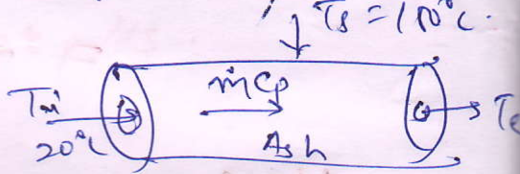
rate of decay depends on the magnitude of $\left(\frac{h A_s}{\dot{m} C_p}\right)$

$$\left(\frac{h A_s}{\dot{m} C_p}\right) = NTU \text{ (number of transfer unit)}$$

is measure of effectiveness of heat transfer system (Non-dimensional no.)

~~Typically~~,

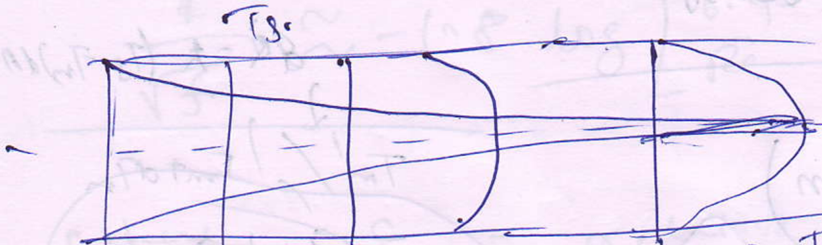


NTU	T_e (°C)
0.01	20.8
0.5	~~23.9~~ 51.5
1	70.6
5	99.5

So, NTU → gives indication of effectiveness

→ Increase beyond 5 ~~or 6 may~~ or length of tube will not add any value.

Internal flow



TE ← Thermal entrance region → Thermally fully developed.

(Entry length, L_t).

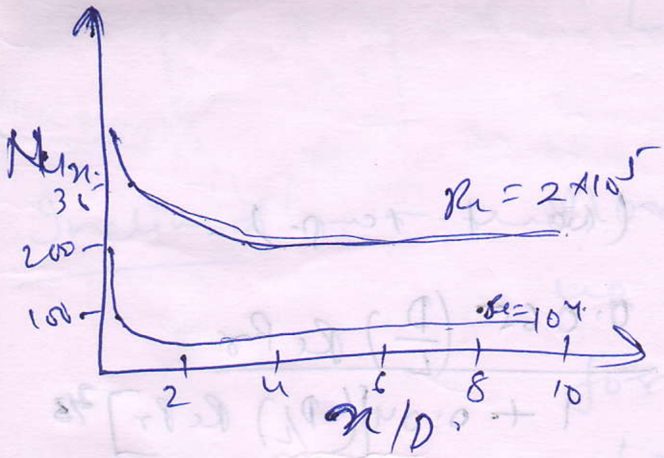
$$L_{t,\,laminar} \approx (0.05\ Re)\,Pr \cdot D$$

($L_{h,\,laminar}$)

for $Re = 20$, $Pr = 1$, $L_t = D$.

$Re = 2300$ (limiting cond.), $L_t = 115\,D$.

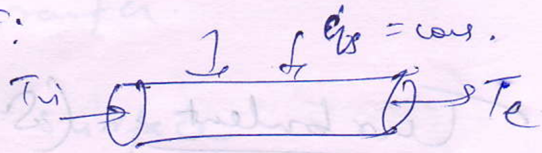
$$L_{t,\,turbulent} \approx 10\,D.$$

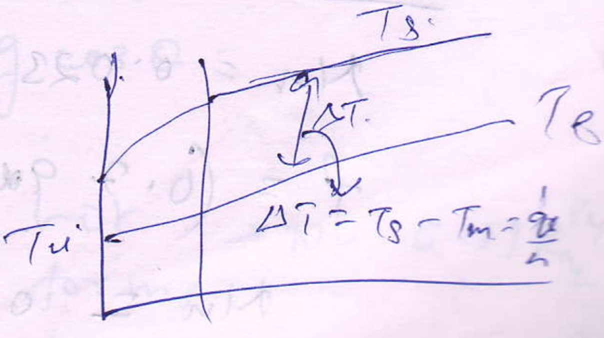


Const. Surface heat flux:

$$\dot{q}_s A_s = \dot{m}\,c_p (T_e - T_i)$$

$$\therefore T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m}\,c_p} \rightarrow \text{linear}$$



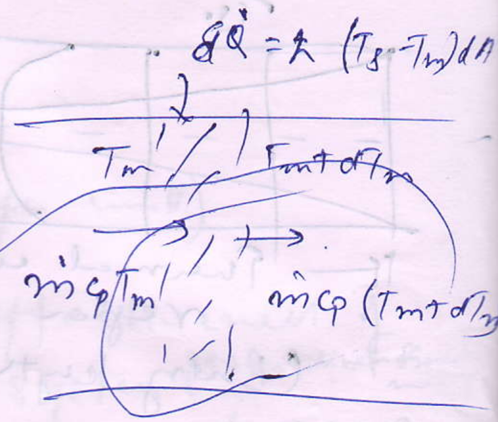


Surf. temp.

$$\dot{q}_s A_s = hA_s(T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

↳ linear.

$\therefore \quad T_s - T_m = \Delta T = \text{const.}$



$$\dot m c_p\, dT_m = \dot q_s (p\,dx)$$

$$\Rightarrow \frac{dT_m}{dx} = \frac{\dot q_s\, p}{\dot m c_p} = \text{const.}$$

Nusselt No. (fully developed). ~ Laminar

Circular tube, const. flux ($\dot q_s$)

$$Nu = \frac{hD}{k} = 4.36.$$

Const. surf. Temp (T_s)

$$Nu = 3.66.$$

Entrance region (const. temp.)

$$Nu = 3.66 + \frac{0.065 \left(\frac{D}{L}\right) Re\, Pr}{1 + 0.04\left[\left(D/L\right) Re\, Pr\right]^{2/3}}$$

Turbulent: (Smooth)

$$Nu = 0.023\, Re^{0.8}\, Pr^{1/3} \qquad \left(\begin{array}{c} 0.7 \le Pr \le 160 \\ Re > 10{,}000 \end{array}\right)$$

$$f = (0.790 \ln Re - 1.64)^{-2} \qquad (3000 < Re < 5\times 10^{6})$$

$$Nu = 0.125\, f\, Re\, Pr^{1/3}.$$

Rough surf.

$$\frac{1}{\sqrt{f}} \simeq -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right].$$

where, $\frac{\varepsilon}{D}$ = relative roughness

= ~~ratio of~~ $\frac{\text{mean height of roughness}}{\text{pipe diameter}}$

Free convection → $Nu = f(Ra_L, Pr)$

Vertical → $Nu = \left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2$

Inclined – More on hot surface facing up than vertical → Plumes forms → reduces boundary layer thickness and resistance to heat transfer.

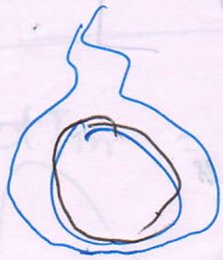
– lower hot surface

$g' = g \cos\theta$

Horizontal – Top surface hot → $Nu = 0.59\, Ra_L^{1/4}$
↳ plumes forms.

– Lower surface hot → $Nu = 0.27\, Ra_L^{1/4}$
↳ low Nu

Horizontal cylinders & spheres



- BLT increases from bottom to top.
- Local Nu at bottom is higher than that at top.

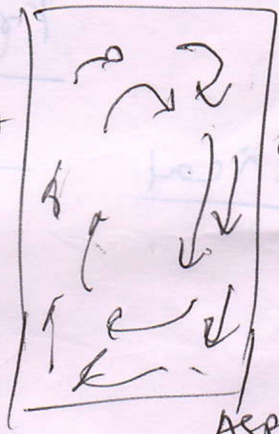
Natural Convection Inside Enclosures

If top is hot.

↳ pure conduction,
→ $Nu \simeq 1$.

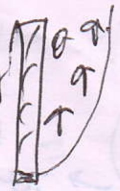
If bottom is hot.

↳ If $Ra_L < 1708$, pure conduction
↳ $Ra_L \geq 1708$ → natural convection sets in

hot  cold

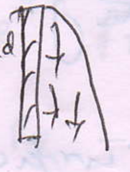
Aspect ratio
(H/L ratio is also imp.)
↓
for vertical inclined plt

Combined Natural & Forced Convection $\left(\frac{Gr}{Re^2}\right)$

hot 

↑↑↑

Assisting flow

cold 

↑↑↑

(opposing flow)

Transverse flow

$$(Nu)_{combined} = \left(Nu^n_{forced} \pm Nu^n_{natural}\right)^{1/n}.$$

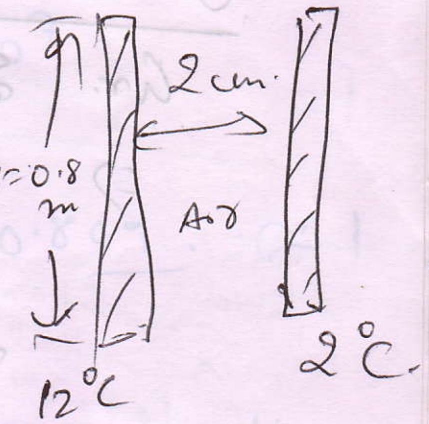
assisting / transverse → +ve

opposing → −ve.

$n \to 3-4.$

Tutorial - 9

Q. ~~Heat~~ loss through Double-pane window



$K = 0.02416\ W/m.K.$

$Pr = 0.7344.$, width = 2 m.

$\nu = 1.4 \times 10^{-5}\ \frac{m^2}{s}$

$\beta = \frac{1}{T_{avg}} = \frac{1}{280}$

$L_c = L = 0.02\ m.$

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr.$$

* Heat loss through radiation

$$= \frac{9.81 \times [1/280][12-2](0.02)^3}{1.4 \times 10^{-5}} (0.7344)$$

$$= 1.05 \times 10^4$$

Aspect Ratio, $\frac{H}{L} = \frac{0.8}{0.02} = 40.$

$$\therefore Nu = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L}\right)^{+0.3}$$

$$= 1.4.$$

$$A_s = H \times W = 0.8 \times 2 = 1.6\,m^2$$

$$\dot{q} = h A_s (T_1 - T_2)$$

$$= \left(\frac{k Nu}{L}\right) A_s (T_1 - T_2)$$

$$= \left(\frac{0.02416 \times 1.4}{0.02}\right) \times 1.6 \times (12-2)$$

$$= \underline{27.1\,W}$$

<u>Cond^n for $Nu = 1$</u>

$$Gr \leq 1708$$

$$\Rightarrow L_c^3 \leq \frac{1708}{\left(\frac{g\beta(T_1-T_2)}{\nu^3}\right)} \qquad \nu = 1.3 \times 10^{-6}$$

$$\Rightarrow L_c < 0.011\,m\,(1.1\,cm)$$

Radiative heat loss

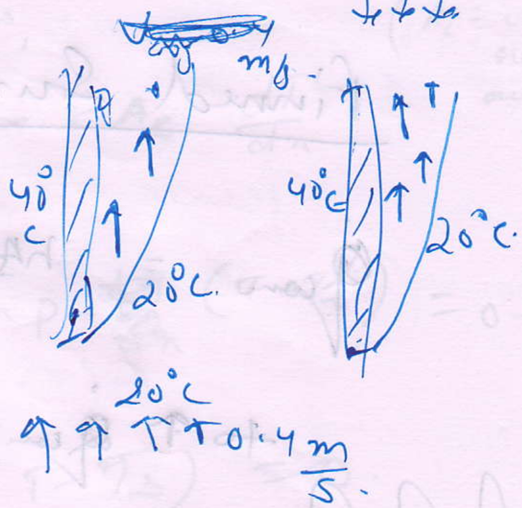
$$= \varepsilon \sigma A_s (T_h^4 - T_c^4)$$

$$= 1 \times 5.67 \times 10^{-8} \times (2 \times 0.8) \times (285^4 - 275^4)$$

$$= \underline{79.68\,W}$$

* Glass acts as insulator for IR radiation

Q. 0.2×0.2 vertical plate.



$K_{air} = 0.02588 \ \frac{W}{m\,K}$

$\nu = 1.608 \times 10^{-5} \ \frac{m^2}{s}$

$Pr = 0.7282$

$\beta = \frac{1}{T_f} = \frac{1}{\left(\frac{313+293}{2}\right)}$

$= 0.0033 \ K^{-1}$

$Re = \frac{VL}{\nu} = \frac{0.4 \times 0.2}{1.608 \times 10^{-5}} = 4975$

$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = 2 \times 10^7$

$\therefore \frac{Gr_L}{Re^2} = \frac{2 \times 10^7}{(4925)^2} = 0.809 \approx 1$

Both natural & forced convection are significant

$Ra = Gr \cdot Pr$

$Nu_{natural} = 0.59 \, Ra_L^{1/4} = 0.59(2 \times 10^7 \times 0.7282)^{1/4}$

$= 36.46$

$Nu_{forced} = 0.664 \, Re^{0.5} \, Pr^{1/3} = 42.44$

$\therefore$ Assisting flow,

$Nu_{combined} = \left(Nu_{nat}^3 + Nu_{for}^3\right)^{1/3}$

$= 29.8 \ W$

Opposing flow

$Nu_{comb} = \left(Nu_{nat}^3 - Nu_{for}^3\right)^{1/3} = 29.8 \ W$

29/10/18

Finned Surface

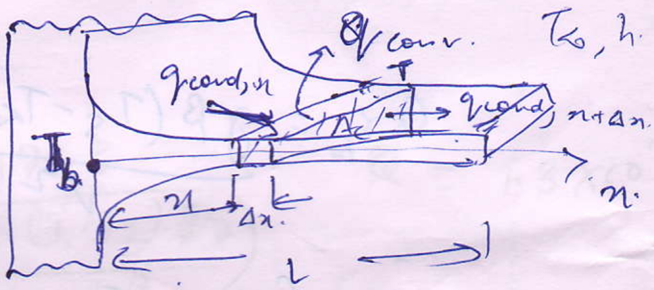
$$\dot{Q}_{conv} = h A_s (T_s - T_\infty)$$

$\underbrace{\quad}_{} \rightarrow$ fixed

to $\uparrow \dot{Q}_{conv} \rightarrow \uparrow h$ or $\uparrow A_s$ $\rightarrow$ Fins.

Too many fins $\rightarrow$ Restricts convective flow

Fin equation



under steady state condition, energy balance

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction into}\\ \text{the element } x\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat cond.}\\ \text{from } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of}\\ \text{heat conv.}\\ \text{from element}\end{pmatrix}$$

$$\Rightarrow \quad q_{cond,x} = q_{cond,x+\Delta x} + q_{conv}$$

where, $q_{conv.} = h(p\Delta x)(T - T_\infty)$.

$$\therefore \frac{q_{cond,\cancel{x+\Delta x}} - q_{cond,x\cancel{+\Delta x}}}{\Delta x} + hp(T - T_\infty) = 0.$$

as $\Delta x \rightarrow 0$,

$$\frac{dq_{conv}}{dx} + hp(T - T_\infty) = 0$$

Now, from Fourier's law, (A_c = cross section area).

$$\cancel{\frac{dq}{dx}} \quad \cancel{+h} \quad q_{cond.} = -KA_c \frac{dT}{dx}$$

$$\Rightarrow \frac{d}{dx}\left(-KA\frac{dT}{dx}\right) + hp \cancel{\frac{d}{dx}} (T - T_\infty) = 0$$

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{hp}{KA_c}(T - T_\infty) = 0$$

Non-dimentionaliing T,

$$(T - T_\infty) = \theta \Rightarrow \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$$

and let, $m = \sqrt{\frac{hp}{KA_c}}$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0 \rightarrow$$ linear, homogenous, 2nd order eqn with cons. coeff.

Sol^n $\Rightarrow$ $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

B.C^n $\rightarrow$ let T_b = temp. of plate attached to fin.

i) at fin base $\therefore$ $\theta(0) = T_b - T_\infty$.

ii) Fin tip.

a) Infinite long fin ($T_L = T_\infty$)

$\therefore$ $\theta(L) = T(L) - T_\infty = 0$. as $L \to \infty$.

But as $x \to \infty$ → $e^{mx} \to \infty$. Not a prospective sol^n.

Hence, C_1 must be '0'.

∴ only sol^n is,

$$\theta(x) = C_2 e^{-mx}$$

∴ ~~$T_x - T_\infty$~~

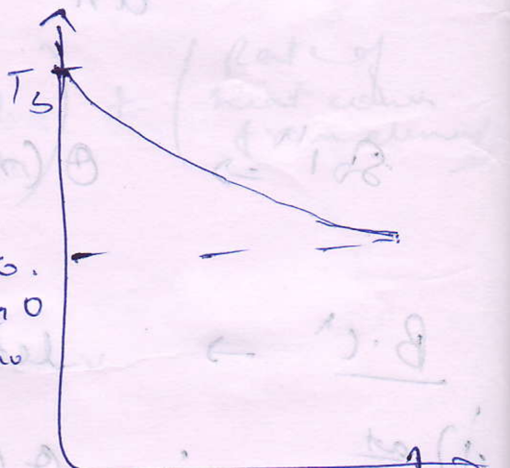
∴ at $x = 0$ → $\theta(0) = T_b - T_\infty$.

∴ $T_b - T_\infty = C_2 e^{-m(0)}$

⇒ $C_2 = T_b - T_\infty$.

∴ $\theta(x) = (T_b - T_\infty) e^{-mx}$

⇒ $$\frac{T_x - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/KA_c}}.$$



Total heat loss from fin

Now, also, at base ($x=0$)

$$q_{\text{long fin}} = -KA_c \left.\frac{dT}{dx}\right|_{x=0} + \cancel{q_{conv}}^{0}$$

$$\left.\frac{dT}{dx}\right|_{x=0} = -(T_b - T_\infty)\sqrt{\frac{hp}{KA_c}} \cdot e^{-m(0)} \to =1.$$

as

$$= -\sqrt{\frac{hp}{KA_c}}\,(T_b - T_\infty) \times -KA_c$$

$$= \boxed{\sqrt{hpKA_c}\,(T_b - T_\infty)}.$$

b) Negligible heat loss from fin tip / Insulated tip

(Adiabatic fin tip, $q_{fin\,tip} = 0$).

↳ fin tip area is negligible compared to total fin area.

↳ fin tip at higher temp. but negligible heat loss.

↳ B.C^n at tip → $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$.

⇒ at ~~tip~~ base → same → $\theta_b = C_1 + C_2$.

& tip → $\left.\frac{d\theta}{dx}\right|_{x=L} = mC_1 e^{mL} - mC_2 e^{-mL} = 0$.

$$\Rightarrow \quad C_1 = \frac{\theta_b}{1+e^{2mL}}, \quad C_2 = \frac{\theta_b}{1+e^{-2mL}}$$

$$\Rightarrow \quad \theta(x) = \frac{\theta_b}{1+e^{2mL}} e^{mx} + \frac{\theta_b}{1+e^{-2mL}} e^{-mx}$$

Can be rearranged as, $\left(\cosh x = \frac{e^x + e^{-x}}{2}\right)$.

$$\boxed{\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}}$$

Total heat transfer rate

$$q_{adia.\,tip} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \cancel{\frac{hp}{kA_c}} \sqrt{hPkA_c}\,(T_b - T_\infty) \tanh mL$$

c) Specified temp. ($T_{fin,tip} = T_L$).

↳ a generalised case of $T_{fin} = T_\infty$?

→ B.C' at tip → $\theta(L) = (T_L - T_\infty)$

we get sol^n as.

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)] \sinh(mx) + \sinh(m(L-x))}{\sinh(mL)}$$

and $$q_{spec.temp} = \sqrt{hpkA_c}\,(T_b - T_\infty)\,\frac{\cosh(mL) - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh(mL)}$$

conv. from fin tip.

↳ many real case scenario

↳ short fins.

$(T...$

$Along$

$\left(\frac{e^x + e^{-x}}{2}\right)$

tip → $-kA_c \frac{dT}{dx}\Big|_{x=L} = hA_c[T_L - T_\infty]$

$+ \left(\frac{h}{mk}\right) \sinh m(L-x)$.

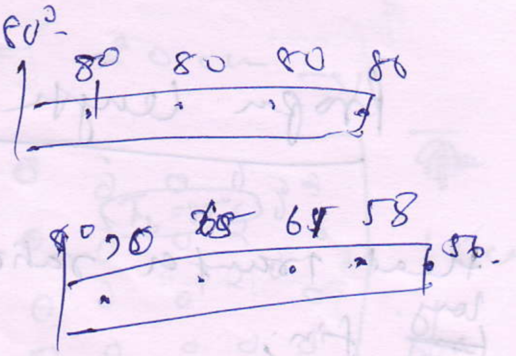
and

$mL + \left(\frac{h}{mk}\right)\sinh(mL)$.

$\sqrt{hpkA_c}(T_b - T_\infty)\tanh mL$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\,\frac{\sinh mL + \left(\frac{h}{mk}\right)\cosh mL}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$$

Fin efficiency :



$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}}$$

$$= \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if entire fin were at base temp.}}$$

eg: Long fin $\rightarrow$ $\eta_{long\text{-}fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$

adiabatic fin $\rightarrow$ $\eta_{adia\text{-}fin} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{hA_{fin}(T_b - T_\infty)} = \frac{\tanh mL}{mL}$

Fin effectiveness :

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\,fin}} = \frac{q_{fin}}{hA_b(T - T_\infty)}$$

$$= \frac{\text{Heat transfer rate from the fin of base Area } A_b}{\text{Heat transfer rate from the surface of area } A_b \text{ (w/o fin)}}$$

long fin with uniform cross section area $(A_b = A_c)$.

$$\varepsilon_{long\text{-}fin} = \frac{q_{fin}}{q_{no\,fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

$\Rightarrow$ $k\uparrow$, $\frac{p}{A_c}\uparrow$ (thin plate, slender pin), $\varepsilon\uparrow$ for low h applications.

more justified to use if h is low

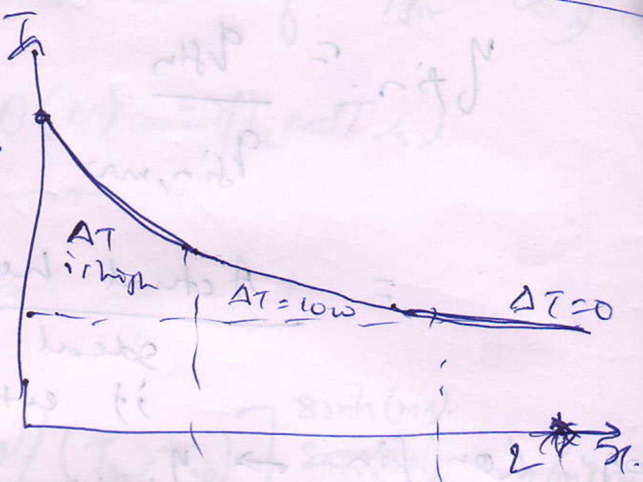
Also - ε-η relation

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\,fin}} = \frac{q_{fin}}{hA_b(T_b - T_\infty)} = \frac{\eta_{fin}\, h A_{fin}(T_b - T_\infty)}{hA_b(T_b - T_\infty)}$$

$$\boxed{\varepsilon_{fin} = \frac{A_{fin}}{A_b}\cdot \eta_{fin}}$$

31/10/18

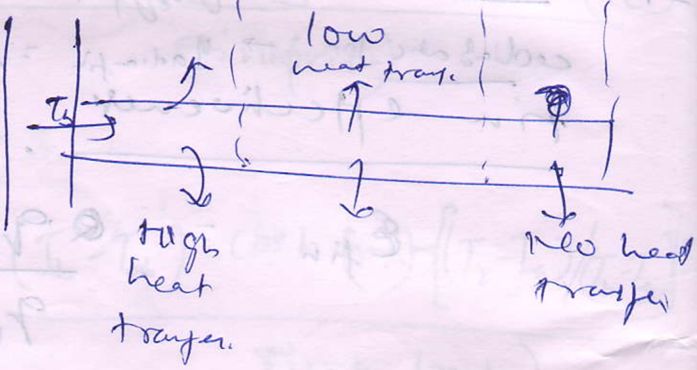
Proper length of fin — Approx. for infinite long fin



Heat transfer ratio for long fin;

$$\frac{q_{fin}}{q_{long\ fin}} = \frac{\sqrt{hpKA_c}\,(T_b - T_\infty)\tanh(mL)}{\sqrt{hpKA_c}\,(T_b - T_\infty)}$$

$$= \tanh(mL)$$



mL	$\tanh(mL)\ \left(=\frac{q_{fin}}{q_{long\ fin}}\right)$
0.1	0.1
0.5	0.462
1	0.762
4	0.999
5	1

fin approaches No heat loss zone at $mL = 5$

Or, $\boxed{L = \frac{5}{m}}$ → length considered to be infinitely long fin beyond which no heat transfer will take place.

Heat exc.

Types of Heat exch.

1. ~~parralle~~ Parallel flow
2. Counter flow
3. Cross flow → un mixed / → mixed.
4. Shell-and-tube type. → One shell & 2-tube pass / → 2 shell & 4 tube pass etc.
5. Plate & frame type.
6. Regenerative type (high specific heat - heat storage - ceramic wool)
7. Condenser | Boiler → Phase change Heat exch. type.
8. Space radiator.

Analysis of heat exchangers.

LMTD.

$\Delta T_{lm} \Rightarrow$

Heat capacity rate.

$$q = \dot{m}_c c_{p_c} (T_{c,out} - T_{c,in})$$
$$= C_c (T_{c,out} - T_{c,in}).$$
$$= \dot{m}_h c_{p_h} (T_{h,\cancel{out}\,in} - T_{h,\cancel{in}\,out})$$
$$= C_h (T_{h,in} - T_{h,out}).$$

C_h & C_c are called heat capacity rate

~~Boi~~ Phase-change heat exchangers.

$\Delta T \to 0 \Rightarrow C \to \infty$

$\longrightarrow$ Limited by C_{min} $\to$ heat transfer.

LMTD.

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}.$$

|| flow $\to$ $\Delta T_1 = T_{h,in} - T_{c,in}.$

$\Delta T_2 = T_{c,out} - T_{c,out}$

counter flow $\to$ $\Delta T_1 = T_{h,in} - T_{c,out}.$

$\Delta T_2 = T_{h,out} - T_{c,in}.$

~~where~~

In counter flow, when $C_c = C_h$

$$\Delta T_{lm} = \frac{0}{0} \Rightarrow \Delta T_{lm} = \Delta T_1 = \Delta T_2.$$

For phase change heat exchanger

ΔT_{lm} is same for $\parallel$ or counter flow

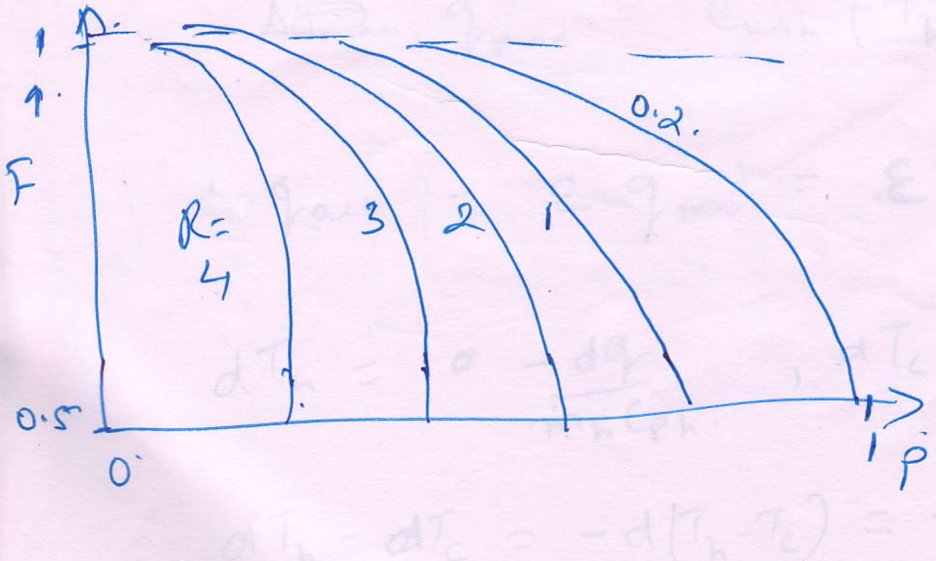
Cross flow, multi pass / shell + tube type heat exchanger

$$\Delta T_{lm} = F \, \Delta T_{lm,CF}$$

where, F = correction factor
= correlated with respect to P & R.

where, $P = \dfrac{t_2 - t_1}{T_1 - t_1}$, $R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{(\dot{m} c_p)_{tube}}{(\dot{m} c_p)_{shell}}$

1 → inlet, T → shell (outside)
2 → outlet. t = tube (inside).



$R \to \infty / 0$ for phase change.
$F = 1$.

Performance Analysis 2/11/18

→ Compactness of Heat Exchangers

Area density, $\beta = \dfrac{\text{Surf. area of heat exchanger}}{\text{Volume of heat exchanger}}$

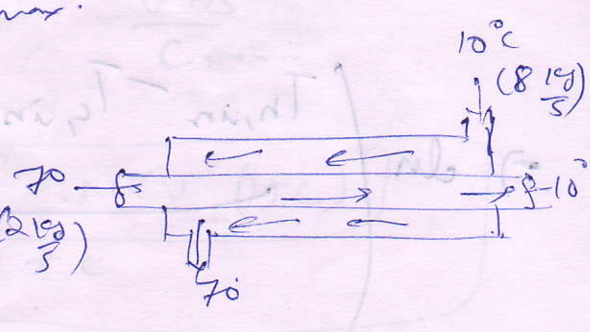
$\beta > 700 \ \dfrac{m^2}{m^3}$ → classified as compact HE.

Car radiators: 1000 m^2/m^3
Human lung, $\beta = 20{,}000 \ m^2/m^3$.

→ Effectiveness - NTU method

Effectiveness, $\varepsilon = \dfrac{q_{actual}}{q_{max}}$

$q_{act} = C_c(T_{c,out} - T_{c,in})$
$= C_h(T_{h,in} - T_{h,out})$



$q_{max} = C_{min}(T_{h,in} - T_{c,in})$

$q_{act} = \varepsilon q_{max} = \varepsilon C_{min}(T_{h,in} - T_{c,in})$

$dT_h = -\dfrac{dq}{\dot m_h c_{ph}}$, $dT_c = \dfrac{dq}{\dot m_c c_{pc}}$

$dT_h - dT_c = -d(T_h - T_c) = -dq\left(\dfrac{1}{\dot m_h c_{pc}} + \dfrac{1}{\dot m_c c_{pc}}\right)$

also, $dq = U(T_h - T_c)\, dA_s$

$$\therefore \frac{d(T_h - T_c)}{(T_h - T_c)} = -U\, dA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

Integrating

$$\Rightarrow \ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right) = U A_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$= \frac{U A_s}{C_c}\left(\frac{C_c}{C_h} + 1\right)$$

Also,

$$C_h(T_{h,in} - T_{h,out}) = C_c(T_{c,out} - T_{c,in})$$

$$\Rightarrow T_{h,out} = T_{h,in} - \frac{C_c}{C_h}(T_{c,out} - T_{c,in})$$

$$\Rightarrow \ln\left(\frac{T_{h,in} - T_{c,in} + T_{c,in} - T_{c,out} - \frac{C_c}{C_h}(T_{c,out} - T_{c,in})}{T_{h,in} - T_{c,in}}\right)$$

$$= \frac{U A_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$\Rightarrow \ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\left(\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}}\right)\right] = \frac{U A_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

Also, $\varepsilon = \dfrac{q}{q_{max}} = \dfrac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})}$

$$\Rightarrow \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \varepsilon \frac{C_{min}}{C_c}$$

$$\Rightarrow \ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\left(\varepsilon \frac{C_{min}}{C_c}\right)\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

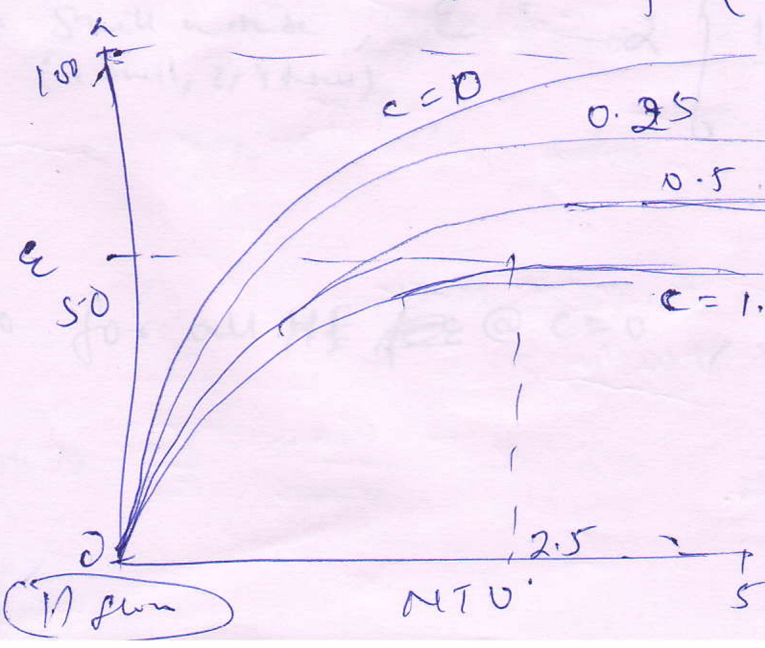
$$\Rightarrow \boxed{\varepsilon_{parallel\,flow} = \frac{1 - \exp\left[-\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)\right]}{\left(1 + \frac{C_c}{C_h}\right)\left(\frac{C_{min}}{C_c}\right)} = \frac{1 - \exp\left(-\frac{UA_s}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right)}{\left(1 + \frac{C_{min}}{C_{max}}\right)}}$$

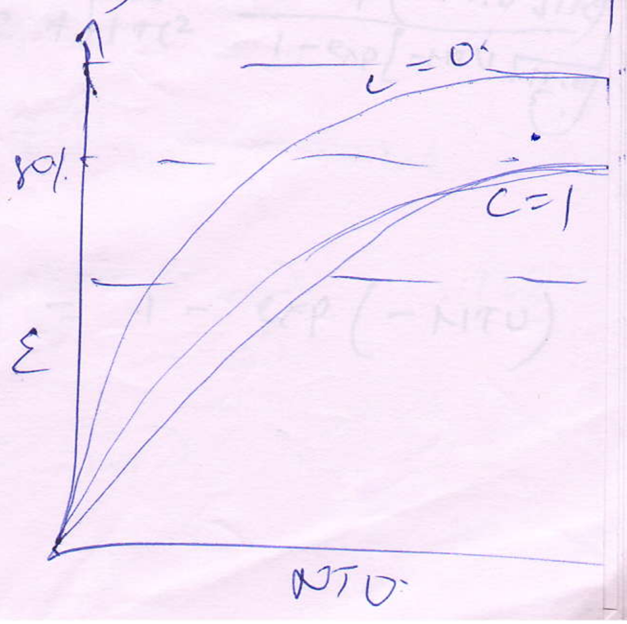
$$NTU = \frac{UA_s}{(\dot{m}C_p)_{min}} = \frac{UA_s}{C_{min}}.$$

$$C = C_{min}/C_{max}$$

$$\Rightarrow \boxed{\varepsilon_{parallel} = \frac{1 - \exp\left[-NTU(1+C)\right]}{1+C}}$$

$$\varepsilon = f(NTU, C)$$





5/11/10

NTU $= \dfrac{UA_s}{C_{min}} = \dfrac{UA_s}{(\dot{m}C_p)_{min}}$

for a specified U & C_{min} NTU represents measure of the heat transfer surface area (A_s)

$\therefore \quad \varepsilon = \cancel{f(UA)}\ f\left(NTU, \dfrac{C_{min}}{C_{max}}\right)$

$= f(NTU, c)$.

eg:,

* ∥ flow, $\varepsilon = \dfrac{1 - \exp(-NTU(1+c))}{1+c}$.

* counter flow, $\varepsilon = \dfrac{1 - \exp[-NTU(1-c)]}{1 - c\exp[-NTU(1-c)]}$ (for $c<1$)

$= \dfrac{NTU}{1+NTU}$ (for $c=1$).

* Shell & tube (1 shell, 2, 4 tubes), $\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\dfrac{1+\exp(-NTU\sqrt{1+c^2})}{1-\exp[-NTU\sqrt{1+c^2}]}\right\}^{-1}$

* for all HE @ $c=0$, $\varepsilon = 1 - \exp(-NTU)$

- ~~E~~ change in ε is negligible beyond $NTU \simeq 3$.

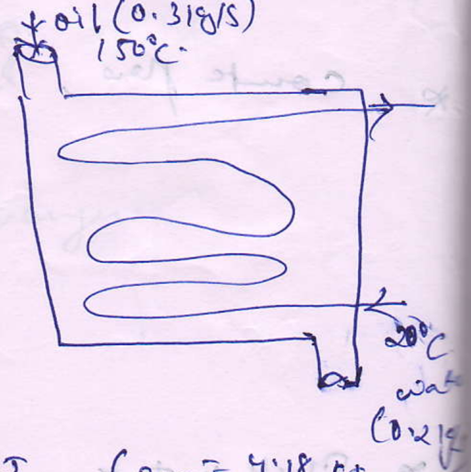
- for a given NTU, $\varepsilon_{counterflow}$ is higher & $\varepsilon_{\parallel flow}$ is lowest.

- For a given heat exchanger type; ε is independent (or converges) ~~f~~ of 'C' for $NTU < 0.3$

- once C & NTU have been evaluated, the ε value can be ~~eo~~ determined from chart or formula / correlation for given type of heat exchanger.

Tutorial - 13

Q. ~~E.Ex~~ 1 shell, 8-tube passes heat exchanger. The tubes are thin walled made of copper.



Tube - $ID = 1.4\,cm$, length of each pass $= 5\,m$.

$U = 310\,W/m^2.K$, $c_{p,oil} = 2.13\,\frac{kJ}{kg\,K}$, $C_{p,w} = 4.18\,\frac{kJ}{kg\,K}$.

Determine, i) q, ii) $T_{w,out}$, iii) $T_{oil,out}$.

Sol. $C_h = \dot m_h c_{p_h} = 0.639\,kW/K$.
$C_c = 0.836\,kW/K$

$\therefore C_{min} = C_h \Rightarrow c = \frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = 0.764$

Selection of HE

→ Heat transfer rate
→ cost
→ Pumping power
→ Size & weight

} ~~effort~~ creating turbulence ~~to creat~~ for enhancing 'h'

→ Type.
→ Materials – more on HTF than interface metals.

~~Refr~~ AC & psychrometrics

6/11/18

Moisture & temp. controll – AC.

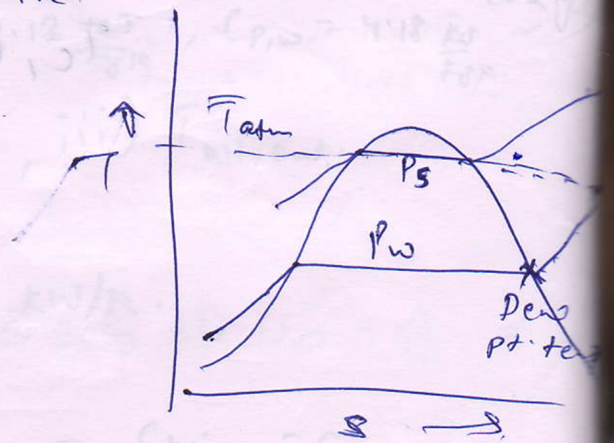
$$P = P_a + P_w$$

Mole fraction of ~~dr~~ water vapour,

$$x_w = \frac{P_w}{P} = P_w \quad (P_a = 1\,atm).$$

Relative Humidity (RH, ϕ) – ratio of partial pr. of wat. vap. (P_w) to the sat. pr. (P_s) at the same temp. of mixture.

$$\therefore RH(\phi) = \frac{P_w}{P_s}.$$



$$P_w V = m_w R_{H_2O} T = n_w \bar{R} T$$

$$\& \quad P_s V = m_s R_{H_2O} T = n_s \bar{R} T.$$

$$\Rightarrow \phi = \frac{P_w}{P_s} = \frac{m_w}{m_s} = \frac{n_w}{n_s} = \frac{x_w}{x_s}$$

Specific Humidity or Humidity ratio (ω)

– mass of wat. vap. (or moisture) per unit mass of dry air in a mix. of air & wat. vap.

∴ if, G = mass of dry air
m = mass of wat. vap.

$$W = \frac{m}{G}$$

$$W_{max} = \frac{m_s}{G} \longrightarrow \text{saturated cond.}$$

Assuming dry air and wat. vap. as ideal gases,

$$P_\omega V = m R_\omega T \Rightarrow m = \frac{P_\omega V}{R_\omega T}$$

$$P_a V = G R_a T \Rightarrow G = \frac{P_a V}{R_a T}$$

$$\Rightarrow W = \frac{m}{G} = \frac{\left(\frac{P_\omega V}{R_\omega T}\right)}{\left(\frac{P_a V}{R_a T}\right)} = \frac{P_\omega}{R_\omega} \times \frac{R_a}{R_a}$$

$$= \left(\frac{P_\omega}{P-P_\omega}\right) \times \left(\frac{R/M_a}{R/M_\omega}\right)$$

$$= \left(\frac{P_\omega}{P-P_\omega}\right) \times \left(\frac{M_\omega}{M_a}\right) = \left(\frac{P_\omega}{P-P_\omega}\right) \times \frac{18}{28.96}$$

$$\boxed{W = 0.622 \left(\frac{P_\omega}{P-P_\omega}\right)}$$

at saturated condition $\rightarrow$ $W_s = 0.622 \times \left(\frac{P_s}{P-P_s}\right)$

Degree of Saturation (μ) → ratio of actual specific humidity & saturated specific humidity both at same temp. T.

$$\therefore \mu = \frac{w}{w_s} = \frac{0.622 \times \frac{p_w}{p-p_w}}{0.622 \times \frac{p_s}{p-p_s}} = \left(\frac{p_w}{p_s}\right)\left(\frac{p-p_s}{p-p_w}\right)$$

Now, if $\phi = 0$ → 0% RH. → $p_w = 0$,

⇒ $\mu = 0$.

if $\phi = 100\%$ → 100% RH → $p_w = p_s$.

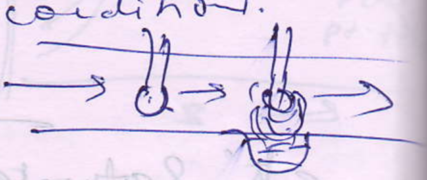
⇒ $\mu = 1$

∴ μ varies between 0 & 1.

Dew point temp. (~~DBT~~)(T_{dpt}) The temp. at which wat. vap. starts condensing (at const. pressure) is called DBT.

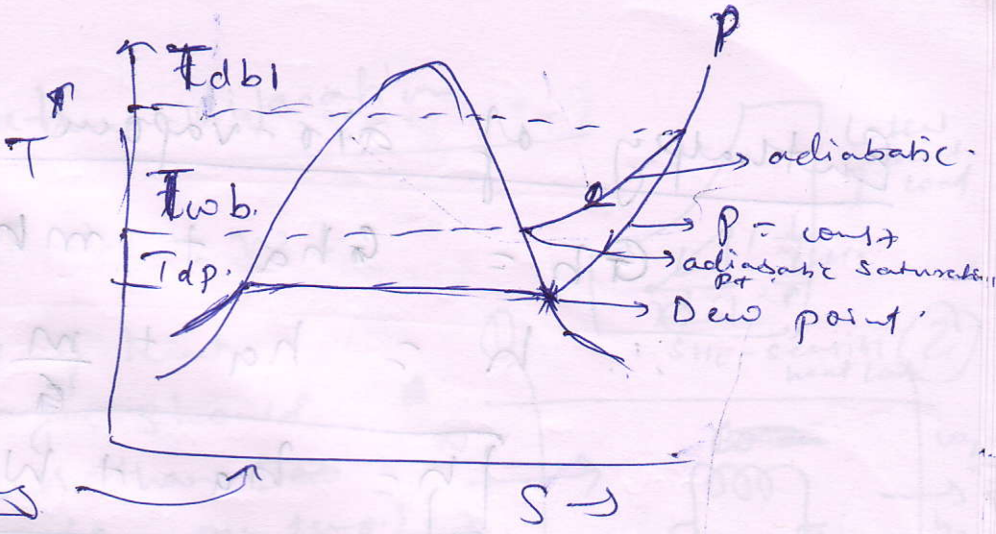
Dry bulb temp. (DBT) → temp. recorded by thermometer at dry bulb.

Wet bulb temp. ⇒ temp. recorded when water vap. is added to unsaturated air with evaporation of water under adiabatic condition.

Psychrometer → 

Adiabatic Saturation temp.

→ ~~The equi~~



Adiabatic cooling

Energy balance

$$G h_{a1} + m_1 h_{w1} + \Delta m \, h_{f2} = G h_{a2} + m_2 h_{w2}$$

$$\Rightarrow G h_{a1} + m_1 h_{g1} + (m_2 - m_1) h_{f2} = G h_{a2} + m_2 h_{g2}$$

div. by G

$$\Rightarrow h_{a1} + w_1 h_{g1} + (w_2 - w_1) h_{f2} = h_{a2} + w_2 h_{g2}$$

$$\Rightarrow w_1 = \frac{(h_{a2} - h_{a1}) + (w_2 h_{g2} - w_2 h_{f2})}{h_{g1} - h_{f2}}$$

$$\boxed{w_1 = \frac{c_{p,a}(T_2 - T_1) + w_2 h_{fg2}}{h_{g1} + h_{f2}}}$$

where, $w_2 = \frac{m_2}{G} = \frac{m_s}{G} = 0.622 \frac{P_s}{P - P_s}$

Psychrometric Chart -

Enthalpy of air-vapour mixture.

$$G h = G h_a + m h_w .$$

$$\therefore h = h_a + \frac{m}{G} h_w$$

$$\boxed{h = h_a + W h_w}$$

→ Enthalpy of mixture per kg ~~of dry~~ air.

Standard Psychrometric Process

(1) **Sensible heating or cooling** (W = const.)

↳ heat addition / removal only.

↳ no change in moisture content.

$G_1 h_1 + Q_{1-2} = G_2 h_2$

$Q_{1-2} = G(h_2 - h_1)$

$= G(h_{a2} + W h_{w2} - h_{a1} - W h_{w1})$

$= G[c_{pa}(T_2 - T_1) - W c_{p,w}(T_2 - T_1)] = G[(c_{pa} + W c_{p,w})(T_2 - T_1)]$

(2) **Adiabatic evaporative cooling.** (h = const.)

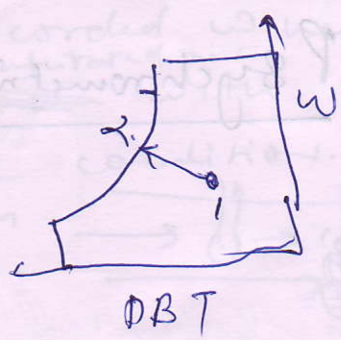
↳ addition of moisture.

↳ enthalpy of air used for evaporation.

$G_1 = G_2 = G$

$\Rightarrow G_1 W_1 + L = G_2 W_2$ (L → amount of water added.)

$\Rightarrow L = G_2 W_2 - G_1 W_1 = G(W_2 - W_1)$

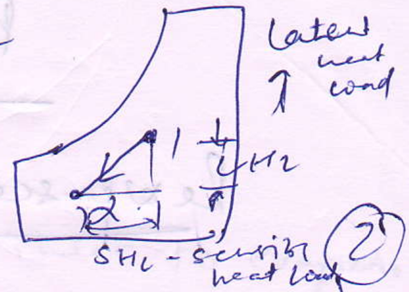


Now, $G_1 h_1 + L h_f = G_2 h_2$.

$\Rightarrow \cancel{G}(h_1 - h_2) + \cancel{G}(W_2 - W_1) h_f = 0$.

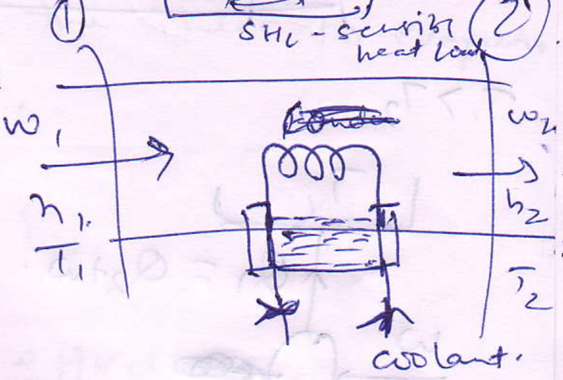
$\Rightarrow h_1 - W_1 h_f = h_2 - W_2 h_f$

(3) Cooling & dehumidification



↳ DBT ↓, w ↓

↳ temp. of heat exchanger should be less than ~~Dew~~ DPT (Dew pt. temp.)



$m_1 = m_2 + L$ → moisture removed

$G w_1 = G w_2 + L$

$\therefore L = G(w_1 - w_2)$.

energy balance,

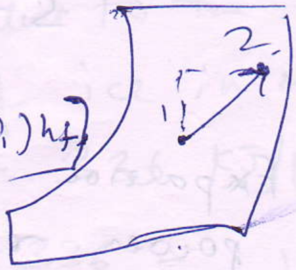
$$G_1 h_1 = G_2 h_2 + Q_{1-2} + L h_{f2}.$$

$$\therefore G h_1 = G h_2 + Q_{1-2} + G(w_1 - w_2) h_{f2}$$

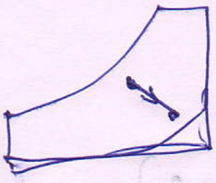
$$\Rightarrow Q_{1-2} = G\left[(h_1 - h_2) - (w_1 - w_2) h_{f2}\right]$$

(4) Heating & humidification.

$$Q_{1-2} = G\left[(h_2 - h_1) - (w_2 - w_1) h_f\right]$$



(5) Chemical dehumidification → Silica gel (Fused Sodium-Silicate & Sulphuric acid)



↳ adsorption of moisture

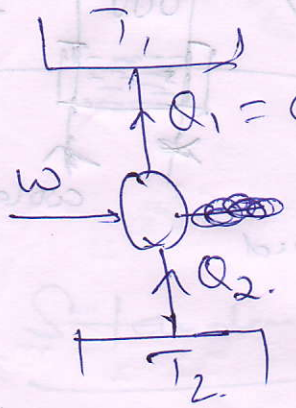
↳ adiabatic (h = const)

Refrigeration

Reversed heat ~~eg~~ engine cycle

→ Heat pump.
→ Refrigeration cycle

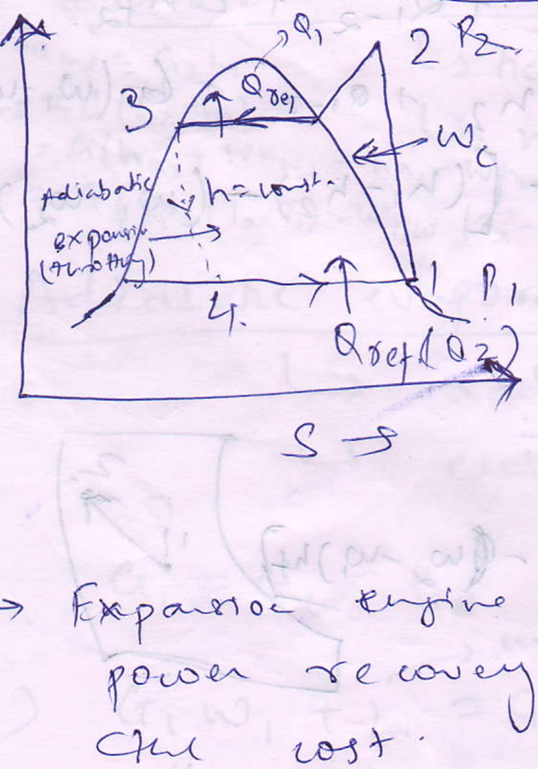
$T_1 > T_2$



$$(COP)_{H.P.} = \frac{Q_1}{W} > 1.$$

$$(COP)_{Ref.} = \frac{Q_2}{W}.$$

Vapour compression ref. cycle



Adiabatic expansion / throttling

$$T\,ds = dh - v\,dp \quad \text{(flow process)}$$

$$\therefore \int ds = -\int_{P_1}^{P_2} \frac{v\,dp}{T} \quad (dh = 0)$$

$$> 0.$$

→ Expansion engine or turbine is not used as power recovery is small & doesn't justify the cost.

Energy analysis

Comp. → $h_2 = h_1 + W_C \Rightarrow W_C = (h_2 - h_1)$

condenser → $Q_1 = (h_2 - h_3)$

Expansion → $h_3 = h_4$.

$$(h_f)_{P_2} = (h_f)_{P_1} + x_4 (h_{fg})$$

$$\therefore x_4 = \frac{(h_f)_{P_2} - (h_f)_{P_1}}{(h_{fg})_{P_1}}$$

quality of ~~a~~ ref. at the inlet to evaporator.
(flash gas fraction).

<u>Evaporator</u>, ~~Q~~ $Q_2 = h_1 - h_4$.

refrigating effect.

$$COP = \frac{Q_2}{W_C} = \frac{h_1 - h_4}{h_2 - h_1}.$$

if, 'w' is the mass flow of ref. in kg/s.
the heat removal rate. $= w(h_1 - h_4)\ \frac{kJ}{s}$

$$= w(h_1 - h_4) \times 3600\ \frac{kJ}{h}.$$

<u>One tonne of ref.</u> → defined as the rate of heat removal from surrounding equivalent to the heat required for ~~melting~~ freezing 1 tonne of ice in one day.
(latent heat of fusion of ice = 336 kJ/kg)

$$\therefore 1 \text{ tonne } \cancel{\text{of ice}} \text{ ice} \equiv \frac{1000 \times 336}{24} = 14000\ \frac{kJ}{h}$$

$\therefore$ Capacity of ref. plant

$$= \frac{w(h_1 - h_4) \times 3600}{14000} \text{ tonnes.}$$