In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import ipysheet as ips
import panel as pn
from scipy import stats
pn.extension('katex', 'mathjax')
```

Tutorial 4

- **solutions for homework problems 1 – 4**
- **tutorial problems on effective conductivity and flow nets**
- **homework problems on effective conductivity and flow nets**

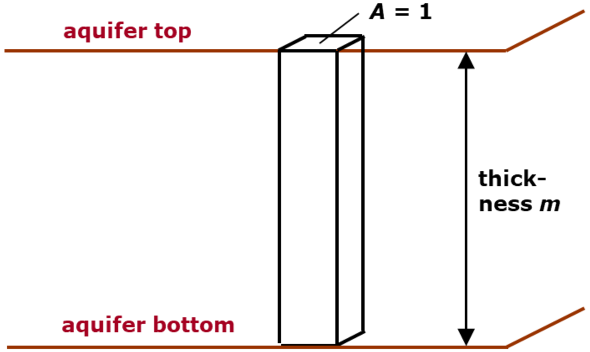
Solutions for Homework Problems 1 – 2

In [2]:
```
#
r1_1 = pn.pane.Markdown("""

### Homework Problem 1 ###

The pressure head in an aquifer extending over 200 km<sup>2</sup> is decreased by 1.60 m.
Determine the loss of groundwater in the aquifer for two scenarios:
A. The aquifer is unconfined (storage coefficient 0.13).
B. The aquifer is confined (storage coefficient 0.0005).

""",width = 800, style={'font-size': '13pt'})

r1_2= pn.pane.PNG("images/T03_H1.png", width=350)
#r1_2 = pn.pane.PNG("images/T03_H1.PNG")

### Tutorial Problem 7 – Solution ###

#<img src="images/T03_H1.PNG" alt="Grosser Garten Map"  width="40%" height="100%" >

r1_3 = pn.pane.Markdown("""
### Solution - Homework Problem 1 ###
<br>
Relevant information can be found in Lecture L03, Slides- 28-30

""",width = 800, style={'font-size': '13pt'})

r1_3b = pn.pane.LaTeX(r"""
<br>
The relevant equations is:<br>
$$
S = \Delta V_w/(A\cdot \Delta H)
$$

""",width = 800, style={'font-size': '13pt'})
pn.Column(r1_1, r1_2, r1_3, r1_3b)
```

Out[2]:

Homework Problem 1

The pressure head in an aquifer extending over 200 km^2 is decreased by 1.60 m. Determine the loss of groundwater in the aquifer for two scenarios: A. The aquifer is unconfined (storage coefficient 0.13). B. The aquifer is confined (storage coefficient 0.0005).



Solution - Homework Problem 1

Relevant information can be found in Lecture L03, Slides- 28-30

The relevant equations is:

$$S = \Delta V_w / (A \cdot \Delta H)$$

In [3]:
```
# Given
A = 200 # km^2, aquifer area
D_h = 1.6 # m, head decrease
S_u = 0.13 # (-), Storativity unconfined aquifer
S_c = 0.0005 # (-) Storage coefficient, confined aquifer

# Solution
DV_wu = A*S_u*D_h * 10**6 # m^3 change in water volume unconfined aquifer
DV_wc = A*S_c*D_h* 10**6  # m^3 change in water volume unconfined aquifer

# output

print("Change in water volume in unconfined aquifer is: {0:1.1e}".format(DV_wu),"m\u00b3 \n")
print("Change in water volume in confined aquifer is: {0:1.1e}".format(DV_wc),"m\u00b3")
```

```
Change in water volume in unconfined aquifer is: 4.2e+07 m³

Change in water volume in confined aquifer is: 1.6e+05 m³
```

Homework Problem 2

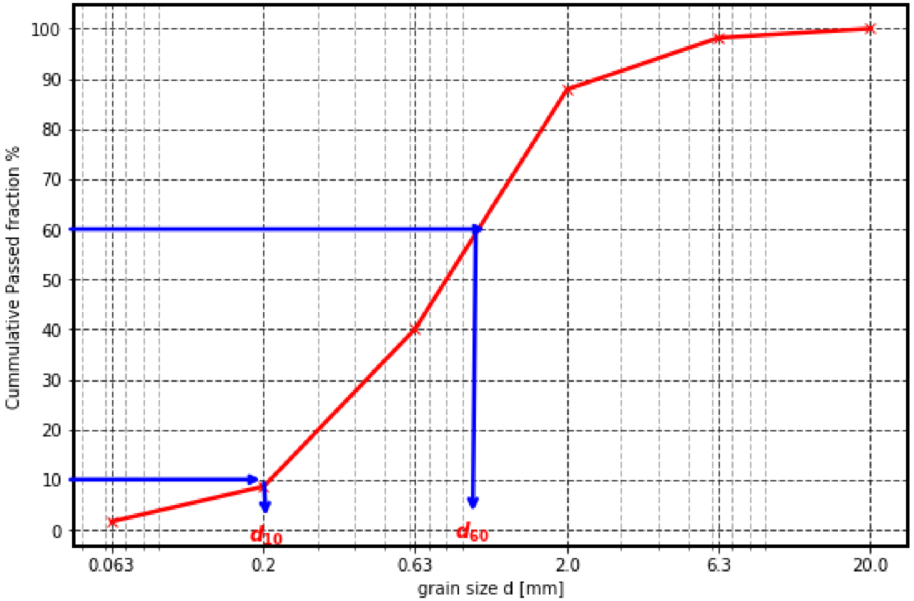
Conduct a sieve analysis for a dried soil sample (see data in the table below)

1. Draw the granulometric curve (cumulative mass distribution) and briefly characterise the sediment with regard to its major constituent(s).
2. What is the coefficient of uniformity?

In [4]:
```
#
title = ["mesh   size  [mm] ", "residue in the sieve [g] ", "∑Retained %", "Commulative Passed %"]
Size = [6.3, 2, 0.63, 0.2, 0.063, "< 0.063 /cup"]
passed = [11, 62, 288, 189, 42, 10]
s2 = ips.sheet(rows=6, columns=4, row_headers=False, column_headers=title)
ips.column(0, Size, row_start=0)
ips.column(1, passed, row_start=0); s2
```

In [5]:
```
# Solution of problem 2

t_sample = np.sum(passed) # g, add the residue column to get total mass
retained_per = passed/t_sample *100 # %, # retain percentage residue/total mass
retain_per_cumsum =np.cumsum(retained_per) # get the cummulative sum of the reatined
passing_cumper = 100 - retain_per_cumsum # substract 100-cummsum to get passing % - the last column

#Output
s3 = ips.sheet(rows=6, columns=4, row_headers=False, column_headers=title)
ips.column(0, Size, row_start=0)
ips.column(1, passed, row_start=0);
ips.column(2, retained_per, row_start=0);
ips.column(3, passing_cumper, row_start=0); s3
```

In [6]:
```
# Plotting granulometric curve

plt.rcParams['axes.linewidth']=2
plt.rcParams['grid.linestyle']='--'
plt.rcParams['grid.linewidth']=1
x = np.append([20], Size[:5]) # adding for all left over.
y = np.append([100],passing_cumper[:5])
fig = plt.figure(figsize=(9,6));
plt.plot(x, y, 'x-', color='red', lw=2.5);
tics=x.tolist()
plt.xscale('log');lw=2.5
plt.grid(which='major', color='k', alpha=0.7)
plt.grid(which='minor', color='k', alpha=0.3)
plt.xticks(x, tics);
plt.yticks(np.arange(0,110,10));
#plt.title('grain size distribution (combined wet sieving and sedimentation analysis)');
plt.xlabel('grain size d [mm]');
plt.ylabel('Cummulative Passed fraction %');

plt.annotate('', xy=(0.20, 10),  xycoords='data', xytext=(0.045, 10), arrowprops=dict(arrowstyle='->', color="b", lw=2.5),ha='righ
t', va='top',)
plt.annotate('', xy=(1.1, 60),  xycoords='data', xytext=(0.045, 60), arrowprops=dict(arrowstyle='->', color="b", lw=2.5),ha='right'
, va='top',)
plt.annotate(r'$d_{60}$', xy=(1, 60),  xycoords="data", xytext=(0.85, -3),color='red',size=12, arrowprops=dict(arrowstyle='<-', col
or="b", lw=2.5),ha='left', va='bottom',)
plt.annotate(r'$d_{10}$', xy=(0.20, 10),  xycoords='data', xytext=(0.235, 1.5),color='red',size=12, arrowprops=dict(arrowstyle='<-'
, color="b", lw=2.5),ha='right', va='top',)
plt.rcParams["font.weight"] = "bold"
#plt.savefig("fig6.png")
mpl_pane = pn.pane.Matplotlib(fig, dpi=144)
```



In [7]:

```
# From the figure
d_10 = 0.22 # mm,approx, diameter 10% passing, see the arrow bottom in x-axis
d_60 = 1.0 # mm, approx diameter 10% passing, see the arrow bottom in x-axis

c_u = d_60/d_10 # [], coefficient of uniformity

#Output
print("The coefficient of uniformity is: {0:1.1f}".format(c_u))
r2_1 = pn.pane.Markdown("""
**Major constituents: coarse sand/medium sand** """, width=600, style={'font-size': '13pt', 'color': 'blue'} )
pn.Row(r2_1)
```

```
The coefficient of uniformity is: 4.5
```

Out[7]:

Major constituents: coarse sand/medium sand

In [8]:

```
# Tutorial Problem 11- Effective Conductivity and flow nets
r5_1 = pn.pane.Markdown("""
#Tutorial Problems on Effective Conductivity and Flow Nets
""", width = 900)

r5_2 = pn.pane.Markdown("""
###Tutorial Problem 11: Effective Hydraulic Conductivity
A sandy layer with a thickness of 2.5 m is embedded between two gravel layers. B
oth gravel layers have a thickness of 1.5 m and a hydraulic conductivity of 3.7·10<sup>-3</sup> m/s.
Steady-state groundwater flow is in parallel to the layering.
A hydraulic gradient of 0.001 and an overall discharge of 1 m³/d per unit width have been determined.
<br><br>
a. Determine the effective hydraulic conductivity.<br><br>
b. What is the hydraulic conductivity of the sand layer?<br><br>
c. Which effective hydraulic conductivity would be obtained if flow was assumed perpendicular to the layering?<br><br>
d. Calculate effective hydraulic conductivity if the angle between the flow direction and the layering equals 45°.
""", style={'font-size': '13pt'})

pn.Column(r5_1, r5_2)
```

Out[8]:

Tutorial Problems on Effective Conductivity and Flow Nets

Tutorial Problem 11: Effective Hydraulic Conductivity

A sandy layer with a thickness of 2.5 m is embedded between two gravel layers. B oth gravel layers have a thickness of 1.5 m and a hydraulic conductivity of $3.7 \cdot 10^{-3}$ m/s. Steady-state groundwater flow is in parallel to the layering. A hydraulic gradient of 0.001 and an overall discharge of 1 m^3/d per unit width have been determined.

a. Determine the effective hydraulic conductivity.

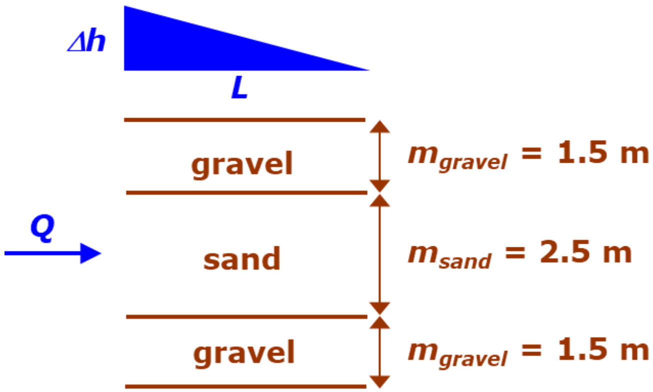
b. What is the hydraulic conductivity of the sand layer?

c. Which effective hydraulic conductivity would be obtained if flow was assumed perpendicular to the layering?

d. Calculate effective hydraulic conductivity if the angle between the flow direction and the layering equals 45°.

In [9]:
```
# Solution of Problem 11
r5_3 = pn.pane.PNG("images/T03_TP11_a.png", width=400)
r5_4 = pn.pane.LaTeX(r"""
Known relationships are (see Lecture 05, Slides 8-13, 22):
$$
Q = WmK\frac{\Delta H}{L}
$$
$$
K = \frac{Q/W}{m\cdot \Delta H \cdot L}
$$
Weighted arithmetic mean to determine hydraulic conductivity for sand:

$$
K = \frac{1}{m}\sum_{i=1}^n m_i\cdot K_i
$$
where $i$ is different layers
""", width = 500, style={'font-size': '13pt'})
spacer2 = pn.Spacer(width=100)

pn.Row(r5_3, spacer2, r5_4)
```

Out[9]:



Known relationships are (see Lecture 05, Slides 8-13, 22):

$$Q = WmK\frac{\Delta H}{L}$$

$$K = \frac{Q/W}{m\cdot \Delta H \cdot L}$$

Weighted arithmetic mean to determine hydraulic conductivity for sand:

$$K = \frac{1}{m}\sum_{i=1}^n m_i\cdot K_i$$

where i is different layers

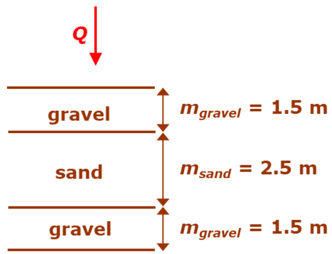
In [10]:
```
#Given Solution of 11 a, b

Q = 1 # m^3/d, discharge
W = 1 # m, per unit width
K_g = 3.7*1E-3# m/s, conductivity of gravel layer
m_g = 1.5 # m, thickness of gravel layer
m_s = 2.5 # m, thickness of sand layer
m = 2*m_g + m_s # m. total thickness of aquifer
Dh_L = 0.001 # (-), hydraulic gradient


#Solution of 11a
Keff_h = (Q/W)/(m*Dh_L) # m/d, conductivity
Keff_hs = Keff_h/(24*3600)# m/s, conductivity unit changed

#Solution of 11b
# K_eff = (2*m_g*K_g + m_s*K_g)/m

K_s = ((m*Keff_hs - 2*m_g*K_g))/m_s

print("Effective horizontal hydraulic conductivity (Keff_h) = {0:1.2f}".format(Keff_h), "m/d\n" )
print("Effective horizontal hydraulic conductivity (Keff_hs) = {0:1.3E}".format(Keff_hs), "m/s\n" )
print("Hydraulic conductivity of sand layer (K_s) = {0:1.1E}".format(K_s), "m/s" )
```

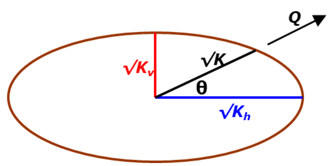
```
Effective horizontal hydraulic conductivity (Keff_h) = 181.82 m/d

Effective horizontal hydraulic conductivity (Keff_hs) = 2.104E-03 m/s

Hydraulic conductivity of sand layer (K_s) = 1.9E-04 m/s
```

In [11]:

```
#Given Solution of 11 c, d

r5_5 = pn.pane.PNG("images/T03_TP11_b.png", width=200)
r5_6 = pn.pane.PNG("images/T03_TP11_c.png", width=200)

r5_7 = pn.Column(r5_5, r5_6)

r5_8 = pn.pane.LaTeX(r"""
Vertical effective conductivity is given by weighted harmoninc mean
$$
K = \frac{m}{2\cdot \frac{m_g}{K_g} + \frac{m_s}{K_s} }
$$
<br>
For inclined aquider the effective conductivity is:

$$
K = \frac{1}{\frac{\cos^2\theta}{K_h} + \frac{\sin^2\theta}{K_v}}
$$

""", style={'font-size': '13pt'})

pn.Row(r5_7,spacer2, r5_8)
```

Out[11]:





Vertical effective conductivity is given by weighted harmoninc mean

$$
K = \frac{m}{2\cdot \frac{m_g}{K_g} + \frac{m_s}{K_s} }
$$

For inclined aquider the effective conductivity is:

$$
K = \frac{1}{\frac{\cos^2\theta}{K_h} + \frac{\sin^2\theta}{K_v}}
$$

In [12]:

```
# Solution of 11c

Keff_v = m/(2*(m_g/K_g)+ (m_s/K_s))

#Given
theta = 45 # theta
theta_r = 45*(np.pi)/180 # degree to radian conversion
K_h = Keff_hs # m/s, solution from 11a
K_v = Keff_v # m/s, solution from 11c

# solution from 11d
Keff_i = 1/((np.cos(theta_r)**2/K_h)+(np.sin(theta_r)**2/K_v))


print("Effective vertical hydraulic conductivity (Keff_v) = {0:1.2E}".format(Keff_v), "m/s\n" )
print("Effective inclined hydraulic conductivity (Keff_i) = {0:1.2E}".format(Keff_i), "m/s" )
```

```
Effective vertical hydraulic conductivity (Keff_v) = 3.93E-04 m/s

Effective inclined hydraulic conductivity (Keff_i) = 6.62E-04 m/s
```

In [13]:

```
#
r6_1 = pn.pane.Markdown("""
### Tutorial Problem 12: Hydrologic Triangle
The figure below shows the position of four groundwater observation wells with measured hydraulic heads in m a.s.l.
<br> <br>
**a.** Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.<br><br>
**b.** Indicate the flow direction.

""",width = 400, style={'font-size': '13pt'})

r6_2 = pn.pane.PNG("images/T03_TP12_a.png", width=400)

pn.Row(r6_1,spacer2, r6_2)
```

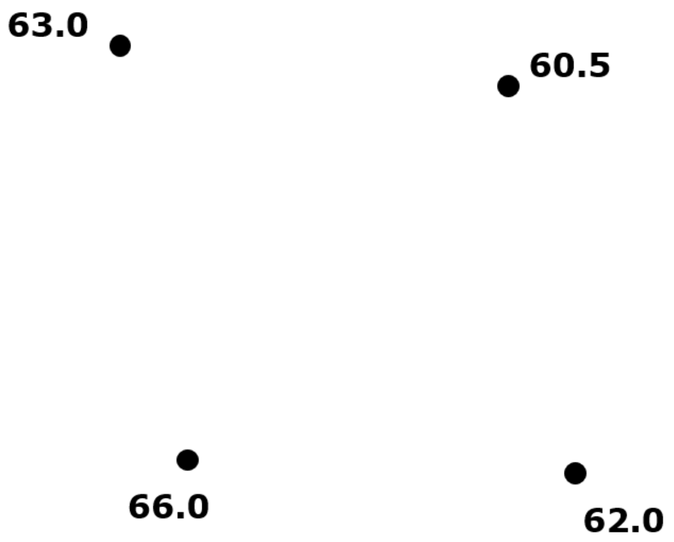
Out[13]:

Tutorial Problem 12: Hydrologic Triangle

The figure below shows the position of four groundwater observation wells with measured hydraulic heads in m a.s.l.

a. Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.

b. Indicate the flow direction.



In [14]:

```
#
r6_3 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 12

Step 1. Connects all the points
""", width=600)

r6_2.object = "images/T03_TP12_b.png"
r6_3
```

Out[14]:

Solution of Tutotrial Problem 12

Step 1. Connects all the points

In [15]:
```
#
r6_4 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 12
Step 2. Divide the connected lines at equal head-level (here = 1 m)
""", width=600)
r6_2.object = "images/T03_TP12_c.png"
```

In [16]:
```
#
r6_4 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 12
Step 3. Join all the equal head lines
""", width=600)
r6_2.object = "images/T03_TP12_d.png"
```

In [17]:
```
r6_4 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 12
Step 4. Mark the flow direction from higher head towards lower head
""", width=600)
r6_2.object = "images/T03_TP12_e.png"
```

In [18]:
```
#
r7_1 = pn.pane.Markdown("""
##Tutorial Problem 13: Flow Nets##

Sketch head isolines and streamlines for the two configurations a) and b) of a well doublette shown below. In both cases flow nets
 should be sketched without and with the uniform flow component.

""",width=800,  style={'font-size': '13pt'})

r7_2 = pn.pane.Markdown("""
 a) withdrawal at both wells:<br><br><br>
""",width=400,  style={'font-size': '13pt'})

r7_3 = pn.pane.PNG("images/T03_TP13_a.png", width=200)

r7_4 = pn.Column(r7_2,r7_3)

r7_5 = pn.pane.Markdown("""
 b) Injection and withdrawl wells:<br><br><br>
""",width=400,  style={'font-size': '13pt'})

r7_6 = pn.pane.PNG("images/T03_TP13_b.png", width=200)

r7_7 = pn.Column(r7_5,r7_6)
r7_8 = pn.Row(r7_4, r7_7)
pn.Column(r7_1, r7_8)
```

Out[18]:

Tutorial Problem 13: Flow Nets

Sketch head isolines and streamlines for the two configurations a) and b) of a well doublette shown below. In both cases flow nets should be sketched without and with the uniform flow component.

a) withdrawal at both wells:

b) Injection and withdrawl wells:

In [19]:
```
r8_1= pn.pane.Markdown("""
#Homework Problems on  Effective Conductivity and Flow Nets <br><br><br>
""", width = 800, style={'font-size': '13pt'})


r8_2= pn.pane.Markdown("""
#There is no obligation to solve homework problems!
""", width = 800, style={'font-size': '13pt', 'color':'red'})

pn.Column(r8_1,r8_2)
```

Out[19]:

Homework Problems on Effective Conductivity and Flow Nets

There is no obligation to solve homework problems!

In [20]:
```
#
r9_1= pn.pane.Markdown("""
###Homework Problem 5: Effective Hydraulic Conductivity
A gravel layer with a thickness of 2.5 m is embedded between two sand layers. Both sand layers have a thickness of
1.5 m and a hydraulic conductivity of 3.7·10<sup>-4</sup> m/s. Steady-state groundwater flow is perpendicular to the layering.
An overall head difference of 5.5 cm and a discharge of 500 l/d per unit area have been determined <br><br>

**a.** Determine the effective hydraulic conductivity.<br><br>
**b.** What is the hydraulic conductivity of the gravel layer?<br><br>
**c.** Which effective hydraulic conductivity would be obtained if flow was assumed to be in parallel with the layering?<br><br>
**d.** Calculate effective hydraulic conductivity if the angle between the flow direction and the layering equals 30°. <br>

""", width = 900, style={'font-size': '13pt'})
r9_1
```

Out[20]:

Homework Problem 5: Effective Hydraulic Conductivity

A gravel layer with a thickness of 2.5 m is embedded between two sand layers. Both sand layers have a thickness of 1.5 m and a hydraulic conductivity of $3.7 \cdot 10^{-4}$ m/s. Steady-state groundwater flow is perpendicular to the layering. An overall head difference of 5.5 cm and a discharge of 500 l/d per unit area have been determined

a. Determine the effective hydraulic conductivity.

b. What is the hydraulic conductivity of the gravel layer?

c. Which effective hydraulic conductivity would be obtained if flow was assumed to be in parallel with the layering?

d. Calculate effective hydraulic conductivity if the angle between the flow direction and the layering equals 30°.

In [21]:

```
#
r10_1= pn.pane.Markdown("""
###Homework Problem 6: Hydrologic Triangle
The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.
 <br><br>

**a.** Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.
<br><br>
**b.** Indicate the flow direction.<br><br>
""", width = 500, style={'font-size': '13pt'})
r10_2 = pn.pane.PNG("images/T03_TH6.png", width=400)

pn.Row(r10_1, r10_2)
```

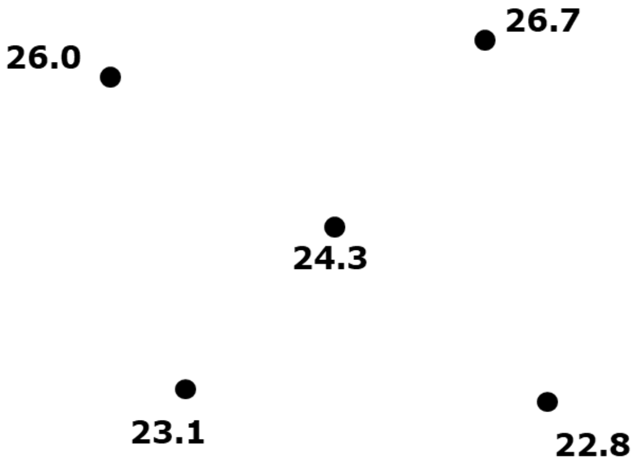
Out[21]:

Homework Problem 6: Hydrologic Triangle

The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.

a. Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.

b. Indicate the flow direction.



In [22]:

```
#
r11_1= pn.pane.Markdown("""
###Homework Problem 7: Flow Nets
Sketch head isolines and streamlines for the well doublette shown below.
In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.
 <br><br><br><br><br><br>
 """, width = 900, style={'font-size': '13pt'})

r11_2 = pn.pane.PNG("images/T03_TH7.png", width=400)

r11_3= pn.pane.Markdown("""
 <br><br><br><br><br><br>
 """, width = 900, style={'font-size': '13pt'})
pn.Column(r11_1, r11_2, r11_3)
```

Out[22]:

Homework Problem 7: Flow Nets

Sketch head isolines and streamlines for the well doublette shown below. In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.