POWERS IN NONSINUSOIDAL SITUATIONS A REVIEW OF DEFINITIONS AND PHYSICAL MEANING

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Abstract

This article expands on the physical meaning of the reactive power in nonsinusoidal situations. It first surveys the sinusoidal waveform case, viewing the active current as a component of identical waveform with the voltage. This approach, when extended to nonsinusoidal waveforms, supports Fryze's model for apparent power, $S^2 = P^2 + Q_F^2$. It is proved that the total reactive power Q_F is composed from four distinctive types of elementary reactive powers. Each one of these basic reactive powers is identified as the amplitude of an oscillation of instantaneous power. The separation of Q_F in Q_1, the reactive power at the system frequency and Q_H, the reactive power at harmonic frequencies is recommended as an effective mean for monitoring filter efficacy and power factor compensation.

Two major recommendations supported by the results of this study are made: 1. To abolish the power models using distortion power. 2. To measure the active power of the system frequency separately from the active power of the harmonics.

Key Words: Nonsinusoidal Systems, Definitions of Powers, Harmonics.

List of Principal Symbols

$D =$ Budeanu's Distortion Power

$F_s(m,n)$, $F_c(m,n) =$ Trigonometric functions representing the elementary reactive power caused by interaction between harmonic voltage of order m and harmonic current of order n.

$i =$ instantaneous current (line current)

$i_a =$ instantaneous current component formed by the in- phase harmonics

i_p, $I_p =$ in-phase instantaneous and rms current. Has the same waveform as the distorted voltage.

$i_q = i - i_p$ (in-quadrature current)

i_r, $I_r =$ instantaneous and rms current formed by the in-quadrature harmonics

$i_{qD} = i_q - i_p$

$p =$ instantaneous power

p_a, $p_p =$ instantaneous power caused by i_a and i_p respectively. Both have the mean value P

$p_q = p - p_p$ (mean value nil)

$p_{qR} = p - p_a$ (mean value nil)

$p_{qD} = p_a - p_p$ (mean value nil)

$P =$ active power

$P_1 =$ system frequency active power

$P_H =$ harmonics active power

$Q =$ reactive power in sinusoidal situations

$Q_1 =$ system frequency reactive power (only fundamental current and voltage contribution)

$Q_F = \sqrt{S^2 - P^2} =$ Fryze's generalized reactive power

$Q_H =$ harmonics reactive power

Q_{Bh}, $Q_{Bmn} =$ elementary reactive powers caused by the in-quadrature components of harmonic order h and m with n, respectively.

Q_{Dh}, $Q_{Dmn} =$ elementary reactive powers caused by the in-phase components of harmonic order h and m with n, respectively.

$S =$ apparent power

v, $V =$ instantaneous and rms voltage

V_h, V_m, $V_n =$ harmonic voltage of order h, m and n. (rms values)

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BACKGROUND

The year was 1888, when for the first time, reference was made to the fact that oscillations of power between an alternating voltage source and the load are caused by the phase angle between voltage and current [1,2]. The explanation given then by Stanley and Shallenberger is, with minor modifications, still found in textbooks and engineering standards.

Nowadays engineers accept the definitions of apparent **S**, active **P** and reactive **Q** powers in sinusoidal systems without reservations. Industry and metrological institutions cooperate harmoniously in the development and use of instrumentation which measure these powers. Energy management programs as well as power flow studies are also dependent on models where **P+jQ** is the complex variable on which the economical optimization is based. After 1888 it required more than forty years and the talents of Steinmetz [3], Houston and Kennely [4], Iliovici [5], Budeanu [6], Emde [7], Knowlton [8] and Fortescue [9] for the rank and file of the profession to fully recognize and accept the importance of power factor and reactive power.

Today the increased use of static power converters, the proliferation of adjustable speed drives and the use of shunt capacitors for minimization of energy costs contribute to the creation of conditions prone to excessive distortion of voltage waveform. While the implications of nonsinusoidal waveforms on the delivery of electric energy were clearly indicated by many early researchers [6,7,10], the engineering community has not reached a consensus yet for a universally accepted definition for the powers in electrical networks with nonsinusoidal situations. Two major models dominate today's approach to the definitions and components of the reactive power: First is the school of Budeanu, which is sanctioned in the ANSI/IEEE Standard 100-1977. Second is the school of Fryze which influenced the International Electrical Commission's position [11].

Recent articles in journals of specialty and conferences attest to the ongoing struggle to recognize and to produce a practical model acceptable to the electric utility and the end-user of electricity. Such a model should yield itself to values with clear physical meaning and lend itself to convenient and accurate measurements [12...17].

This article is meant to help in the understanding of the physical meaning of instantaneous power components and to relate the characteristic values of these components (amplitude, oscillation frequency and phase) to the basic models for apparent power. This study has two parts: First reviews the energetical factors in systems with sinusoidal voltage. On the second part, the method of analysis for sinusoidal system is extended to nonsinusoidal voltage situations and the two schools of thought are compared.

SYSTEMS WITH SINUSOIDAL VOLTAGE, $v = \sqrt{2}V \sin \omega t$

The resolution of the current $i = \sqrt{2}I \sin(\omega t + \theta)$ in an active (in-phase) and a reactive (quadrature) current, Fig. 1, leads to two

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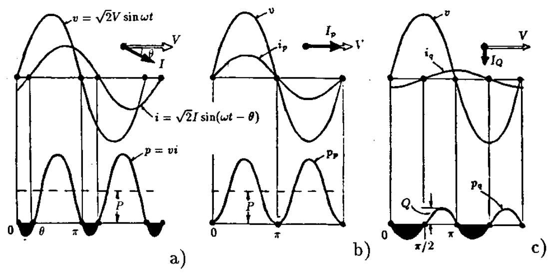

Fig. 1 Instantaneous Powers in Sinusoidal Situations

a) Current and Voltage Waves. The Total Instantaneous Power, p
b) In-Phase Current and the Intrinsic Instantaneous Power, p_p
c) Reactive Current and the Double Frequency Power Oscillation, p_q

distinctive instantaneous power components:

$$p = vi = p_p + p_q \tag{1}$$

where

$$p_p = P(1 - \cos 2\omega t) \ ; \qquad P = VI\cos\theta \tag{2}$$

$$p_q = Q\sin 2\omega t \ ; \qquad Q = VI\sin\theta \tag{3}$$

P is the active power, also called average, effective or real power. The component p_p has the average value P, and is a unidirectional pulsating power, fluctuating between 0 and 2P. This fluctuation being inherent to an alternating voltage source, the term p_p may be called **intrinsical instantaneous power.**

The component p_q is an oscillation of power at double supply frequency. This component transfers energy back and forth between the sources and the linear loads. **The amplitude of the oscillation p_q is the reactive power** Q, Fig. 1.

In the literature it is customary to use the terms **delivered** and **received** to designate the flow directions of the reactive power [18]. The fact is that the time-average energy transferred by p_q is always nil, and the oscillations are due to the property of the capacitors and inductors to store energy in the electric and magnetic field respectively. Since the power oscillations caused by capacitors and inductors are 180° out of phase it is convenient to use a complex apparent power representation, i.e. the visualization of a four-quadrant power flow [18].

To follow more in depth the physical meaning of the reactive power we shall now consider a lossless synchronous generator supplying a single-phase linear inductor or capacitor. The instantaneous mechanical power delivered by the prime mover is

$$p_m = p_q + J\Omega(d\Omega/dt) \tag{4}$$

when J = momentum of inertia
Ω = mechanical angular velocity
$p_q = p$ = instantaneous electrical power

Since the system is assumed lossless $p_m = 0$, (once the rotor is brought to synchronous speed the steam to the turbine can be cut) and (4) can be written

$$Q\sin 2\omega t = -J\Omega d\Omega/dt \tag{5}$$

The angular velocity will oscillate around a synchronous value with an excursion $\Delta\Omega$. The maximum kinetic energy delivered or received is obtained from (5)

$$J\Delta\Omega^2/2 = Q/\omega \tag{6}$$

This equation emphasizes the energetical meaning of Q as a value proportional with the maximum energy transferred back and forth between the prime mover and the load.

If a perfectly balanced three-phase purely reactive load is supplied by a **p**-poles alternator the mechanical torque developed by the prime mover (see Appendix) has the equation

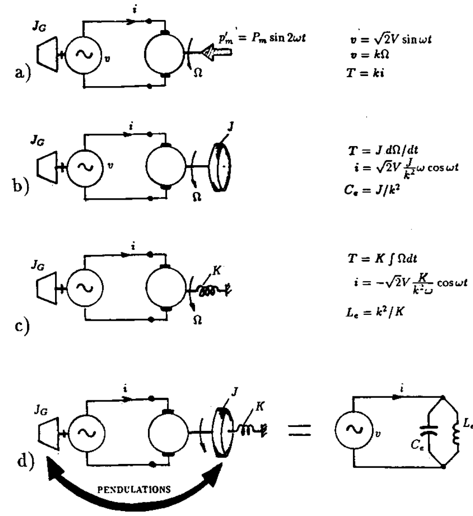

Fig. 2 Generation of Reactive Power in Systems with Mechanical Components.

a) dc Machine Driven with a Hypothetical Shaft Power $P_m \sin 2\omega t$
b) dc Machine Coupled to a Flywheel
c) dc Machine Coupled to a Torsion Bar
d) dc Machine Driving a Flywheel and a Torsion Bar

$$\mathbf{T} = Q\frac{\mathbf{p}}{2\omega}[\sin 2\omega t + \sin 2(\omega t - 2\pi/3) + \sin 2(\omega t + 2\pi/3)] = 0$$

Here Q is the reactive power "flowing" in one phase. This result shows that in the case of balanced loads the rotors of turbine and alternators do not deliver/receive kinetic energy under steady-state operation. The oscillations of p_q take place among the three reactive loads via the inductive coupling of the armature and field windings.

An enlightening set of examples, shown in Fig. 2, will further reinforce the physical meaning of the instantaneous rate of energy transfer, p_q. In Fig. 2a, a hypothetical dc machine is coupled with a prime mover which delivers the mechanical power $p'_m = P_m \sin 2\omega t$. Assuming the dc machine lossless.

$$v = k\Omega \ ; \qquad \Omega = \Omega_m \sin\omega t \ ; \qquad \Omega_m = \sqrt{2}V/k \tag{7}$$

and the torque is

$$T = p'_m/\Omega = ki \tag{8}$$

The current supplied by the source v obtained from (7) and (8) is

$$i = p'_m/k\Omega = (2P_m/k\Omega_m)\cos\omega t$$

and the instantaneous rate of energy transfer becomes

$$p_q = vi = (2\sqrt{2}VP_m/k\Omega_m)\sin\omega t\ \cos\omega t = P_m\sin 2\omega t$$

Thus $\mathbf{Q = P_m}$. This simple result tells that we are dealing with a pendulum where the energy is changed back and forth between the "prime-mover" delivering p_m and the kinetic energy stored in the masses turning with the alternator's shaft. The amplitude of the instantaneous power is Q. To illustrate further the physical meaning of this concept the power source p'_m is replaced with a flywheel, Fig. 2b, or a torsion bar, Fig. 2c, or a combination, Fig. 2d. (Kapp oscillator [19])

In the general case, Fig. 2d, the torque is

$$T = ki = Jd\Omega/dt + K\int\Omega dt \tag{9}$$

Substitution of (7) in (9) yields

$$i = \sqrt{2}V(\omega J/k^2 - K/k^2\omega)\cos\omega t \tag{10}$$

This proves that the equivalent circuit of the electromechanical system includes a capacitor C_e in parallel with an inductor L_e,

$$C_e = J/k^2 \ ; \qquad L_e = k^2/K$$

and the expression of the p_q obtained from (10) shows that the amplitude of the oscillations of power is

$$Q = V^2(\omega J/k^2 - K/k^2\omega) = \omega(W_J - W_K) \tag{11}$$

where

$$W_J = J\Omega_m^2/2 = J(V/k)^2 = C_eV^2$$
$$W_k = K\alpha_m^2/2 = K(V/k\omega)^2 = L_eI^2$$

are the maximum energies stored in the flywheel and spring respectively in the process of mechanical pendulation.

The definition of Q as the amplitude of an instantaneous rate of energy transfer with average energy transfer nil can be extended to any type of energy conversion. Consider for example a time-varying resistor with a conductance

$$g(t) = g\cot(\omega t) \tag{12}$$

the current supplied by the source is

$$i = gv = \sqrt{2}Vg\cos\omega t$$

and the instantaneous power

$$p_q = V^2g\sin 2\omega t$$

with the amplitude $Q = V^2g$ as a measure of the reactive power. In this case the oscillations of power do not take place with the help of a capacitor, inductor, or mechanical component. The energy supplied by the source is stored in the form of thermal energy in a heat sink when $g > 0$ (for $0 < \omega t < \pi/2$) and returned from the heat sink to the source when $g < 0$ (for $\pi/2 < \omega t < \pi$) and the resistor "generates" electric energy. It is difficult to find such a resistor in nature, nevertheless this example paves the road for the next paragraphs where more realistic time-varying devices are considered.

SYSTEMS WITH SINUSOIDAL VOLTAGE, $v = \sqrt{2}V\sin\omega t$ AND NONSINUSOIDAL CURRENT

To help observe the contribution of a nonsinusoidal current to the process of energy transfer we will assume first a current

$$i = \sqrt{2}I_h\sin(h\omega t + \theta_h)\,, \qquad h \neq 1 \text{ and integer}$$

The instantaneous power in this case is

$$p = vi = p_q = p_{D1h} + p_{B1h}$$

where

$$p_{D1h} = Q_{D1h}F_c(1,h)\,; \qquad p_{B1h} = Q_{B1h}F_s(1,h) \tag{13}$$

and $F_c(1,h)$, $F_s(1,h)$ are the functions

$$F_c(1,h) = \cos[(h-1)\omega t] - \cos[(h+1)\omega t] \tag{14}$$

$$F_s(1,h) = -\sin[(h-1)\omega t] + \sin[(h+1)\omega t] \tag{15}$$

The instantaneous power components p_{D1h} and p_{B1h} are nonsinusoidal oscillations. However each of these oscillations is in turn composed out of two elementary sinusoidal oscillations of equal amplitudes at the frequencies $(h\pm1)f$. The amplitudes of the elementary power oscillations are

$$Q_{D1h} = VI_h\cos\theta_h \quad \text{and} \quad Q_{B1h} = VI_h\sin\theta_h$$

The two **elementary components** of Q_{1h} are in quadrature

$$Q_{1h}^2 = Q_{D1h}^2 + Q_{B1h}^2 \tag{16}$$

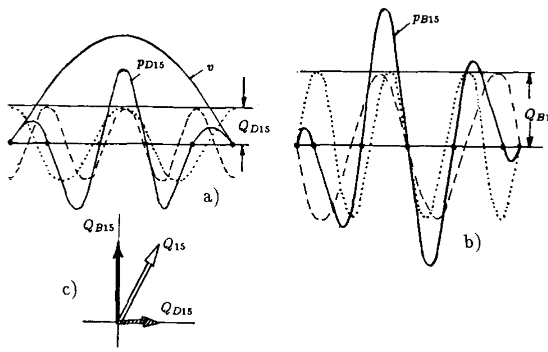

Fig. 3 Example of Reactive Power due to a Sinusoidal Voltage $v = \sqrt{2}V\sin\omega t$ and a Current Harmonic $i = \sqrt{2}I_5\sin(5\omega t + 63°)$

a) The Component p_{D15} and the Amplitude Q_{D15}

b) The Component p_{B15} and the Amplitude Q_{B15}

c) Four-Quadrant Representation of Q_{15}

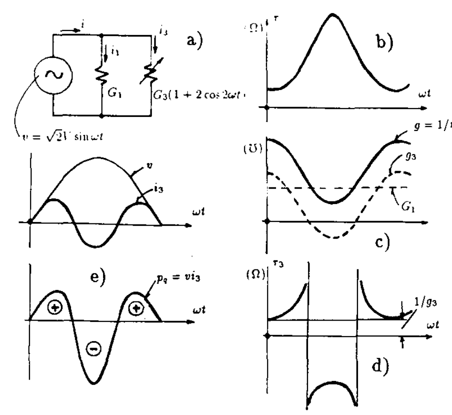

Fig. 4 Generation of Reactive Power by Means of a Time-Varying Resistor.

a) The Considered Circuit

b) Equivalent Resistance Variation

c) Conductances G_1, g_3 and g

d) The Fictitious Time-Varying Resistor r_3

e) Oscillograms of Voltage, Third Harmonic Current and p_q

Since $Q_{D1h}, Q_{B1h} \lesseqgtr 0$, it is possible to designate a four-quadrant flow direction (Fig. 3c) to the **elementary reactive power** Q_{1h}. It is of interest to note that by making $h = 1$, p_{B1h} oscillation becomes of the form (3) encountered in the sinusoidal current case.

An example where $h = 5$ and $\theta_5 = 63°$, Fig. 3, shows the functions $F_s(5,1)$ and $F_c(5,1)$.

To investigate the nature of p_{D1h} and p_{B1h} we shall address a case where the time-varying resistor has the conductance

$$g = G_1 + G_3(1 + 2\cos 2\omega t) \tag{17}$$

This particular load (Fig. 4), yields a current with a fundamental and a 3rd harmonic

$$i = vg = \sqrt{2}I_1 \sin\omega t + \sqrt{2}I_3 \sin 3\omega t \tag{18}$$

where

$$I_1 = VG_1\,; \qquad I_3 = VG_3 \tag{19}$$

and the instantaneous power $p = p_p + p_q$, where

$$\text{where} \quad \begin{aligned} p_p &= P(1 - \cos 2\omega t) \\ p_q &= V^2 G_3 F_c(1,3) \end{aligned} \tag{20}$$

Comparing (20) with (13) results that $Q = Q_{D13} = V^2 G_3 > 0$, i.e. delivered. The *nonsinusoidal oscillations* of p_q are caused by the time-varying conductance $G_3(1 + 2\cos 2\omega t)$, which can be viewed as a reversible electro-thermal energy converter similar to the resistor described by (12) in parallel with a constant resistor of conductance G_3.

The practical implications of the time-varying resistor (18) for filtering applications is evident when we analyze a simple but realistic nonlinear resistor with the i/v characteristic

$$\begin{aligned} i &= Av^3 = 2\sqrt{2}AV^3 \sin^3\omega t \\ &= \sqrt{2}I_1' \sin\omega t - \sqrt{2}I_3' \sin 3\omega t \end{aligned} \tag{21}$$

where

$$I_1' = 3AV^3/2\;; \qquad I_3' = I_1'/3 \tag{22}$$

Comparing (18) with (21) results that a nonlinear resistor can be viewed as a time-varying element. Moreover, by adjusting $G_3 = AV^3/2$, the third harmonic generated by the nonlinear resistor can be cancelled by the third harmonic produced by the time varying resistor (17).

In a similar way the reactive power of the type p_{B1h} is produced when a nonlinear inductor is energized. For example assuming a flux-current characteristic $\psi = ai^{1/3}$ and keeping in mind that

$$\psi = \int v dt = -\left(\sqrt{2}V/\omega\right) \cos\omega t$$

results

$$i = -\sqrt{2}I_1 \cos\omega t - \sqrt{2}I_3 \cos 3\omega t \tag{23}$$

$$I_1 = \frac{3}{2}(V/a\omega)^3\;; \qquad I_3 = I_1/3$$

The instantaneous power in this case is

$$p_q = -Q_1 \sin 2\omega t - Q_{B13}F_s(3,1) \tag{24}$$

and the powers are

$$S^2 = Q^2 = V^2 I^2 = VI_1^2 + VI_3^2 = Q_1^2 + Q_{B13}^2$$

The **fundamental reactive power** Q_1 and the elementary harmonic power Q_{B13} are amplitudes of totally different frequencies of oscillations and cannot be added algebraically. The elementary reactive power Q_{B13} caused by the nonlinear inductor can be cancelled in the following way:

A time-varying resistor is connected in parallel with the nonlinear inductor. The resistor's conductance is described by the equation

$$g = G_1'' - G_3'' \sin 2\omega t\;; \tag{25}$$

and yields the current

$$i = \sqrt{2}I_p'' \sin\omega t - \sqrt{2}I_{q1}'' \cos\omega t + \sqrt{2}I_3'' \cos 3\omega t$$

where

$$I_p'' = VG_1'\;;\; I_{q1}'' = VG_3''/2 \text{ and } I_3'' = VG_3''/2$$

The conductance $-G_3'' \sin 2\omega t$, may be thought of as an equivalent inductance $L_e = 2/G_3''\omega$ in parallel with a 3rd harmonic current source or a time-varying conductance $(\cos 3\omega t/2\sin\omega t)G_3''$.

This particular time-varying resistor is producing the instantaneous power

$$p_q = P'(1 - \cos 2\omega t) - Q_1'' \sin 2\omega t + Q_{B13}''F_s(3,1)$$

If $Q_{B13}'' = V^2 G_3'' = Q_{B13} = VI_3$, then the 3rd harmonic current I_3 in (23) will be cancelled by the current I_3''. This observation reinforces the usefulness of a unified theory on the nature of reactive powers.

THYRISTORIZED CIRCUITS WITH SINUSOIDAL VOLTAGE

A triac in series with a resistor R, Fig. 5, is triggered at $\omega t = \beta$. In this circuit the fundamental current is lagging with respect to the voltage and a fundamental reactive power Q_1 as well as harmonic reactive power are created. The analysis of the circuit will be carried in the spirit of the previous examples. This circuit has an equivalent time-varying conductance

$$\begin{aligned} g = 0 \text{ for } \quad & 0 < \omega t < \beta\,; \quad \pi < \omega t < \pi + \beta \\ g = 1/R = G \text{ for } \quad & \beta < \omega t < \pi\,; \quad \pi + \beta < \omega t < 2\pi \end{aligned}$$

which can be represented by the Fourier series

$$g = G\left[(\pi - \beta)/\pi - \sum_{n=1,2,3\cdots} (a_n \cos 2n\omega t + b_n \sin 2n\omega t)\right] \tag{26}$$

$$a_n = (\sin 2n\beta)/n\pi\,; \qquad b_n = (1 - \cos 2n\beta)/n\pi$$

Computations of line current $i = vg$ yield

$$i = \sqrt{2}VG\left\{\left(\frac{\pi - \beta}{\pi} + \frac{\sin 2\beta}{2\pi}\right)\sin\omega t - \frac{1 - \cos 2\beta}{2\pi}\cos\omega t - \sum_{h=3,5,7,\cdots} (B_h \cos h\omega t - D_h \sin h\omega t)\right\} \tag{27}$$

where

$$B_h = (a_n - a_{n+1})/2\,; \qquad D_h = (b_n - b_{n+1})/2\,; \qquad h = 2n + 1$$

The instantaneous power

$$p = p_p + p_{Q_1} + \sum_h (p_{B1h} + p_{D1h})$$

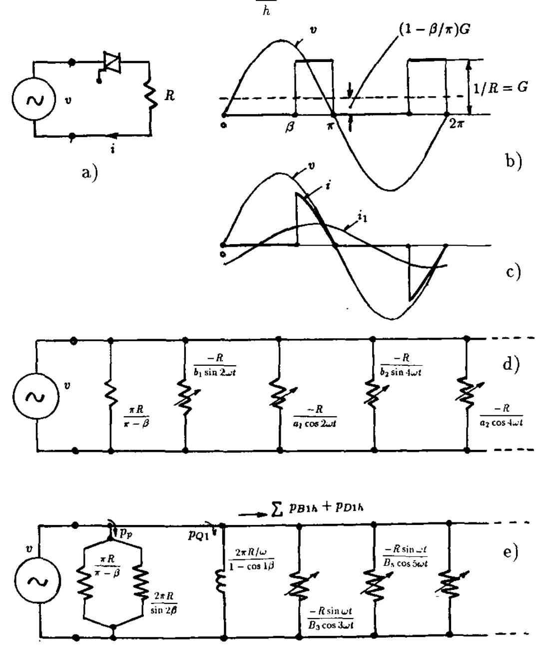

Fig. 5 Generation of Reactive Powers in a Circuit with a Triac Controlled Resistor

a) Circuit
b) Equivalent Conductance-Time-Variation
c) Voltage, Current and Fundamental Current
d) Equivalent Circuits based on eq. (26)
e) Equivalent Circuit based on eq. (27)

where

$$p_p = P(1-\cos 2\omega t); \quad P = V^2G\left[(\pi-\beta)+(\sin 2\beta)/2\right]/\pi$$
$$p_{Q_1} = Q_1 \sin 2\omega t; \quad Q_1 = V^2G(1-\cos 2\beta)/2\pi$$
$$p_{B1h} = Q_{B1h}F_s(1,h); \quad Q_{B1h} = -V^2GB_h$$
$$p_{D1h} = Q_{D1h}F_c(1,h); \quad Q_{D1h} = V^2GD_h$$

and the functions $F_c(1,h)$ and $F_s(1,h)$ were defined in (14) and (15).

For this purely resistive circuit there are no components able to store and return energy. Nevertheless the equivalent circuits derived from (26), Fig. 5d, or from (27), Fig. 5e, show that besides a constant resistor, there are an infinite number of fictitious time-varying resistors all connected in parallel. The active power is dissipated by the constant resistor. Each fictitious time-varying resistor has a time-average dissipated thermal energy zero. The time-varying resistors however, are the sources of power oscillations of types p_{B1h} and p_{D1h}. The fundamental reactive power is caused by a fictitious equivalent inductor, Fig. 5e, obtained in an identical manner as in (25).

SYSTEMS WITH NONSINUSOIDAL VOLTAGE $v = \sqrt{2}\sum V_h \sin(h\omega t + \alpha_h)$

The general expressions of the current and active power are

$$i = \sqrt{2}\sum I_h \sin(h\omega t + \beta_h) \tag{28}$$

and

$$P = \sum V_h I_h \cos\theta_h\,; \qquad \theta_h = \alpha_h - \beta_h \tag{29}$$

The active power P has two components

$$P = P_1 + P_H$$
$$P_1 = V_1 I_1 \cos\theta_1 \tag{30}$$

is the system frequency active power and

$$P_H = \sum_{h\neq 1} p_h = \sum_{h\neq 1} V_h I_h \cos\theta_h \tag{31}$$

is the harmonics active power.

The current i is resolved in two components. Budeanu [6] recommended the division

$$i = i_a + i_r \tag{32}$$

where

$$i_a = \sqrt{2}\sum(I_h\cos\theta_h)\sin\sigma_h$$
$$i_r = \sqrt{2}\sum(I_h\sin\theta_h)\cos\sigma_h; \qquad \sigma_h = h\omega t + \alpha_h \tag{33}$$

The instantaneous power

$$p = p_a + p_{qR} \tag{34}$$

where

$$p_a = P - \sum_{h=1} P_h\cos 2\sigma_h + \sum_{\substack{m,n=1\\ m\neq n}} (V_m I_n \cos\theta_m)F_c(m,n) \tag{35}$$

$$p_{qR} = \sum_{h=1}(V_h I_h \sin\theta_h)\sin 2\sigma_h + \sum_{\substack{m,n=1\\ m\neq n}} (V_m I_n \sin\theta_m)F_s(m,n) \tag{36}$$

$$F_s(m,n) = \sin(\sigma_m+\sigma_n)+\sin(\sigma_m-\sigma_n) \tag{37}$$

$$F_c(m,n) = \cos(\sigma_m-\sigma_n)-\cos(\sigma_m+\sigma_n) \tag{38}$$

$$\sigma_m = m\omega t + \alpha_m\,; \qquad \sigma_n = n\omega t + \alpha_n$$

and $F_s(m,n)$, $F_c(m,n)$ are generalized forms of functions (14) and (15).

Kusters and Moore [13], Page [14] and Czarnecki [15] resolve the current i in two different components than (32). The first component i_p is the **in-phase** component, having exactly the same waveform as the distorted voltage. Assuming a scale factor $\mathbf{K}$

$$i_p = \sqrt{2}\mathbf{K}\sum V_h\sin\sigma_h$$

and from

$$P = \frac{1}{T}\int_0^T v i_p dt = 2\frac{\mathbf{K}}{\mathbf{T}}\int_0^T(\sum_h V_h\sin\sigma_h)^2 dt = \mathbf{K}V^2$$

results $\mathbf{K} = P/V^2$, where $V = \sqrt{\sum V_h^2}$ =rms voltage and

$$i_p = \sqrt{2}(P/V^2)\sum_h V_h\sin\sigma_h \tag{39}$$

with an rms current $I_p = (P/V^2)\sqrt{\sum V_h^2} = P/V$.

The second component, called the quadrature component, is $i_q = i - i_p$ and can be subdivided [15] in two components with harmonics 90° out of phase,

$$i_q = i_r + i_{qD}$$

where i_r was defined in (33) and

$$i_{qD} = \sqrt{2}\sum_h\left(I_h\cos\theta_h - \frac{P}{V^2}V_h\right)\sin\sigma_h \tag{40}$$

with respective rms values

$$I_R = \sqrt{\sum_{h=1}(I_h\sin\sigma_h)^2} \tag{41}$$

$$I_{QD} = \sqrt{\sum_{h=1}\left[I_h\cos\theta_h - (P/V^2)V_h\right]^2} \tag{42}$$

The instantaneous power is

$$p = p_p + p_q = p_p + p_{qR} + p_{qD} \tag{43}$$

where

$$p_p = P - P\sum_{h=1}(V_h/V)^2\cos 2\sigma_h + P\sum_{\substack{m,n=1\\ n\neq n}}(V_mV_n/V)^2F_c(m,n) \tag{44}$$

p_{qR} was defined in (36) and

$$p_{qD} = \sum_{h=1}\left[P(V_h/V)^2 - P_h\right]\cos 2\sigma_h - \sum_{\substack{m,n=1\\ m\neq n}}\left[P(V_mV_n/V^2) - V_mI_n\cos\theta_m\right]F_c(m,n) \tag{45}$$

Comparing (45) with (35) one learns that both instantaneous powers p_p and p_a have the same average power and oscillations frequencies. The amplitudes of the oscillations, however, are not the same. This difference stems from the fact that in Budeanu's definition p_{qD} is included into the p_a. The subtraction of (35) from (43) gives

$$p_p - p_a = -p_{qD} \tag{46}$$

Realizing that p_p in (43) has an identical definition as the one used for (1), where the in-phase current happens to be sinusoidal and therefore has the same waveform as the voltage, results that p_a in (34) is not the intrinsic power. This fact also hints that the definition of reactive power obtained from (32) may be deficient as far as a physical meaning is concerned. Further comparison between the two methods requires an elaboration on the definitions of the powers.

The rms current computed from (39), (41) and (42) is

$$I^2 = I_p^2 + I_R^2 + I_{QD}^2 \tag{47}$$

and the apparent power resolution will be based on the equation

$$S^2 = V^2I^2 = P^2 + Q_R^2 + Q_D^2 \tag{48}$$

where

$$Q_R^2 = V^2 I_R^2 = \sum_h Q_{Bh}^2 + \sum_{m \neq n} Q_{Bmn}^2 \quad (49)$$

$$Q_D^2 = V^2 I_{QD}^2 = \sum_h Q_{Dh}^2 + \sum_{m \neq n} Q_{Dmn}^2 \quad (50)$$

$$Q_{Bh} = V_h I_h \sin\theta_h \quad (51)$$

$$Q_{Bmn} = V_m I_n \sin\theta_n \quad (52)$$

$$Q_{Dh} = V_h I_n \cos\theta_h - P(V_h/V)^2 \quad (53)$$

$$Q_{Dmn} = V_m I_n \cos\theta_n - P V_m V_n / V^2 \quad (54)$$

Every one of the **elementary reactive powers** defined in (51), (52), (53) and (54) are recognized as the amplitudes of the oscillations in the instantaneous power (p_q). From (43), (45) and (36) results

$$\begin{aligned} p_q = & \sum_{h=1} Q_{Bh} \sin 2\sigma_h + \sum_{\substack{m,n=1 \\ m \neq n}} Q_{Bmn} F_s(m,n) + \\ & \sum_{h=1} Q_{Dh} \cos 2\sigma_h + \sum_{\substack{m,n=1 \\ m \neq n}} Q_{Dmn} F_c(m,n) \end{aligned} \quad (55)$$

The total compensation of p_q will lead to a unity power factor. The oscillations with the amplitude Q_{Bh} and Q_{Bmn} are typical for linear or nonlinear inductors or capacitors and can be compensated, in theory, with the help of simple shunt reactances (linear capacitors or inductors). The terms with the oscillations of amplitude Q_{Dh} and Q_{Dmn} are typical for nonlinear resistors or lossy nonlinear reactances and can be compensated only with the help of time-varying impedances designed as harmonic cancellation devices or active filters.

The current division given in (32) leads to the widely accepted apparent power division

$$S^2 = P^2 + Q_B^2 + D^2 \quad (56)$$

where

$$Q_B^2 = \left(\sum_{h=1} Q_{Bh} \right)^2 \quad (57)$$

with Q_{Bh} defined in (51) and

$$D^2 = \sum_{\substack{m,n=1 \\ m \neq n}} V_m^2 I_n^2 + V_n^2 I_m^2 - 2 V_m V_n I_m I_n \cos(\theta_m - \theta_n) \quad (58)$$

Inspecting the instantaneous power p_{qR} given in (36), one will conclude that the components of term D cannot be recognized in the amplitudes of the oscillations, therefore no physical meaning can be attributed to D. Only the terms Q_{Bh} can be recognized in the amplitudes of the oscillations. Moreover Q_B may be nil while $|Q_{Bh}| \neq 0$ and minimization of Q_B does not mean a reduction of the oscillations of p_{qR}. Thus the equation (58) is misleading giving the impression that Q_B can be partially or totally cancelled while in reality the oscillations of power will take place.

CONCLUSIONS

1. In nonsinusoidal situations the apparent power can be defined from

$$S^2 = (P_1 + P_H)^2 + Q_F^2 \quad (59)$$

where

$$Q_F^2 = Q_B^2 + D^2 \quad (60)$$

or

$$Q_F^2 = Q_R^2 + Q_D^2 \quad (61)$$

The definition (60) lacks physical meaning and the measured value of Q_B provides useless information for the purpose of harmonic cancellation or power factor improvement.

2. No direction of flow can be attributed to the reactive power Q_F. The term Q_F can be divided in a multitude of elementary reactive powers for which complex (four-quadrant) representation becomes possible. This is evident when (55) is rewritten in the form where

$$\begin{aligned} p_q = & \sum_{h=1} |Q_{Dh} + jQ_{Bh}| \sin(2\sigma_h + \psi_h) + \\ & \sum_{\substack{m,n=1 \\ m \neq n}} |Q_{Dmn} + jQ_{Bmn}| F_s(m,n,\psi) \end{aligned}$$

$$F_s(m,n,\psi) = \sin(\sigma_m - \sigma_n - \psi_{mn}) + \sin(\sigma_m + \sigma_n + \psi_{mn})$$

$$\tan\psi_{mn} = Q_{Dmn}/Q_{Bmn} \; ; \qquad \tan\psi_h = Q_{Bh}/Q_{Dh}$$

The complex representation is self explanatory. The usefulness of this complex representation is questionable. Advanced instrumentation which will monitor separately for each harmonic the complex elementary reactive powers may not be practical due to the large number of significant harmonics.

3. The harmonic power P_H, may be dissipated or generated by the load. Under certain circumstances some harmonic powers are received while others are generated by the same load. When P_H is absorbed by ac motors the energy transferred by P_H is entirely converted in heat loss and no mechanical power will be produced by P_H. Significant losses due to to eddy currents in transformer windings, cables and shieldings may be included in P_H. In case of lighting and resistive heating P_H represents useful power.

   It makes sense to develop wattmeters able to measure P_H separately. When $P_H < 0$ such instruments indicate to what extent the end-user is polluting the power network with harmonics. When $P_H > 0$ and the dominating loads are ac motors this information may provide economical justification for the installation of filters or the use of other mitigating techniques at the busses where $P_H < 0$.

4. The term $Q_1 = V_1 I_1 \sin\theta_1$ fundamental (system frequency) reactive power, is usually the dominant elementary reactive power component. It has a strong impact on the voltage profile and on the rms value of the line current. It is recommended to be measured separately from the rest of reactive power terms.

5. Based on the fact that the physical nature of power oscillations with zero transfer of energy is the same for linear reactive loads for nonlinear and for the time-varying loads, it is recommended to use reactive power meters which measure only Q_F. Such meters are useful in situations where the impact of harmonics on the load performance is non-consequential. For situations where the flow of powers should be closely monitored the main power component to be measured are found in the expression

$$S^2 = (P_1 + P_H)^2 + Q_1^2 + Q_H^2$$

where

$$Q_H^2 = Q_F^2 - Q_1^2.$$

6. **The power factor improvement by means of linear capacitors at busses with distorted voltages, is not always effective due to the possibility of resonance. In the near future the methods of harmonic cancellation will be based on active or time-varying devices similar to static VAr compensation. The performance of such devices is not dependent on Thevenin impedance, is immune to resonances and will not receive (sink) harmonics generated elsewhere in the network. More important such "active filters" are able to cancel Q_R as well as Q_D. Therefore**

separate monitoring of Q_D and Q_R does not seem to offer a significant advantage.

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APPENDIX—IDEAL THREE-PHASE ALTERNATOR SUPPLYING A THREE-PHASE REACTIVE LOAD

The total energy stored in the magnetic field of the **p**-poles cylindrical rotor synchronous machine is

$$W = \frac{L}{2}\left(i_a^2 + i_b^2 + i_c^2\right) + M\left(i_a i_b + i_a i_c + i_b i_c\right) + I_f\left(M_{af} i_a + M_{bf} i_b + M_{cf} i_c\right) + \frac{1}{2} L_f I_f^2$$

where L = self inductance of each phase of the armature
M = mutual inductance between phases
L_f = self-inductance of the field
I_f = field current
$M_{af} = M\cos\omega t$; $M_{bf} = M\cos(\omega t - 2\pi/3)$; $M_{cf} = M\cos(\omega t + 2\pi/3)$

are the mutual inductances of the phases and the field winding. The torque delivered to the generator is

$$\mathbf{T} = \partial W/\partial\Omega = -I_f M \tfrac{\mathbf{p}}{2}\left[i_a \sin\omega t + i_b \sin(\omega t - 2\pi/3) + i_c \sin(\omega t + 2\pi/3)\right]$$

Since the load is reactive

$$i_a = \sqrt{2} I \cos\omega t\ ,\ i_b = \sqrt{2} I(\omega t - 2\pi/3), i_c = \sqrt{2} I \cos(\omega t + 2\pi/3).$$

Substitution in the torque equation yields

$$\mathbf{T} = -I\frac{I_f M \mathbf{p}}{4}\sqrt{2}\left[\sin 2\omega t + \sin 2(\omega t - 2\pi/3) + \sin 2(\omega t + 2\pi/3)\right] = 0$$

The power

$$p = \Omega\mathbf{T} = 2\omega\mathbf{T}/\mathbf{p} = 0$$

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From 1963 to 1969, he was on the staff of the Electrical Engineering Department at Technion-Israel Institute of Technology, first as a Teaching Assistant and later as a Lecturer. From 1969 to 1974, he worked for the High Voltage Power Corporation as Senior Research and Development Engineer. In 1974 he joined Worcester Polytechnic Institute where he presently holds the George I. Alden Chair of Engineering. His expertise is in power electronics, electromechanical energy conversion, and high-voltage technology.

Dr. Emanuel is a Registered Professional Engineer in the State of Massachusetts. He is a member of CIGRE, Sigma Xi, Eta Kappa Nu, Tau Beta Pi, and the Societe Royale Belge des Electriciens.

He is the 1982 recipient of the WPI Trustees Award for Outstanding Teaching and the 1986 recipient of the WPI Trustees Award for Outstanding Research.

Discussion

J. L. Willems, (University of Gent, Gent, Belgium): The author is to be commended for a careful discussion of the concept of reactive power in non-standard situations. As illustrated by the bibliography of the paper the discussion of the reactive power concept is almost exactly 100 years old; it has received much renewed attention because of the distortion in the power system currents and voltages due to the increasing use of static converters. Professor Emanuel is well known for his many interesting contributions to the concept of the characterization of reactive and distorted currents and the corresponding powers.

The deficiency of the widespread definition of reactive power for periodic waveforms

$$Q_B = \Sigma_h V_h I_h \sin \Theta_h$$

as proposed by Budeanu, is convincingly shown in the paper. An elaborate discussion of the drawbacks of Budeanu's reactive power concept was also recently given by Czarnecki [A]. In the discussor's opinion one of the fundamental aspects which is not taken into account in Budeanu's reactive power definition, is that negative and positive reactive powers do compensate if they correspond to the same frequency, but this is not the case if they correspond to different frequencies. To clarify this point somewhat more, we consider a (linear) capacitor and a (linear) inductor connected in parallel to a sinusoidal voltage source. Oscillating power transfer at twice the supply frequency is needed to realize the fluctuating energy $Cv^2/2$ stored in the capacitor, with C denoting the capacitance and v the voltage. Similarly, oscillating power transfer at twice the supply frequency is required to realize the fluctuating energy $Li^2/2$ stored in the inductor, with L the inductance and i the current. Since the inductor and the capacitor currents are in quadrature with respect to the voltage, but the capacitor current is leading and the inductor current is lagging, the energy in the capacitor increases when the energy in the inductor is decreasing, and vice-versa. The voltage source therefore only has to supply the *difference* of the oscillating power components. This is also shown by expression (11) of Emanuel's paper. Consider now a nonsinusoidal source with a voltage consisting of a fundamental frequency component and a third harmonic. It is connected to a reactive element which has a negative or capacive reactance at the fundamental frequency and a positive or inductive reactance at the third harmonic frequency. The fluctuations of the stored capacitive and inductive energies are not synchronous in this case, such that the pulsating power to be delivered by the source does not correspond to the difference of both energy components. Therefore there is no justification for simply adding the reactive powers corresponding to different frequencies, as is done in Budeanu's reactive power concept. Moreover, there are also cross-terms in the stored energy expression, corresponding e.g. to the product of the fundamental current and the third harmonic current in the expression of the inductive energy.

If we are looking for a generalized definition of the reactive power concept we should consider the following questions:

(i) Which features make the concept of reactive power in sinusoidal situations so interesting for the analysis of practical power system operation?
(ii) How can the concept of reactive power be generalized to non-sinusoidal situations in such a way that these properties or at least some of them are retained?

With respect to question (i) the following interesting properties of reactive power in sinusoidal situations may be stated:

(a) The reactive power is the amplitude of the fluctuating component of the supplied power which can be compensated by a parallel reactive element.
(b) The reactive power corresponds to the apparent power of the compensating reactive element which realizes unity power factor.
(c) Zero reactive power is equivalent to unity power factor. It corresponds to the smallest line currently supplying the same active power.
(d) For an inductive line the voltage drop in the line is (approximately) proportional with the transmitted reactive power.

Moreover the measurement of reactive power is easily realized. An additional interesting property of reactive power is that it satisfies the conservation property, as is the case for the instantaneous and the active power.

No quantity has yet been found which has the same properties in the periodic nonsinusoidal case; probably it does not exist. It is however clear that Budeanu's generalization is not a good one. It is even difficult to generalize the reactive power concept with respect to the above considerations taken separately. Kusters and Moore were mainly concerned with feature (a) and defined the capactive (inductive) reactive current as the maximal current which can be cancelled by a linear capacitor (inductor). Consideration (d) leads to reactive power concept similar to Budeanu's, but where the reactive power contributions of the harmonics are multiplied by the harmonic order.

As pointed out by the author of the paper and also by Czarnecki [A] the power fluctuation, which is not related to the active power transfer, consists of a large number of components at different frequencies. The main distinction between their approaches to generalize the reactive power concept is that Emanuel characterizes the oscillations by powers, whereas Czarnecki concentrates on currents. The current components are the sources of the power oscillations, so that both points view agree very much. Indeed, the current decomposition given by (39) and (41), leading to the power definitions, is also proposed by Czarnecki [B] who further decomposes the current i_{qD} into the scattered current and the generalized current.

We want to emphasize an aspect which may look confusing. In the sinusoidal case the power fluctuation is decomposed into $p \cos 2\omega t$ and $Q \sin 2\omega t$, where the symbols of (1)–(3) are used. The former fluctuation is not related to the reactive power, only the latter component determines the reactive power. The reactive power does not characterize the total power fluctuation [C]. This distinction between the *intrinsic fluctuation* and the redundant fluctuation of the power transfer should also be retained in the non-sinusoidal situation.

Professor Emanuel also considers time-varying resistors and inductors. This is very interesting although I suspect that these considerations would become rather complex for non-sinusoidal voltages. It is also interesting to note that the fundamental frequency component in (18) is not solely determined by the average value of the conductance g.

I assume that the non-sinusoidal voltage and currents in the paper do not contain d.c. components. Would their presence affect the analysis very much?

I am not really convinced by the argument below (46) as an indication that the definition of p_p in (43) is justified, becasue—if I understand it well—it reduces to the commonly used quantity in the sinusoidal case. It is exactly this argument which has been used to justify Budeanu's definition of reactive power, whose deficiencies are now widely recognized. Since as well p_a as p_p reduce to the same quantity in the sinusoidal case, the argument could be used for the justification of both quantities and corresponding decompositions (25) and (44).

It is clear that the quantities Q_R and Q_D do not completely characterize the power oscillations. Indeed different power oscillations in (51)–(54) may lead to the same results in (49) and (50). To what extent are the power quantities Q_R and Q_D interesting in practice?

I want to commend the author once more for this interesting paper and I am looking forward to his comments on my remarks.

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Jacques L. Willems, University of Gent, Gent, Belgium.

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Tongyi Tang, (Tsinghua University, Peking PRC): Dr. Emanuel has given a comprehensive review and appraise on the work done by many celebrated authors. We agree with him on the viewpoint that there is every reason to divide the active power P further into P_1 and P_H (see eq. (30)), as we can judge whether the load is a harmonic source or a sink by the sign of P_H. As for the reactive power, to subdivide Q_F into Q_1 and Q_H, i.e. $Q_F^2 = Q_1^2 + Q_H^2$ is a practical suggestion, and to emphasize the unified theory on the nature of the reactive powers is also very suggestive.

From the engineering point of view, compensation usually takes place in the three-phase systems, we would like to hear more from him in this respect.

J. H. R. Enslin (University of Pretoria, Pretoria, South Africa): Prof. Emanuel should be commended for his fine paper. This paper gives a good fundamental background on the aspects, approaches and physical meaning of power quantities in electric power systems in the frequency and time domains. The physical interpretation of the generation of reactive power, as indicated in fig. 2, is very practical and should be noted. This physical meaning is however not followed through in the view of nonsinusoidal current and/or voltages, which can be addressed further. The approach of using nonlinear resistors in the equivalent circuit for the purpose of analyses of the power components is interesting, they should however be described as *loss-less,* and thus the name *resistor* may not be relevant for the interpretation of power flow in electric power systems, as shown in figure 5.

The majority of the problems of power definitions arises when the voltage is nonsinusoidal as elaborated by Prof. Emanuel. The physical meaning of Q_B and D in the case of a nonsinusoidal voltage is problematic and it is quite true that Q_B may be nil while $|Q_{Bh}| \neq 0$, and no reduction of p_{qR} oscillations will result. It is however felt that Q_1 and Q_H is of the same nature, and generated by the same means, and is therefore not orthogonal as indicated by $Q_F^2 = Q_1^2 + Q_H^2$. Q_F should be divided in 2 components by a summation, $Q_F = Q_1 + Q_H$, as in $P = P_1 + P_H$. When this results, it is still necessary for the orthogonal component D, shown in eq. 56. The interpretation difference between Q and D in nonsinusoidal voltages should however still be receiving attention. It is believed that these two components are *generated* on a different method and should therefore be distinguished from each other.

The observation in the conclusions 6, that active filters are able to cancel Q_R as well as Q_D is quite correct, however it is not cost effective to cancel both components with an active power filter since this implies an active filter size of the same magnitude, or sometimes larger, than the total nonlinear load to be compensated. Active power filters are capable of compensating fast changing distortion conditions, and should be used to compensate these components alone. Other quasi-dynamic compensation systems, i.e. static VAR compensators can compensate large amounts of fundamental reactive power more cost effectively than active power filters.

The other observations which are made, are quite true and should be noted by electrical engineers interested in power system analysis.

PIOTR S. FILIPSKI, National Research Council of Canada, Ottawa, Canada.

The author has to be congratulated for his in-depth review of the nonsinusoidal power theories. This review clearly shows that so called "time-domain" and "frequency domain" approaches constitute merely different mathematical expressions of the same processes.

In the conclusions the paper gives clear guidance to a designer of measurement instrumentation. One can only add that from the practical point of view it is as easy to measure all the harmonic components as it is to measure P_1, P_H, Q_1, Q_H. A digital watt/var meter displaying only these quantities would be discarding most of the acquired information, with only little gain in the processing time.

The different nonsinusoidal power theories invariably start with the definitions of apparent power S and the active power P. The geometric difference between these two powers is either called the reactive power [10] or is further subdivided, with the compensation in mind. In the simplest approach [10] the reactive power is expressed in terms of the reactive current $I_{r,rms}$, $Q_F = \sqrt{S^2 - P^2} = V_{rms} I_{r,rms}$. If a reactive current was delivered to the load by an ideal lossless active compensator then the load-compensator system would present itself to the source as a purely resistive load, with unity power factor. This is the rationale behind introduction of the nonsinusoidal reactive current.

Part of the reactive current can be usually compensated by a passive reactive shunt element, e.g. capacitor C or inductor L. For this reason Kusters and Moore [13] proposed to further subdivide the reactive current to distinguish a component compensatable by an "optimal" C or L and call it capacitive/inductive reactive current. The power associated with this optimal element was called the capacitive/inductive reactive power. In [14] this approach was extended to a two element L-C shunt. In the consequent proposal [15] current/power which could be compensated by a shunt being an optimal, arbitrarily complicated, L-C network was called the reactive current/power.

Each of these approaches has its merits, discussed in the appropriate references. Generally, however, they all leave a certain residual part of the load current and a residual power component, which cannot be compensated by a passive shunt elements but would have to be compensated by an active compensator. Physical interpretation of this residual power, expressed partially in terms of cross-products of different harmonics of voltage and current, is not clear.

Similarly, when the paper defines elementary nonsinusoidal oscillations p_{D1h}, p_{B1h} (13) it seems to be guided by a mathematical convenience rather than by description of actual physical processes.

One can therefore distinguish in the apparent power components those which one can and cannot attach a physical interpretation. In trying to explain this problem one can ask a more fundamental question, whether the apparent power in nonsinusoidal situation has a deeper physical meaning or is it merely a formal product of V_{rms} and I_{rms}? In the latter one cannot expect to be able to attach physical interpretation to all the power components. In other words, the traditional definition of apparent power is the main source of confusion in the nonsinusoidal power theory.

By questioning in Conclusion 2 the practical advantage of distinguishing different elementary power components the paper indirectly supports the above hypothesis. What is, in the author's opinion, the physical meaning of the apparent powers in a nonlinear resistive circuit as shown in Fig.4a, calculated on both sides of the switch ?

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Y. Baghzouz (University of Nevada, Las Vegas, NV): The author is commended for reviving this debatable subject of vital importance to the power-engineering community. He provided an in depth meaning of reactive power in sinusoidal cases and an excellent review of 'non-active' power in nonsinusoidal cases. It is hoped that further work in this area would lead to new power definitions in nonsinusoidal systems.

The discusser will appreciate the author's comments on the items that follow:

1. Under the presence of sinusoidal voltages, the idea of mitigating the third harmonic of a nonlinear inductor by a nonlinear shunt resistor is an interesting one. However, nonlinear resistors tend to have a v/i characteristic described by $v = i^3$ instead of $v = i^{1/3}$. In such cases, e.g., magnetizing inductance and core loss resistance of a transformer, it is not possible to cancel the third harmonic component. Furthermore, the power loss in the nonlinear resistor does not permit an economical filtering technique.
2. Passive linear loads absorb fundamental active power P_1 and harmonic active power P_h at any harmonic frequency. Does this statement hold true for passive nonlinear loads? If so, the separation of P into P_1 and P_h as suggested by the author, will be of little use since most electrical loads are passive.
3. The engineering community is slowly being convinced that the reactive and distortion powers defined by Budeanu, i.e., Q_B and D in equations (57) and (58), lack physical meaning and cannot be used when optimizing power factor. Furthermore, the distortion power D is not related to waveform distortion as proved in Ref [A]. The author made a step forward by decomposing the total non-active power into the sum of four elementary reactive powers (Eqns. (51)–(54)). Each of these reactive powers represents the amplitude of an alternating component. However, it is important to keep in mind that this physical meaning is lost right after summing the squares of these elementary

components, i.e., Q_R and Q_D in Eqns. (49) and (50). Note that Q_R and Q_D correspond, respectively, to the inductive (capacitive) reactive power and risidual inductive (capacitive) component defined in Ref. [13] of the paper.

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The paper presented by Prof.Emanuel can be considered as a proof of how tangled is the problem of energy flow in nonsinusoidal circuits, since it was written by one of the most experienced scientists in this area in the world, involved in a study of power properties of such systems for many years. Despite that there is a number of a very controversial claims in his paper.

Because more and more energy is utilized now at non-sinusoidal waveforms,the comprehension of power properties of such circuits is not only a pure academic problem,but a problem with important technical and economic implications.This imposes rigid requirements upon definition and interpretation of powers in such systems. Therefore,the discusser does not agree with some claims expressed in the paper discussed.

Despite this, the discusser would like to express his appreciation to Prof. Emanuel for his long-lasting commitment to nonsinusoidal systems problems, as well as for turning the electrical engineer's attention with the paper presented to the lasting confusion regarding powers in nonsinusoidal systems definition.

To keep this discussion in order, particular comments are numbered, which would facilitate their referencing.

1.The Author provides interesting bibliography of old attempts of defining the reactive power. It is a pity, however, that definitions themselves, as the phrase "Review of Definition" in the title suggests, were not presented. Comprehension of the present state of study on powers in nonsinusoidal systems requires that contributions made by Kimbark, Depenbrock, Sharon,Van Wyk, Slonim, [1-5] are taken into account, even if they are considered out of date now. It seems that also results published in papers [7-12] contributed to the problem discussed and should be referenced.

2.Despite the claim that the concept presented supports Fryze's approach, it seems to me that it deforms the very essence of it.Fryze objected Budeanu's definition of reactive power

$$Q_B \triangleq \sum_{n=1}^{\infty} U_n I_n \sin\phi_n$$

because harmonics were used for that purpose.He stressed that such a fundamental quantity should be defined directly, in the time-domain. By 1930 Fryze managed to define only the power

$$Q_F \triangleq \|u\| \|i_b\| = \sqrt{Q_B^2 + D_B^2}$$

in the time-domain. He defined [6] Budeanu's reactive Q_B and distortion power D_B in the time-domain just before his death in 1964. Emanuel's approach, based on harmonic decomposition, seems to have very little in common with Fryze's approach.

3.Emanuel's suggestions contradict Fryze's approach in one other essential point,namely,the new powers like Q_{Bmn}, Q_{Dmn}, Q_{Dh} suggested are not based on the current, but rather on instantaneous power decomposition. Most technical and economical effects of the load changes are attributed to the change of the current rms value, rather then voltage, which remains usually close to its rating value. Therefore, both Fryze's and other major attempts of defining powers, suggested by Depenbrock, Shepherd, Kusters and Moore, compiled in [9],or my proposals [8,11] relay on the current decomposition.

4.As long as the firing angle remains constant, the thyristor circuit shown in Fig.5a,is only a non-linear, but not a time-varying, circuit.Its parameters vary not because of time passage, but because of supply voltage changes.When the source voltage is constant,the circuit parameters are constant as well. The frequency of equivalent circuit parameters is not an independent variable as it is in time-variant systems. Superposition principle can not be applied to it, which is possible in circuits with time-varying parameters.

5.The explanation of how the energy can be stored in a resistive circuit with a controlled thyristor, in order to find the reason of instantaneous power oscillation, is wholly artificial. It suggests that the circuit can be seen as containing resistors that are sources of electric power. Such resistors could not exist,however, since it would breach thermodynamic laws. "Negative resistances" known in electronics are built as circuits which contain internal supply sources. The power equation of the circuit should take into account, however, all sources of electric power.Therefore,the explanation of how the energy is stored in the resistive circuit to enable instantaneous power oscillation is incredible. The Author tries to explain, however, a phenomenon that simply does not exist. The instantaneous power at the source terminals in the circuit shown in Fig.5a. $p(t) = u(t)i(t)$, is always positive,so that there is not any reciprocating flow of energy that would require the electrical energy to be stored in the load.

6.The observations compiled above justifies the following conclusion.The oscillating components of instantaneous power p_{B1h} and p_{D1h} in the paper discussed have occurred as a result of an artificial decomposition of non-oscillating instantaneous power. They have not been related to any physical phenomenon in the circuit.

7.If the source of sinusoidal voltage of fundamental frequency

$$v \triangleq \sqrt{2}\, V \sin\omega t$$

is loaded only with the harmonic current of frequency different than ω, namely

$$i_h \triangleq \sqrt{2}\, I_h \sin(h\omega t+\theta_h)$$

as it is considered in the paper discussed, then the apparent power of the source is equal to the product

$$S = VI_h$$

which is not affected by the phase angle θ_h. Therefore, there is not any rationale for using that angle for the power properties description, and decomposition of the apparent power S into components Q_{B1h} and Q_{D1h} seems to be artificial and redundant. Source apparent power VI_h is reduced only if current I_h is reduced, so that building two separate compensators for Q_{B1h} and Q_{D1h} does not seem to have any practical sense.

8.Functions $F_s(m,n)$ and $F_c(m,n)$ do not have dimension of power, therefore, referencing them as powers in the "List of Principal Symbols" is rather confusing.

9.Suggestion that a time-variant resistor can be used as a compensator does not seem to be either feasible or useful, or at least the example presented does not show any advantages of such compensators. As it results from eqn.(21),the resistor suggested compensates power Q_{D13}, but simultaneously it loads the source with the active power P which is three times higher than the power compensated. If the resistor given with eqn.(25) is used for this purpose, it doubles, moreover, the reactive power of fundamental harmonic.

10. If neither physical phenomena in the circuit nor practical reasons regarding compensation do justify the need of reactive power decomposition into components Q_{B1h} and Q_{D1h} in the case when only the current is distorted, the same holds true when also the voltage is nonsinusoidal. Thus, the paper discussed does not contain any credible rationale for decomposition given by eqn. (49) and (50).

11. The current i_g decomposition, given without sequential number between eqn.(39) and (40), can be referenced as being introduced by discusser in paper [8] provided the load is linear. Decomposition of the current i_g into the reactive i_r and scattered i_s components, in the paper referenced, holds true only on the condition that the load is linear and time-invariant, while such constrains on the load properties were not imposed in the paper discussed. Decomposition suggested for nonlinear loads is presented in paper [7] and it differs substantially from Emanuel's decomposition.

12. Considerations in the section on systems with nonsinusoidal voltage seem to not take into account such a situation where the set of current harmonics differs from the voltage harmonics set, which may happen in the case of circuits with ideal voltage sources. How in such a case, for example, is the angle θ_h in eqn.(33) calculated? The difference between these two sets affects substantially the power properties. Thus, ought not this difference affect the powers definitions, if it affects the power properties? One might say, that when the source impedance is not equal to zero, then these two sets are identical, but in such a case the powers suggested can not be applied to power flow analysis in circuits with ideal sources, which is not acceptable.

13. The claim (placed below eqn.(55)) that the power Q_{Bmn} *can be compensated with shunt reactors does not seem* to be right. If there is lack of n-order harmonic in the source voltage, then a reactor connected at the source terminals does not affect the n-order harmonic of the source current and the product $V_m I_n \sin\theta_m \triangleq Q_{Bmn}$ remains unchanged. This is visible in the example of a circuit shown in Fig.5a, where it is not possible to change the power Q_{B13} with a linear reactance connected at the source terminals, since its current can not contain any harmonic other than the fundamental one.

14. While discussing the term Q_{Dh} it should be noticed that it occurs not only in non-linear circuits, as it is claimed in the paper discussed, but in linear circuits [8], if their conductance varies with frequency.

15. I do not agree with the claim that the "terms... Q_{Dmn} ...can be compensated only with the help of time-varying impedances designed as harmonic cancellation devices or active filters". This terms occurs as a result of harmonic generation in non-linear and/or time-varying loads. Any device that protects the power system against harmonic injection, by creating a harmonic current divider, reduces or at least affects the value of the power discussed. Harmonic filters, well known and used in power systems, fulfill just such a function.

16. The claim that "..the physical nature of power oscillation with zero transfer of energy is the same for linear reactive loads and for nonlinear ...loads,..." is not correct. Therefore, the conclusion drawn from that claim that only power Q_F should be measured is not properly justified, since this power has very little in common with these oscillations. In the circuit shown in Fig.5 there is not any oscillation of the instantaneous power, despite *non zero power* Q_F. *Four quite different* phenomena contribute to Fryze's Q_F power value:

(i) presence of harmonics reactive powers Q_n,
(ii) change of the load conductance with frequency,
(iii) generating of current harmonics in the load non-linearity and/or due to its time-variance,
(iv) asymmetry of the load, in the case of multi-phase systems.

However, only some of them, like (i), causes power oscillations. Moreover, each of them requires quite different technical means to reduce their impact upon the power Q_F value. Thus, the value Q_F itself does not provide sufficient information regarding how it can be compensa ted by reactive compensators and to what extent. Even such a compensator like a capacitor bank cannot be designed based on Q_F value at distorted waveforms. This is true, as it was shown in [8], even for linear loads, even most so for non-linear or three-phase[12] systems.

17. The idea of the power decomposition into the active and reactive powers of fundamental frequency and the power of higher harmonics as suggested in the paper discussed was articulated for the first time, as to my knowledge, by Depenbrock [2]. It seems this should have been recognized in Emanuel's paper. Depenbrock's decomposition for English language readers was presented in paper [9]. *Also Budeanu's equation (56) should not* be described as "widely accepted" while the paper "What is Wrong with the Budeanu Concept of Reactive and Distortion Powers and Why It Should be Abandoned" [10] was neglected.

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CLOSURE

ALEXANDER E. EMANUEL (Worcester Polytechnic Institute, Worcester, MA.): The author is grateful to each discussor for his valuable comments and contributions. Similar observations addressed by two or more discussors will be answered first. We will start with the important question on what happens when the set of current harmonics differs from the voltage harmonics set, and how the presence of dc components affects the resolution proposed? This question asked by Professors Willems and Czarnecki (comment #12) can be answered with the help of the basic circuit illustrated in Fig. 1A.

The sinusoidal voltage v, supplies a "black box" containing a direct voltage E and a resistance R. The current i has the expression

$$i = \frac{\sqrt{2}V}{R}\sin\omega t - \frac{E}{R}$$

In this case the powers are

$$S = \frac{V^2}{R}\sqrt{1+\left(\frac{E}{V}\right)^2}; \qquad P = \frac{V^2}{R}; \qquad Q_H = \frac{VE}{R}$$

To investigate the nature of Q_H by the mathematical means explained in this paper, one will first identify the two basic components of the current:

The in-phase current $i_p = \frac{\sqrt{2}V}{R}\sin\omega t$

and the in-quadrature component $i_q = -E/R$, which can be expressed with the equation

$$i_q = \sqrt{2}\frac{E}{R}\sin\left[(0)t - \pi/4\right].$$

In this case $h = 0$ and $\theta_h = \pi/4$. (this will address Czarnecki comment #7). The fictitious angle $-\pi/4$ is a crucial value in the process of modeling the oscillations of energy. The elementary reactive powers according to (13) are

$$Q_{D10} = \frac{VE}{R}\cos(-\pi/4) = \frac{VE}{\sqrt{2}R} \qquad ; \qquad Q_{B10} = \frac{-VE}{\sqrt{2}R}$$

The functions F_c, F_s describe or represent the time variations of the instantaneous powers (Czarnecki #8), and are

$$F_c(1,0) = \cos(-\omega t) - \cos(\omega t) = 0$$
$$F_s(1,0) = -\sin(-\omega t) + \sin(\omega t) = 2\sin\omega t$$

Thus the instantaneous reactive power is

$$p_q = vi_q = p_{D10} + p_{B10}$$
$$= \frac{VE}{\sqrt{2}R}\left[\cos(-\omega t) - \cos(\omega t)\right] - \frac{VE}{\sqrt{2}R}\left[-\sin(-\omega t) + \sin(\omega t)\right]$$

This result shows that when dc components are present, the two elementary oscillations of $F_c(n,0)$ do "exist" but will cancel each other, i.e. $p_{D10} = 0$, nevertheless $Q_{D10} \neq 0$. (This approach should remind the reader of the analysis of single phase alternating magnetic fields where the stationary field is viewed as a result of two equal but opposite rotating fields, Czarnecki #6). Equation (16) yields

$$Q_{10} = \sqrt{Q_{D10}^2 + Q_{B10}^2} = \frac{VE}{R} = Q_H$$

and this is the correct result.

This article strove to bring the resolution of S to a par with the advances made in the last decade by the power electronics engineers. In a recent paper Grady, Samotyj and Noyola [C1]

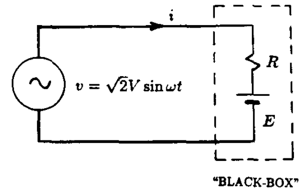

Fig. 1A Circuit With dc Component

presented an extensive literature survey of key publications on active power filters and conditioners. These devices inject equal but 180° out of phase harmonic currents which cancel the harmonic currents generated by the nonlinear loads. The above authors predict that these types of compensators have considerable potential to correct wave distortion. Active power filters started to emerge from the stage of 1kVA laboratory prototypes, larger units are already in the process of being built. (200kVA, EPRI, Power Electronics Application Center). The observation made by Dr. Filipski and Prof. Enslin to conclusions of the paper is well taken and supports the comments presented in [C1]. Active harmonic cancellation devices are circuits capable of compensating Q_H, Q_1 and P_H simultaneously. The compensation can be selective; Q_H only or, Q_H and part of Q_1. This means that the scattered power (Czarnecki ref [8] and comment #11) can be compensated. In Fig. 2A is shown a bus supplying a nonlinear load equipped with a capacitance and a set of tuned filters for power factor compensation. The bus harmonic voltage V_h and the load h-harmonic current phasor I_h, with its three basic components sketched in Fig. 2Ab. The component I_{hr} (reactive), can be compensated by the passive filters. The component I_{hs} (scattered), being in-phase with V_h cannot be compensated by a shunt reactance. Since active filters have the capability to generate harmonic current phasors of any desired phase, they can compensate I_{hs}. The property of the scattered power, $\sum V_h I_{hs} = 0$, indicates that theoretically no active power will be dissipated or generated by the active conditioners. It is of importance to realize that both currents I_p, eq. (47), and I_a (rms value of i_a, eq. (32)), "carry" the same active power, nevertheless $I_p \leq I_a$. If a load is perfectly compensated the rms line current $I = I_p$. The best compensation is not necessarily obtained at unity power factor when the line current has exactly the same waveform as the distorted voltage. It can be argued that the best strategy is to compensate the dominant nonlinear loads in such a way that the respective line currents are sinusoidal. Another approach is to expend the use of the active conditioners and let them "sink"a part of the harmonics generated by nonlinear loads located at more remote busses. Such sophisticated methods of adaptive control will certainly cause some changes in the future decision-making process for the most economic power factor correction strategy. In conclusion 2 of this paper is shown that by using the complex

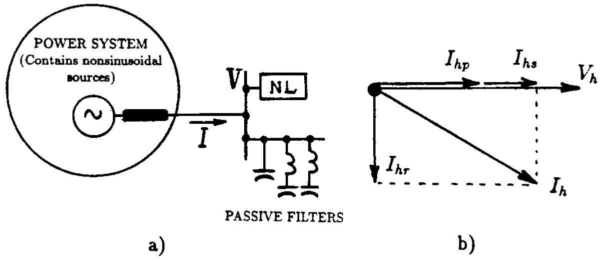

Fig. 2A The "Scattered" Current I_{hs} Cannot be Compensated with Passive Filters

representation of the elementary powers Q_{Bh}, Q_{Dh}, Q_{Bmn} and Q_{Dmn} one can eventually define delivered and received powers. These observations do not support the statements presented in Czarnecki #13, 14 and 15. The author agrees with Dr. Czarnecki that Q_F (or Q_B and Q_H) do not represent oscillations of energy (comment #16). The elementary reactive powers, (51) to (54), represent power oscillations amplitudes and so does Q_1, therefore since we have a device which can help generate Q_1 as well as other elementary reactive powers there is no need to keep them separated if the goal of the measurement is only to find out how much "non-active power" (reference [C2] calls it fictitious power) flows through the feeder.

Any practicing engineer is aware of the importance of measuring and controlling Q_1. This leaves Q_H as a value which can be used in the overall evaluation of harmonic flow. The author envisions relatively simple panel meters able to monitor P_H and Q_H. For special diagnosis there are commercially available advanced real time data acquisition systems and for calibration most accurate instrumentation capable of measuring any component or subcomponent (elementary powers) of S, P, Q can be constructed [C3].

Probably the equivalent circuit of the thyristorized resistor, Fig. 5, and its interpretation is the most controversial aspect of this paper, as pointed out by Profs. Willems and Enslin and strongly criticized in Czarnecki #5. The author is in total agreement with the discussors that the instantaneous power p flows in one direction only, from source to load. However, due to the synchronized switching the power p is modulated and this enables the mathematical separation in intrinsic power p_p with an average equal to the active power and what is left, the power p_q, which is made out of oscillations of elementary reactive powers. The behavior of a thyristorized resistor circuit can be simulated with the help of a temperature dependent resistor, heated during the first 90° of each half-cycle and cooled during the next 90° of each cycle. Evidently in this case the heat flow has two components: First is heat flowing continuously from the resistor to the surrounding media. Second is an oscillation of heat which can be controlled with a reversible heat-pump. Such a system can be easily built and studied if the alternating voltage has a very low frequency. The expected current waveform is shown in Fig. 3A. If the resistor-heat-exchanger system is hidden in a black-box, an observer located at the voltage source may describe the load as lossy and containing an imperfect electrothermal energy conversion system causing the generation of p_q. The thyristorized resistor representation with the help of an equivalent circuit containing a constant resistance in parallel or series with components which produce the same p_q is correct as long as one is aware that the equivalent circuit is used for the study of the electric circuit outside the black-box.

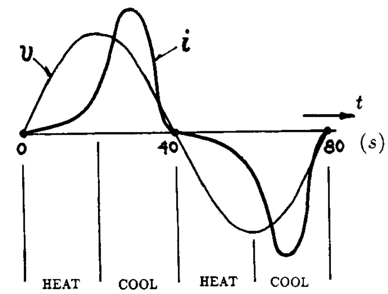

Fig. 3A Production of Reactive Power by Heating/Cooling a Resistor

Professor Tongyi Tang's concern about polyphase definitions cannot be properly answered in such a limited space. The main hurdle stems from the definition of the apparent power S, in unbalanced circuits. A comprehensive survey of this subject was presented by Curtis and Silsbee [C2] and more recently by Czarnecki, (ref [11] in his discussion). The resolution presented in this paper can be extended to the general case of an unbalanced polyphase circuit when the polyphase system is separated in single-phase circuits or, by the use of symmetrical components.

The answer to Dr. Filipski's last question sheds some light on the interpretation of S. The apparent power supplied to the resistor equals P. The apparent power supplied to the thyristor equals Q_F.

Professor Bagzouz is right in his remark that Q_R and Q_D have no physical meaning. The "elementary powers" however, do represent amplitudes of oscillating powers. The nonlinear resistor paragraph was presented only to help elucidate the mechanism of the p_q generation. (Bagzouz #1).

Finally, I sincerely regret that many valuable contributors were left unmentioned (Czarnecki #17). A review of Depenbrock's work as presented by Czarnecki in his ref [9] page 144 shows that the S is resolved in

$$S^2 = P^2 + Q_1^2 + V^2 + N^2$$

Reading carefully the comments Dr. Czarnecki made about the powers V and N in [9] results very clearly that neither him or Depenbrock recommended the metering of P_H or to lump the powers V and N in one entity Q_H. Neither is ever mentioned in their works the need to the separate of Q_H in elementary reactive powers.

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