

Mathematics

Grade 7

Government of Nepal
Ministry of Education, Science and Technology
Curriculum Development Centre
Sanothimi, Bhaktapur

Publisher: Government of Nepal
Ministry of Education, Science and Technology
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Sanothimi, Bhaktapur

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First Edition: 2022 AD

Price:

Printed at: Janak Education Materials Centre Ltd.
Sanothimi, Bhaktapur

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Preface

School education is the foundation for preparing the citizen who are loyal to the nation and nationality, committed to the norms and values of federal democratic republic, self-reliant and respecting the social and cultural diversity. It is also remarkable for developing a good moral character with the practical know-how of the use of ICT along with the application of scientific concept and positive thinking. It is also expected to prepare the citizens who are moral and ethical, disciplined, social and human value sensitive with the consciousness about the environmental conversation and sustainable development. Moreover, it should be helpful for developing the skills for solving the real life problems. This textbook 'Mathematics, Grade 7' is fully aligned with the intent carried out by the National Curriculum Framework for School Education, 2076 and is developed fully in accordance with the new Basic Level Mathematics Curriculum (Grade 6-8), 2077.

This textbook, initially written by Ms. Anupama Sharma, Mr. Dev Narayan Yadav, Mr. Medani Prasad Kadel and Mr. Jagannath Adhikari, has been revised by a team of experts Mr. Lok Nath Bhattacharai, Mr. Santosh Shrestha, Mr. Ganesh Prasad Sapkota, Ms. Pramila Bakhati and Mr. Jagannath Adhikari. It has been translated by a team of experts Mr. Naba Raj Pathak, Mr. Daya Raj Acharya, Ms. Pramila Bakhati, Mr. Ram Chandra Dhakal and Mr. Jagannath Adhikari. The contribution made by Director General Ana Prasad Neupane, Prof. Dr. Ramjee Prasad Pandit, Mr. Keshab Raj Phulara, Mr. Ram Hada and Ms. Nirmala Gautam is remarkable in bringing the book in this form. The language of this book was edited by Mr. Nabin Kumar Khadka. The layout was designed by Mr. Nawaraj Puri. The Curriculum Development Centre extends sincere gratitude to all of them.

The textbook is a primary resource for classroom teaching. Considerable efforts have been made to make the book helpful in achieving the expected competencies of the curriculum. Curriculum Development Centre always welcomes constructive feedback for further betterment of its publications.

2079 B.S.

Curriculum Development Centre
Samothimi, Bhaktapur

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1.0 Review

Discuss in pairs and write the following sets in set notation:

- (a) The set of seven days of a week
- (b) The set of vowel letters in English alphabet
- (c) The set of colors used in the national flag of Nepal
- (d) The set of square numbers up to 50
- (e) The set of prime numbers up to 15

1.1 Types of Set

1.1.1 Null Set

Activity 1

Study the following set. How many elements are there? Discuss and find out the conclusions:

- (a) The set of female Prime Minister of Nepal
- (b) The set of boy-student studying in Kanya School
- (c) The set of natural numbers between 1 and 2

There is no element in the above three set. A set which does not contain any element is called an empty set or null set or void set. It is denoted by the Greek symbol \emptyset . \emptyset is read as 'Phi'

A set which does not contain any element is called an empty set or Null set or void set. It is denoted by \emptyset or { }.

1.1.2 Finite and Infinite Sets

Activity 2

Observe the following pictures and discuss by constructing the required sets:



- (a) How many colors are there in the set of colors in rainbow?
- (b) How many fish are there in the set of fish inside the aquarium?
- (c) How many numbers are there in the set of natural numbers?

From the above pictures, it can be said with certainty that the colors of the rainbow and the number of fish in the aquarium can be counted. But the number of elements in the set of natural numbers is not finite.

- (a) A set is said to be finite set if it contains finite or fixed number of elements.
- (b) A set that has no definite number of elements is called infinite set.

Activity 3

1. Study the following sets and discuss whether they are finite or infinite.

$$N = \{\text{natural numbers}\}$$

$$W = \{\text{whole numbers less than } 30\}$$

$$W = \{\text{even numbers}\}$$

$$S = \{\text{square numbers up to } 30\}$$

- (a) What are the elements of the set N, W, E and S?
- (b) Is it possible to say that the number of elements of the set N, W, E and S is definitely fixed?

The first element of both Set N and E are known as the first elements but the last elements are not known. It is not possible to say for sure that there are so many elements when the last element is not known. Both sets have infinite elements. So both N and E are infinite sets. The elements of both sets W and S are definite. So, W and S are finite sets.

Example 1

State whether the following sets are finite or infinite. Write the number of elements if they are finite.

- (a) The set of whole numbers
- (b) The set of factors of 24

Solution

Here,

- (a) Let $W =$ The set of whole numbers,

$$W = \{0, 1, 2, 3, \dots\}$$

There are infinite number of elements in the set W. So W is infinite set.

- (b) Let $F =$ The set of factors of 24.

$$F = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

There are 8 elements in the set F. So, F is finite set.

1.1.3 Equivalent and Equal Sets

Activity 4

The sets are $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$.

What are the number of elements in set A and set B? Are the number of elements in set A and set B equal? Discuss.

Here, the number of elements in set A and set B are equal. So, set A and set B are equivalent sets.

Two sets are said to be equivalent if the number of elements of the sets are equal. Equivalent sets are written as $A \sim B$.

Activity 5

Observe the set given below and discuss the questions asked:

$$A = \{\triangle, \bigcirc, \square, \square\}$$

$$B = \{\square, \triangle, \bigcirc, \square\}$$

- How many elements are there in set A and set B?
- Are the number of element in set A and set B equal?
- Are the elements in set A and set B same?

There are four elements in both set A and set B.

Every element of set A are in set B and every element of set B are in set A. So, set A and set B are equal sets.

- Two sets are said to be equal, if the number of elements in the sets are equal and same. In symbol it is written as $A = B$.
- All equal sets are equivalent but all equivalent sets may not be equal sets.

Example 2

If A and B are two sets, then answer the following questions:

A = {natural numbers greater than 1 and less than 4}

B = {prime factors of 6}

- Write the set A and set B by listing method.
- Write the numbers of elements of the both set A and set B.
- Is each element of set A in set B?
- Is each element of set B in set A?
- What type of sets can be called to set A and set B?

Solution

Here,

- $A = \{2, 3\}$, $B = \{2, 3\}$
- The number of elements of the both set A and set B are 2.
- Each element of set A is in set B.
- Each element of set B is in set A.
- Set A and set B are equal sets.

Exercise 1.1

1. Write the following sets by listing method and also write the numbers of elements of these sets:

- $A = \{ \text{prime numbers up to } 13 \}$
- $B = \{x; x \text{ is a multiple of } 4 \text{ less than } 40\}$
- $C = \{\text{counting numbers greater than } 2 \text{ and less than } 7\}$
- $A = \{\text{factors of } 20\}$

2. Tick (✓) if the following sets are empty and cross (✗) if they are others than empty.

- (a) The set of natural numbers less than 1
- (b) The set of even prime numbers
- (c) The set of grade 6 students taller than 6 feet
- (d) The set of odd numbers divisible by 2

3. Separate the finite and infinite set from the following sets. If they are finite write the numbers in elements of these sets:

- (a) $A = \{\text{odd numbers greater than } 20\}$
- (b) $P = \{\text{prime numbers less than } 20\}$
- (c) $C = \{\text{composite numbers less than } 20\}$
- (d) $W = \{\text{whole numbers less than } 1\}$

4. State which of the following sets are equal? If they are equal, write them using "=". sign.

- (a) $P = \{\text{prime numbers from } 2 \text{ to } 9\}$, $Q = \{\text{odd numbers from } 1 \text{ to } 8\}$
- (b) $A = \{\text{natural numbers less than } 20 \text{ which is divisible by } 2\}$, $B = \{\text{even number less than } 10\}$
- (c) $C = \{l, u, r, e\}$, $D = \{r, u, l, e\}$,
- (d) $A = \{\text{factors of } 16\}$, $B = \{\text{natural numbers up to } 16 \text{ which is divisible by } 4\}$

5. Separate the equivalent set from the following sets:

- (a) $A = \{g, o, d\}$, $B = \{d, o, t\}$
- (b) $A = \{a, b, c, \dots, x\}$, $Y = \{1, 2, 3, \dots, 12\}$
- (c) $C = \{l, i, n, e\}$, $D = \{f, i, l, e\}$
- (d) $G = \{1, 2, 3, 4, 5\}$, $H = \{x : x \text{ is a natural number less than } 6\}$

6. If $A = \{0, 1, 2, 3\}$, $B = \{1, 2, 3\}$, $C = \{0\}$, $D = \{\}$, $E = \{1, 2, 3\}$ and $F = \{1\}$ are given sets then answer the following questions:
- Write the number of elements of each set.
 - Which sets, have equal numbers of elements?
 - Identify the sets which are not equal even though the number of elements is equal?
 - Which sets are equal? Write.
7. Are the sets $A = \{4, 5\}$ and $B = \{c, d\}$ equivalent? Write with reason.
8. If $A = \{a, b, c, d\}$ and $B = \{a, b, c, d, e\}$ then,
- Are the set A and set B equal?
 - What should be done to set A and set B for making them equal?

Project Work

Prepare a list of 10 items that are in your classroom. Create different sets based on the same properties from the same item list. Observe the set, write their name and types and present it in the classroom.

Answer

Show the answers to your teacher.

1.2 Universal Set

Activity 1

Divide all the students into appropriate number groups, form the set of the following by discussing in the group.

- (a) E = set of even numbers
- (b) O = set of odd numbers
- (c) S = set of square numbers
- (d) C = set of cube numbers
- (e) P = set of prime numbers
- (f) A = set of composite numbers

Which single set can cover all the qualities or characteristics of the above sets (a to f)?

Can all the above sets be discussed in the set of natural number i.e.
 $N = \{1, 2, 3, \dots\}$

Similarly, make the set of girl-student in your class (G), set of boy-student (B) and set of students wearing cap (C).

Which definite set can possess all the qualities or characteristics of these three sets?

Does the set $S = \{\text{set of total students of the class}\}$ contain all the above three sets?

A certain set is said to be a universal set if all the sets under discussion are the subsets of the certain set. The universal set is denoted by U.

Activity 2

Be divided into appropriate number of groups, read the dialogue below. Discuss what can be its universal set. Present it in the class.

- Rupesh : Odd numbers up to 13.
Norbu : It doesn't have the number 16 and counting number greater than 16.
Manju : Even numbers up to 14.
Saddam : Does it have the prime numbers too?

Based on the above discussion, find the universal set that Bimal wrote on the board.

Here, all of the above sets can be discussed on the set of natural numbers up to 15, i.e. $(N) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$.

So, the set N can be the universal set here.

Example 1

Here $U = \{\text{natural numbers less than } 15\}$ is a universal set. Now, write the following sets by listing method:

- (a) $A = \{\text{even numbers less than } 15\}$
(b) $B = \{\text{prime numbers less than } 15\}$

Solution

Here, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

- (a) $A = \{\text{even numbers less than } 15\}$
 $\therefore A = \{2, 4, 6, 8, 10, 12, 14\}$
- (b) $B = \{\text{prime numbers less than } 15\}$
 $\therefore B = \{2, 3, 5, 7, 11, 13\}$

Exercise 1.2

1. If $U = \{\text{natural numbers less than } 25\}$ is a universal set then write the following sets by listing method.
 - (a) $A = \{\text{square numbers less than } 25\}$
 - (b) $O = \{\text{odd numbers}\}$
 - (c) $C = \{\text{composite numbers}\}$
2. $U = \{\text{the set of whole numbers from } 5 \text{ to } 20\}$ is a universal set. Make the following sets by listing method based on the given universal set:
 - (a) $E = \{\text{even numbers}\}$
 - (b) $M_3 = \{\text{multiples of } 3\}$
 - (c) $P = \{\text{prime numbers}\}$
3. Write an appropriate universal set for the following sets:
 $A = \{2, 4, 6, 8, 10, 12, 14\}$ and $B = \{1, 3, 5, 7, 9\}$
4. Based on the following discussions among the students of class 7 in a school, find the universal set:

Dolma : Considering M_4 , here $M_4 = \{4, 8, 12\}$

Arya : There is no zero in this set.

Krishna : There is not 13 in this set too.

Zakir : Ah! It doesn't have fraction either .

Solmu : There is not any even number in it.

Jitu : It doesn't have decimal numbers too.

Answer

Show the answers to your teacher.

1.3 Subset

Activity 1

Be seated in groups of appropriate number. Consider the following set of tools in an instrument box as a universal set.

$$U = \{\text{protractor, set square, compass, ruler}\}$$

Now, each group make the following sets from the above universal set:

1. Sets having only one element from universal set

$$A = \{\text{protractor}\}, \quad B = \{\text{set square}\}$$

$$C = \{\text{compass}\}, \quad D = \{\text{ruler}\}$$

2. Sets having two elements

$$E = \{\text{protractor, set square}\}$$

$$F = \{\text{protractor, compass}\}$$

$$G = \{\text{protractor, ruler}\}$$

$$H = \{\text{set square, ruler}\}$$

$$I = \{\text{set square, compass}\}$$

$$J = \{\text{compass, ruler}\}$$

3. Sets having three elements.

$$K = \{\text{protractor, set square, compass}\}$$

$$L = \{\text{protractor, compass, ruler}\}$$

$$M = \{\text{set square, compass, ruler}\}$$

$$N = \{\text{protractor, set square, ruler}\}$$

4. Set having four elements

$$O = \{\text{protractor, set square, compass, ruler}\}$$

5. An empty set

$$P = \{\quad\}$$

Now discuss the following questions with your friends based on the above set:

- Are the elements of set A also the elements of set U?
- Are all the elements of sets B, C, D also the elements of set U?
- Are all the elements of sets E, F, G, H, I and J also the elements of set U?
- Are all the elements of sets K, L, M and N also the elements of set U?
- Are all the elements of set O also the elements of set U?

If all the elements of the first set are also the elements of the second set, then the first set is said to be subset of the second set.

1.3.1 Proper and Improper Subsets

Activity 2

Every student make the one/one set each. e.g. $P = \{1, 2, 3\}$

Now, make the subset from the set you formed and discuss the following questions:

$$A = \{1\}, B = \{2\} C = \{3\}$$

$$D = \{1, 2\}, E = \{1, 3\} F = \{2, 3\}$$

$$G = \{1, 2, 3\}, H = \{\quad\}$$

- What is the subset formed with all the elements of P? What types of subset is this?
- What are the subsets made up of some elements of P? What are these subsets called? What types of subset are these subsets?
- Can an empty set be called proper subset of any set?
- How many subsets can be formed from set P? How many of them are proper subsets and improper subset?
- Are equal sets improper subset to each other?
- How are the proper subset and improper subset denoted in mathematical symbols?
- How many subsets can be formed from P?

1. If a subset B is formed by taking some elements from the given set A then the subset B is said to be a proper subset of the given set A. It is written as $B \subset A$.
2. If a subset B is formed by taking all the elements from the given set A then the subset B is said to be an improper subset of the given set A. It is written as $B \subseteq A$.
3. Empty set is proper subset of every non-empty set.
4. Equal sets are improper subset to each other.

Example 1

Separate the proper subsets and improper subsets from the following subsets formed from the universal set given below.

$$U = \{a, e, i, o, u\}$$

$$A = \{a, e\}$$

$$B = \{e, o, u\}$$

$$C = \{a, e, i, o, u\}$$

Solution

Here, subsets A and B are proper subsets of U.

Similarly, subset C is improper subset of U.

Exercise 1.3

1. If a set $F = \{\text{Marigold, Rose, Rhododendron}\}$, then
 - (a) Make the subsets taking only one element from set F and name them.
 - (b) Make the subsets taking two elements from set F and name them.
 - (c) Make the subset having all elements from set F and name it.
 - (d) Make the subset having no element and name it.
2. Separate the proper subset and the improper subset from the subsets formed in Question (1).

3. Among the sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$ and $C = \{4, 3, 2, 1\}$, which is the proper subset of A?
4. Make all the possible subsets from the set $A = \{a, b, c\}$.
5. Write down all possible subsets that can be formed from each of the following sets:
(a) $P = \{a, b\}$ (b) $Q = \{4, 5\}$ (c) $R = \{p, q, r\}$
6. Copy the following table and fill in the blanks:

Sets	Subsets	Proper subsets	Improper subsets
$\{1\}$			
$\{1, 2\}$			
$\{1, 2, 3\}$			

7. Write any one set by listing method. Write all the possible subsets of that set. Separate the proper and improper subsets and present them in the class.

Project Work

Make a list of items in your classroom. Make the sets by their characteristics. Make all the possible subsets of each of those sets and present in the class.

Answer

Show the answers to your teacher.

Miscellaneous Exercise

1. If set $A = \{\text{factors of } 6\}$ and $B = \{\text{prime numbers less than } 10\}$, then
 - a) Write the set A and set B by listing method and set-builder method.
 - b) Are the set A and set B equivalent or equal sets, why?
2. If the set $L = \{\text{natural numbers from } 1 \text{ to } 10 \text{ exactly divisible by } 2\}$, $M = \{\text{multiples of } 2 \text{ upto } 10\}$ and $N = \{\text{natural numbers from } 12 \text{ to } 16\}$, then
 - a) Write the set L, M and N by listing method.
 - b) Which sets are equal and equivalent? Write with the reason.
3. If the set $A = \{\text{even numbers greater than } 10\}$ and $B = \{\text{even numbers less than } 10\}$ then,
 - a) Write the set A and set B by listing method and set-builder method.
 - b) What are the set A and set B finite or infinite? Why? Write with reason.

Answer

Show the answers to your teacher.



2.0 Review

Look at the pictures, fill out the table given below and discuss.

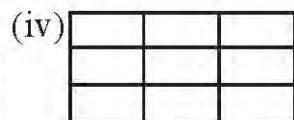
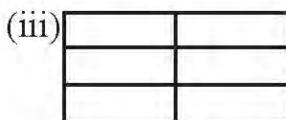
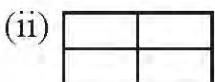


Figure no.	Number of boxes along the length	Number of boxes along the breadth	Total number of boxes
(i)			
(ii)			
(iii)			
(iv)			

- (a) How do we know the total numbers of boxes without counting?
- (b) Which pictures have equal number of square boxes along length and breadth?
- (c) Which pictures are of square shapes?
- (d) What does the total number of boxes in a square figure and the number of boxes along the length denote?

- If a number is multiplied by itself, the product so obtained is called square of that number.
- Any square number has two equal factors, one of those factors is called the square root of that number. It is written using the symbol ' $\sqrt{\quad}$ ' to denote the square root of a number.
For example: $\sqrt{64} = \sqrt{8^2} = 8$.

2.1 Square and Square root of the Number

Activity 1

Take a square number, such as: 64

Now subtract the successive odd numbers starting from 1,3,5,7,9,11 from that number. The number of steps obtained to get the result 0 is the square root of the given number.

$$\text{Step 1: } 64 - 1 = 63$$

$$\text{Step 2: } 63 - 3 = 60$$

$$\text{Step 3: } 60 - 5 = 55$$

$$\text{Step 4: } 55 - 7 = 48$$

$$\text{Step 5: } 48 - 9 = 39$$

$$\text{Step 6: } 39 - 11 = 28$$

$$\text{Step 7: } 28 - 13 = 15$$

$$\text{Step 8: } 15 - 15 = 0$$

In how many times did the result come zero?

Since the result is zero in the eighth time, the square root of 64 is 8.

2.1.1 Square Root by Prime Factorization Method

Activity 2

Look at the following examples and discuss the questions asked below.

Finding the prime factor of 6 and 36,

$$\begin{array}{r} 2 \longdiv{6} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \longdiv{36} \\ 2 \longdiv{18} \\ 3 \longdiv{9} \\ 3 \end{array}$$

$$6 = 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$

- What is the relationship between the number 6 and 36?
- How many 2 are there among the prime factors of 6?
- How many 3 are there among the prime factors of 6?
- How many 2 and 3 are among the prime factors of 36?
- What difference did you find between the square number and the prime factors of its square root?

Now, finding the square root of 36,

$$\begin{aligned}
 &= \sqrt{36} \\
 &= \sqrt{2 \times 2 \times 3 \times 3} \\
 &= \sqrt{2^2 \times 3^2} \\
 &= 2 \times 3 \\
 &= 6
 \end{aligned}$$

Therefore, the square root of 36 is 6.

Procedure for finding the square root by prime factorization method:

- Find the prime factors of the given number,
- Put the prime factors in root ($\sqrt{}$) sign,
- Write the number of pairs in exponents form,
- Write one number of each pair outside the root ($\sqrt{}$) sign and multiply.
- The product so obtained is the square root of that number.

Example 1

Find the square root of 1225 by prime factorization method:

Solution

Here, finding the prime factors of 1225,

$$\begin{array}{r}
 5 \quad | \quad 1225 \\
 5 \quad | \quad 245 \\
 \hline
 7 \quad | \quad 49 \\
 \hline
 \quad \quad \quad 7
 \end{array}$$

Now, finding the square root of 1225,

$$\begin{aligned}&= \sqrt{1225} \\&= \sqrt{5 \times 5 \times 7 \times 7} \\&= \sqrt{5^2 \times 7^2} \\&= 5 \times 7 \\&= 35\end{aligned}$$

Example 2

If the area of a squared seminar hall is 625 m^2 , find its length

Solution

Here, the area of the squared seminar hall (A) = 625 m^2

Length (l) = ?

Now,

$$A = 625 \text{ m}^2$$

$$\text{or, } l^2 = 625 \text{ m}^2$$

$$\text{or, } l = \sqrt{625}$$

$$\begin{aligned}&= \sqrt{5 \times 5 \times 5 \times 5} \\&= \sqrt{5^2 \times 5^2} \\&= 5 \times 5 \\&= 25\end{aligned}$$

$$\begin{array}{r} 5 \mid 625 \\ \hline 5 \mid 125 \\ \hline 5 \mid 25 \\ \hline 5 \end{array}$$

∴ The length of the seminar hall is 25 m.

2.1.2 Square Root by Division Method

Activity 3

Observe the method of finding the square root of 1764 by division method and discuss:

- How is the digits of 1764 paired?
- Why was the number divided by 4 at first?
- Why was it added by putting 4 below 4 again?
- Should we take the same number in divisor and quotient when dividing always? Find its conclusion.

		4 2
4	17 64	
+ 4	- 16	↓
82	1 64	
+ 2	- 1 64	
84	0	

Procedure:

- Begin on the right mark off the digits in pairs from ones places or right to left. For example: $\overline{17} \overline{64}$
- Considering the first pair 17 as a square number, the smaller than 17 but the largest perfect square number is 16. Its square root is calculated to be 4.
- When multiplying by 4 with putting top as quotient and also in the division, the product should be subtracted by putting below 17. Double 4, the number in the quotient and place the double that is 8 as next divisor.
- Bring down the next pair 64 with the remainder 1. Now the dividend is 164.
- Now you have to put a number 2 in the quotient to the right of 4, and to the right of 8 in the divisor, multiply the number formed by the added number 82 by 2 and place the product 164 below the dividend (164).
- Now subtract it, the remainder is 0, If divisible, you can calculate by putting it to the right of the number is large. So, the square root of 1764 becomes 42.

Example 4

Find the square root of 95481 by division method.

Solution

Here,

		3 0 9
3	9 54 81	
+ 3	- 9	
60	0 54	
+ 0	- 0	
609	54 81	
+ 9	- 54 81	
618	0	

Therefore, the square root of 95481 is 309.

Example 5

Find the square root of $\frac{144}{169}$.

Solution

Here, finding the square root of $\frac{144}{169}$

$$\begin{aligned} &= \sqrt{\frac{144}{169}} \\ &= \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 3 \times 3}{13 \times 13}} \\ &= \sqrt{\frac{2^2 \times 2^2 \times 3^2}{13^2}} \\ &= \frac{2 \times 2 \times 3}{13} \\ &= \frac{12}{13} \end{aligned}$$

- Here, pairing from ones places to 9 5481 was $\bar{9} \bar{54} \bar{81}$.
- Now the first dividend is 9.
- Dividing 9 by divisor 3, the remainder became 0.
- Bringing down 54 to the right of 0 and it became 054.
- Adding 3 to divisor 3, it became 6.
- Any number placed on the right side of 6 is greater than 54. So put the 0 right side of 6 it becomes 60.
- Multiplying 60 by 0 became 0 which is placed down of 054.
- Subtracting 0 from 054 is 054.
- 81 was brought down to the right of 054.
- Now the dividend was 05481.
- Placing 9 to the right of 60 became 609.
- Multiplying 609 by the added number 9 became 5481. Now subtracting it the remainder become 0, so the calculation was correct. The square root of 95481 became 309.

- The square root of the numerator and the denominator must be calculated separately.
- The answer should be written with simplifying the fraction.

Example 6

Is 12675 a square number? If not so, what is the number by which 12675 is divided to make the quotient square number?

Solution

Here, $12675 = 3 \times 5 \times 5 \times 13 \times 13$

Making pairs of the two same numbers,

$$2675 = 3 \times 5^2 \times 13^2$$

3	12675
5	4225
5	845
13	169

13

When making pairs of the two same number, there is not pair of 3.

So, 12675 is not a perfect square number. The number that comes from dividing 12675 by 3 is the squared number.

Hence, the required number = 3

Exercise 2.1

1. Find the square of the given numbers:

- (a) 19 (b) 20 (c) 18 (d) 35 (e) 54 (f) 63

2. Tick (✓) if the following statements are true and cross (✗) if they are false.

- (a) If any number has 0, 1, 4, 5, 6 and 9 in the ones place, then that number is a square number.
(b) If there are zeros at the end of any even number, then that number is square number.
(c) 169000 is perfect square number.

3. Find the square root of the following numbers by prime factorization method:

- (a) 169 (b) 324 (c) 1225 (d) 5625 (e) 121×196

4. Find the square root of the following numbers by division method:

- (a) 2304 (b) 8836 (c) 9801 (d) 11025 (e) 95481

5. Prove that:

(a) $\sqrt{25} \times \sqrt{36} = \sqrt{25 \times 36}$ (b) $\frac{\sqrt{625}}{\sqrt{25}} = \sqrt{\frac{625}{25}}$

6. Find the square root of the following numbers:

(a) $\frac{625}{1024}$ (b) $\frac{49}{81}$ (c) $\frac{324}{1225}$ (d) $\frac{1225}{2916}$
(e) 121×196 (f) 144×169 (g) $1\frac{91}{2025}$

7. Is 500 a square number? Test by prime factorization method.
8. Is 325 a square number? If not, find out by which least number should 325 be multiplied to get a perfect square number.
9. Find the area of a squared land having length 37 m.
10. If the area of a squared seminar hall is 729 m^2 , find its length.
11. The social teacher has taken the class 7 students to visit the Hanumandhoka. The amount for lunch eaten by the student is Rs. 15625. Find out the number of students if all the students have eaten lunch equal to the number of students.
12. What is the square root of 196? Find out from the Procedure of continuous subtraction.
13. Class 7 students have donated the total of Rs 2,500 for flood relief. Each student gave as much as the number of students in the class. Find out how many students are there in the class.
14. What is the least number to be added to 1021 so that the sum will be square?
15. What is the least number to be subtracted from 18227 so that the result will be square?
16. Is 7200 a square number? If not, find out by which least number should 7200 be divided to get the quotient square number.

Project Work

Make a note of the number of students in your school. Test whether the number is square or not. If it's not a square number, what is the least number by which the number multiplied to make a square number? Find out and present in the class.

Answer

1 and 2 : Show the answers to your teacher.

3. (a) 13 (b) 18 (c) 35 (d) 75 (e) 154

4. (a) 48 (b) 94 (c) 99 (d) 105 (e) 309

5. Show the answers to your teacher.

6. (a) $\frac{25}{32}$ (b) $\frac{7}{9}$ (c) $\frac{18}{35}$ (d) $\frac{35}{54}$

(e) 154 (f) 156 (g) $1\frac{1}{45}$ 7. No

8. No, Multiplying by 13 becomes the square number

9. 1369 m^2 10. 27 11. 125 12. 14 13. 50 persons

14. 3 15. 2 16. 2

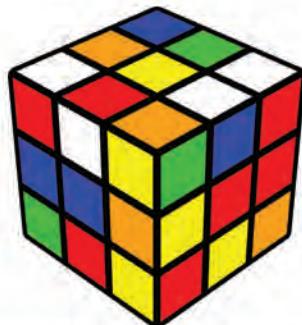
2.2 Cube and Cube Roots of Numbers

Activity 1

Observe the Rubik's Cube in the given picture. How many unit cubes are there in the picture? Discuss.

On the upper surface: $3 \times 3 = 9$ unit cubes

Being 3 surfaces = $9 \times 3 = 27$ unit cubes



The product of three of the same number is called the cube number, for example, if any number is 3, then it can be written as the cube number of 3 is $3 \times 3 \times 3 = 27$.

Example 1

Find the cube number of 5.

Solution

$$\begin{aligned}\text{Here, the cube number of } 5 &= 5 \times 5 \times 5 \\ &= 125\end{aligned}$$

Here, the cube number of 5 is = 125

Example 2

A tank has 6 feet length, 6 feet breadth and 6 feet height which is being built underground to store drinking water in your home. How many cubic feet of underground pit is required to construct the tank?

Solution

Here the cube number of 6 is the required number.

$$\begin{aligned}\text{The volume of the underground pit (V)} &= 6 \text{ ft} \times 6 \text{ ft} \times 6 \text{ ft} \\ &= 36 \text{ ft}^2 \times 6 \text{ ft}\end{aligned}$$

Therefore, the required volume of the pit is 216 cubic feet.

2.2.1 Cube Root by Prime Factorization Method

Activity 2

Fill in the chart given below and discuss the conclusion:

Number	Product of multiplying a number by itself three times	Cube number of this number	One of the three same factors
1	$1 \times 1 \times 1 = 1$	1	1
2	$2 \times 2 \times 2 = 8$	8	2
3	$3 \times 3 \times 3 = 27$	27	3
4			
5			
6			
7			
8			
9			

One of the three same factors of a cube number is the cube root of the number. For example: 2 is cube root of $8 = 2 \times 2 \times 2$ or $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

Example 3

Find the cube root of 512.

Solution

Here, finding the prime factors of 512.

$$512 = 2 \times 2$$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

Now, cube root of 512,

$$\begin{aligned} &= \sqrt[3]{512} \\ &= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{2 \times 2 \times 2} \\ &= \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3} \\ &= \sqrt[3]{2^3 \times 2^3 \times 2^3} \\ &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

Procedure:

- Find the prime factors of the given number,
- Put the prime factors in the sign $\sqrt[3]{}$,
- Collect three of the same number in a group,
- Take one/ one number for each collection and multiply,
- The product obtained is the cube root of that number.

Example 4

If the volume of a cubical tank is 4096 m^3 , find the height of this tank:

Solution

Here, Volume of tank (V) = 4096 m^3 ,

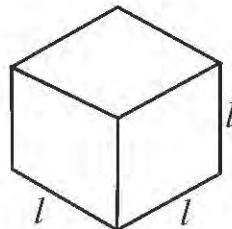
Height of the tank (l) = ?

To find the height of the tank, we have to find the cube root of 4096.

Height of the tank (l) = $\sqrt[3]{4096}$

$$\begin{aligned} &= \sqrt[3]{2 \times 2 \times 2} \\ &= \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 2^3} \\ &= \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3} \\ &= 2 \times 2 \times 2 \times 2 \\ &= 16 \end{aligned}$$

Therefore, the height of the cubical tank is 16 m.



2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

Example 5

Is 1323 a cubic number? If not, find out by which least number should be multiplied to make it a perfect cube:

Solution

Here, $1323 = 3 \times 3 \times 3 \times 7 \times 7$
 $= 3^3 \times 7^2$

3	1323
3	441
3	147
7	49
	7

There are only two 7 among the prime factors of 1323. Another number 7 is needed to get 1323 perfect cube.

Therefore, the required number is 7 which is multiplied by 1323, the product becomes perfect cube number.

Exercise 2.2

1. Find the cube of the following numbers:

- (a) 7 (b) 12 (c) 13 (d) 15
(e) 21 (f) 30 (g) 42

2. Find the cube root of the following numbers.

- (a) 8 (b) 343 (c) 1000 (d) 8000

3. What is the volume of a cubic room whose length is 25 m?
4. The Drinking Water Project has constructed a 27-meter-long cubical tank. What is the capacity of the tank? ($1\text{m}^3 = 1000\text{ l}$)
5. Is 392 a cube number? If not, find out by which least number should 325 be multiplied to get a cube number. Find by using prime factorization method.
6. Is 1728 a cube number or not? Test by using prime factorization method.

7. Prove that:

- (a) $\sqrt[3]{27} \times \sqrt[3]{125} = \sqrt[3]{27 \times 125}$ (b) $\sqrt[3]{\frac{512}{64}} = \frac{\sqrt[3]{512}}{\sqrt[3]{64}}$
8. Is 2916 a cube number? If not, find out by which number should 2916 be divided to get the quotient a perfect cube number?

9. Which least number should 3993 be divided to get the quotient is cube number, find out the cube number of quotient.

10. Find the cube of natural numbers from 1 to 10. Test and write as given below:

 - (a) Is the cube number of natural odd numbers also odd?
 - (b) Is the cube of natural even number also even?

Project Work

Search and write about at least five examples of cube and cube numbers used in our daily life and present in the class.

Answer

2.3 Highest Common Factor

2.3.0 Review

Activit 3

Fill in the blanks given below and discuss:

Factors of 20 = {.....}

Factors of 35 = {.....}

Common factors of 20 and 35 = {.....}

Common factors of 20 and 35 = {.....}

Greatest common factor of 20 and 35 =

HCF of 20 and 35 =

The greatest common factor among all the common factors of the given number is called the highest common factor.

2.3.1 Methods for Finding Highest Common Factor (HCF)

2.3.1.1 HCF by Prime Factorization Method

Study the following conditions and discuss the questions asked in the group:

Activity 4

On her birthday, Harimaya thought to distribute 30 apples and 40 bananas to old fathers and mothers living in old age home.

- Find the greatest number of people to whom she can distribute those apples and bananas equally.
- How much will each one get?
- What will be the mathematical Procedure to find it out?

To do this, you need to find the greatest number that exactly divides the given numbers 30 and 40.

$$\begin{array}{r} 2 \mid 30 \\ 3 \mid 15 \\ \hline 5 \end{array} \quad \begin{array}{r} 2 \mid 40 \\ 2 \mid 20 \\ \hline 2 \mid 10 \\ \hline 5 \end{array}$$

$$30 = 2 \times 3 \times 5$$

$$40 = 2 \times 2 \times 2 \times 5$$

$$30 \div 10 = 3$$

$$40 \div 10 = 4$$

Product of common prime factors $= 2 \times 5 = 10$

The greatest number that exactly divides 30 and 40 is 10. So Harimaya can distribute apples and bananas equally to 10 people. She can distribute $\frac{3}{3}$ apples and $\frac{4}{4}$ bananas to each.

The greatest number which divides exactly the given number is HCF

Example 1

Find the HCF of the numbers 100, 125, 200 by prime factorization method.

Solution

Here

$$\begin{array}{r} 2 \mid 100 \\ 2 \mid 50 \\ 5 \mid 25 \\ \hline 5 \end{array} \quad \begin{array}{r} 5 \mid 125 \\ 5 \mid 25 \\ \hline 5 \end{array} \quad \begin{array}{r} 2 \mid 200 \\ 2 \mid 100 \\ 2 \mid 50 \\ \hline 5 \mid 25 \\ \hline 5 \end{array}$$

$$100 = 2 \times 2 \times 5 \times 5$$

$$125 = 5 \times 5 \times 5$$

$$200 = 2 \times 2 \times 2 \times 5 \times 5$$

Product of common prime factors
 $= 5 \times 5 = 25$

$$\therefore \text{HCF} = 25$$

Procedure

- Find the prime factors of the given numbers.
- Take common prime factors among them.
- Find the product of common prime factors.
- This product is the required HCF

2.3.1.2 HCF by Division Method

Activity 2

In the case of given large numbers, it is considered appropriate to find HCF by Division method. In this method, divide the largest number by the smallest number among the given numbers until the remainder is zero.

Observe the following example and discuss how to find HCF by division method in group.

Which is the greatest number that divides the numbers 100, 125 and 200?

Here $100) 125 (1$

$$\begin{array}{r} - 100 \\ \hline 25) 100 (4 \\ - 100 \\ \hline 0 \end{array}$$

Now, dividing the third number 200 by 25.

$$\begin{array}{r} 25) 200 (8 \\ - 200 \\ \hline 0 \end{array}$$

Therefore, the greatest number that divides the numbers 100, 125 and 200 is 25.

Procedure

- Divide greatest number by smaller one of the given numbers into the next largest number. (Until the remainder is smaller than the divisor)
- Next, divide to the divisor by the remainder. Go on repeating the Procedure of dividing the proceeding by the remainder last obtained, till the remainder is zero.
- Then the last, divisor is the required HCF which is exactly divisible to the given numbers.
- Now, we should divide the third number by HCF of other given numbers.

Example 2

How many students at most can 125 oranges, 150 sweet lemons, and 225 guavas be divided equally and how many fruits of each will each student get?

Solution

Here, the required number is the HCF of 125, 150, and 225.

Dividing each number by HCF, the quotient is the number of fruits that each student gets equally.

Here, finding the HCF of 125 and 150,

$$\begin{array}{r} 125) 150 (1 \\ \underline{- 125} \\ 25) 125 (5 \\ \underline{- 125} \\ 0 \end{array}$$

Now, dividing the another number 225 by the divisor 25

$$\begin{array}{r} 25) 225 (9 \\ \underline{- 225} \\ 0 \end{array}$$

Therefore, HCF is 25.

Therefore, 25 students at most can be divided equally.

Therefore, it can be distributed 125 oranges, 150 sweet lemons, and 225 guavas equally to at most 25 students.

Each student gets $125 \div 25 = 5$ oranges, $150 \div 25 = 6$ lemons and $225 \div 25 = 9$ guavas.

Example 3

Find the greatest number from which subtracting 10, the difference exactly divides 558, 700 and 840.

Solution

Finding the number that divides 558, 700 and 840 exactly.

$$\begin{array}{r} 588) 700 (1 \\ \underline{- 588} \\ 112) 588 (5 \\ \underline{- 560} \\ 28) 112 (4 \\ \underline{- 112} \\ 0 \end{array}$$

again,

$$\begin{array}{r} 28) 840 (30 \\ \underline{- 840} \\ 0 \end{array}$$

HCF of the numbers 558, 700 and 840 = 28

Let the required number = x

According to questions,

$$\begin{aligned} x - 10 &= 28 \\ \text{or, } x &= 28 + 10 \\ \text{or, } x &= 38 \end{aligned}$$

Hence, the required number = 38

Exercise 2.3

- 1. Find the HCF of the following numbers by prime factorization method:**
(a) 21 and 28 (b) 26 and 52 (c) 9, 18, and 36
(d) 12, 18 and 36 (e) 20, 35 and 55

- 2. Find the HCF of the following numbers by division method:**
(a) 144 and 312 (a) 500 and 625 (a) 120, 320 and 480
(a) 80, 90 and 120 (a) 144, 384 and 432

3. How many people at most can 72 copies and 99 pencils be divided equally and how many copies and pencils will each person get? Find.

4. One of the organizations has donated 125 kg pulses, 150 kg seeds and 235 kg rice as relief to Covid. How many families can these foods be distributed equally? How much quantity of pulses, seeds (gedagudui) and rice does each family get?

5. One organization has managed for the distribution of 80 blankets, 90 sweaters and 120 warm jackets for the flood victims. At most, how many families can be divided equally? Find out how many clothes of each type each family receives.

6. On the occasion of his birthday, Prabin distributed 60 apples, 72 oranges and 108 bananas to the adult in old age home. How many adult at most were these fruits distributed equally? Find out how many fruits each person received.

7. Which is the greatest number that can divide 275, 440 and 715 exactly?

8. Find the greatest number to which adding 10, the sum divides 225, 375 and 525 exactly.

Answer

1. (a) 7 (b) 26 (c) 9 (d) 6 (e) 5
2. (a) 24 (b) 125 (c) 40 (d) 10 (e) 48
3. 9 people, 8 copies, 11 pencils
4. 25 families, 5 kg pulses, 9 kg seeds and 11 kg rice
5. 10 families 10 blankets, 90 sweaters and 120 warm jackets
6. 12 people, 5 apples, 6 oranges, 9 bananas
7. 55 8. 74

2.4 Lowest Common Multiple

2.4.0 Review

Activity 1

Fill in the blanks given below and discuss:

LCM defines the least number which is exactly divisible by two or more numbers. Or

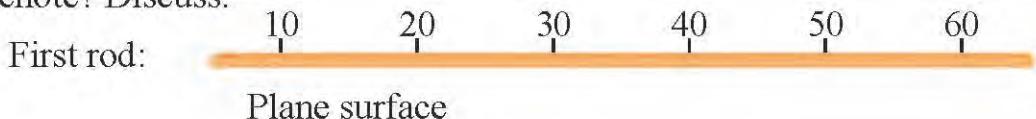
LCM of two or more numbers is the least number that can be exactly divisible by those numbers.

Activity 2

There are three long rods with the length of 10 cm, 20 cm and 30 cm in the given figure.



As the three rods are measured together on a plane surface, how many cm. of shortest distance can be measured? What does this distance denote? Discuss.



$$M_{10} = \{10, 20, 30, 40, 50, 60, \dots\}$$

$$M_{20} = \{20, 40, 60, \dots\}$$

$$M_{30} = \{30, 60, \dots\}$$

The shortest distance measured by all three rods is 60 cm. It is called LCM

Therefore, LCM = 60 cm

2.4.1 Methods for Finding Lowest Common Multiple (LCM)

Method 1: LCM by Prime Factorization Method

Activity 1

Discuss about the questions asked by observing the given examples:

Example 1

Find the LCM of 18, 24 and 36 by prime factorization method.

- Find the prime factors of the numbers 18, 24 and 36.
- Find the common factor of all the numbers. Then find the common factor of 2/2 numbers.
- Also, take the remaining prime factors.
- Find the product of the common and the remaining prime factors.
- What is called to this product? Write the conclusion.

Solution

$$\begin{array}{c} 2 \mid 18 \\ 3 \mid 9 \\ \quad 3 \end{array} \qquad \begin{array}{c} 2 \mid 24 \\ 2 \mid 12 \\ \quad 2 \mid 6 \\ \quad 3 \end{array} \qquad \begin{array}{c} 2 \mid 36 \\ 2 \mid 18 \\ 3 \mid 9 \\ \quad 3 \end{array}$$

$$\text{Factors of } 18 = 2 \times \boxed{3} \times \boxed{3}$$

$$\text{Factors of } 24 = 2 \times \boxed{2} \times \boxed{2} \times 2 \times \boxed{3} \times \boxed{3}$$

$$\text{Factors of } 36 = 2 \times \boxed{2} \times \boxed{3} \times \boxed{3} \times \boxed{3}$$

$$\text{Common factors of } 18, 24 \text{ and } 36 = 2 \times 3 = 6$$

$$\text{Common factors of } 18 \text{ and } 36 = 3$$

$$\text{Common factors of } 24 \text{ and } 36 = 2$$

$$\text{Remaining factors} = 2$$

$$\text{LCM} = \text{Common factors} \times \text{Remaining factors}$$

$$= 6 \times 3 \times 2 \times 2$$

$$= 72$$

Therefore, LCM of the numbers 18, 24 and 36 = 72

Method 2: LCM by Division Method

By combining the given numbers 18, 24 and 36, the LCM can be found in the following methods according to the Procedure of prime factorization:

Solution

Here,

2	18, 24, 36
3	9, 12, 18
3	3, 4, 6
2	1, 4, 2
	1, 2, 1

$$\therefore \text{LCM} = 2 \times 3 \times 3 \times 2 \times 2 \\ = 72$$

Procedure

- Write all the given numbers in rows using commas in any order.
- Continue dividing the smallest common prime factor which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible.
- Keep dividing if it divides exactly only two numbers too.
- This Procedure is continued till none of these two of the given number are divisible by a same number.
- Find the product of all divisors prime factors and the remaining factors of the last row. This product is LCM of the given numbers .

Example 2

Find the least number which is exactly divisible by 24, 36 and 56

Solution

Here, The least number which is exactly divisible by the given number is LCM of the given numbers.

Finding the LCM of 24, 36 and 56,

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \\ = 504$$

2	24, 36, 56
2	12, 18, 28
2	6, 9, 14
3	3, 9, 7
	1, 3, 7

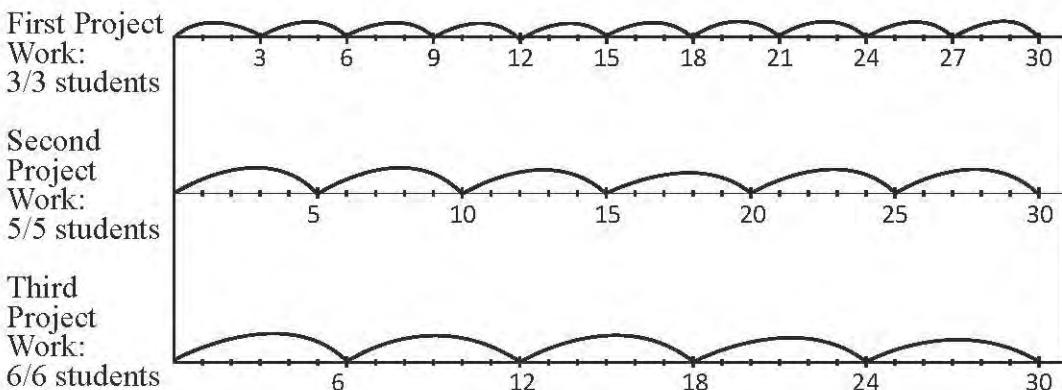
Therefore, 504 is the least number which is exactly divisible by 24, 36 and 56.

Example 3

The mathematics teacher has decided to assign 3 Project Works to the students of class 7. When the first Project Work is assigned into the groups of 3 students, the second into the groups of 5 students and the third into groups of 6 students, there are no students left. Now, find out at least how many students are there in the class 7.

Solution

Here:



Minimum number of students required to form a group so that there is no student remained for each task = Project Work can be done by forming a group of 30 students.

$$\therefore \text{LCM} = 30$$

\therefore There are at least 30 students in class 7.

Another Method

Here,

The number of students in each group in the three Project Works is 3, 5 and 6 respectively,

Now, finding the LCM of 3, 5 and 6,

$$\therefore \text{LCM} = 3 \times 5 \times 2 = 30$$

$$\begin{array}{r} 3 \\ \hline 3, 5, 6 \\ 1, 5, 2 \end{array}$$

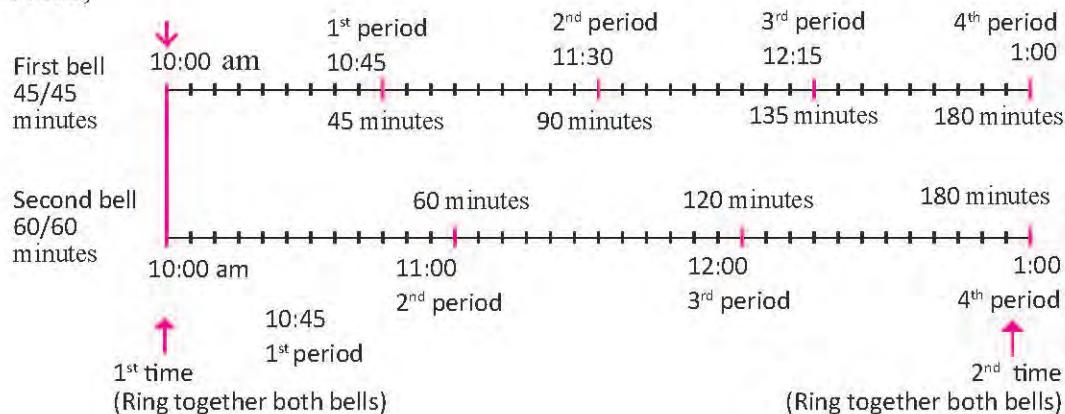
Therefore, if there are at least 30 students, the first Project Work can be done in groups of 3/3 students, the second Project Work in groups of 5/5 students and the third Project Work in groups of 6/6 students.

Example 4

Two bells ring at an interval of 45 minutes and 60 minutes. If two bells ring at the same time at 10:00 am first, find out what time they will ring together the next time?

Solution

Here,



∴ The second time, at 1:00 o'clock, both bells will ring at the same time.

Another Method

Here, for this, finding the LCM of 45 and 60,

$$\begin{array}{r} 5 \mid 45, 60 \\ \hline 3 \mid 9, 12 \\ \hline 3, 4 \end{array}$$

$$\therefore \text{LCM} = 5 \times 3 \times 3 \times 4$$

$$= 180 \text{ minutes}$$

$$= 3 \text{ hours}$$

The second time, the bell rings together = 10:00 o'clock + 3 hours
= 1 o'clock

Example 5

Find the least number from which when 5 is subtracted, the difference will be exactly divisible by 18, 24 and 36:

Solution

Here

The LCM of 18, 24 and 36 is the number obtained by subtracting 5 from the required number.

2	18, 24, 36
3	9, 12, 18
3	3, 4, 6
2	1, 4, 2
	1, 2, 1

So, finding the LCM of 18, 24 and 36.

$$\therefore \text{LCM} = 2 \times 3 \times 3 \times 2 \times 2 \\ = 72$$

According to the question, let the required number = x , then,

$$x - 5 = 72 \\ \text{or } x = 72 + 5 \\ \text{or } x = 77$$

Hence, the required number is 77.

Example 6

Find the least number, with which when 5 is added, the sum will be exactly divisible by 32, 64 and 192.

Solution

Here, the LCM of 32, 64 and 192 is the number obtained by adding 5 in the required number.

So, finding the LCM of 32, 64 and 192,

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \\ = 192$$

Let the required number = x

According to question,

$$x + 5 = 192$$

$$\text{or } x = 192 - 5$$

$$\text{or } x = 187$$

Hence, the required least number is = 187

2	32, 64, 192
2	16, 32, 96
2	8, 16, 48
2	4, 8, 24
2	2, 4, 12
2	1, 2, 6
	1, 1, 3

Exercise 2.4

- Find the LCM of the following numbers by prime factorization method:
 - (a) 28, 42 and 56 (b) 3, 11 and 33
 - (c) 6, 15 and 21 (d) 20, 36 and 44
 - (e) 50, 75 and 125 (f) 210, 280 and 420
- Find the least number which is exactly divisible by 42, 49 and 63.
- The sports teacher divided the students into groups of 15 in the first game, 20 in the second game and 25 in the third game. Find out the least number of students from which the group for each game can be formed without leaving any students.
- In the three watches, the alarm rang at the interval 10, 15 and 20 minutes respectively. All the alarms rang at 10 o'clock in the morning. Find out at what other time the alarm of the three watches will ring again at the same time.
- Find the least number from which when 10 is subtracted, the difference will be exactly divisible by 15, 22, 45.

6. Find the least number, with which when 3 is added the sum is exactly divisible by 8, 12 and 14.

Project Work

If the paper strip or the rod is of the size 3 units, 11 units and 33 units, what is the LCM of 3, 11 and 33? Show and present it in the class.

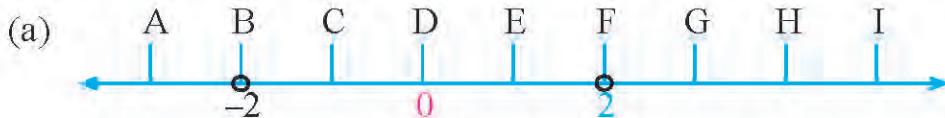
Answer

- | | | | |
|------------|---------|---------------|------------------|
| 1. (a) 168 | (b) 33 | (c) 210 | (d) 1980 |
| (e) 750 | (f) 840 | | |
| 2. 882 | 3. 300 | 4. 11 o'clock | 5. 190 6. 165 |



3.0 Review

Study the condition given below and discuss it in the group.



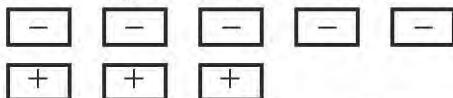
- 2, 0 and 2 are represented by B, D and F in the number line. Which integers are represented by A, C, G and I?
- (b) How can integers 8, -2, 3, 0 and 6 be represented in the number line?
- (c) How can 5 units to the right of place A and 5 units to the left of B be shown from the origin by considering any one point as the point of origin in the number line?

3.1 Addition of Integer

Activity 1

- Take two rectangular pieces of paper of different colors. Write '+' sign on a piece of 1st type colored paper. Write '-' sign on pieces of 2nd colored paper.
- Take any two numbers from the set of integers and what is the sum? Find out by using the pieces of paper.

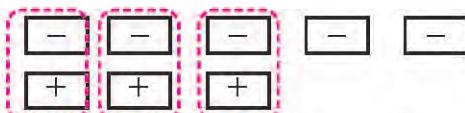
For example: $(-5) + (+3)$



Number of '-' is 5.

Number of '+' is 3.

- Now, remove the piece of paper with written '+' and '−' by making pairs.



Two pieces of paper remained consisting '−' sign.
So, it became $(-5) + (+3) = -2$.

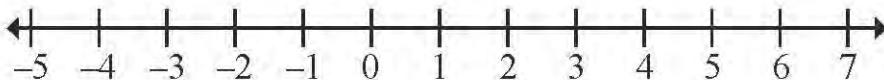
Activity 2

Addition of integers through the number lines

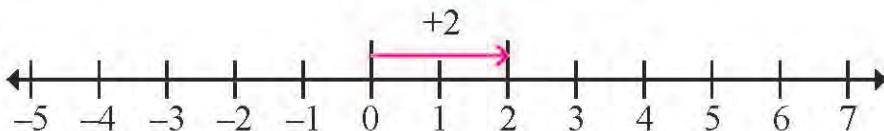
Discuss in groups by observing the number lines given below:

(a) $(+2) + (+4) = ?$

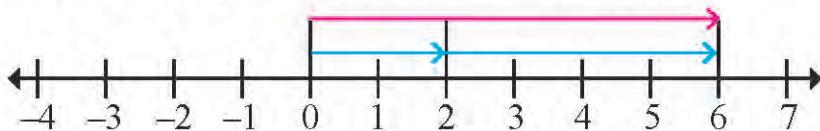
- (i) Draw the number line.



- (ii) Move 2 units to the right from the origin (zero).



- (iii) Now, move 4 units to the right from that point.



- (iv) At what number did you reach? Note down in your copy.

$$(+2) + (+4) = 6$$

(b) $(+6) + (-2) = ?$

Moving 6 units to the right from the origin and returning 2 units to the left from that point, which point is reached? Note down by making the number line.

(c) $(+2) + (-7) = ?$

Moving 2 units to the right from the origin and 7 units to the left from that point, which point is reached? Note down by making the number line.

(d) $(-3) + (-5) = ?$

Moving 3 units to the left from the origin and 5 units to the left again from that point, which point is reached? Show it in the number line.

Properties of Addition of Integers

1. Closure Property

Take any two numbers from the set of integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. Find the sum of these numbers. What is the result? Discuss.

For example, $0 + 1 = 1$

$$-2 + 1 = -1$$

$$-2 - 3 = -5$$

The sum of any two integers is also an integer.

If a and b are two integers, then $a+b$ is also an integer. This is called the closure property of addition.

2. Commutative Property

Take any two numbers from the set of integers. Change the order of the numbers and find the sum. Discuss what the result is.

For example, $2 + 3 = 3 + 2 = 5$

$$-1 + 1 = 1 - 1 = 0$$

$$-2 + (-3) = (-3) + (-2) = -5$$

If we add the integers in any order, then the result becomes the same integer.

If a and b are any two integers, then $a + b = b + a$. This is called Commutative law of addition.

3. Associative Property

Take any three numbers from the set of integers. Put the numbers in any order and add up the sum of the first two integers and add the third integer to the sum. What is the result? Discuss.

For example, In -3 , -2 and -5

$$\begin{aligned} [(-3) + (-2)] + (-5) &= (-3) + [(-2) + (-5)] \\ -5 - 5 &= -3 - 7 \\ \therefore -10 &= -10 \end{aligned}$$

If a , b and c are any integers then there is $(a+b) + c = a + (b+c)$. This is called associative property of addition.

4. Additive Inverse

$$-5 + (+5) = ?$$

$$(+2) + (-2) = ?$$
 Discuss.

If the sum of any two integers is zero, then they are additive inverse to each other.

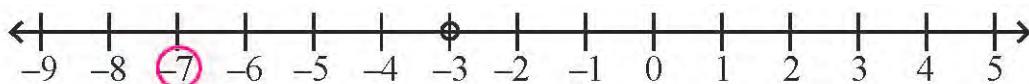
For any integer a , its negative integer is $(-a)$, where $a + (-a) = (-a) + a = 0$. So, a and $-a$ are additive inverse to each other.

Example 1

Write the integer which lies 4 unit left from -3 .

Solution

Here,



The integer which lies 4 unit left from -3 is (-7) .

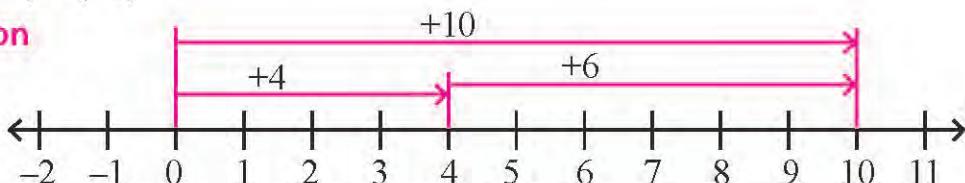
Example 2

Add by using number line:

(a) $(+4) + (+6)$

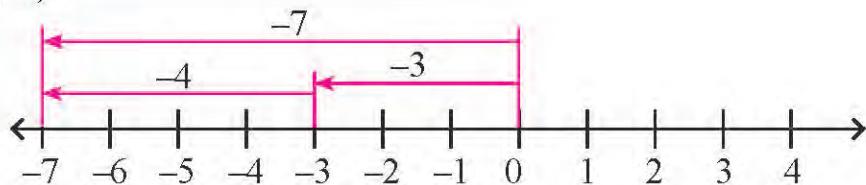
Solution

Here,



Therefore, $(+4) + (+6) = +10$

(b) $(-3) + (-4)$



Therefore, $(-3) + (-4)$

$$= -3 - 4$$

$$= -7$$

Example 3

Test the associative law from the integers (-7), (-2) and (+6) :

Solution

Here,

$$\begin{aligned} & (-7) + (-2) + (+6) \\ &= [(-7) + (-2)] + (+6) \\ &= -9 + 6 \\ &= -3 \end{aligned}$$

Again,

$$\begin{aligned} & (-7) + [(-2) + (+6)] \\ &= (-7) + (+4) \\ &= -7 + 4 \\ &= -3 \end{aligned}$$

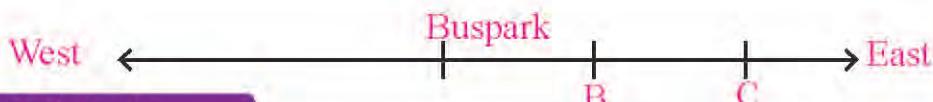
Therefore, $[(-7) + (-2)] + (+6) = (-7) + [(-2) + (+6)] = -3$

3.2 Subtraction of Integers

Activity 3

Ram went to the place C to the east 20 km from the bus park. Then he returned 13 km west to the place B from the same route.

If the bus park is to be considered as origin and the distance to the east is taken as positive value, how many kilometers is Ram away from the bus park? Discuss by observing the given number line.

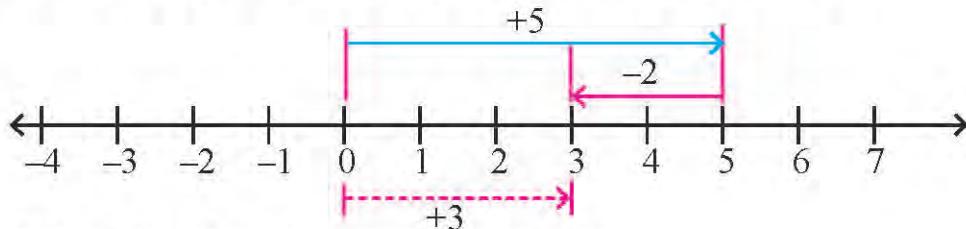


Activity 4

Observe and discuss about the examples of subtraction of integers from the number line given below:

1. $(+5) - (+2) = ?$

Moving 5 units from 0 (zero) to the right and again 2 units to the left, in which point is reached? Note down from the number line.

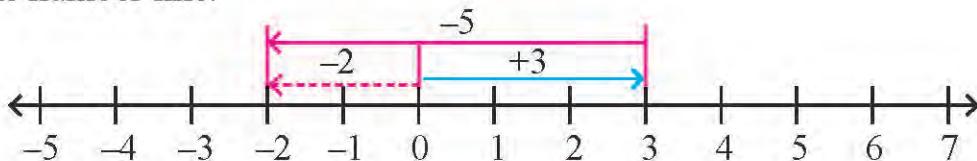


We reach at the point 3 unit right or $(+3)$.

Therefore, $(+5) - (+2) = (+3)$

2. $(+3) - (+5) = ?$

Move 3 units to the right from the origin (0). Return 5 units to the left again from that point. In which point is reached? Discuss by drawing the number line.



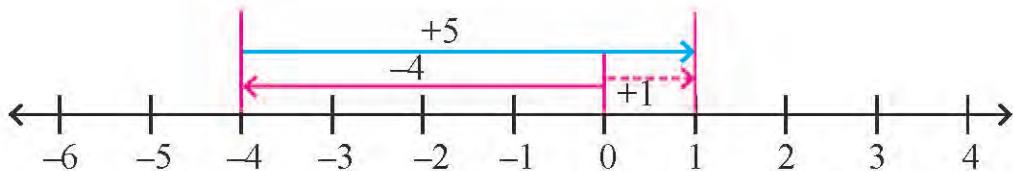
We reach at the point 2 unit left or (-2) .
Therefore, $(+3) - (+5) = (-2)$

Example 4

(a) Show the number $(-4) - (-5)$ in the number line.

Solution

$$\begin{aligned} \text{Here, } & (-4) - (-5) \\ &= (-4) + 5 \\ &= 1 \end{aligned}$$

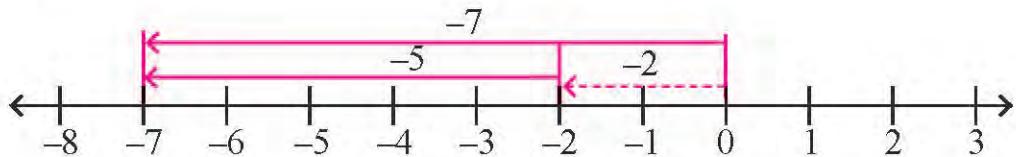


Therefore, $(-4) - (-5) = (+1)$

(b) Show the number $(-2) - (+5)$ in the number line.

Solution

$$\begin{aligned} \text{Here, } & (-2) - (+5) \\ &= -2 - 5 \\ &= -7 \end{aligned}$$



$\therefore (-2) - (+5) = -7$.

Example 5

Simplify:

$$(+250) + (-275) - (+148) + (+207) - (175)$$

Solution

$$\begin{aligned} \text{Here, } & (+250) + (-275) - (+148) + (+207) - (175) \\ & = 250 - 275 - 148 + 207 - 175 \\ & = (-25) + 59 - 175 \\ & = -25 + 59 - 175 \\ & = 34 - 175 \\ & = -141 \end{aligned}$$

3.3 Absolute Value of Integers

Discuss by observing the number line given below:



Here, the bus park is at origin. There is a place A 4 km right from the bus park and there is place B which is 4 kilometer left from the bus park. What is the distance from the place A to the place B?

Is $4 \text{ km} + (-4) \text{ km} = 0$? Discuss.

The distance from place A to B is $4 \text{ km} + 4 \text{ km} = 8 \text{ km}$. Distance is never negative. Therefore, the absolute value of both $+4$ and -4 is 4 . -4 and 4 are opposite of integers.

The positive numerical value of any integer is called absolute value. So, $|+a| = |-a| = a$.

The integer lying at the same distance from origin but in opposite direction from the origin in which the given integer lie, is called the opposite of the given integer.

Exercise 3.1

1. Write the integers which lie 8 unit right from the given number:
(a) (-2) (b) (-6) (c) 0 (d) $(+3)$ (e) $(+5)$
2. Write the integers which lie 8 unit left from the given number:
(a) (-3) (b) (-4) (c) 0 (d) $(+2)$ (e) $(+7)$
3. Write the opposite integer of the given integers:
(a) $(+3)$ (b) $(+4)$ (c) (-8) (d) 0 (e) (-2)
4. Write the absolute value of the given integers:
(a) $|+10|$ (b) $|-6|$ (c) $|-5|$ (d) $|+4|$ (e) $|-9|$
5. Find the sum by using number lines:
(a) $(+3) + (+4)$ (b) $(-4) + (-3)$
(c) $(+5) + (-2)$ (d) $(-5) + (+2)$
6. Subtract by using number lines:
(a) $(-8) - (-3)$ (b) $(+9) - (-4)$
(c) $(+4) - (+5)$ (d) $(+7) - (+2)$
7. Find the sum from the both ways using the commutative law of the integers given below.
(a) $(+3)$ and $(+6)$ (b) $(+4)$ and (-3)
(c) $(+5)$ and (-3) (d) (-3) and (-1)
8. Find the sum from the both ways using the associative law of the integers given below.
(a) $(+2), (-3)$ and (-5) (b) $(+4), (-3)$ and $(+6)$
(c) $(-5), (+4)$ and (0) (d) $(-2), (-5)$ and $(+8)$
9. What is the sum of $(+9)$ and its opposite integer?
10. What is the sum of (-30) and $(+30)$?
11. What should be subtracted from (-25) to get (-20) ?

12. The sum of two integers is (-115) , if one integer is 175 . Find the next integer.
13. Two buses departed from the same place at the same time. One bus traveled 125 kilometers to the east and the other 120 kilometers to the west. Find the distance traveled by the two buses.

14. Simplify:

- (a) $(-30) - (-40) - (-20) + (+2)$
 (b) $(+75) - (-14) - (-10) + (+1)$
 (c) $(-40) + (-25) + (+60) + (-5)$
 (d) $(-30) - (-40) - (-20) - (-10)$

15. Find the sum of each rows, each columns and the diagonals from the two tables given below. Which table gets the results in the same sum of rows, columns and diagonals? Write.

Table 1

-5	-1	-4
-5	-2	7
0	3	-3

Table 2

1	-10	0
-4	-3	-2
-6	4	-7

Answer

- | | | | | |
|------------|---------|--------|-------------------------|---------|
| 1. (a) +6 | (b) +2 | (c) +8 | (d) +11 | (e) +13 |
| 2. (a) -11 | (b) -12 | (c) -8 | (d) -6 | (e) -1 |
| 3. (a) -3 | (b) -4 | (c) +8 | (d) No opposite integer | (e) +2 |
| 4. (a) 10 | (b) 6 | (c) 5 | (d) 4 | (e) 9 |

5-8. Show the answers to your teacher.

9. 0 10. -60 11. -5

12. -290 13. 245 km 14. (a) +32 (b) +100 (c) -10

(d) +40 15. Show the answers to your teacher.

3.4 Multiplication of Integers

Discuss by observing the following condition:



How much distance can a bus travel in 8 hours from west to east at the rate of 40 km per hour?

$$\begin{aligned}\text{The distance covered by the bus in 8 hours} &= 8 \times 40 \text{ km} \\ &= 320 \text{ km}\end{aligned}$$

Activity 1

Complete the multiplication table given below. What is the product of the number asked? Find out by discussing.

\times	3	2	1	0	-1	-2	-3
3	9	6	3	0	-3	-6	-9
2	6	4	2	0	-2	-4	-6
1	3	2	1	0	-1	-2	-3
0							
-1							
-2							
-3							

(a) $(-3) \times (-2) = \underline{\hspace{2cm}}$

(b) $(+2) \times (-1) = \underline{\hspace{2cm}}$

(c) $(+1) \times 0 = \underline{\hspace{2cm}}$

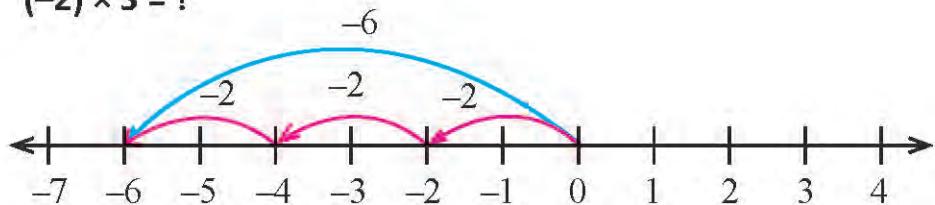
Having same sign: $\rightarrow \left\{ \begin{array}{l} \text{Positive integer} \times \text{positive integer} = \text{positive integer} \\ \text{Negative integer} \times \text{negative integer} = \text{positive integer} \end{array} \right.$

Having opposite sign: $\rightarrow \left\{ \begin{array}{l} \text{Positive integer} \times \text{negative integer} = \text{negative integer} \\ \text{Negative integer} \times \text{positive integer} = \text{negative integer} \end{array} \right.$

Activity 2 Multiplication by using number lines

Observe and discuss the multiplication of numbers using the number line given below.

(a) $(-2) \times 3 = ?$



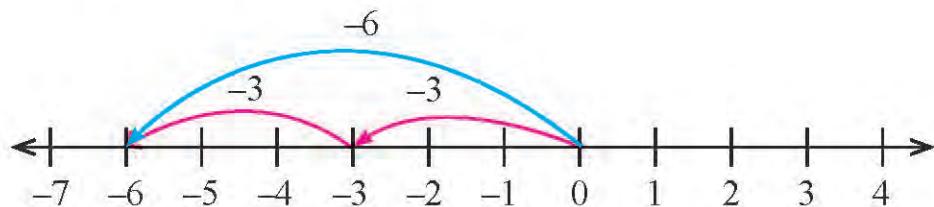
Which point is reached by moving 2 units left 3 times from the origin?
Discuss:

$$\begin{aligned} \text{Here, } & (-2) \times 3 \\ &= -6 \end{aligned}$$

$$\text{Hence, } (-2) \times 3 = -6$$

(b) $2 \times (-3) = ?$

Go 3 units left 2 times from the origin. At what point do you reach?
Note down by drawing number line.



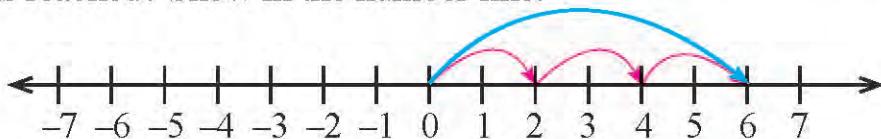
Here,

$$\begin{aligned} 2 \times (-3) \\ &= -6 \end{aligned}$$

$$\text{Hence, } 2 \times (-3) = -6$$

(c) $(-2) \times (-3) = ?$

$(-2) \times 3$ represents the point reached by moving 2 units with 3 times to the left from the origin as in activity 3 (a). The '-' sign in front of 3 means to go in the opposite direction of the previous direction. So, count how many units there are from the origin to the point given by $(-2) \times 3$ and go the equal unit to the right from the origin. In which point is reached? Show in the number line.



Here, $(-2) \times (-3)$

$$= 6$$

Therefore, $(-2) \times (-3) = 6$

Properties of Multiplication of Integers

1. Closure Property

Take any two numbers from the set of integers. Find the product of the numbers. Discuss what the result is.

The product of any two integers is also an integer. It is called closure property of multiplication.

If a and b are two integers, then $a \times b$ is also integer.

For example: $(-5) \times (+4) = (-20)$

$$(-4) \times (-2) = (+8)$$

$$1 \times 0 = 0$$

2. Commutative Property

Take any two numbers from the set of integers. Find the product by changing the order of those numbers. What is the result? Discuss.

For example: $3 \times 2 = 2 \times 3 = 6$

$$(-8) \times (+3) = (+3) \times (-8) = -24$$

$$1 \times 0 = 0 \times 1 = 0$$

If the product of any two integers is equal to the product of their changed order, it is called commutative property of multiplication.

If a and b are two integers, then $a \times b = b \times a$.

3. Associative Property

Place any three integers in different order. Find the product of the first two and multiply by the third. What comes the product? Discuss.

For example, In three integers 2, 3 and -4 ,

$$[2 \times 3] \times (-4) = 2 \times [3 \times (-4)]$$

$$6 \times (-4) = 2 \times (-12)$$

$$\therefore -24 = -24$$

The product of three integers remains unchanged even if the order in which they are grouped is altered.

If a , b and c are three integers, then

$$(a \times b) \times c = a \times (b \times c)$$

4. Distributive Property

For example, in three integers $(+6)$, $(+3)$ and (-2) ,

$$+6 [(+3) + (-2)] = (+6) \times (+3) + (+6) \times (-2)$$

$$\text{or, } +6(+1) = 18 - 12$$

$$\text{or, } +6 = +6$$

If a , b and c are three integers, then

$$a(b + c) = a \times b + a \times c$$

5. Multiplicative Property of 1

$$(-5) \times 1 = (-5)$$

If a be a integers, then

$$1 \times (+6) = +6$$

$$a \times (+1) = (+1) \times a = a$$

6. Multiplicative Property of Zero

$$2 \times 0 = 0$$

$$0 \times 2 = 0$$

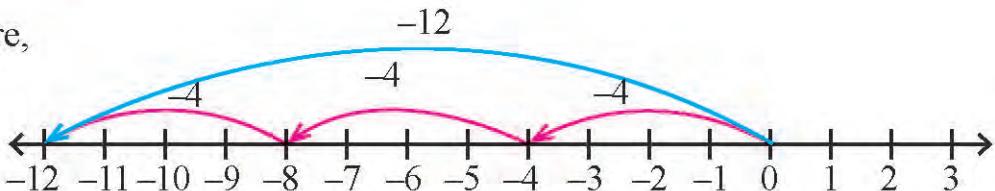
If a be a integers, then $a \times 0 = 0 \times a = 0$

Example 1

Multiply by using number line: $(-4) \times 3$.

Solution

Here,



Therefore, $(-4) \times 3 = (-12)$

Example 2

Multiply $(+12) \times (-8) \times (+2)$

Solution

Here, $(+12) \times (-8) \times (+2)$

$$\begin{aligned} &= (-96) \times (+2) \\ &= -192 \end{aligned}$$

Example 3

Multiply $(+5)$, $(+6)$ and (-7) from both ways using the Associative property of multiplication.

Solution

Here, $(+5) \times (+6) \times (-7)$

$$\begin{aligned} &= [(+5) \times (+6)] \times (-7) \\ &= (+30) \times (-7) \\ &= (-210) \end{aligned}$$

Again, $(+5) \times [(+6) \times (-7)]$

$$\begin{aligned} &= (+5) \times (-42) \\ &= (-210) \end{aligned}$$

$$\therefore [(+5) \times (+6)] \times (-7) = (+5) \times [(+6) \times (-7)] = (-210)$$

Example 4

Simplify using the distributive property of multiplication:

(a) $(-5) \times [(+24) - (-6)]$

Solution

Here, $(-5) \times [(+24) - (-6)]$
= $(-5) \times (+24) - (-5) \times (-6)$
= $(-120) - (+30)$
= $-120 - 30$
= -150
 $\therefore [(-5) \times [(+24) - (-6)]] = -150$

3.5 Division of Integers

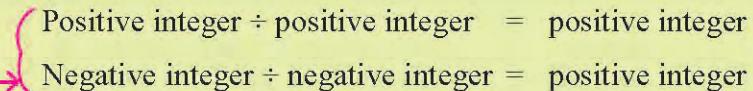
Discuss by observing the following examples:

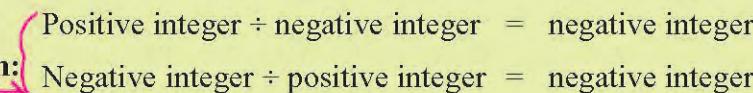
$$(-8) \times (-4) = 32$$

$$32 \div (-8) = ?$$

$$32 \div (-4) = ?$$

Here is $32 \div (-8) = (-4)$ and $32 \div (-4) = (-8)$

Having same sign: 
Positive integer \div positive integer = positive integer
Negative integer \div negative integer = positive integer

Having opposite sign: 
Positive integer \div negative integer = negative integer
Negative integer \div positive integer = negative integer

Exercise 3.2

1. Fill in the blanks with appropriate number:

- (a) $\boxed{\quad} \div (-6) = 4$
- (b) $81 \div \boxed{\quad} = (-9)$
- (c) $19 \times \boxed{\quad} = 0$
- (d) $-20 \times \boxed{\quad} = -20$
- (e) $\boxed{\quad} + (-45) = 1$

2. Multiply by using number line:

- (a) $(+3) \times (+2)$
- (b) $(-5) \times (+3)$
- (c) $(+2) \times (-6)$
- (d) $(-4) \times (-3)$
- (e) $(+5) \times (-4)$

3. Find the product in two different way, using the associative property:

- (a) $(+3) \times (+4) \times (+5)$
- (b) $(+7) \times (-5) \times (-3)$
- (c) $(-2) \times (-2) \times (-2)$
- (d) $(+4) \times (+8) \times (-5)$

4. Simplify by using the distributive property of multiplication:

- (a) $(+6) \times [(-8) + (+30)]$
- (b) $(-9) \times [(+24) - (-6)]$
- (c) $(+7) \times [(-12) - (+8)]$
- (d) $(-8) \times [(-3) + (-5)]$

5. Find the quotient:

- (a) $(+36) \div (+6)$
- (b) $(-45) \div (+5)$
- (c) $(+54) \div (-6)$
- (d) $(-95) \div (-1)$

6. Simplify the given numbers.

- (a) $[(+7) \times (+8) \times (-6)] \div (-3)$
- (b) $[(+12) \times (-8)] \div [(+2) \times (-1)]$
- (c) $[(+6) \times (+4)] \div [(-3) \times (-2)]$
- (d) $(+5) \times (-4) \times (-8) \times (-3)$

7. The product of two integers is $(+63)$. If one of them is $(+7)$, find the other integer.
 8. Find the number for which if multiplied by (-5) the product is 90 .
 9. Find the number which divides $(+56)$ and the quotient is $(+70)$, find.
 10. How many times is (-12) multiplied to get the product (-144) ?
 11. How many times is (-13) multiplied to get the product (-169) ?
- 12. In a quiz contest, a rule is made to provide $(+5)$ for each correct answer, (-2) for each wrong answer and (0) if not answered.**
- (a) If group A gave 4 correct answers and 5 incorrect answers, how much marks did they get?
 - (b) How much marks did group B get if they gave 5 correct answers and 5 incorrect answers?
 - (c) Which group scored the highest marks? How much more marks did the group obtain? Find out.
- 13. In a quiz contest, rules are made to provide $(+3)$ for each correct answer and (-2) for each incorrect answer.**
- (a) Group A scored 18. if they answered 12 questions incorrectly, find out how many questions they answered correctly?
 - (b) Group B obtained (-5) marks. If they answered 7 questions were answered incorrectly and find out how many questions they answered correctly.
 - (c) Which group answered most of the questions correctly? How many more questions did the group answer?

Project Work

Show the mathematical operation between the integers given below in number line and present it in the class.

1. $(+3) \times (+2)$
2. $(+3) \times (-2)$
3. $(-3) \times (+2)$
4. $(-3) \times (-2)$

Answer

1. (a) -24 (b) -9 (c) 0 (d) +1 (e) +46

2-4. Show the answers to your teacher.

5. (a) +6 (b) -9 (c) -9 (d) +95

6. (a) +112 (b) +48 (c) +4 (d) -480

7. +9 8. -18 9. +8 10. 12 11. 13

12. (a) 10 (b) 15 (c) group B, 5

13. (a) 14 (b) 3 (c) group A, 11.

3.6 Simplification of Integers

We have already discussed in the previous lesson the problems of addition, subtraction, multiplication and division of integers. Now, we will discuss about the simplification of integers.

Activity 1

Discuss by observing the questions asked about the mathematical problem given below.

In a quiz contest, rules are made for each correct answer (+5), and for each incorrect answer (-3).

- If annapurna house won with a total of 104 marks, in which 25 questions were answered correctly, how many questions did they answer incorrectly?
- What mathematical operations should be done to solve this problem?

Here, writing the given problem in mathematical sentences,

$$25 \times (+5) + \boxed{\quad} \times (-3) = 104$$

Let, number of questions answered incorrectly = x

$$\text{Now, } 25 \times (+5) + x \times (-3) = 104$$

$$\text{or, } 125 - 3x = 104$$

$$\text{or, } -3x = 104 - 125$$

$$\text{or, } -3x = -21$$

$$\therefore x = 7$$

7 questions were answered incorrectly.

Example 1

Simplify:

$$\begin{aligned}&(+12) + (-5) + (+25) \div (-5) - (-6) \times (+7) \\&= (+12) + (-5) + (-5) - (-6) \times (+7) \\&= (+12) + (-5) + (-5) - (-42) \\&= (+7) + (-5) + 42 \\&= +2 + 42 \\&= 44\end{aligned}$$

- The problem consisting addition, subtraction and multiplication signs, first treat the operation of multiplication.
- The problem consisting addition subtraction and divisions signs, first treat the operation of division.
- The problem consisting multiplication and division signs, first treat the operation which comes first from left to right.

Exercise 3.3

1. Simplify:

- $(-6) \times (-4) \div (+4) + (-5) - (-1)$
- $(-15) \div (+5) \times (-4) + (-10) - (+7)$
- $(-12) + (+16) \times (-27) \div (-9)$
- $(-3) \times (+16) - \{(+12) \div (+6) + (-10)\}$

2. In a school quiz contest, rules were made to provide (+10) for each correct answer and (-5) for each incorrect answer.

- If the Blue House scored 60 marks, in which 2 questions were answered incorrectly then how many questions were answered correctly? Find out.
- If the Yellow House obtained 20 marks, in which 4 questions were answered incorrectly, how many questions were answered correctly? Find out.

- (c) Which group answered more questions correctly? How many more questions did the group answer correctly than the group who answered less? State.
3. At the beginning of the month of Baishakh, there was Rs. 5,000 in Ram's bank account. Rs. 30,000 was credited as a salary to his account on 7th Baisakh. He paid Rs. 945 as an electricity tariff from the account on 9th Baisakh. Rs. 500 is also recharged in his mobile phone from his account on 11th Baisakh. He paid Rs. 10,000 to the ABC food store from the account on 15th Baisakh. Now find out how much money is left in his account.
4. The first integer is 2 more than five times of the second integer. If the first integer is 77, then find the second integer.
5. A tank of 1000 liter capacity has 500 liters of water. Two taps are fitted to it. In 1 minute, tap A fills 25 liters of water and tap B drains out 15 liters of water. How many liters of water are left in the tank if both the taps are opened for 6 minutes?
6. Write down five examples of integer using addition and subtraction in our daily life and present them in the class.

Project Work

Write down five examples of integer using addition and subtraction in our daily life and present them in the class.

Answer

- | | | | |
|--------------|--------|-------------------|----------|
| 1. (a) +2 | (b) -5 | (c) +36 | (d) -40 |
| 2. (a) 7 | (b) 4 | (c) blue house, 3 | |
| 3. Rs. 23555 | 4. | 15 | 5. 560 l |

4.0 Review

Take any two numbers from the set of integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. Which operation would be possible of those numbers among addition, subtraction, multiplication and division? Discuss.

4.1 Introduction to Rational Number**Activity 1**

Study the given condition and discuss the questions asked on the basis of number lines.

Two numbers 3 and -4 are taken from the set of integers. Among them addition, subtraction, multiplication and division operation have been done.

$$3 + (-4) = -1$$

$$3 - (-4) = 3 + 4 = 7$$

$$3 \times (-4) = -12$$

$$3 \div (-4) = \frac{3}{-4}$$



- Is the result always integer when two integers are added, subtracted and multiplied?
- What is the quotient between the integers?
- Is $\frac{3}{-4}$ in the number line of the integers?

Adding, subtracting and multiplying any two integers always gives the integer. But dividing one integer by another integer may not always be integer.

For example: $\frac{2}{3}, \frac{1}{2}, \frac{4}{5}$ are not integers. These come in the form of $\frac{a}{b}$. Such numbers are rational numbers. The set of rational numbers is denoted by Q .

$$Q = \{\dots, -4, -3, -\frac{5}{2}, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{2}{3}, \frac{1}{2}, 1, \dots\}$$

If any number can be expressed in the form of $\frac{a}{b}$ then such number is called rational number. Here a and b are integers and $b \neq 0$.

Activity 2

Discuss the following questions in the group with friends and present the conclusion in the class.

- Are all natural numbers rational numbers?
- Are all integers rational numbers?

Since all integers belong to the set of rational numbers, the set of integers is proper subset of the set of rational numbers.
So it can be written as $\mathbb{Z} \subset \mathbb{Q}$.

Properties of Rational Numbers

Discuss by observing the characteristics of the rational numbers given below.

1. Identity Property

Identity property of addition	Identity property of multiplication
$\frac{1}{2} + 0 = 0 + \frac{1}{2} = \frac{1}{2}$	$\frac{1}{2} \times 1 = 1 \times \frac{1}{2} = \frac{1}{2}$
$-\frac{2}{3} + 0 = 0 + -\frac{2}{3} = -\frac{2}{3}$	$-\frac{2}{3} \times 1 = 1 \times -\frac{2}{3} = -\frac{2}{3}$
Adding zero (0) to any rational number gives the same number. It is called the identity property of addition.	Multiplying any rational number by 1 gives the same number. It is called the identity property of multiplication.

2. Inverse Property

Inverse property of addition	Inverse property of multiplication
$-1 + 1 = 0$	$2 \times \frac{1}{2} = 1$
$\frac{-1}{2} + \frac{1}{2} = 0$	$\frac{2}{3} \times \frac{3}{2} = 1$
In any rational number $\frac{a}{b}$ adding $\frac{-a}{b}$ becomes zero, then it is called the inverse property of the addition. $\frac{a}{b}$ and $\frac{-a}{b}$ are inverse to each other.	If any rational number $\frac{a}{b}$ multiplied by $\frac{b}{a}$ becomes 1, then it is called inverse property of multiplication. $\frac{a}{b}$ and $\frac{b}{a}$ are considered to be the inverse to each other.

3. Commutative Property

Commutative property of addition	Commutative property of multiplication
$\frac{1}{2} + \frac{2}{3} = \frac{2}{3} + \frac{1}{2}$	$\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{2}$
If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then $\frac{b}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ is called the commutative property of the addition.	If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ is called the commutative property of the multiplication.

4. Associative Property

Associative property of addition	Associative property of multiplication
$\frac{1}{2} + \left(\frac{2}{3} + \frac{3}{5}\right) = \left(\frac{1}{2} + \frac{2}{3}\right) + \frac{3}{5}$ If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers then $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$ is called the associative property of the addition.	$\frac{1}{2} \times \left(\frac{2}{3} \times \frac{3}{5}\right) = \left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{3}{5}$ If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers then $\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$ is called the associative property of the multiplication.

5. Closure Property

Closure property of addition	Closure property of multiplication
$\frac{1}{2}$ and $\frac{2}{3}$ are rational numbers. $\frac{1}{2} + \frac{2}{3} = \frac{3+4}{6} = \frac{7}{6}$ is also a rational number.	$\frac{1}{2}$ and $\frac{2}{3}$ are rational numbers. $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ is also a rational number.
The sum of two rational numbers is also a rational number. It is called closure property of addition.	The product of two rational numbers is also a rational number. It is called closure property of multiplication

4.2 Decimal and Rational Number

Activity 3

Fill in the blanks by converting the fractions given below in the table into decimal numbers and discuss the findings.

S.N.	Fractions	Decimal Numbers
1	$\frac{1}{2}$	0.5
2	$\frac{1}{3}$	0.333
3	$\frac{2}{7}$	0.285714285714
4	$\frac{2}{3}$...
5	$\frac{3}{10}$...
6	$\frac{5}{13}$...

When converting fractions into decimals, it can be expressed as terminating, non-terminating and recurring decimals.

1. Terminating Decimal

Discuss by observing the example given below.

$$\frac{1}{4} = 0.25, \quad \frac{1}{8} = 0.125, \quad \frac{1}{3} = 1.5$$

Converting $\frac{1}{4}$, $\frac{3}{8}$, $\frac{3}{2}$ into the decimal number, the numbers after the decimal end in a certain place.

When the denominator of the rational number divides the numerator, after the decimal number terminates or ends in the quotient, then such a number is called terminating decimal numbers.

2. Non Terminating Recurring Decimal

Convert the fractions $\frac{1}{3}$, $\frac{2}{9}$, $\frac{4}{11}$ into decimal numbers. Discuss what the conclusion comes based on the results.

$$\frac{1}{3} = 0.3333\dots$$

$$\frac{2}{9} = 0.2222\dots$$

$$\frac{4}{11} = 0.363636\dots$$

$$\frac{2}{7} = 0.285714285714\dots$$

0.333..... can be written as 0.3 too.

Converting the above fractions into decimal numbers, the decimal parts does not terminate or end. The same digit or block of digits are repeated. Such a number is called non-terminating recurring decimal number.

Terminating decimal numbers and non-terminating decimals numbers are called rational numbers.

Note: If the denominators of the rational numbers have a multiples of 2 or 5, then that numbers are terminating decimal number. For example, $\frac{1}{2}, \frac{1}{5}, \frac{7}{10}, \frac{7}{25}, \dots$

If there are numbers other than 2 and 5 in the denominator of the rational number, then that numbers are non-terminating recurring decimal number. For example, $\frac{1}{3}, \frac{2}{3}, \frac{5}{7}, \dots$

4.3 Conversion of Decimal into Fraction

I. Conversion of terminating decimal numbers into fraction

Discuss how 0.75 can be converted into the fraction.

0.75 means 75th part of hundredths.

$$\text{So, } \frac{\cancel{75}}{\cancel{100}} \frac{3}{4}$$

0.75 has two numbers after the decimal. So,
0.75 has to be multiplied and divided by 100.

Another method: converting 0.75 into the fraction

$$\begin{aligned} & \frac{0.75 \times 100}{100} \\ &= \frac{75}{100} \\ &= \frac{3}{4} \end{aligned}$$

II. Conversion of non-terminating recurring decimal numbers into fraction

Discuss how we can convert $0.\overline{3}$ into the fraction.

$$\text{Let, } x = 0.\overline{3}$$

$$x = 0.33 \dots \quad (\text{i})$$

Multiplying the equation (i) by 10, we get,

$$10x = 3.33 \dots \quad (\text{ii})$$

There is same number 3 repeated after the decimal in $0.\overline{3}$, so it should be multiplied by 10.

Now, subtracting the equation (i) from the equation (ii),

$$10x - x = 3.33\dots - 0.33\dots$$

$$\text{or, } 9x = 3$$

$$\text{or, } x = \frac{3}{9}$$

$$\text{or, } x = \frac{1}{3}$$

$$\text{Therefore, } 0.\overline{3} = \frac{1}{3}$$

Example 1

Convert the following fraction into decimals and separate terminating and non-terminating recurring decimal numbers.

(a) $\frac{5}{8}$

(b) $\frac{2}{3}$

Solution

(a) $\frac{5}{8}$

$$= 0.625$$

0.625 is a terminating decimal number.

$$\begin{array}{r} 0.625 \\ 8 \overline{) 50} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

(b) $\frac{2}{3}$

$$= 0.666\dots$$

$$= 0.\bar{6}\dots$$

$0.\bar{6}$ is a non-terminating recurring decimal number.

$$\begin{array}{r} 0.666 \\ 3 \overline{) 20} \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

Example 2

Convert the following decimal numbers into fractions.

(a) 0.35 (b) 0.4̄

Solution

(a) 0.35

$$= \frac{0.35 \times 100}{100} = \frac{35}{100} = \frac{7}{20}$$

(b) $0.\overline{41}$

Let, $x = 0.\overline{41}$

$$x = 0.4141\dots \quad (\text{i})$$

Multiplying the equation (i) by 100, we get,

$$100x = 41.4141\dots \quad (\text{ii})$$

Now, subtracting the equation (i) from the equation (ii),

$$100x - x = 41.4141\dots - 0.4141\dots$$

$$\text{or, } 99x = 41$$

$$\text{or, } x = \frac{41}{99}$$

Therefore, $0.\overline{41} = \frac{41}{99}$

Example 3

Find any two rational numbers lying between $\frac{1}{2}$ and $\frac{5}{6}$.

Solution

To find any two rational numbers between $\frac{1}{2}$ and $\frac{5}{6}$

Any one number = $\frac{\frac{1}{2} + \frac{5}{6}}{2}$ (Finding the average value of $\frac{1}{2}$ and $\frac{5}{6}$)

$$= \frac{3+5}{6} \times \frac{1}{2} = \frac{5}{6} \times \frac{1}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$$

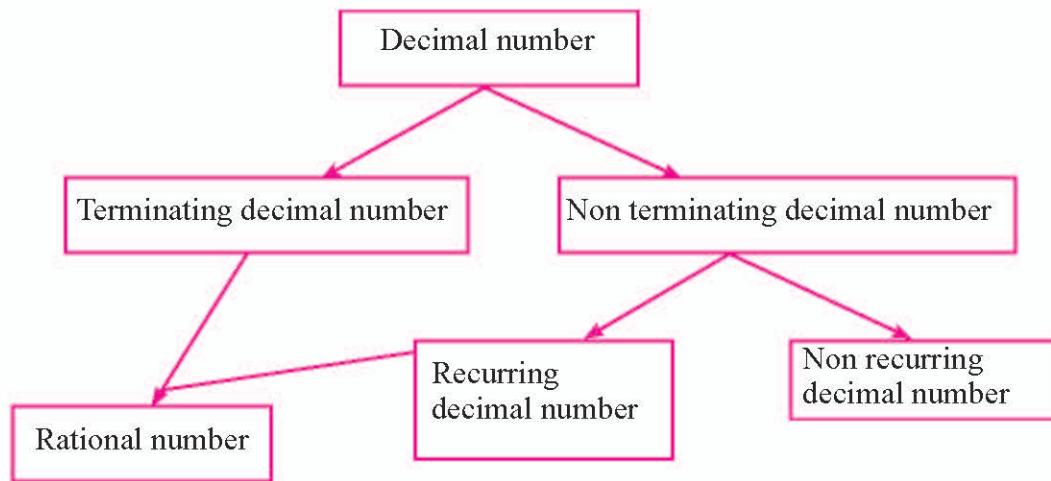
Another number = average value of $\frac{1}{2}$ and $\frac{2}{3}$

$$= \frac{\frac{1}{2} + \frac{2}{3}}{2}$$

$$= \frac{3+4}{6} \times \frac{1}{2} = \frac{7}{12}$$

Any two rational numbers between $\frac{1}{2}$ and $\frac{5}{6}$ are $\frac{2}{3}$ and $\frac{7}{12}$

The decimal number can be shown from the chart given below:



Exercise 4.1

1. Convert the following fraction into decimals and separate terminating and non-terminating recurring decimal numbers:

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{2}{7}$ (d) $\frac{15}{2}$ (e) $\frac{17}{13}$
(f) $\frac{55}{10}$ (g) $\frac{37}{20}$ (h) $\frac{25}{17}$ (i) $\frac{12}{25}$

2. Write the additive inverse of the following numbers:

(a) $\frac{2}{5}$ (b) $\frac{-5}{7}$ (c) $\frac{22}{12}$ (d) $\frac{12}{7}$ (e) $\frac{-11}{8}$

3. Write the multiplicative inverse of the following numbers:

(a) $\frac{3}{4}$ (b) $\frac{25}{10}$ (c) $\frac{-2}{3}$ (d) $\frac{22}{12}$ (e) $\frac{1}{8}$

4. Convert the following decimal numbers into fraction:

(a) 0.5 (b) $0.\overline{7}$ (c) $0.\overline{24}$ (d) $0.2\overline{7}$
(e) $1.\overline{57}$ (f) 2.35 (g) 7.025

5. Write the any two rational numbers lying between $\frac{1}{2}$ and $\frac{3}{4}$.
 6. Find any two rational numbers lying between $\frac{1}{3}$ and $\frac{3}{4}$.
7. **Write the answers of the following questions with reasons:**
- (a) Are all the natural numbers rational numbers?
 - (b) Are all the whole numbers rational numbers?
 - (c) Are all the integers rational numbers?
 - (d) Is zero (0) a rational number?
 - (e) Are all the rational numbers integers?

Project Work

Show the relationship among natural numbers, whole numbers, integers and rational numbers in the figure and present them in the class.

Answer

Show answers to your teachers.



5.1 Fraction

5.1.0 Review

Discuss in a group and calculate the problem related to fraction given below:

$$(a) \frac{1}{5} + \frac{1}{7} \quad (b) 5\frac{5}{6} - 1\frac{2}{3} \quad (c) 6 \times \frac{2}{5} \quad (d) 12 \div \frac{2}{5}$$

5.1.1 Simplification of Fractions

Activity 1

Study the following mathematical problem and discuss the questions asked:

The monthly income of Sirjana is Rs. 24,000. She spent one-third of her monthly income on education. Similarly, if she spent half of income on food,

- What part of her total income did she spend?
- How much did she spend in total?

Here, one-third of the income spent on education = one part out of three part (one third) = $\frac{1}{3}$

Half of the income spent on food = one part out of 2 part (half) = $\frac{1}{2}$

Total expenditure = ?

To find out the total expenditure, both expenditure must be added.

$$\text{So, total expenditure} = \frac{1}{3} + \frac{1}{2}$$

$$= \frac{1}{3} \times \frac{2}{2} + \frac{1}{2} \times \frac{3}{3}$$

$$= \frac{2}{6} + \frac{3}{6}$$

Making equal denominators
to unequal denominators

Total expenditure = $\frac{5}{6}$ (writing in lowest term)

Now, Total expenditure amount = $\frac{5}{6}$ of total income.

$$\begin{aligned} &= \frac{5}{6} \times \text{Rs. } 24000 \\ &= 5 \times \text{Rs. } 4000 \\ &= \text{Rs. } 20,000 \end{aligned}$$

Another Method (by Model Drawing)

Since one-third of the income is spent on education, dividing her income into three equal parts,

Sirjana's income = Rs. 24000		
x	x	x

$$3x = 24000$$

$$\text{or, } x = \frac{24000}{3}$$

$$\text{or, } x = 8000$$

Hence, expenditure on education = Rs. 8000

Again, half of the income is spent on food,

Sirjana's income = Rs. 24000	
y	y

$$2y = 24000$$

$$\text{or, } y = \frac{24000}{2}$$

$$\text{or, } y = 12000$$

Hence, expenditure on food = Rs. 12000

Therefore, total expenditure = expenditure on education + expenditure on food

$$\begin{aligned} &= 8000 + 12000 \\ &= \text{Rs. } 20,000 \end{aligned}$$

$$\text{Part of total expenditure} = \frac{\text{total expenditure}}{\text{total income}} = \frac{\text{Rs. } 20000}{\text{Rs. } 24000} = \frac{5}{6}$$

Example 1

There are 48 students in class 7 of Shanti Niketan School. Two-thirds ($\frac{2}{3}$) of them are boys and the rest are girls,

- What is the number of girls?
- What is the number of boys? Find out.

Solution

Here, total number of students = 48

$$\begin{aligned}\text{Number of girls} &= \frac{2}{3} \text{ part of total students} \\ &= \frac{2}{3} \times 48 \\ &= 32\end{aligned}$$

$$\begin{aligned}\text{The number of boys} &= \text{Total number of students} - \text{number of girls} \\ &= 48 - 32 \\ &= 16\end{aligned}$$

Hence, there are 32 girls and 16 boys.

Another Method (by Model Drawing)

The number of girls is two-thirds,

48		
Girls	Boys	
x	x	x

$$3x = 48$$

$$x = \frac{48}{3}$$

$$\therefore x = 16$$

Girls = 2 part
Boys = 1 part

Numbers of girls = 2 part

$$= 2 \times 16$$

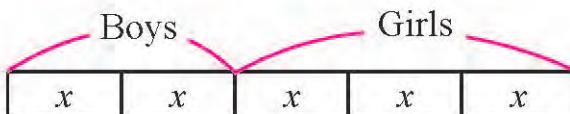
$$= 32$$

Numbers of boys = $48 - 32 = 16$

Example 2

In a basic school, $\frac{2}{5}$ part of the total students are boys. If there are 90 girls, how many boys are there? How many students are there in total?

Solution



$$\text{Here, } 3x = 90$$

$$\text{Or, } x = \frac{90}{3}$$

$$\text{Or, } = 30$$

$$\begin{aligned}\text{Numbers of boys } & 2x = 30 \times 2 \\ & = 60\end{aligned}$$

$$\text{Total numbers of students} = 5x = 5 \times 30 = 150$$

Example 3

Prabin bought a cake on his birthday. Among his friends, Kripa ate $\frac{1}{2}$ portion, Aman ate $\frac{1}{3}$ portion and Sandeep ate $\frac{1}{6}$ portion. Find out who ate the most portion of cakes.

Solution

$$\text{Portion of the cake eaten by Kripa} = \frac{1}{2}$$

$$\text{Portion of the cake eaten by Aman} = \frac{1}{3}$$

$$\text{Portion of the cake eaten by sandeep} = \frac{1}{6}$$

Now, find the LCM of the denominator of the three fractions.

$$\text{LCM} = 6$$

Multiples of 2 = 2, 4, 6, 8, ...

Multiples of 3 = 3, 6, 9, ...

Multiples of 6 = 6, 12, ...

Lowest common multiples = 6

Now, make the denominator 6 of all fractions.

$$\frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

$$\frac{1}{6} = \frac{1}{6} \times \frac{1}{1} = \frac{1}{6}$$

Here 3 times of $\frac{1}{6}$ is $\frac{3}{6}$, 2 times of $\frac{1}{6}$ is $\frac{2}{6}$

So, $\frac{3}{6}$ is the greatest fraction among the 3 fractions.

∴ Kripa ate $\frac{1}{2}$ portion of the cake that is the most.

Exercise 5.1

1. Simplify:

$$(a) \left(11\frac{1}{13} - 1\frac{3}{8}\right) + 2\frac{7}{8}$$

$$(b) 13\frac{1}{9} - \left(4\frac{8}{9} - 6\frac{5}{11}\right)$$

$$(c) \frac{1}{2} + \frac{1}{3} \times \frac{6}{2} - \frac{4}{5} \div \frac{1}{2}$$

$$(d) \left(\frac{1}{2} + \frac{2}{3}\right) \div \frac{3}{5} \times \frac{4}{5} + \frac{5}{9} - 2$$

2. Subtract $(6\frac{1}{8} - 8)$ from 30 and add $1\frac{1}{4}$

3. Sandeep's monthly income is Rs 27,000. He has spent $\frac{1}{5}$ part of his income on food. If $\frac{1}{10}$ part spent on clothing and $\frac{2}{5}$ part spent on transportation, then:

(a) How many parts did he spend in total?

(b) How many parts did he save?

(c) Find out how much money he has saved.

4. At the fun fair, Rema spent $\frac{1}{5}$ of her money on entertainment and $\frac{1}{2}$ on food. Find out in which title she has spent the most amount.

5. Out of 40 students in a class, $\frac{1}{5}$ students like English subject. $\frac{2}{5}$ Students like mathematics. The rest of the students like science subjects.
- Find the number of students who like English.
 - Find the number of students who like Mathematics.
 - Write in the form of fraction to the students who like science.
6. Ranju's father gave her Rs 60,000. She bought books for one-third of her money. She bought clothes with one-fourth of the rupees. One-fifth of the rupees is spent on her travel.
- How much did she spend?
 - How much money had been spent in total?
 - Find out how much money she saved.
7. Ritu ate $\frac{3}{5}$ parts of an apple and the rest was eaten by his brother, Suman.
- Write in fraction the part of the apple that Suman ate.
 - Find out who ate the most part of apple.
8. Mohan gave $\frac{2}{3}$ parts of his money to his wife. He gave $\frac{1}{5}$ part of the remaining amount to his son and $\frac{1}{3}$ parts to his daughter. If he gave Rs. 60,000 to his wife then,
- How much money did he give to his son?
 - How much money did he give to his daughter?
 - How much more money was given between son and daughter?
9. Prakriti has spent $\frac{1}{4}$ parts of the money. She has lost $\frac{2}{5}$ parts from the remaining amount. If the lost money is Rs. 900, find out how much money he had at the beginning.

10. There are $\frac{4}{5}$ part girls in one school. The school's picnic program includes $\frac{1}{4}$ part girls and $\frac{1}{3}$ part boys. If 190 students went to the picnic program, find out the total number of students in the school

Project Work

How much time do you spend on the following activities out of the time you spend in a day at school? Present in the class by writing the form of the fraction.

- (a) Morning prayer
- (b) Lunch time
- (c) Reading time
- (d) Time for other activities

Answer

1. (a) $12 \frac{15}{26}$ (b) $14 \frac{67}{99}$ (c) $\frac{-5}{2}$ (d) $\frac{1}{9}$

2. $33 \frac{1}{8}$

3. (a) $\frac{7}{10}$ part (b) $\frac{3}{10}$ part (c) Rs. 8,100

4. Show answers to your teachers.

5. (a) 8 (b) 16 (c) $\frac{2}{5}$ part

6. (a) $\frac{47}{60}$ part (b) Rs. 4,700 (c) Rs. 1300

7. (a) $\frac{2}{5}$ (b) Ritu $\frac{1}{5}$

8. (a) Rs. 6000 (b) Rs. 10,000 (c) to daughter: Rs. 4000

9. Rs. 3000

10. 600

5.2 Decimal

5.2.0 Review

Calculate of the decimal number given below:

(a) $8.97 + 23.2$

(b) 3.6×5.8

(c) $7.7 - 2.8$

(d) $17.40 \div 4$

What should be considered when adding, subtracting, multiplying and dividing of decimal number? Discuss in pairs and find out the conclusion.

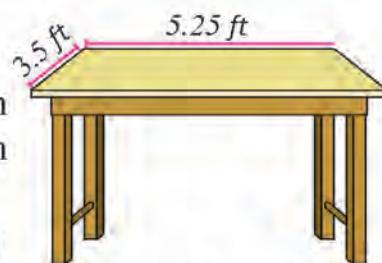
5.2.1 Simplification of Decimal

Activity 1

Isha was making some patterns to decorate the classroom. It requires 4.5 cm length squared paper. He has a rectangular paper with 54 cm length and 4.5 cm breadth. How many pieces can be made having length 4.5 cm from that paper? She divided 54 by 4.5 to find the pieces of paper. Is this right? Discuss and conclude it.

Activity 2

The rectangular surface of the table shown in the picture has the length 5.25 ft and breadth 3.5 ft.



- What is the perimeter of the table?
- Which mathematical operation should be used to find the perimeter? Discuss.

Here to find the perimeter of the table,

Both the length and the breadth of the table should be added twice.

Or, perimeter of the table = $2(l + b)$

$$\begin{aligned}\text{So, perimeter of the table} &= 2(5.25 \text{ ft} + 3.5 \text{ ft}) \\ &= 2 \times 8.75 \text{ ft} \\ &= 17.5 \text{ ft}\end{aligned}$$

Example 1

How can the following mathematical problem be simplified?
Conclude by discussing the questions asked.

Simplify:

$$7.5 + \{6.72 \div 2.8 (3.59 - 1.49)\}$$

- Is it possible to divide 6.72 by 2.8 without subtracting 1.49 from 3.59?
- Which operation should be done within brackets at first?
- What is the order in brackets when simplifying?

When simplifying, the operation of division, multiplication, addition and subtraction is done respectively, but here, first of all, we have to subtract 1.49 from 3.59. The result should be multiplied by 2.8, then we should divide to 6.72. Therefore $(3.59 - 1.49)$ is placed in the small bracket and $\{6.72 \div 2.8 (3.59 - 1.49)\}$ is placed in the middle bracket.

Solution

$$\begin{aligned} & \text{Here } 7.5 + \{6.72 \div 2.8 (3.59 - 1.49)\} \\ &= 7.5 + \{6.72 \div 2.8 (2.1)\} \\ &= 7.5 + \{6.72 \div 5.88\} \\ &= 7.5 + 1.14 \\ &= 8.64 \end{aligned}$$



Simplifying the decimals with four mathematical operations (+, -, ×, ÷) and brackets, as in the simplification of whole numbers, should be done after doing the rest of the operations, included in the brackets at first. The brackets used in simplification should be followed by operations consisting of small bracket (), middle bracket { } and large bracket [] respectively.

Exercise 5.2

1. Simplify:

- (a) $1.44 \div 1.2 + 6.2$
- (b) $12.75 - \{4.38 - (2.4 \times 4.32 \div 3.6 - 0.85)\}$
- (c) $\frac{1.2 \times 1.2 - 0.4 \times 0.4}{2.4 - 1.6}$
- (d) $\frac{4.5 \times 4.5 - 2.1 \times 2.1}{4.5 + 2.1} - 1$
2. Area of the triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. If the base of a triangle is 25.75 cm and the height is 30.15 cm then find the area.
3. The length of a rectangular garden is 22.66 cm, and breadth is 15.65 cm.
 - (a) What is the area of the garden?
 - (b) Find out the perimeter of the garden.
4. The perimeter of a squared table is 24.4 ft. Find the length of the table.
5. The area of a rectangular field is 215.66 m^2 . If the field is 67.35 m long, find out how wide it is.
6. If the length of a squared field is 8.45 m, then find the perimeter of that field.

Project Work

Search today's exchange rates of currency from magazines or the internet. Note down the buying and selling rates of currencies of any five countries and find the differences among them and present it in the class.

Answer

- | | | | |
|--------------------------|-----------------------------|-------------|---------|
| 1. (a) 7.4 | (b) 10.4 | (c) 1.6 | (d) 2.4 |
| 2. 388.18 cm^2 | 3. (a) 354.62 m^2 | (b) 76.62 m | |
| 4. 6.1 ft | 5. 3.20 m | 6. 33.8 m | |

6.1 Ratio

6.1.0 Review

The weight of Anuja is 30 kg and weight of Ramesh is 60 kg. Discuss how the weight of Anuja and Ramesh can be compared.

As difference,

Difference between weight of Anuja and Ramesh = $(60 - 30)$ kg = 30 kg

As quotient,

$$\frac{\text{Weight of Anuja}}{\text{Weight of Ramesh}} = \frac{30 \text{ kg}}{60 \text{ kg}} = \frac{1}{2}$$

The difference itself refers to how many digits are small or large and quotient refers to how many times small or large.

6.1.1 Introduction to Ratio

Activity 1

Observe the given table and discuss the questions asked:

Item	Price
	Rs. 10
	Rs. 70
	Rs. 5

- (a) How many times is the price of copy than the price of pencil?
- (b) How many times is the price of eraser than the price of pencil?

$$\frac{\text{price of copy}}{\text{price of pencil}} = \frac{\text{Rs. } 70}{\text{Rs. } 10} = \frac{7}{1}$$

The price of copy is 7 times the price of pencil.

$$\frac{\text{price of eraser}}{\text{price of pencil}} = \frac{\text{Rs. } 5}{\text{Rs. } 10} = \frac{1}{2}$$

The price of eraser is half the price of pencil.

Ratio is the comparison on the basis of one quantity to another quantity.

The ratio having the same unit of two quantity a and b is $\frac{a}{b}$ or $a:b$, where, $b \neq 0$. $a:b$ can be read as a is to b where a and b are the terms of ratio.

Simplest form of a Ratio

Activity 2

The length of the pencil shown below is 18 cm and diameter is 6 mm. What is the ratio of the diameter of a pencil to its length?



Can be written as $\frac{18 \text{ cm}}{6 \text{ mm}} = \frac{3}{1}$?

Is the length of the pencil only three times its diameter? Discuss.

The length of the pencil is not only three times of the diameter, how to do?



If the units of the given quantities are different then we should compare them by changing into the same unit.



Changing the length of the pencil in millimeter,

$$18 \text{ cm} = 18 \times 10 \text{ mm} = 180 \text{ mm}$$

So, ratio of the diameter of pencil and its length can be written as

$$= \frac{10 \text{ mm}}{180 \text{ mm}} = \frac{1}{18}$$

- Ratio is the comparison of two quantities having the same unit.
- Ratio has no unit.

Relation of Ratio and Fraction

Activity 3

Draw a circle. Divide it into five equal parts.

- Write in fraction to the shaded part.
- Find the ratio of the shaded part and non-shaded part.
- Discuss what are the differences between fraction and ratio.

Here,

Here, fraction of the shaded part = $\frac{3}{5}$

The ratio of shaded and non-shaded part = $\frac{2}{3} = 2:3$

In fraction $\frac{3}{5}$ refers to 3 parts out of 5 parts.

In the ratio 2:3 refers in total there are $2 + 3 = 5$ parts.



Example 1

Write the ratio of the capacity of the two buckets shown in the figure.



Capacity: 9 l



Capacity: 15 l

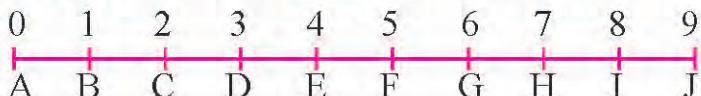
Solution

Here $\frac{\text{capacity of the first bucket}}{\text{capacity of the second bucket}} = \frac{9\text{ l}}{15\text{ l}} = \frac{3 \times 3\text{ l}}{3 \times 5\text{ l}} = \frac{3}{5} = 3:5$

Therefore, the capacity of first bucket: the capacity of second bucket = 3:5

Example 2

Each part represents 1 cm in the number line given below.



What is the ratio of AC and AF?

Solution

Here $AC : AF$

$$\begin{aligned}&= \frac{AC}{AF} \\&= \frac{2 \text{ cm}}{5 \text{ cm}} = \frac{2}{5} = 2:5\end{aligned}$$

Example 3

Bishnu obtained 75 marks out of 100 full marks in mathematics. In science, he obtained 50 marks out of 75 full marks. In which subject did he obtain better marks?

Solution

Here, fraction of marks obtained in mathematics = $\frac{75}{100} = \frac{3}{4}$

Fraction of marks obtained in science = $\frac{50}{75} = \frac{2}{3}$

Making equal to the denominator,

Fraction of marks obtained in mathematics = $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

Fraction of marks obtained in science = $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

$\therefore \frac{9}{12} > \frac{8}{12}$ So, Bishnu obtained better marks in mathematics.

Example 4

The height of Anju and Aman are 145 cm and 165 cm respectively. Find the ratio of their height.

Solution

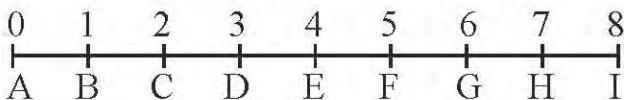
$$\begin{aligned}\text{Here, the ratio of the height of Anju and Aman} &= \frac{145 \text{ cm}}{165 \text{ cm}} \\ &= \frac{5 \times 29}{5 \times 33} = \frac{29}{33} = 29 : 33\end{aligned}$$

Exercise 6.1

1. Write the ratio of each of the following and convert it to the lowest term:

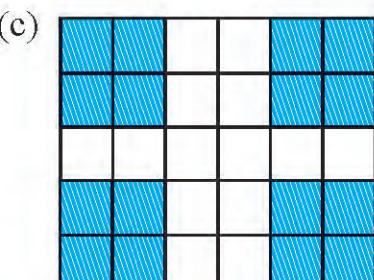
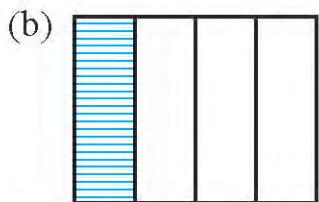
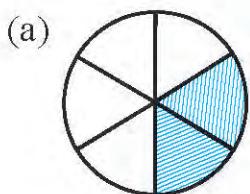
- (a) 10 cm and 100 cm (b) Rs. 180 and Rs. 240 (c) 10 kg and 2 kg
(d) 8 hours and 24 hours (e) 250 ml and 1000 ml (f) 2.5 kg and 7.5 kg

2. Each part represents 1 cm in the number line given below. Find the ratio of the distance asked:



- (a) AB : AG (b) AC : DI (c) CF : CH
(d) BG : BI (e) AF : AI

3. Find the ratio of the shaded part to non-shaded part from the given figure:



- 4.** There are 450 students in a school, among them 180 are girls then,
- find the ratio of girls and total students.
 - find the ratio of boys and total students.
 - find the ratio of girls and boys.
- 5.** There are 25 teachers and 500 students in a school, then find the ratio of teachers and students.
- 6.** The height of Pramesh is 165 cm and the height of Pramila is 150 cm then,
- find the ratio of the height of Pramesh and Pramila.
 - find the ratio of the height of Pramila and Pramesh.
- 7.** Rahul obtained 40 marks out of 50 full marks in English, 20 marks out of 30 full marks in Nepali and 13 marks out of 20 full marks in Mathematics.
- Which result is better in the subjects English and Nepali?
 - Which result is better in the subjects Nepali and Mathematics?
 - Which is the best result in these subjects? Compare by finding the ratio.
- 8.** Find out how much money will Alisa and Dipesh get while dividing Rs 60,000 in the ratio of 5:4.

Project Work

Note down the number of boys and girls of grade 5 to 10 in your school. With class-wise for each class,

- Find the ratio of girls and boys.
- Find the ratio of girls and total students.
- Find the ratio of boys and total students and present the conclusion in the classroom,

Answer

1-3 Show the answers to your teacher.

4. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{3}$ 5. 1:20 6. (a) 11:10 (b) 10:11

7. Show the answers to your teacher. 8. Rs. 2500, Rs. 3500

6.2 Proportion

6.2.0 Review

Complete the given table so that there are equivalent fractions of $\frac{2}{5}$.

$\frac{2}{5} = \frac{4}{10}$	$\frac{2}{5} = \frac{10}{\square}$
$\frac{2}{5} = \frac{6}{\square}$	$\frac{2}{5} = \frac{\square}{30}$
$\frac{2}{5} = \frac{14}{\square}$	$\frac{2}{5} = \frac{20}{\square}$

Are all fractions $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$ equivalent? Conclude after discussion.

6.2.1 Introduction to Proportion

Activity 1

Take two sheets of squared paper of equal size. Divide the first in four equal parts and the second into 16 equal parts.

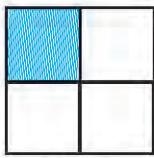


Fig. I

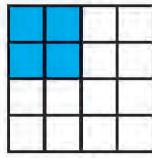


Fig. II

Find the ratio of the shaded part and non-shaded part from the Fig. I.

$$\frac{\text{shaded part}}{\text{non-shaded part}} = \frac{1}{3}$$

Now find the ratio of the shaded part and non-shaded part from the Fig. II

$$\frac{\text{shaded part}}{\text{non-shaded part}} = \frac{4}{12} = \frac{1}{3}$$

So, $\frac{1}{3}$ and $\frac{4}{12}$ are equal ratios.

Equal ratios are called proportion.

Activity 2

Dipendra obtained 25 marks out of 30 in the examination of mathematics. Narmada obtained 20 marks in the examination out of 24 full marks. Whose result is better? What is the ratio of their score? Discuss how it can be compared.

$$\text{Ratio of obtained marks and full marks of Dipendra} = \frac{25}{30} = \frac{5}{6}$$

$$\text{Ratio of obtained marks and full marks of Narmada} = \frac{20}{24} = \frac{5}{6}$$

Since, the ratio of $\frac{25}{30}$ and $\frac{20}{24}$ are equal, the result of both seems to be equal. So, here $\frac{25}{30} = \frac{20}{24}$

Since the two ratios are equal, these ratios are called proportions. Here, 25 is called the first term, 30 the second term, 20 the third term and 24 is called the fourth term.

Among the four numbers a , b , c and d , if the ratio of a and b and the ratio of c and d are equal then, a , b , c and d are said to be in proportion. It is written as $a : b :: c : d$ or $a \times d = b \times c$.

In the above example $\frac{25}{30}$ and $\frac{20}{24}$ are proportional.

It is also written as $25 : 30 :: 20 : 24$



The two outer terms are called extremes. The two inner terms are called means. Here, 25 and 24 are extremes and 30 and 20 are means.

$$25 \times 24 = 30 \times 20 = 600$$

The product of extremes and product of means is equal separately.

Example 1

Are these numbers 3, 4, 9 and 12 in proportion?

Solution

Here, product of extremes = $3 \times 12 = 36$

Product of means = $4 \times 9 = 36$

Since product of extremes and product of means are equal, 3, 4, 9 and 12 are in proportion.

Another method

Ratio of the first and second numbers = $\frac{3}{4}$

Ratio of the third and fourth numbers = $\frac{9}{12} = \frac{3}{4}$

Since both the ratios are equal, 3, 4, 9 and 12 are in proportion.

Exercise 6.2

1. Which of the following ratios are equal? Write with reason:

(a) $\frac{2}{3}$ and $\frac{4}{5}$

(b) $\frac{8}{4}$ and $\frac{2}{1}$

(c) $\frac{4}{5}$ and $\frac{12}{20}$

(d) $\frac{3}{5}$ and $\frac{9}{15}$

2. Check whether the numbers given below are in proportion or not.

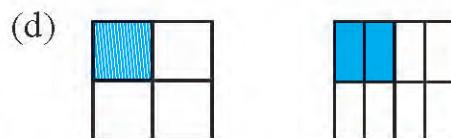
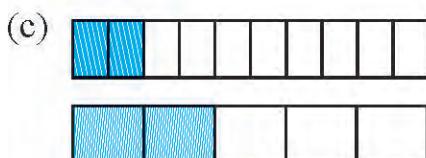
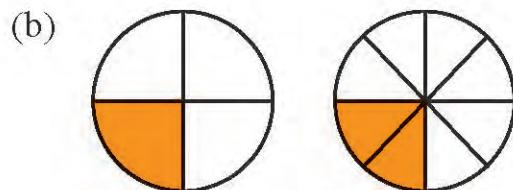
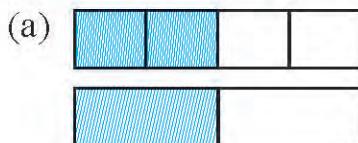
(a) 2, 3, 12, 20

(b) 7, 8, 14, 20

(c) 5 m, 3 m, 25 m, 35 m

(d) 3 ft, 8 ft, 12 ft, 32 ft

- 3.** Write the ratio of shaded part and non-shaded part from the pair of figure given below. Write with reason if these figures are in proportion or not.



- 4.** Tick (✓) for the right answers and cross (✗) for the wrong ones.

- (a) $16 : 24 :: 20 : 30$
- (b) $8 : 9 :: 24 : 27$
- (c) $32 \text{ m} : 64 \text{ m} = 6 \text{ sec} : 12 \text{ sec}$
- (d) $45 \text{ km} : 60 \text{ km} = 12 \text{ hours} : 15 \text{ hours}$

- 5.** Write extremes and means separately from the proportions given below.

- (a) $40 : 200 :: 15 : 75$
- (b) $3 : 7 :: 21 : 49$
- (c) $10 : 55 :: 2 : 11$
- (d) $25 : 15 = 5 : 3$

- 6.** The ratio of the score of Anita is $10 : 12$ in Mathematics and Science. If her score of mathematics is 80, then what is her score in science?

7. If the price of 6 kg oranges is Rs. 540, then find out the price of 8 kg oranges.
8. A restaurant made lemonade by putting 200 ml of honey in 3 liters of hot water for 15 people. Later 3 people were added. How many ml of honey should be mixed in 600 ml of water to make the same quality of lemonade?

Project Work

Take two sheets of squared paper, divide them into different parts so that the ratio of these papers are equal and present them in the class.

Answer

- | | |
|--|----------|
| 1-5. Show the answers to your teacher. | 6. 96 |
| 7. Rs. 720 | 8. 40 ml |



7.0 Review

Discuss the following questions:

- Subas bought a bag for Rs. 750. How much actual profit or loss did he get if he sold it at Rs. 800?
- Kamala bought a bag for Rs. 1000. After seeing minor defect in the bag, she sold it at Rs. 900. How much actual profit or loss did Kamala get?

Subas has bought the bag at low prices and sold them at high prices. So there was actual profit. Similarly, Kamala bought at high price and sold at low price. So there was loss.

Actual profit is the difference between selling price and cost price.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

$$\text{Loss} = \text{cost price} - \text{selling price}$$

7.1 Problems of Profit and Loss with Percentage

Activity 1

Study and discuss the following condition:

Mankaji is a businessman. He bought each t-shirt from a wholesaler at the rate of Rs 1000. The first t-shirt was sold at Rs. 1200. Similarly, after seeing it slightly torn in the second t-shirt, he sewed it and sold at Rs. 950.

- How much profit was made on the first T-shirt?
- What was loss on the second T-shirt?
- How can we find the actual profit on the first T-shirt in percentage?
- What was the loss percentage on the second T-shirt? How can it be found?

In the first t-shirt,

Cost price = Rs. 1000

Selling price = Rs. 1200

$$\begin{aligned}\text{Actual Profit} &= \text{Rs. } 1200 - \text{Rs. } 1000 \\ &= \text{Rs. } 200\end{aligned}$$

$$\text{Profit } x = \frac{\text{actual profit}}{\text{CP}} \times 100\%$$

$$\begin{aligned}\text{Now, } \frac{\text{Rs. } 200}{\text{Rs. } 1000} \times 100\% \\ = 20\% \text{ (It is actual profit percent.)}\end{aligned}$$

In the second t-shirt,

Cost price = Rs. 1000

Selling price = Rs. 950

$$\begin{aligned}\text{Loss} &= \text{Rs. } 1000 - \text{Rs. } 950 \\ &= \text{Rs. } 50\end{aligned}$$

$$\text{Loss } x = \frac{\text{actual loss}}{\text{CP}} \times 100\%$$

$$\begin{aligned}\text{Now, } \frac{\text{Rs. } 50}{\text{Rs. } 1000} \times 100\% \\ = 5\% \text{ (It is loss percent.)}\end{aligned}$$

$$\text{Profit percentage} = \frac{\text{actual profit}}{\text{CP}} \times 100\% \rightarrow \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100\%$$

$$\text{Loss percentage} = \frac{\text{actual loss}}{\text{CP}} \times 100\% \rightarrow \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100\%$$

Example 1

Rajesh has bought a watch for Rs. 1200 and he sold at Rs. 1500. What would be the profit or loss? Find in percentage.

Solution

Here, cost price of the watch = Rs. 1200

Selling price of the watch = Rs. 1500

The selling price is more than the cost price, so there was profit.

Now, actual profit = selling price – cost price

$$\begin{aligned}&= \text{Rs. } 1500 - \text{Rs. } 1200 \\ &= \text{Rs. } 300\end{aligned}$$

$$\text{Actual profit percentage} = \frac{\text{actual profit}}{\text{CP}} \times 100\%$$

$$\begin{aligned}&= \frac{\text{Rs. } 300}{\text{Rs. } 1200} \times 100\% \\ &= 25\%\end{aligned}$$

∴ Rajesh got 25% actual profit by selling this watch.

Example 2

Jyoti decided to sell an electric heater at 10 percent profit which was bought for Rs 2,500. How much should she sell the heater?

Solution

Here, cost price of the heater = Rs. 2500

Actual profit percentage = 10%

Selling price of the heater = ?

Adding 10% actual profit in the cost price Rs. 2500 of the heater is the selling price.

Actual profit = 10% of Rs. 2500

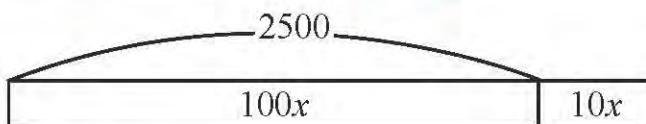
$$\begin{aligned} &= \frac{10}{100} \times \text{Rs. } 2500 \\ &= \text{Rs. } 250 \end{aligned}$$

Now, selling price = cost price + profit

$$\begin{aligned} &= \text{Rs. } 2500 + \text{Rs. } 250 \\ &= \text{Rs. } 2750 \end{aligned}$$

Therefore, Jyoti has to sell the heater at Rs. 2750 to earn 10 % profit.

Next method (By model drawing)



$$100x = \text{Rs. } 2500$$

$$x = 25$$

$$\text{Now, } 110x = 25 \times 110$$

$$= \text{Rs. } 2750$$

Example 3

By selling a mobile for Rs 21,000, a mobile seller gained 5%.

- (a) What is the cost price of the mobile?
- (b) At what price should the mobile be sold to receive 10 % profit?

Solution

Here, selling price of the mobile= Rs. 21000

Profit percentage = 5%

Cost price of the mobile= ?

- (a) Let, cost price of the mobile = Rs. x

Now, selling price = cost price + profit

$$\text{So, Rs. } 21,000 = x + 5\% \text{ of } x$$

$$\text{or, Rs. } 21000 = x + \frac{x \times 5}{100}$$

$$\text{or, } \text{Rs. } 21000 = \frac{100x + 5x}{100}$$

$$\text{or, } \text{Rs. } 21000 = \frac{105x}{100}$$

$$\text{or, } 105x = \text{Rs. } 21000 \times 100$$

$$\text{or, } x = \text{Rs. } \frac{21000 \times 100}{105}$$

$$\text{or, } x = \text{Rs. } 20,000$$

So the mobile was bought for Rs 20,000.

- (b) Calculating the selling price to receive 10% profit,

Now, selling price = cost price + profit

$$= \text{Rs. } 20,000 + \text{Rs. } 20,000 \text{ of } 10\%$$

$$= \text{Rs. } 20,000 + \text{Rs. } 20,000 \times \frac{10}{100}$$

$$= \text{Rs. } 20,000 + \text{Rs. } 2,000$$

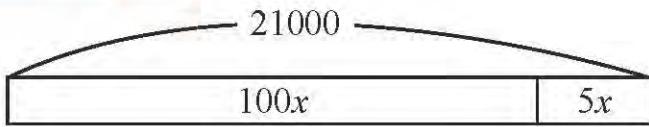
$$= \text{Rs. } 22,000$$

So to receive 10 % profit, the mobile has to be sold for Rs 22,000.

Next method (By model drawing)

Here,

$$105x = \text{Rs. } 21,000$$
$$x = 200$$



Now, CP = $100x$ = Rs. 20,000

$$\text{SP} = 110x$$

$$110x = 110 \times 200$$
$$= \text{Rs. } 22,000$$

Exercise 7.1

- What is the profit or loss and their percentage in the following condition? Find out.

Cost price (Rs.)	Selling price (Rs.)
300	350
550	500
7000	7720
5000	2750
12000	15000

- Ajaya bought a television for Rs 30,500 and sold it for Rs 29,000. Find out what his loss percentage is.
- Find out what is the cost price of a sari if it is sold for Rs. 30,500 by gaining 10% profit.
- What will be the selling price of a mobile bought at Rs. 30,000 by gaining 10% profit?
- A fruit seller bought 10 kg oranges at the rate of Rs. 55 per kg. If he sold all the oranges at the rate of Rs. 60 per kg, what would be the profit percentage?

6. Sukman bought 50 bulbs at the rate of Rs 40 to each bulb. When he opened the packet of bulbs, 15 bulbs were broken. What would be the profit or loss? Find it in percentage.
7. When a book was sold to a bookseller for Rs. 575, the profit was 15 %. How much money was paid for the book? Find out the actual profit amount.
8. Rajendra bought a refrigerator for Rs 32,500. He spent Rs. 500 to bring the goods. Find out the profit percentage if he sold the refrigerator for Rs 33,500.
9. Aaitman bought the old house for Rs. 95,00,000. He spent Rs. 200,000 for house repairing. If the house was sold for Rs. 1,10,00,000, what would be the profit or loss, find in percentage.
10. Basmati bought 20 dozen bananas at the rate of Rs. 120 per dozen. Among them, 3 dozen bananas were spoiled. Find out at what rate the rest of the bananas should be sold to get 10 % profit.

Project Work

Note down the price of any five items from two different shops. What is the percentage of profit you got from which shop? Present the findings in the classroom.

Answer

- | | | |
|-----------------------|-------------------|---------------------------|
| 1. (a) Profit: 16.66% | (b) Loss: 9.09% | (c) Profit: 10.28% |
| (d) Loss 45% | (e) Profit 25% | |
| 2. Loss: 4.91% | 3. Rs. 3,000 | 4. Rs. 33,000 |
| 5. 9.09% | 6. Profit: 5% | 7. Rs. 500, Rs. 75 |
| 8. 1.51% | 9. Profit: 13.40% | 10. Rs. 155.30 per dozen. |

8.0 Review

Laxmi bought 30 bananas for Rs. 225 and brought them home. These bananas are not enough for the guests who have come to the house. So, 12 bananas have to be added again.

- (a) How much money does she need to buy 12 bananas now?
- (b) What should be done to find out the price of 1 banana?
- (c) Discuss how to find out the price of 12 bananas.

The rule of computing the value of the same number of objects on the basis of given value of one unit of object and finding the value of a object from the values of the same number of objects is called unitary method.

8.1 Direct Variation

Activity 1

The number of copies and their price are given in the table below, fill in the prices based on the given price in the table below:

Number of copies	1	2	3	4	5	6
Total price (Rs).	50	100	150			

Discuss the relationship between the number of copies and the price.

From the above table, as the number of copies increases, the price of copy is also increasing and as the number of copies decreases, the price of copy is also decreasing. Therefore, there is direct variation between the number of copies and the price.

When one of the two quantities decreases or increases, the other quantity decreases or increases in the same proportion, then the relation of those quantities is called direct variation.

Example 1

If the price of 3 copies is Rs. 270, then what is the price of 5 copies?

If you know the price of 3 copies, how to calculate the price of 5 copies?



Listen! Find the price of one copy from the price of the first 3 copies. Then the price of each copy can be found.



Solution

Here, the price of 3 copies = Rs. 270

$$\text{The price of 1 copy} = \frac{\text{Rs. } 270}{3} = \text{Rs. } 90$$

$$\begin{aligned}\text{The price of 3 copies} &= \text{Rs. } 90 \times 5 \\ &= \text{Rs. } 450\end{aligned}$$

Next method:

This problem can be solved by using ratios too.

Numbers of copies	Price (Rs.)
3	270
5	x (Let)

Since there is a direct variation between the number of copies and their price, it can be written in proportion as follows.

$$\text{So, } \frac{3}{5} = \frac{270}{x}$$

$$\text{or, } 3x = 270 \times 5$$

$$\text{or, } x = \frac{270 \times 5}{3}$$

$$\therefore x = 450$$

Since both variables decrease and increase in direct variation, this ratio can be written as

$$\frac{5}{3} = \frac{x}{270}$$

Therefore, the price of 5 copies is Rs. 450.

8.2 Indirect Variation

Activity 2

Discuss the question by observing the table given below:

Number of workers	1	2	5
Time to complete the work	10	5	2

What is the relationship between the number of workers and the number of days to complete the work?

In the above table, the number of working days seems to decrease as the number of workers increases. This is considered to be indirect variation.

When one of the two quantities decreases, the other quantity increases in the same proportion, and when one quantity decreases, the other quantity increase in the same proportion, then the relation between those quantities is said to be indirect variation.

Example 2

If 12 men can dig a field in 20 days, how many days does it take for 8 men to dig the same field?

Solution

There are 8 men out of 12 men. (The number of men has been decreased. So it takes a long time to complete the work.)

12 men can dig a field in 20 days.

1 man can dig a field in 20×12 days.

8 men can dig a field in $\frac{20 \times 12}{8} = 30$ days.

\therefore 8 men can dig a field in 16 days.

Next method:

Here

Numbers of men	Working days
12	20
8	x (Let)

There is indirect variation between the working day and the number of men. Therefore, if there are few men, many working days are needed and if there are many men, few working days are needed.

$$\text{So, } \frac{20}{x} = \frac{8}{12}$$

$$\text{or, } 8x = 20 \times 12$$

$$\text{or, } x = \frac{20 \times 12}{8}$$

$$\text{or, } x = 30$$

Being indirect variation, it can also be written $\frac{x}{20} = \frac{12}{8}$

\therefore 8 men can dig a field in 16 days.

Exercise 8

1. Tick (v) if the following statements are correct and cross (x) if they are incorrect.
 - (a) If the number of items increases, so price of the items also increases. It is a direct variation.
 - (b) There is a direct variation between the number of people and time taken for the work.
 - (c) There is a direct variation between taken time for a bus to travel a certain distance and the distance traveled by the bus.
 - (d) There is indirect variation between the water filling capacity of the drinking water pipe and the time taken to fill.
 - (e) There is a direct variation between the number of hands and fingers.
2. If the costs of 5 kg sugar is Rs. 450, then what is the cost of 3 kg sugar?
3. A motorcycle can travel 320 kilometers with 8 liters of petrol. How many liters of petrol is required to travel 50 kilometers?
4. If 15 people take 16 days to dig a field, how many days will it take for 8 people to dig the same field?
5. 30 men can plant the crops in 17 days. If Amar wants to complete the same work in 10 days, how many workers will be needed?
6. If a machine of the industry can fill 6600 bottles of beverages in 3 hours, how many bottles can it fill in 8 hours?

- Rama buys 3 kg lentil and 2 kg sugar for Rs 540. If the cost of 1 kg sugar is Rs. 90, what is the cost of 1 kg lentil?
- If a cyclist can travel the distance of 15 kilometers per hour, how many meters per minute would his speed be?
- In a camp, 50 people have enough food for 54 days. How many days will the food last for 60 people?
- The weight of rice and its cost is shown in the table below. Fill in the blanks based on its weight and cost. (Also show the Procedure.)

S. N.	Weight of rice (kg)	Cost of rice (Rs.)
(a)	10	1250
(b)	1	
(c)		375
(d)	9	
(e)		3125

Project Work

Write down the five conditions of direct and indirect variation that are used in our daily life by searching with your senior or from the internet. Present them in the classroom.

Answer

- Show the answer to your teacher.
- Rs. 270
- $1.25 l$
- 30 days
- .51
- 17600
- Rs. 120
- 250 minutes
- 45 days
- (a) Rs.125 (b) 3 (c) 1125 (d) 25

Miscellaneous Exercise

- 1. In a programme hall, 400 chairs are arranged in rows and columns in the square form.**
 - (a) How many chairs are there in each row?
 - (b) If 2 / 2 chairs are added in each row and column, how many chairs are required to re-arrange in the square form?
 - (c) What percentage of seats were added to each row and column?
 - (d) What percentage of chairs needs to be added to arrange in the square form?
- 2. The road traffic lights at three different places are changing every 48 seconds, 72 seconds and 108 seconds respectively. If they all change together at 9 o'clock in the morning, find out when they will change the next time again?**
- 3. Bimala bought 150 eggs in five crates and brought them in cartons. When she came home, she opened the carton of eggs and found 30 eggs were broken.**
 - (a) If she sells the remaining eggs each at Rs. 15, she will loss 10 %, how much did buy the 150 eggs?
 - (b) If 5 % profit to get, at what rate should the remaining eggs be sold?
 - (c) If only 25 eggs were broken, how much would Bimala have to sell each egg to get 5 % profit?
- 4. A school building is being painted. 6 workers can paint 520.2 meters of wall in one hour.**
 - (a) How many meters of wall can 7 workers paint in one hour?
 - (b) How many hours does it take for 7 workers to paint the 3641.4 meter wall?

- 5.** *a, b, and c are three integers. If $a = -25$, $b = 8$ and $c = -4$ then prove that:*
- $a + (b + c) = (a + b) + c$
 - $a \times (b + c) = ab + ac$
- 6.** *Find any two integers:*
- Whose sum is -5 .
 - Whose difference is -7 .
 - Whose difference is 0 .
- 7.** Angel scored 150 marks in the first terminal examination out of 200 full marks. Roshani scored 180 marks out of 300 full marks in this exam. Compare who gets better results.
- 8.** *In the entrance exam, there were a rule to provide (+5) for each correct answer, (-2) for each wrong answer and (0) for failing to answer.*
- Ram solved 7 questions in total, out of which 4 are with correct answers and 3 are with wrong answers. How many marks did he get in total?
 - Ruchita solved the 8 questions in total, out of which 4 are correct and 4 are wrong. How many marks did she get in total?
 - Who gets more marks? And how many does he/she get more? Find out.
- 9.** *In a quiz contest of the school rules were made to provide (+5) for each correct answer and (-2) for each wrong answer.*
- The Red House got 30 marks in which 10 questions were answered incorrectly. Find out how many questions have been answered correctly.
 - The Green House got (-12) marks out of which 16 questions were answered incorrectly. Find out how many questions have been answered correctly.
 - Which group answered the most questions correctly? How many more questions did the group answer than the group who answered fewer? Write.

10. On the first day, Anuja read $\frac{1}{5}$ part of a book. She read 40 pages on the second day. If she read $\frac{7}{10}$ parts of the book in two days, find out how many pages does the book have in total.

11. Write any two rational numbers which lie between $\frac{1}{4}$ and $\frac{2}{5}$.

Answer

9.0 Review



The solid objects given above are of different sizes. Discuss the following questions in appropriate groups about their size, length of the sides, area and length of the surrounding edges.

- Write the name of each shape.
- Find the perimeter of the upper surface of each solid object.
- How can the surface area of each solid object be calculated?

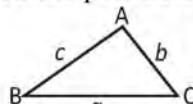
9.1 Direct Variation

Activity 1

Sit in groups with appropriate numbers. Measure the length of the three edges of a set square in your geometry box. Find out the perimeter of the set square.



The sum of the edges around the triangle is called the perimeter of the triangle.



Here, perimeter of $\triangle ABC$,
 $P = BC + CA + AB$
or, $p = a + b + c$



Example 1

Find the perimeter of the given triangle:

Solution

Here, side of triangle (AB) = 8 cm

Side of triangle (BC) = 12 cm

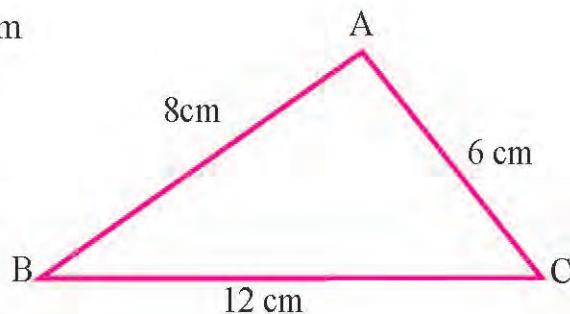
Side of triangle (AC) = 6 cm

Perimeter of the triangle (P) = ?

Now, $P = AB + BC + CA$

$$= 8 \text{ cm} + 12 \text{ cm} + 6 \text{ cm}$$

$$= 26 \text{ cm}$$



Therefore, perimeter of the triangle (P) = 26 cm

Example 2

If Anupama bought triangular land having its measurement as 15 m, 17 m and 12 m, find out how long wall should she build around it.

Solution

Here, first edge of the land (a) = 15 m

Second edge of the land (b) = 17 m

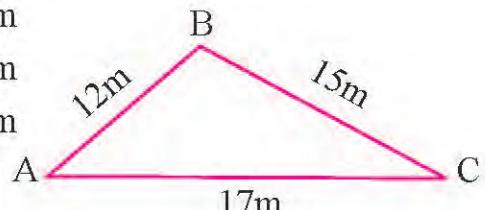
Third edge of the land (c) = 12 m

Perimeter of the land (P) = ?

Now, $P = a + b + c$

$$= (15 + 17 + 12) \text{ m}$$

$$= 44 \text{ m}$$



The length of the wall around the land is 44 meters.

Example 3

Find out the length of the second edge of the triangular park whose total length around it is 400 meters and the length of the first edge is 120 m and the length of the third edge is 180 m.

Solution

Here, perimeter of the park (P) = 400 m

First edge of the park (a) = 120

Third edge of the park (c) = 180 m

Second edge of the park (b) = ?

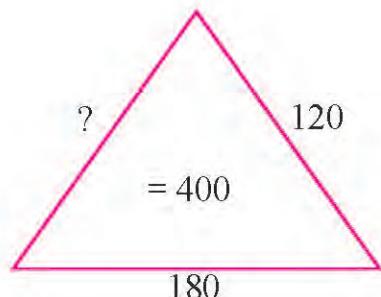
Now, $P = a + b + c$

$$\text{or, } 400 = 120 + b + 180$$

$$\text{or, } 400 - 300 = b$$

$$\text{or, } b = 100 \text{ m}$$

Therefore, the length of the second edge of the park is 100 m



Example 4

Find out how much Diya spent if she used wire costing Rs. 20 per meter for fencing 5 times in triangular vegetable field which has the length edges of 10 m, 12 m and 14 m.

Solution

The first edge of the vegetable field (a) = 10 m

The second edge of the vegetable field (b) = 12 m

The third edge of the vegetable field (c) = 14 m

Now, $P = a + b + c$

$$= (10 + 12 + 14) \text{ m} = 36 \text{ m}$$

The length of the wire used to enclose the fence at a time = 36 m

\therefore Length of wire used for fencing at 5 times = $36 \times 5 \text{ m} = 180 \text{ m}$

Cost of the wire per meter = Rs. 20

Cost of the wire of 180 m = 180×20 m = Rs. 3,600

Therefore, she spent Rs. 3,600 for fencing with wire.

Example 5

The shape as shown in the given picture is made by using small sticks. If $AD + BE = 7$ cm, and $AB = BC = CA = 7.5$ cm, then find the total length of the sticks.

Solution

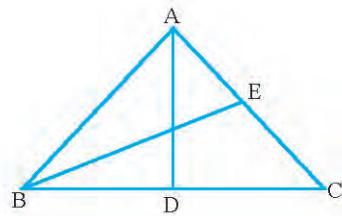
Here,

$AD = BE = 7$ cm and $AB = BC = CA = 7.5$ cm

Total length of the sticks

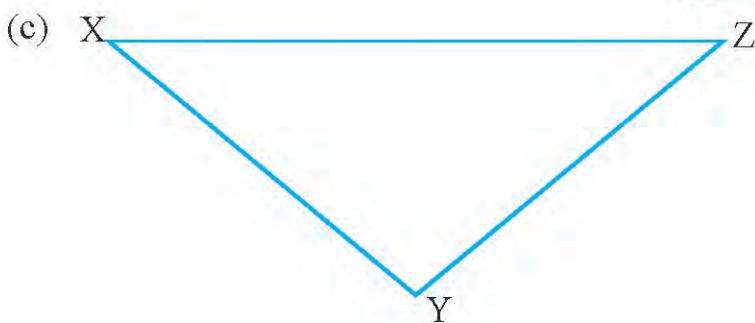
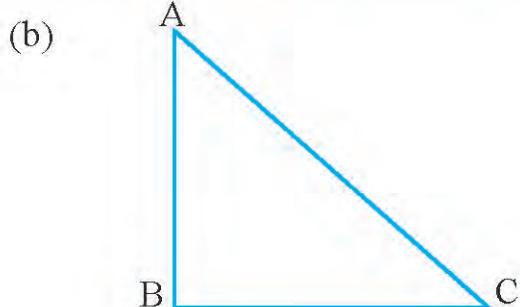
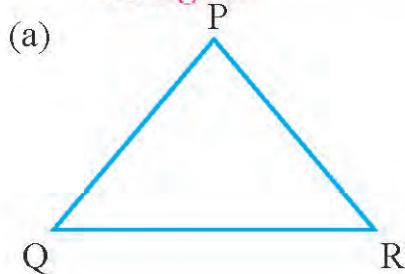
$$\begin{aligned} &= AB + BC + CA + AD + BE \\ &= 7.5 + 7.5 + 7.5 + 7 + 7 \\ &= 36.5 \text{ cm} \end{aligned}$$

∴ Total length of these sticks is 36.5 cm.

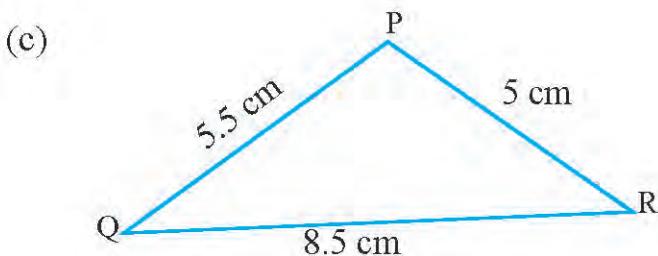
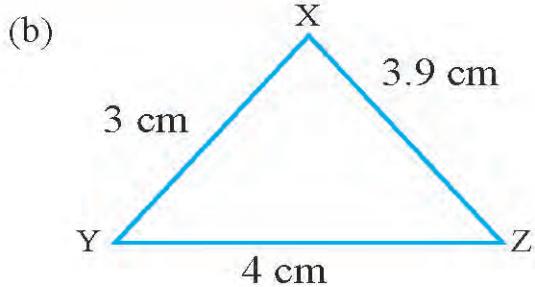
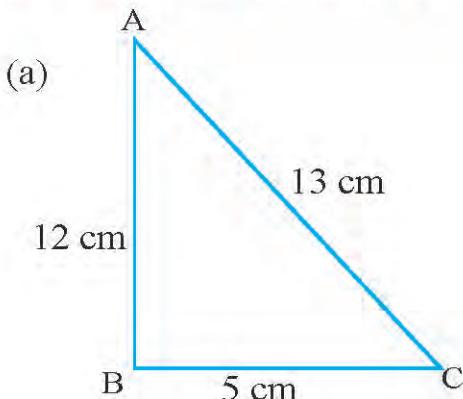


Exercise 9.1

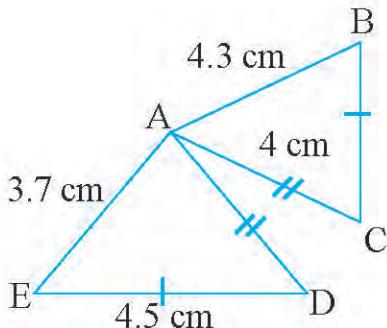
1. Find the perimeter by measuring the sides of the following triangles:



2. Find the perimeter of the following triangles:

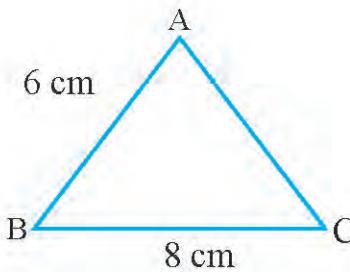


3. Find the perimeter of the following triangles:

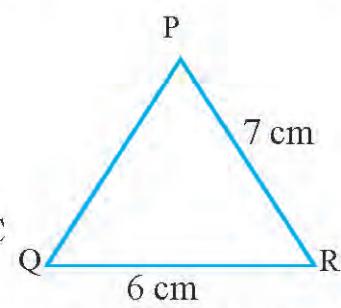


4. When Ram makes a paper triangle with the length of 40 cm around it, find the length of the third side. If the two sides are 14 cm and 16 cm respectively.
5. Find out how many times it can be fenced with a 540 m long wire to an equilateral triangular land with 18 meters side.

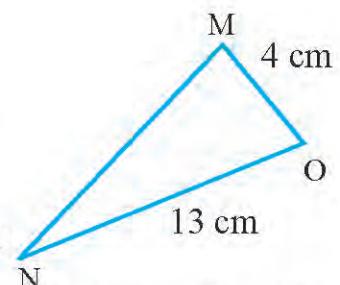
6. Among the three edges of a triangular garden, the length of the first edge is 12 ft and the length of the other two edges of equal size is 7 ft. How much does it cost to enclose the triangular garden 20 times if a plastic rope costs 60 paisa per foot?
7. Find the unknown length of the following triangles.



$$\text{Perimeter (P)} = 19 \text{ cm}$$



$$\text{Perimeter (P)} = 23 \text{ cm}$$



$$\text{Perimeter (P)} = 28 \text{ cm}$$

Project Work

Set the three wooden sticks upright at some distance on the school's ground and make a triangular shape using the rope. Find the perimeter of the shape and present it in the classroom.

Answer

1. Show answers to your teacher
2. (a) 30 cm (b) 10.9 cm (c) 19 cm
3. 12.2 cm, 12.8 cm
4. 10 cm 5. 10 times 6. Rs. 312
7. (a) 5 cm (b) 10 cm (c) 11 cm

9.2 Cuboid and Cube

9.2.1 Total Surface Area of Cuboid

Activity 1

Take a cuboid and separate its length, breadth and height. Discuss the rectangular surfaces formed in the cuboid. The rectangular surfaces in the cuboid are shown ABCD, ABGF, ADEF, BCMG, CDEM, GFEM respectively.

Here

$$\begin{aligned}\text{Area of rectangle } ABCD (A_1) &= CD \times AD \\ &= b \times h = bh\end{aligned}$$

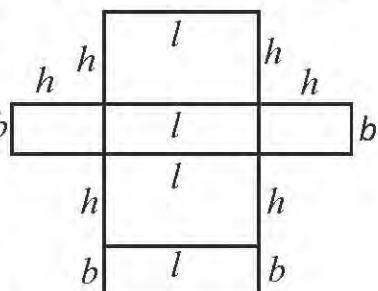
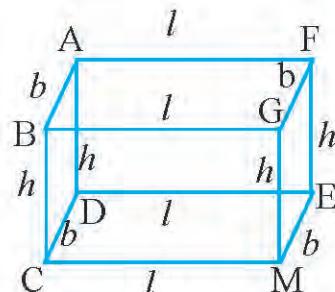
$$\begin{aligned}\text{Area of rectangle } ABGF (A_2) &= AB \times AF \\ &= b \times l = lb\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } ADEF (A_3) &= AD \times AF \\ &= h \times l = hl\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } BCMG (A_4) &= BG \times BC \\ &= l \times h = lh\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } CDEM (A_5) &= CD \times DE \\ &= b \times l = bl\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } GFEM (A_6) &= GF \times GM \\ &= b \times h = bh\end{aligned}$$



$$\begin{aligned}\text{Hence, total surface area of cuboid } (A) &= A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \\ &= bh + lb + lh + lh + lb + bh \\ &= 2lb + 2bh + 2lh \\ &= 2(lb + bh + lh)\end{aligned}$$

The total surface area of cuboid is $= 2(lb + bh + lh)$

Example 1

Find the total surface area of a cuboid having 4 cm length, 3 cm breadth and 2 cm height.

Here, length (l) = 4 cm

breadth (b) = 3 cm

height (h) = 2 cm

Total surface area (A) = ?

Now, total surface area of a cuboid (A) = $2(lb + bh + lh)$

$$= 2(4 \times 3 + 3 \times 2 + 4 \times 2)$$

$$= 2(12 + 6 + 8)$$

$$= 2 \times 26$$

$$= 52 \text{ cm}^2$$

Therefore, total surface area of cuboid is 52 cm^2 .

Example 2

If a geometry box has length (l) = 15 cm, breadth (b) = 7 cm and height (h) = 3 cm then find the total surface area of the box.

Solution

Here, length (l) = 15 cm

breadth (b) = 7 cm

height (h) = 3 cm

Total surface area of the geometry box (A) = ?

Now, according to formula,

Total surface area of the geometry box (A) = $2(lb + bh + lh)$

$$= 2(15 \times 7 + 7 \times 3 + 15 \times 3)$$

$$= 2(105 + 21 + 45)$$

$$= 345 \text{ cm}^2$$

∴ Total surface area of the geometry box is 345 cm^2 .

Example 3

Find the breadth of a box having its length of 42 cm, height of 28 cm and total surface area of 7812 cm².

Solution

Here, length (l) = 42 cm

breadth (b) = ?

height (h) = 28 cm

Total surface area of the box (A) = 7812 cm²

Now, according to formula,

Total surface area of the box (A) = $2(lb + bh + lh)$

$$\text{or, } 7812 = 2(42b + 28b + 42 \times 28)$$

$$\text{or, } 7812 = 2(70b + 1176)$$

$$\text{or, } 7812 = 140b + 2352$$

$$\text{or, } 7812 - 2352 = 140b$$

$$\text{or, } \frac{5460}{140} = b$$

$$\text{or, } b = 39 \text{ cm}$$

Therefore, breadth of the box is 39 cm.

9.2.2 Total Surface Area of a Cube

Activity 2

Take a dice and measure the length of the edge of the dice. You will get the length of all its equal sides.

Here is length (l) = breadth (b) = height (h).

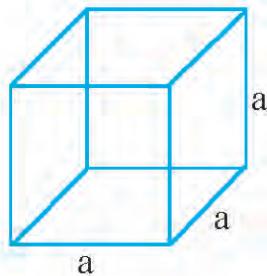
A cuboid having equal length, breadth and height is called a cube.

Now, let $l = b = h = a$

Total surface area of cuboid (A) = $2(lb + bh + lh)$

$$\begin{aligned}
 &= 2(a \cdot a + a \cdot a + a \cdot a) \\
 &= 2(a^2 + a^2 + a^2) \\
 &= 6a^2
 \end{aligned}$$

Therefore, total surface area of cube is $(A) = 6a^2$ square unit.



Next method

Area of a surface of the cube $= a \times a = a^2$

Area of 6 surface of the cube $= 6a^2$

\therefore Total surface area of cube $(A) = 6a^2$

Example 1

The length of one edge of a solid paper toy is 3.5 cm. Find the surface area of the toy.

Solution

Here, length of a side of cube (a) $= 3.5$ cm

Total surface area of the box (A) $= ?$

Now, according to formula,

Total surface area of the cube (A) $= 6a^2$

$$= 6 \times (3.5)^2$$

$$= 6 \times 12.25$$

$$= 73.50 \text{ cm}^2$$

\therefore Total surface area of the toy is $(A) = 73.50 \text{ cm}^2$

Example 2

If the surface area of a cubical solid is 54 m^2 , find the length of one edge of the solid.

Solution

Total surface area of the cubical solid (A) = 54 m^2

Length of a edge (a) = ?

Now, according to formula, total surface area of the box (A) = $6a^2$

$$\text{or, } 54 = 6a^2$$

$$\text{or, } \frac{54}{6} = a^2$$

$$\text{or, } 9 = a^2$$

$$\text{or, } a = \sqrt{9}$$

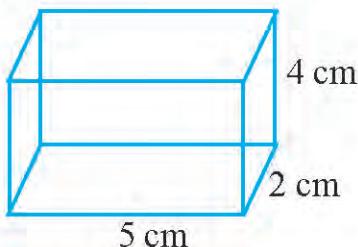
$$\text{or, } a = 3 \text{ m}$$

∴ The length of one edge of the solid is (a) = 3 m

Exercise 9.2

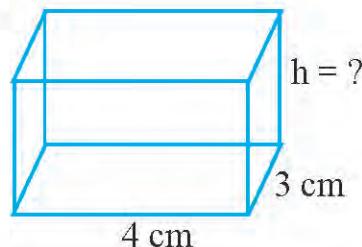
1. Find the unknown edge or surface area of each solid object given below:

(a)

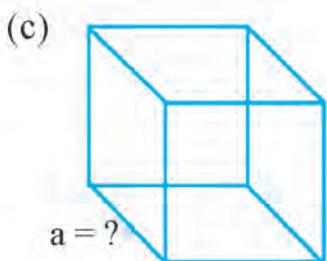


Total surface area (A) = ?

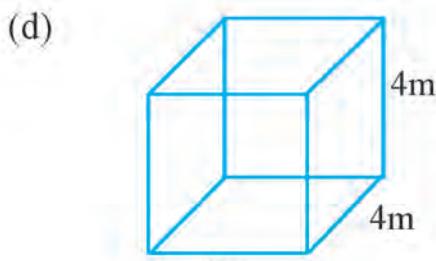
(b)



Total surface area (A) = 45 cm^2



$$\text{Total surface area } (A) = 216 \text{ cm}^2$$



$$\text{Total surface area } (A) = ?$$

2. (a) If the cuboid has length (l) = 10 cm, breadth (b) = 8 cm and total surface area (A) = 376 cm^2 , find the height (h) of the cuboid.
- (b) The total surface area of one cartoon biscuit is 9400 cm^2 . If its length is 50 cm and breadth is 30 cm, find its height.
3. (a) If the total surface area of a solid is 726 cm^2 , find the length of one edge of the solid. What is the area covered by the solid on the ground, find out.
- (b) If the total surface area of a solid is 864 cm^2 , find the length of the side of the solid. When removing its lid, find out how much area can be painted on the outside of it.
4. **A duster having 7 cm length and 8 cm breadth is placed on the table and covered 80 cm^2 of the surface of the table.**
 - a) Find the length of the duster.
 - b) Find the total surface area of the duster.
5. **Measure the length of the edges of the surface of cuboid and cubical solids as given below and find the total surface area of the solid**



Project Work

Make a cuboid from the chart paper. Find the total surface area by measuring the edge of the cuboid and present it in the classroom.

Answer

1. (a) 76 cm^2 (b) 1.5 cm (c) 96 cm^2 (d) 6 cm
2. (a) 6 cm (b) 40 cm
3. (a) 11 cm, 121 cm^2 (b) 12 cm, 720 cm^2
4. (a) 10 cm, 412 cm^2
5. Show answers to your teacher.

9.3 Volume of Cuboid and Cube

Activity 1

If the edge of a cuboid shape container have the length (l) = 15 cm, breadth (b) = 14 cm and height (h) = 13 cm, how much colors can this colour pot contain? Discuss with your classmates.

Here, length (l) = 15 cm,

$$\text{breadth } (b) = 14 \text{ cm}$$

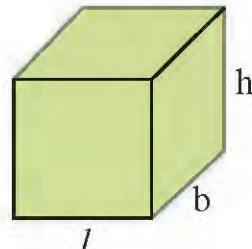
$$\text{height } (h) = 13 \text{ cm}$$

$$\text{Now, Volume } (V) = l \times b \times h$$

$$= (15 \times 14 \times 13)$$

$$= 2730 \text{ cm}^3$$

\therefore It contains 2730 cm^3 .



Activity 2

The length of an edge of a cubical pot of rice is 60 cm. How much rice does this pot contain? Discuss with classmates.

Here, since all the edges of a cubical pot are equal, each side is 60 cm in length, breadth and height.

Now, capacity of this pot = length × breadth × height

$$= 60 \times 60 \times 60$$

$$= 216000 \text{ cm}^3$$

Therefore, the volume of this pot (V) = 216000 cm^3

Hence, the volume of cubical solid object is $(V) = l \times l \times l$

$$V = l^3$$

Example 1

The length, breadth and height of a cuboid shape sugar pot is 3 m, 2 m and 1m respectively. Find out how much sugar this sugar pot contains.

Solution

Here, length (l) = 3 m,

breadth (b) = 2 m

height (h) = 1 m

$$\begin{aligned}\text{Now, Volume of the pot (V)} &= l \times b \times h \\ &= 3 \times 2 \times 1 \\ &= 6 \text{ m}^3\end{aligned}$$

∴ It contains 6 m^3 sugar.

Example 2

A rectangular tank contains 600 l of water. Find the breadth if the length of the tank is 200 cm and height is 5 cm.

Solution

Here, volume (V) = 600 l

$$= \frac{600}{1000} = 0.6 \text{ m}^3$$

$$\left[\because 1 \text{ l} = \frac{1}{1000} \text{ m}^3 \right]$$

length (l) = 200 cm = 2 m

height (h) = 50 cm = 0.5 m

breadth (b) = ?

Now, according to formula (V) = l × b × h

$$\text{or, } 0.6 = 2 \times b \times 0.5$$

$$\text{or, } 0.6 = b$$

$$\text{or, } b = \frac{6}{10} = 0.6 \text{ m} = 60 \text{ cm}$$

$$\text{or, } b = 60$$

Therefore, length of the container is 60 cm.

Example 3

The length of cuboid is three times of its breadth. If its height and volume are 8 cm and 864 cm^3 respectively, then find its total surface area.

Solution

Here, volume (V) = 864 cm^3

height (h) = 8 cm

let, breadth (b) = x cm

\therefore length (l) = $3x$ cm

Now, according to formula, $V = l \times b \times h$

$$\text{or, } 864 = 3x \times x \times 8$$

$$\text{or, } 864 = 24x^2$$

$$\text{or, } x^2 = \frac{864}{24} = 36$$

$$\text{or, } x = 6$$

$$\text{or, } b = 6 \text{ cm}$$

$$\text{or, } l = 3x = 3 \times 6 = 18 \text{ cm}$$

Now, the total surface area $= 2(lb + bh + lh)$

$$= 2(18 \times 6 + 6 \times 8 + 18 \times 8)$$

$$= 2(108 + 48 + 144)$$

$$= 600 \text{ cm}^2$$

Therefore, total surface of the cuboid is 600 cm^2 .

Example 4

If the length of one edge of a cubical water tank is 1.5 m, then find out how many liters of water it can hold.

Solution

Here, the length of one edge of a cubical water tank (a) = 1.5 m

Volume (V) = ?

Now, according to formula (V) = a^3

$$= (1.5)^3$$

$$= 3.375 \text{ m}^3$$

$$= 3.375 \times 1000 \text{ l}$$

$$\boxed{\therefore 1 \text{ m}^3 = 1000 \text{ l}}$$

$$\therefore V = 3375 \text{ l}$$

Therefore, this pot holds 3375 l water.

Example 5

A cubic tank holds 216,000 liters of water. Find the surface area of its surface.

Solution

Here, volume of cube (V) = 216000 l

$$= \frac{216000}{1000} \text{ m}^3 = 216 \text{ m}^3$$

$$\left[\because 1 \text{ l} = \frac{1}{1000} \text{ m}^3 \right]$$

Length of the side of cube (a) = ?

Now, according to formula (V) = a^3

$$\text{or, } 216 \text{ m}^3 = a^3$$

$$\text{or, } a = \sqrt[3]{216}$$

$$\text{or, } a = 6 \text{ m}$$

Now, total surface area (A) = $6a^2$

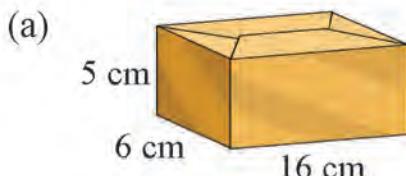
$$= 6 \times 6^2$$

$$= 216 \text{ m}^2$$

Therefore, the total surface area of the cube is 216 m².

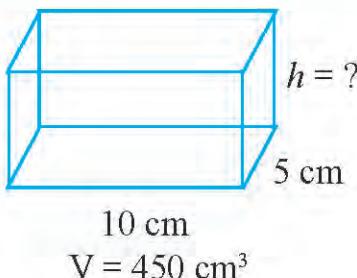
Exercise 9.3

1. Find the volume of the solid objects given below:

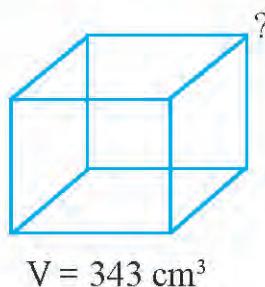


2. Find the unknown edges in the solid objects given below:

(a)



(b)



3. A room is 5 m long, 4 m wide and 3 m high. Find the volume of the room.
4. The length of a seminar hall is twice its height. If its length is 8 m and volume 576 m^3 . Find the total surface area of the room.
5. The length of one edge of a dice is 9 cm. Find the volume of the dice.
6. (a) The volume of a cubical box is 512 cm^3 . Find the total surface area of the box.
 (b) The volume of a cubical box is 125 cm^3 . Find the length of an edge and total surface area of the solid.
7. **A rectangular piece of gold measuring 4 cm in length, 2 m in width and 1 cm in height is melted down to a cube then**
 - (a) Find the volume of a cubical piece of gold.
 - (b) Find the total surface area of a cubical piece of gold.
8. The milk pot is 32 cm long, 16 cm wide and 8 cm high. How many times does it take to make it empty when it is pulled out by a 8 cm long cubical pot? Find out.
9. (a) When constructing a rectangular tank containing 64,000 l of water for a village, the length is 8 m and breadth 4 m. Find out, how deep should the tank be made?
 (b) When constructing a cubical tank of water with a capacity of 216,000 l for a school, what is the area under the base of the water tank?

Project Work

Find the volume of cubical solid by collecting from around your home and discuss the results of your work in the classroom.

Answer

- | | | |
|---------------------------|---------------------------|--|
| 1. (a) 480 cm^3 | (b) 26.25 cm^3 | 2. (a) 9 cm |
| (b) 7 cm | | 3. 60 m^3 4. 432 m^2 |
| 5. 729 cm^3 | 6. (a) 384 cm^2 | (b) 5 cm, 150 cm^2 |
| 7. (a) 8 cm^3 | (b) 24 cm^2 | 8. 8 times |
| 9. (a) 2 m | (b) 36 m^2 | |

9.4 Relation between Circumference and Diameter of Circle and its Uses

Activity 1

Take a coin and tie the coin around with a thread and measure the length of the thread or turn a coin on scaled surface one round and measure its distance.



Now, find the distance between the point A and B in the picture.

Here, the distance between points A and B = 2 cm

Length of the thread (l) = 6.28 cm

Now dividing the length of the thread CD by AB is $\frac{6.28}{2} = 3.14$

Dividing the circumference of both by their diameter becomes approximately 3.14.

or, $c = \pi d = 2\pi r$ [\because diameter (d) = $2 \times$ radius.]



Similarly, take a bangle. Find its diameter and circumference and its ratio.

Is the value of the circumference of the coin divided by its diameter equal to the value of the circumference of the coin divided by its diameter?

3.14 is a constant value. It is denoted by Greek letter ' π '.

Therefore, $\frac{c}{d} = \pi$

Example 1

If the diameter of a circle is 14 cm, then find the radius and circumference of the circle. ($\pi = \frac{22}{7}$)

Solution

Here, diameter of the circle (d) = 14 cm

$$\therefore \text{Radius of the circle (r)} = \frac{d}{2}$$

$$\begin{aligned} &= \frac{14}{2} \\ &= 7 \end{aligned}$$

$$\because d = 2r$$

Now, circumference of the circle (c) = $2\pi r$

$$\begin{aligned} &= 2 \times \frac{22}{7} \times 7 \\ &= 44 \text{ cm} \end{aligned}$$

Therefore, radius of the circle = 7 cm and length of the circle = 44 cm.

Example 2

The circumference of a circular ring of gold is 7 cm. Find the diameter of the ring. ($\pi = \frac{22}{7}$)

Solution

Here, circumference of the circle (C) = 7 cm

Diameter of the circle (d) = ?

According formula,

$$c = \pi d$$

$$\begin{aligned}d &= \frac{c}{\pi} \\&= \frac{7}{\frac{22}{7}} \\&= \frac{7 \times 7}{22} \\&= 2.23 \text{ cm}\end{aligned}$$

Therefore, the diameter (d) = 2.23 cm

Example 3

The diameter of a circular fish pond is 56 m. Find out how long wires it takes to wrap it around 5 times. ($\pi = \frac{22}{7}$)

Solution

Diameter of the fish pond (d) = 56 m

Circumference of the fish pond (C) = ?

According to the formula,

$$\begin{aligned}c &= \pi d \\&= \frac{22}{7} \times 56 \text{ m} \\&= 176 \text{ m}\end{aligned}$$

Therefore, radius of the circle = 7 cm and length of the circle = 44 cm.

The wire needed to encircle the barbed wire once = 176 m

$$\therefore \text{Wire required to enclose barbed wire 5 times} = 176 \times 5 \text{ m} \\ = 880 \text{ m}$$

Therefore, length of the wire is 880 m.

Example 4

When Ram crosses a distance of 440 meters from a bicycle, the wheel turns 100 times. Find the radius of the wheel. ($\pi = \frac{22}{7}$)

Solution

Here, the distance covered by turning the wheel 100 times = 440 m

Radius of the wheel (r) = ?

$$\text{Circumference of the wheel (C)} = \frac{440}{100} = 4.4 \text{ m} = 4.4 \times 100 \text{ cm} = 440 \text{ cm.}$$

Now, according to the formula,

$$C = \pi d$$

$$\text{or, } 440 = \frac{22}{7} \times d$$

$$\text{or, } d = \frac{440 \times 7}{22}$$

$$\text{or, } 2r = 140$$

$$\text{or, } r = \frac{140}{2} = 70 \text{ cm}$$

Therefore, radius of the circle = 70 cm

Exercise 9.4

1. Find the circumference of the circle based on the measurements given below: ($\pi = \frac{22}{7}$)
(a) $r = 3.5$ cm (b) $r = 49$ cm (c) $r = 10.5$ cm
(d) $d = 70$ m (e) $d = 17.5$ cm (f) $d = 56$ m
2. Find the radius of the circle from the length of the circumference of the circle given below: ($\pi = \frac{22}{7}$)
(a) $c = 176$ cm (b) $c = 308$ cm (c) $c = 616$ cm
(d) $c = 660$ m (e) $c = 242$ cm (f) $c = 330$ m
3. (a) If a goat tied with a 14 meter long rope stretches the rope and walks around, how far does the goat walk in 5 times? Find out.
(b) How many meters does the wheel of a 77 cm diameter of car travel at 50 times? Find out.
4. (a) How many times does the wheel of a bicycle with a radius of 35 cm have to be rolled to cover a distance of 44 meters? Find out.
(b) Find out how many times can cover a distance of 17.6 km by running around in a circular pond with a diameter of 140 meters.
5. (a) If the wheel of a bus travels a distance of 44 meters by rolling 20 times, find the radius of the wheel.
(b) Find out the approximate diameter of the pond if Sita walks a distance of 1 km 980 m while walking 15 times around a circular pond.

Project Work

Find the length of the circumference by measuring the diameter of the circular objects around your house. Also find the ratio of circumference and diameter and discuss the results in the class.

Answer

- | | | | |
|---------------|------------|-----------|-----------|
| 1. (a) 22 cm | (b) 308 cm | (c) 66 cm | (d) 220 m |
| (e) 55 m | (f) 176 m | | |
| 2. (a) 28 cm | (b) 49 m | (c) 98 m | (d) 105 m |
| (e) 38.5 cm | (f) 52.5 m | | |
| 3. (a) 440 m | (b) 121 m | | |
| 4. (a) 20 k6s | (b) 40 k6s | | |
| 5. (a) 35 cm | (b) 42 cm | | |

Miscellaneous Exercise

- If Ram bought a triangular piece of land with 30 feet, 24 feet and 27 feet edges, find out the length around the land.
- If the perimeter of an equilateral triangle is 23.25 cm, find the length of the one side.
- Ram bought a 440 meter long net for Rs 35,200. He used the net to fence his triangular land where the length of the three sides of the land is 9 m, 8 m and 5 m respectively. How many times has he surrounded the land? Find out how much it costs to fence the once.
- (a) If a cubical tank holds 343,000 l of water, then calculate the total surface area of the tank.
(b) If the total surface area of a cubical tank is 1776 m^2 , find out how many liters of water it can hold.

5. The length of the cuboid is double of its breadth. If its height is 10 cm and volume is 8000 cm^3 then find the total surface area.
6. If the length of a room is 8 m and height is 4 m, total surface area is 304 m^2 , find the volume of the room.
7. The length of a cuboid is twice its breadth. If the height of the cuboid and the area of the total surface area are 10 cm and 736 cm^2 respectively. Find the volume of the cuboid.
8. **The length of a room is three times of its breadth. If the height of the room is 2 m and the volume is 96 m^3 then,**
 - (a) Find the length and breadth of the room.
 - (b) Find the total surface area of the room.
9. A 44 meter long wire is bent to form a triangle. If the length of the two edges is 16 m and 17 m, find the length of the remaining edge.
10. If the total surface area of a cube is 600 cm^2 , find the volume of the cube.
11. The volume of a cube is 1728 cm^3 . The total surface area of a cube is equal to the total surface area of the cuboid. Find the length, if the cuboid is 8 cm and the breadth is 12 m.
12. The total surface area of a cubical piece of silver is 150 cm^2 . If it is melted to make a cuboid with 5 cm breadth and 4 cm height, find the length of the cuboid.
13. Find the radius of the wheel, if the wheel of a truck travels a distance of 2.2 km in 400 times.
14. A rectangular tank filled with water has 1 m length, 90 cm breadth and 60 cm height. Find out, how many times the tank becomes completely empty when the water from the tank is pumped out by a cubical vessel with a 30 cm edge.

15. If a room is 8 meters long, 6 meters wide and 2 meters high,

- Find out how much it costs to paint the total surface of the room at the rate of Rs. 50 per square meter.
- If 0.5 m^2 space is given per student, find out how many students are occupied in that room.

Answer

- | | | |
|---|-------------------------|------------------------|
| 1. 81 ft | 2. 7.75 cm | 3. 20 times |
| 4. (a) 294 m^2 | (b) 2744000 l | |
| 5. (a) 2800 cm^2 | (b) 320 m^3 | 6. 1280 cm^3 |
| 7. $l = 12 \text{ m}$, $b = 4 \text{ m}$, $h = 2 \text{ m}$ and $A = 160 \text{ m}^2$ | | |
| 8. 11 m | 9. 1000 m^3 | 10. 16.8 cm |
| 11. 6.25 cm | 12. 87.5 cm | 13. 20 times |
| 14. | 15. (a) Rs. 7600 | (b) 192 |