In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import panel as pn
from scipy import stats
pn.extension('katex', 'mathjax')
```

Tutorial 6 - Tutorial Problems on Flow in Confined/Unconfined Aquifer ¶

- **solutions for homework problems 3 – 4**
- **tutorial problems on flow in confined and unconfined aquifers**
- **homework problems on flow in confined and unconfined aquifers**

Solutions for Homework Problems 3 – 4

Homework Problem 3

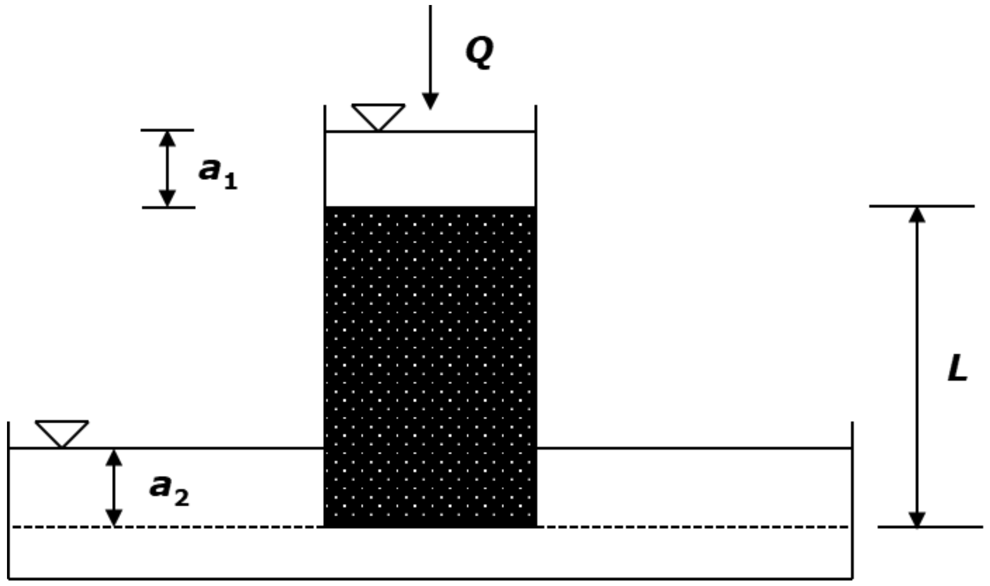
In [3]:
```python
#
r3_1 = pn.pane.Markdown("""
## ##


**A**. Derive an expression for hydraulic conductivity K for the constant-head permeameter shown in the figure.<br><br>
**B**. The hydraulic conductivity of a sample (length 10 cm, diameter 4 cm) is to be determined.
The water depths a<sub>1</sub> and a<sub>2</sub> equal 6 cm and 3 cm, resp. A water volume of 250 ml
passed the sample during an experimental period of 36 s. <br> <br>
**C**. Which material could be contained in the sample?
""", style={'font-size': '12pt'})
spacer2=pn.Spacer(width=50)
r3_2 = pn.pane.PNG("images/T06_TH3.png", width=500)
pn.Row(r3_1,spacer2, r3_2)
```

Out[3]:

A. Derive an expression for hydraulic conductivity K for the constant-head permeameter shown in the figure.

B. The hydraulic conductivity of a sample (length 10 cm, diameter 4 cm) is to be determined. The water depths a_1 and a_2 equal 6 cm and 3 cm, resp. A water volume of 250 ml passed the sample during an experimental period of 36 s.

C. Which material could be contained in the sample?



Solution of the Homework Problem 3

In [6]:

```
#
r3_3 = pn.pane.Markdown("""
###  """, width = 700, style={'font-size': '13pt'})

r3_4 = pn.pane.LaTeX(r"""
General formula for constant-head permeameter:
$$
K = \frac{QL}{A(h_{in}-h_{out})}
$$
<br>
The column outlet is chosen as the reference level
$z = 0$ with the $z-$axis pointing up-ward. As a consequence, we have $z = L$ at the inlet. <br> <br> <br> <br>
""", style={'font-size': '12pt'})

r3_5 = pn.pane.LaTeX(r"""
At the Outlet: <br>
pressure head =  $a_2$  <br>
elevation head = 0 <br>
$h_{out}$ = $a_2$ <br>
""", width = 300, style={'font-size': '12pt'})

r3_6 = pn.pane.LaTeX(r"""
At the Inlet: <br>
pressure head = a<sub>1</sub> <br>
elevation head = $L$<br>
$h_{in}$ = $a_1+ L$ <br>

""", width = 300, style={'font-size': '12pt'})

r3_7 = pn.pane.LaTeX(r"""
head difference: <br>
$$h_{in}-h_{out} = a_{1} + L - a_{2}$$<br>
hydraulic conductivity:
$
K = \frac{QL}{A (a_1 + L - a_2)}
$
""", width = 800, style={'font-size': '12pt'})

C1 = pn.Row(r3_5, r3_6)

r3_2.object = "images/T06_TH3a.png"

pn.Column(r3_3, r3_4, C1, r3_7)
```

Out[6]:

General formula for constant-head permeameter:

$$K = \frac{QL}{A(h_{in} - h_{out})}$$

The column outlet is chosen as the reference level $z = 0$ with the $z-$axis pointing up-ward. As a consequence, we have $z = L$ at the inlet.

At the Outlet:
pressure head = a_2
elevation head = 0
h_{out} = a_2

At the Inlet:
pressure head = a_1
elevation head = L
h_{in} = $a_1 + L$

head difference:

$$h_{in} - h_{out} = a_1 + L - a_2$$

hydraulic conductivity: $K = \frac{QL}{A(a_1+L-a_2)}$

In [10]:
```python
# Problem 3b, Given are:
L = 10# cm, length of column
a1 = 6# cm, pressure head at 1
a2 = 3# cm, pressure head at 2
d = 4 # cm, diameter of the column
V = 250 # mL, volume
A = np.pi*(d/2)**2 # cm^2 Area of the column
t = 36 # s, time

# interim calculation
Q = V/t # mL/s, discharge

#calculation
K = (Q*L)/(A*(a1+L-a2))# cm/s, Conductivity

#output
print('\033[1m' + 'Results are:' + '\033[0m \n')
print("The conductivity of the column is:{0:1.3f}".format(K), "cm/s \n")
print("The conductivity of the column is:{0:1.2e}".format(K/100), "m/s\n")

r3_8 = pn.pane.Markdown("""
The sample in the column is: **Coarse sand - Fine gravel**
""", width=400)

pn.Row(r3_8)
```

```
Results are:

The conductivity of the column is:0.425 cm/s

The conductivity of the column is:4.25e-03 m/s
```

Out[10]:

The sample in the column is: **Coarse sand - Fine gravel**

Homework Problem 4

In [19]:
```python
# given data - you may change the number

t = np.array([0, 5, 18, 23, 27, 29]) # min, given time
Dh = np.array([36.9, 33.6, 26.3, 23.9, 22.1, 21.3]) # cm, head difference

# creating data table
data = {"Time (min)": t, "Δh (m)": Dh}
df = pd.DataFrame(data)
df1 = df.T


r_h4 = pn.pane.Markdown("""
A Darcy experiment is performed by a falling-head permeameter using water at 20°C.
Length and diameter of the sample are 20 cm and 6 cm, resp. The inner tube diameter is 4 cm.
The following data are available for the time-dependent hydraulic head difference :
""",width = 600, style={'font-size': '12pt'})


spacer2=pn.Spacer(width=50)

pn.Column(r_h4,spacer2, df1)
```

Out[19]:

A Darcy experiment is performed by a falling-head permeameter using water at 20°C. Length and diameter of the sample are 20 cm and 6 cm, resp. The inner tube diameter is 4 cm. The following data are available for the time-dependent hydraulic head difference :

	0	1	2	3	4	5
Time (min)	0.0	5.0	18.0	23.0	27.0	29.0
Δh (m)	36.9	33.6	26.3	23.9	22.1	21.3

In [18]:

```
r_h4c = pn.pane.Markdown("""
**A.** Convert times to seconds and plot the logarithm of the ratios of head differences ln(Δh(0)/Δh(t)) vs. time t.
(Use the coordinate system on next page). <br><br>
**B.** Determine the slope of the corresponding regression line.<br><br>
**C.** Determine hydraulic conductivity K.<br><br>
**D.** Determine intrinsic permeability k.<br>
""", width=500, style={'font-size': '12pt'})

r_h4e = pn.pane.PNG("images/T06_TH4b.png", width=350)
spacer2=pn.Spacer(width=50)


pn.Row(r_h4c, spacer2, r_h4e)
```

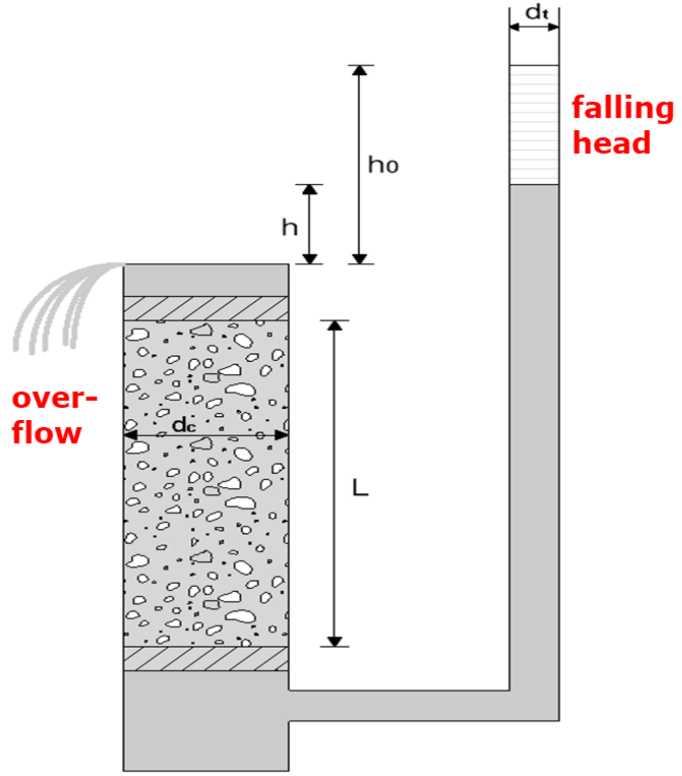
Out[18]:

A. Convert times to seconds and plot the logarithm of the ratios of head differences ln(Δh(0)/Δh(t)) vs. time t. (Use the coordinate system on next page).

B. Determine the slope of the corresponding regression line.

C. Determine hydraulic conductivity K.

D. Determine intrinsic permeability k.



Solution of Homework Problem 4

In [26]:
```
#
r4_1 = pn.pane.LaTeX(r"""
The formula for variable-head permeameter (See Tutorial 3, Problem Nr. 10):
$$K = \frac{d_t^2 L}{d_t^2 t} \cdot \ln \frac{ h_{in}(0)-h_{out}}{h_{in}(t) - h_{out}} = \frac{d_t^2 L}{d_e^2 t}\cdo
t \ln\frac{\Delta h(0)}{\Delta h(t)}$$
Rearrangement shows that the natural logarithm of $\Delta h(0)/\Delta h(t)$ depends linearly on time $t$:
$$
\ln\frac{\Delta h(0)}{\Delta h(t)} = \frac{K \cdot d_e^2 L}{L\cdot d_t^2} \cdot t
$$
$$\text{slope} =\frac{K\cdot d_e^2}{L\cdot d_t^2}$$
$$K = L \frac{d_t^2 }{d_e^2}\cdot \text{slope}$$
""",width=800, style={'font-size': '12pt'} )

r4_1
```

Out[26]:

The formula for variable-head permeameter (See Tutorial 3, Problem Nr. 10):

$$K = \frac{d_t^2 L}{d_t^2 t} \cdot \ln \frac{ h_{in}(0)-h_{out}}{h_{in}(t) - h_{out}} = \frac{d_t^2 L}{d_e^2 t}\cdot \ln\frac{\Delta h(0)}{\Delta h(t)}$$

Rearrangement shows that the natural logarithm of $\Delta h(0)/\Delta h(t)$ depends linearly on time t:

$$\ln\frac{\Delta h(0)}{\Delta h(t)} = \frac{K \cdot d_e^2 L}{L\cdot d_t^2} \cdot t$$

$$\text{slope} =\frac{K\cdot d_e^2}{L\cdot d_t^2}$$

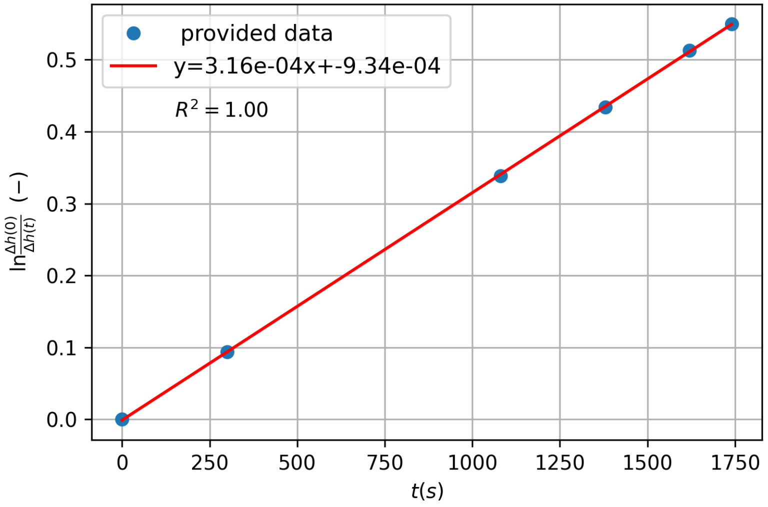
$$K = L \frac{d_t^2 }{d_e^2}\cdot \text{slope}$$

In [29]:

```
#Caclulation

t_s = t*60 # s, time in second
Dh0_Dht = Dh[0]/Dh # (-), Delta h(0)/Delta h(t)
ln_Dhodht = np.log(Dh0_Dht)# (-), ln(Delta h(0)/Delta h(t))

slope, intercept, r_value, p_value, std_err = stats.linregress(t_s, ln_Dhodht) # linear regression


# result table
data2 = {"Time (s)": t_s, "Δh(0)/Δh(t)":Dh0_Dht,"ln (Δh(0)/Δh(t)": ln_Dhodht}
df2 = pd.DataFrame(data2)

fig = plt.figure()
plt.plot(t_s, ln_Dhodht, 'o', label=' provided data');
pred = intercept + slope*t_s
plt.plot(t_s, pred, 'r', label='y={:0.2e}x+{:0.2e}'.format(slope,intercept)) ;
plt.xlabel(r"$t (s)$");
plt.ylabel(r"$\ln\frac{\Delta h (0)}{\Delta h (t)}\;\:(-)$");
plt.grid();
plt.legend(fontsize=11)
plt.text(150, 0.42,'$R^2 = {:0.2f}$'.format(r_value) )
plt.close() # otherwise we have 2 figure
r4_2 = pn.pane.Matplotlib(fig, dpi=300)

pn.Row(df2,spacer2,r4_2)
```

Out[29]:

	Time (s)	Δh(0)/Δh(t)	ln (Δh(0)/ Δh(t)
0	0	1.000000	0.000000
1	300	1.098214	0.093685
2	1080	1.403042	0.338643
3	1380	1.543933	0.434333
4	1620	1.669683	0.512634
5	1740	1.732394	0.549504



In [31]:
```
#Solution of 4C
# Given
L = 20 # cm, Length of the column
d_t = 4 # cm, diameter of the tube
d_c = 6# cm, diameter of the column
slope = slope # 1/s, from fitting see plot

K = L*(d_t**2/d_c**2)*slope # cm/s, conductivity calculated using eqn from previous slide
K_m = K/100 # m/s, conductivity

#output
print('\033[1m' + 'Results are:' + '\033[0m \n')
print("The conductivity in the column is: {0:1.2e}".format(K), "cm/s\n")
print("The conductivity in the column is: {0:1.2e}".format(K_m), "m/s\n")

#Solution of 4D
# Given
rho_w = 998.2 # kg/m^3, density of water
eta_w = 1.0087E-3# kg/(m-s), viscocity of water
g = 9.81 # m/s^2, accl. due to gravity

k = K/100*eta_w/(rho_w*g)# m^2, K = k*ρ/n
k_D = k/0.987E-12 # D, 1D = 0.987E10-12 m^2

print("The permeability of the media is: {0:1.2e}".format(k), "m\u00b2 \n")
print("The permeability of the media in Darcy's unit is: {0:1.2f}".format(k_D), "D")
```

```
Results are:

The conductivity in the column is: 2.81e-03 cm/s

The conductivity in the column is: 2.81e-05 m/s

The permeability of the media is: 2.89e-12 m²

The permeability of the media in Darcy's unit is: 2.93 D
```

Tutorial Problems

Tutorial Problem 16- flow in confined aquifer

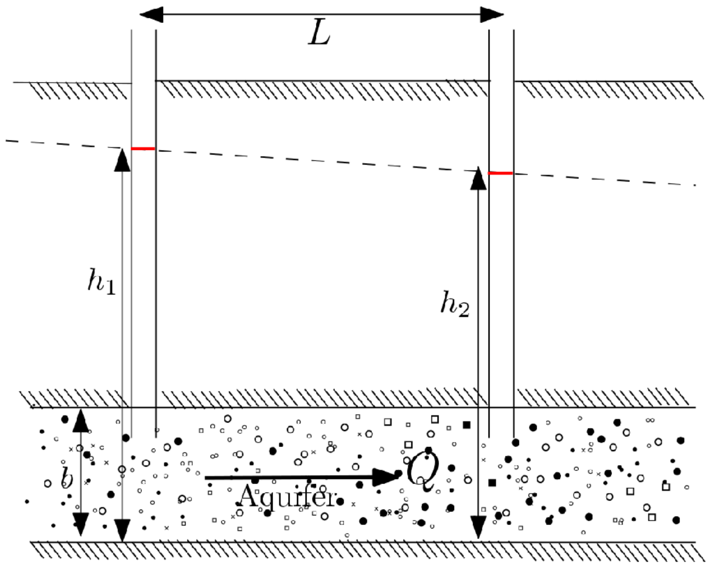
In [34]:
```
r16_2 = pn.pane.LaTeX(r"""
A confined aquifer is 30 m thick and 5 km wide. Two observation wells are located 1.5 km apart
in the direction of flow. The head in well 1 is 90 m and in well 2 it is 85.0 m. The hydraulic conductivity
is 1.5 m/d (figure is not to the scale).
<br><br>
1. What is the total daily flow of water through the aquifer?
<br><br>
2. What is the elevation of the potentiometric surface at a point located 0.5 km from well $h_1$
and 1 km from well $h_2$?
""",width = 600, style={'font-size': '12pt'})

r16_3 = pn.pane.PNG("images/T06_TP16.png", width=350)

r16_4 = pn.pane.Markdown("""
### Solution of Problem 20:###
""",width = 600, style={'font-size': '12pt'})


pn.Row(r16_2, r16_3)
```

Out[34]:

A confined aquifer is 30 m thick and 5 km wide. Two observation wells are located 1.5 km apart in the direction of flow. The head in well 1 is 90 m and in well 2 it is 85.0 m. The hydraulic conductivity is 1.5 m/d (figure is not to the scale).

1. What is the total daily flow of water through the aquifer?

2. What is the elevation of the potentiometric surface at a point located 0.5 km from well h_1 and 1 km from well h_2?



Solution of Problem 16

In [37]:

```
# solution
r16_6 = pn.pane.LaTeX(r"""
From Darcy Law:
$$ q' = K m \frac{dh}{dL} \qquad\qquad \text{(eq. 1A)}$$
where,
$q'$ is the flow per unit width [L$^2$T$^{-1}$], <br>
$m$ is aquifer thicknes [L]<br>
$K$ is Hydraulic Conductivity [LT$^{-1}$]<br>
and $\frac{dh}{dl}$ = hydraulic gradient [-]<br><br>

Since the thickness of the aquifer is uniform, any hydraulic head between two known
heads ($h_1$ and $h_2$) can be obtained by rearranging the above equation, from
$$
h_2 = h_1 - \frac{q'}{Km}x
\qquad\qquad \text{(eq. 1B)}$$
where $x$ is the distance from $h_1$
""",width = 900, style={'font-size': '12pt'})

r16_6
```

Out[37]:

From Darcy Law:

$$ q' = K m \frac{dh}{dL} \qquad\qquad \text{(eq. 1A)}$$

where, q' is the flow per unit width [L^{2}T^{-1}],
m is aquifer thicknes [L]
K is Hydraulic Conductivity [LT^{-1}]
and $\frac{dh}{dl}$ = hydraulic gradient [-]

Since the thickness of the aquifer is uniform, any hydraulic head between two known heads (h_1 and h_2) can be obtained by rearranging the above equation, from

$$h_2 = h_1 - \frac{q'}{Km}x \qquad\qquad \text{(eq. 1B)}$$

where x is the distance from h_1

In [40]:
```
# Given are:
m_1 = 30 # m, uniform thinckness of aquifer
w_1 = 5 # km, width of the aquifer
d_l = 1.5 # km, distance between wells
hy1_w1 = 90 # m, head in well 1
hy1_w2 = 85 # m, head in well 2
K_1 = 1.5 # m/d, conductivity in aquifer
d_x = 0.5 # km, distance from head 1

# interim calculation
w_1m = w_1*1000 # m, widht of the aquifer
d_lm = d_l*1000 # m, distance between wells
d_xm = 0.5*1000 # m, distance from head 1

#Solution 1
dh_y1 = (hy1_w1 - hy1_w2)/d_lm # (-), head gradient
Q_y1 = K_1*m_1*dh_y1*w_1m # m^3/day, discharge using the first eq. above.

#Solution 2
q_1 = Q_y1/w_1m # m^2/d, flow per unit width
h_y1 = hy1_w1-(q_1/(K_1*m_1))*d_xm # head at 0.3 Km from Well 1, using the second equation


#output
print('\033[1m' + 'Results are:' + '\033[0m \n')
print("The daily discharge from the aquifer is: {0:1.2f}".format(Q_y1), "m\u00b3/d\n")
print("The head at 0.5 Km from well 1 is : {0:1.2f}".format(h_y1), "m")
```

```
Results are:

The daily discharge from the aquifer is: 750.00 m³/d

The head at 0.5 Km from well 1 is : 88.33 m
```

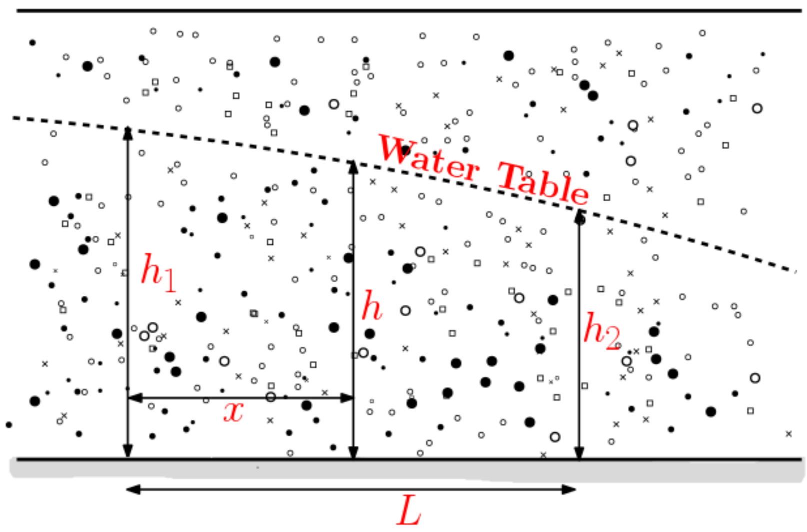
Tutorial Problem 17- flow in unconfined aquifer

In [41]:

```python
r17_1 = pn.pane.LaTeX(r"""
Discharge from an unconfined aquifer presented in the figure below in which $h_1 = 20$ m, $h_2 = 10$ m, and $L = 50$ m
is to be obtained. Other information available are that the aquifer is 30 m wide and has a uniform conductivity
$K = 5 \times 10^{-6}$ m/s. Also known are that the Duipuit assumptions applies to this unconfined aquifer.
""",width = 500, style={'font-size': '12pt'})

r17_2 = pn.pane.PNG("images/T06_TP17.png", width=400)

pn.Row(r17_1, r17_2)
```

Out[41]:

Discharge from an unconfined aquifer presented in the figure below in which $h_1 = 20$ m, $h_2 = 10$ m, and $L = 50$ m is to be obtained. Other information available are that the aquifer is 30 m wide and has a uniform conductivity $K = 5 \times 10^{-6}$ m/s. Also known are that the Duipuit assumptions applies to this unconfined aquifer.



Solution Tutorial Problem 17

In [44]:

```
r17_4 = pn.pane.LaTeX(r"""
As Dupuit assumptions are valid, the discharge per unit width of aquifer ($q'$)
can be obtained from
$$
q' = -Kh\frac{dh}{dx} \qquad\qquad \text{eq. (3A)}
$$
where $h$ is saturated thickness of aquifer located at $x$ distance from $h_1$ end.
From figure, at $x = 0$, $h = h_1$ and at $x = L$, $ h = h_2$. Based on this
differential equation eq. (3A) can be directly integrated after separation of variable to obtain $q'$, i.e.,
$$
\int_0^L q'dx = -K\int_{h_1}^{h_2}h dh
$$
Integration leads to
$$
q'x\Big|^L_0 = -K\frac{h^2}{2}\Big|_{h_1}^{h_2}
$$
resulting to
$$
q'L = -K\Bigg( \frac{h_2^2}{2} - \frac{h_1^2}{2}\Bigg)
$$

and $q'$ is then obtained from

$$
q' = -\frac{1}{2}K\Bigg( \frac{h_2^2 -h_1^2 }{L}\Bigg) \qquad\qquad \text{eq. (3B)}
$$
""",width = 700, style={'font-size': '12pt'})

r17_4
```

Out[44]:

As Dupuit assumptions are valid, the discharge per unit width of aquifer (q') can be obtained from

$$q' = -Kh\frac{dh}{dx} \qquad \text{eq. (3A)}$$

where h is saturated thickness of aquifer located at x distance from h_1 end. From figure, at $x = 0$, $h = h_1$ and at $x = L$, $h = h_2$. Based on this differential equation eq. (3A) can be directly integrated after separation of variable to obtain q', i.e.,

$$\int_0^L q'dx = -K\int_{h_1}^{h_2} hdh$$

Integration leads to

$$q'x\Big|_0^L = -K\frac{h^2}{2}\Big|_{h_1}^{h_2}$$

resulting to

$$q'L = -K\left(\frac{h_2^2}{2} - \frac{h_1^2}{2}\right)$$

and q' is then obtained from

$$q' = -\frac{1}{2}K\left(\frac{h_2^2 - h_1^2}{L}\right) \qquad \text{eq. (3B)}$$

In [45]:
```
#Solution of Tutorial Problem 22:
# Given

h3_1 = 20 # m, aquifer head at point 1
h3_2 = 10 # m, aquifer head at point 1
K3 = 5 * 10**-6 # m/s uniform conductivity of aquifer
L3 = 50 # m, length of the aquifer
W3 = 30 # m, width of the aquifer

#Calculation
q3 = -1/2*K3*((h3_2**2 - h3_1**2)/L3) # m^2/s, unit width discharge using eq. 3B
Q3 = q3 * W3 # m^3/s, total dischage from given width

#output
print('\033[1m' + 'Results are:' + '\033[0m \n')
print("Discharge per unit width of aquifer is: {0:1.2e}".format(q3), "m\u00b2/s \n")
print("Discharge from the given width of aquifer is: {0:1.2e}".format(Q3), "m\u00b3/s")
```

```
Results are:

Discharge per unit width of aquifer is: 1.50e-05 m²/s

Discharge from the given width of aquifer is: 4.50e-04 m³/s
```

Homework Problems Flow problems

These problems will require you to make some research. I will suggest that you check the **Groundwater** book by **R. Allan Freeze** and **John A. Cherry**. The book is now freely available at [https://gw-project.org/books/groundwater/ (https://gw-project.org/books/groundwater/)](https://gw-project.org/books/groundwater/)

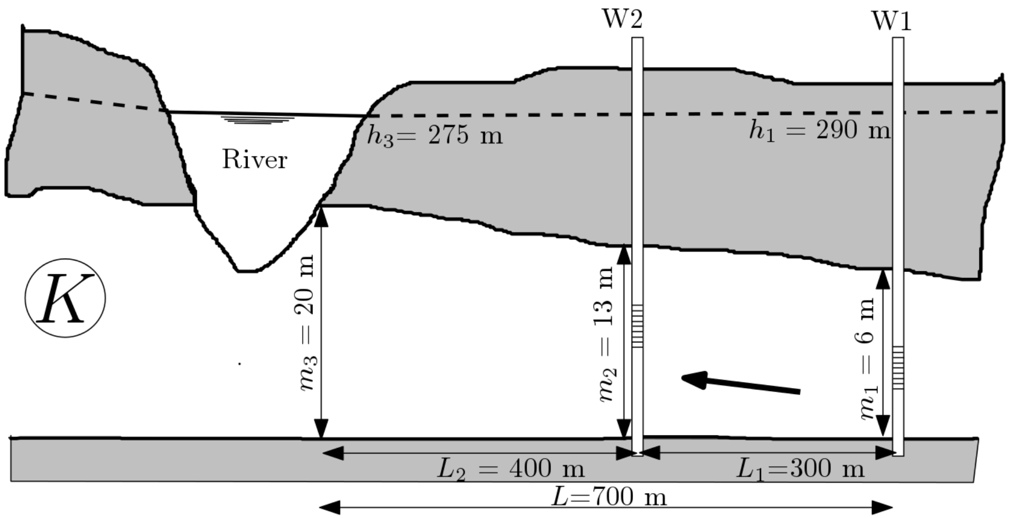
There is no-obligation to submit the homework problems

Homework Problem 8 - confined aquifer

In [46]:
```
rh8_2 = pn.pane.LaTeX(r"""
Presented below in the figure is the available information of an aquifer cross-section.
The aquifer is confined and of variable thickness across the cross-section. It has a uniform conductivity
$5.6 \times 10^{-5}$ m/s. The total Discharge from the aquifer of width 500 m is required to be obtained.
""",width = 400, style={'font-size': '12pt'})

rh8_3 = pn.pane.PNG("images/T06_HP8.png", width=500)

pn.Row(rh8_2, rh8_3)
```

Out[46]:

Presented below in the figure is the available information of an aquifer cross-section. The aquifer is confined and of variable thickness across the cross-section. It has a uniform conductivity 5.6×10^{-5} m/s. The total Discharge from the aquifer of width 500 m is required to be obtained.



Homework Problem 9 - unconfined aquifer

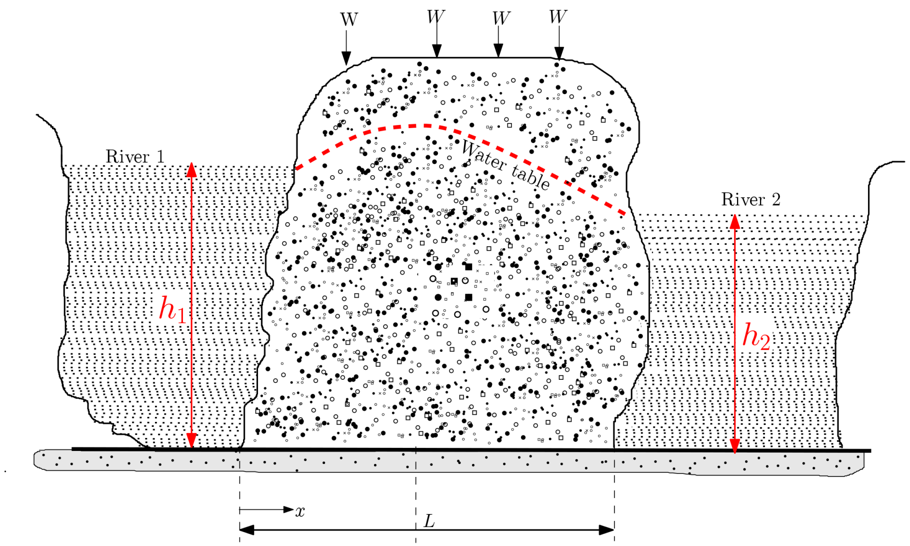
In [47]:
```
rh9_2 = pn.pane.LaTeX(r"""
In a schematic below an unconfined aquifer is found to divide 2 rivers of differnt stages $h_1 = 30$ m and $h_2 = 10
$ m.
The aquifer of length $L = 50$ m and with uniform conductivity $K = 5\times 10^{-6}$ m/s is found to receive recharg
e at
the rate ($w$) of 0.01 m/d. <br>
a) What will be the hydraulic head and discharge per unit width ($q'$) in the aquifer at 5 m
from the left river. <br>
b) What will the head at the same location when aquifer receives no recharge.

""",width = 900, style={'font-size': '12pt'})

rh9_3 = pn.pane.PNG("images/T06_HP9.png", width=450)

pn.Column(rh9_2, rh9_3)
```

Out[47]:

In a schematic below an unconfined aquifer is found to divide 2 rivers of differnt stages $h_1 = 30$ m and $h_2 = 10$ m. The aquifer of length $L = 50$ m and with uniform conductivity $K = 5\times 10^{-6}$ m/s is found to receive recharge at the rate (w) of 0.01 m/d.

a) What will be the hydraulic head and discharge per unit width (q') in the aquifer at 5 m from the left river.

b) What will the head at the same location when aquifer receives no recharge.



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