In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import ipysheet as ips
import panel as pn
from scipy import stats
pn.extension('katex')
```

Tutorial 7

solutions of homework problems 5 – 7

tutorial problems on Flow in natural systems

1. Flow in confined Aquifers

2. Flow in unconfined Aquifers

Solution of Homework Problems 5 – 7

In [2]:
```
#Homework Problem 5
h5_1= pn.pane.Markdown("""
###Homework Problem 5:
""", width = 900, style={'font-size': '13pt'})

s3=pn.Spacer(width=150)

h5_2= pn.pane.LaTeX(r"""
In this problem we consider the roughness of the inner-surface of the facture
that can affect the conductivity of water (at 9$^\circ C$) in the rock matrix. In this example we consider
a composite rock matrix with average fracture aperature of 30 $\mu$m and the average
spacing between fractures to be 0.5 m.  Further, we will consider a general relative roughness
of the inner surface ($\zeta$) of the fracture to be 0.4 and neglect the influence of non-fractured conductivity ($K_{mat}$).
We find the effect of surface roughness on conductivity.
""", width = 900, style={'font-size': '13pt'})

h5_3= pn.pane.Markdown("""
###Hint for solving homework problem 5:
""", width = 900, style={'font-size': '13pt'})

h5_4= pn.pane.LaTeX(r"""
With surface roughness in consideration, the conductivity of rock matrix can be obtained from:
$$
K_t = \frac{g \rho e^3}{12 C F_d \mu} + K_{mat}
$$


With $ C = (1+ 8.8\zeta^{1.5})$ describes the fracture roughness for depending on relative roughness $\zeta$
""", width = 900, style={'font-size': '13pt'})
pn.Column(h5_1, s3, h5_2, s3, h5_3, s3, h5_4)
```

Out[2]:

Homework Problem 5:

In this problem we consider the roughness of the inner-surface of the facture that can affect the conductivity of water (at 9$^{\circ}C$) in the rock matrix. In this example we consider a composite rock matrix with average fracture aperature of 30 μm and the average spacing between fractures to be 0.5 m. Further, we will consider a general relative roughness of the inner surface (ζ) of the fracture to be 0.4 and neglect the influence of non-fractured conductivity (K_{mat}). We find the effect of surface roughness on conductivity.

Hint for solving homework problem 5:

With surface roughness in consideration, the conductivity of rock matrix can be obtained from:

$$K_t = \frac{g\rho e^3}{12CF_d\mu} + K_{mat}$$

With $C = (1 + 8.8\zeta^{1.5})$ describes the fracture roughness for depending on relative roughness ζ

In [3]:
```
# Solution Homework Problem 5
#Given
B_5 = 30 * 10**-6 # m, aperature
F_d5 = 0.5 # m, average spacing between fractures
z_a = 0.4# (), relative roughness
mu_5 = 0.0013465 # N-s/m^2, dynamic visocity of water at 9°C
rho_5 = 999.73 # kg/m^3, density of water at 9°C
g_5 = 9.81 # N/kg, gravitational constant
K_mat5 = 0 # m/s, matrix conductivity neglected in this problem

#interim calculation
C = 1+8.8*z_a**1.5 # (), C calculation

#Solution
K_t5 = g_5*rho_5*B_5**3/(12*C*F_d5) + K_mat5 # m/s, conductivity with inclusion of roughness factor
K_t5b = g_5*rho_5*B_5**3/(12*F_d5) + K_mat5 # m/s, conductivity without roighness consideration
dif_K = K_t5b/K_t5 # m/s, ratio of conductivities without and with roughness

#output
print("The conductivity with inclusion of roughness factor is: {0:1.3e}".format(K_t5), "m/s \n")
print("The conductivity without inclusion of roughness factor is :{0:1.3e}".format(K_t5b), "m/s \n")
print("The ratio of conductivities without and with roughness is:{0:1.2f}".format(dif_K), "\n")
print("Over 3 times higher conductivity is found when roughness is not considered.\n This will also lead to same increase in linear velo
city")
```

```
The conductivity with inclusion of roughness factor is: 1.368e-11 m/s

The conductivity without inclusion of roughness factor is :4.413e-11 m/s

The ratio of conductivities without and with roughness is:3.23

Over 3 times higher conductivity is found when roughness is not considered.
 This will also lead to same increase in linear velocity
```

In [5]:

```
#Homework Problem 6

h6_1 = pn.pane.Markdown("""

### Homework Problem 6: Hydrologic triangle ###

The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.

<br>
a) Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.
<br>

b) Indicate the flow direction.<br> <br>

""",width = 700, style={'font-size': '13pt'})

h6_2 = pn.pane.PNG("images/T05_2a.png", width=300)

pn.Column(h6_1, h6_2)
```

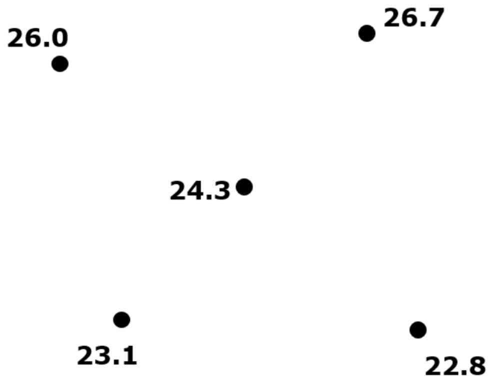
Out[5]:

Homework Problem 6: Hydrologic triangle

The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.

a) Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.

b) Indicate the flow direction.



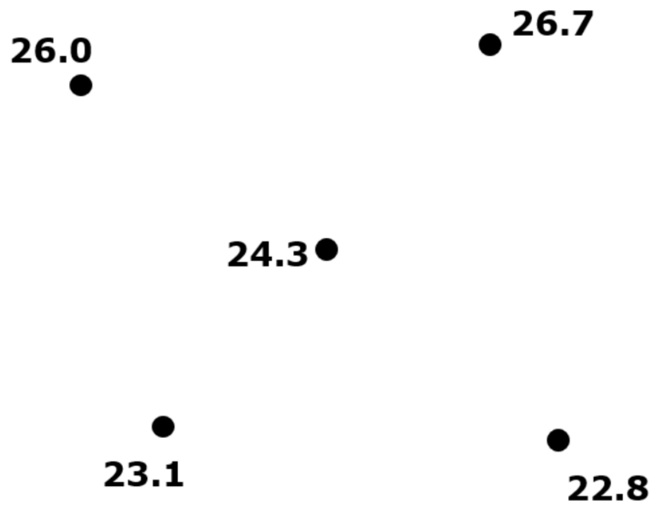
In [4]:

```
#Solution Homework Problem 6:
h6_3 = pn.pane.Markdown("""
### Solution of Homework Problem 6 ###
**See Lecture 06 Slides 8--10 for more information**

""",width = 700, style={'font-size': '13pt'})

h6_4 = pn.pane.PNG("images/T05_2a.png", width=400)

pn.Column(h6_3, h6_4)
```

Out[4]:

Solution of Homework Problem 6

See Lecture 06 Slides 8–10 for more information



In [6]:

```
h6_4.object = "images/T05_2b.png"
```

In [7]:

```
h6_4.object = "images/T05_2c.png"
```

In [8]:

```
h6_4.object = "images/T05_2c.png"
```

In [9]:

```
h6_4.object = "images/T05_2d.png"
```

In [6]:

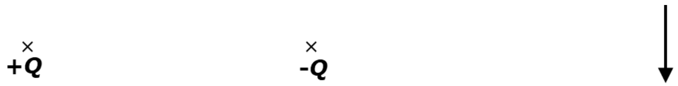
```
h6_4.object = "images/T05_2e.png"
```

In [7]:
```python
# Homework Problem 7

h7_1= pn.pane.Markdown("""
###Homework Problem 7: Flow Nets
Sketch head isolines and streamlines for the well doublette shown below.
In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.
 <br><br><br><br><br><br>
 """, width = 900, style={'font-size': '13pt'})

h7_2 = pn.pane.PNG("images/T03_TH7.png", width=400)

h7_3= pn.pane.Markdown("""
 <br>
 """, width = 900, style={'font-size': '13pt'})

pn.Column(h7_1, h7_2, h7_3)
```

Out[7]:

Homework Problem 7: Flow Nets

Sketch head isolines and streamlines for the well doublette shown below. In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.



In [8]:

```
# Solution of Homework Problem 7

h7_4= pn.pane.Markdown("""
### Solution of Homework Problem 7: Flow Nets ###
The flownet is presented in the figure below.

**See  lecture 06, slides 17--19 for more information**

The solution provided in qualitative.

 """, width = 900, style={'font-size': '13pt'})

h7_5 = pn.pane.PNG("images/T07_H7.png", width=700)



pn.Column(h7_4, h7_5)
```

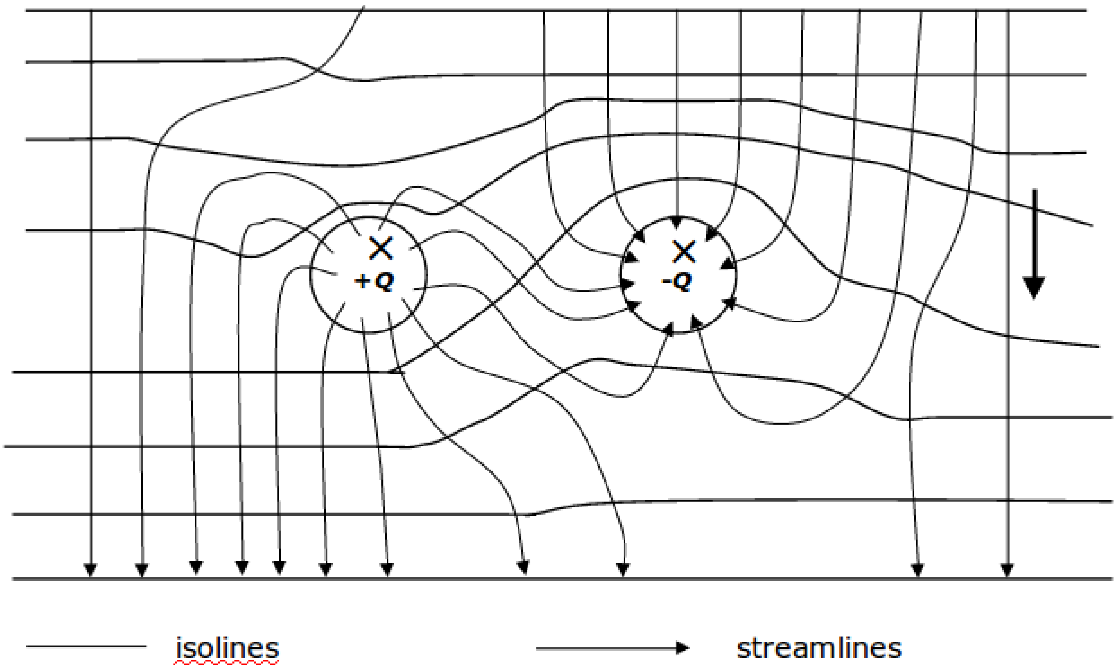
Out[8]:

Solution of Homework Problem 7: Flow Nets

The flownet is presented in the figure below.

See lecture 06, slides 17–19 for more information

The solution provided in qualitative.



Tutorial Problems on flow in natural systems

1. Flow in Unconfined Aquifers

In [27]:

```
# Tutorial problem 20
r1_1 = pn.pane.Markdown("""
### Tutorial Problem 20: Flow in Confined Aquifer with a Uniform Thickness ###
""",width = 500, style={'font-size': '13pt'})

r1_2 = pn.pane.LaTeX(r"""
A confined aquifer is 30 m thick and 5 km wide. Two observation wells are located 1.5 km apart
in the direction of flow. The head in well 1 is 90 m and in well 2 it is 85.0 m. The hydraulic conductivity
is 1.5 m/d.
<br><br>
1. What is the total daily flow of water through the aquifer?
<br><br>
2. What is the elevation of the potentiometric surface at a point located 0.5 km from well $h_1$
and 1 km from well $h_2$?
""",width = 600, style={'font-size': '13pt'})

r1_3 = pn.pane.PNG("images/T06_Y1.png", width=350)

r1_4 = pn.pane.Markdown("""
### Solution of Problem 20:###
""",width = 600, style={'font-size': '13pt'})


r1_5 = pn.Column(r1_1, r1_2, r1_4)
pn.Row(r1_5, r1_3)
```

Out[27]:

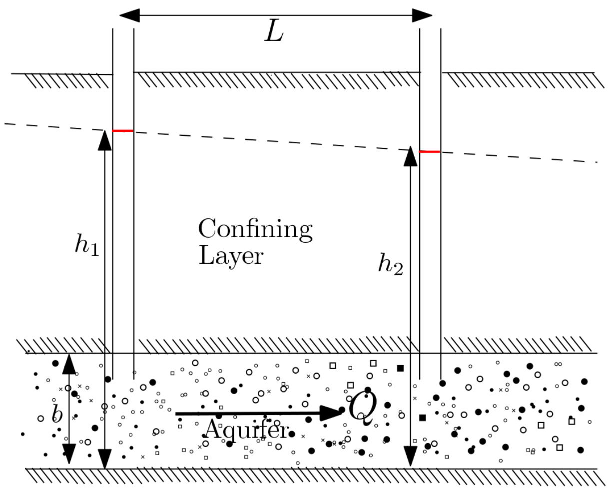
Tutorial Problem 20: Flow in Confined Aquifer with a Uniform Thickness

A confined aquifer is 30 m thick and 5 km wide. Two observation wells are located 1.5 km apart in the direction of flow. The head in well 1 is 90 m and in well 2 it is 85.0 m. The hydraulic conductivity is 1.5 m/d.

1. What is the total daily flow of water through the aquifer?

2. What is the elevation of the potentiometric surface at a point located 0.5 km from well h_1 and 1 km from well h_2?

Solution of Problem 20:



In [10]:

```
# solution
r1_6 = pn.pane.LaTeX(r"""
From Darcy Law:
$$ q' = K b \frac{dh}{dL} \qquad\qquad \text{(eq. 1A)}$$
where,
$q'$ is the flow per unit width [L$^2$T$^{-1}$], <br>
$b$ is aquifer thicknes [L]<br>
$K$ is Hydraulic Conductivity [LT$^{-1}$]<br>
and $\frac{dh}{dl}$ = hydraulic gradient [-]<br><br>

Since the thickness of the aquifer is uniform, any hydraulic head between two known
heads ($h_1$ and $h_2$) can be obtained by rearranging the above equation, from
$$
h_2 = h_1 - \frac{q'}{Kb}x
\qquad\qquad \text{(eq. 1B)}$$
where $x$ is the distance from $h_1$
""",width = 900, style={'font-size': '13pt'})

r1_6
```

Out[10]:

From Darcy Law:

$$ q' = K b \frac{dh}{dL} \qquad\qquad \text{(eq. 1A)}$$

where, q' is the flow per unit width [L^{2}T^{-1}],
b is aquifer thicknes [L]
K is Hydraulic Conductivity [LT^{-1}]
and $\frac{dh}{dl}$ = hydraulic gradient [-]

Since the thickness of the aquifer is uniform, any hydraulic head between two known heads (h_1 and h_2) can be obtained by rearranging the above equation, from

$$h_2 = h_1 - \frac{q'}{Kb}x \qquad\qquad \text{(eq. 1B)}$$

where x is the distance from h_1

In [11]:
```
# Given are:
m_1 = 30 # m, uniform thinckness of aquifer
w_1 = 5 * 1000 # m, width of the aquifer
d_l = 1.5 * 1000 # m, distance between wells
hy1_w1 = 90 # m, head in well 1
hy1_w2 = 85 # m, head in well 2
K_1 = 1.5 # m/d, conductivity in aquifer


#Solution 1
dh_y1 = (hy1_w1 - hy1_w2)/d_l # (-), head gradient
Q_y1 = K_1*m_1*dh_y1*w_1 # m^3/day, discharge using the first eq. above.

#Solution 2
w_2 = 0.5 *1000 # m, distance from well 1
q_1 = Q_y1/w_1 # m^2/d, flow per unit width
h_y1 = hy1_w1-(q_1/(K_1*m_1))*w_2 # head at 0.3 Km from Well 1, using the second equation


#output
print("The daily discharge from the aquifer is: {0:1.2f}".format(Q_y1), "m\u00b3/d")
print("The head at 0.3 Km from well 1 is : {0:1.2f}".format(h_y1), "m")
```

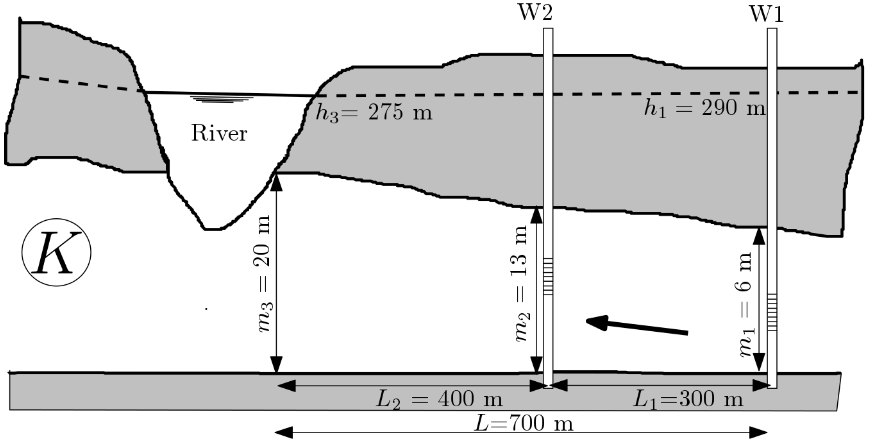
```
The daily discharge from the aquifer is: 750.00 m³/d
The head at 0.3 Km from well 1 is : 88.33 m
```

In [15]:
```
# Tutorial Problem 21
r2_1 = pn.pane.Markdown("""
### Tutorial Problem 21 ###
""",width = 800, style={'font-size': '13pt'})

r2_2 = pn.pane.LaTeX(r"""
Presented below in the figure is the available information of an aquifer cross-section.
The aquifer is confined and of
variable thickness across the cross-section. It has a uniform conductivity $5.6 \times 10^{-5}$ m/s
The total Discharge from the aquifer of width 500 m is required to be obtained.
""",width = 600, style={'font-size': '13pt'})

r2_3 = pn.pane.PNG("images/T06_Y2.png", width=500)

pn.Column(r2_1, r2_2, r2_3)
```

Out[15]:

Tutorial Problem 21

Presented below in the figure is the available information of an aquifer cross-section. The aquifer is confined and of variable thickness across the cross-section. It has a uniform conductivity 5.6×10^{-5} m/s The total Discharge from the aquifer of width 500 m is required to be obtained.



In [16]:
```
#solution Problem 21

r2_4 = pn.pane.LaTeX(r"""
The aquifer is confined but of variable thickness, hence $T = Kb$, cannot be used. In this case we need to
find a representative aquifer thickness. This is simple if it can be assumed that a slope decreases linearly through-out
its length. So, we can write for a representative $m$
$$
m = \frac{m_3-m_1}{L}x + m_1  \qquad\qquad \text{(eq. 2A)}
$$
where $x$ is a lengthwise distance of $m$ from $m_1$. Equation above is simply
an equation of straight line $y = ax + c$, in which slope $a = \frac{m_3-m_1}{L} $ and $c = m_1$.

Now Darcy law can be used to obtain discharge per unit width ($q'$).

$$
q' = -bK\frac{dh}{dx} \qquad\qquad \text{(eq. 2B)}
$$

with conductivity $K$ and hydraulic head gradient $dh/dx$. We substitute $b$ from eq. (2A) to eq. (2B)
$$
q' = -\Bigg(\frac{m_3-m_1}{L}x + m_1\Bigg) \cdot K\frac{dh}{dx}  \qquad\qquad \text{(eq. 2C)}
$$

Rearranging eq. (2C) we get
$$
-dh = \frac{q'}{K}\cdot \frac{dx}{\frac{m_3 - m_1}{L}x+m_1}
\qquad\qquad \text{(eq. 2D)}
$$

Differential equation Eq. (2D) has to be solved to obtain the discharge.

""",width = 900, style={'font-size': '13pt'})
r2_4
```

Out[16]:

The aquifer is confined but of variable thickness, hence $T = Kb$, cannot be used. In this case we need to find a representative aquifer thickness. This is simple if it can be assumed that a slope decreases linearly through-out its length. So, we can write for a representative m

$$m = \frac{m_3 - m_1}{L}x + m_1 \qquad \text{(eq. 2A)}$$

where x is a lengthwise distance of m from m_1. Equation above is simply an equation of straight line $y = ax + c$, in which slope $a = \frac{m_3 - m_1}{L}$ and $c = m_1$. Now Darcy law can be used to obtain discharge per unit width (q').

$$q' = -bK\frac{dh}{dx} \qquad \text{(eq. 2B)}$$

with conductivity K and hydraulic head gradient dh/dx. We substitute b from eq. (2A) to eq. (2B)

$$q' = -\left(\frac{m_3 - m_1}{L}x + m_1\right) \cdot K\frac{dh}{dx} \qquad \text{(eq. 2C)}$$

Rearranging eq. (2C) we get

$$-dh = \frac{q'}{K} \cdot \frac{dx}{\frac{m_3 - m_1}{L}x + m_1} \qquad \text{(eq. 2D)}$$

Differential equation Eq. (2D) has to be solved to obtain the discharge.

In [17]:

```
#Tutorial Problem 21 – Continued
r2_5 = pn.pane.LaTeX(r"""
Eq. (2D) is a variable separated differential equation, so direct integration can be done with the following hydraulic (boundary) condit
ions:
$$
\text{for } x = 0,  \; h = h_1 \qquad \text{and}\qquad \text{for } x = L, \; h = h_3
$$

i.e.,
$$
-\int_{h_1}^{h_3} dh = \frac{q'}{K}\cdot\int_0^L \frac{dx}{\frac{m_3-m_1}{L}x + m_1}
\qquad \qquad \text{eq. (2E)}$$

The integral on the right hand side of eq. (2E) is an elementary integral the solution of which is of the form:
$$
\int\frac{dx}{ax + b} = \frac{1}{a}\ln(ax+b) + C
$$
based on this, the solution of eq. (2E) will be
$$
h_1 - h_3 = \frac{q'}{K} \frac{1}{\frac{m_3-m_1}{L}}\cdot\Bigg[\ln\Bigg(\frac{m_3-m_1}{L}L + m_1 \Bigg) - \ln\Bigg(\frac{m_3-m_1}{0}0 +
 m_1 \Bigg)\Bigg]
$$
Simplifying which we get
$$
h_1 - h_3  = \frac{q'}{K}\cdot \frac{L}{m_3-m_1}\cdot \ln\frac{m_3}{m_1}
$$
Then, $q'$ the unit aquifer width discharge can be obtained from
$$
q' = K \frac{h_1- h_3}{L}\cdot\frac{m_3 - m_1}{\ln\frac{m_3}{m_1}}
\qquad \qquad \text{eq. (2F)}$$
""",width = 900, style={'font-size': '13pt'})
r2_5
```

Out[17]:

Eq. (2D) is a variable separated differential equation, so direct integration can be done with the following hydraulic (boundary) conditions:

$$\text{for } x = 0,\ h = h_1 \qquad \text{and} \qquad \text{for } x = L,\ h = h_3$$

i.e.,

$$-\int_{h_1}^{h_3} dh = \frac{q'}{K} \cdot \int_0^L \frac{dx}{\frac{m_3 - m_1}{L} x + m_1} \qquad\qquad \text{eq. (2E)}$$

The integral on the right hand side of eq. (2E) is an elementary integral the solution of which is of the form:

$$\int \frac{dx}{ax + b} = \frac{1}{a} \ln(ax + b) + C$$

based on this, the solution of eq. (2E) will be

$$h_1 - h_3 = \frac{q'}{K} \frac{1}{\frac{m_3 - m_1}{L}} \cdot \left[\ln\left(\frac{m_3 - m_1}{L} L + m_1 \right) - \ln\left(\frac{m_3 - m_1}{0} 0 + m_1 \right) \right]$$

Simplifying which we get

$$h_1 - h_3 = \frac{q'}{K} \cdot \frac{L}{m_3 - m_1} \cdot \ln \frac{m_3}{m_1}$$

Then, q' the unit aquifer width discharge can be obtained from

$$q' = K \frac{h_1 - h_3}{L} \cdot \frac{m_3 - m_1}{\ln \frac{m_3}{m_1}} \qquad\qquad \text{eq. (2F)}$$

In [18]:
```
#Tutorial Problem Y2 – Continued
# given
h2_1 = 290 # m, head in Well 1
h2_3 = 275 # m, head in the river end
m2_1 = 6 # m, aquifer thickness at well 1
m2_3 = 20 # m, aquifer thickness near river end
K2 = 5.6 * 10**-6 # m/s, conductivity of aquifer
L2 = 700 # m, length of the river cross-section
W2 = 500 # m, Width of aquifer

# solution
# Discharge per unit width using eq. 2F
q2 = K2*((h2_1 - h2_3)/L2)*(m2_3 - m2_1)/np.log(m2_3/m2_1)
Q2 = q2*W2

#output
print("Discharge per unit width of aquifer is: {0:1.2e}".format(q2), "m\u00b2/s \n")
print("Discharge from the given width of aquifer is: {0:1.2e}".format(Q2), "m\u00b3/s")
```

```
Discharge per unit width of aquifer is: 1.40e-06 m²/s

Discharge from the given width of aquifer is: 6.98e-04 m³/s
```

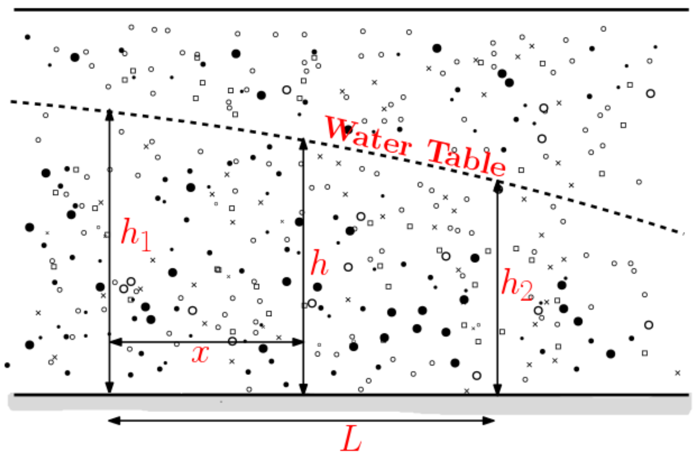
2. Tutorial Problems on Flow in Unconfined Aquifers

In [19]:
```
#Tutorial Problem 22 –
r3_1 = pn.pane.Markdown("""
### Tutorial Problem 22 ###
""",width = 600, style={'font-size': '13pt'})


r3_2 = pn.pane.LaTeX(r"""
Discharge from an unconfined aquifer presented in the figure below in which $h_1 = 20$ m, $h_2 = 10$ m, and $L = 50$ m
is to be obtained. Other information available are that the aquifer is 30 m wide and has a uniform conductivity
$K = 5 \times 10^{-6}$ m/s. Also known are that the Duipuit assumptions (check here: https://en.wikipedia.org/wiki/Dupuit-Forchheimer_as
sumption) applies to this unconfined aquifer.
""",width = 800, style={'font-size': '13pt'})

r3_3 = pn.pane.PNG("images/T06_Y3.png", width=400)

pn.Column(r3_1, r3_2, r3_3 )
```

Out[19]:

Tutorial Problem 22

Discharge from an unconfined aquifer presented in the figure below in which $h_1 = 20$ m, $h_2 = 10$ m, and $L = 50$ m is to be obtained. Other information available are that the aquifer is 30 m wide and has a uniform conductivity $K = 5 \times 10^{-6}$ m/s. Also known are that the Duipuit assumptions (check here: https://en.wikipedia.org/wiki/Dupuit-Forchheimer_assumption) applies to this unconfined aquifer.



In [20]:
```
#Solution Tutorial Problem 22

r3_4 = pn.pane.LaTeX(r"""
As Dupuit assumptions are valid, the discharge per unit width of aquifer ($q'$)
can be obtained from
$$
q' = -Kh\frac{dh}{dx} \qquad\qquad \text{eq. (3A)}
$$
where $h$ is saturated thickness of aquifer located at $x$ distance from $h_1$ end.
From figure, at $x = 0$, $h = h_1$ and at $x = L$, $ h = h_2$. Based on this
differential equation eq. (3A) can be directly integrated after separation of variable to obtain $q'$, i.e.,
$$
\int_0^L q'dx = -K\int_{h_1}^{h_2}h dh
$$
Integration leads to
$$
q'x\Big|^L_0 = -K\frac{h^2}{2}\Big|_{h_1}^{h^2}
$$
resulting to
$$
q'L = -K\Bigg( \frac{h_2^2}{2} - \frac{h_1^2}{2}\Bigg)
$$

and $q'$ is then obtained from

$$
q' = -\frac{1}{2}K\Bigg( \frac{h_2^2 -h_1^2 }{L}\Bigg) \qquad\qquad \text{eq. (3B)}
$$
""",width = 700, style={'font-size': '13pt'})

r3_4
```

Out[20]: As Dupuit assumptions are valid, the discharge per unit width of aquifer (q') can be obtained from

$$q' = -Kh\frac{dh}{dx} \qquad \text{eq. (3A)}$$

where h is saturated thickness of aquifer located at x distance from h_1 end. From figure, at $x = 0$, $h = h_1$ and at $x = L$, $h = h_2$. Based on this differential equation eq. (3A) can be directly integrated after separation of variable to obtain q', i.e.,

$$\int_0^L q' dx = -K\int_{h_1}^{h_2} hdh$$

Integration leads to

$$q'x\Big|_0^L = -K\frac{h^2}{2}\Big|_{h_1}^{h^2}$$

resulting to

$$q'L = -K\left(\frac{h_2^2}{2} - \frac{h_1^2}{2}\right)$$

and q' is then obtained from

$$q' = -\frac{1}{2}K\left(\frac{h_2^2 - h_1^2}{L}\right) \qquad \text{eq. (3B)}$$

In [23]:

```
#Solution of Tutorial Problem 22:
# Given

h3_1 = 20 # m, aquifer head at point 1
h3_2 = 10 # m, aquifer head at point 1
K3 = 5 * 10**-6 # m/s uniform conductivity of aquifer
L3 = 50 # m, length of the aquifer
W3 = 30 # m, width of the aquifer

#Calculation
q3 = -1/2*K3*(h3_2**2 - h3_1**2)/L3 # m^2/s, unit width discharge using eq. 3B
Q3 = q3 * W3 # m^3/s, total dischage from given width

#output
print("Discharge per unit width of aquifer is: {0:1.2e}".format(q3), "m\u00b2/s \n")
print("Discharge from the given width of aquifer is: {0:1.2e}".format(Q3), "m\u00b3/s")
```

```
Discharge per unit width of aquifer is: 1.50e-05 m²/s

Discharge from the given width of aquifer is: 4.50e-04 m³/s
```

In [24]:
```
#Tutorial Problem 23:

r4_1 = pn.pane.Markdown(r"""
###Tutorial Problem 23 ###
""",width = 900, style={'font-size': '13pt'})

r4_2 = pn.pane.LaTeX(r"""
In a schematic below an unconfined aquifer is found to divide 2 rivers of diffrent stages $h_1 = 30$ m and $h_2 = 10$ m.
The aquifer of length $L = 50$ m and with uniform conductivity $K = 5\times 10^{-6}$ m/s is found to receive recharge at
the rate ($w$) of 0.01 m/d. <br>
a) What will be the hydraulic head and discharge per unit width ($q'$) in the aquifer at 5 m
from the left river. <br>
b) What will the head at the same location when aquifer receives no recharge.


""",width = 900, style={'font-size': '13pt'})

r4_3 = pn.pane.PNG("images/T06_Y4.png", width=500)

pn.Column(r4_1, r4_2, r4_3)
```

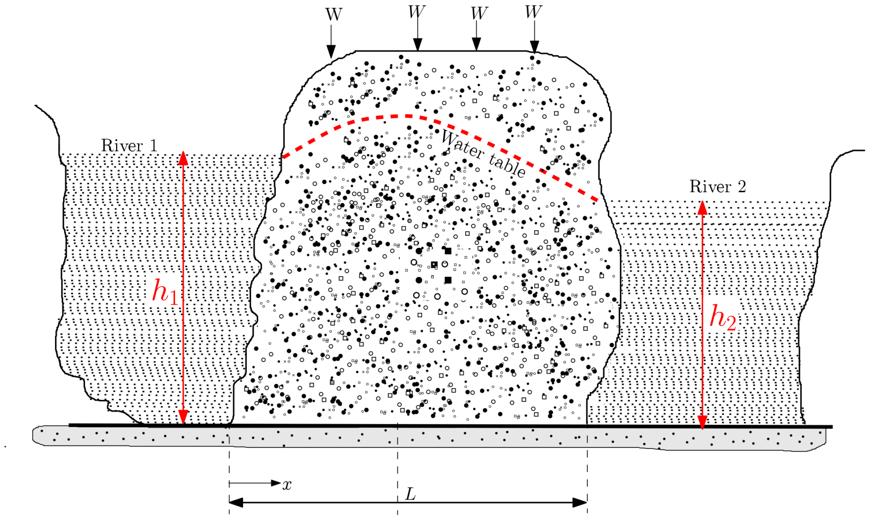
Out[24]:

Tutorial Problem 23

In a schematic below an unconfined aquifer is found to divide 2 rivers of diffrent stages $h_1 = 30$ m and $h_2 = 10$ m.
The aquifer of length $L = 50$ m and with uniform conductivity $K = 5\times 10^{-6}$ m/s is found to receive recharge at
the rate (w) of 0.01 m/d.

a) What will be the hydraulic head and discharge per unit width (q') in the aquifer at 5 m from the left river.

b) What will the head at the same location when aquifer receives no recharge.



In [25]:
```
# Solution of Tutorial Problem 23

r4_2 = pn.pane.LaTeX(r"""

For an unconfined aquifer, the case here, the water table = hydraulic head. For a condition as in this problem the height
of the water table $h$ as a function of position $x$ can be obtained from the following expression provided in Fetter (2014):
$$
h = \sqrt{h_1^2 - \frac{(h_1^2- h_2^2)x}{L} + \frac{w}{K}(L-x)x}
\quad\quad \text{eq. (4A)}
$$

From whhich discharge per unit width $(q')$ can be obtained by differentiating eq. (4A) with respect to $x$,
as from Darcy Law $q' = -Kh (dh/dx)$ for unconfined aquifer. Thus we get (from Fetter (2014),
$$
q'(x) = \frac{K (h_1^2 - h_2^2)}{2L}- w\Bigg(\frac{L}{2}-x\Bigg)
\quad\quad \text{eq. (4B)} $$

For the case when $w= 0$, the last term under the square root in eq. (4A) becomes 0, and then $h$ is
$$
h = \sqrt{h_1^2 - \frac{(h_1^2- h_2^2)x}{L}}
\quad\quad \text{eq. (4C)}
$$

""",width = 900, style={'font-size': '13pt'})


r4_2
```

Out[25]:

For an unconfined aquifer, the case here, the water table = hydraulic head. For a condition as in this problem the height of the water table h as a function of position x can be obtained from the following expression provided in Fetter (2014):

$$h = \sqrt{h_1^2 - \frac{(h_1^2 - h_2^2)x}{L} + \frac{w}{K}(L-x)x} \qquad \text{eq. (4A)}$$

From whhich discharge per unit width (q') can be obtained by differentiating eq. (4A) with respect to x, as from Darcy Law $q' = -Kh(dh/dx)$ for unconfined aquifer. Thus we get (from Fetter (2014),

$$q'(x) = \frac{K(h_1^2 - h_2^2)}{2L} - w\left(\frac{L}{2} - x\right) \qquad \text{eq. (4B)}$$

For the case when $w = 0$, the last term under the square root in eq. (4A) becomes 0, and then h is

$$h = \sqrt{h_1^2 - \frac{(h_1^2 - h_2^2)x}{L}} \qquad \text{eq. (4C)}$$

In [26]:

```
#Solution of Tutorial Problem 23:
# Given

h4_1 = 30 # m, River 1 stage
h4_2 = 10 # m, River 2 stage
K4 = 5 * 10**-4 # m/s uniform conductivity of aquifer
L4 = 50 # m, length of the aquifer
w4 = 10**-4 # m/d recharge rate in the aquifer
x4 = 5 # m, loaction at which water table is to be found

#Calculation part a
h4_w = np.sqrt(h4_1**2 - ((h4_1**2 - h4_2**2)*x4)/L4 + (w4/K4)*(L4-x4)*x4) # head at x = 5 m from eq. 4A
q4 = K4*((h4_1**2- h4_2**2)/2*L4) - w4*((L4/2)-x4)     # m^2/s, total dischage from given width

#Calculation part b
h4_nw = np.sqrt(h4_1**2 - ((h4_1**2 - h4_2**2)*x4)/L4)


#output
print("The water table at the required  location (x) with recharge is: {0:1.2f}".format(h4_w), "m \n")
print("Discharge per unit width from the aquifer is: {0:1.2f}".format(q4), "m\u00b2/s \n")
print("The water table at the required  location (x) without recharge is: {0:1.2f}".format(h4_nw), "m ")
```

```
The water table at the required  location (x) with recharge is: 29.41 m

Discharge per unit width from the aquifer is: 10.00 m²/s

The water table at the required  location (x) without recharge is: 28.64 m
```