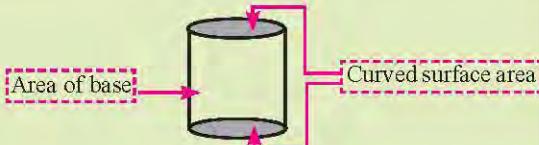
The total surface area of the cylinder = curved surface area of cylinder + area of two circles.

$$\begin{aligned}&= 2\pi rh + 2\pi r^2 \quad [\text{area of one circle} = \pi r^2] \\&= 2\pi r(h + r)\end{aligned}$$

The side figure with one curved surface and two parallel circular bases is called Cylinder.

In cylinder

- Area of the base (A) = πr^2
- Curved Surface Area = $2\pi rh$
- Total Surface Area = $2\pi r(r + h)$
 $= C(r + h)$ where, $C = 2\pi r$



Example 1

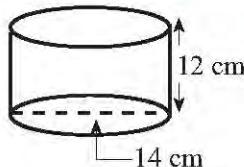
Find the curved surface area and total surface area (TSA) of cylinder.

Solution,

Here,

$$\text{Diameter } (d) = 14 \text{ cm}$$

$$\text{Height } (h) = 12 \text{ cm}$$



$$\text{Curved surface area of cylinder} = ?$$

$$\text{Total surface area} = ?$$

$$\text{Radius } (r) = \frac{d}{2} = \frac{14}{2} \text{ cm} = 7 \text{ cm.}$$

Now,

$$\text{Curved surface area of cylinder (CSA)} = 2\pi rh$$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times 7 \times 12 \\&= 528 \text{ cm}^2\end{aligned}$$

$$\text{Total surface area of cylinder (TSA)} = 2\pi r(r + h)$$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times 7 (7 + 12) \text{ cm}^2 \\&= 44 \times 19 \text{ cm}^2 \\&= 836 \text{ cm}^2\end{aligned}$$

Example 2

The curved surface area of a cylindrical box is 1232 cm^2 . If the radius of the base and height of the cylinder are equal, find the area of circular bases and total surface area of cylinder. Also compare curved surface area and the circular base.

Solution,

Here,

$$\text{Curved surface area of cylinder (CSA)} = 1232 \text{ cm}^2$$

$$\text{Radius } (r) = \text{height } (h)$$

$$\text{Area of the circular base} = ?$$

$$\text{Total surface area} = ?$$

We know that,

$$\text{Curved surface area of cylinder} = 2\pi r h$$

$$\text{or, } 1232 = 2 \times \frac{22}{7} \times r \times r \quad [\because r = h]$$

$$\text{or, } \frac{1232 \times 7}{2 \times 22} = r^2$$

$$\text{or, } 196 = r^2$$

$$\text{or, } r = \sqrt{196} = 14$$

$$\therefore \text{Radius } (r) = 14 \text{ cm}$$

$$\text{Now, radius } (r) = \text{height } (h) = 14 \text{ cm}$$

$$\begin{aligned}\text{Area of the circular base} &= 2\pi r^2 \\ &= 2 \times \frac{22}{7} \times (14 \text{ cm})^2 \\ &= 1232 \text{ cm}^2\end{aligned}$$

Again,

$$\begin{aligned}\text{Total surface area (TSA)} &= \text{Curved surface area} + \text{Area of the circular base} \\ &= (1232 + 1232) \text{ cm}^2 \\ &= 2464 \text{ cm}^2\end{aligned}$$

Here, curved surface area of cylinder and area of the circular base are equals.

Example 3

The total surface area of cylinder is 2992 cm^2 . If the sum of height and radius is 34 cm , then find:

- (a) Radius of the base of cylinder.
- (b) Area of the base .
- (c) Curved surface area.

Solution,

$$\text{Total surface area (TSA)} = 2992 \text{ cm}^2$$

$$\text{Here, sum of radius and height } (r + h) = 34 \text{ cm}$$

- (a) Radius of the base (r) =?

By formula,

$$\text{Total surface area of cylinder (TSA)} = 2\pi r(r + h)$$

$$\text{or, } 2992 = 2 \times \frac{22}{7} \times r \times 34$$

$$\text{or, } \frac{2992 \times 7}{2 \times 22 \times 34} = r$$

$$\text{or, } r = 14 \text{ cm}$$

- (b) Area of the base (A) = πr^2

$$\begin{aligned} &= \frac{22}{7} \times (14)^2 \\ &= 616 \text{ cm}^2 \end{aligned}$$

- (c) Curved surface area = $2\pi r h$

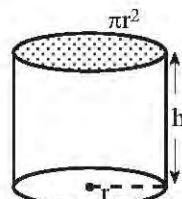
$$\begin{aligned} &= 2 \times \frac{22}{7} \times 14 \times (34 - r) \quad [\because r + h = 34 \text{ cm}] \\ &= \frac{44}{7} \times 14 \times (34 - 14) \\ &= 1760 \text{ cm}^2 \end{aligned}$$

Volume of cylinder

Activity 2

Make an appropriate group in your class. Take cylindrical radish or soft cylindrical solid objects and measure their radius and height.

Cut the cylindrical radish or soft cylindrical solid objects as shown in the following figure and make a cuboid from the pieces of the cylinder.



Now, compare the cylinder and cuboid and find the cuboid's length, breadth, and height. Then find the volume of the cuboid.

Here, the length of the cuboid = πr

The breadth of the cuboid = r

Height of the cuboid = h

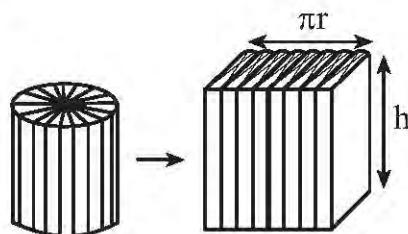
Volume of the cuboid = $l \times b \times h = \pi r \times r \times h = \pi r^2 h$

Here, the volume of the cylinder = volume of the cuboid

$$= \pi r^2 h$$

$$= \pi r^2 \times h$$

$$= A \times h \quad [\text{a} \text{ngfsf] cfwf/ j[Q x'g] ePsfn] cfwf/sf] If]qkmn (A) = \pi r^2 \times G5 .]$$



The volume of the cylinder (V) = $\pi r^2 h = A \times h$, where, A is the area of the base of the cylinder

Example 4

If radius of base and height of a cylinder are 14 cm and 20 cm respectively, find volume of the cylinder.

Solution,

Here,

Radius of the base (r) = 14 cm

height of cylinder (h) = 20 cm

Volume of cylinder (V) = ?

By formula,

Volume of cylinder (V) = $\pi r^2 h$

$$= \frac{22}{7} \times (14)^2 \times 20 \text{ cm}^3$$

$$= \frac{22}{7} \times 196 \times 20 \text{ cm}^3$$

$$= 12,320 \text{ cm}^3$$

Example 5

The capacity of cylindrical water tank is 539 l. If the height of cylinder is 1.4 m, then find area of the base.

Solution,

Here,

$$\text{Capacity of cylinder (Volume of cylinder) } (V) = 539 \text{ l}$$

$$\begin{aligned} &= \frac{539}{1000} \text{ m}^3 & [\because 1 \text{ m}^3 = 1000 \text{ l}] \\ &= 0.539 \text{ m}^3 \end{aligned}$$

$$\text{height (h)} = 1.4 \text{ m}$$

$$\text{Area of the base (A)} = ?$$

By formula,

$$\text{Volume of cylinder (V)} = A \times h$$

$$\text{or, } 0.539 \text{ m}^3 = A \times 1.4 \text{ m}$$

$$\text{or, } \frac{0.539}{1.4} \text{ m}^2 = A$$

$$\text{or, } A = 0.385 \text{ m}^2$$

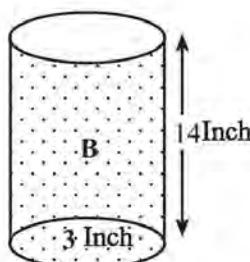
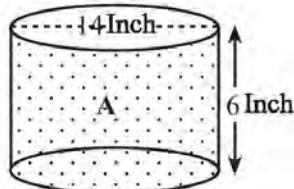
$$A = 0.385 \times 100 \times 100 \text{ cm}^2 = 3850 \text{ cm}^2$$

Therefore, area of base of cylinder (A) = 3850 cm²

Example 6

Rajan sold the soybean putting them into paper pots as shown in the above figures. The diameter of pot A is 14 inches and the height is 6 inches. Likewise, the radius of pot B is

3 inches, and the height is 14 inches. Saroj and Vishal go to the market to buy soyabean. At that time Saroj said the quantity of soybean on both pots is equal, so the price of both pots must be equal. Again, Bishal said the quantity of soybean on both pots is not equal, so the price of both pots may not be equal. Find, who is correct and who is wrong on the basis of their arguments.



Solution,

Here, Both the pictures given above are cylinder

In cylinder A

$$\text{diameter } (d) = 14 \text{ inch}$$

$$\text{height } (h_1) = 6 \text{ inch}$$

$$\text{Now, Radius } (r_1) = \frac{d}{2} = \frac{14}{2} \text{ inch} = 7 \text{ inch}$$

$$\text{Volume of cylinder A } (V_1)$$

$$= \pi r_1^2 h_1$$

$$= \frac{22}{7} \times (7)^2 \times 6$$

$$= 924 \text{ cubic inch}$$

Similarly, In cylinder B

$$\text{radius } (r) = 3 \text{ inch}$$

$$\text{height } (h_2) = 14 \text{ inch}$$

$$\text{Volume of cylinder B } (V_2)$$

$$= \pi r^2 h_2$$

$$= \frac{22}{7} \times (3)^2 \times 14 \text{ cubic inch}$$

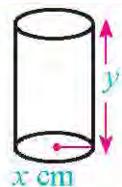
$$= 396 \text{ cubic inch}$$

Among them Bishal is correct because volume of cylinder A is more than B. So, the rate of soyabean present inside A must be more than that of B.

Exercise 7.1

1. If radius and height of cylinder is $x \text{ cm}$ and $y \text{ cm}$:

- (a) Circumference of base b) Area of the base
- (c) Curved surface area d) Total surface area
- (e) Volum



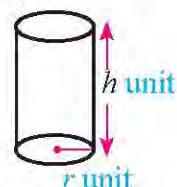
2. If raius of cylinder is $x \text{ cm}$ and height is $y \text{ cm}$, then find curved surface area and volume of the prism.

3. The circumference of the base of a cylinder is $c \text{ cm}$ and height $h \text{ cm}$. Find curved surface area of the cylinder.

4. Find the volume of the cylinder of area A sq. feet and height h feet.

5. If the radius of the cylinder is r unit and height h unit, find:

- | | |
|------------------------|-------------------------|
| (a) Volume | (b) Curved surface area |
| (c) Plane surface area | (d) Total surface area |

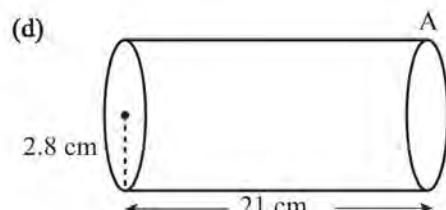
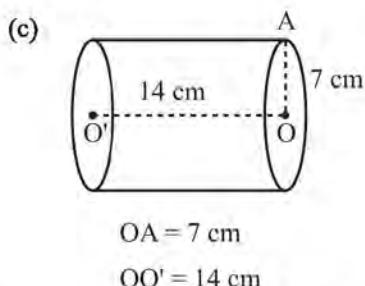
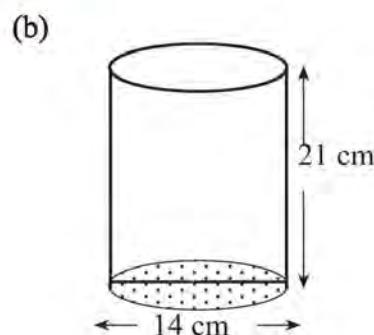
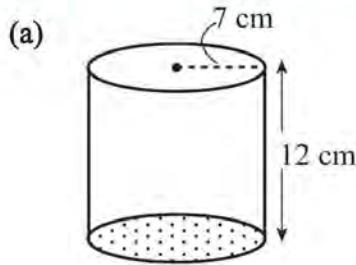


6. The circumference of the base of a cylinder is C unit and the sum of radius and height is S unit, find the total surface area of the cylinder.

7. Find the plane surface area, curved surface area, and volume of the following prism:

- (a) radius (r) = 7 cm and height (h) = 5 cm
- (b) radius (r) = 3.5 m and height (h) = 6 m
- (c) radius (r) = 2 ft and height (h) = 7 ft

8. Find the area of the base, plane surface area, curved surface area, total surface area, and volume of the following prism:



9. If the circumference of the base of a cylinder is 176 cm, and the height is 30 cm, find the curved surface area and total surface area of the cylinder.

10. If the circumference of the base of a cylinder is 88 cm, and the sum of radius and height is 24 cm, then find:

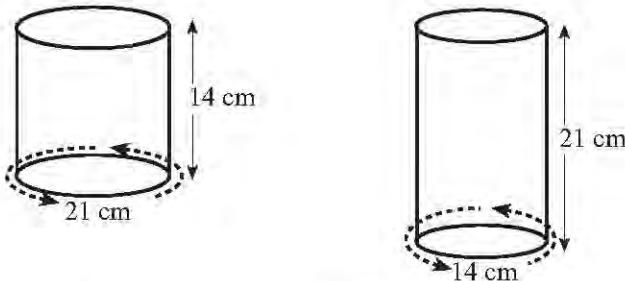
- (a) Area of the base. (b) Curved surface area sum of radius and height is 24 cm
- (c) Total surface area (d) Volume

11. If the sum of the radius and height of the cylinder is 34 cm, and the total surface area is 2992 cm^2 then find the volume of the prism.

12. The sum of the diameter and height of the cylinder is 28 cm, and the curved surface area is 462 cm^2 , find the total surface area.

13. The ratio of the radius of the base and height of a cylinder is 1:3 and their sum is 56 cm, find the curved surface area and total surface area of the cylinder.

14. If the ratio of the radius of the base and height of a cylinder is 1:3 and its curved surface area is 924 cm^2 , find the area of the base and volume of the cylinder.
15. The curved surface area of a cylindrical log is three times more than the area of the base. If the sum of the radius of the base and height of the log is 25 cm, find the volume of the cylindrical log.
16. The inner circumference of the following cylindrical pots are 21 cm and 14 cm and the heights are 14 cm and 21 cm respectively. Find how much water can be filled in these pots.



17. The students in a school are requested to participate in the competition of making cylindrical pencil cases. The radius and height of each pencil case must be 3 cm and 10.5 cm respectively. If 35 students participated in the competition, find how much cardboard paper was used in that competition.
18. A hospital provides mushroom soup on a cylindrical glass with a 7 cm radius for his patient each day. If the mushroom soup filled the 6 cm height of the glass, find how much soup is prepared by the hospital daily for their 250 patients.
19. Mankumari planned to prepare a cylindrical bucket from the mat to store crops. If the breadth of the mat is 1.1 meter, then how long mat is needed to store 1.4 cu. meter crops.

Project Work

Sit in the appropriate number of groups. Search different cylindrical objects like; water tanks, drums, buckets to store crops, glasses, etc and measure their radius of the base and height. Find the curved surface area, total surface area, and volume of each object and present the result in your class.

Answers

- 1-6. Show to your teacher.
7. (a) 308 cm^2 , 220 cm^2 , 770 cm^3
(b) 77 cm^2 , 132 cm^2 , 231 cm^3
(c) 25.14 ft^2 , 88 ft^2 , 88 ft^3
8. (a) 154 cm^2 , 308 cm^2 , 528 cm^2 , 836 cm^2 , 1848 cm^3
(b) 154 cm^2 , 308 cm^2 , 924 cm^2 , 1232 cm^2 , 3234 cm^3
(c) 154 cm^2 , 308 cm^2 , 616 cm^2 , 924 cm^2 , 2156 cm^3
(d) 2464 cm^2 , 4928 cm^2 , 3696 cm^2 , 8624 cm^2 , 51744 cm^3
9. 5288 cm^2 , 10208 cm^2
10. (a) 616 cm^2 (b) 880 cm^2 (c) 2112 cm^2 (d) 6160 cm^3
11. 12320 cm^3
12. 539 cm^2 if 1617 cm^2
13. 3696 cm^2 , 4928 cm^2
14. 154 cm^2 , 3234 cm^3
15. 4714.28 cm^3
16. 0.162635 l
17. 7919.8 cm^2
18. 57.75 l
19. 4 m

7.2.1 Surface area of sphere

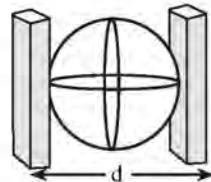
What do you mean by sphere? Is it like a circle? Can we make a circle into our copy? Yes, we can construct a circle in our copy because it is a closed plane figure made by joining the points' equidistance from a fixed point. The fixed point is called the center of the circle and the equal distance is called the radius of the circle. Balls and marbles are the spheres.



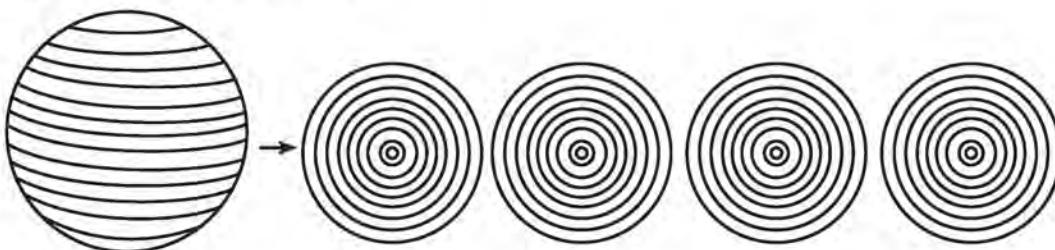
Activity 1

Make an appropriate number of groups and take a rubber ball for each group.

Thumb a pin at any point on the surface of the ball. Cover the ball by thread with the help of a pin and glue on the overall surface of the ball. Now, remove the thread from the ball.



Find the diameter of the rubber ball. For this, put the ball in the middle of two pieces of cardboard paper as shown in the above figure. The distance between two pieces of cardboard paper is the diameter of the sphere. The radius is half of the diameter. Using the radius, construct four circles in your copy and paste the thread on the surface of all circles.



Present the result of the above Activity in your class.

Here, the thread covered by the surface of the ball also covers the surface of four circles where the radius of all circles is equal to the radius of the sphere.

So, the surface area of the sphere = area of the four circles.

$$\begin{aligned} &= \pi r^2 + \pi r^2 + \pi r^2 + \pi r^2 \\ &= 4\pi r^2 \end{aligned}$$

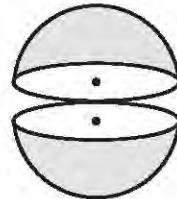
Therefore, the surface area of the sphere (A) = $4\pi r^2$, where r is the radius of the sphere.

7.2.2 Surface area of hemisphere

Activity 2

Make necessary groups on the basis of students' numbers. Take one lemon for each group. Cut the lemon into two equal parts from its center and discuss the following questions:

- What are these two pieces called?
- How many surfaces are there?
- What is the curved surface area of the hemisphere?
- What is the total surface area of the hemisphere?



The two equal parts of the sphere made by cutting from its center are called the hemispheres. There are two surfaces in a hemisphere. They are called curved surfaces and plane surfaces. The curved surface area of the hemisphere is half of the curved surface area of the sphere.

The curved surface area of the hemisphere = $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Likewise, the total surface area of the hemisphere

$$\begin{aligned}&= \text{curved surface area} + \text{plane surface area.} \\&= 2\pi r^2 + \pi r^2 && [\because \text{the plane surface area is the area of the circle}] \\&= 3\pi r^2\end{aligned}$$

Where r is the radius of the hemisphere which is equal to the radius of the sphere.

Example 1

Find the surface area of a sphere having radius 3.2 cm.

Solution,

Here, radius of sphere (r) = 3.2 cm

Surface area of sphere = ?

From formula,

$$\begin{aligned}\text{Surface area of the sphere (A)} &= 4\pi r^2 \\&= 4 \times \frac{22}{7} \times (3.2 \text{ cm})^2 \\&= 128.73 \text{ cm}^2\end{aligned}$$

Example 2

If the circumference of the great circle of a water tank is 22 ft, calculate the surface area of water tank. ($\pi = \frac{22}{7}$)

Solution,

Here, circumference of the great circle (C) = 22 ft

Surface area of the sphere =?

We know that,

- a) Circumference of the circle (C) = $2\pi r$

$$22 \text{ ft} = 2 \times \frac{22}{7} \times r$$

$$\text{or, } \frac{22 \times 7}{2 \times 22} \text{ ft} = r$$

$$\text{or, } r = 3.5 \text{ ft} = 3 \text{ ft } 6 \text{ inch}$$

- b) Surface area of the sphere (A)

$$\begin{aligned} &= 4 \pi r^2 = 4 \times \frac{22}{7} \times (3.5)^2 \\ &= 154 \text{ ft}^2 \end{aligned}$$

So, surface area of the sphere (A) = 154 ft²

Example 3

Find the curved surface area, total surface area and plain surface area of a hemisphere with diameter 28 cm.

Solution,

Here, diameter of the hemisphere (d) = 28 cm

Radius of hemisphere (r) = 14 cm

Plain surface area =?

Curved surface area =?

Total surface area =?

Using formula,

a) Plain surface area = $\pi r^2 = \frac{22}{7} (14 \text{ cm})^2 = 616 \text{ cm}^2$

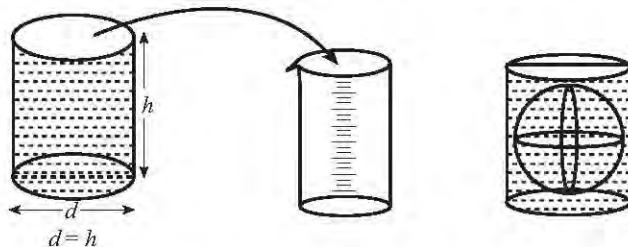
b) Curved surface area (CSA) = $2\pi r^2 = 2 \times \frac{22}{7} \times (14 \text{ cm})^2 = 1232 \text{ cm}^2$

c) Total surface area (TSA) = $3\pi r^2 = 3 \times \frac{22}{7} \times (14 \text{ cm})^2 = 1848 \text{ cm}^2$

7.2.3 Volume of sphere

Activity 3

Sit into the appropriate number of groups. In each group, take a cylinder having a diameter of the base equal to the diameter of the sphere. The diameter and height of the cylinder must be equal. Now, fill the water in the cylinder and note down the quantity of the water in the cylinder using a measuring cylinder.



Now put the sphere into the cylinder as shown in the picture. Then put the water in the cylinder with the sphere from the measuring cylinder. Again note down the quantity of the water in the cylinder using a measuring cylinder. Now compare the quantity of water with and without putting the sphere in the cylinder and present the result in the class.

Here, the quantity of the water by putting the sphere in the cylinder is one-third of the water without putting the sphere in the cylinder. The two-third of the water was replaced by the sphere.

$$\text{Volume of the sphere (V)} = \frac{2}{3} \text{ volume of cylinder}$$

$$= \frac{2}{3} (\text{base area of the cylinder} \times \text{height})$$

$$= \frac{2}{3} (\pi r^2 \times h) = \frac{2}{3} (\pi r^2 \times d) \quad [\because d = h]$$

$$= \frac{2}{3} \pi r^2 \times 2r = \frac{4}{3} \pi r^3$$

$$\text{Volume of cylinder (V)} = \frac{4}{3} \pi r^3 \text{ where } r = \text{radius of the sphere.}$$

7.2.4 Volume of hemisphere

Activity 4

Does the volume of hemisphere is half of the volume of sphere? Discuss in your group.

Here, volume of hemisphere is half of the volume of sphere.

$$\begin{aligned}\text{Volume of hemisphere (V)} &= \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \\ &= \frac{2}{3} \pi r^3\end{aligned}$$

Example 4

Find the volume of sphere with diameter 21 cm:

Solution,

Here, diameter (d) = 21 cm

volume of sphere (V) = ?

$$\text{Now, radius (r)} = \frac{d}{2}$$

$$= \frac{21}{2} \text{ cm}$$

Now,

$$\begin{aligned}\text{Volume of sphere (V)} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2} \right)^3 \text{ cm}^3 \\ &= 4851 \text{ cm}^3\end{aligned}$$

So, Volume of sphere (V) = 4851 cm³

Example 5

The diameter of metallic sphere is 4.2 cm. If $1 \text{ cm}^3 = 8.9 \text{ gm}$, find the mass of the sphere:

Solution,

Here, diameter of sphere (d) = 4.2 cm

$$1 \text{ cm}^3 = 8.9 \text{ gm}$$

mass of sphere =?

$$\text{Now, radius } (r) = \frac{d}{2} = \frac{4.2}{2} \text{ cm} = 2.1 \text{ cm}$$

$$\text{By formula, volume of sphere } (V) = \frac{4}{3}\pi r^3$$

$$\begin{aligned} &= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \text{ cm}^3 \\ &= 38.808 \text{ cm}^3 \end{aligned}$$

We know that,

$$1 \text{ cm}^3 = 8.9 \text{ gm}$$

$$\begin{aligned} 38.808 \text{ cm}^3 &= 8.9 \times 38.808 \text{ gm} \\ &= 345.39 \text{ gm} \end{aligned}$$

∴ Mass of the sphere (mass) = 345.39 gm

Example 6

The radius of a hemispherical urn is 3.5 cm. If the urn is filled with water completely, find the volume of water.

Solution,

Here, radius of the hemisphere (r) = 3.5 cm

volume of hemisphere = volume of water

By formula,

volume of the water in urn

$$\begin{aligned} (V) &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 \text{ cm}^3 = 89.8 \text{ cm}^3 \end{aligned}$$

So, volume of water (V) = 89.8 cm³

Example 7

The radius of the sphere and hemisphere with equal areas are r_1 and r_2 , respectively. Find the ratio of their radius.

Solution,

Here, radius of sphere = r_1

radius of hemisphere = r_2

$$\frac{r_1}{r_2} = ?$$

We know that,

$$\text{Surface area of sphere } (A_1) = 4\pi r_1^2$$

$$\text{Surface area of hemisphere } (A_2) = 3\pi r_2^2$$

By question,

$$A_1 = A_2$$

$$4\pi r_1^2 = 3\pi r_2^2$$

$$\text{or, } \frac{r_1^2}{r_2^2} = \frac{3\pi}{4\pi}$$

$$\text{or, } \frac{r_1}{r_2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore r_1 : r_2 = \sqrt{3} : 2$$

Example 8

Three solid metallic spheres with ratio 1 cm, 6 cm and 8 cm are melted and made a sphere. Find the radius of large sphere.



Solution,

Here, radius of first sphere (r_1) = 1 cm

radius of second sphere (r_2) = 6 cm

radius of third sphere (r_3) = 8 cm

Suppose, V_1 , V_2 , V_3 are the volume of three small sphere and V is the volume of new sphere.

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$$

Let r is the radius of new sphere, then

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\text{or, } \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_1^3 + r_2^3 + r_3^3)$$

$$\text{or, } r^3 = [(1)^3 + (6)^3 + (8)^3] \text{ cm}^3$$

$$\text{or, } r^3 = (1 + 216 + 512) \text{ cm}^3$$

$$\text{or, } r^3 = 729 \text{ cm}^3$$

$$\text{or, } r = \sqrt[3]{729} \text{ cm}$$

$$\text{or, } r = 9 \text{ cm}$$

So, radius of new sphere (r) = 9 cm

Exercise 7.2

- 1. If the radius of a sphere is x unit, then find:**

- 5. If the diameter of a sphere is d cm, find:**

- (a) Circumference of the large circle
 (b) Area of the large circle
 (c) Volume of the sphere.
 (d) Surface area of the sphere

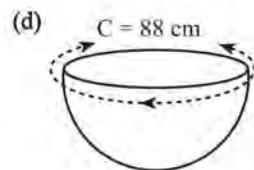
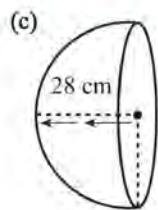
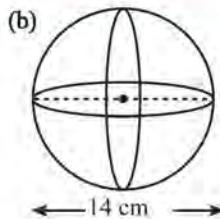
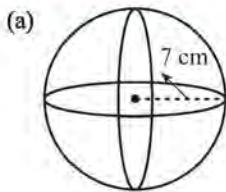
6. Find, how many materials (TPE Elastomer Lather) are needed to make given in the picture.



volleyball
 $r = 10.5 \text{ cm}$

7. Kazakhstan Pavilion and Science of Noor Alam, Kazakhstan is in a spherical shape. A science exhibition of 100 countries was organized in this hall in 2017. If the diameter of the spherical hall is 80 meters and the outer surface of the hall is covered by glass, find the surface area of the hall covered by the hall.

8. Find the surface area and volume of the given spheres and hemispheres:



9. The diameter of a spherical ball is 35 cm, find the surface area and volume of the ball.
10. The circumference of the large circle of a hemisphere is 44 cm. Find the total surface area of the hemisphere.
11. The total surface area of the spherical solid object is 2464 cm^2 . Find the diameter of the object.
12. If the volume of a sphere is 38808 m^3 . Find its radius.
13. (a) If the total surface area of the hemisphere is $243\pi \text{ cm}^2$, find its volume.
(b) If the volume of a sphere is $2304\pi \text{ cm}^3$, find the surface area of the sphere.
14. The diameter of the moon is approximately one-fourth of the diameter of the sun. Find the ratio of their surface areas.
15. The ratio of surface area and volume of a sphere is 1:3. Find the diameter and volume of the sphere.
16. If the radius of the tennis ball is doubled on its original radius, find the total changes in their volume.
17. If the radius of a spherical balloon is increased from 7cm to 14cm, find the ratio of the surface areas.
18. Three spheres with radius of 2 cm, 12 cm, and 16 cm respectively are melted and formed into a single sphere. Find the diameter of the new sphere.
19. The total surface area of a hemispherical object is $243\pi \text{ cm}^2$, find the length of the perimeter and volume of the object.

Project Work

- Take a plastic ball and cut it from the middle such that it would be divided into two equal parts. Now, construct a cylinder of diameter and height equal to the diameter of the ball using thick paper. Fill the water into the hemispherical ball and pour the water into the cylinder. Find out how many times the cylinder is filled with the water. Find the ideas related to the formula of volume of sphere and hemisphere on the basis of the formula of the cylinder.
- Take a spherical lemon. Cut the lemon into two equal parts. Trace four circles using the great circle of hemispherical lemon. Now paste the cover of the lemon on the circle. Present the conclusion of these Activity in your class.
- Make different groups in appropriate numbers. Collect the football, volleyball, and basketball from your school for each group. Find the total surface area and volume of the football, volleyball, and basketball. Present the result in your class and make a report including the suggestions of the class.

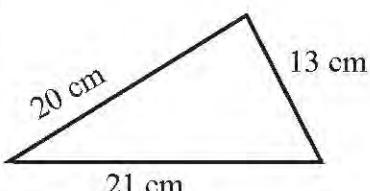
Answers

- | | | |
|------|--|---|
| 1-5. | Show to your teacher. | |
| 6. | 1386 cm^2 | $7. 20114.29 \text{ m}^2$ |
| 8. | (a) 616 cm^2 , 1437.34 cm^3
(c) 7392 cm^2 , 45994.67 cm^3 | (b) 616 cm^2 , 1437.34 cm^3
(d) 1848 cm^2 , 5749.34 cm^3 |
| 9. | 3850 cm^2 , 22458.34 cm^3 | 10. 462 cm^2 11. 28 cm |
| 12. | 21 m | 13. (a) 1527.43 cm^3 |
| | (b) 1810.29 cm^2 or $576\pi \text{ cm}^2$ | 14. 16: 1 |
| 15. | 18 cm, 3054.85 cm^3 | 16. 8 times greater |
| 17. | 1: 4 18. 36 cm | 19. 56.57 cm , 1527.43 cm^3 |

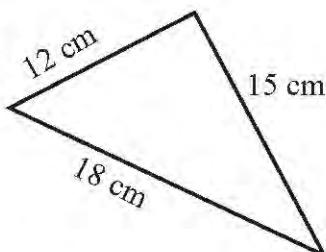
Miscellaneous Exercise

1. Find the areas of the following triangles:

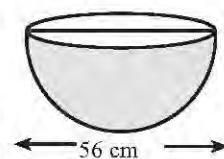
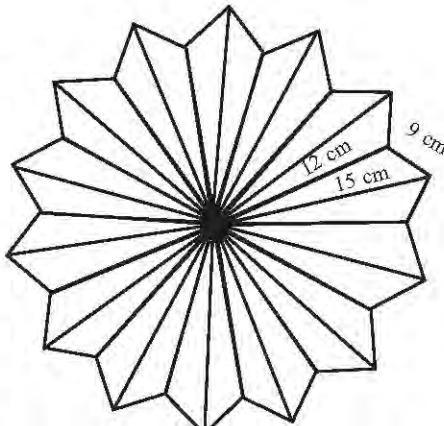
a)



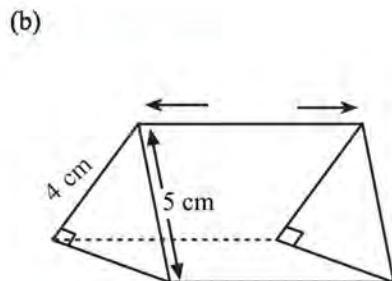
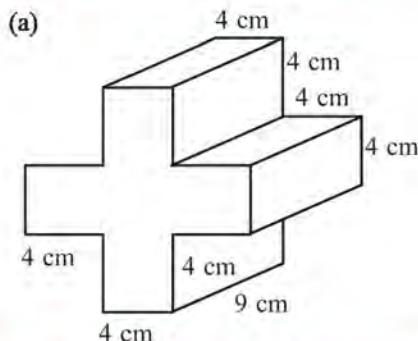
b)



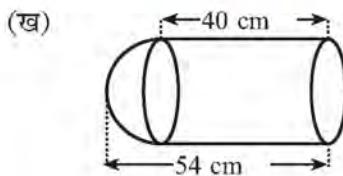
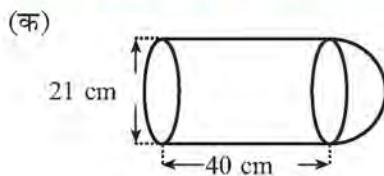
2. A businessman made a pattern on the floor of a living room of his house as shown in the given figure. Twenty-eight triangular tiles of sides 9cm, 12cm, and 15cm were used to make such pattern. Find out the total cost of laying tiles if the per-unit centimeter cost of the tile is 10 paisa.
3. In a solid cylinder, three times the curved surface area is equal to two times the total surface area of the same cylinder. If the curved surface area of the cylinder is 616 cm^2 , find the circumference of the base and height of the cylinder.
4. The difference between the total surface area and curved surface area of a cylinder is 308, find the diameter of the base and perimeter of the cylinder
5. The plane surface area of a cylinder is 1232cm^2 . If the radius of the base is equal to the height of the cylinder, then find the volume of the cylinder.
6. The total surface area of a sphere is 616 cm^2 , find the diameter of the great circle of the given sphere.
7. Find the total surface area of the given hemispherical solid object.
8. There are one door of size $2\text{m} \times 1.5 \text{ m}$ and four windows of size $1 \text{ meter} \times 1.5 \text{ meters}$ in a room of size 12 meters and 10 meters. The total cost for coloring on the four walls at the rate of Rs 180 is Rs.30,060, find the height of the rooms.



9. Find total surface area and volume of the given prisms:



10. Find the cost of colouring the following solid objects, if the cost of colouring per sq. centimeter is Rs. 180:



Answers

- | | | |
|--|--------------------------|--|
| 1. (a) 126 cm^2 | (b) 89.29 cm^2 | 2. Rs. 15,120 |
| 3. 44 cm, 14 cm | 4. 14 cm, 44 cm | 5. 8624 cm^3 |
| 6. 14 cm | 7. 7392 cm^3 | 8. 4 m |
| 9. (a) 592 cm^2 , 720 cm^3 | | (b) 132 cm^2 , 60 cm^3 |
| 10. (a) Rs. 662310 | | (b) Rs. 966240 |

8.1 Review

Answer the given questions discussing with your friends:

- (i) What are the next terms in the given pattern?
 - (a) 2, 4, 6, 8, ...
 - (b) 1, 4, 9, 16, ...
- (ii) The length of the perimeter of the wheels of a cycle is 1.5 m. the distance covered by the wheels in each round up to the 6th round is given below. Now, find the distance covered by the wheels on the 7th round.
1.5 m, 3 m, 4.5 m, 6 m, 7.5 m, 9 m, ...
- (iii) Pemba deposits Rs 500 at the interest rate of 10% annually in a bank. The interest at the end of the first, second, third, and fourth-year is given below. Find the interest of the fifth and sixth years by observing the following pattern.
Rs. 50, Rs. 100, Rs. 150, Rs. 200, ...
Observe the relation between the numbers, the difference between the two terms, and the pattern of the numbers.

8.1.1 Introduction to sequence



The above pattern of marbles is prepared by class IX students in a playground.

- (a) Find the number of marbles in each pattern
- (b) Add another new pattern.
- (c) Do these patterns present a fixed sequence? Discuss in group.

If the group of numbers is presented in a fixed pattern, then it is called a sequence.

The number of marbles in the above patterns 1, 3, 6, 10, 15, ... is an example of the sequence.

If the number of terms in a sequence is fixed, then it is called a finite sequence and if the terms are not fixed, then it is called an infinite sequence. In other words, the

sequence is said to be finite if we can calculate the last term and infinite if we can not calculate its last term.

Here, 1, 3, 5, 7, ..., 15 is finite sequence, and 10, 20, 30, 40, ... is infinite sequence.

Example 1

Find out the 5th and 6th term in a sequence of 4, 7, 10, 13, ...

Solution,

Here 4, 7, 10, 13, ... the sequence of the numbers gives a special pattern; where the coming number is more than 3 than it's previous number. So, the 5th and 6th terms are $13 + 3 = 16$ and $16 + 3 = 19$ respectively.

8.1.2 General term

Activity 1

4, 8, 12, 16, ... is a sequence. Now discuss on the following questions:

What is the 25th term of the sequence?

What is the 99th term of the sequence?

Activity 2

What is the n^{th} term of the following sequence when n stated from 1? Discuss.

Sequence	formula for n^{th} term
1, 3, 5, 7, 9, 11, ...	
1, 4, 9, 16, 25, ...	
1, 8, 27, ...	

The n^{th} term of any sequence is calculated by observing the terms of sequence. The n^{th} term is called General term of the sequence.

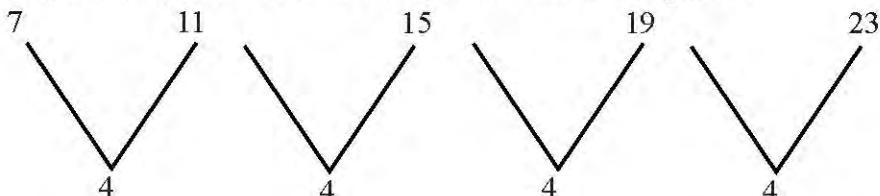
Example 2

Find the n^{th} term of the given sequence:

7, 11, 15, 19, 23, ...

Solution,

Here, Observe the differences between each terms of the sequence:



The common difference between the terms is 4, so the n^{th} term is in the form of $4n + 3$.
Observe the following pattern:

$$\text{First term } (t_1) = 7 = 4 \times 1 + 3 = 7$$

$$\text{Second term } (t_2) = 11 = 4 \times 2 + 3 = 11$$

$$\text{Third term } (t_3) = 15 = 4 \times 3 + 3 = 15$$

$$\text{Fourth term } (t_4) = 19 = 4 \times 4 + 3 = 19$$

$$\text{Fifth term } (t_5) = 23 = 4 \times 5 + 3 = 23$$

⋮ ⋮

$$n^{\text{th}} \text{ term } (t_n) = 4 \times n + 3 = 4n + 3$$

∴ To observe the given pattern, we found n^{th} term $t_n = 4n + 3$.

Example 3

If the general term $(t_n) = 2n - 1$, find the first five terms of the sequence; where n represent the natural number.

Solution,

Here, $t_n = 2n - 1$

Put $n = 1$ we have, $t_1 = 2 \times 1 - 1 = 2 - 1 = 1$

Put $n = 2$ then, $t_2 = 2 \times 2 - 1 = 4 - 1 = 3$

Put $n = 3$ then, $t_3 = 2 \times 3 - 1 = 6 - 1 = 5$

Put $n = 4$ then, $t_4 = 2 \times 4 - 1 = 8 - 1 = 7$

Put $n = 5$ then, $t_5 = 2 \times 5 - 1 = 10 - 1 = 9$

∴ The given sequence of the numbers is 1, 3, 5, 7, 9.

Exercise 8.1

1. Find two terms more in each sequences:

- | | |
|-----------------------------|--|
| (a) 1, 2, 3, 4, ... | (b) 2, 4, 6, 8, 10, ... |
| (c) 5, 10, 15, 20, 25, ... | (d) 1, 2, 4, 8, 16, ... |
| (e) 22, 20, 18, 16, 14, ... | (f) -8, -6, -4, ... |
| (g) 2, 5, 9, 14, 20, ... | (h) $\frac{1}{3}, \frac{4}{5}, \frac{7}{7}, \frac{10}{9}, \dots$ |

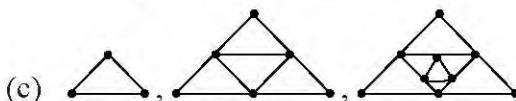
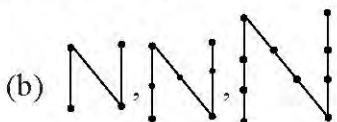
2. Find the general terms (t_n) of the following sequences:

- | | |
|--|---|
| (a) 4, 6, 8, 10, ... | (b) 7, 11, 15, 19, 23, ... |
| (c) 2, 6, 10, 14, 18, ... | (d) 25, 22, 19, 16, ... |
| (e) $\frac{1}{3}, \frac{4}{5}, \frac{7}{7}, \frac{10}{9}, \dots$ | (f) $\frac{2}{7}, \frac{5}{8}, \frac{8}{9}, \frac{11}{10}, \dots$ |
| (g) 40, 38, 36, 34, ... | (h) $\frac{2}{5}, \frac{4}{8}, \frac{6}{11}, \frac{8}{14}, \dots$ |

3. From given general terms, find first 5 terms and present it in the form of sequence when ' n ' represent the natural number:

- | | |
|------------------------------|--------------------------|
| (a) $t_n = 2n + 4$ | (b) $t_n = 3n - 1$ |
| (c) $t_n = 3^n$ | (d) $t_n = n^2 - 1$ |
| (e) $t_n = (-1)^n \cdot n^2$ | (f) $t_n = n^2 + 2n + 3$ |
| (g) $t_n = 3n^2 - 5$ | |

4. Observe the following patterns and find two patterns more then find the general term (t_n):



Answers

1. (a) 5, 6 (b) 12, 14 (c) 30, 35 (d) 32, 64 (e) 12, 10
(f) -2, 0 (g) 27, 35 (h) $\frac{13}{11}, \frac{16}{13}$
2. (a) $2n + 2$ (b) $4n + 3$ (c) $4n - 2$ (d) $28 - 3n$ (e) $\frac{3n - 2}{2n + 1}$
(f) $\frac{3n - 1}{n + 6}$ (g) $42 - 2n$ (h) $\frac{2n}{3n + 2}$
3. (a) 6, 8, 10, 12, 14, ... (b) 2, 5, 8, 11, 14, ...
(c) 3, 9, 27, 81, 243, ... (d) 0, 3, 8, 15, 24, ...
(e) -1, 4, -9, 16, -25, ... (f) 6, 11, 18, 27, 28, ...
(g) -2, 7, 22, 43, 70, ...
4. Show your teacher.

8.2 Introduction to series

Discuss the following questions in a group

- (a) What is the sum of the first five terms of the series whose general term is $(t_n) = 2n + 1$?
- (b) Can we represent these five terms by $t_1 + t_2 + t_3 + t_4 + t_5$? What do is it called?

If all the terms of any sequence is expressed in the form of summation, then it is called series. It is represented by the symbol ' Σ ' (Sigma or Summation). E.g. Here, 7, 11, 15, 19, 23, ... is a sequence. The corresponding series is $7 + 11 + 15 + 19 + 23 + \dots$

Example 1

If the general term of the sequence is $(t_n) = 2n + 3$:

- (a) Find the value of first five terms
- (b) Find the sum of first five terms:
- (c) Write this sequence using the sign ' Σ '.

Solution,

(a) Put the value of $n = 1, 2, 3, 4, 5$ in the general term $(t_n) = 2n + 3$, we get

$$t_1 = 2 \times 1 + 3 = 2 + 3 = 5$$

$$t_2 = 2 \times 2 + 3 = 4 + 3 = 7$$

$$t_3 = 2 \times 3 + 3 = 6 + 3 = 9$$

$$t_4 = 2 \times 4 + 3 = 8 + 3 = 11$$

$$t_5 = 2 \times 5 + 3 = 10 + 3 = 13$$

(b) $t_1 + t_2 + t_3 + t_4 + t_5$
= $5 + 7 + 9 + 11 + 13$
= 45

(c) We can write the series using sign ' \sum ' by $\sum_{n=1}^5 2n + 3$ x'G5 .

Example 2**Find the value of:**

$$\sum_{n=1}^6 (2^n + 1)$$

Solution,

Here, $\sum_{n=1}^6 (2^n + 1) = (2^1 + 1) + (2^2 + 1) + (2^3 + 1) + (2^4 + 1) + (2^5 + 1) + (2^6 + 1)$
= $(2 + 1) + (4 + 1) + (8 + 1) + (16 + 1) + (32 + 1) + (64 + 1)$
= $3 + 5 + 9 + 17 + 33 + 65$
= 132

Exercise 8.2

1. (a) Define sequence with a suitable example.
(b) What is the difference between sequence and series.
2. **Find sequence and series from the given expressions:**

(a) 3, 6, 9, 12, 15, ...	(b) 2 + 4 + 6 + 8 + 10 + ...
(c) 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$	(d) $\sum_{n=1}^3 (3n + 1)$
(e) 1 + 4 + 9 + 16 + ...	(f) $\frac{1}{5}, \frac{3}{8}, \frac{5}{11}, \frac{7}{14}$
3. **Find the value of:**

(a) $\sum_{n=0}^4 (2n - 1)$	(b) $\sum_{n=2}^6 (3n + 2)$	(c) $\sum_{n=1}^5 (n^2 + 1)$
(d) $\sum_{n=1}^3 (n^2 + 2n + 1)$	(e) $\sum_{n=1}^{10} 5n$	(f) $\sum_{n=5}^{10} n^2$
(g) $\sum_{n=3}^8 (n^2 - 2)$	(h) $\sum_{n=1}^5 \left(\frac{2n+1}{n} \right)$	(i) $\sum_{n=0}^4 \left(\frac{n}{n+1} \right)$
4. **Write the given series using symbol \sum :**

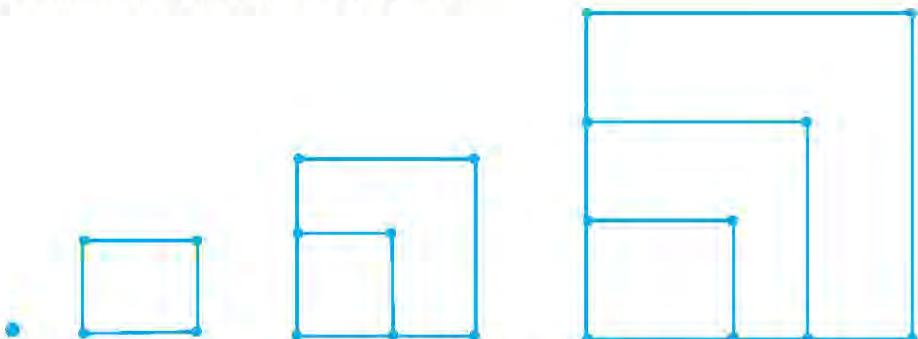
(a) 5 + 7 + 9 + 11 + ... + 21	(b) 2 + 4 + 6 + 8 + 10 + 12
(c) 30 + 25 + 20 + ... + 5	(d) 1 + 5 + 9 + 13 + 17 + 21
(e) 1 + 4 + 9 + 16	(f) $a + ab^1 + ab^2 + ab^3 + ab^4$

Answers

- 1 - 2. Show to your teacher.
3. (a) 15 (b) 70 (c) 60 (d) 29 (e) 275
(f) 355 (g) 187 (h) $\frac{73}{60}$ (i) $\frac{163}{60}$
4. (a) $\sum_{n=1}^9 (2n + 3)$ (b) $\sum_{n=1}^6 2n$ (c) $\sum_{n=1}^6 (35 - 5n)$
(d) $\sum_{n=1}^6 (4n - 3)$ (e) $\sum_{n=1}^4 n^2$ (f) $\sum_{n=1}^5 (ab^{n-1})$

8.3 Arithmetic sequence and series

Study the pattern in the following figures:



Now, discuss the following questions based on the above figures:

- How can we write the dots in a sequence?
- How does the number of dots increase? Can we find a certain rule?
- How can we find different terms without making a figure(eg. tenth term)?

The dots on the four figures are 1, 4, 7, 10 respectively.

This can be written in the form of sequence as; 1, 4, 7, 10. Each term of the sequence is increased by three. So, the common difference is 3. Eg. $10 - 7 = 3$, $7 - 4 = 3$, $4 - 1 = 3$

Likewise, find the common difference in the sequences 5, 10, 15, 20, 25, and 50, 45, 40, 35, 30.

So, if each term of any sequence is increased or decreased by a fixed number, then its previous terms of the sequence is called an arithmetic sequence. The difference between the successive terms is called the common difference. For example, in the common difference sequences 15, 25, 35, 45, 55 is 10, and the common difference in the sequence 100, 90, 80, 70 is -10.

The common difference in an arithmetic sequence is denoted by d and the first term, second term, third term,... n^{th} terms are denoted by $t_1, t_2, t_3, \dots, t_n$ respectively.

If, $t_1, t_2, t_3, t_4, \dots, t_{n-1}, t_n$ is an arithmetic sequence,

the common difference (d) = $t_2 - t_1, t_3 - t_2, t_n - t_{n-1}$

The sum of the n terms of the above arithmetic sequence is $S_n = t_1 + t_2 + t_3 + \dots + t_n$

Generally, $d = t_2 - t_1$.

8.3.1 General term of arithmetic sequence

If, $t_1, t_2, t_3, t_4, \dots, t_{n-1}, t_n$ are in arithmetic sequence,

Common difference is $(d) = t_2 - t_1$.

Where, first term $(t_1) = a = a + (1 - 1) \times d$

Second term $(t_2) = a + d = a + (2 - 1) \times d$

Third term $(t_3) = a + 2d = a + (3 - 1) \times d$

⋮ ⋮ ⋮

n^{th} term $(t_n) = a + (n - 1) \times d$

\therefore General term of arithmetic sequence is $(t_n) = a + (n - 1) d$.

Example 1

The following numbers is the distance travelled by a pedestrian in each step in feet:

3, 6, 9, 12, 15, 18, ...

Find the distance travelled by the pedestrian in the n^{th} step.

Solution,s

Here,

The sequence is 3, 6, 9, 12, 15, 18, This is arithmetic sequence.

where, first term $(t_1) = a = 3$

Common difference $(d) = t_2 - t_1 = 6 - 3 = 3$

Now,

$$\begin{aligned} n^{\text{th}} \text{ term } (t_n) &= a + (n - 1) \times d \\ &= 3 + (n - 1) \times 3 \\ &= 3 + 3n - 3 \\ &= 3n \end{aligned}$$

So, the pedestrian travelled $3n$ feet distance in any n step.

8.4 Geometric sequence and series

First of all, observe the following sequence:

2, 4, 8, 16, 32, 64, ...

2, 6, 18, 54, ...

27, 9, 3, 1, $\frac{1}{3}$, ...

Now, discuss the following questions:

- What is the pattern of the sequences?
- Does the difference between two successive terms equal?
- What is the ratio between two successive terms?
- What is the common rule to find the next term on the basis of its previous term?

In the above sequence, the difference between two successive terms are not equal.

Now, find the ratio between two successive terms.

$$\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = \frac{32}{16} = \dots = 2$$

$$\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \dots = 3$$

Here, the ratio between two successive terms is equal. So, the sequences are called geometric sequences. The series related to the geometric sequence is called geometric series.

E.g. $2 + 4 + 8 + 16 + 32 + 64 + \dots$

If the n th term of the geometric sequence is (t_n) and the $(n-1)$ term is t_{n-1} , the

common difference is $\frac{t_n}{t_{n-1}}$

$t_1 + t_2 + t_3 + \dots + t_n$ is a geometric series. Where t_1, t_2, \dots, t_n are in a geometric sequence.

8.4.1 General term of geometric sequence

If, $t_1 + t_2 + t_3 + \dots + t_{n-1}, t_n$ are in geometric sequence,

Common ratio (r) = $\frac{t_2}{t_1} = \frac{t_n}{t_{n-1}}$

First term (t_1) = a

Second term (t_2) = $ar = ar^{2-1}$

Third term (t_3) = $ar^2 = ar^{3-1}$

$\vdots \quad \vdots \quad \vdots$

n^{th} term (t_n) = ar^{n-1}

\therefore General term of the geometric sequence is $(t_n) = ar^{n-1}$.

Example 2

A person deposits Rs. 200 in a bank for ten years. He got 10% interest annually and added on the principal. What is total amount after 10 years?

Solution,

The total amount at the end of each year with 10% interest can be written in the following sequence:

[Where principal is Rs. 200]

200, 220, 242, ...

The common ratio is (r) = $\frac{220}{200} = \frac{242}{220} = \frac{11}{10}$

The total amount after ten years is

$$\begin{aligned}t_{10} &= 200 \left(\frac{11}{10}\right)^{10-1} \\&= \text{Rs. } 200(1.1)^9 \\&= \text{Rs. } 200 \times 2.36 \\&= \text{Rs. } 472\end{aligned}$$

Example 3

Write whether the following sequence are arithmetic or geometric with a reason.

- (a) 4, 7, 10, 13, ... (b) 3, 6, 12, 24, ...

Solution,

(a) Here,

First term (t_1) = 4

Second term (t_2) = 7

Third term (t_3) = 10

$$\therefore t_2 - t_1 = 7 - 4 = 3$$

$$\text{Again, } t_3 - t_2 = 10 - 7 = 3$$

In this sequence, the difference between two consecutive terms are same. So, it is arithmetic sequence.

(b) Here,

First term (t_1) = 3

Second term (t_2) = 6

Third term (t_3) = 12

$$\therefore \frac{t_2}{t_1} = \frac{6}{3} = 2$$

$$\text{Again, } \frac{t_3}{t_2} = \frac{12}{6} = 2$$

Here, the ratio between two consecutive terms are same. So, it is geometric sequence.

Example 4

Which of the term of the arithmetic sequence 2, 7, 12, ... have value 62?

Solution,

(a) Here,

$$\text{First term } (t_1) = 2$$

$$\text{Second term } (t_2) = 7$$

Let n^{th} term $(t_n) = 62$

Number of terms $(n) = ?$

$$\text{Common difference } (d) = t_2 - t_1 = 7 - 2 = 5$$

We know that,

$$\text{general term } (t_n) = a + (n - 1)d$$

$$\text{or, } 62 = 2 + (n - 1) \times 5$$

$$\text{or, } 62 - 2 = (n - 1) \times 5$$

$$\text{or, } \frac{60}{5} = n - 1$$

$$\text{or, } 12 = n - 1$$

$$\text{or, } n = 12 + 1 = 13$$

\therefore The 13th terms of the arithmetic sequence 2, 7, 12, ... have value 62.

Example 5

If the 2nd term and 5th term of a geometric sequence are 6 and 162 respectively, find 10th term of the sequence.

Solution,

Here,

$$\text{second term } (t_2) = 6$$

$$\text{fifth term } (t_5) = 162$$

$$10^{\text{th}} \text{ term } (t_{10}) = ?$$

We know that,

$$\text{general term } (t_n) = a \cdot r^{n-1}$$

$$\therefore t_2 = ar$$

$$\text{or, } 6 = ar \dots\dots \text{(i)}$$

$$t_5 = ar^4$$

$$\text{or, } 162 = ar^4 \dots\dots \text{(ii)}$$

Dividing equation (ii) by (i) we get,

$$\frac{ar^4}{ar} = \frac{162}{6}$$

$$\text{or, } r^3 = 27$$

$$\therefore r = 3$$

put $r = 3$, in equation (i),

$$6 = 3a$$

$$\therefore a = 2$$

Now,

$$\begin{aligned} 10^{\text{th}} \text{ term } (t_{10}) &= ar^9 \\ &= 2 \cdot (3)^9 \\ &= 2 \times 19683 \\ &= 39,366 \end{aligned}$$

Exercise 8.3

1. Write which of the following sequence are arithmetic or geometric give reasons.
- (a) $4, 2, 1, \frac{1}{2}, \dots$ (b) $4, 10, 16, \dots$
(c) a, ab, ab^2, \dots (d) $6 + 12 + 24 + \dots$
(e) $30 + 27 + 24 + \dots$
2. The third term and ninth term of an arithmetic series are 9 and 33 respectively, find the fifth term of the series.
3. Find the 10th term and common difference of the arithmetic sequences 2, 6, 10, 14, 18,
4. If the fourth term and seventh term of an arithmetic series are 20 and 35 respectively, find its tenth term.
5. The first three terms of an arithmetic sequence are $x - 1$, $x + 2$, and $3x$, find the value of x . Also, find the first term and fifth term of the sequence.
6. If the fifth term and eighth term of an arithmetic series are 17 and 26 respectively, then which term of the series has a value of 44?
7. Find the common ratio and tenth term of the geometric sequence 12, 6, 3,
8. If the second term and fifth term of a geometric series are 6 and 162 respectively, find its ninth term.
9. The first three terms of a geometric sequence are $z + 6$, z , and $z - 3$, find the value of z . Also, find the fifth term of the sequence.

Answers

1. (a) Geometric (b) Arithmetic (c) Geometric
(d) Geometric (e) Arithmetic
2. $t_5 = 17$ 3. $d = 4$, $t_{10} = 38$ 4. $t_{10} = 50$
5. $x = \frac{5}{2}$, $a = \frac{3}{2}$, $t_5 = \frac{27}{2}$ 6. $n = 14$ 7. $r = \frac{1}{2}$, $t_{10} = \frac{3}{128}$
8. 13, 122 9. $z = 6$, $t_5 = \frac{3}{4}$