Previous Lecture

- **groundwater and aquifers**
- **pressure**
- **storage properties**
- **questions?**

Today

- **Darcy‘s law (1D)**
- **hydraulic conductivity**
- **intrinsic permeability**
- **velocities, travel time and pore volume**

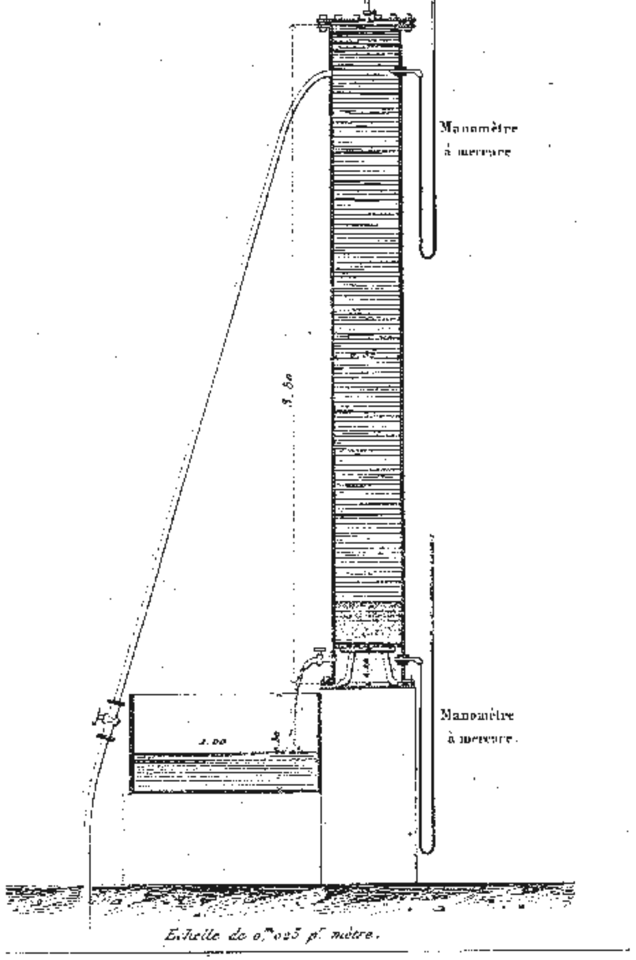
Darcy`s Law (1D)

Darcy’s Experiment

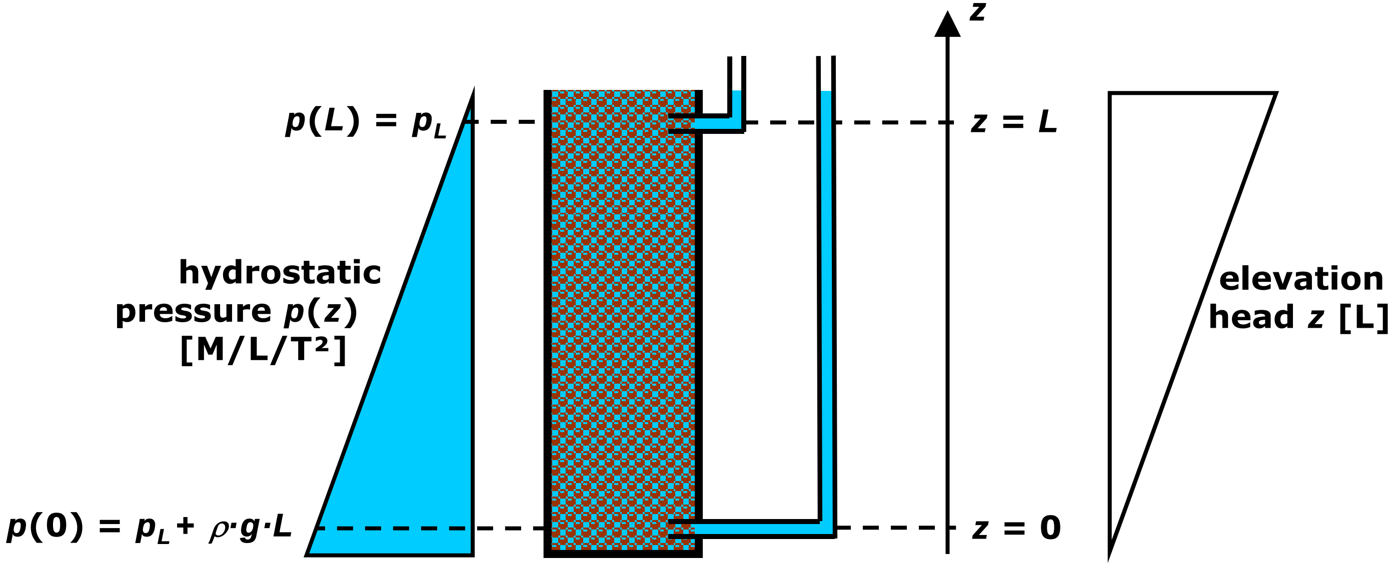
- **column with constant cross section (*A* = const.)**
- **sand as filling material (“porous medi-um“)**
- **voids between sand grains completely filled with water (saturated conditions)**
- **constant discharge through the column (*Q* = const.)**
- **Purpose:**
  to quantify the relationship between dis-charge, flow velocity, pressure differen-ce, flow distance, and cross section

Fig. 3.

Appareil destiné à déterminer la loi de l'écoulement de l'eau à travers le sable



A Quick Reminder on Hydrostatic Pressure



- **From previous considerations we know that hydrostatic pressure differences will not allow to fully quantify water flow.**
- **In fact, elevation head differences have to be considered as well.**
- **In the above figure, the increase of hydrostatic pressure head with depth is compensated by an according decrease of elevation head. As a consequence, there is no water flow.**
- **Water flow requires changes of the *sum* of both heads.**

Hydraulic Head I

- **Hydraulic head or piezometric head is the sum of hydrostatic pressure head and elevation head:**

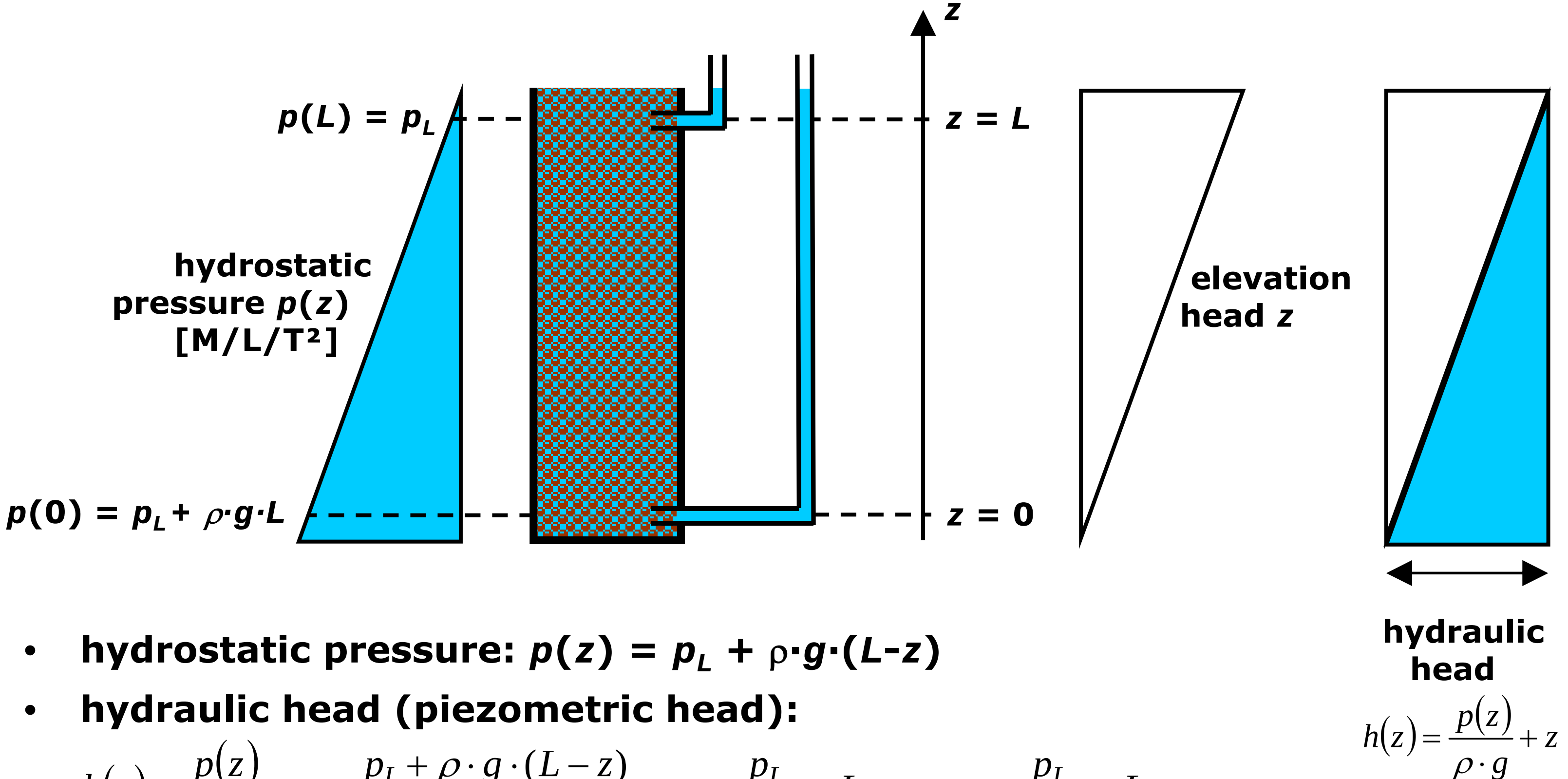
$$h(z) = \frac{p(z)}{\rho \cdot g} + z$$

- **In this equation it is assumed that the *z*-axis is oriented upward.**
- **If the *z*-axis points downward, we have:**

$$h(z) = \frac{p(z)}{\rho \cdot g} - z$$

- **Water flow is governed by differences in hydraulic head (*not* by differences in pressure head *alone*!).**

Hydraulic Head II



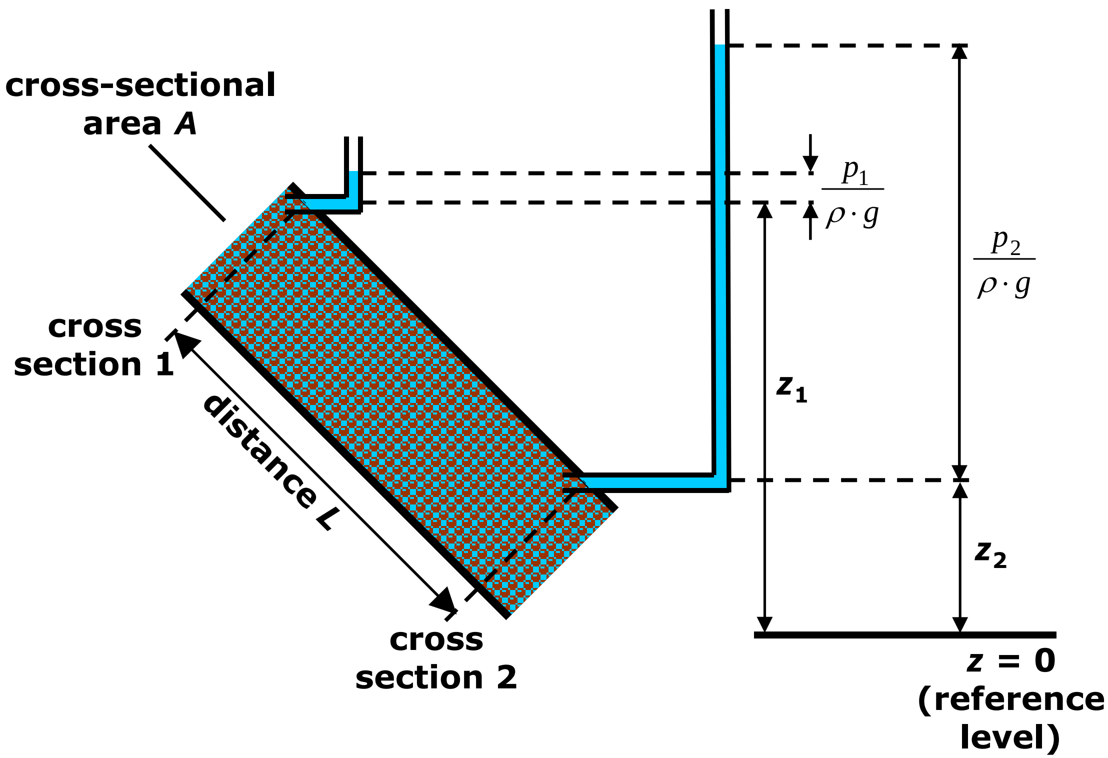
- **hydrostatic pressure: $p(z) = p_L + \rho \cdot g \cdot (L - z)$**
- **hydraulic head (piezometric head):**

$$h(z) = \frac{p(z)}{\rho \cdot g} + z = \frac{p_L + \rho \cdot g \cdot (L - z)}{\rho \cdot g} + z = \frac{p_L}{\rho \cdot g} + L - z + z = \frac{p_L}{\rho \cdot g} + L = \text{const.}$$

- **hydraulic head difference between observation points: $\Delta h = 0$**

Hydraulic Heads in Darcy's Experiment

- **column with water flow ($Q \neq 0$), arbitrarily oriented**
- **measurement points ("standpipes" or "piezometers") for hydrostatic pressure *p* and hydraulic head (piezometric head) *h* at distance *L***

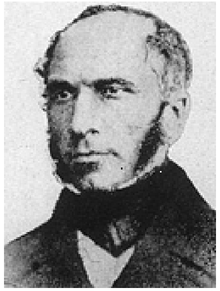


measured hydraulic heads:

$$h_1 = \frac{p_1}{\rho \cdot g} + z_1 \qquad h_2 = \frac{p_2}{\rho \cdot g} + z_2$$

- **here: $h_1 < h_2$**
- **Thus, flow is from cross section 2 towards cross section 1.**
- ***Differences* of hydraulic head are independent from the position of the origin of the *z*-axis!**

Darcy's Law for 1D Groundwater Flow



Henry Darcy
(1803 – 1858)

- **The volumetric flow rate or discharge is**
  - proportional to the cross-sectional area,
  - proportional to the difference in hydraulic head,
  - inversely proportional to the travel distance.
- **Groundwater flow is directed from regions with high hydrau-lic head to regions with low hydraulic head.**
- **Equation (Darcy, 1856):**

$$Q = -AK\frac{\Delta h}{L}$$

Hydraulic Conductivity

Some Basics

- **The ratio $\Delta h/L$ is called <u>hydraulic gradient</u>.**
- **The hydraulic gradient is dimensionless [-].**
- **The value of hydraulic conductivity corresponds to the volumetric flow rate through a unit cross section under a unit hydraulic gradient.**
- **The dimension of hydraulic conductivity is L/T.**
- **Hydraulic conductivity depends on properties of the fluid (density, viscosity, temperature) *and* on properties of the porous medium (effective porosity, grain size distribution).**
- **typical values for hydraulic conductivity:**

gravel	10^{-2} m/s – 10^{-1} m/s
coarse sand	≈ 10^{-3} m/s
medium sand	10^{-4} m/s – 10^{-3} m/s
fine sand	10^{-5} m/s – 10^{-4} m/s
silty sand	10^{-7} m/s – 10^{-5} m/s
silt	10^{-9} m/s – 10^{-6} m/s
clay	$< 10^{-9}$ m/s

Obtaining Hydraulic Conductivities

- **In general, hydraulic conductivity can be obtained in the laboratory or in the field.**
- **Laboratory methods can be based on**
  – the evaluation of sieve analysis data ("indirect method"),
  – some version of Darcy's experiment ("direct method").
- **Field methods include pump tests which will be covered later in this semester.**
- **Advantages of laboratory methods:**
  controlled conditions, small sample size (easy handling), lower costs, larger number of experiments
- **Disadvantages of laboratory methods:**
  disturbed samples, additional pathways at column walls, small sample size (randomly high or low *K*), flushing of fine material
- **Field experiments are much more complicated and expensive than laboratory tests. Resulting hydraulic conductivities represent averages over an aquifer volume which is covered by the experiment. The size of this volume depends on subsurface properties and on the experimental method used.**

Estimating Hydraulic Conductivity from Sieve Analysis Data

- **Sieve analysis data can be evaluated to estimate hydraulic conductivity of unconsolidated media.**
- **There are several empirical methods.**
- **The simplest one dates back to Hazen (1892):** $K = 0.0116 \cdot d_{10}^2$

  with
  ***K* = hydraulic conductivity (m/s)**
  d_{10} = grain diameter (mm) corresp. to 10 % of cumulative mass fraction
- **NOTE:**
  Hazen's equation is only valid for the indicated units. Conversion of units may be necessary before using this formula.
- **In an extended version (see Excel sheet entitled "sieve analysis") the effect of temperature θ is considered:**

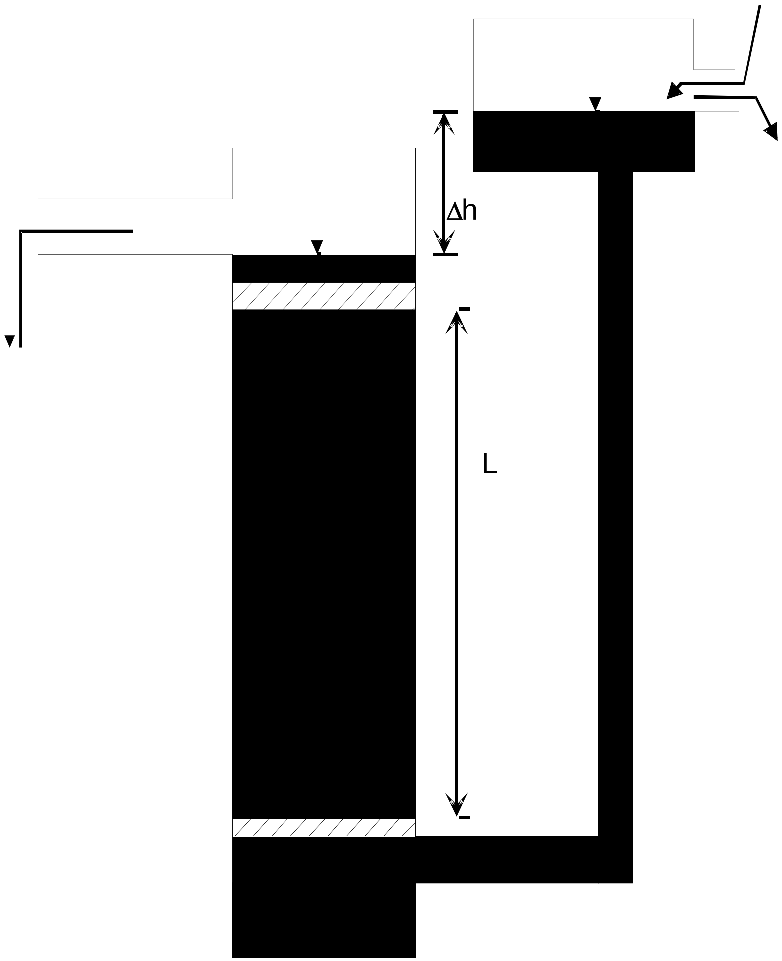
$$K = 0.0116 \cdot d_{10}^2 \cdot (0.7 + 0.03 \cdot \theta)$$

  with
  θ = temperature (°C)
- **NOTE:**
  This equation is only valid for the indicated units. Conversion of units may be necessary before using this formula.

Permeameter

- **Permeameter = instrument to determine hydraulic conductivity**
- **The design of permeameters is based on Darcy's experiment.**
- **Permeameters are used to determine hydraulic conductivities of soil samples in the laboratory.**
- **Two types:**
  – constant-head permeameter
  – falling-head permeameter
- **Constant-head permeameter:**
  Hydraulic heads at inflow and outflow of the Darcy column are constant in time. As a consequence, the discharge is not changing with time.
- **Falling-head permeameter:**
  The hydraulic head at the outflow of the Darcy column is not changing, but the hydraulic head at the inflow is decreasing with time. As a result, the discharge also decreases with time.
- **Larger experimental time periods are needed for the falling-head permeameter, in particular if hydraulic conductivity is low. On the other hand, no measurement of discharge or water volume is required.**

Constant-Head Permeameter



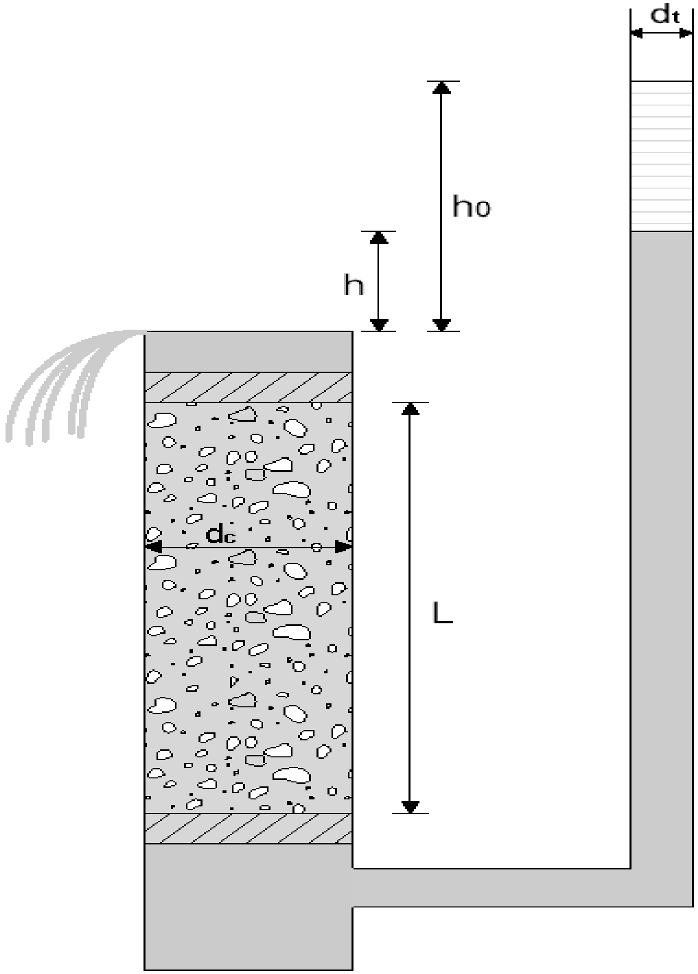
$$K = \frac{QL}{A(h_{in} - h_{out})}$$

with

Q = discharge [L³/T]
L = length of sample [L]
A = cross-sectional area of sample [L²]
h_{in} = hydraulic head at column inlet [L]
h_{out} = hydraulic head at column outlet [L]

h_{out} can be set equal to zero as only head *differences* are important.

Falling-Head Permeameter



$$K = \frac{d_t^2 L}{d_c^2 t} \cdot \ln \frac{h_{in}(0) - h_{out}}{h_{in}(t) - h_{out}}$$

with

L =	**length of sample [L]**
d_c =	**diameter of sample cylinder [L]**
d_t =	**tube diameter [L]**
t =	**time interval [T]**
$h_{in}(0)$=	**initial hydraulic head at column inlet [L]**
$h_{in}(t)$=	**final hydraulic head at column inlet [L]**
h_{out} =	**hydraulic head at column outlet [L]**

h_{out} can be set equal to zero as only head *differences* are important.

Intrinsic Permeability

Hydraulic Conductivity and Intrinsic Permeability

- **As indicated before, hydraulic conductivity depends on properties of the fluid (density, viscosity, temperature) *and* on properties of the porous medium (effective porosity, grain size distribution).**
- **When fluids other than water have to be considered (e.g., in oil exploration) it is important to deal with a parameter which only depends on properties of the porous medium. To this end, other impacts on hydraulic conductivity were quantified.**
- **Experimental results showed that hydraulic conductivity is**
  – proportional to fluid density,
  – proportional to gravitational acceleration,
  – inversely proportional to dynamic fluid viscosity:

$$K = k \cdot \frac{\rho \cdot g}{\eta}$$

Density and Viscosity of Water

- **The table below provides densities and viscosities of water for different temperatures (10 °C ≈ field conditions, 20 °C ≈ laboratory conditions in regions with moderate climate):**

	10 °C	20 °C
density (kg/m³)	999.7	998.2
kinematic viscosity (m²/s)	$1.3101 \cdot 10^{-6}$	$1.0105 \cdot 10^{-6}$
dynamic viscosity (Pa·s)	$1.3097 \cdot 10^{-3}$	$1.0087 \cdot 10^{-3}$

- **Kinematic and dynamic viscosities are related by:**

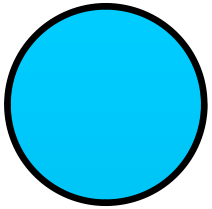
$$\eta = \rho \cdot \nu$$

Properties of Intrinsic Permeability

- **Intrinsic permeability only depends on properties of the porous medium.**
- **The dimension of intrinsic permeability is L^2.**
- **Intrinsic permeability of unconsolidated porous media is roughly proportional to the square of the pore diameter.**
- **The value of intrinsic permeability of a porous medium equals 1 m^2 if a fluid with dynamic viscosity of 1 Pa·s can pass through the porous medium at a Darcy velocity of 1 m/s under a hydrostatic pressure gradient of 1 Pa/m (horizontal flow).**
- **The unit "Darcy" (symbol: D) is sometimes used for intrinsic permeability. Conversion to SI units: 1 D = $0.987 \cdot 10^{-12}$ m^2**
- **The intrinsic permeability of a porous medium equals 1 D if a fluid with dynamic viscosity of 1 mPa·s can pass through the porous medium with a Darcy velocity of 1 cm/s under a hydrostatic pressure gradient of 1 atm/cm.**
- **Intrinsic permeability for**
  – weakly permeable aquifers: 10^{-4} – 10^{-1} D,
  – well permeable aquifers: 10^{-1} – 10^{2} D,
  – highly permeable aquifers: > 10^{2} D.

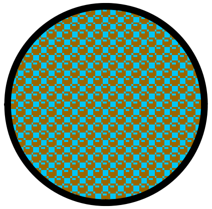
Velocities,
Travel Time
and Pore Volume

Cross-sectional Area



column without porous medium – pipe flow

identical total cross-sectional area *A*



column with porous medium – Darcy experiment

- **Pipe flow: The cross-sectional area of flow coincides with the total cross-sectional area *A*.**
- **Darcy experiment: Only part of the total cross-sectional area *A* is available or flow.**
- **Cross-sectional area of flow in a porous medium = $n_e \cdot A$**

Darcy Velocity and Linear Velocity

- **In groudwater hydraulics, the ratio Q/A is termed Darcy velocity or specific discharge. Frequently, the symbols v_f or q are used.**
- **The Darcy velocity is different from the actual velocity of water in the pore space.**
- **The average flow velocity of water in the pore space is termed (average) linear velocity.**
- **Linear velocity v is related to the cross-sectional area of flow via**

$$v = \frac{Q}{n_e \cdot A}$$

- **As a consequence, the relationship between Darcy velocity and linear velocity is given by $v = v_f / n_e$.**
- **Linear velocity is never smaller than Darcy velocity.**
- **Sometimes the symbol v_a is used to denote linear velocity.**

Some Values for Linear Velocity

- **Typical values for linear velocities in unconsolidated aquifers:**
  sand: 0.5 m/d – 1 m/d
  gravel: 30 m/d – 300 m/d
- **Roughly speaking, linear velocity in unconsolidatd aquifers is increasing with grain size.**
- **Linear velocities in fractured or karstified aquifers can be rather high along fractures or conduits:**
  fractures: 200 m/d – 1.2 km/d
  karst conduits: 3 km/d – 14 km/d
- **On the contrary, linear velocities are very low in the rock matrix of consolidated aquifers (1 cm/d or even less).**

Travel Time and Pore Volume

- **The travel time of water through a column of length L is given by $t = L/v$. (The linear velocity v has to be used here, not the Darcy velocity!)**
- **The travel time may also be called residence time.**
- **The travel time through a column is termed pore volume. It can be understood as the time needed to replace the water in the column.**
- **In this sense, the pore volume is not a volume but a time!**
  1 PV corresponds to the ratio L/v.
- **The pore volume (PV) is frequently used for normalisation purposes in order to better compare column experiments conducted under different flow velocities. This is mostly done for studying the transport behaviour of chemicals dissolved in water and their arrivals at the column outlets.**