In [96]:

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import ipysheet as ips
import panel as pn
from scipy import stats
pn.extension('katex', 'mathjax')
```

Tutorial 6

Homework Problem on Permeameters ¶

In [62]:

```
#
r1_1 = pn.pane.Markdown("""
## Homework Problem 3 ##


**A**. Derive an expression for hydraulic conductivity K for the constant-head permeameter shown in the figure.<br><br>
**B**. The hydraulic conductivity of a sample (length 10 cm, diameter 4 cm) is to be determined.
The water depths a<sub>1</sub> and a<sub>2</sub> equal 6 cm and 3 cm, resp. A water volume of 250 ml passed the sample during an expe
**C**. Which material could be con-tained in the sample?
""", style={'font-size': '13pt'})

spacer2=pn.Spacer(width=50)

r1_2 = pn.pane.PNG("images/T02_TH3.png", width=500)
pn.Row(r1_1,spacer2, r1_2)
```

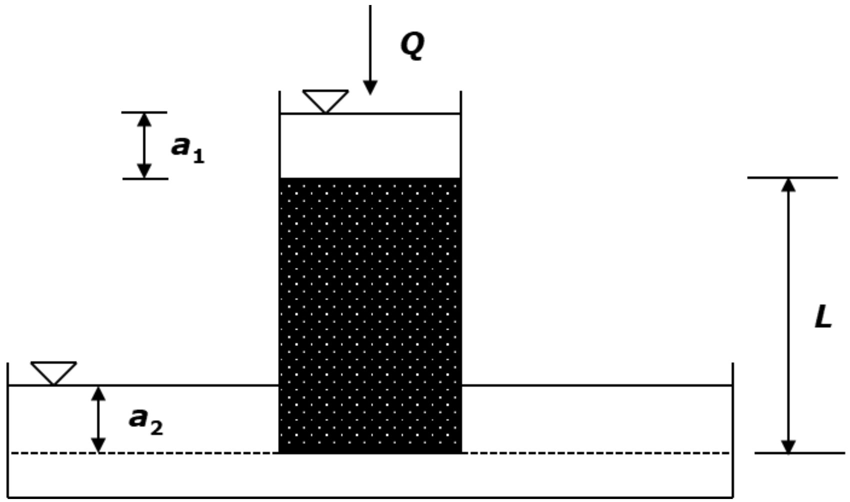
Out[62]:

Homework Problem 3

A. Derive an expression for hydraulic conductivity K for the constant-head permeameter shown in the figure.

B. The hydraulic conductivity of a sample (length 10 cm, diameter 4 cm) is to be determined. The water depths a_1 and a_2 equal 6 cm and 3 cm, resp. A water volume of 250 ml passed the sample during an experimental period of 36 s.

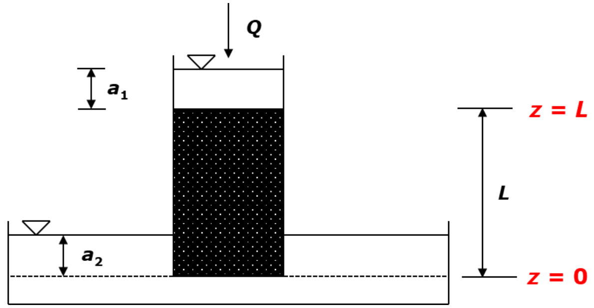
C. Which material could be con-tained in the sample?



In [63]: #↔

Out[63]:

Solution of the Homework Problem 3



For more information check L04 slide 14-15

General formula for constant-head permeameter:

$$K = \frac{QL}{A(h_{in} - h_{out})}$$

The column outlet is chosen as the reference level $z = 0$ with the z −axis pointing up-ward. As a consequence, we have $z = L$ at the inlet.

at the outlet:
pressure head = a_2
elevation head = 0
h_{out} = a_2

at the inlet:
pressure head = a_1
elevation head = L
h_{in} = a_1 + L

head difference: $h_{in} - h_{out} = a_1 + L - a_2$

hydraulic conductivity: $K = \frac{QL}{A(a_1+L-a_2)}$

In [64]:

```python
# Problem 3b and c, Given are:
L = 10# cm, length of column
a1 = 6# cm, pressure head at 1
a2 = 3# cm, pressure head at 2
d = 4 # cm, diameter of the column
V = 250 # ml, volume of water passed
t = 36 # s, time required


#interim calculation
A = np.pi*(d/2)**2 # cm^2 Area of the column
Q = V/t # mL/s, discharge


#calculation
K = Q*L/(A*(a1+L-a2))# cm/s, Conductivity (note: 1cm³ = 1 mL)

#output
print("The conductivity of the column is:{0:1.3f}".format(K), "cm/s")
print("The conductivity of the column is:{0:1.3E}".format(K/100), "m/s")

r1_8 = pn.pane.Markdown("""
**The sample in the column is: _Coarse sand - Fine gravel_**
""", width=400)

pn.Row(r1_8)
```

```
The conductivity of the column is:0.425 cm/s
The conductivity of the column is:4.251E-03 m/s
```

Out[64]:

The sample in the column is: *Coarse sand - Fine gravel*

In [65]: #↔

Out[65]:

A Darcy experiment is performed by a falling-head permeameter using water at 20°C. Length and diameter of the sample are 20 cm and 6 cm, resp. The inner tube dia-meter is 4 cm. The following data are available for the time-dependent hydraulic head difference :

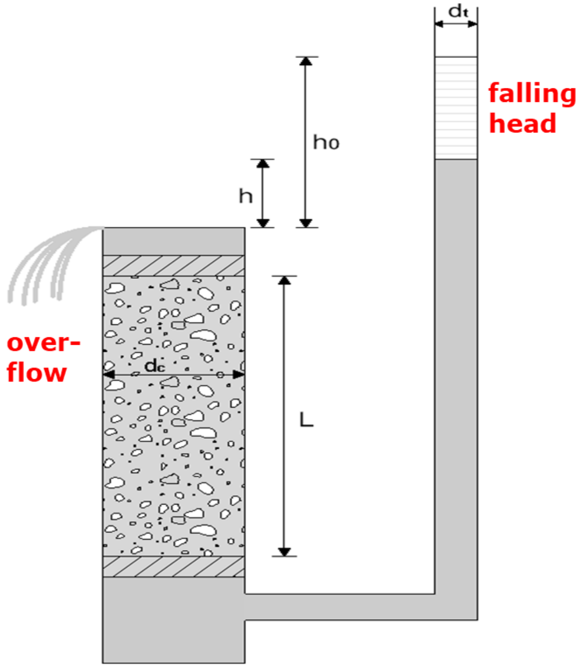
***t* (min)**	**0**	**5**	**18**	**23**	**27**	**29**
Δ*h* (cm)	**36.9**	**33.6**	**26.3**	**23.9**	**22.1**	**21.3**

A. Convert times to seconds and plot the logarithm of the ratios of head differences ln(Δh(0)/Δh(t)) vs. time t. (Use the coordinate system on next page).

B. Determine the slope of the correspon-ding regression line.

C. Determine hydraulic conductivity K.

D. Determine intrinsic permeability k.



Solution of Homework Problem 4

More information on this topic - L04 slide 16

```
In [66]: #↔
```

Out[66]:

The formula for variable-head permeameter:

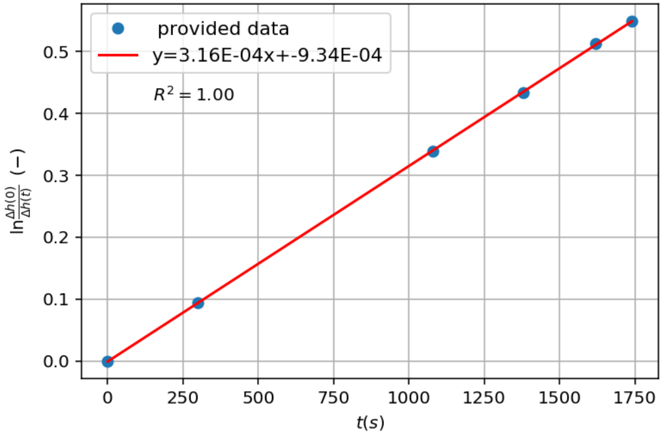
$$K = \frac{d_t^2 L}{d_t^2 t} \cdot \ln \frac{h_{in}(0) - h_{out}}{h_{in}(t) - h_{out}} = \frac{d_t^2 L}{d_c^2 t} \cdot \ln \frac{\Delta h(0)}{\Delta h(t)}$$

Rearrangement shows that the natural logarithm of $\Delta h(0)/\Delta h(t)$ depends linearly on time t:

$$\ln \frac{\Delta h(0)}{\Delta h(t)} = \frac{K \cdot d_c^2}{L \cdot d_t^2} \cdot t$$

$$\text{slope} = \frac{K \cdot d_c^2}{L \cdot d_t^2}$$

$$K = L \frac{d_t^2}{d_c^2} \cdot \text{slope}$$



In [67]:

```python
#
#Solution of 4C
# Given
L = 20 # cm, Length of the column
d_t = 4 # cm, diameter of the tube
d_c = 6# cm, diameter of the column
slope = slope # obtained from the fit equation

K = L*(d_t**2/d_c**2)*slope # cm/s, conductivity calculated using eqn from previous slide

print("The conductivity in the column is: {0:1.2E}".format(K), "cm/s\n")
print("The conductivity in the column is: {0:1.2E}".format(K/100), "m/s\n")

#Solution of 4D
# Given
rho_w = 998.2 # kg/m^3, density of water
eta_w = 1.0087E-3# kg/(m-s), viscocity of water
g = 9.81 # m/s^2, accl. due to gravity

k = K/100*eta_w/(rho_w*g)# m^2, K = k*ρ/n
k_D = k/0.987E-12 # D, 1D = 0.987E10-12 m^2

print("The permeability of the media is: {0:1.2E}".format(k), "m\u00b2 \n")
print("The permeability of the media in Darcy's unit is: {0:1.2f}".format(k_D), "D")
```

```
The conductivity in the column is: 2.81E-03 cm/s

The conductivity in the column is: 2.81E-05 m/s

The permeability of the media is: 2.89E-12 m²

The permeability of the media in Darcy's unit is: 2.93 D
```

Tutorial 6

Tutorial Problems on Wells

In [68]:

```
# Tut problem 18
r3_1 = pn.pane.Markdown("""

### Tutorial Problem 18 ###

A pumping test was conducted with a constant water withdrawal rate of 9 m³/h. The table shows the time-drawdown series recorded at an
Determine the storage coefficient, the transmissivity and the hydraulic conducti-vity by using the Theis method.
To this end, it is necessary to complete the table on the right such that data are made available for further steps (see next page).
<br><br><br>

## Solution of Problem 18 ##
**See L07 - slides 29-33 for more information on this problem**
""",width = 600, style={'font-size': '13pt'})
#data
df_t1 =np.array([1, 2,    3,    4,    5,    7,    9,   12,   18,   23,   33, 41,   56,  126,  636, 1896])
df_s1 = np.array([0.01, 0.03, 0.05, 0.06, 0.07, 0.09, 0.12, 0.14, 0.16, 0.17, 0.18, 0.19, 0.2 , 0.22, 0.3 , 0.32])


d = {'time [min]': df_t1, 'drawdown [m]': df_s1}
df = pd.DataFrame(data=d, index=None)
pn.Row(r3_1, df)
```

Out[68]:

Tutorial Problem 18

A pumping test was conducted with a constant water withdrawal rate of 9 m^3/h. The table shows the time-drawdown series recorded at an observation well which is located 9.85 m apart from the pumping well. The aquifer thickness is 5 m. Determine the storage coefficient, the transmissivity and the hydraulic conducti-vity by using the Theis method. To this end, it is necessary to complete the table on the right such that data are made available for further steps (see next page).

Solution of Problem 18

See L07 - slides 29-33 for more information on this problem

	time [min]	drawdown [m]
0	1	0.01
1	2	0.03
2	3	0.05
3	4	0.06
4	5	0.07
5	7	0.09
6	9	0.12
7	12	0.14
8	18	0.16
9	23	0.17
10	33	0.18
11	41	0.19
12	56	0.20
13	126	0.22
14	636	0.30
15	1896	0.32

```
In [69]:
```

```python
# Problem 18 solution contd.
r3_2 = pn.pane.Markdown("""
As a first approach, the graphical solution of the problem is to be determined, i.e. double-logarithmic data and type curve sheets ar
<br> <br>

1. Plot data for _s_ vs. _t/r<sup>2</sup>_ in the data sheet.<br>

2. Determine coordinates of the match point _A_ on the type curve sheet, e.g. _1/u<sub>A</sub>_ = 1 and _W<sub>A</sub>_ = 1.<br>

3. Put the data sheet on top of the type curve sheet and shift it in parallel to the coordinate axes until data points
fall on the type curve as close as possible.<br>
4. Determine coordinates of the match point _A_ on the data sheet:
""",width = 600, style={'font-size': '13pt'})

#given
r = 9.85 # m, observation well distance
t_s = df_t1*60 # s, converting time in s
t_r2 = t_s/r**2# s/m^2,  finding t/r^2

#output
d2= {'time [min]': df_t1, 'drawdown [m]': df_s1, "t/r2 (s/m^2)": t_r2}
df2 = pd.DataFrame(data=d2, index=None)

pn.Row(r3_2, df2)
```

Out[69]:

As a first approach, the graphical solution of the problem is to be determined, i.e. double-logarithmic data and type curve sheets are compared as follows:

1. Plot data for *s* vs. t/r^2 in the data sheet.
2. Determine coordinates of the match point *A* on the type curve sheet, e.g. $1/u_A = 1$ and $W_A = 1$.
3. Put the data sheet on top of the type curve sheet and shift it in parallel to the coordinate axes until data points fall on the type curve as close as possible.
4. Determine coordinates of the match point *A* on the data sheet:

	time [min]	drawdown [m]	t/r2 (s/m^2)
0	1	0.01	0.618413
1	2	0.03	1.236827
2	3	0.05	1.855240
3	4	0.06	2.473653
4	5	0.07	3.092066
5	7	0.09	4.328893
6	9	0.12	5.565719
7	12	0.14	7.420959
8	18	0.16	11.131439
9	23	0.17	14.223505
10	33	0.18	20.407637
11	41	0.19	25.354943
12	56	0.20	34.631142
13	126	0.22	77.920070
14	636	0.30	393.310830
15	1896	0.32	1172.511531

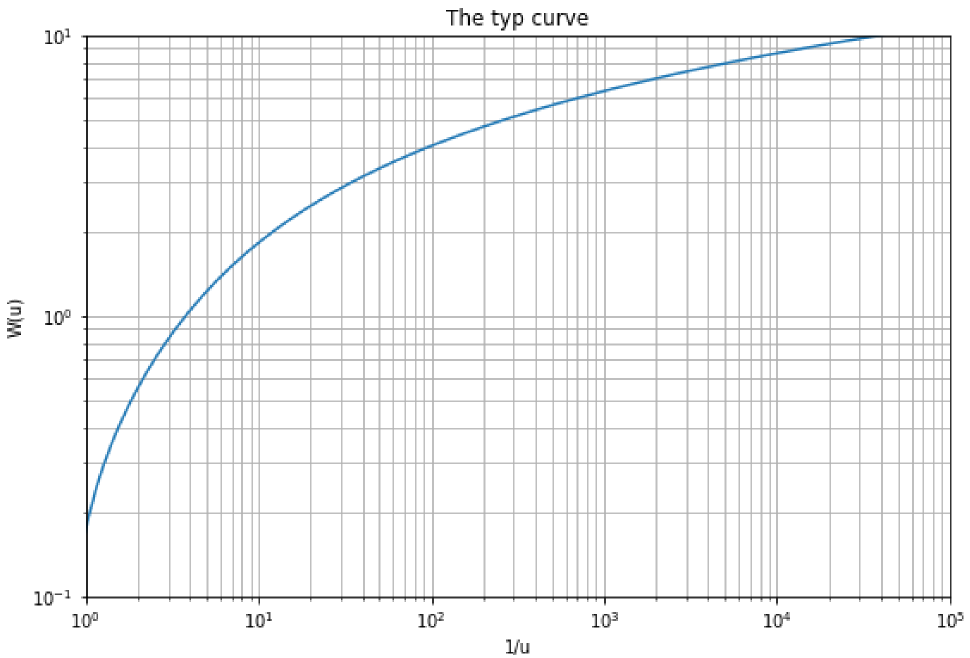
In [70]:

```python
# Given in this problem

Q = 9 # m^3/h, discharge
i_ua = 1 # (-), 1/U_a
W_a = 1 #, (-), Well function W(u)
t_r2m = 0.6 # s/m^2, t/r^2 obtained from data matching with the typ curve
s = 0.06 # m, drawdown, obtained from data matching with the typ curve
m = 5 # m, aquifer thickness

# Compute
T = Q*W_a/(3600*(4*np.pi*s)) # m^2/s, transmissivity, T= Q.Wa/(4.pi.s). /3600 for hr-s
S = (4*T*t_r2m)/i_ua #(-), Storage coeff.
K = T/m # m/s, conductivity

#output
print("The Transmissivity at the site is: {0:1.2E}".format(T), "m\u00b2/s\n")
print("The Storage coefficient at the site is: {0:1.2E}".format(S),"\n")
print("The Conductivity at the site is: {0:1.2E}".format(K), "m/s")
```

```
The Transmissivity at the site is: 3.32E-03 m²/s

The Storage coefficient at the site is: 7.96E-03

The Conductivity at the site is: 6.63E-04 m/s
```

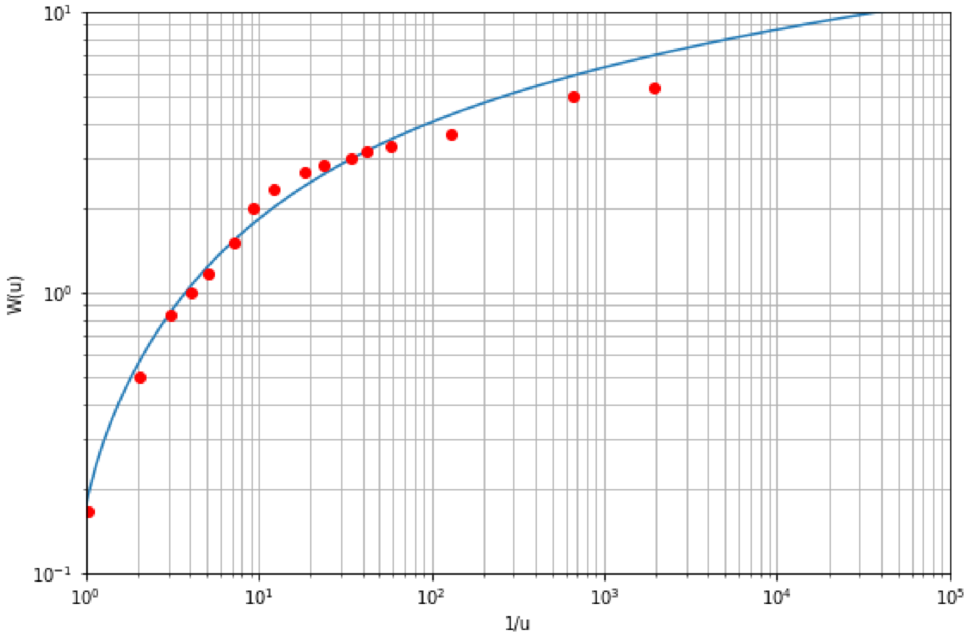
In [71]:

```
#
# Typ curve
#code to find W(u) (see L07/S-31) using the infinite series W(u) = -0.5772-log(u)+u-u^2/(2*2!)+u^3/(3*3!)-... (100 terms)
# It is possible to use: from scipy.special import expi def W(u):  return  -expi(-u)

def W(u):
    w = -0.5772 -np.log(u) + u
    a = u
    for n in range(1, 100):
        a = -a * u * n / (n+1)**2 # new term (next term)
        w += a  # w = w+a
        return w

u_1 = np.logspace(10,-1,250, base=10.0) # setting the value of u
w_u =W(1/u_1) # finding W(1/u) : as we use 1/u in the typ curce

plt.figure(figsize=(9,6))
plt.loglog(u_1, w_u)
plt.title("The typ curve"); plt.ylim((0.1, 10)); plt.xlim(1, 1e5)
plt.grid(True, which="both",ls="-"); plt.ylabel(r"W(u)");plt.xlabel(r"1/u") ;
```



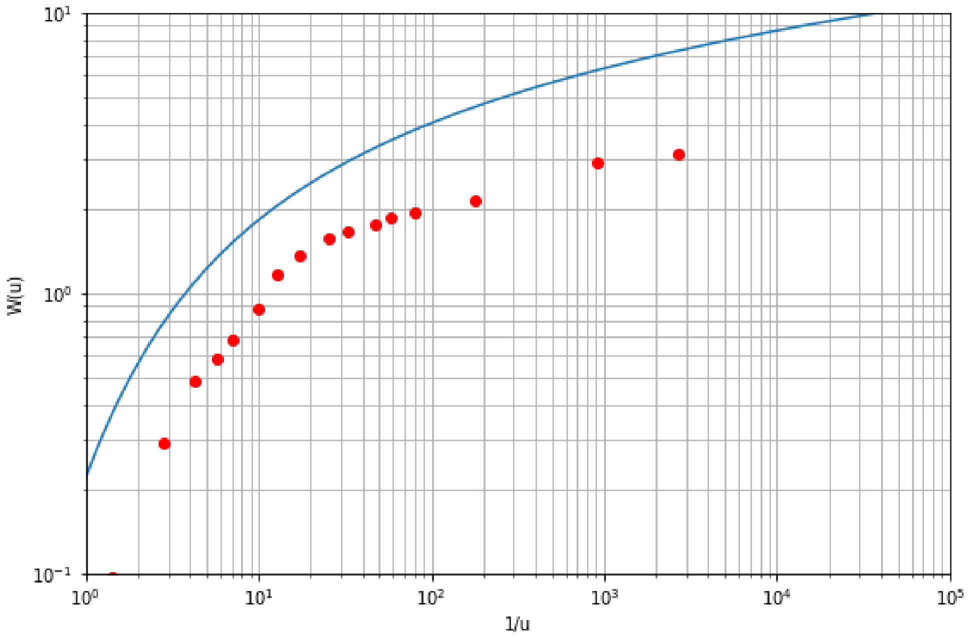
In [78]:

```python
# Solution from typ curve (extra code)

# Data to fit
p_r = 9/3600 # m^3/s, pumping rate /3600 for unit /h- /s
r = 9.85 # m, distance of observation well
m = 5 # m, aquifer thickness
d_s = df_s1 # m, drawdown data

# Obtained from graphical method
S_C = 7.97e-03 # (-), storage coeff.
T = 0.00332 # m^2/s, transmissivity
K_aq = T/m

#Calculations
u_1d = 4*T*t_s/(S_C*r**2)
w_ud = 4*np.pi*d_s*T/p_r

# plots
u_1 = np.logspace(10,-1,250, base=10.0)
w_u =W(1/u_1)

plt.figure(figsize=(9,6))
plt.loglog(u_1, w_u)
plt.loglog(u_1d, w_ud, "o", color="red"  )
plt.ylim((0.1, 10))
plt.xlim(1, 1e5)
plt.grid(True, which="both",ls="-")
plt.ylabel(r"W(u)");
plt.xlabel(r"1/u") ;
```



In [98]:

```
# Additional Code  interactive one

from ipywidgets import interact # for interactive plot with slider
from scipy.special import expi # easily obtain well function

def W(u):
    return  -expi(-u)

def f(T, S_C, r, Q):

    #data that can be changed
    d_s = np.array([0.01, 0.03, 0.05, 0.06, 0.07, 0.09, 0.12, 0.14, 0.16, 0.17, 0.18,
       0.19, 0.2 , 0.22, 0.3 , 0.32])

    t_s = np.array([60, 120,    180,    240,    300,    420,    540,    720,
                    1080,   1380,   1980,   2460,   3360,   7560,  38160, 113760])

    # calculated function see L07-slide 31
    u_1d = 4*T*t_s/(S_C*r**2) # calculating 1/u
    w_ud = 4*np.pi*d_s*T/Q   # well function

    # plots


    u_1 = np.logspace(10,-1,250, base=10.0)
    w_u =W(1/u_1)

    plt.figure(figsize=(9,6));plt.loglog(u_1, w_u); plt.loglog(u_1d, w_ud, "o", color="red"  )
    plt.ylim((0.1, 10));plt.xlim(1, 1e5)
    plt.grid(True, which="both",ls="-")
    plt.ylabel(r"W(u)");plt.xlabel(r"1/u")

f(0.00233, 8e-3, 7, 0.003)
interactive_plot = interact(f, T=(0.0005, 0.05, 0.002), S_C=(0.00005, 0.05, 0.0001), r= (1.0,15.0, 1 ), Q=(0.0005, 0.05, 0.002))
```

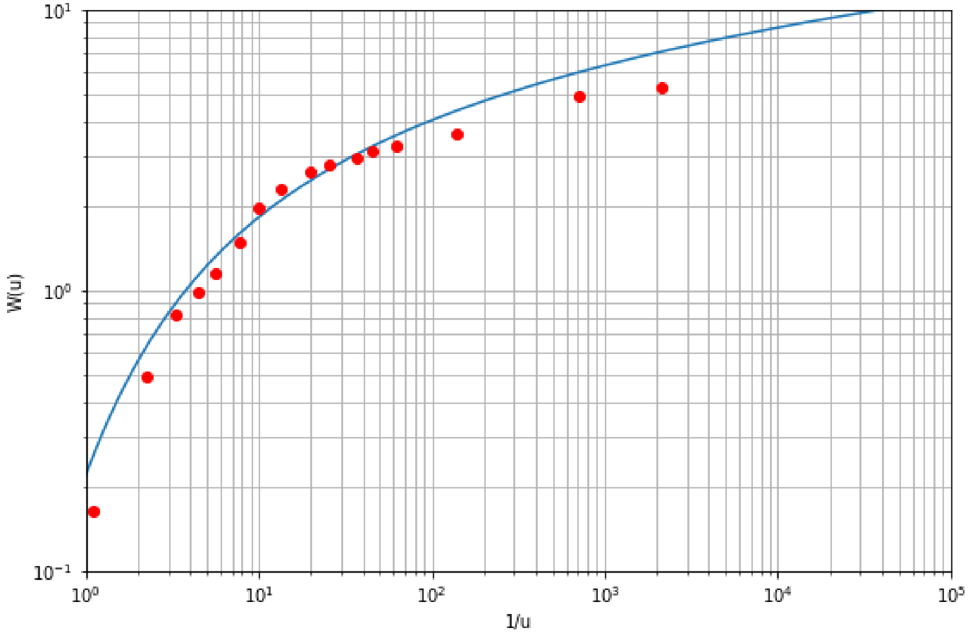


T 0.01

S_C 0.02

r 9.00

Q 0.01



In [89]:
```
# Tutorial Problem 19↔
```
Out[89]:

Tutorial Problem 19

A pumping test is conducted in a confined aquifer with thickness m = 14.65 m. The pumping rate is kept constant at Q = 50 m^3/h and the corresponding drawdown s is recorded in an observation well at a distance r = 251.32 m from the pumping well (see table).

1. Determine storage coefficient S, transmissivity T and hydraulic conductivity K by employing the Theis method (graphical solution).

2. What is the drawdown in the pumping well (radius including gravel pack: r_W = 0.3 m) after 500 min? (Hint: Use the approximation W(u) ≈ –0.5772 – lnu + u which is valid for u << 1)

3. How big is the radius of influence according to Siechardt's equation?

	Time [min]	Drawdown [m]
0	1	0.01
1	2	0.03
2	3	0.05
3	4	0.06
4	5	0.07
5	7	0.09
6	9	0.12
7	12	0.14
8	18	0.16
9	23	0.17
10	33	0.18
11	41	0.19
12	56	0.20
13	126	0.22
14	636	0.30
15	1896	0.32

Solution of Problem 19

See L07 - slides 29-33 for more information on this problem

In [86]:

```
#Solution 19.  steps
r4_2 = pn.pane.Markdown("""

### Solution Steps
1. Determine storage coefficient _S_,  transmissivity _T_ and hydraulic conductivity _K_ by employing the
Theis method (graphical solution).
<br><br>
2. What is the drawdown in the pumping well (radius including gravel pack: _r<sub>w</sub>_ = 0.3 m) after 500 min?
(Hint: Use the approximation _W(u) ≈ –0.5772 – lnu + u_,  which is valid for _u_ << 1)
<br><br>
3. How big is the radius of influence according to Siechardt's equation?

""",width = 600, style={'font-size': '13pt'})

#given
r = 251.32  # m, observation well distance
t19_s = d19_tm*60 # s, converting time in s
t19_r2 = t19_s/r**2# s/m^2,  finding t/r^2

#output
d19_2= {'time [min]': d19_tm, 'drawdown [m]': d19_s, "t/r2 (s/m^2)": t19_r2}
df19_2 = pd.DataFrame(data=d19_2, index=None)

r4_3 = pn.pane.Markdown("""

### Solution of 15-1
1. Determine storage coefficient _S_,  transmissivity _T_ and hydraulic conductivity _K_ by employing the Theis method (graphical sol

""",width = 600, style={'font-size': '13pt'})
r4_4= pn.Column(r4_2, r4_3)
pn.Row(r4_4, df19_2)
```

Out[86]:

Solution Steps

1. Determine storage coefficient *S*, transmissivity *T* and hydraulic conductivity *K* by employing the Theis method (graphical solution).

2. What is the drawdown in the pumping well (radius including gravel pack: r_w = 0.3 m) after 500 min? (Hint: Use the approximation *W(u) ≈ –0.5772 – lnu + u*, which is valid for *u* << 1)

3. How big is the radius of influence according to Siechardt's equation?

	time [min]	drawdown [m]	t/r2 (s/m^2)
0	1	0.01	0.000950
1	2	0.03	0.001900
2	3	0.05	0.002850
3	4	0.06	0.003800

Solution of 15-1

1. Determine storage coefficient *S*, transmissivity *T* and hydraulic conductivity *K* by employing the Theis method (graphical solution).

In [75]:

```
# Given information
Q_19 = 50 # m^3/h, discharge per hours
m_19 = 14.65 # m , aquifer thickness
r_19  = 251.32 # m, distance to observation well

# Match point obtained from the type curve sheet (Manually done, you could also do by fitting)
i_u19 = 1 # (-), 1/ua
W_19a = 1 # (-), W(u)
t19_r2m = 0.004 # s/m^2, t/r^2
s_19 = 0.8 # m, drawdown

#Calculations
T_19 = (Q_19/3600)*W_19a/(4*np.pi*s_19) # m^2/s, transmissivity
S_19 = (4*T_19*t19_r2m)/i_u19 # (-), storage coefficient
K_19 = T_19/m_19 # m/s, hydraulic conductivity of the aquifer

#output
print("The Transmissivity at the site is: {0:1.2E}".format(T_19), "m\u00b2/s\n")
print("The Storage coefficient at the site is: {0:1.3E}".format(S_19), "\n")
print("The Conductivity at the site is: {0:1.1e}".format(K_19), "m/s")
```

```
The Transmissivity at the site is: 1.38E-03 m²/s

The Storage coefficient at the site is: 2.210E-05

The Conductivity at the site is: 9.4e-05 m/s
```

Solution of 19 -2

2. What is the drawdown in the pumping well after 500 min? (Hint: Use the approximation formula $W(u) \approx -0.5772 - \ln u + u$, which is valid for $u << 1$)

In [87]:

```
# Given
r_w = 0.3 # m, radius of the weell
t_19_2 = 500*60 # s, given time in min, converted to s.
Q_19s = Q_19/3600 # m^3/s, discharge unit converted

#Calculations
u_19 = (S_19*r_w**2)/(4*T_19*t_19_2)
W_19b = -0.5772 - np.log(u_19)+u_19
s_19b = (Q_19s*W_19b)/(4*np.pi*T_19) # see L07 - slide 32

# Solution of 15C:
#How big is the radius of influence according to Siechardt's equation? (L07, slide 27)
R_19 = 3000*s_19b*np.sqrt(K_19)

#output
print("u = {0:1.2E}".format(u_19), "\n")
print("W(u)= {0:1.2f}".format(W_19b), "\n")
print("The drawdonw at the site is: {0:1.2f}".format(s_19b), "m\n")
print("The radius of influence is is: {0:1.2f}".format(R_19), "m")
```

```
u = 1.20E-08

W(u)= 17.66

The drawdonw at the site is: 14.13 m

The radius of influence is is: 411.62 m
```

HOME WORK PROBLEMS

Effective Conductivity and Wells

There is no obligation to submit the homework

Pls. submit within two weeks if you wish to.

In [88]:

```
#Homework Problem 8↔
```

Out[88]:

Homework Problem 8: Effective Hydraulic Conductivity

A gravel layer with a thickness of 2.5 m is embedded between two sand layers. Both sand layers have a thickness of 1.5 m and a hydraulic conductivity of $3.7 \cdot 10^{-4}$ m/s. Steady-state groundwater flow is perpendicular to the layering. An overall head difference of 5.5 cm and a discharge of 500 l/d per unit area have been determined

a. Determine the effective hydraulic conductivity.

b. What is the hydraulic conductivity of the gravel layer?

c. Which effective hydraulic conductivity would be obtained if flow was assumed to be in parallel with the layering?

d. Calculate effective hydraulic conductivity if the angle between the flow direction and the layering equals 30°.

Homework Problem 8: Pumping Test Evaluation

A pumping test is conducted to determine hydraulic properties of a confined aquifer of thickness $m = 10$ m. For this purpose, a constant pumping rate of 1219 m³/d is established and drawdown is recorded in an observation well located at $r = 120$ m from the well. The problem is to be solved with the Theis method (see Problem 19) and the same time and drawdown data from Problem 19 is to be used.

Based on these data determine the storage coefficient S, the transmissivity T and the hydraulic conductivity K of the aquifer.