Previous Lecture

- **quantification of three-dimensional groundwater flow**
- **two-dimensional groundwater flow in confined aquifers**
- **two-dimensional groundwater flow in unconfined aquifers**
- **complete formulation of groundwater flow problems**
- **questions?**

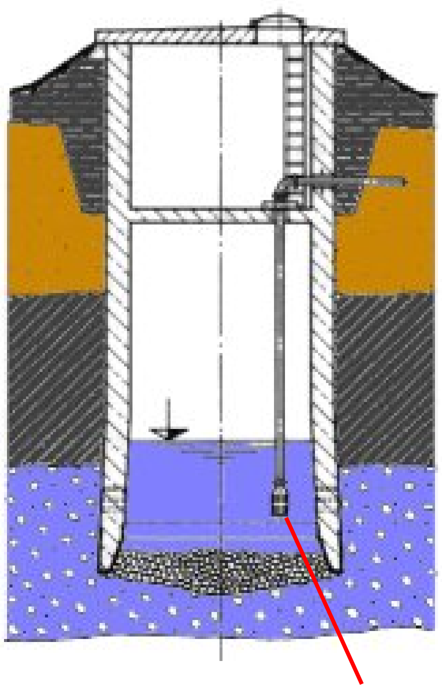
Today:

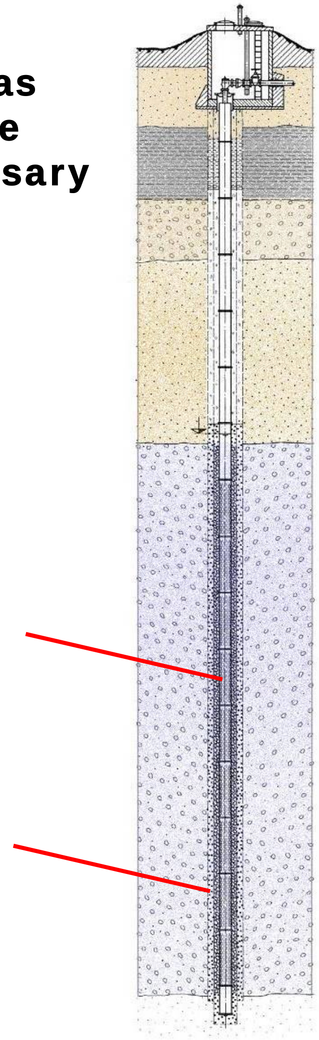
- **wells – overview**
- **groundwater flow near wells operated at steady state**
- **aquifer characterisation by pumping tests**

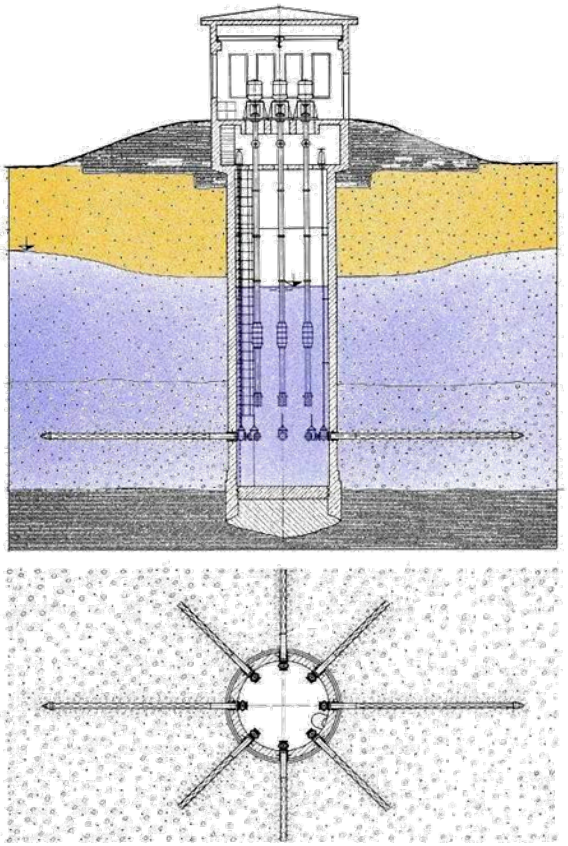
Wells – Overview

What is a Well?

A <u>well</u> is a shaft or a hole that has been sunk, dug or drilled into the earth to extract water (Int. Glossary of Hydrology).







Int. Glossary of Hydrology: http://www.cig.ensmp.fr/~hubert/glu/HINDENT.HTM

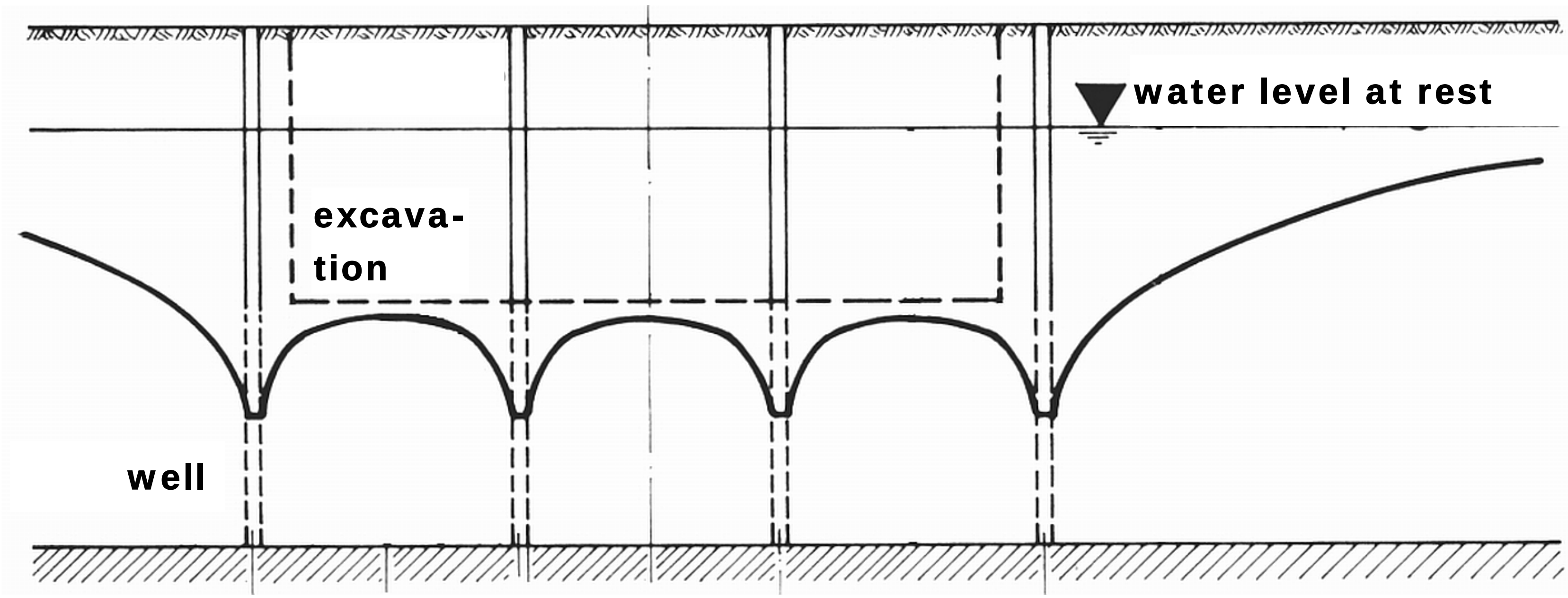
Using Wells

Wells can be used for

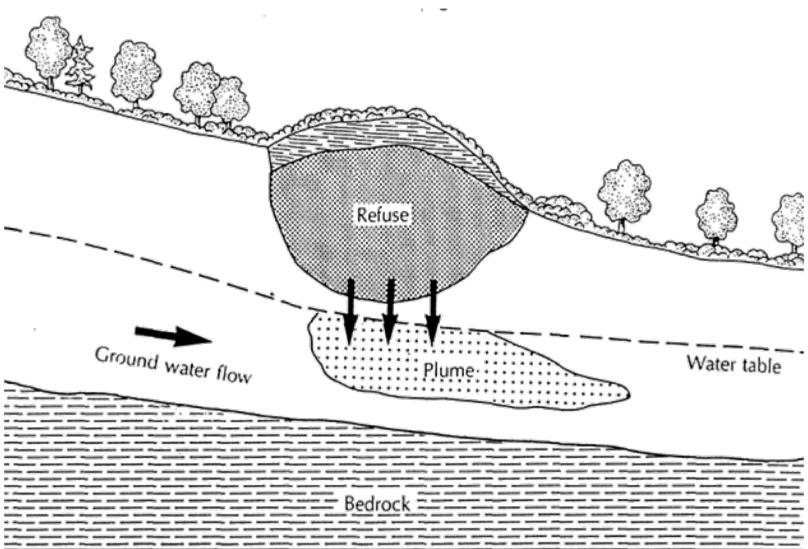
- **water supply (house-holds, agriculture, indu-stry)**
- **lowering the groundwa-ter level (excavations, open-pit mining)**
- **remediation of aquifer contamination ("pump and treat")**
- **aquifer characterisation (pumping tests)**

Apart from aquifer charac-terisation, wells are usual-ly operated at steady state (constant pumping rate).

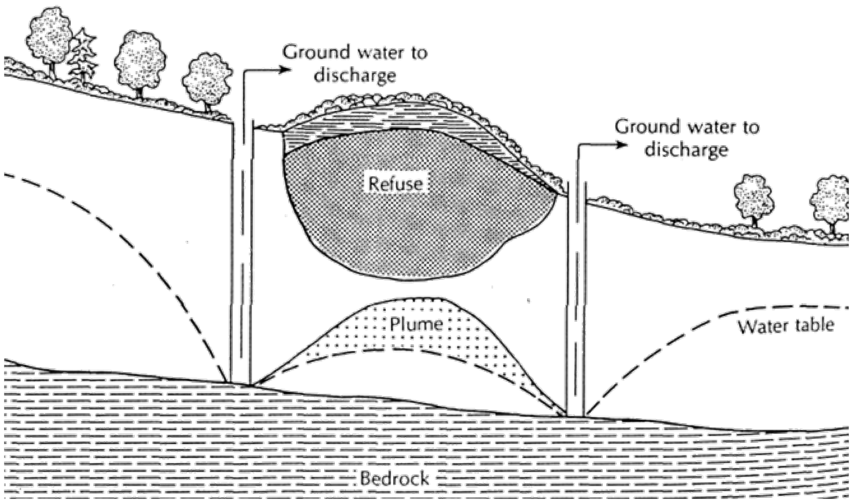
Example: Lowering the Groundwater Level



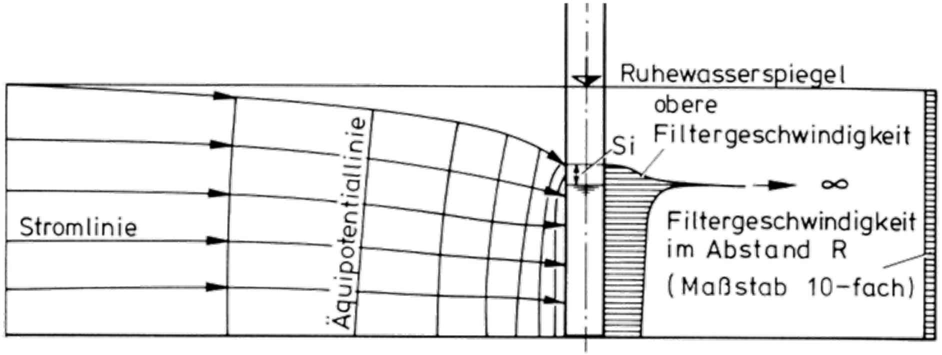
Example: Pump and Treat at Contaminated Sites



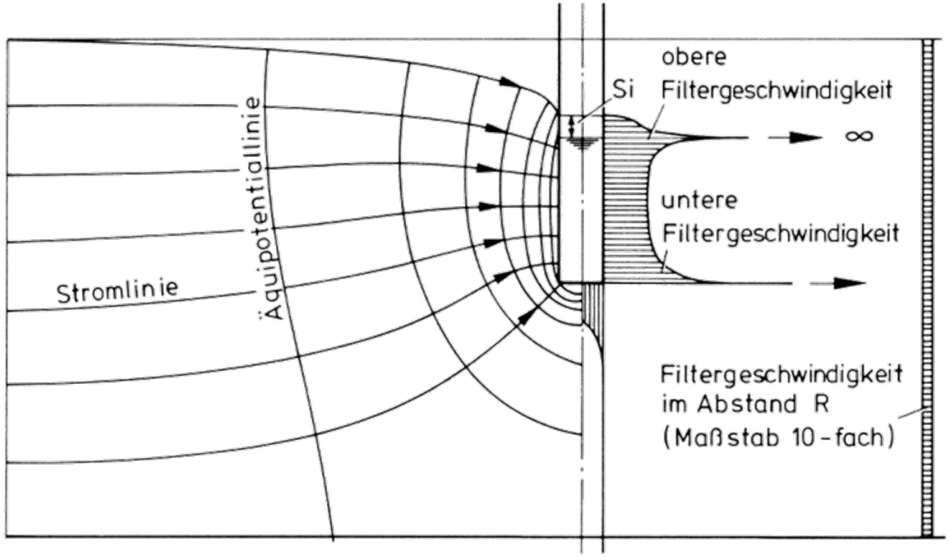
Before pumping



Fully vs. Partially Penetrating Wells



<u>fully penetrating well</u> =
well which extends through the whole saturated depth of an aquifer and is constructed in such a manner that water is permitted to enter the well screen over its length (Int. Glossary of Hydrology)



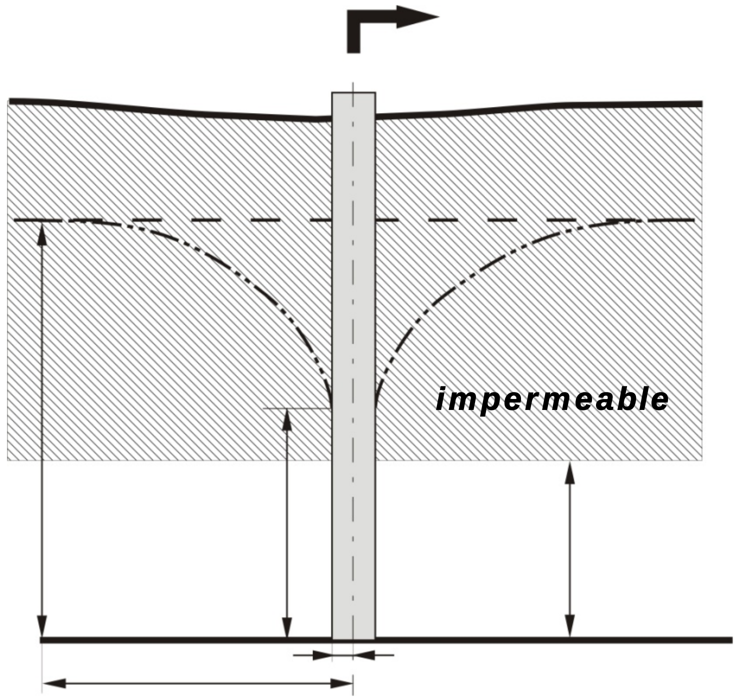
Groundwater Flow Near Wells Operated at Steady State

Basic Questions and General Approach

- **Which relevant quantities are needed to describe steady-state flow towards a well?**
- **What is the quantitative relationship between the hydraulic parameters under steady-state conditions?**

- **In order to answer these questions the law of continuity (conservation of volume) and Darcy‘s law have to be applied.**
- **It is convenient to study the confined and the unconfined case separately.**

Cone of Depression in a Confined Aquifer

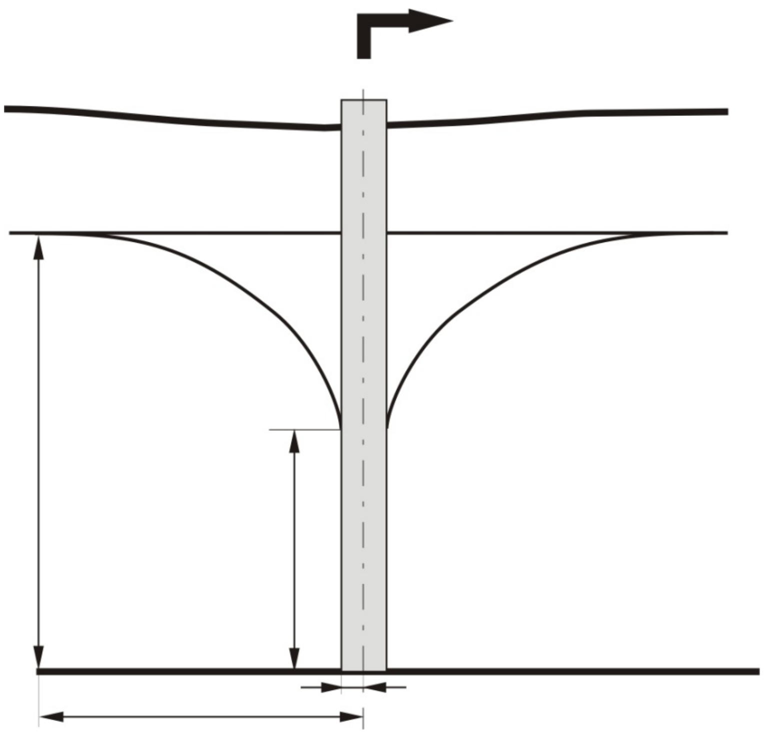


Relevant quantities:

- **pumping rate** Q **[L³ / T]**
- **thickness** m **[L]**
- **hydraulic conductivity** K **[L / T]**
- **water level at rest** H **[L]**
- **water level in the pumping well** h **[L]**
- **radius of influence** R **[L] ***
- **well radius** r_w **(incl. gravel pack!) [L]**
- **drawdown** $s = H - h$ **[L]**

*** The radius of influence is the distance from the well axis at which the drawdown becomes negligible or unobservable.**

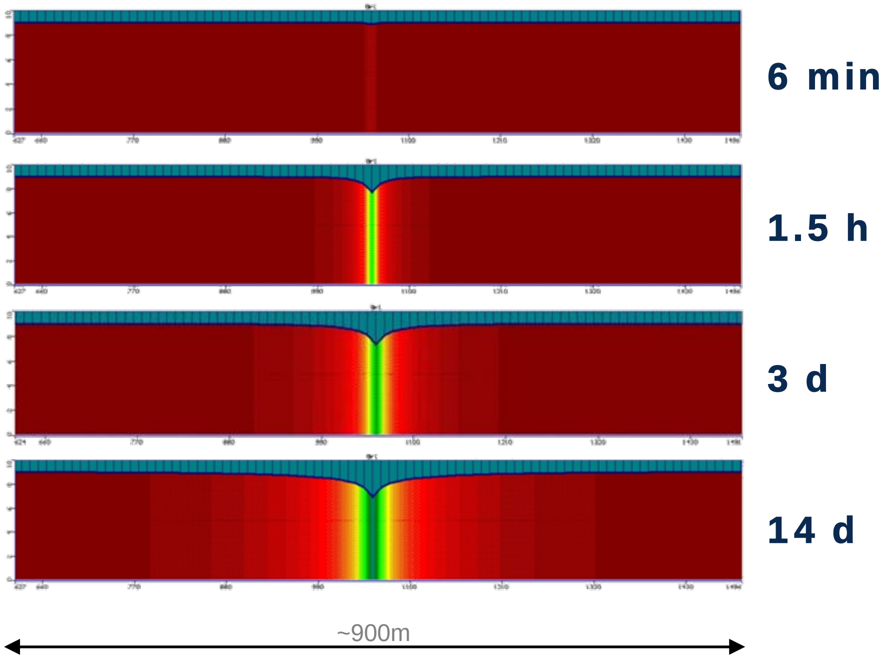
Cone of Depression for an Unconfined Aquifer



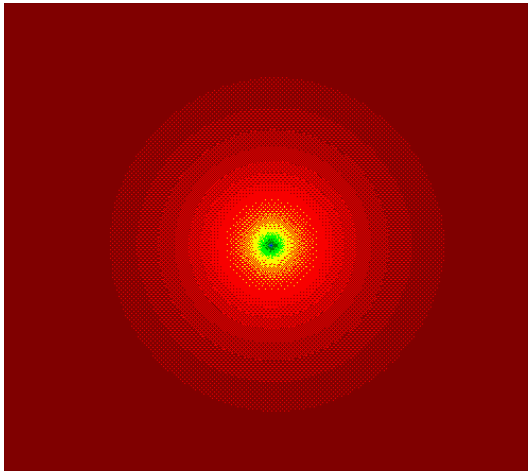
Relevant quantities:

- **pumping rate Q [L³ / T]**
- **hydraulic conductivity K [L / T]**
- **water level at rest H [L]**
- **water level in the pumping well h [L]**
- **radius of influence R [L]**
- **well radius r_w (incl. gravel pack!) [L]**
- **drawdown $s = H - h$ [L]**

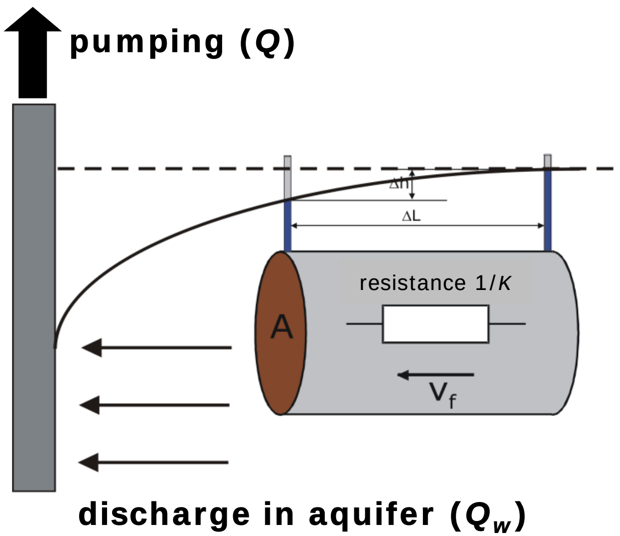
Computer Simulation of a Cone of Depression

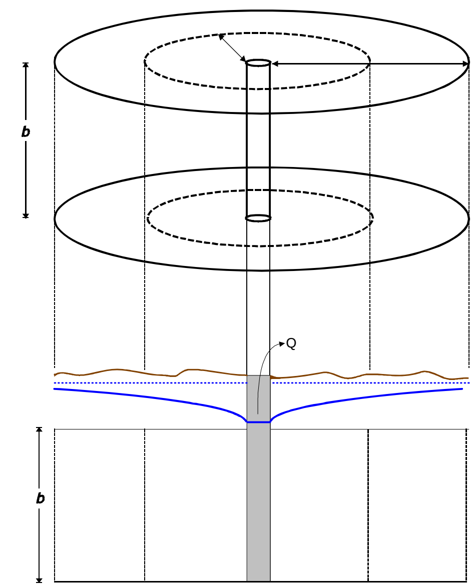


top view (14 d)



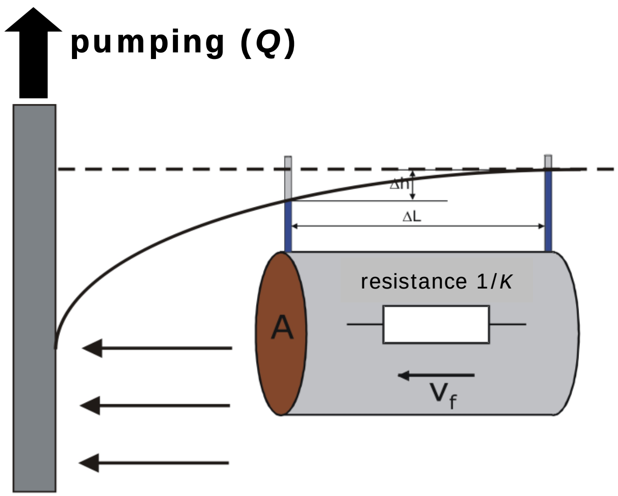
Law of Continuity





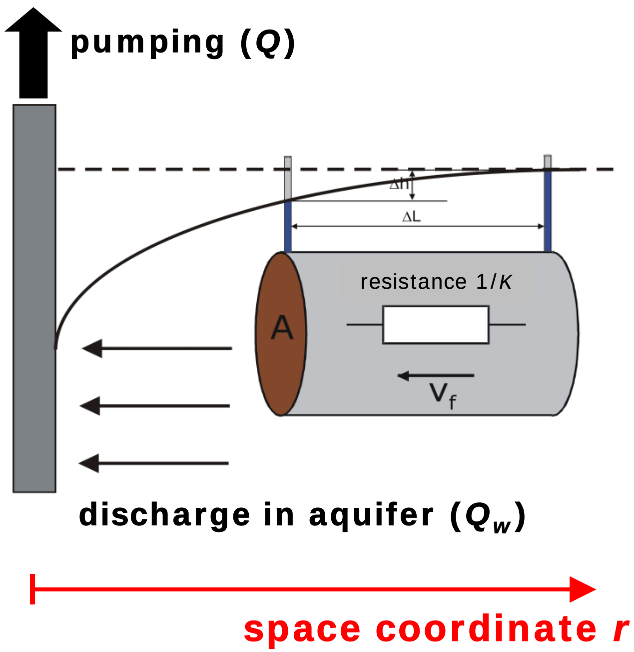
- Under steady-state conditions, the law of continuity implies that the pumping rate Q corresponds to the discharge Q_w near the well.
- In addition, under steady-state conditions the law of continuity implies that there is the same discharge at all cross sections which completely surround the well ("mantle of a cylinder").
- Mathematically speaking: Q_w = const.

Darcy's Law for Flow Towards a Well I



- The coordinate r is introduced to denote the radial distance from the well axis.
- In general, Darcy's law relates the Darcy velocity v_f and the hydraulic gradient i.
- If the hydraulic gradient was constant in space, we would obtain: $i = \Delta h / \Delta r$.
- However, the hydraulic gradient i depends on the distance r within a cone of depression (see figure).
- Qualitatively speaking, i is decreasing with increasing distance from the well.
- If the hydraulic gradient is not constant in space, the ratio $\Delta h / \Delta r$ has to be replaced by the derivative dh / dr.

Darcy‘s Law for Flow Towards a Well II



hydraulic gradient:

$$i(r) = \frac{dh}{dr}(r)$$

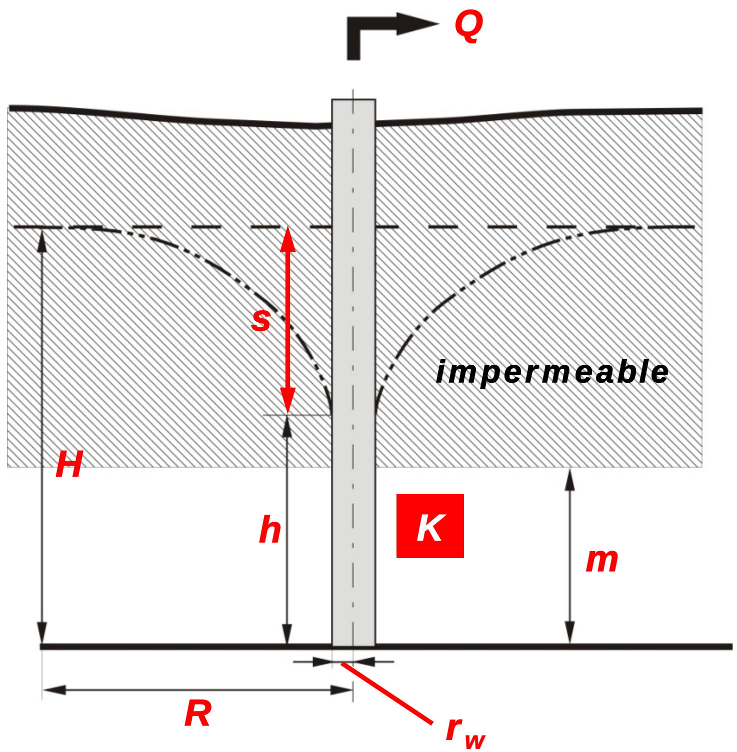
Darcy‘s law:

$$v_f(r) = -K \cdot i(r) = -K \cdot \frac{dh}{dr}(r)$$

discharge in the aquifer:

$$Q_w = A \cdot v_f = -A \cdot K \cdot \frac{dh}{dr}(r)$$

Flow Towards a Well in a Confined Aquifer I



discharge in the aquifer:

$$Q_w = A \cdot v_f = -A \cdot K \cdot \frac{dh}{dr}(r)$$

area *A* of a cross section at distance *r* from the well axis:

$$A = 2 \cdot \pi \cdot r \cdot m$$

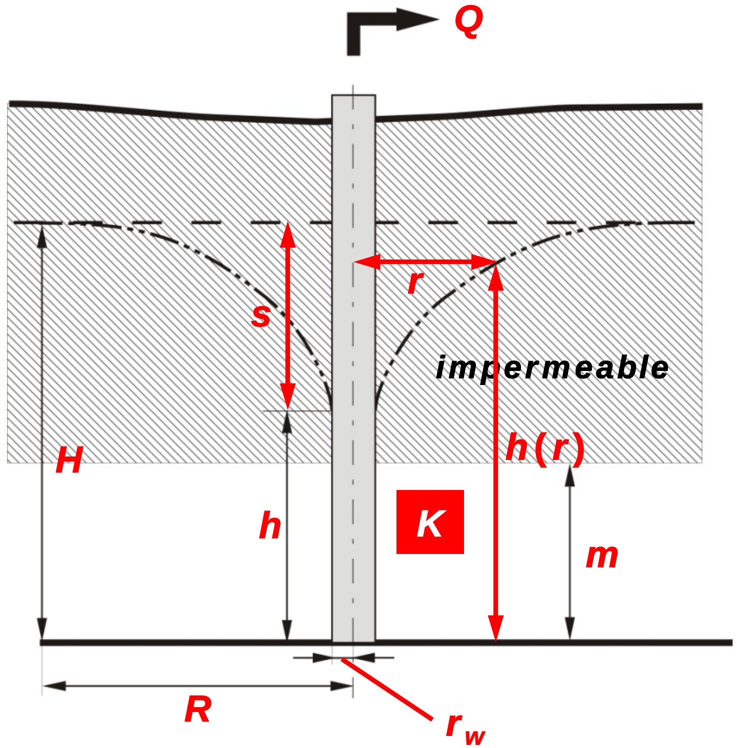
insert in the equation for the discharge:

$$Q_w = -2 \cdot \pi \cdot r \cdot m \cdot K \cdot \frac{dh}{dr}(r)$$

This relationship is a first-order differential equation for hydraulic head *h*(*r*). It can be solved by separation of variables:

$$dh(r) = -\frac{Q_w}{2 \cdot \pi \cdot m \cdot K} \cdot \frac{dr}{r}$$

Flow Towards a Well in a Confined Aquifer II



after separation of variables:

$$dh(r) = -\frac{Q_w}{2\cdot\pi\cdot m\cdot K}\cdot\frac{dr}{r}$$

integration from r_w to R (right-hand side) and from h to H (left-hand side):

$$\int_h^H dh(r) = -\frac{Q_w}{2\cdot\pi\cdot m\cdot K}\cdot\int_{r_w}^R\frac{dr}{r}$$

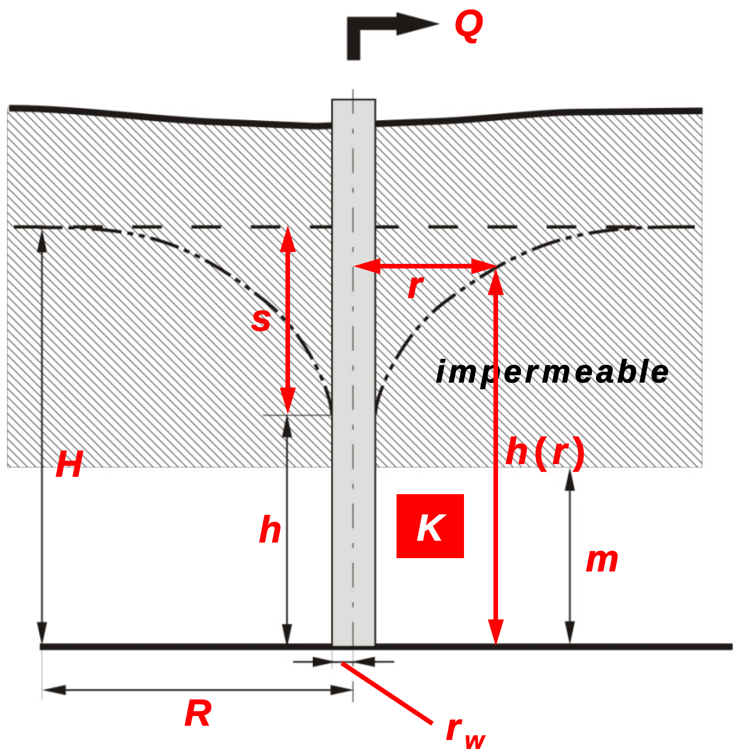
indefinite integrals:

$$\left[h(r)\right]_h^H = -\frac{Q_w}{2\cdot\pi\cdot m\cdot K}\cdot\left[\ln r\right]_{r_w}^R$$

insert limits of integration:

$$H-h = -\frac{Q_w}{2\cdot\pi\cdot m\cdot K}\cdot\left(\ln R-\ln r_w\right)$$

Flow Towards a Well in a Confined Aquifer III



after inserting the limits of integration:

$$H - h = -\frac{Q_w}{2\cdot\pi\cdot m\cdot K}\cdot(\ln R - \ln r_w)$$

solving for Q_w:

$$Q_w = -\frac{2\cdot\pi\cdot m\cdot K\cdot(H-h)}{\ln R - \ln r_w}$$

The negative sign on the right-hand side indicates that flow is anti-parallel to the direction of the coordinate axis.

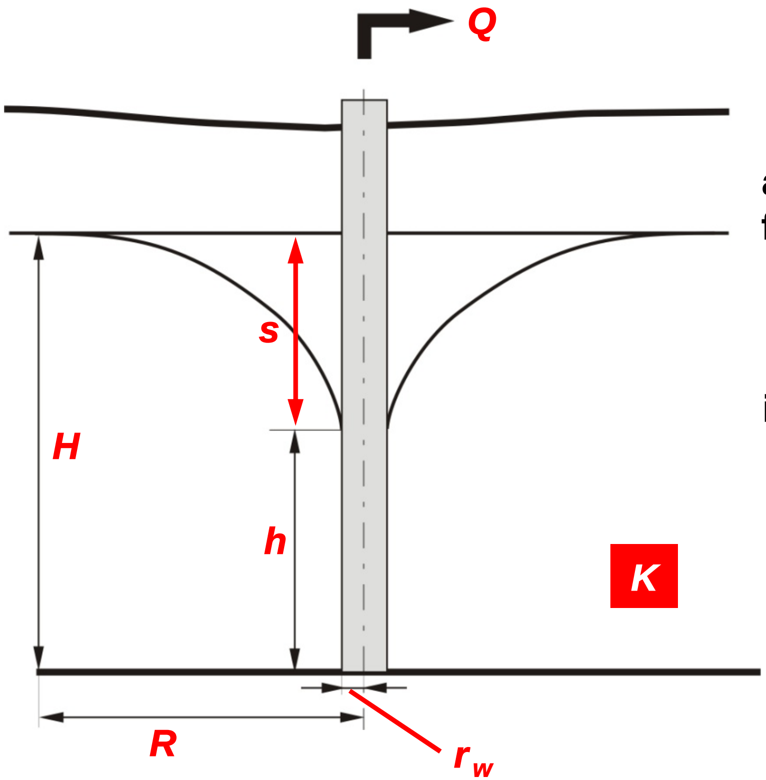
Frequently, the pumping rate Q is used instead of Q_w and the negative sign is omitted:

$$Q = \frac{2\cdot\pi\cdot m\cdot K\cdot(H-h)}{\ln R - \ln r_w} = \frac{2\cdot\pi\cdot m\cdot K\cdot(H-h)}{\ln(R/r_w)}$$

with decadic logarithm:

$$Q = \frac{2\cdot\pi\cdot m\cdot K\cdot(H-h)}{2.3\cdot(\lg R - \lg r_w)} = \frac{2\cdot\pi\cdot m\cdot K\cdot(H-h)}{2.3\cdot\lg(R/r_w)}$$

Flow Towards a Well in an Unconfined Aquifer I



discharge in the aquifer:

$$Q_w = A \cdot v_f = -A \cdot K \cdot \frac{dh}{dr}(r)$$

area *A* of a cross section at distance *r* from the well axis:

$$A = 2 \cdot \pi \cdot r \cdot h(r)$$

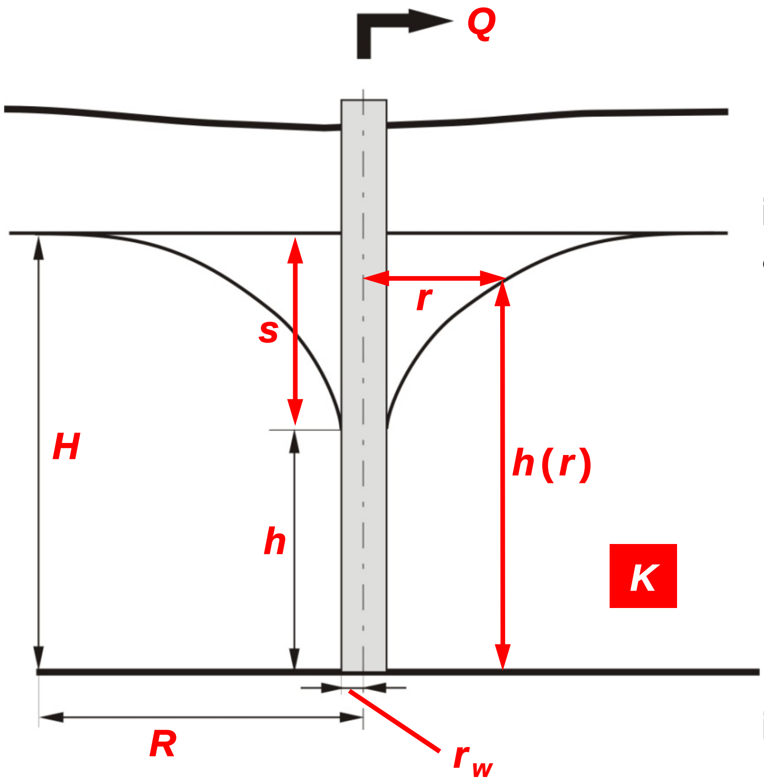
insert in the equation for the discharge:

$$Q_w = -2 \cdot \pi \cdot r \cdot h(r) \cdot K \cdot \frac{dh}{dr}(r)$$

This relationship is a first-order differential equation for hydraulic head *h*(*r*). It can be solved by separation of variables:

$$h(r) \cdot dh(r) = -\frac{Q_w}{2 \cdot \pi \cdot K} \cdot \frac{dr}{r}$$

Flow Towards a Well in an Unconfined Aquifer II



after separation of variables:

$$h(r)\cdot dh(r) = -\frac{Q_w}{2\cdot\pi\cdot K}\cdot\frac{dr}{r}$$

integration from r_w to R (right-hand side) and from h to H (left-hand side):

$$\int_h^H h(r)\cdot dh(r) = -\frac{Q_w}{2\cdot\pi\cdot K}\cdot\int_{r_w}^R\frac{dr}{r}$$

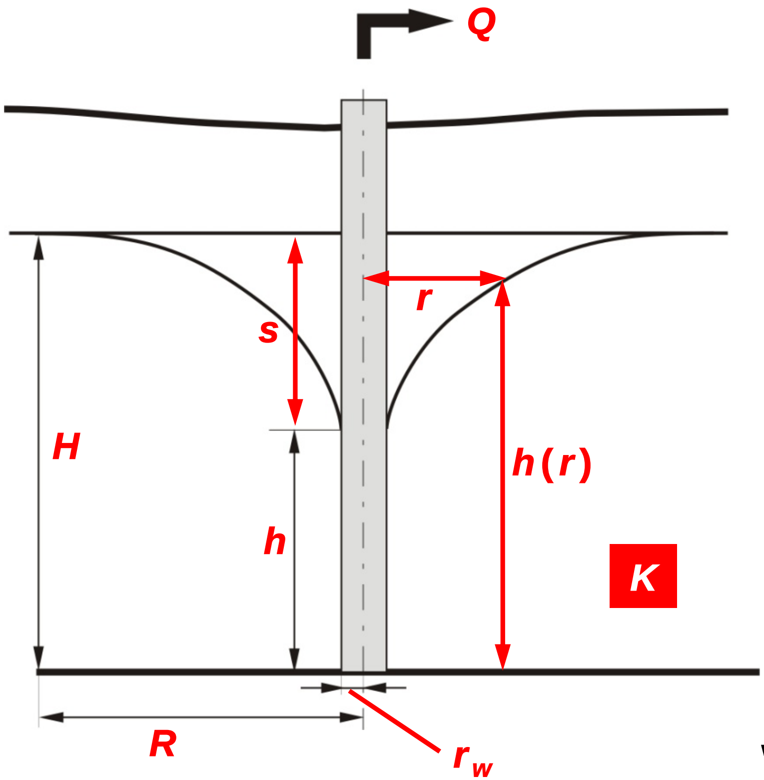
indefinite integral:

$$\left[\frac{1}{2}\cdot h(r)^2\right]_h^H = -\frac{Q_w}{2\cdot\pi\cdot K}\cdot\left[\ln r\right]_{r_w}^R$$

insert limits of integration:

$$\frac{1}{2}\cdot\left(H^2 - h^2\right) = -\frac{Q_w}{2\cdot\pi\cdot K}\cdot\left(\ln R - \ln r_w\right)$$

Flow Towards a Well in an Unconfined Aquifer III



after inserting limits of integration:

$$H^2 - h^2 = -\frac{Q_w}{\pi \cdot K} \cdot \left(\ln R - \ln r_w\right)$$

solving for Q_w:

$$Q_w = -\frac{\pi \cdot K \cdot \left(H^2 - h^2\right)}{\ln R - \ln r_w}$$

The negative sign on the right-hand side indicates that flow is anti-parallel to the orientation of the coordinate axis.

Frequently, the pumping rate Q is used instead of Q_w and the negative sign is omitted:

$$Q = \frac{\pi \cdot K \cdot \left(H^2 - h^2\right)}{\ln R - \ln r_w} = \frac{\pi \cdot K \cdot \left(H^2 - h^2\right)}{\ln\left(R / r_w\right)}$$

with decadic logarithm:

$$Q = \frac{\pi \cdot K \cdot \left(H^2 - h^2\right)}{2.3 \cdot \left(\lg R - \lg r_w\right)} = \frac{\pi \cdot K \cdot \left(H^2 - h^2\right)}{2.3 \cdot \lg\left(R / r_w\right)}$$

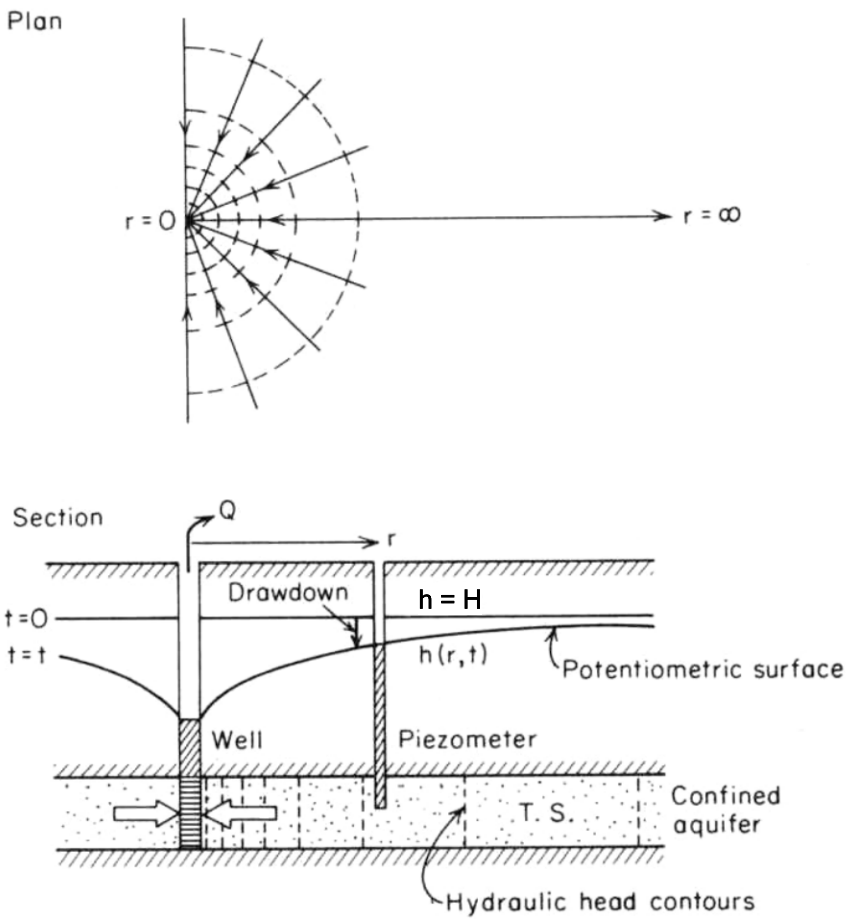
Radius of Influence

Lembke (1886, 1887)	$R = H\ (K\ /\ 2\ N)^{1/2}$
Weber (Schultze, 1924)	$R = 2.45\ (H\ K\ t\ /\ S)^{1/2}$
Kusakin (Aravin and Numerov, 1953)	$R = 1.9\ (H\ K\ t\ /\ S)^{1/2}$
Siechardt (Certousov, 1962)	$R = 3000\ s\ K^{1/2}$
Kusakin (Certousov, 1949)	$R = 575\ s\ (H\ K)^{1/2}$
s = drawdown in pumping well, t = pumping time, N = groundwater recharge, S = storage coefficient, K = hydraulic conductivity, H = water level at rest (unconfined aquifer). In confined aquifers, H has to be replaced by the aquifer thickness m.	

- There are several (semi-)empirical formulae to estimate the radius of influence. (The above table is incomplete.)
- Practitioners appear to prefer the formulae of Siechardt and Kusakin.
- In both formulae, K has to be expressed in m/s and all other quantities in m!
- R depends on drawdown $s = H - h$ in both formulae. Try-and-error or iterative strategies have to be used to determine R and h.

Aquifer Characterisation by Pumping Tests

Some General Remarks on Pumping Tests



(Freeze und Cherry, 1979)

- **Pumping tests are used to estimate aquifer properties like hydraulic conductivity (*K*), transmissivity (*T*) or storativity (*S*).**
- **Pumping results in an evolving cone of depression.**
- **The decrease in hydraulic head (or increase in drawdown) with time is recorded in one or more observa-tion wells (and sometimes also in the pumping well itself).**
- **A variety of different schemes exist to evaluate pumping test data. The appropriate method has to be se-lected according to the specific setting (confined or unconfined, layered system, horizontal or incli-ned aquifer bottom etc.).**
- **A well known approach to derive *T* and *S* from pumping test data was developed by Theis.**

Applicability of the Theis Method

Charles V. Theis
(1900 – 1987)

Pumping test data can be evaluated according to Theis (1935) if the following assumptions are (approximately) justified:

- The aquifer is confined, homogeneous and isotropic.
- The aquifer thickness is uniform.
- The aquifer bottom is horizontal.
- The well is fully penetrating.
- The well radius is very small as compared to the radius of influence.
- The pumping rate is constant within the measurement period.
- There is no vertical flow component.
- The evolution of the cone of depression is not influenced by other hydraulic factors (surface water, impermeable boundaries etc.).

Drawdown According to Theis

The time-dependent drawdown *s* in an observation well, which is a distance *r* apart from the pumping well, is given by

$$s(r,t) = \frac{Q}{4\pi T} \cdot W(u)$$

with the well function

and the dimensionless variable $u = \frac{Sr^2}{4Tt}$

Logarithmic Representation

The decadic logarithm of the equations

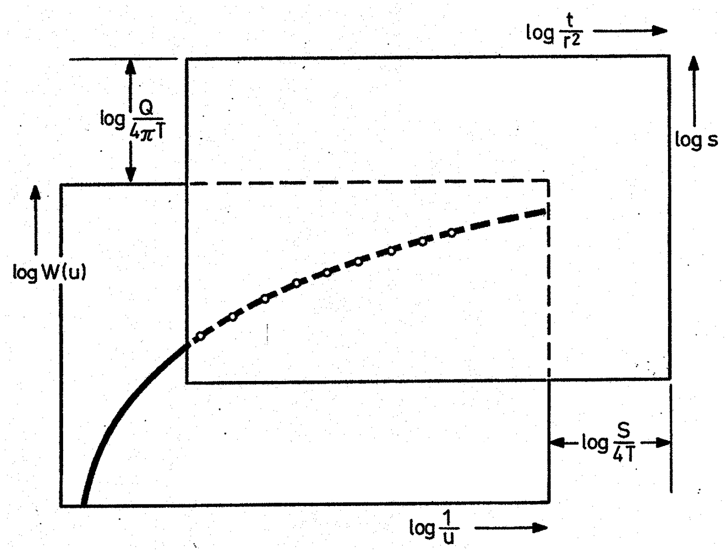
$$s(r,t) = \frac{Q}{4\pi T} \cdot W(u) \qquad u = \frac{Sr^2}{4Tt}$$

is taken. This yields, together with a slight rearrangement of the second equation:

$$\lg s = \lg \frac{Q}{4\pi T} + \lg W(u) \qquad \lg \frac{t}{r^2} = \lg \frac{S}{4T} + \lg \frac{1}{u}$$

These two equations are used to derive *T* and *S* from drawdown data.

This can either be done by applying special computer software or manually by a graphical method which is illustrated in the figure.

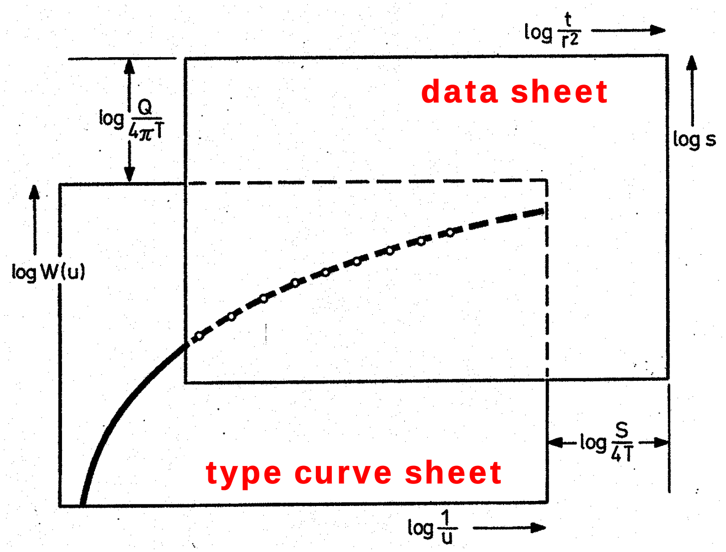


Manual Comparison of Data and Type Curve

- **The logarithm of drawdown, lgs, is plotted against lg(*t / r²*) in the data sheet.**
- **The logarithm of the well function, lg*W*, is plotted against lg(1 / u) in a type curve sheet. (This curve is independent from aquifer properties!)**
- **Both sheets are put on top of each other such that the data coincide with some part of the type curve.**
- **The shifts along the vertical and the horizontal axes correspond to the constant terms in the equations**

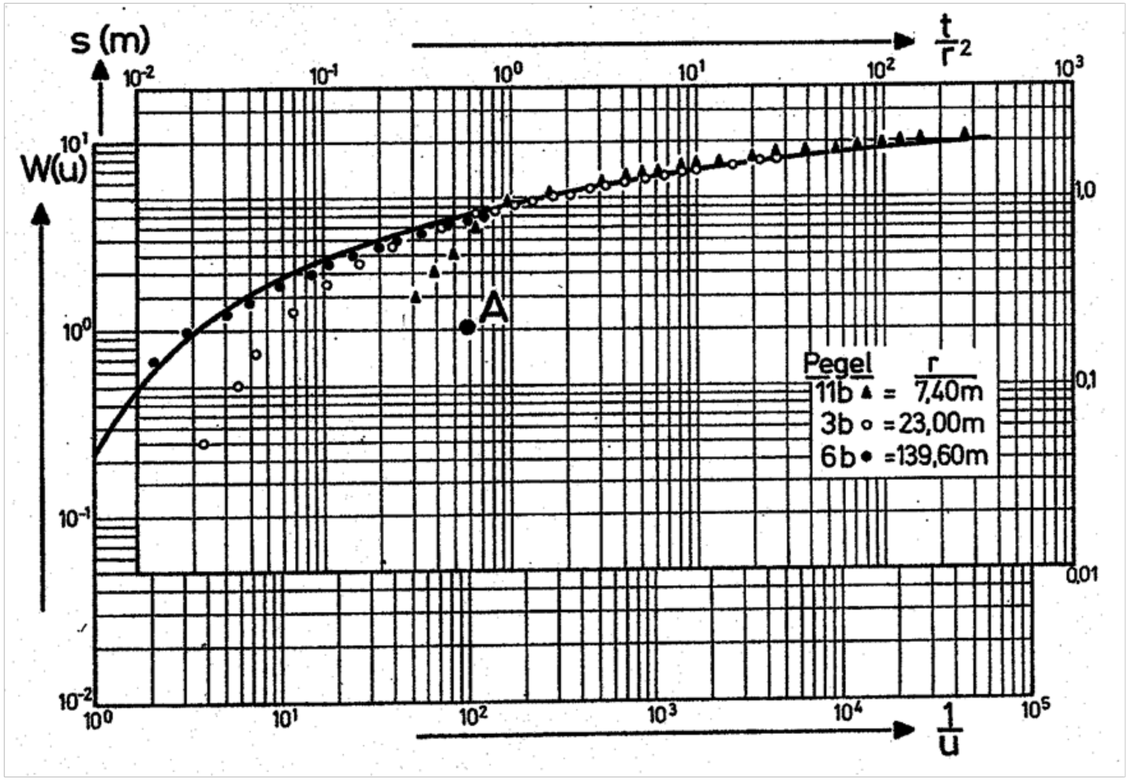
$$\lg s = \lg \frac{Q}{4\pi T} + \lg W(u)$$

$$\lg \frac{t}{r^2} = \lg \frac{S}{4T} + \lg \frac{1}{u}$$



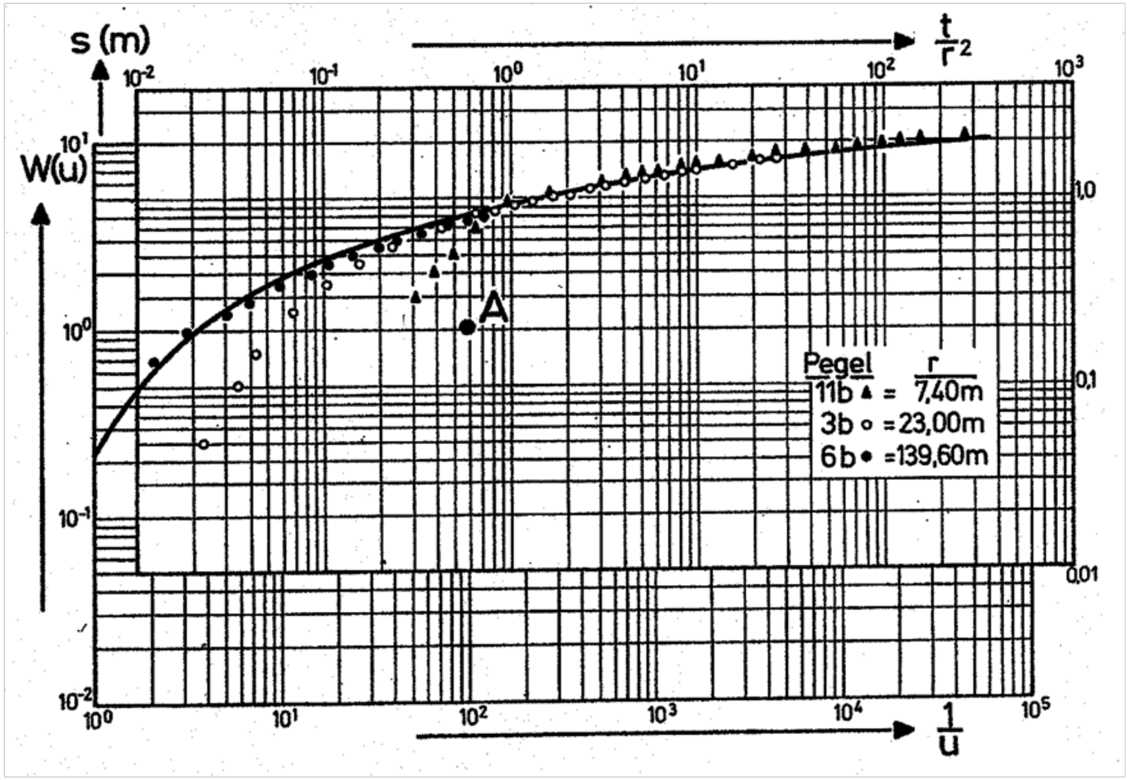
- **The constant term in the upper equation can then be solved for *T*.**
- **Finally, the constant term in the lower equation can be solved for *S*.**

Example



- The practical application of the Theis method is facilitated by selecting a <u>match point</u> in the range of the data such that corresponding values W_A and $1 / u_A$ are "simple".
- In the example:
  $W_A = 10^0 = 1$
  $1 / u_A = 10^2 = 100$
- Next, values for s and t / r^2 at the match point are determined.
- In the example:
  $s = 0.2$
  $t / r^2 = 0.57$

Example – Continued



- **Previous results:**
  $W_A = 10^0 = 1$
  $1/u_A = 10^2 = 100$
  $s = 0.2$
  $t/r^2 = 0.57$
- **Next, we get T via**

$$T = \frac{Q}{4\pi s} \cdot W_A$$

- **For $Q = 26.7$ l/s we obtain $T = 1.06 \cdot 10^{-2}$ m²/s.**
- **Finally, S is obtained from**

$$S = \frac{4T \cdot t / r^2}{1/u_A}$$

- **In the example: $S = 2.42 \cdot 10^{-4}$**

Computer-Based Comparison of Data and Type Curve

- **Of course, measured data and type curve can be favourably compared with the help of computer programs.**
- **These programs work pretty much the same as shown for the manual comparison. In particular, results are again displayed in a double-logarithmic diagram.**