$$\Rightarrow \boxed{\theta = A e^{-\lambda^2 \tau} \cos(\lambda X)}.$$

2nd B.C

$$\frac{\partial \theta(1,\tau)}{\partial X} = -Bi \cdot \theta(1,\tau).$$

$$\Rightarrow \cancel{-} \cancel{A e^{-\lambda^2 \tau}} \cdot \lambda \sin\lambda = \cancel{-} Bi \cdot (\cancel{A e^{-\lambda^2 \tau}} \cos(\lambda))$$

$$\Rightarrow \lambda \frac{\sin\lambda}{\cos\lambda} = Bi$$

$$\Rightarrow \boxed{\lambda \tan\lambda = Bi}. \longrightarrow$$ Characteristic equation or Eigen fnc.

$\tan\lambda \rightarrow$ periodic in π.

$\rightarrow$ eqn. has root between 0 & π but will repeat at π to 2π & 2π to 3π and so on

Characteristic eqn. is implicit and its roots (chara. soln or eigen values) needs to be determined numerally.

Infinite no. of soln. of form $A e^{-\lambda^2 \tau} \cos(\lambda X)$

$$\text{or } \theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X).$$

Fourier Number $= \frac{\alpha}{L^2} \cdot t = \frac{K}{(\rho C_p L^2)/t}$

→ Ratio of diffusive or conductive heat rate to the quantity storage rate

$$\tau = \frac{\alpha t}{L^2} = \frac{K L^2 (1/L)}{\left(\frac{\rho C_p L^3}{t}\right)} \frac{(\Delta T)}{(\Delta T)} = \frac{(K L) \cdot \frac{\Delta T}{\Delta T}}{\left(\frac{\rho C_p L^3}{t}\right)}$$

= the rate at which heat is conducted across a body of thickness L and normal area L^2 (and the volume L^3)

The rate at which the heat is stored in a body of vol. (L^3)

→ Measure of ~~the~~ heat conducted through a body relative to heat stored.

→ Large value of τ (F_o) indicates faster propagation of heat through a body.

9/10/18.

B.C

$$-KA\left(\frac{\partial T}{\partial x}\right)_{x=(L-dx)} - hA(T_{x=L} - T_\infty) = \rho C_p (A\,dx)\left(\frac{\partial T}{\partial t}\right)_{x=L}$$

as $dx \to 0$.

$$-k\cancel{A}\left(\frac{\partial T}{\partial x}\right)_{x=L} = h\cancel{A}\left(T_{x=L} - T_\infty\right).$$

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X). \rightarrow \text{linea combin of all solution}$$

↓

decay func. → as $n\uparrow \rightarrow$ RHS term ↓.

when A_n & λ_n are func. of Bi.

~~Experience~~ when $\tau > 0.2$, only taking the first term ~~gives~~ & neglecting others give 98% accuracy

∴ Solution with 1-term approximation ($\tau > 0.2$)

$$\theta = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2$$

$$\theta_{center, x=0} = A_1 e^{-\lambda_1^2 \tau}.$$

Long Cylinder :-

$$\theta_{cyl.} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_0)$$

$$\theta_{cyl,center} = A_1 e^{-\lambda_1^2 \tau}$$

Sphere

$$\theta_{sphere} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda r/r_0)}{(\lambda r/r_0)}$$

$$\theta_{center,sphere} = A_1 e^{-\lambda_1^2 \tau}$$

10/10/18

Transient heat conduction in multidimensional Systems.

Superposition approach → Product Solution

$$\left(\frac{T(x,y,z,t) - T_\infty}{T_i - T_\infty}\right)_{rect.\ bar} = \theta(x,y,z,t) = \theta(x,t) \cdot \theta(y,t) \cdot \theta(z,t).$$

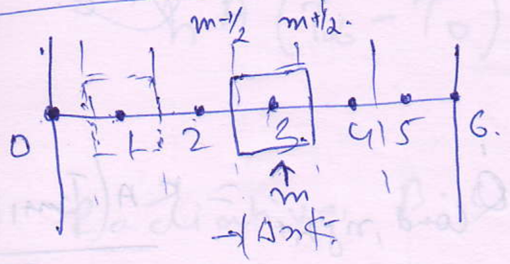
Cylinder

Infinite cyl. → $\theta(r,t) = \theta(r,t)_{cyl}$

Short cyl. → $\theta(x,r,t) = \theta(r,t)\,\theta(x,t)$.

Numerical methods in heat conduction

Finite difference.



$$\frac{df(x)}{dx} = \lim_{x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

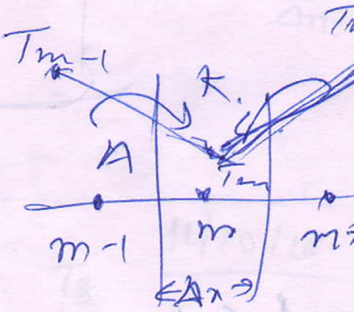
$$\frac{dT}{dx} \simeq \left(\frac{T_{m+1} - T_m}{\Delta x}\right)$$

$$\left.\frac{dT}{dx}\right|_{m-1/2} \cong \frac{T_m - T_{m-1}}{\Delta x}, \quad \left.\frac{dT}{dx}\right|_{m+1/2} \simeq \frac{T_{m+1} - T_m}{\Delta x}$$

$$\left.\frac{d^2T}{dx^2}\right|_m = \frac{\left.\frac{dT}{dx}\right|_{m+1/2} - \left.\frac{dT}{dx}\right|_{m-1/2}}{\Delta x} = \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2}.$$

1-D

Steady State heat conduction



$$\frac{d^2T}{dx^2} + \frac{\bar{q}}{K} = 0$$

$$\Rightarrow \left(\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2}\right) + \frac{\bar{q}_m}{K} = 0.$$

Approach → Energy Balance method

$$\left(\begin{array}{c}\text{Rate of heat} \\ \text{conduction at} \\ \text{left surf.}\end{array}\right) + \left(\begin{array}{c}\text{Rate of} \\ \text{heat cond.} \\ \text{at right surf.}\end{array}\right) + \left(\begin{array}{c}\text{Rate of} \\ \text{heat gen.} \\ \text{inside the} \\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of chag} \\ \text{of enthalpy}\end{array}\right)$$

or

$$\dot{Q}_{cond,left} + \dot{Q}_{cond,right} + \dot{E}_{gen,elem} = \cancel{\frac{\Delta E_{elem}}{\Delta t}}$$

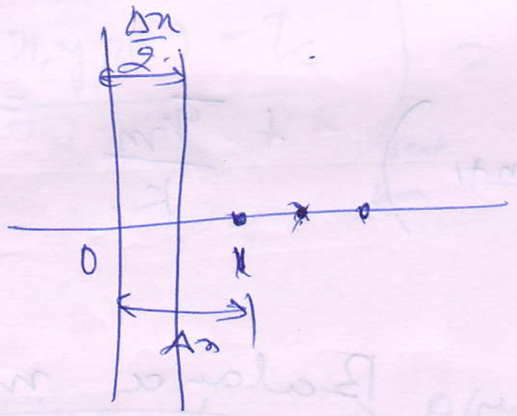
(Steady state)

$$\dot{Q}_{cond,left} = KA\frac{(T_{m-1} - T_m)}{\Delta x}, \quad \dot{Q}_{cond,right} = KA\frac{(T_{m+1} - T_m)}{\Delta x}$$

$$\therefore KA\frac{(T_{m-1} - T_m)}{\Delta x} + KA\frac{(T_{m+1} - T_m)}{\Delta x} + \bar{q}\, A\Delta x = 0$$

$$\Rightarrow \boxed{\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\bar{q}}{K} = 0}$$

B.C's -



$$\sum_{\text{all sides}} \dot{Q} + \dot{E}_{gen,elem} = 0$$

1) Specified heat flux :

$$q_0 A + KA\frac{T_1 - T_0}{\Delta x} + \bar{q}\left(A\frac{\Delta x}{2}\right) = 0$$

Insulated wall/Boundary $\rightarrow$ $q_0 = 0$

2) Convect Boundary Cond

$$hA(T_\infty - T_0) + KA\frac{(T_1 - T_0)}{\Delta x} + \bar{q}\left(A\frac{\Delta x}{2}\right) = 0$$

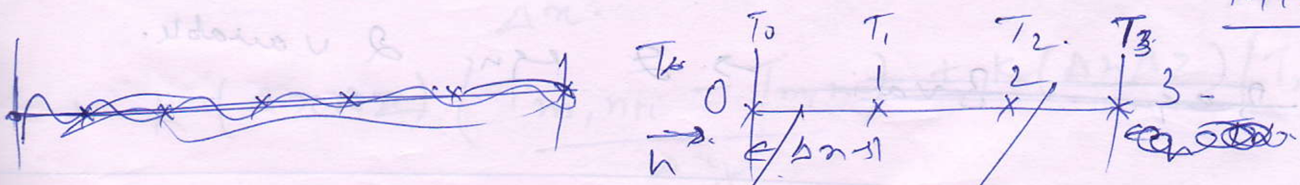
3) Radiative B.C's.

$$\varepsilon\sigma(T^4_{surr} - T^4_0) + KA\frac{(T_1 - T_0)}{\Delta x} + \bar{q}\left(A\frac{\Delta x}{2}\right) = 0$$

4) Combined Convection, Rad. & Heat flux.

$$\dot{q}_0 A + hA(T_\infty - T_0) + \varepsilon\sigma A(T^4_{surr} - T^4_0) + KA\frac{(T_1 - T_0)}{\Delta x}$$
$$+ \bar{q}\left(A\frac{\Delta x}{2}\right) = 0$$

11/10/18.



$$\frac{T_0 - 2T_1 + T_2}{\Delta x^2} + \frac{\bar{q}}{k} = 0 \Rightarrow 2T_1 - T_0 - T_2 = \bar{q}/k\,\Delta x^2$$

$$\Rightarrow 2T_2 - T_1 - T_3 = \bar{q}/k\,\Delta x^2$$