Algorithm 2 3-ADMM-H mixed-binary heuristic

Require: Initial choice of $x_0, \bar{x}_0, y_0, \lambda_0$. Choice of $\varrho, \beta, c > 0$, tolerance $\epsilon > 0$, and maximum number of iterations $K_{\max}$.

1: **while** $k < K_{\max}$ **and** $\|A_0 x^k + A_1 \bar{x}^k - y_k\| > \epsilon$, **do**

2: First block update (QUBO):

$$x_k = \underset{x \in \{0,1\}^n}{\arg\min}\ q(x) + \frac{c}{2}\|Gx - b\|_2^2 + \\ + \lambda_{k-1}^{\intercal} A_0 x + \frac{\varrho}{2}\|A_0 x + A_1 \bar{x}_{k-1} - y_{k-1}\|^2 \tag{17}$$

3: Second block update (Convex):

$$\bar{x}_k = \underset{\bar{x} \in \mathbb{R}^m}{\arg\min}\ f_1(\bar{x}) + \lambda_{k-1}^{\intercal} A_1 \bar{x} + \\ \frac{\varrho}{2}\|A_0 x_k + A_1 \bar{x} - y_{k-1}\|^2 \tag{18}$$

4: Third block update (Convex+quadratic):

$$y_k = \underset{y \in \mathbb{R}^n}{\arg\min}\ \frac{\beta}{2}\|y\|_2^2 - \lambda_{k-1}^{\intercal} y + \frac{\varrho}{2}\|A_0 x_k + A_1 \bar{x}_k - y\|^2$$

5: Dual variable update:

$$\lambda_k = \lambda_{k-1} + \varrho(A_0 x_k + A_1 \bar{x}_k - y_k)$$

6: Compute merit value:

$$\eta_k = q(x_k) + \phi(\bar{x}_k) + \\ + \mu(\max(g(x_k), 0) + \max(l(x_k, \bar{x}_k), 0)) \tag{19}$$

7: **end while**

8: **return** $x_{k^*}, \bar{x}_{k^*}, y_{k^*}$, with $k^* = \min_k \eta_k$.

Problem Statement :-

$$\min_{x \in \{0,1\}^3} \quad v + w + t + 5u^2 - 20u + 20$$

$$\text{s.t} \quad v + 2w + t + u \leq 3$$

$$v + w + t \geq 1$$

$$v + w = 1$$

$$x = [v \;\; w \;\; t]^T$$

$\rightarrow$ $q(x) = v + w + t$

$\rightarrow$ $Gx = b \Rightarrow [1 \;\; 1]\begin{bmatrix} v \\ w \end{bmatrix} = 1$

$$G = [1 \;\; 1]$$

$$b = 1$$

$\rightarrow$ $A_0 x + A_1 \bar{x} = y$

$\ell(z,u) \leq 0 \rightarrow v + 2w + t + u \leq 3 \Rightarrow v' + 2w' + t' + u - 3 \leq 0.$

where $z = [v' \, w' \, t']^T$, s.t $x = z$.

$g(z) \leq 0 \rightarrow 1 - v - w - t \leq 0 \Rightarrow 1 - v' - w' - t' \leq 0$

$$\bar{X} = \{(z \in \mathbb{R}^3, u \in \mathcal{U}) \mid g(z) \leq 0, \ell(z,u) \leq 0\}$$

$f_1(\bar{x}) = 5u^2 - 20u + 20$

$\rightarrow$ $A_0 x + A_1 \bar{x} = y \rightarrow x = [v \;\; w \;\; t]^T \quad \bar{x} = [v' \; w' \; t' \; u]^T$

$$A_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times 3} \quad A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}_{3\times 4}$$

$$A_0 x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ t \end{bmatrix} = \begin{bmatrix} v \\ w \\ t \end{bmatrix}$$

$$A_1 \bar{x} = \underset{3\times 4}{\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}} \begin{bmatrix} v' \\ w' \\ t' \\ u \end{bmatrix}_{4\times 1} = \begin{bmatrix} -v' \\ -w' \\ -t' \end{bmatrix}$$

$$y = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Steps:-

$\rho = 1001$, $\beta = 1000$, $C = 900$, $K_{max} = 100$,

$\epsilon = 10^{-6}$ ↳ tolerance factor

Step 1 (QUBO, Quantum):-

$$x_k = \underset{x \in \{0,1\}^3}{\operatorname{argmin}} \Big[v_k + w_k + t_k + 450 \| v_k + w_k - 1 \|^2 + \lambda_{k-1}^T A_0 x_k + \frac{\rho}{2} \| A_0 x_k + A_1 \bar{x}_{k-1} - y_{k-1} \|^2$$

Step 2 (Convex, Classical):-

$$\bar{x}_k = \underset{\bar{x} \in \mathbb{R}^4}{\operatorname{argmin}} \underbrace{5u^2 - 20u + 20}_{f_1(\bar{x})} + \lambda_{k-1}^T A_1 \bar{x} + \frac{\rho}{2} \| A_0 x + A_1 \bar{x} - y_{k-1} \|^2$$

Step 3 (Convex + Quadratic, Classical):-

$$y_k = \underset{y \in \mathbb{R}^3}{\operatorname{argmin}} \ \frac{\beta}{2} \|y\|_2^2 - \lambda_{k-1}^T y + \frac{\rho}{2} \|A_0 x_k + A_1 \bar{x}_k - y\|^2$$