```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import pandas as pd
        import ipysheet as ips
        import panel as pn
        from scipy import stats
        pn.extension('katex', 'mathjax')
```

Tutorial 5

- **Tutorial Problem on Hydraulic Conductivities in Complex Systems**

1. **Unsaturated Zone**
2. **Consolidated Media**
3. **Flow nets**

- **homework problems on Hydraulic Conductivities and flow nets**

Tutorial Problem on Hydraulic Conductivities in Unsaturated Zone

Tutorial Problem 12

From the laboratory test the degree of saturation(θ) of the unsaturated core (temperature = 9$^\circ C$) sample was found to be 30% and relative permeability (k_r) is assumed to be 0.1. From the grain analysis the sample was determined to be predominantly medium sand (intrinsic permeability, $k = 1.61 \times 10^{-7}$ cm^2). Provided that density (ρ) and dynamic viscosity of water (μ) at 9$^\circ C$ is 999.73 kg/m^3 and 0.0013465 N⋅s/m^2 respectively, find the conductivity of the sample. What will be the conductivity of the same sample when the moisture content is 1% ($k_r \approx 0.001$) and 80% ($k_r \approx 0.4$). Explain the effect of moisture content on the sample.

Solution of Tutorial Problem 12

Lecture contents on the topic in L02- slides 02, 22 & 26

Hydraulic conductivity of the unsaturated sample ($\theta < 100\%$) can be obtained from the following expression:

$$K(\theta) = \left(\frac{k\rho g}{\mu}\right) k_r(\theta)$$

In [2]:

```
# Given
kr_30  = 0.05 # (-), relative permeability for moist. cont. 30%
i_p = 1.61 * 10**-7 # cm^2, intrinsic permeability
rho = 999.73 # kg/m^3, Sample density
mu = 0.0013467 #  N-s/m^2, dynamic visc.
g_c = 9.81 # N/kg, force unit used for gravitational constant

# Solutions 1
i_pm = i_p/10000 # m^2 unit conversion for int. permeab.
K_30 = (i_p*rho*g_c/mu)*kr_30

# Solution 2  when moisture content is 1% and 80%
kr_1 = 0.001 # (-), relative permeability for moist. cont. 1%
kr_80 = 0.4 # (-), relative permeability for moist. cont. 80%
K_1 = (i_p*rho*g_c/mu)*kr_1
K_80 = (i_p*rho*g_c/mu)*kr_80

# output

print("The conductivity of water when moisture content is 30%  is: {0:1.1e}".format(K_30),"m/s \n")
print("The conductivity of water when moisture content is 1%  is: {0:1.1e}".format(K_1),"m/s \n")
print("The conductivity of water when moisture content is 80%  is: {0:1.1e}".format(K_80),"m/s \n")
print("The conductivity of media increases very rapidly with increase of moisture content")
```

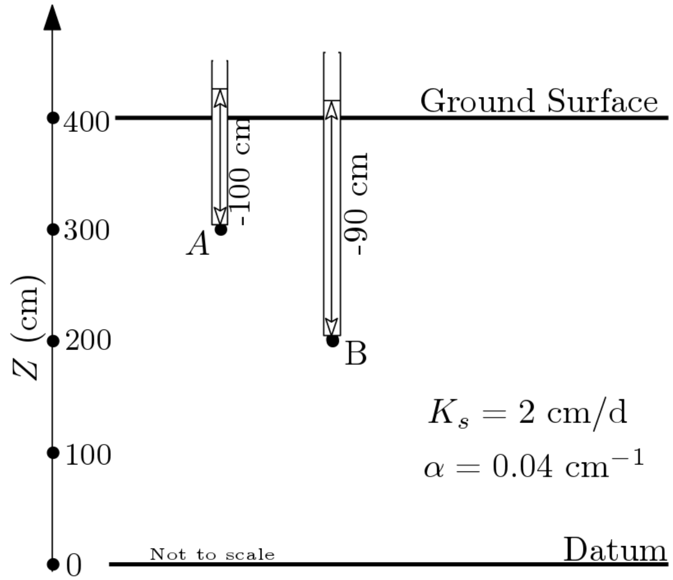
```
The conductivity of water when moisture content is 30%  is: 5.9e-02 m/s

The conductivity of water when moisture content is 1%  is: 1.2e-03 m/s

The conductivity of water when moisture content is 80%  is: 4.7e-01 m/s

The conductivity of media increases very rapidly with increase of moisture content
```

In [3]:
```
# Tutorial Problem 13

r2_1 = pn.pane.Markdown("""
### Tutorial Problem 13 """, width = 650, style={'font-size': '13pt'})

r2_2 = pn.pane.LaTeX(r"""


From the analysis of laboratory results the unsaturated hydraulic conductivity fits the following exponential
model as a function of pressure head ($\psi$): $K(\psi) = K_s \exp(\alpha\cdot \psi)$,
with $K_s$ [LT$^{-1}$] the saturated hydraulic conductivity and $\alpha$ [L$^{-1}$] a fit parameter.
For the pressure head measurements and the data provided in the figure below, find $K(\psi)$.
Also, find the Darcy velocity for this case.
""", width = 900, style={'font-size': '13pt'})

r2_3 =pn.pane.PNG("images/T04_a_1.png", width=400)


r2_4 = pn.pane.Markdown("""<br>
### Solution Tutorial Problem 13 """, width = 700, style={'font-size': '13pt'})

pn.Column(r2_1, r2_2, r2_3, r2_4)
```

Out[3]:

Tutorial Problem 13

From the analysis of laboratory results the unsaturated hydraulic conductivity fits the following exponential model as a function of pressure head (ψ): $K(\psi) = K_s \exp(\alpha \cdot \psi)$, with K_s [LT^{-1}] the saturated hydraulic conductivity and α [L^{-1}] a fit parameter. For the pressure head measurements and the data provided in the figure below, find $K(\psi)$. Also, find the Darcy velocity for this case.



Solution Tutorial Problem 13

In [4]:
```
# Given

K_s = 2 # cm/d # saturated conductivity
al_a = 0.04 # 1/cm, fit constant
Ph_a = -100 # cm, pressure head at A
Ph_b = -90 # cm, pressure head at B
Z_a = 300 # cm, elevation head at A from datum
Z_b = 200 # cm, elevation head at B from datum

# Solution 1
Ph_m = (Ph_a+Ph_b)/2 # mean pressure head
K_psi = K_s*np.exp(al_a*Ph_m)

#Solution 2
H_A = Ph_a+Z_a # cm, hydraulic head at A
H_B = Ph_b+Z_b # cm, hydraulic head at B
dh_dz = (H_B - H_A)/(Z_b - Z_a) # (-), hydraulic head gradient
q_z = -K_psi*dh_dz # cm/d, Darcy velocity

print("The unsaturated conductiviy of the sample is: {0:1.3f}".format(K_psi), "cm/d")
print("The Darcy velocity is: {0:1.3f}".format(q_z), "cm/d")
print("The negative sign indicates the direction opposite to increase in z.")
```

```
The unsaturated conductiviy of the sample is: 0.045 cm/d
The Darcy velocity is: -0.040 cm/d
The negative sign indicates the direction opposite to increase in z.
```

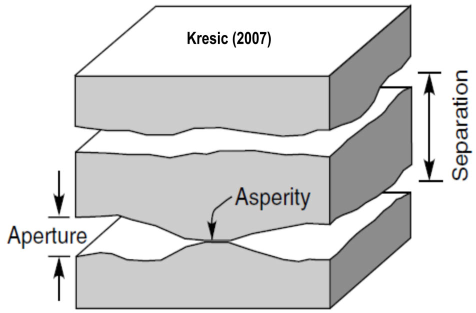
Tutorial Problem on Hydraulic Conductivities in Consolidated Media

In [5]:
```
# Tutorial Problem 14
r3_1 = pn.pane.Markdown("""
## Tutorial Problem 14 """, style={'font-size': '13pt'})
spacer2=pn.Spacer(width=50)

r3_2 = pn.pane.LaTeX(r"""
Discharge of water at 9$^\circ C$ through the  fractured rock with a uniform fracture aperature $e = 0.1$ cm and width 1
 m  is to be
obtained. For simplicity, only a single fracture is considered (see figure below) and a hydraulic gradient = 0.001 is ass
umed.
Additionally, the flow in the fracture is assumed to be laminar or Darcy conditions are valid. Available water propertiec
at 9$^\circ C$ are:
dynamic viscosity $\mu$ =   0.0013465 N$\cdot$s/m$^2$ and density $\rho$ = 999.73 kg/m$^3$.
""", width = 900, style={'font-size': '13pt'})

r3_3 = pn.pane.PNG("images/T04_a_2.png", width=300)

spacer3=pn.Spacer(width=150)
r3_4 = pn.pane.Markdown("""<br>
### Solution Tutorial Problem 14
<br> **Check Lecture L02 slide 7 for more information**
""", width = 700, style={'font-size': '13pt'})
r3_5 = pn.pane.LaTeX(r"""
The conductivity $(K_s)$ in the single fracture can be obtained from:
$$
K_s = \frac{g \rho e^2}{12 \mu}
$$
where, $g =$ gravitational constant, $\rho =$ density of fluid, $e=$ fracture aperature and $\mu =$ dynamic viscocity

""", width = 900, style={'font-size': '13pt'})

pn.Column(r3_1,spacer2, r3_2, spacer2, r3_3, spacer3, r3_4, r3_5)
```

Out[5]:

Tutorial Problem 14

Discharge of water at 9$^{\circ}C$ through the fractured rock with a uniform fracture aperature $e = 0.1$ cm and width 1 m is to be obtained. For simplicity, only a single fracture is considered (see figure below) and a hydraulic gradient = 0.001 is assumed. Additionally, the flow in the fracture is assumed to be laminar or Darcy conditions are valid. Available water propertiec at 9$^{\circ}C$ are: dynamic viscosity μ = 0.0013465 N·s/m^2 and density ρ = 999.73 kg/m^3.



Solution Tutorial Problem 14

Check Lecture L02 slide 7 for more information

The conductivity (K_s) in the single fracture can be obtained from:

$$K_s = \frac{g\rho e^2}{12\mu}$$

where, $g =$ gravitational constant, $\rho =$ density of fluid, $e =$ fracture aperature and $\mu =$ dynamic viscocity

In [11]:
```
# Solution Problem 14

# Given
e_p = 0.01 # cm, Fracture aperature
W = 1 # m, fracture width
mu_3 = 0.0013465 # N-s/m^2, dynamic visocity of water at 9°C
rho_3 = 999.73 # kg/m^3, density of water at 9°C
g_3 = 9.81 # N/kg, gravitational constant
i_3 = 0.001 # (), hydraulic head

#Solution 1
B_m = 0.1/100# m, unit conversion for B
K_3 = e_p**2*rho_3*g_3/(12*mu_3) # m/s, Conductivity of rock media
Q_3 = W*e_p*K_3*i_3 # Q = KiA - as Darcy's law is valid

print("The conductiviy of the fracture is: {0:1.3f}".format(K_3), "m/s")
print("The discharge from the rock is: {0:1.3f}".format(Q_3), "m\u00b3/s")
```

```
The conductiviy of the fracture is: 60.697 m/s
The discharge from the rock is: 0.001 m³/s
```

Tutorial Problem 15

The effective porosity of individual matrix blocks within a fractured aquifer is 1.5 % and the hydraulic conductivity K_{matrix} is 10^{-8} m/s. The average aperture of fractures is 35 μm with an average distance between fractures of 0.8 m. Water temperature is 9$^{\circ}C$.

a) Calculate the hydraulic conductivity of an individual fracture.

b) How much is the total hydraulic conductivity?

c) Calculate the average linear velocity (in m/a) within fractures and matrix blocks respectively under consideration of a hydraulic gradient i = 0.001

Solution of Tutorial Problem 15

For the composite (fracture + matrix), the conductivity (K_t) is obtained from:

$$K_t = \frac{e}{F_d}K_s + K_{mat}$$

which is equivalent to

$$K_t = \frac{g\rho e^3}{12F_d\mu} + K_{mat}$$

where, K_{mat} = matrix conductivity, and F_d = average fracture distance

In [ ]:
```
# Solution 15,

#Given are:
e_4 = 35*10**-6 # m, aperature
F_d = 0.8 # m, average fracture distance
K_mat = 10**-8# m/s, Hyd. Conductivity
n_e = 1.5/100# (), effective porosity in number
g_4 = 9.81 # N/kg, gravitational constant (known)
i_4 = 0.001

#Water properties at 9°C
mu_4 = 0.0013465 # N-s/m^2, dynamic visocity of water
rho_4 = 999.73 # kg/m^3, density of water

#Solution (a), (b) and (c)
K_f = e_4**2*rho_4*g_4/(12*mu_4) # m/s, individual hydraulic conductivity see problem 14
K_o = e_4/F_d*K_f+ K_mat # m/s, total Hydraulic conductivity of mass
q_mat = K_mat*i_4 # m/s Darcy velocity in total matrix
v_mat = q_mat/n_e # m/s, linear velocity in total matrix
q_f = K_f*i_4 # Darcy's velocity in single fracture
v_f = q_f/F_d # Linear velocity in single fracture


#output
print("The conductivity of the single fracture is: {0:1.3e}".format(K_f), "m/s")
print("The conductivity of the total rock matrix is: {0:1.3e}".format(K_o), "m/s")
print("Linear velocity in total rock matrix is: {0:1.3e}".format(v_mat), "m/s")
print("Linear velocity in single fracture system is: {0:1.3e}".format(v_f), "m/s")
```

Tutorial Problem on Flow-nets

In [19]:
```
# Tutorial Problem 16
r5_1 = pn.pane.Markdown("""
### Tutorial Problem 16: Hydrologic Triangle
The figure below shows the position of four groundwater observation wells with measured hydraulic heads in m a.s.l.
<br> <br>
**a.** Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.<br><br>
**b.** Indicate the flow direction.

""",width = 400, style={'font-size': '13pt'})

r5_2 = pn.pane.PNG("images/T03_TP12_a.png", width=400)

pn.Row(r5_1,spacer2, r5_2)
```

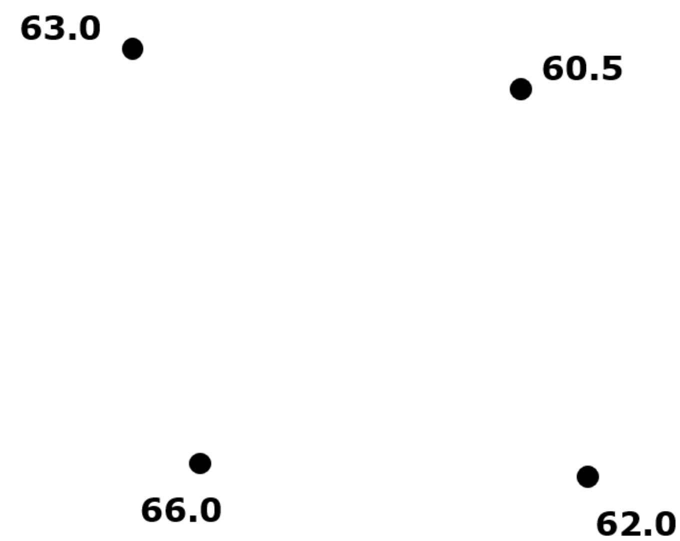
Out[19]:

Tutorial Problem 16: Hydrologic Triangle

The figure below shows the position of four groundwater observation wells with measured hydraulic heads in m a.s.l.

a. Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.

b. Indicate the flow direction.



In [13]:
```
#
r5_3 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 16

Step 1. Connects all the points
""", width=600)

r5_2.object = "images/T03_TP12_b.png"
r5_3
```

Out[13]:

Solution of Tutotrial Problem 16

Step 1. Connects all the points

In [15]:
```
#
r5_4 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 16
Step 2. Divide the connected lines at equal head-level (here = 1 m)
""", width=600)
r5_2.object = "images/T03_TP12_c.png"
r5_4
```

Out[15]:

Solution of Tutotrial Problem 16

Step 2. Divide the connected lines at equal head-level (here = 1 m)

In [16]:
```
#
r5_5 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 16
Step 3. Join all the equal head lines
""", width=600)
r5_2.object = "images/T03_TP12_d.png"
r5_5
```

Out[16]:

Solution of Tutotrial Problem 16

Step 3. Join all the equal head lines

In [18]:
```
#
r5_6 = pn.pane.Markdown("""
### Solution of Tutotrial Problem 16
Step 4. Mark the flow direction from higher head towards lower head
""", width=600)
r5_2.object = "images/T03_TP12_e.png"
r5_6
```

Out[18]:

Solution of Tutotrial Problem 16

Step 4. Mark the flow direction from higher head towards lower head

In [17]:
```
# Tutorial Problem 17
r6_1 = pn.pane.Markdown("""
##Tutorial Problem 17: Flow Nets##

Sketch head isolines and streamlines for the two configurations a) and b) of a well doublette shown below. In both cases
 flow nets should be sketched without and with the uniform flow component.

""",width=800,  style={'font-size': '13pt'})

r6_2 = pn.pane.Markdown("""
 a) withdrawal at both wells:<br><br><br>
""",width=400,  style={'font-size': '13pt'})

r6_3 = pn.pane.PNG("images/T03_TP13_a.png", width=200)

r6_4 = pn.Column(r6_2,r6_3)

r6_5 = pn.pane.Markdown("""
 b) Injection at both wells:<br><br><br>
""",width=400,  style={'font-size': '13pt'})

r6_6 = pn.pane.PNG("images/T03_TP13_b.png", width=200)

r6_7 = pn.Column(r6_5,r6_6)
r6_8 = pn.Row(r6_4, r6_7)
pn.Column(r6_1, r6_8)
```

Out[17]:

Tutorial Problem 17: Flow Nets

Sketch head isolines and streamlines for the two configurations a) and b) of a well doublette shown below. In both cases flow nets should be sketched without and with the uniform flow component.

a) withdrawal at both wells:

b) Injection at both wells:

HOMEWORK PROBLEMS

There is no obligation to submit the homework

You are encouraged to submit the homework as ipynb file to my email.

Pls. submit within the next 2 weeks times.

In [8]:
```
#Homework Problem 5
r7_1= pn.pane.Markdown("""
###Homework Problem 5:
""", width = 900, style={'font-size': '13pt'})

s3=pn.Spacer(width=150)

r7_2= pn.pane.LaTeX(r"""
In this problem we consider the roughness of the inner-surface of the facture
that can affect the conductivity of water (at 9$^\circ C$) in the rock matrix. In this example we consider
a composite rock matrix with average fracture aperature of 30 $\mu$m and the average
spacing between fractures to be 0.5 m.  Further, we will consider a general relative roughness
of the inner surface ($\zeta$) of the fracture to be 0.4 and neglect the influence of non-fractured conductivity ($K_{ma
t}$).
We find the effect of surface roughness on conductivity.
""", width = 900, style={'font-size': '13pt'})

r7_3= pn.pane.Markdown("""
###Hint for solving homework problem 5:
""", width = 900, style={'font-size': '13pt'})

r7_4= pn.pane.LaTeX(r"""
With surface roughness in consideration, the conductivity of rock matrix can be obtained from:
$$
K_t = \frac{g \rho e^3}{12 C F_d \mu} + K_{mat}
$$


With $ C = (1+ 8.8\zeta^{1.5})$ describes the fracture roughness for depending on relative roughness $\zeta$
""", width = 900, style={'font-size': '13pt'})
pn.Column(r7_1, s3, r7_2, s3, r7_3, s3, r7_4)
```

Out[8]:

Homework Problem 5:

In this problem we consider the roughness of the inner-surface of the facture that can affect the conductivity of water (at 9$^{\circ}C$) in the rock matrix. In this example we consider a composite rock matrix with average fracture aperature of 30 μm and the average spacing between fractures to be 0.5 m. Further, we will consider a general relative roughness of the inner surface (ζ) of the fracture to be 0.4 and neglect the influence of non-fractured conductivity (K_{mat}). We find the effect of surface roughness on conductivity.

Hint for solving homework problem 5:

With surface roughness in consideration, the conductivity of rock matrix can be obtained from:

$$K_t = \frac{g\rho e^3}{12CF_d\mu} + K_{mat}$$

With $C = (1 + 8.8\zeta^{1.5})$ describes the fracture roughness for depending on relative roughness ζ

In [9]:
```
#
#
r10_1= pn.pane.Markdown("""
###Homework Problem 6: Hydrologic Triangle
The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.
 <br><br>

**a.** Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.
<br><br>
**b.** Indicate the flow direction.<br><br>
""", width = 500, style={'font-size': '13pt'})
r10_2 = pn.pane.PNG("images/T03_TH6.png", width=400)

pn.Row(r10_1, r10_2)
```

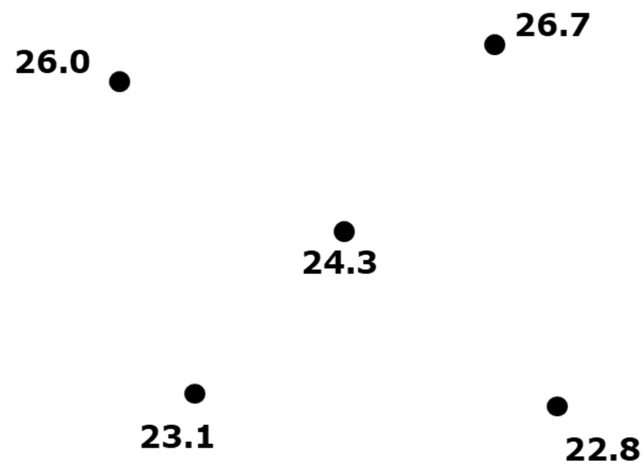
Out[9]:

Homework Problem 6: Hydrologic Triangle

The figure below shows the position of five groundwater observation wells with measured hydraulic heads in m a.s.l.

a. Sketch head isolines for intervals of 1 m by applying the hydrologic triangle method.

b. Indicate the flow direction.



In [10]:
```
#
r11_1= pn.pane.Markdown("""
###Homework Problem 7: Flow Nets
Sketch head isolines and streamlines for the well doublette shown below.
In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.
 <br><br><br><br><br><br>
 """, width = 900, style={'font-size': '13pt'})

r11_2 = pn.pane.PNG("images/T03_TH7.png", width=400)

r11_3= pn.pane.Markdown("""
 <br><br><br><br><br><br>
 """, width = 900, style={'font-size': '13pt'})
pn.Column(r11_1, r11_2, r11_3)
```

Out[10]:

Homework Problem 7: Flow Nets

Sketch head isolines and streamlines for the well doublette shown below. In this case, injection and withdrawal of groundwater is superimposed to a uniform flow component.