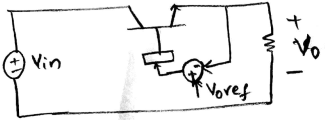
Linear regulator vs switched-mode DC-DC converter

Linear regulator



- → $V_{in} > V_o$
- → Excellent regulation of V_o
- → Very low ripple in V_o
- → Poor efficiency

Switched-mode DC-DC converter

- → $V_{in} > V_o$ or $V_{in} < V_o$
- → Regulated V_o
- → Low Ripple in V_o ⇒ large capacitor
- → 100% efficiency ideally

DC-DC converters: Analysis in periodic steady-state

Analysis of DC-DC converters in periodic steady-state:

① Assumptions
(i) switches have no ON-state voltage drops ⇒ No conduction loss
(ii) Switches change their states instantaneously ⇒ No switching loss
(iii) Winding resistance of inductor = 0
(iv) Equivalent series resistance (ESR) of capacitor = 0

② Approximations: Small-ripple approximation
Output voltage is constant ; $v_o \approx V_o$
current thro' inductor is constant $i_L \approx I_L$ (continuous conduction mode)

③ Principles
(i) Volt-sec balance across inductor
(ii) Amp-sec (charge) balance across capacitor
(iii) Power balance $P_{in} = P_{out}$
(iv) KCL, KVL

(v) Flux continuity
(vi) Volt-sec bal across each winding
(v) & (vi) for isolated DC-DC converters

DC-DC converters: Design of circuit components

Design & choice of circuit components

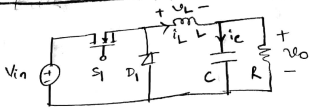
Component	Design/ choice based on
MOSFET	Blocking voltage, Peak current, RMS current, ON-state resistance $R_{DS\,ON}$, Transition times t_{on} & t_{off}
Diode	Blocking voltage, Average current, Forward voltage drop, Reverse recovery charge
Inductor	Peak-to-peak ripple current, average current, switching frequency, worst-case duty cycle, power loss
Capacitor	Voltage rating, peak-to-peak voltage ripple, RMS ripple current, ESR

Usual Specifications of a DC-DC converter

$\underbrace{\boxed{V_{in\,max},\ V_{in\,min}}}_{\text{Range of input voltage}}$, $\boxed{V_o}$ → output voltage, $\boxed{\Delta i_L}$ → Peak-to-peak inductor current, $\boxed{\Delta v_o}$ → output voltage ripple, $\boxed{R_{max},\ R_{min}}$, $\boxed{CCM \text{ or } DCM}$ → Range of load

Buck (step-down) converter

Buck Converter



$0 < t < DT_s$: S_1 ON D_1 OFF

$DT_s < t < T_s$: S_1 OFF D_1 ON

$V_o = D\,V_{in}$

$T_s = \dfrac{1}{f_{sw}}$

$\Delta i_L = \dfrac{V_o(1-D)T_s}{L}$

$\Delta v_o = \dfrac{\Delta i_L\, T_s}{8C}$

$T_o = 2\pi\sqrt{LC}$ | $f_o = \dfrac{1}{T_o}$

$\Delta v_o \approx V_o$

$\Rightarrow T_s \ll T_o$

$f_{sw} \gg f_o$

$i_{S_1}\,pk = I_L + \dfrac{\Delta i_L}{2}$

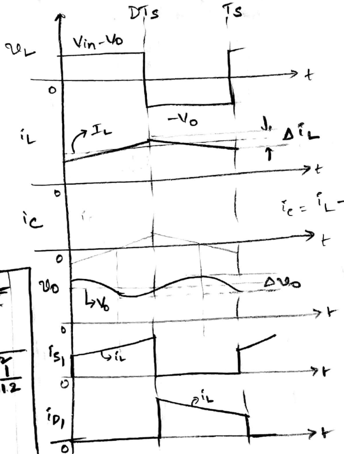
i_{S_1} RMS

$= I_L\sqrt{D}\sqrt{1 + \left[\dfrac{\Delta i_L}{I_L}\right]^2 \dfrac{1}{12}}$

$\approx I_L\sqrt{D}$

i_{D_1}, av

$= (1-D)I_L$



$i_c = i_L - I_L$

$I_L = \dfrac{V_o}{R}$

$i_{S_1} = i_{in}$

↓

Input current is pulsating!

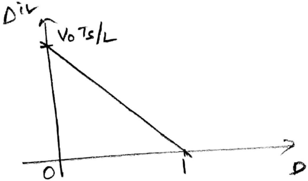
$i_{in}\,av = D\,I_L = i_{S_1}\,av.$

Buck (step-down) converter

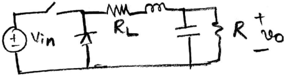
Buck Converter:

$$\Delta i_L = \frac{V_o(1-D)T_s}{L}$$

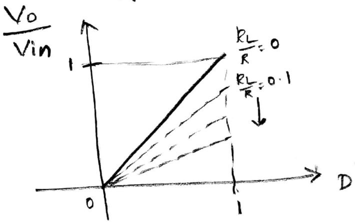
for given V_o, T_s & L



Effect of one non-ideality

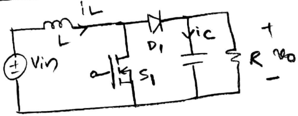


$$V_o = \frac{D\,V_{in}}{1 + \frac{R_L}{R}}$$



Boost (step-up) converter

Boost Converter



$0 < t < DT_s$: S_1 ON D_1 OFF

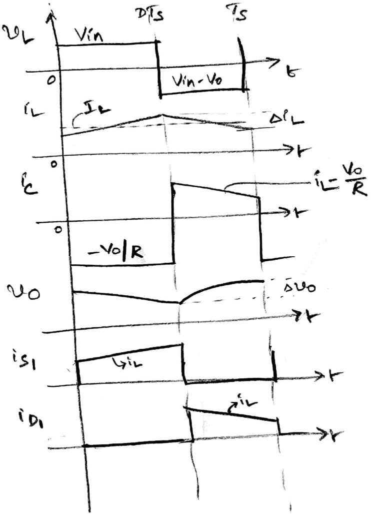
$DT_s < t < T_s$: S_1 OFF D_1 ON

$$V_o = \frac{V_{in}}{1-D}, \quad 0 < D < 1$$

$$I_L = \frac{V_o}{(1-D)R}$$

$$\Delta i_L = \frac{V_o D(1-D)T_s}{L}$$

$$\Delta v_o = V_o \frac{DT_s}{RC}$$



$i_{in} = i_L$

↓

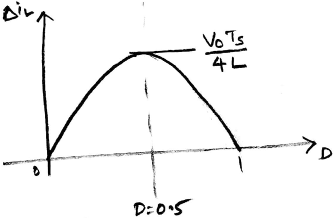
Input current is smooth.

Boost (step-up) converter

Boost converter

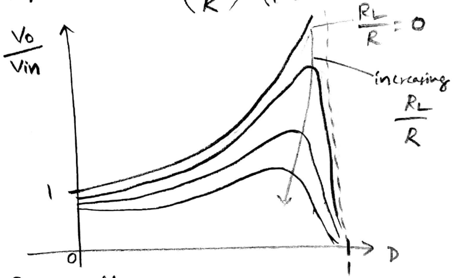
$$\Delta i_L = \frac{V_o D (1-D) T_s}{L}$$

for given V_o, T_s & L



Effect of one non-ideality:

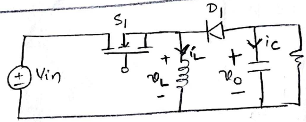
$$\frac{V_o}{V_{in}} = \frac{1}{1-D} \cdot \frac{1}{1 + \left(\frac{R_L}{R}\right)\frac{1}{(1-D)^2}}$$



Practically $D_{max} = \frac{1}{2}$ or $\frac{2}{3}$.

Buck-boost converter

Buck-boost converter



$0 < t < DT_S$: S_1 ON D_1 OFF

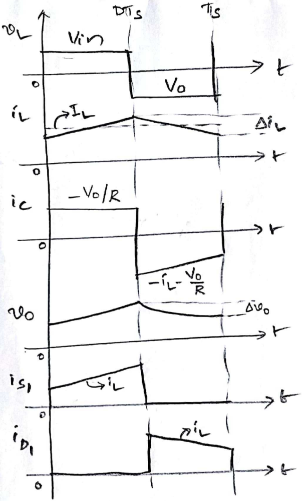
$DT_S < t < T_S$: S_1 OFF D_1 ON

$$V_o = \frac{-D}{1-D} V_{in} \quad 0 < D < 1$$

$$\Delta i_L = \frac{-V_o (1-D) T_S}{L}$$

$$\Delta v_o = \frac{-V_o D T_S}{RC}$$

$$I_L = \frac{-V_o}{R(1-D)}$$



$i_{S_1} = i_{in}$

↓

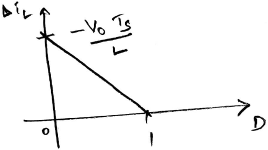
Input current is pulsating

Buck-boost converter

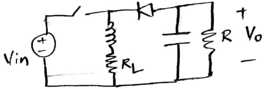
Buck-boost converter

$$\Delta i_L = \frac{-V_o(1-D)T_s}{L}$$

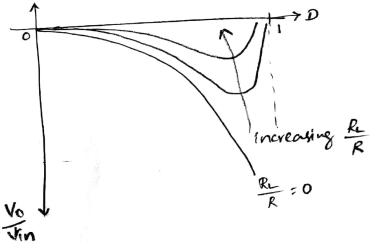
for given V_o, T_s, L



Effect of one non-ideality

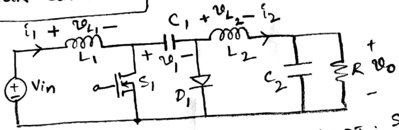


$$\frac{V_o}{V_{in}} = \frac{-D}{(1-D)\left[1+\frac{1}{(1-D)^2}\frac{R_L}{R}\right]}$$



Ćuk converter

Ćuk Converter



$0 < t < DT_s$: S_1 ON, D_1 OFF

$DT_s < t < T_s$: S_1 OFF, D_1 ON

$$v_1 \approx V_1 = \frac{Vin}{1-D}$$

$$v_o \approx V_o = \frac{-D}{1-D} Vin$$

$$I_1 = \left[\frac{D}{1-D}\right]^2 \frac{Vin}{R} = \frac{-D}{1-D} I_2$$

$$I_2 = \frac{-D}{1-D} \frac{Vin}{R} = \frac{V_o}{R}$$

$\frac{V_o}{Vin}$ same as buck-boost

Advantages of Ćuk converter compared to buck-boost

→ Non-pulsating input current

→ Isolated gate drive not needed

$$\Delta i_1 = \frac{Vin\, DT_s}{L_1}$$

$$\Delta i_2 = \frac{Vin\, DT_s}{L_2}$$

$$\Delta v_1 = \frac{Vin\, D^2 T_s}{2(1-D) R C_1}$$

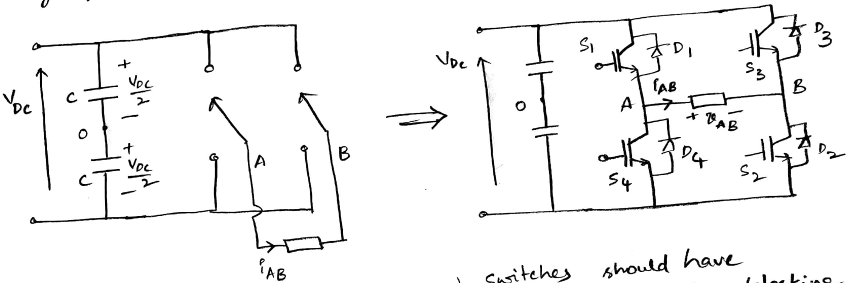
$$\Delta v_o = \frac{\Delta i_2 T_s}{8C}$$

Module 4: Summary

- Linear regulator vs switched-mode DC-DC converter
- Principles of steady-state analysis of DC-DC converters (non-isolated and isolated)
- Non-isolated DC-DC converters: Buck, Boost, Buck-Boost and Ćuk
- Switch-realization and design of the various circuit components with typical specifications
- Examples of the effect of non-idealities

1-ph 2-level voltage source inverter (VSI)

Single-phase two-level voltage source inverter (VSI):



Pole voltages v_{AO}, v_{BO} can have two levels $+\frac{V_{DC}}{2}$ or $-\frac{V_{DC}}{2}$

$\Rightarrow$ 2-level VSI

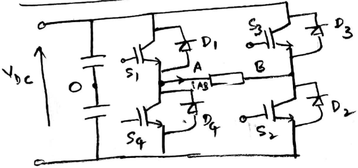
Switches should have
Unidirectional voltage blocking
Bidirectional current carrying capabilities

$i_{AB} > 0 \Rightarrow S_1$ or D_4 conduct

$i_{AB} < 0 \Rightarrow S_4$ or D_1 conduct

1-ph 2-level VSI: Square-wave operation

1-ph 2-level VSI:



Peak value of the fundamental component of v_{AB}, denoted by

$$V_{AB,1} = \frac{4 V_{DC}}{\pi}$$

Square-wave operation gives the maximum possible output voltage

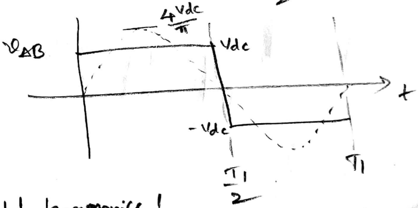
Square wave operation

Desired frequency of fundamental component of output v_{AB}

$= f_1$; $T_1 = \frac{1}{f_1}$

S_1 & S_2 are ON for $0 < t < \frac{T_1}{2} \Rightarrow v_{AB} = V_{DC}$

S_3, S_4 are ON for $\frac{T_1}{2} < t < T_1 \Rightarrow v_{AB} = -V_{DC}$



But v_{AB} contains all odd harmonics!

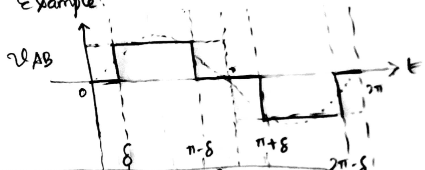
For fixed V_{DC}, amplitude of $v_{AB,1}$ is Constant

1-ph 2-level VSI: Pulse width modulation (PWM)

Pulse Width Modulation (PWM):

Required
- → To control the output voltage with fixed DC input → 1
- → To reduce the harmonic content in the output voltage → 2

Example:



$i_{AB} > 0$	S_1, D_3	S_1, S_2	S_1, D_3	D_4, S_2	D_4, D_3	D_4, S_2	Conducting
$i_{AB} < 0$	D_1, S_3	D_1, D_2	D_1, S_3	S_4, D_2	S_4, S_3	S_4, D_2	

$$V_{AB,1} = \frac{4V_{DC}}{\pi}\cos\delta$$

vary $\delta \Rightarrow$ 1 achieved

$$V_{AB,3n} = \frac{4V_{DC}}{\pi}\cos 3n\delta$$

No triplen

$\delta = 30^\circ \Rightarrow V_{AB,3n} = 0 \Rightarrow$ 2 achieved

To achieve both 1 & 2, create multiple transitions, $\delta_1, \delta_2, \ldots$ Maintaining HWS QWS.

1-ph 2-level VSI: Selective harmonic elimination PWM

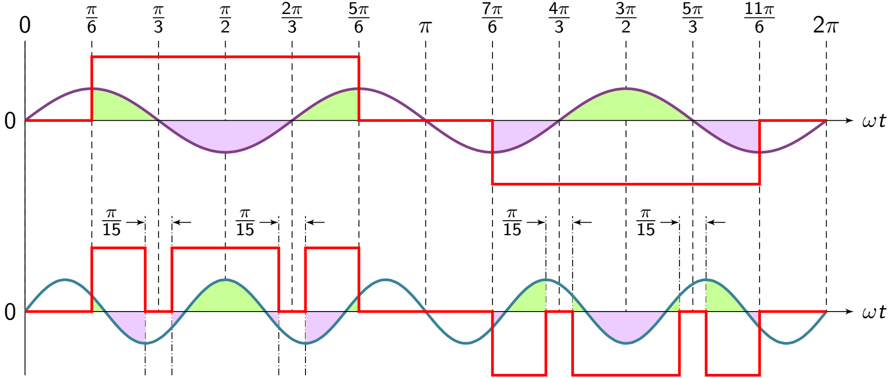
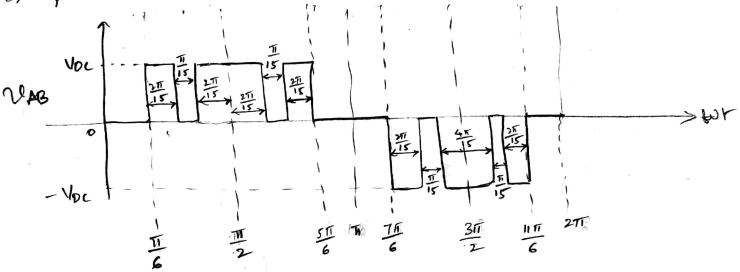


Illustration of harmonic elimination

1-ph 2-level VSI: Selective harmonic elimination PWM

Selective Harmonic elimination PWM (SHEPWM):

Example



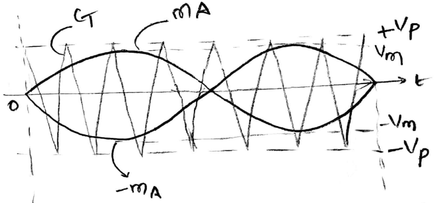
v_{AB} is free from 3n and 5^{th} harmonics.

SHEPWM is OFF-LINE PWM $\Rightarrow$ switching instants need to be calculated separately before implementation.

1-ph 2-level VSI: Sine-triangle PWM

Real-time PWM : Sine-triangle PWM (STPWM)

1 ph-VSI : Two types of STPWM
- → Bipolar Voltage switching
- → Unipolar Voltage switching



$m_A = V_m \sin \omega t$ → Modulating wave

C_T → Triangular carrier

$m_A > C_T \Rightarrow S_1$ and S_2 ON
- $i_{AB} > 0 \Rightarrow S_1 S_2$ conduct
- $i_{AB} < 0 \Rightarrow D_1 D_2$ conduct

otherwise S_3 and S_4 ON
- $i_{AB} > 0 \Rightarrow D_4 D_3$ conduct
- $i_{AB} < 0 \Rightarrow S_4 S_3$ conduct

→ Switch / diode conduction determined by the polarity of i_{AB}

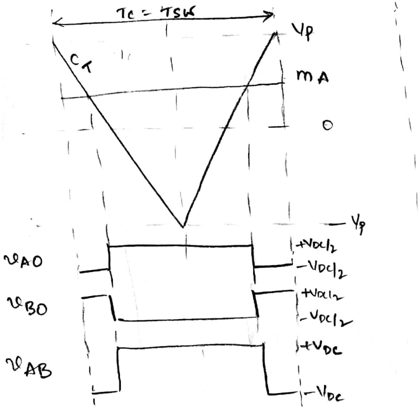
Both legs are switched independently

$m_A > C_T \Rightarrow S_1$ ON otherwise S_4 ON

$-m_A > C_T \Rightarrow S_3$ ON otherwise S_2 ON

1-ph 2-level VSI: Sine-triangle PWM

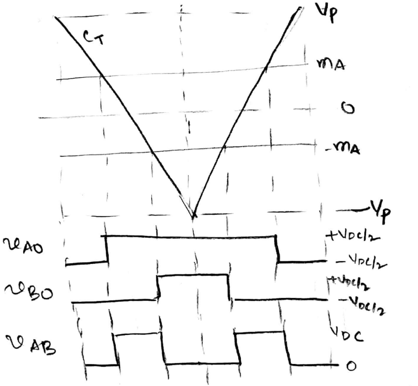
1-ph VSI : Bipolar Voltage switching



$$v_{AO(av)} = \frac{V_{DC}}{2}\frac{m_A}{V_p} \qquad v_{BO(av)} = \frac{-V_{DC}}{2}\frac{m_A}{V_p}$$

$$v_{AB(av)} = v_{AO(av)} - v_{BO(av)} = V_{DC}\frac{m_A}{V_p}$$

Unipolar voltage switching



$$v_{AO(av)} = \frac{V_{DC}}{2}\frac{m_A}{V_p} \qquad v_{BO(av)} = \frac{-V_{DC}}{2}\frac{m_A}{V_p}$$

$$v_{AB(av)} = V_{DC}\frac{m_A}{V_p}$$

1-ph 2-level VSI: Sine-triangle PWM

1 ph VSI : ST PWM

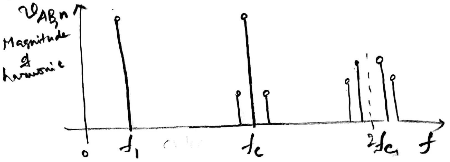
Bipolar Voltage Switching | Unipolar Voltage Switching

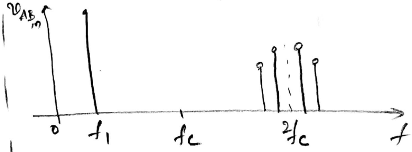
Same fundamental component of $v_{AB} \Rightarrow v_{AB,1} = m_A \cdot \frac{V_{DC}}{V_P}$

$= V_{DC} \cdot \frac{V_m}{V_P} \sin \omega_1 t$

$\omega_1 = 2\pi f_1$

Harmonics are different





Dominant harmonic in v_{AB} is around $2f_c$ for unipolar volt. switching

Same value of load inductance offers more impedance at $2f_c$

$\Rightarrow$ current waveform i_{AB} is smoother in case of unipolar volt. switching

1-ph 2-level VSI: Sine-triangle PWM vs square-wave operation

1-ph STPWM:

Fundamental voltage at output $v_{AB,1} = V_{DC} \frac{V_m}{V_p} \sin \omega t$

$\frac{V_m}{V_p}$: Modulation index → Ratio of the peak value of modulating signal to that of carrier signal

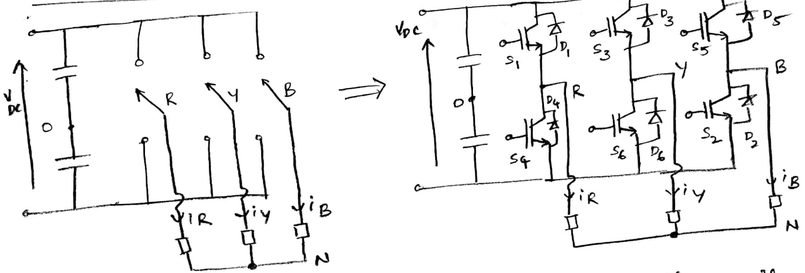
$\frac{V_m}{V_p} \leq 1.0 \Rightarrow$ linear modulation

Maximum possible amplitude of the fundamental component of output voltage, in linear modulation = V_{DC}
Harmonics are at frequencies far from f_1

Square wave operation yields a fundamental component $\frac{4V_{DC}}{\pi} = 1.27\, V_{DC}$
But v_{AB} has all odd harmonics

3-phase 2-level VSI

3-ph 2-level VSI:



$$\left.\begin{matrix} v_{RO} \\ v_{YO} \\ v_{BO} \end{matrix}\right\} \pm \frac{V_{DC}}{2} \;; \quad \begin{aligned} v_{RY} &= v_{RO} - v_{YO} \\ v_{YB} &= v_{YO} - v_{BO} \;; \\ v_{BR} &= v_{BO} - v_{RO} \end{aligned}$$

$$v_{RN} = \frac{v_{RY} - v_{BR}}{3} = \frac{2v_{RO} - v_{YO} - v_{BO}}{3}$$

$$v_{YN} = \frac{v_{YB} - v_{RY}}{3} = \frac{2v_{YO} - v_{BO} - v_{RO}}{3}$$

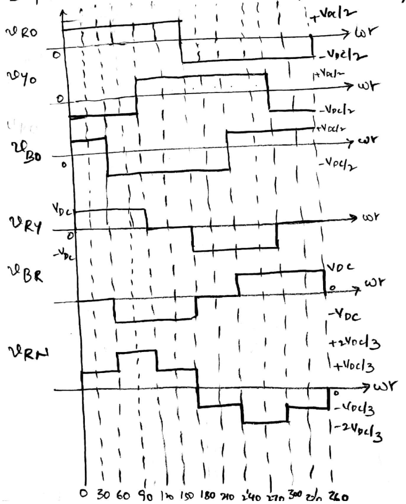
$$v_{BN} = \frac{v_{BR} - v_{YB}}{3} = \frac{2v_{BO} - v_{RO} - v_{YO}}{3}$$

Load is balanced; $v_{RN} + v_{YN} + v_{BN} = 0.$

$i_R + i_Y + i_B = 0$

3-phase 2-level VSI: Square-wave or six-step operation

3-ph 2-level VSI: Square-wave operation → Also known as six-step operation



Fundamental component of v_{RO}

$$v_{RO,1} = \frac{4}{\pi}\frac{V_{DC}}{2} = \frac{2V_{DC}}{\pi}$$

$$v_{RY,1} = \frac{4}{\pi} V_{DC} \cos 30 = \sqrt{3} \cdot \frac{2V_{DC}}{\pi}$$

$$v_{RN,1} = \frac{2V_{DC}}{\pi}$$ } Max. possible amplitude of phase voltage at output

$$v_{RO,1} = v_{RN,1}$$

v_{RO}, v_{YO}, v_{BO} } contain all odd harmonics

v_{RY}, v_{RN}, v_{YB}, v_{YN}, v_{BR}, v_{BN} } Contain only non-triplen odd harmonics.

3rd harmonics of v_{RO}, v_{YO}, v_{BO} are in phase get cancelled in v_{RY}, v_{YB}, v_{BR}.

3-phase 2-level VSI: Sine-triangle PWM

Sine-triangle PWM : 3ph- 2 level VSI

$$v_{RO(av)}^{T_c} = \left.\begin{array}{l} v_{RO,1} \\ = v_{RN,1} \end{array}\right\} = \frac{V_{DC}}{2}\frac{m_R}{V_p} = \frac{V_{DC}}{2}\frac{V_m}{V_p}\sin\omega_1 t$$

$$v_{YO(av)}^{T_c} = \left.\begin{array}{l} v_{YO,1} \\ v_{YN,1} \end{array}\right\} = \frac{V_{DC}}{2}\frac{m_Y}{V_p} = \frac{V_{DC}}{2}\frac{V_m}{V_p}\sin(\omega_1 t - 120^\circ)$$

$$v_{BO(av)}^{T_c} = \left.\begin{array}{l} v_{BO,1} \\ v_{BN,1} \end{array}\right\} = \frac{V_{DC}}{2}\frac{m_B}{V_p} = \frac{V_{DC}}{2}\frac{V_m}{V_p}\sin(\omega_1 t + 120^\circ)$$

Maximum amplitude of fundamental phase voltage at output $= \frac{V_{DC}}{2} = 0.5\,V_{DC}$
($v_{RN,1}$) in linear modulation

Harmonics are at frequencies far from f_1

Square wave operation gives fundamental component ph-voltage $\Big\}\ \frac{2V_{DC}}{\pi} = 0.637\,V_{DC}$

But v_{RN} has all odd non-triplen harmonics

All legs are controlled independently

$m_R > C_T \Rightarrow v_{RO} = +\frac{V_{DC}}{2}$ else $-\frac{V_{DC}}{2}$

$m_Y > C_T \Rightarrow v_{YO} = +\frac{V_{DC}}{2}$ else $-\frac{V_{DC}}{2}$

$m_B > C_T \Rightarrow v_{BO} = +\frac{V_{DC}}{2}$ else $-\frac{V_{DC}}{2}$

Module 5: Summary

- Switch realization in DC-AC inverters
- Single-phase two-level voltage source inverter (VSI)
  - Square-wave operation
  - Concept of pulse width modulation (PWM)
  - Selective harmonic elimination PWM (off-line PWM)
  - Real-time PWM: sine-triangle PWM
  - Simulation examples
- Three-phase two-level VSI
  - Square-wave or six-step operation
  - Sine-triangle PWM