EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Faculty of Sciences
Department of Geosciences

Research Group: Geo- and Environment sciences · Hydrogeochemisty

Water-nexus modelling and Modelling of MAR schemes

I. Mathematical background

Yadav, P. K.[1], Köhler, A.[1], Tripathi, M.[2]

[1]University of Tübingen, Germany
[2]Indian Institute of Technology Delhi (IITD), India

prabhas.yadav@uni-tuebingen.de

26.11.2025, Roorkee

Funded by:

Deutsche Forschungsgemeinschaft
German Research Foundation

DAAD
Deutscher Akademischer Austauschdienst
German Academic Exchange Service

HTWD
Hochschule für Technik und Wirtschaft Dresden
University of Applied Sciences

Motivation - Why Model?

- For **Predictions** (mostly forward)
- For better **Planning**
- Optimum output (Economy)

Image Source: LanGeek.co (https://shorturl.at/TaOtP)

The sky is dark.
Is it is going to rain?

Motivation - Why Model?

- For **Predictions** (mostly forward)
- For better **Planning**
- Optimum output (Economy)

Image Source: LanGeek.co (https://shorturl.at/TaOtP)

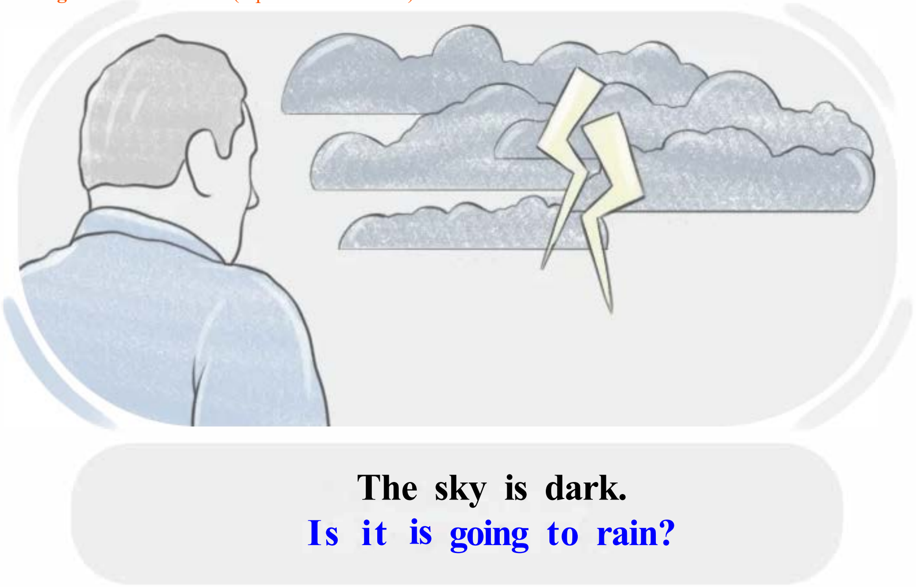
The sky is dark.
Is it is going to rain?

Motivation - Why Model?

- For **Predictions** (mostly forward)
- For better **Planning**
- Optimum output (Economy)

Groundwater is a fundamental resources - proper **planning** and **Optimum** use is essential

Image Source: LanGeek.co (https://shorturl.at/TaOtP)



How to know if it will „**very likely**“ rain?

MODEL IT!

Contents

1. Mathematical aspects: Theory and methods (This Session)
2. Numerical modelling: MODFLOW-ModeMuse (Second Session)
3. Modelling MAR scenarios (The Last Session)

The materials for the second and last sessions can be found at Here

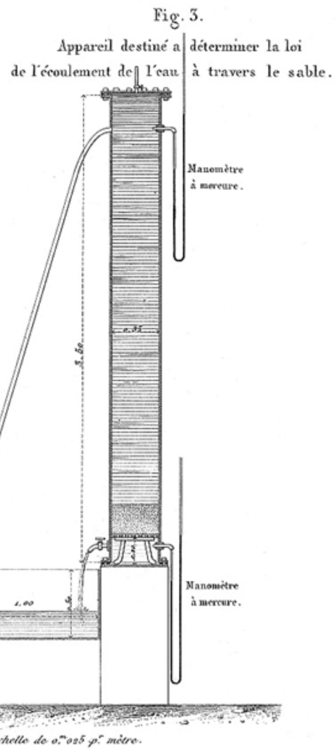
Please **download** the `*.exe` files and install in your (**Windows**) system.

Darcy Law: The starting point

$$\boxed{Q \propto \frac{A\,\Delta h}{x}}$$

with : Q [LT^{-3}] = *water discharge*
A [L^2]= *Area*
Δh [L] = *hydraulic head difference*
x [L] = *distance*

Henry Darcy (1803 - 1858)



Source: https://doi.org/10.1029/2001WR000727

Generalized 1D Darcy's Law

$$\frac{Q}{A} = q = -K\frac{\mathrm{d}h}{\mathrm{d}x}$$

with: q [L] = *specific discharge*
K [LT^{-1}] = *Hydraulic Conductivity*
minus $(-)$: *flow towards low head*

K characterizes domain physical properties and $\frac{\mathrm{d}h}{\mathrm{d}x} = I[-]$, also called head gradient, is actually an energy gradient

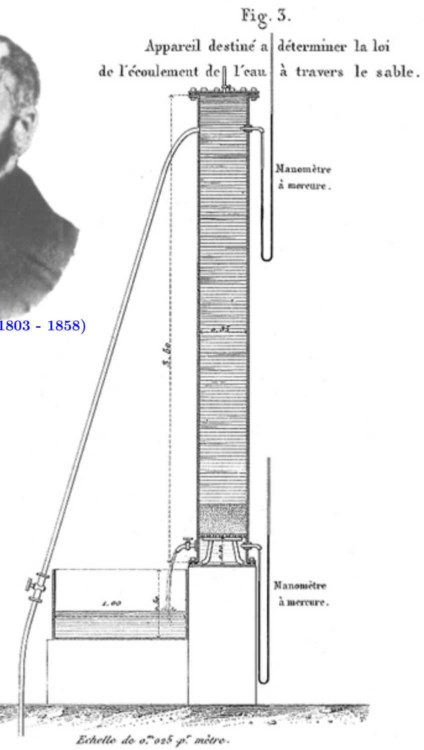
K and I deserves a further attention

Darcy Law: The starting point

$$\boxed{Q \propto \frac{A\,\Delta h}{x}}$$

with : Q [LT^{-3}] = *water discharge*
A [L^2]= *Area*
Δh [L] = *hydraulic head difference*
x [L] = *distance*

Henry Darcy (1803 - 1858)



Source: https://doi.org/10.1029/2001WR000727

Generalized 1D Darcy's Law

$$\frac{Q}{A} = q = -K\frac{\mathrm{d}h}{\mathrm{d}x}$$

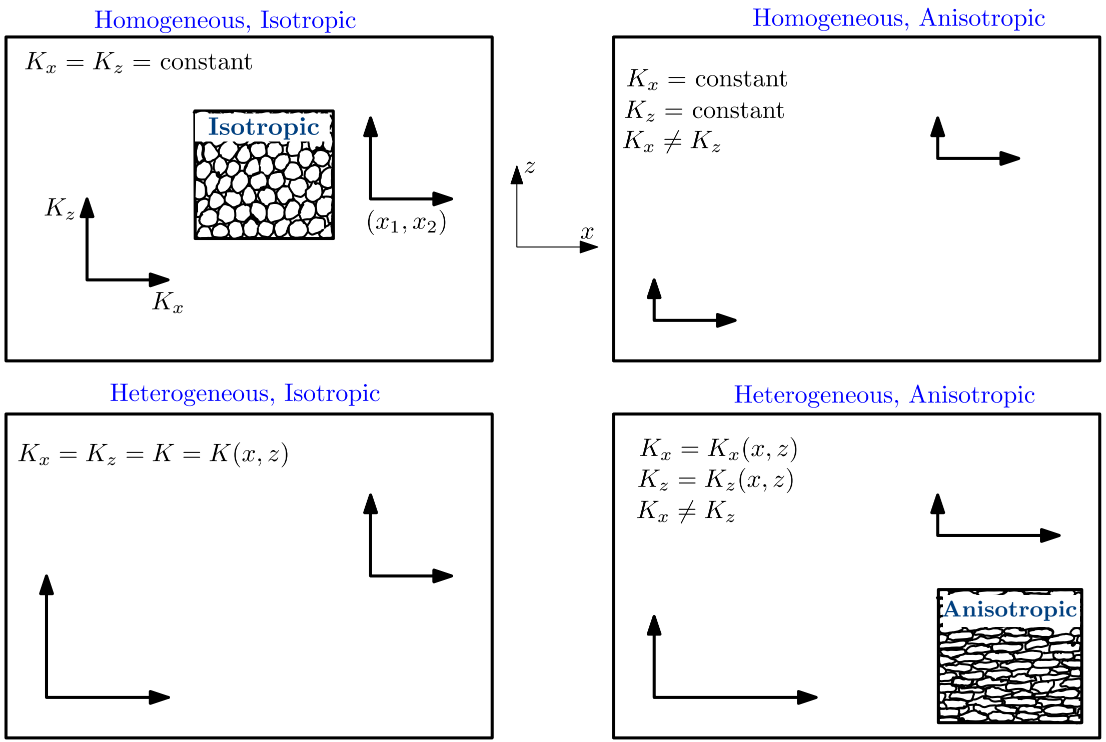
with: q [L] = *specific discharge*
K [LT^{-1}] = *Hydraulic Conductivity*
minus $(-)$: *flow towards low head*

K characterizes domain physical properties and $\frac{\mathrm{d}h}{\mathrm{d}x} = I[-]$, also called head gradient, is actually an energy gradient

K and I deserves a further attention

The Conductivity (K)

- ***K*** is a **composite property** of porous media and fluid
- Groundwater (GW) **normally** has uniform fluid properties
- Porous media is **generally** is very heterogeneous.
- **Primarily**, GW problem is homogeneity - heterogeneity and isotropic - anisotropic pairs or any of their combination



The hydraulic head gradient ($I = \frac{\mathrm{d}h}{\mathrm{d}x}$)

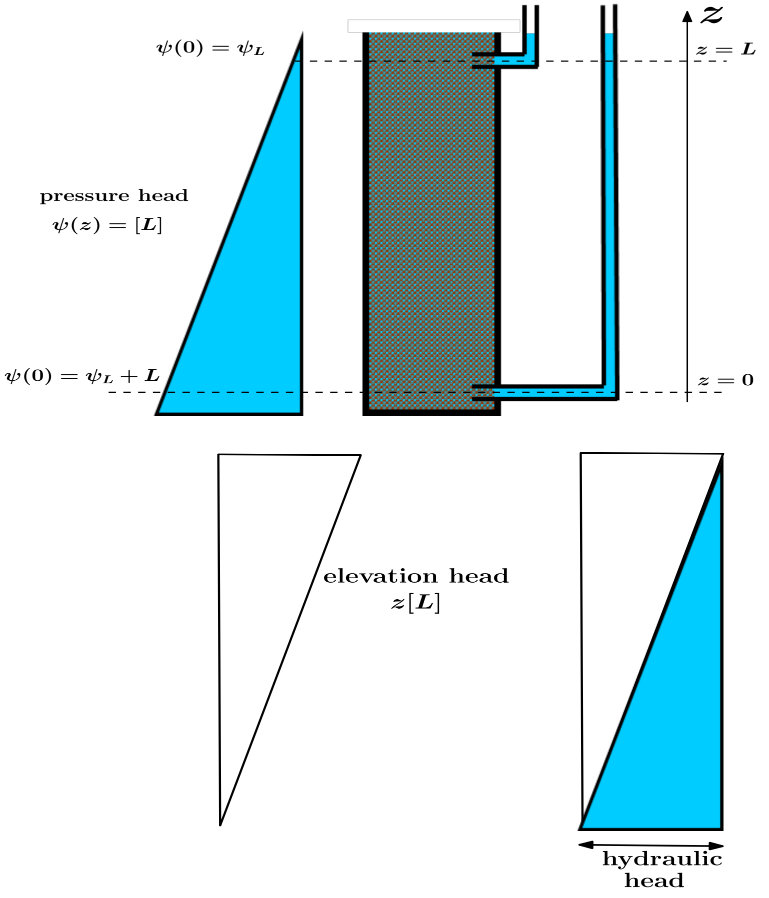
- h, also called **piezometric head**, sums the energy available for the GW flow.
- Only considered are **Potential** (pressure) and **Elevation** energy.
- Thus:

$$h = z + \frac{p}{\rho g}$$

with:

p [M L^{-1} T^{-2}]: *Pressure*
ρ [ML^{-3}]: *density*
g [LT^{-2}]: *gravity*

- GW flows only when $\boxed{\Delta h \neq 0}$
- Flow direction: higher to lower head



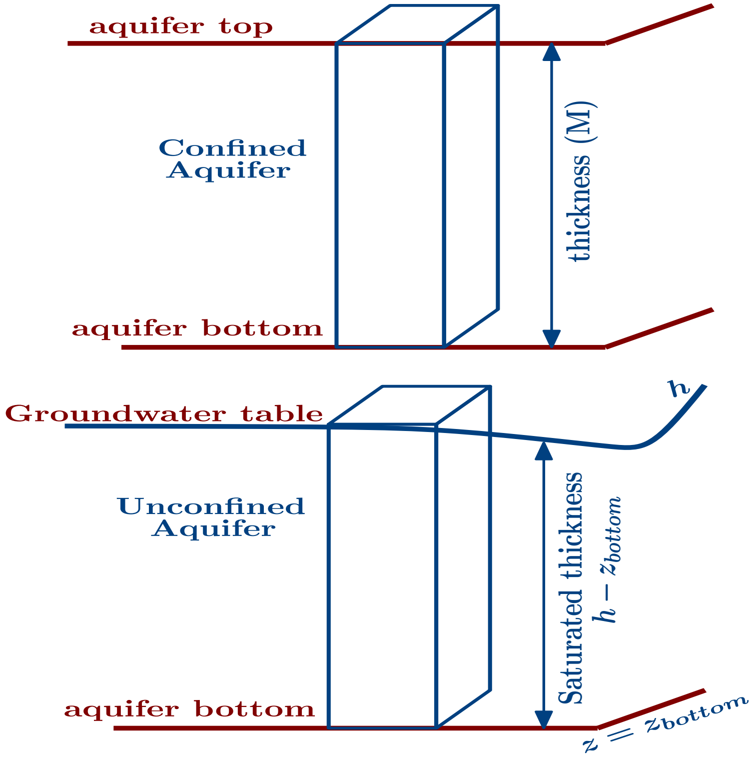
Other important aquifer quantities

- **Specific Storage (S_s)** [L^{-1}] quantifies water storage capacity of an Aquifer from:

$$S_s = \frac{\Delta V_w}{V_T \cdot \Delta \psi}$$

with:
ΔV_w [L^3]: *changed Water volume*
$\Delta \psi$ [L]: *change in Hydrostatic pressure head*
V_T [L^3]: Total aquifer volume

- **Confined Aquifer** - Aquifer bounded in both end: $\boxed{S = S_s \cdot M}$, with Storativity (S) [-] and Aquifer thickness (M)[L]
- **Unconfined Aquifer** - Aquifer top is groundwater table: $\boxed{S = \eta_e}$ with effective porosity (η_e)[-]



GW flow is normally a 2D problem due a small thickness compared to length/width

Darcy's law in 2D and 3D

- $\boxed{K}$ and $\boxed{h}$ are space-dimension dependent
- They appear in the Darcy's law
- Thus in 3D, h is a **column vector**:

$$\text{grad } h = \nabla h = \begin{bmatrix} \frac{\mathrm{d}h}{\mathrm{d}x} \\ \frac{\mathrm{d}h}{\mathrm{d}y} \\ \frac{\mathrm{d}h}{\mathrm{d}z} \end{bmatrix}$$

- For 2D, the third dimension can be omitted
- The Darcy's law in 3D is:

$$\boxed{\boldsymbol{q = -K \cdot \nabla h}}$$

- K **tensors**:

3D and **anisotropic** & **heterogeneous**:

$$\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

Principal axis aligned to the domain:

$$\begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}$$

- Other simplified form of K can be obtained
- The most **general** Darcy's law:

$$q = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} \frac{\mathrm{d}h}{\mathrm{d}x} \\ \frac{\mathrm{d}h}{\mathrm{d}y} \\ \frac{\mathrm{d}h}{\mathrm{d}z} \end{bmatrix}$$

Darcy's law in 2D and 3D

- $\boxed{K}$ and $\boxed{h}$ are space-dimension dependent
- They appear in the Darcy's law
- Thus in 3D, h is a **column vector**:

$$\text{grad } h = \nabla h = \begin{bmatrix} \frac{\mathrm{d}h}{\mathrm{d}x} \\ \frac{\mathrm{d}h}{\mathrm{d}y} \\ \frac{\mathrm{d}h}{\mathrm{d}z} \end{bmatrix}$$

- For 2D, the third dimension can be omitted
- The Darcy's law in 3D is:

$$\boxed{\boldsymbol{q = -K \cdot \nabla h}}$$

- K **tensors**:

3D and **anisotropic** & **heterogeneous**:

$$\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

Principal axis aligned to the domain:

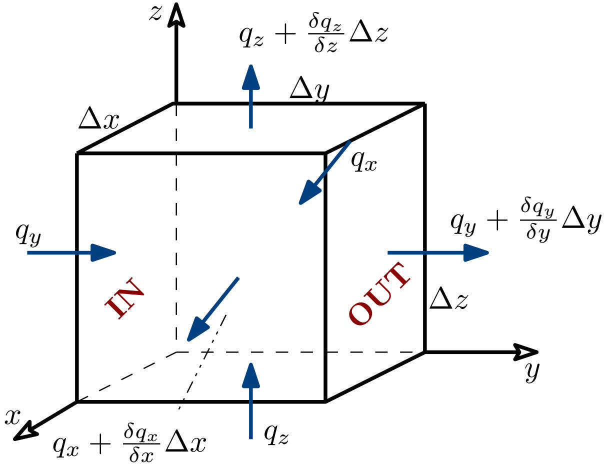
$$\begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}$$

- Other simplified form of K can be obtained
- The most **general** Darcy's law:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{K_{xx}} & \boldsymbol{K_{xy}} & \boldsymbol{K_{xz}} \\ \boldsymbol{K_{yx}} & \boldsymbol{K_{yy}} & \boldsymbol{K_{yz}} \\ \boldsymbol{K_{zx}} & \boldsymbol{K_{zy}} & \boldsymbol{K_{zz}} \end{bmatrix} \cdot \begin{bmatrix} \frac{\mathrm{d}h}{\mathrm{d}x} \\ \frac{\mathrm{d}h}{\mathrm{d}y} \\ \frac{\mathrm{d}h}{\mathrm{d}z} \end{bmatrix}$$

Groundwater flow system equations

- Consider a unit volume $(\Delta x, \Delta y, \Delta z)$



- **Mass (water) and budget (storage)** and applying **Darcy's law** generates the Groundwater flow equation

- 3D Transient, Homogeneous and Isotropic

$$\boxed{S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)}$$

- 3D Transient, Homogeneous and Anisotropic with Source/Sink (***N***) [T^{-1}]

$$\boxed{S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right) + N}$$

- 3D Steady-State, Homogeneous and Isotropic

$$\boxed{\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right) = 0}$$

Special groundwater flow system equations

- **The Laplace equation**:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- 3D, Steady-state, homogeneous, isotropic, and confined aquifer **without** source/sink

Pierre-Simon Laplace (1749 - 1827)

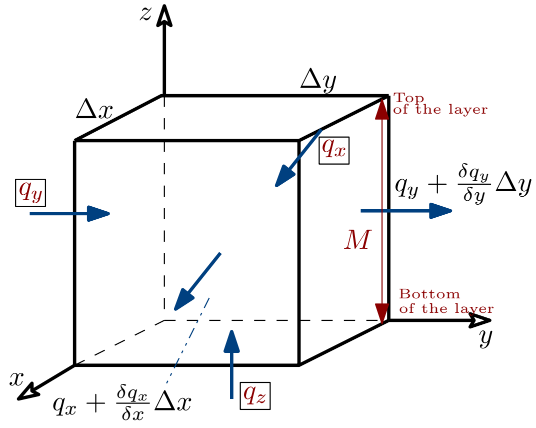
- **The Poisson equation**:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = -\frac{N}{K}$$

- 3D, steady-state, homogeneous, isotropic and confined aquifer **with** source/sink

Siméon Denis Poisson (1781–1840)

Special groundwater flow system equations: The 2D cases



- Vertical GW flow to layering are negligible, i.e., $\boxed{q_x, q_y \gg q_z}$
- Therefore, GW flow problems are **often** considered as **2D** problems

- Transmissivity (T) [L^2T^{-1}] is then used to **sum** the vertical flow component:

$$T = K \cdot M$$

with: M [L] Aquifer thickness
K [L T^{-1}] Hyd. Conductivity

- Thus, for **anisotropic aquifers**:

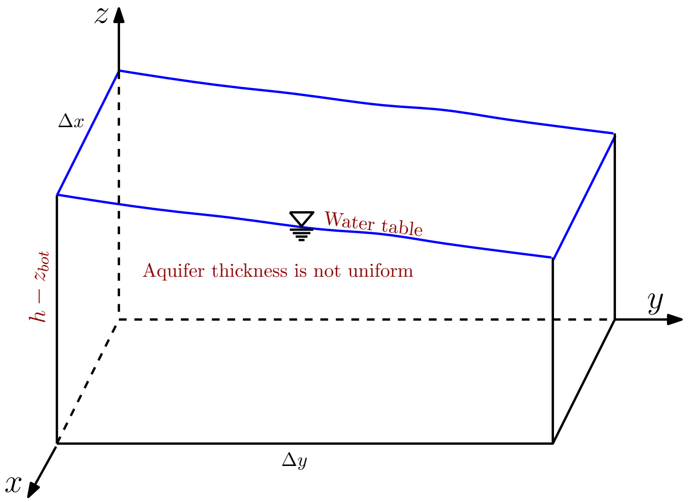
$$\boxed{T_x = K_x \cdot M} \text{ and } \boxed{T_y = K_y \cdot M}$$

- for horizontally **Isotropic** confined aquifers:

$$\boxed{T = K \cdot M}$$

The special 2D flow equations: Unconfined aquifers

- Aquifer thickness is a variable



- Thus:

$$\boxed{T_x = K_x \cdot (h - Z_{bot})}$$

$$\boxed{T_y = K_y \cdot (h - Z_{bot})}$$

- The 2D GW flow equation for **unconfined** and **anisotropic** with **source/sink** is:

$$\boxed{S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(T_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y\frac{\partial h}{\partial y}\right) + N}$$

with Stortavity (S) [-]

- Same equation is for the **confined** case with T calculated as in the previous slide
- This equation is also called the **Boussinesq equation**

Joseph Valentin Boussinesq (1842 – 1929)

Complete formulation of Groundwater flow equation

Step I: Specify the geometric properties of the region of interest (dimension, shape…)

Step II: Characterize Aquifer - K, S_s, S or T, by considering heterogeneity and Anisotropy

Step III: Select the appropriate flow equation, and provide additional data (e.g., Wells, river)

Step IV: Specify the initial condition(IC):

A. head value at time $t = 0$

B. **Not required** for steady-state problems.

Step IV: Specify boundary conditions (**BC**):

A. BCs have to be given along the complete boundary (also at infinity)

B. BCs may be time-dependent

C. There are **three** major types of BCs .

Step V: Select the appropriate **Solver**

What are the BCs & how they are set?

The 3 Boundary conditions

Dirichlet BC:

- Also called first type BC
- The head is specified, e.g., $h = h_0$

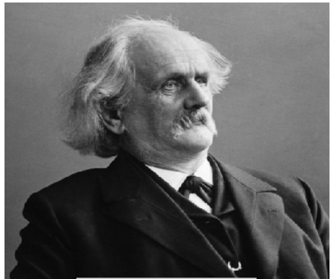
Neumann BC:

- Also called second type BC
- The head **gradient** is specified, e.g., $\frac{dh}{x} = h_0$
- **Special case** - No flow BC: $\frac{dh}{x} = 0$

Cauchy or Robin BC:

- Also called Third type or Mixed BC
- Combines other BCs, e.g., $q = C(h - h_b)$
  with Conductance C [LT^{-1}] *and external head* h_b [L]

Peter Gustav Lejeune Dirichlet (1805 - 1859)



Carl Neumann (1832 - 1925)

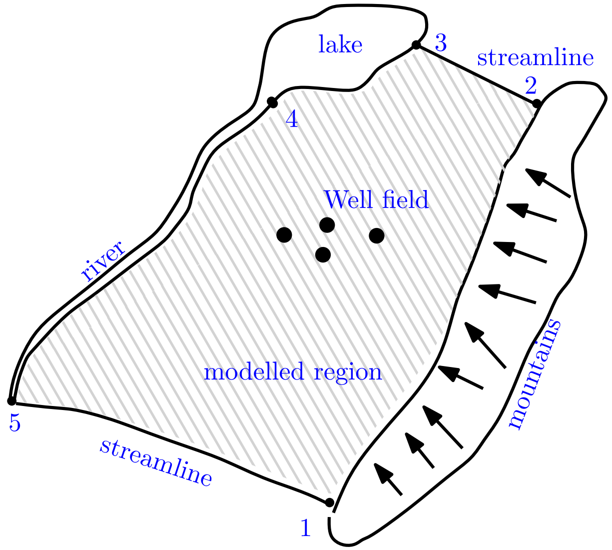
Augustin-Louis Cauchy (1789 - 1857)

Summarizing the mathematical formulation of GW flow problem

Aquifer/Flow property	Mathematical Formulation
Transient	with time derivative
Confined	linear partial differential equation
Anisotropic	$T_x \neq T_y$ or $K_x \neq K_y$ (tensor)
Heterogeneous (inhomogeneous)	coefficients depends on space coordinate(s)
With sources/sinks	inhomogeneous differential equation (contains a term without h)
Fixed-head boundary condition	boundary condition of the first kind (Dirichlet)
Flux BC (e.g.: no flow)	boundary condition of the second kind (Neumann)

Implementing boundary condition to the GW flow problem

- BCs must be set based on geological and geographical setting.
- An e.g., GW model domain

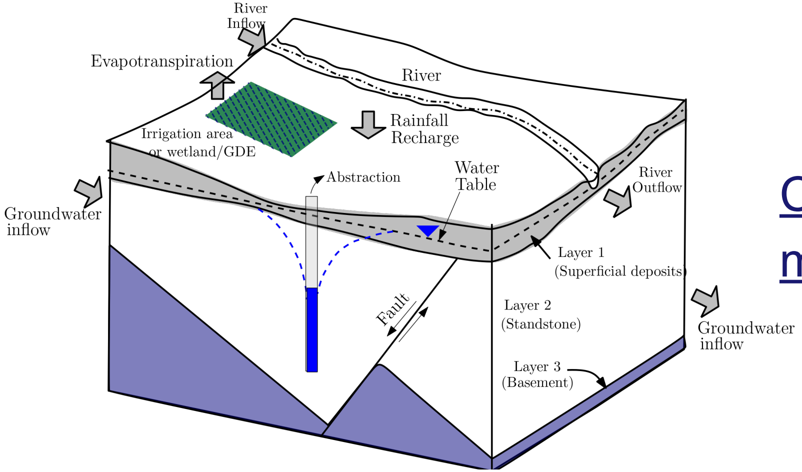


Boundary 1-2: Specified flux (non-zero)
Boundary 2-3: Specified zero flux
Boundary 3-4: Specified head
Boundary 4-5: Semipervious
Boundary 5-1: Zero flux

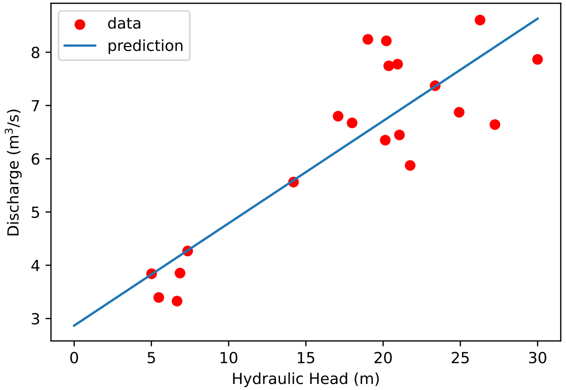
- A **complete** GW flow problem: **System equation + fully defined BCs**

Modeling GW flow problems

- A **Model** is a representation of a **real** or **conceptual** system
- Model can be:
  - Conceptual model (*qualitative or graphic representation*)
  - Process-based and empirical models (*data or experiment-based*)
  - Mathematical model (*Conservation laws based*)



Conceptual model



Empirical model

Solving mathematical models of GW flow problems

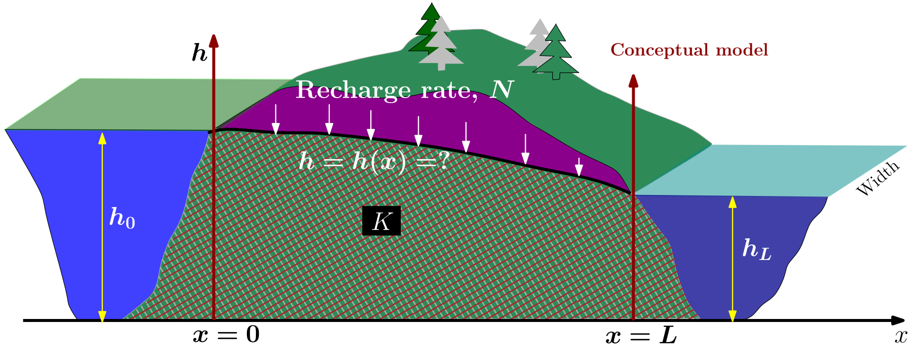
- The mathematical models are generally solved either:
  - **Analytically** (exact solution), or
  - **Numerically** (approximate solution)
- Hybrid methods, e.g., **Analytic Element Method** (AEM)
- Numerical method is a **universal** one, although not always economical

Solving mathematical models of GW flow problems

- The mathematical models are generally solved either:
  - **Analytically** (exact solution), or
  - **Numerically** (approximate solution)
- Hybrid methods, e.g., **Analytic Element Method** (AEM)
- Numerical method is a **universal** one, although not always economical

Analytical Models

- Can solve only simple problems, e.g., homogeneous, steady-state



The solution (Streamlit Web-App by Alvin):

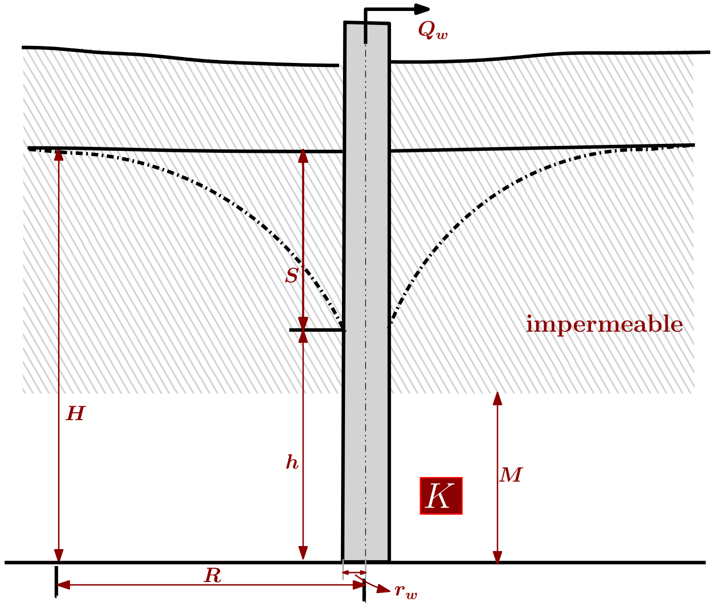
$$h(x) = \sqrt{h_o^2 - (h_o^2 - h_L^2)\cdot \frac{x}{L} + \frac{N}{K}\cdot x \cdot (L - x)}$$

Analytical (Thiem) solution for steadily operating Well

- For wells in **Confined** Aquifer:
  - In **Homogeneous** and **Isotropic** aquifer
  - Operating at **Steady-State** condition
  - For a **Fully Penetrating** well
  - GW flow is **Horizontal** and **Radial**
- Thiem (1906) Solution:

$$Q_w = \frac{2 \cdot \pi \cdot M \cdot K \cdot (H - h)}{2.3 \cdot \log(R/r_w)}$$

with: *Well radius* r_w [L], *Confined thickness* M [L], *well discharge*(Q_w) [L^3T^{-1}], *water level in aquifer at rest* H[L], *water level* in the well h [L], *drawdown* $S = H - h$ [L]



what is

Radius of Influence (*R*) [L]?

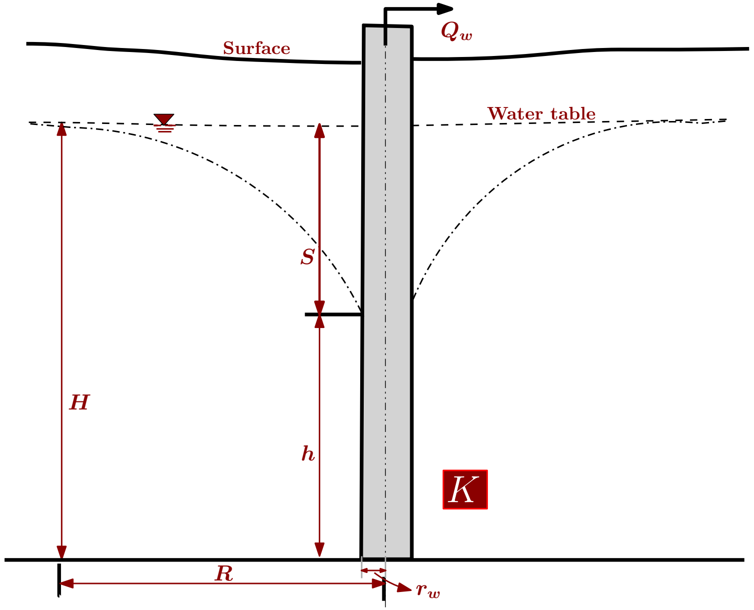
Analytical (Thiem) solution for steadily operating Well

- For a well in an **Unconfined** Aquifer
- Other assumptions as in the **confined** case

$$Q_w = \frac{\pi \cdot K \cdot (H^2 - h^2)}{2.3 \log(R/r_w)}$$

- ***R*** is obtained from Empirical formula (e.g.,):

Source	Formula
Kyrieleis and Sichardt (1930)	$R = 3000 \cdot s\sqrt{K}$
Arvin and Numerov (1953)	$R = 1.9\sqrt{(H \cdot K \cdot t/s)}$ *with pumping time (t)* [T]



How about transient wells?

Analytical (Theis) solution for transiently operating wells

- Theis (1935) requires:
  - $r_w \gg R$ in **Confined** aquifer
  - Others as is in (Thiem 1906) solution
- Theis (1935) Solution:

$$s(r,t) = \frac{Q}{4\pi T} \cdot W(u)$$

where r [L] *is distance from well to the observation well*

with: $\boxed{W(u) = \int_u^\infty \frac{e^{-\tilde{u}}}{\tilde{u}}\, d\tilde{u}}$ and $\boxed{u = \frac{Sr^2}{4Tt}}$

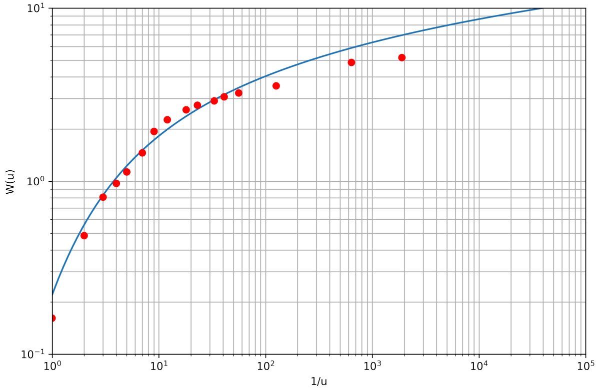
- The **plot** of **log-transformed** form of eqs. $s(r,t)$ and $W(u)$ leads to so-called the **Type-Curve**

- log transformed forms:

$$\boxed{\log s(r,t) = \log \frac{Q}{4\pi T} \cdot \log W(u)}$$

$$\boxed{\log \frac{t}{r^2} = \log \frac{S}{4T} + \log \frac{1}{u}}$$

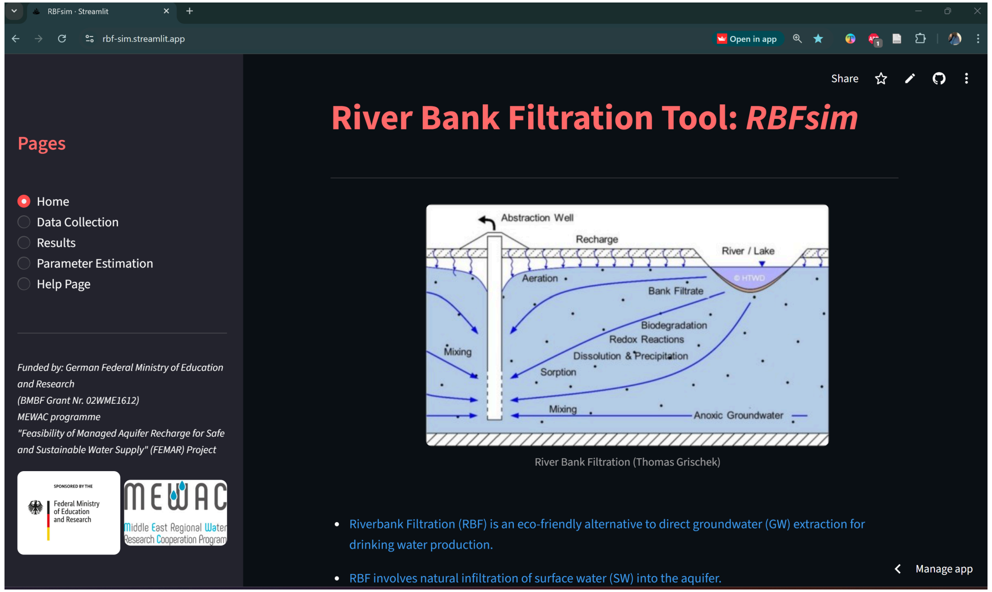
- Type curve allows to quantify S and T



Check the Web-App(Web-App by Alvin)

Hybrid solution for River Bank Filtration cases

- **Hybrid solution methods** attempts to combine Analytical and/or Numerical methods
- **Hybrid Method** can economically provide to some GW flow problems
- **RBFsim** implements Analytic Element Method (AEM) to solve several Riverbank filtration scenario.
- **RBF*sim*** can be simulated on-line from **https://rbf-sim.streamlit.app/**

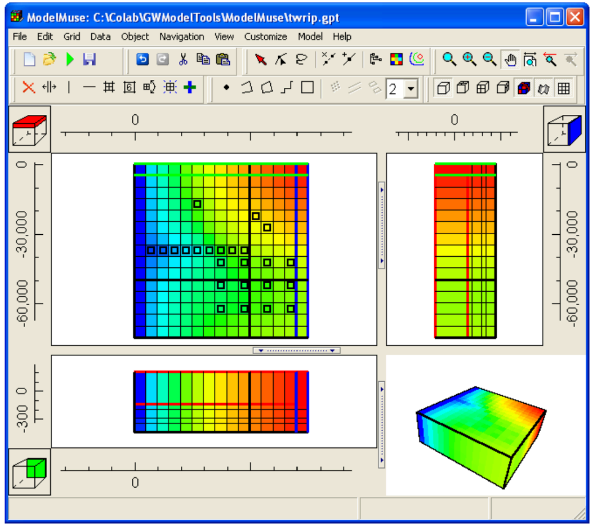


Source: Yadav et al. (2024)

Numerical methods for solving GW flow problems

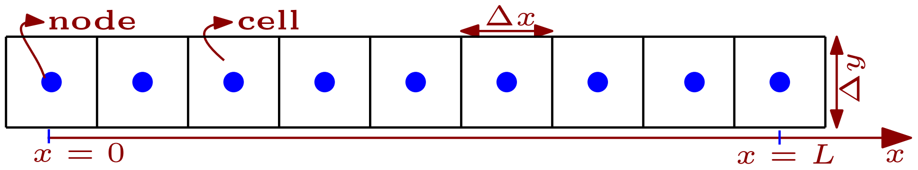
- **Numerical** methods can solve almost all types of GW flow problems
- Finite Different Method (FDM) is very generally the numerical method of choice, due to:
  - Simplified **adaptation** of the problem
  - **Economical** also for large domains
  - Sufficiently **accurate** and **stable**
- Other (to FDM) methods e.g., **Finite Element Method**, **Finite Volume Method** are also actively developed and used.

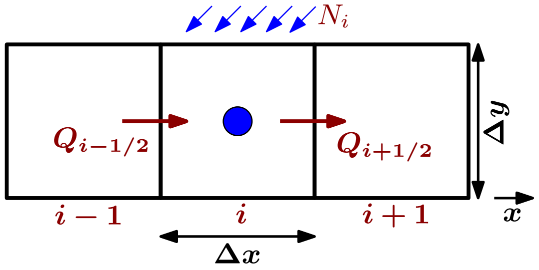
- MODFLOW - an open-source FDM software has been an industry-standard for decades



- ModelMuse - the GUI for MODFLOW simplifies the use of MODFLOW.

MODFLOW (FDM) methods - the 1D case





- Based on two fundamental principles:
  - conservation of volume and
  - Darcy‘s Law
- Space is discretized ($\Delta x, \Delta y \cdots$) in **Grids**
- Hydraulic head is computed at the **node**, which here is **block-centred**

- Volume conservation:

$$Q_{i-\frac{1}{2}} + N_i \cdot \Delta y \cdot \Delta x = Q_{i+\frac{1}{2}}$$

- Darcy’s law:

$$Q_{i-\frac{1}{2}} = -T \cdot \Delta y \cdot \frac{(h_i - h_{i-1})}{\Delta x}$$

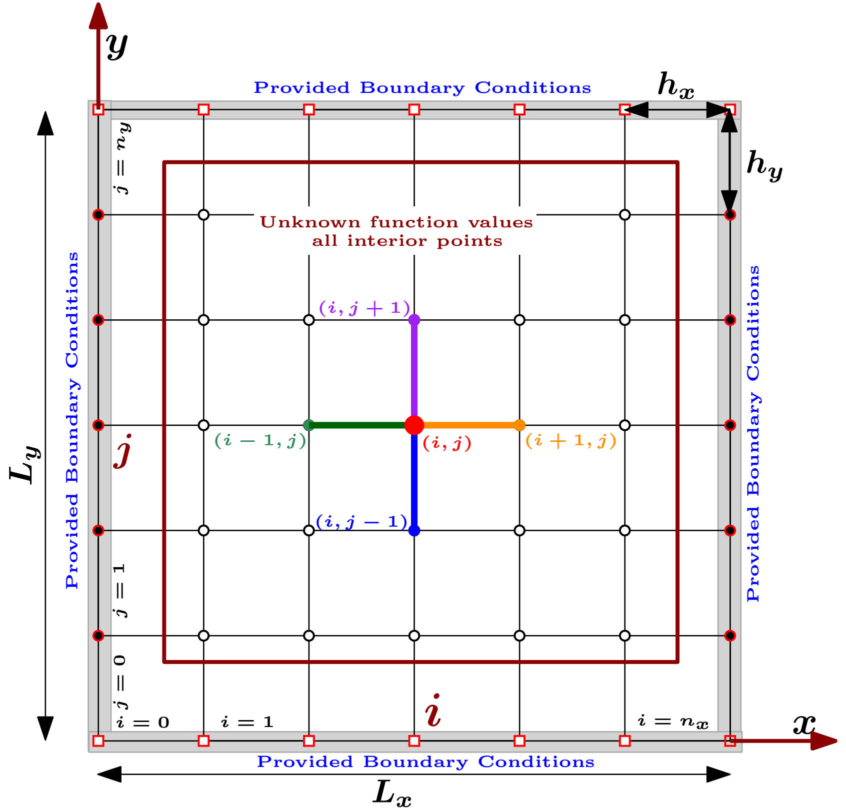
$$Q_{i+\frac{1}{2}} = -T \cdot \Delta y \cdot \frac{(h_{i+1} - h_i)}{\Delta x}$$

- Leading to FD equation (fixed ***h*** BCs):

$$h_i = \frac{h_{i-1} + h_{i+1}}{2} + \frac{N_i}{2T} \cdot \Delta x^2$$

MODFLOW (FDM) methods - the 2D Case

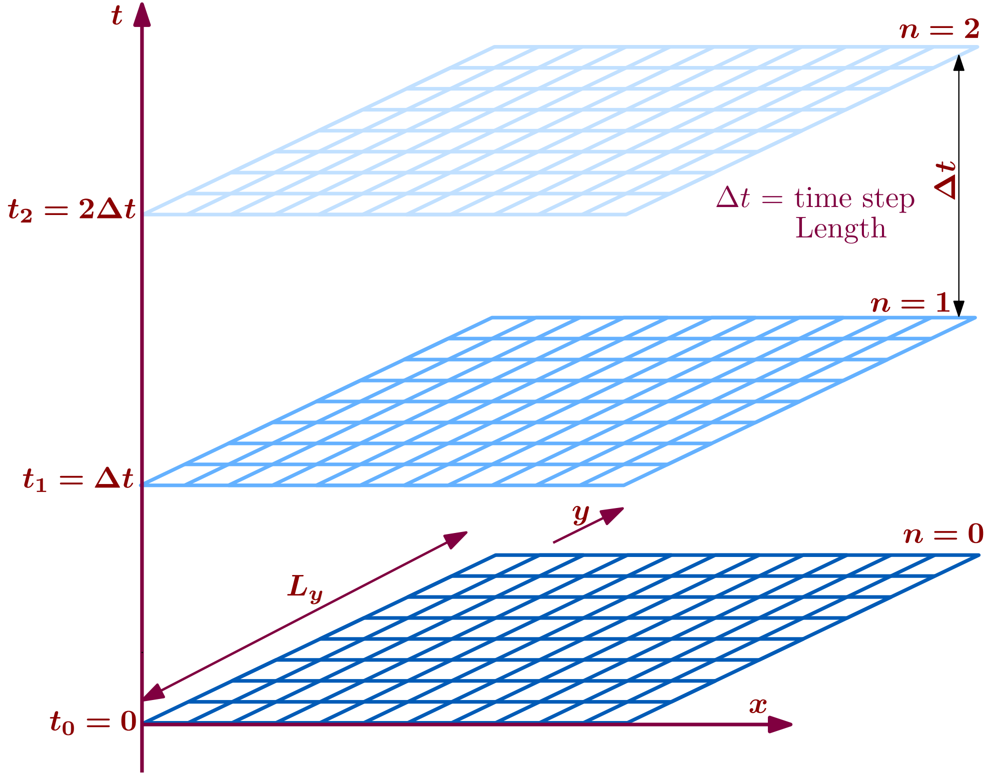
- GW flow is **generally** modelled as a 2D system
- 2D system normally is a single layer
- Multi-layer systems can be developed by varying ***K*** in each layer
- ***T*** or ***K*** are calculated at the cell edges
- The 2D (or 3D) FD equations are developed similar to 1D case
- FD equation for interior cells depends on BCs and processes



MODFLOW (FDM) methods - time derivative

- Transient problem requires discretization of **time**
- For **stability** of the solution it is required that: $\boxed{C = v\frac{\Delta t}{\Delta x} < 1}$

  with Courant number **C** *[-] and GW flow velocity* $\boldsymbol{v}$ *[LT^{-1}]*
- Internally, the **time-stepping** (in MODFLOW) follows the *fully implicit backward Euler* method

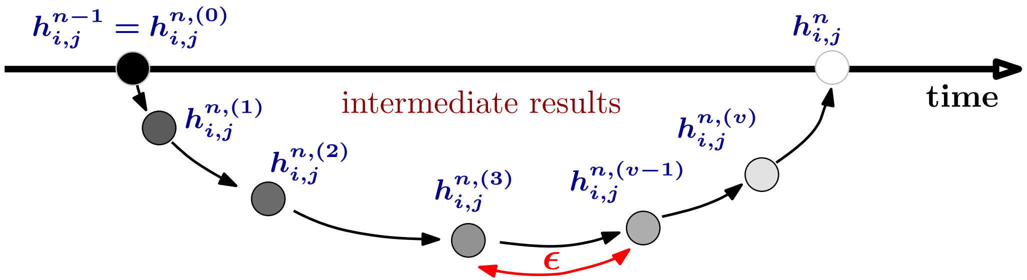


MODFLOW (FDM) methods - Iteration and Solver

- MODFLOW iteratively solves the **well-posed** GW flow problem
- Each numbered (*in bracket*) iterative time-step improves the solution until **acceptable error-tolerance** or **maximum iteration** (*max-itr*) is reached.
- Error criterion ε:

$$\max \left| h_{ij}^{n,(v)} - h_{ij}^{n,(v-1)} \right| < \varepsilon$$

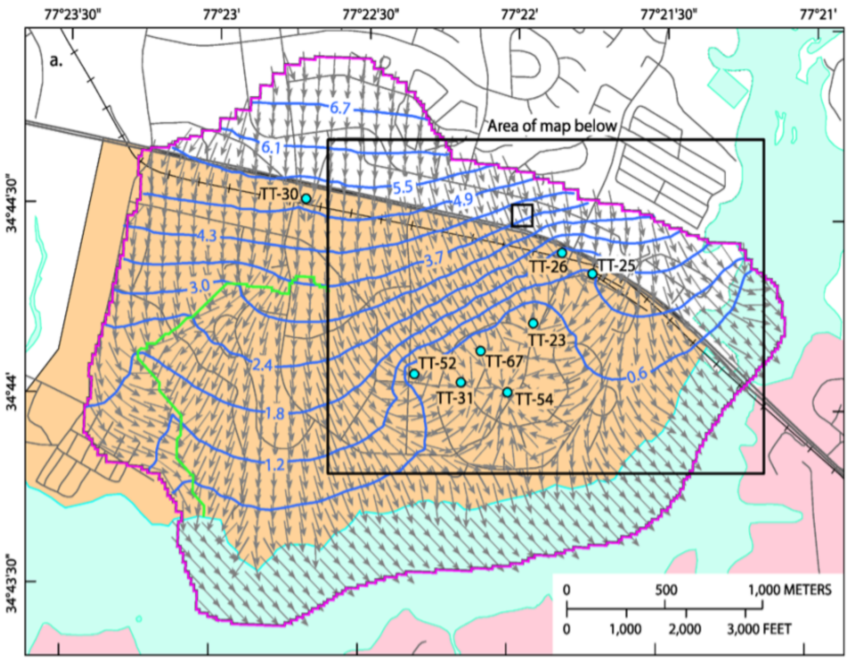
- ε or **max-itr** can be problem-type specific



Solvers	Problem-type
Preconditioned Conjugate Gradient (**PCG**)	Confined aquifer
Geometric Multigrid Solver (**GMG**)	Large 3D Grid
Newton Formulation (**NWT**)	Complex nonlinear model

Post-Processing GW flow results

- Visualization of MODFLOW (or numerical) results normally requires use of complex codes
- Generally (for GW flow problems):
  - head countour lines are plotted
  - flowlines (vector) provide flow direction in the domain
  - streamlines, tangent to flowlines, provide speed of flow
- Visualization can be computationally intensive



1.8

Simulated potentiometric contour. The contour intervals are 0.6 m and 0.7 m.

Simulated direction of groundwater flow

Site: U.S. Marine Corps Base Camp Lejeune, North Carolina

Source: Maslia et al. (2009)

The figure will be replaced by Own figure from Code

Computing interface for GW flow Problems

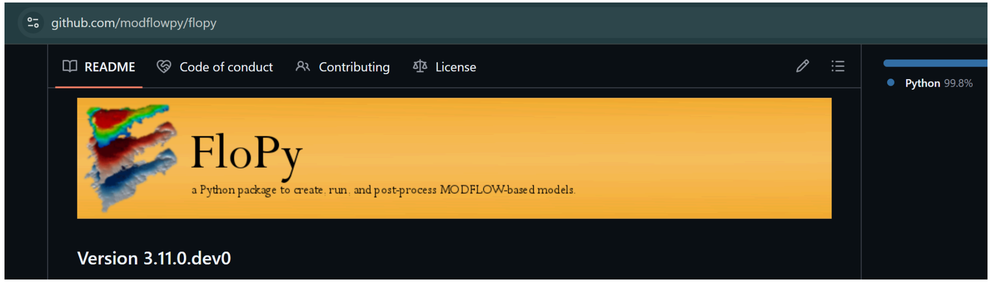
- GW flow problem using FD method have a simple code structure
- For simple problems computational codes can be easily developed
- More complex problems will require MODFLOW-type code
- FloPy allows scripting MODFLOW
  - Scripting helps in automatization
  - Not suitable for problems (e.g.) with complex geometeries
  - Requires programming skills

1D short code will be added here

python

```
fn main() {
    println!("Hello World!");
}
```

FloPy has been developed since 2001

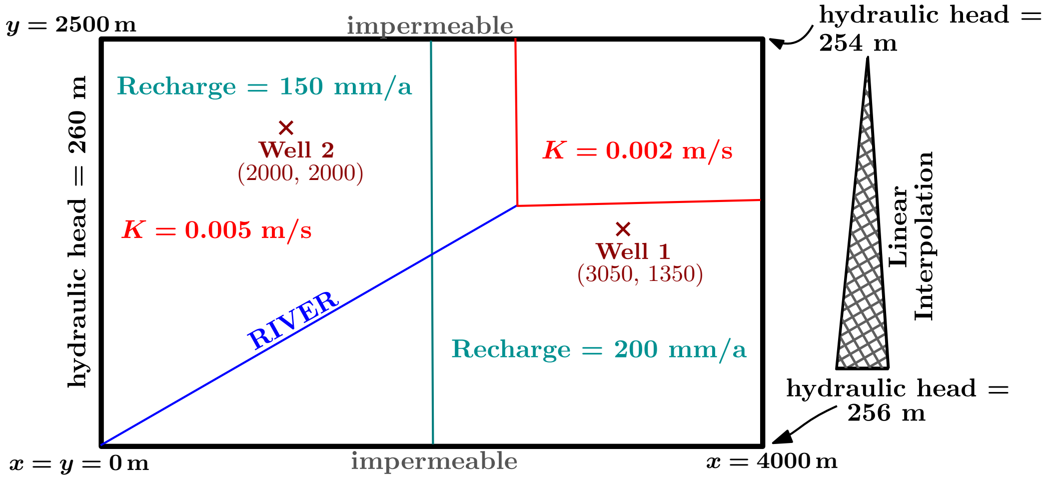


Modeling GW flow with MODFLOW and ModelMuse

- ModelMuse provides an excellent user-interface for MODFLOW
  - It is Open-source code (flexibility)
  - Developed by USGS since 2006
  - Extensively documented and cost-free

- In the next two sessions we use MODFLOW with ModelMuse to
  - 2D horizontal GW flow model
  - 2D vertical GW flow model
  - 3D GW flow model
  - Include Riverbank filtration scenario

- The first 2D model conceptual setup



Download the required files from here

References

Arvin, V. I., and S. N. Numerov. 1953. *Theory of Motion of Liquids and Gases in Undeformable Porous Media*. Gostekhizdat, Moscow, USSR.

Kyrieleis, W., and W. Sichardt. 1930. *Grundwasserabsenkung Bei Fundierungsarbeiten*. Julius Springer, https://books.google.co.in/books?id=UJdBAAAAIAAJ.

Theis, C. V. 1935. The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-water Storage. *Eos Trans. AGU* 16, no. 2: 519–24, 10.1029/TR016i002p00519.

Thiem, G. 1906. Hydrologische Methoden [Hydrological Methods]. PhD thesis, Universität Stuttgart, Stuttgart, Germany.

Yadav, P. K., V. Batheja, A. Köhler, V. Cantarella, M. Tufail, C. Werth, and T. Grischek. 2024. RBFsim – A Tool for Early Planning Stage of Riverbank Filtration Systems. *Sustain. Water Resour. Manag.* 10, no. 5: 167, 10.1007/s40899-024-01137-9.

Thank You !!!

Acknowledgements(not in order):

🤝 DFG/DAAD for funds

👍 Alvin for web-apps

Slides coded using:

typst