

Experiment no. 1

Power measurement in balanced 3 phase circuits and power factor improvement

1. Power in Single Phase Circuits

Let $v = V_m \cos(\omega t) = \sqrt{2} \bar{V} \cos(\omega t)$ is the voltage applied to a R-L circuit and $i = I_m \cos(\omega t - \theta) = \sqrt{2} \bar{I} \cos(\omega t - \theta)$ is the current flowing in it (V and I are the rms values of the voltage and current respectively, θ being the phase difference between the voltage and current).

The power at any instant of time is

$$p = vi = V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

$$p = \frac{V_m I_m}{2} \cos \theta + \frac{V_m I_m}{2} \cos(2\omega t - \theta)$$

$$p = \frac{V_m I_m}{2} \cos \theta + \frac{V_m I_m}{2} \cos(2\omega t) \cos \theta + \frac{V_m I_m}{2} \sin \theta \sin(2\omega t)$$

A careful study of the above equation reveals the following characteristics:

- The **average** value of power = $P = \frac{V_m I_m}{2} \cos \theta = V I \cos \theta$ **Watt**.
In AC circuits, when we talk of power rating, we are usually referring to the average power. For example, a “100 W” bulb implies that **average** power output of the bulb is 100 W.
- The frequency of instantaneous power is **twice** the frequency of the voltage and current. Note that instantaneous power may be negative for a portion of each cycle, even if the load is passive. In a passive network, negative power at a given instant implies that energy stored in inductors or capacitors is being given back to the supply at that instant. The fact that the instantaneous power varies with time explains why some single phase motor driven appliances such as refrigerators, which are fed from single phase supply experience vibration, and hence require resilient mountings to prevent excessive vibration of the equipment.
- If the load is purely resistive, the voltage and current are *in phase*, which means that $\theta = 0$. The above equation reduces to $p = P + P \cos(2\omega t)$. In this case instantaneous real power can never be negative, i.e., power cannot be given back to the supply from a purely resistive network.
- If the circuit is purely reactive (either capacitive or inductive, $\theta = \pm 90^\circ$), the expression for instantaneous power is $p = \frac{V_m I_m}{2} \sin(2\omega t)$. The average power is zero. Therefore in a purely reactive circuit there is no average power dissipation. Energy is drawn from the supply for a part of a cycle is stored, and given back in another part of a cycle. However, since there is a current flow, the supply system must be designed to source such currents. Therefore it is useful to define a quantity called “**reactive power**” as follows:

$$Q = \frac{V_m I_m}{2} \sin \theta = V I \sin \theta$$

- Dimensionally P and Q are same. However, in order to distinguish between average (also called “**real power**”) and reactive power, we use the term ‘**VAR**’ for reactive power. **Q is positive for inductive load and negative for capacitive load**. In other words inductors demand or absorb reactive power while capacitors furnish or deliver the reactive power. For **purely resistive circuits**, $Q=0$.

If v and i are represented by phasors \bar{V} and \bar{I} , then it is easily seen that

$$\bar{S} = P + jQ = \bar{V} \bar{I}^*$$

$$\cos \theta = \frac{P}{\sqrt{P^2 + Q^2}}$$

\bar{S} is called the **complex power** and $\cos \theta$ is called the **power factor**. Power factor is said to be leading if θ is negative (capacitive circuits), and lagging for if θ is positive (inductive circuits).

Dimensionally, complex power is the same as real and reactive power. However, in order to distinguish complex power from real and reactive power, we use the term ‘volt amps’ or ‘VA’. The magnitude of complex power ($|\bar{S}| = S = |\bar{V} \bar{I}^*| = \sqrt{P^2 + Q^2} = VI$), is referred to as **apparent power**.

S indicates the capacity of the source required to supply the load. The apparent power specifications of source and load equipment can be more important than the average power specifications. This is because any electrical equipment requires appropriate design of conductor size, amount of dielectric (insulation) and magnetic material to handle both current and voltage *magnitudes*. Thus, S has a direct bearing on *size and cost* of any electrical equipment. Since average power represents the useful output of the load equipment (essentially energy converting devices like motors, heaters, lighting), operating such equipment close to unity power factor is desirable (i.e, P should be as close to S as possible, or Q should be minimized).

The vector diagram for a lagging load is shown in Fig.1 (a). The current vector lags the voltage vector by angle θ . In Fig. 1(b) the current phasor is resolved along x and y axes and in Fig.1(c) these components are multiplied by V . The resulting triangle is known as power triangle. The sides of the triangle are P , Q and S , where:

$$P = VI \cos \theta \text{ Watt}$$

$$Q = VI \sin \theta \text{ VAR}$$

$$S = VI \text{ VA}$$

1.1 Measurement of Power in single phase circuits

Power consumed by single phase load can be measured using a wattmeter (or a Power analyser). This meter typically comprises of the following:

- **Two identical current coils.** Depending upon the magnitude of load current these coils are connected either in series or in parallel (if the current rating of one coil is less than the load current, these coils are connected in parallel, otherwise they are connected in series). The two ends of these coils are marked as ‘M’ which stands for Mains, and ‘L’ for load. The current coils are connected in series with the load.
- **A voltage coil with different voltage tapings.** Depending upon the voltage applied to the load suitable taping is used. The two ends of this coil are marked as ‘C’ which stands for common and ‘V’ stands for voltage. This coil is connected across the load. The terminals C and M are connected together.

The reading of this meter is proportional to

$$V_{app} I_{Fl} \cos \angle (\bar{I}_{Fl} \text{ \& } \bar{V}_{app})$$

where, V_{app} is the voltage applied to voltage coil, & I_{Fl} is current flowing through the current coil.

2. Three Phase Systems

The generation, transmission, distribution and utilization of large blocks of electric power are accomplished by means of three-phase circuits. A **balanced** three phase system consists of three sinusoidal voltages of **identical amplitudes and frequency** but are out of phase with each other by 120° . In discussing three-phase circuits, standard practice is to refer the three phase as A, B, and C (or R, Y, B). Furthermore, the A-phase (or R-phase) is almost always used as the *reference* phase. Because the phase voltages are out of phase by 120° , two possible phase relationships can exist between the A phase (or R phase) voltage and the B and C phase voltages (Y and B). One possibility is for the B phase to lag the A phase by 120° (phase C will be lagging B phase by 120° , or lead A phase by 120°). In this case the phase relationship is known as **ABC (or RYB) phase sequence**. The only other possibility is for the C phase to lag A phase (or B phase to lead A phase by 120°). This phase relationship is known as **ACB sequence**. In phasor notation, two possible sets of balanced phase voltages are:

$$\bar{V}_a = V \angle 0; \quad \bar{V}_b = V \angle -120^\circ; \quad \bar{V}_c = V \angle -240^\circ;$$

and $\bar{V}_a = V \angle 0; \quad \bar{V}_c = V \angle -120^\circ; \quad \bar{V}_b = V \angle -240^\circ = V \angle +120^\circ.$

An important characteristic of balanced three phase voltages is that the sum of the voltages is **zero** ($\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$). Three phase voltage sources can be either Y (star) **or** delta connected. These connections are shown in Fig.2 (a) and (b) respectively. Let V_{an} , V_{bn} and V_{cn} are the phase voltages (the **voltages** between line and neutral). Now there are 4 wires - three for three lines and fourth one for the neutral. All the single phase loads are connected between any one of the lines and neutral. The relationship between the line-line voltage (\bar{V}_{ab} , \bar{V}_{bc} and \bar{V}_{ca}) and phase voltage can be derived as follows:

Referring to Fig.2(a), the line-line voltages for ABC sequence are:

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn} = V \angle 0 - V \angle -120^\circ = \sqrt{3}V \angle 30^\circ$$

Similarly, $\bar{V}_{bc} = \sqrt{3}V \angle -90^\circ \quad \bar{V}_{ca} = \sqrt{3}V \angle -210^\circ$

In a three phase system the load is said to be **balanced** if the **magnitude and phase** of the load impedance connected to each phase is the same. For the Fig.2 (b), the phase current (the current in each phase of the load) is

$$\begin{aligned} \bar{I}_A &= \frac{\bar{V}_{an}}{Z_{an}} = \frac{V \angle 0}{Z \angle \theta} = I \angle -\theta \\ \bar{I}_B &= \frac{\bar{V}_{bn}}{Z_{bn}} = \frac{V \angle -120^\circ}{Z \angle \theta} = I \angle -(120 + \theta) \\ \bar{I}_C &= \frac{\bar{V}_{cn}}{Z_{cn}} = \frac{V \angle -240^\circ}{Z \angle \theta} = I \angle -(240 + \theta) \end{aligned}$$

Line current is the current in each phase of the line. In *star connected* system, phase currents & line currents are identical. The complete vector diagram for a three phase system feeding a star connected load is shown in fig.3(a).

For *delta connected* system shown in Fig. 2(b), line to line voltage is the same as phase voltage. However, the relationship between the phase currents and line currents can be determined as follows: The phase currents are given by:

$$\begin{aligned} \bar{I}_{ab} &= \frac{\bar{V}_{ab}}{Z_{ab}} = \frac{V \angle 0}{Z \angle \theta} = I \angle -\theta \\ \bar{I}_{bc} &= \frac{\bar{V}_{bc}}{Z_{bc}} = \frac{V \angle -120^\circ}{Z \angle \theta} = I \angle -(120 + \theta) \\ \bar{I}_{ca} &= \frac{\bar{V}_{ca}}{Z_{ca}} = \frac{V \angle -240^\circ}{Z \angle \theta} = I \angle -(240 + \theta) \end{aligned}$$

We can write the line currents in terms of the phase currents by direct application of KCL:

$$\begin{aligned}\bar{I}_A &= \bar{I}_{ab} - \bar{I}_{ca} = I\angle-\theta - I\angle-(240 + \theta) = \sqrt{3}I\angle-(30 + \theta) \\ \bar{I}_B &= \bar{I}_{bc} - \bar{I}_{ab} = \sqrt{3}I\angle-(150 + \theta) \quad \& \quad \bar{I}_C = \bar{I}_{ca} - \bar{I}_{bc} = \sqrt{3}I\angle-(270 + \theta)\end{aligned}$$

The complete vector diagram for a three phase system feeding a delta connected load is shown in fig.3(b).

2.1 Power in three phase circuits

In a balanced three phase system, the magnitude of each line-to-neutral voltage (also line-line) is the same, as is the magnitude of each phase current. Therefore, the expression for total power in terms of phase current and phase voltage is given by:

$$P = 3 V I \cos \theta \quad \text{Watt}, \quad \theta = \angle(\bar{I} \& \bar{V})$$

In-terms of line voltage & line current the above expression becomes:

$$P = \sqrt{3} V_L I_L \cos \theta \quad \text{Watt}, \quad \theta = \angle(\bar{I} \& \bar{V})$$

In the above expression V and I are the rms values of the phase voltage and phase current respectively. We can also calculate reactive power (Q) and complex power (S). For balanced load, the expressions for Q and S are given by:

$$Q = \sqrt{3} V_L I_L \sin \theta \text{ VAR}, \quad \theta = \angle(\bar{I} \& \bar{V}), \quad S = \sqrt{3} V_L I_L \text{ VA}$$

All the above equations are **valid for both** the connections (Y or Delta).

2.2 Measurement of Power in a three phase circuit

Real power in **balanced (or unbalanced)** 3 phase circuits can be measured by using **two wattmeters** connected in any two lines of a three-phase three wire system. In Fig.4(a) the current coils are connected in lines A and C, with the voltage coil reference connections in line B. The current flowing through the current coil of the wattmeter connected in line A is \bar{I}_A and that in line C is \bar{I}_C . Similarly, the voltage applied to the voltage coils is \bar{V}_{ab} and \bar{V}_{cb} (and not V_{bc}) respectively. Their readings will be:

$$W_A = V_{ab} I_A \cos \angle(\bar{I}_A \& \bar{V}_{ab}) \quad \text{and} \quad W_B = V_{cb} I_C \cos \angle(\bar{I}_C \& \bar{V}_{cb})$$

From Fig.3 (a or b) angle between V_{ab} and I_A is $(30 + \theta)$ and between V_{cb} and I_C is $(30 - \theta)$. Therefore the wattmeter readings will be:

$$W_1 = \sqrt{3} V I_A \cos(30 + \theta) \quad W_2 = \sqrt{3} V I_C \cos(30 - \theta)$$

$$W_1 + W_2 = 3 V I \cos \theta = \sqrt{3} V_L I_L \cos \theta \text{ Watt}$$

Note that if θ is greater than 60° , W_1 will be negative. In actual power measurement it is possible to encounter such a situation when one of the wattmeters has negative reading. The point to remember is that the total power in three phase circuit is obtained by adding the two wattmeter readings, taking the signs of the readings into account.

It may be worthwhile to identify the following cases:

- If θ is zero (load is purely resistive), $W_1 = W_2$ and both are positive.
- For $\theta = 60^\circ$ lag, $W_1 = 0$. Similarly for $\theta = 60^\circ$ lead, $W_2 = 0$.

Note that you will be using a **Power analyzer** to measure power. It is also possible to measure other quantities such as voltage, current, Q, S, power factor etc. as well using this equipment.

3. Power factor Improvement

Electrical service to industrial customers is given as a three-phase supply as opposed to the single phase power supplied to residential and small commercial customers. Most electric loads are reactive in nature and have lagging power factor (less than unity). Transmission lines, transformers and generators of the electric power utility have to carry the lagging reactive power of the load. Transformers, the distribution systems and the generators are all rated in kVA or MVA. An improvement in the power factor leads to a reduction in KVA for the same real (average) power supplied. This leads to a release in some of the generation and transmission capability so that it can be used to serve more customers. Generally, power is distributed through transmission lines (in the city of Mumbai, power is distributed through underground cables) and the voltage at one end of these lines is maintained constant. These transmission lines have finite resistance and inductance. From Fig.1 it can be inferred that as the power factor falls, for a fixed amount of power, the magnitude of current flowing through these lines increases. As a result the voltage at the load end drops. Also, loss in the transmission line increases thereby decreasing the overall efficiency.

In order to discourage the bulk consumers drawing a large amount of reactive power from the source, the utilities charge higher tariff if the power factor falls below a specified value. Hence, to improve the power factor, capacitors in three-phase banks are connected to the system, such that the combination of the plant load and the capacitor banks presents a load to the serving utility which is nearer to unity power factor. This is explained with the help of Fig.5. In the Fig.5(a), a R-L load is being supplied by the source and a capacitor is connected across the load. The vector diagram for this combination is shown in Fig.5(b). It is known that the capacitor draws a leading current (or supplies lagging current) from the source. If the magnitude of the current drawn by the capacitor is equal to the quadrature component of the load current ($I_y = I_L \sin \theta$), the source will then supply only the active component of current (I_x). In that case there is a significant reduction in the current flowing through the transmission line (without the capacitor the current flowing in the transmission line is ' I_L ' and with the capacitor it is $I_L \cos \theta$). This results in the decrease in voltage drop and power loss in the transmission line.

(**Note to TAs/RAs :** Explain the following:

- **Power analyzer / wattmeter connections**
- **various features of power analyzer)**

4 Laboratory Work

4.1 Part - I: Star Connected Load:

- **Note down the ratings of the meters and the 3 phase variac.** Connect the incandescent bulb load in Y and complete the circuit connection as shown in Fig. 4(a).
 - **After getting the connections checked** from the TAs, set the output voltage of the autotransformer (also called as variac) to **zero** and switch on the 3 phase 415V, 50 Hz supply.
 - **Slowly increase** the output voltage of the variac by turning the regulating handle till the voltmeter reads **200V**. (The corresponding voltage across each bulb will be 115V).

(Note that one can directly apply 400 V to this star connected load. We carry out the above procedure just as a precautionary measure). Note down the 2 power analyzer readings (voltages, currents and powers) and determine the load power factor using the formula for 2 wattmeter method.

- Observe the current and voltage waveforms (line-line) on the power analyzer and measure the angle between these two waveforms. Note down the waveforms. Repeat the readings for **400V.(230V)**

- Reduce the output voltage of the variac to **zero** and only then switch off the supply.

4.2 Part - II: Delta Connected Load:

- Connect the bulbs in delta *without* disturbing other part of the circuit. **After getting the connections checked** from the TAs, switch on the three phase supply
- Slowly increase the output voltage of the variac by turning the regulating handle till the voltmeter reads **115V**. Note down the 2 power analyzer readings (voltages, currents and powers). Determine the power factor and compare the meter reading with those of Part -1. Note down the current and voltage waveforms from the power analyzer. Repeat the readings for **230V**.
- Reduce the output voltage of the variac to **zero** and only then switch off the supply. In the report, comment on the differences between the two sets of readings.

4.3 Part - III: Powerfactor Improvement:

- A 3 phase induction motor will be used as the lagging load for this part. **Note down the ratings of the motor and the name plate details. Note the ratings of the 3 phase autotransformer or variac as well.** Connect the circuit as shown in Fig.6. Ensure that the switches S1, S2 and S3 are switched off initially.
- Switch on the 3 phase supply. Slowly increase the voltage applied to the motor by rotating the handle of the 3 phase variac, till the voltage reading of the power analyzers read **230V** (You will observe initially a high current flows through the motor. As the motor accelerates this current reduces automatically.) Note down the 2 power analyzer readings (voltages, currents and powers). Also note down the current and voltage waveforms from the power analyzer. Also note down the motor speed using the *tachometer*.
- Reduce the applied voltage to **zero** by turning the handle of the 3 phase variac.
- There are three delta connected capacitor banks each of $10\mu\text{F}$ per phase. These banks can be connected across the load by closing S1, S2 and S3. Firstly, connect the capacitor bank across the load by **closing S1**.
- Gradually increase the output voltage of the variac to 230V. Repeat the readings (voltages, currents, powers and waveforms and the motor speed). Then, reduce the applied voltage to **zero**.
- Close S2 and S3 in steps and for each case repeat the above step. Please ensure that the applied voltage to the load is zero in each case initially, before closing the switches.
- **After all cases, reduce the voltage applied to zero. Open S1, S2 and S3. Shut down the 3 phase supply and disconnect the circuit.**
- In the report determine the source powerfactor for all the cases using the formula for 2 wattmeter method and comment on the values.

5. Questions to be answered in report

- With all three capacitor banks connected across the load, the source powerfactor might be leading. How can one infer this from the wattmeter readings? Are there any advantages of overcompensating the load?
- You might have observed the voltage & current waveforms on the power analyzer (section 4.1). Why is the angle between these two waveforms is 30° even though the load is purely resistive?
- What is the reason for reducing the voltage to zero every time before switching on the capacitors for the power factor improvement experiment?

D. Power needs to be measured without wattmeters or power analyzer. If you have been given thick and thin wires for connections. Which one will you use for connecting :

(i) an ammeter and (ii) a voltmeter ? Justify your answer.

E. During the late hours of the night you might have observed the intensity of an incandescent bulb to be much higher compared to that during 7-8pm. What could be the reason ?

F. Why do the single phase motor driven appliances experience a vibration ?

G. You might have observed power sockets with two pins while some of them have three pins. What is the difference between these power sockets ? Is it for single phase and 3 phase cases respectively?

H. Utilities use energy meters to measure the energy consumed by consumers. Energy is given by:

$$E = \int P \, dt$$

$$E = \int (VI \cos \theta) \, dt$$

where P is the power consumed by the load. From Fig. 1 it can be inferred that though the consumer is drawing ' I ' A of current, he/she is being charged only for $I \cos \theta$. In other words there is no apparent advantage of improving the power factor to unity. Is this correct ? Justify your answer.

I. Suppose a $(3+j4) \, kVA$ load is being supplied at $230 \, V$ (load voltage) and the transmission line has an impedance of $(1 + j1) \, \Omega$. Determine the following :

(a) Voltage at the source terminals.

(b) Power loss in the transmission line.

(c) The required $kVAR$ rating of the capacitor to compensate the load fully (source supplies only the active component of current).

(d) The source current, drop in the transmission line, power loss in the transmission line after compensation.

(e) If the same were a 3 phase case and the supply voltage was $415V$ (l-l) determine the value of the capacitors needed for the following capacitor connections, assuming load to be connected in star:

i) Star

ii) Delta

J. The variac or autotransformer used for adjusting the voltage is also known as a *dimmerstat*. Why ?

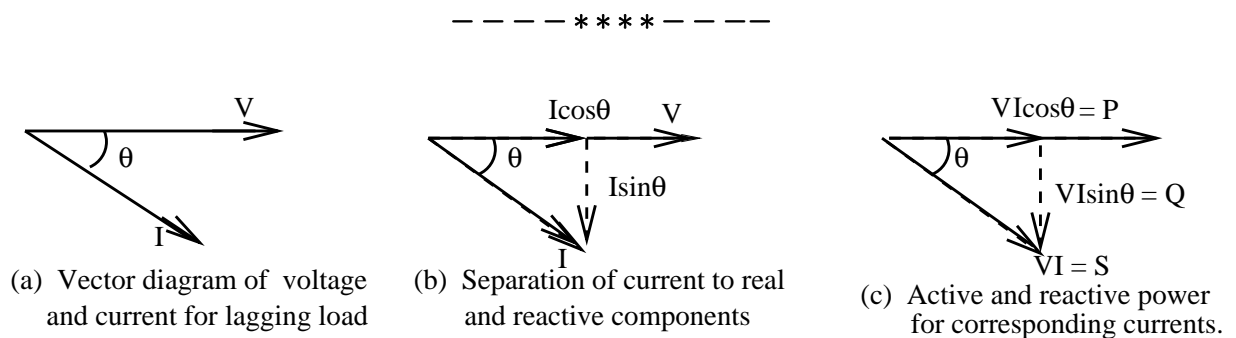


Figure 1: Vector Diagram for an Inductive load

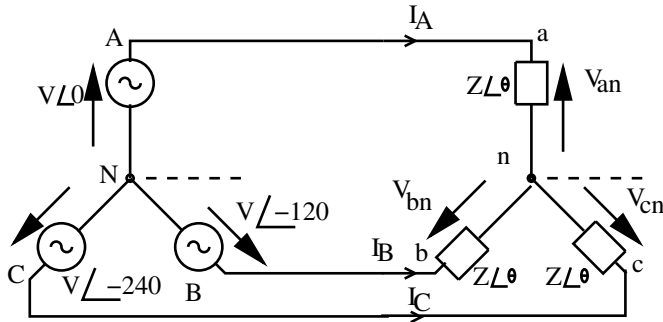


Fig. 2(a) Star – Star System

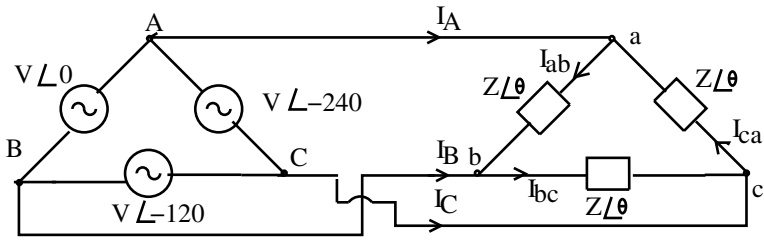


Fig. 2(b) Delta – Delta System

Figure 2: Schematic configuration of star – delta supply and load system

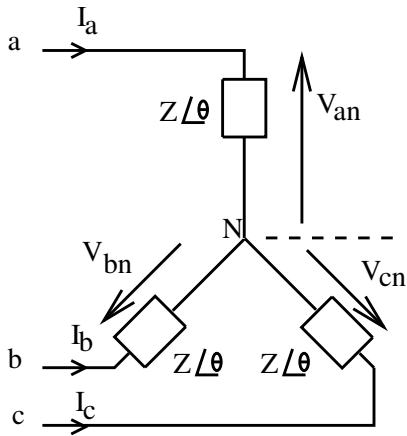


Fig. 3(a) Vector Diagram for a 3–phase system supplying a balanced star connected load

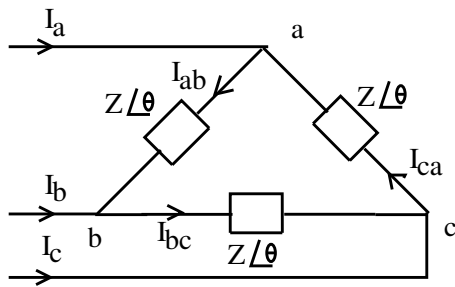
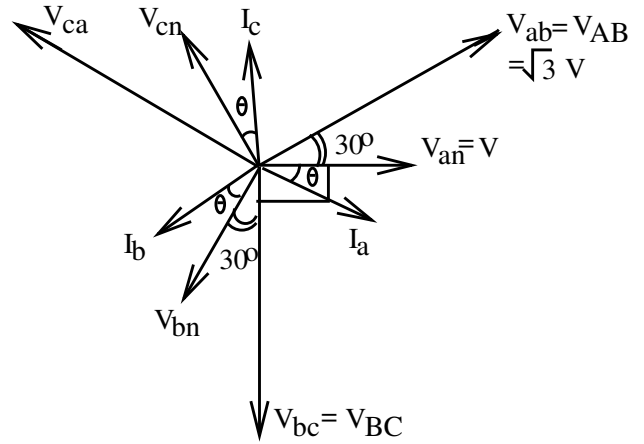
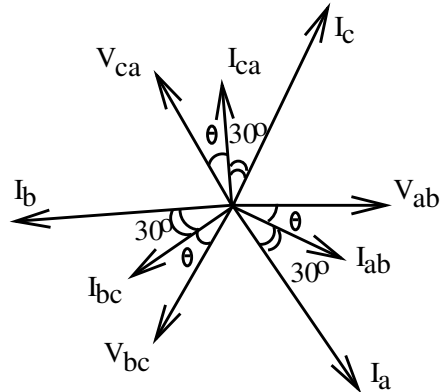


Fig. 3(b) Vector Diagram for a 3–phase system supplying a balanced delta connected load



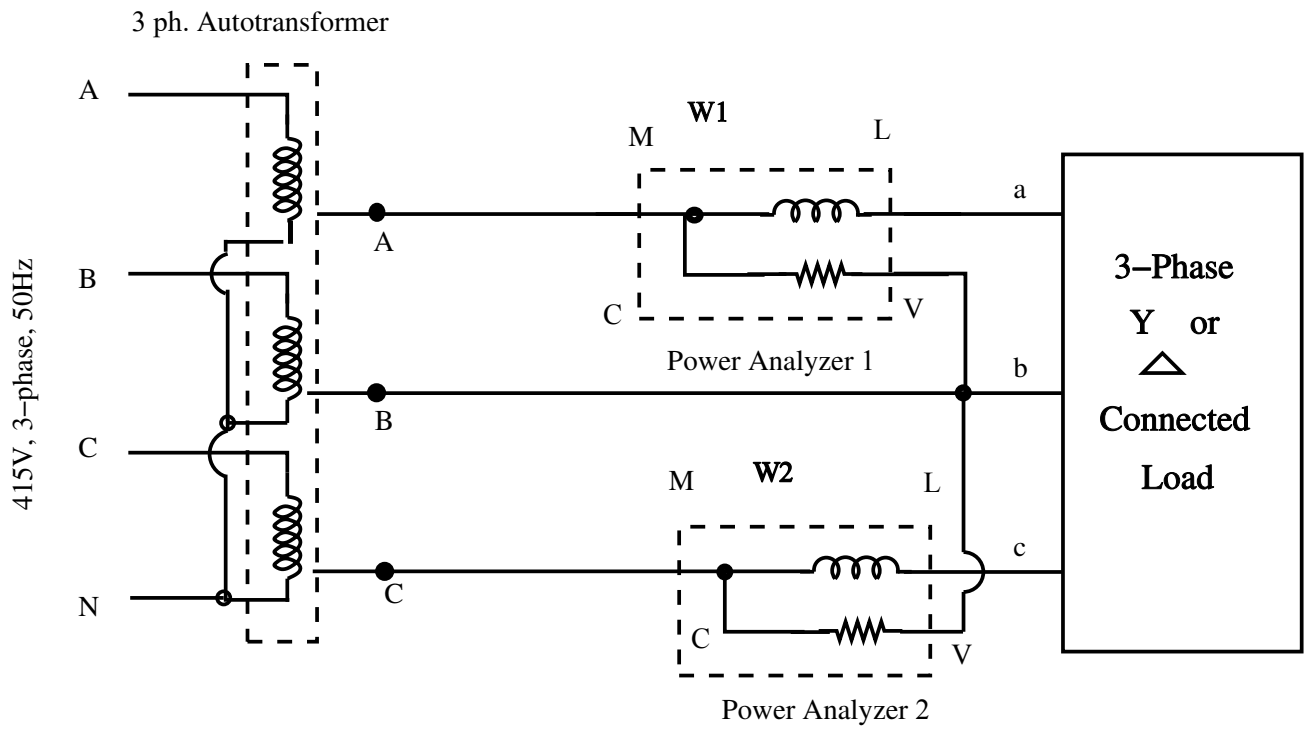


Fig. 4 (a) Circuit Diagram to measure power consumed by 3-phase Load using 2-Wattmeter method (M: MAINS L:LOAD C:COMMON V:VOLTAGE)

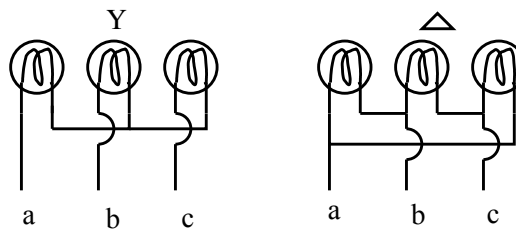


Fig. 4(b) 3-Phase Load connection: Star & Delta

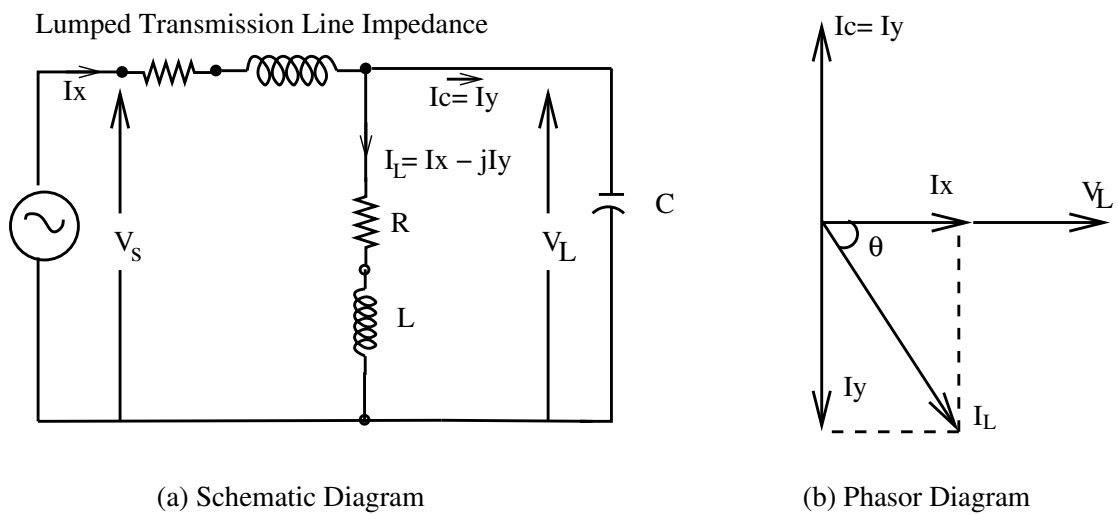


Figure 5 Power Factor Improvement

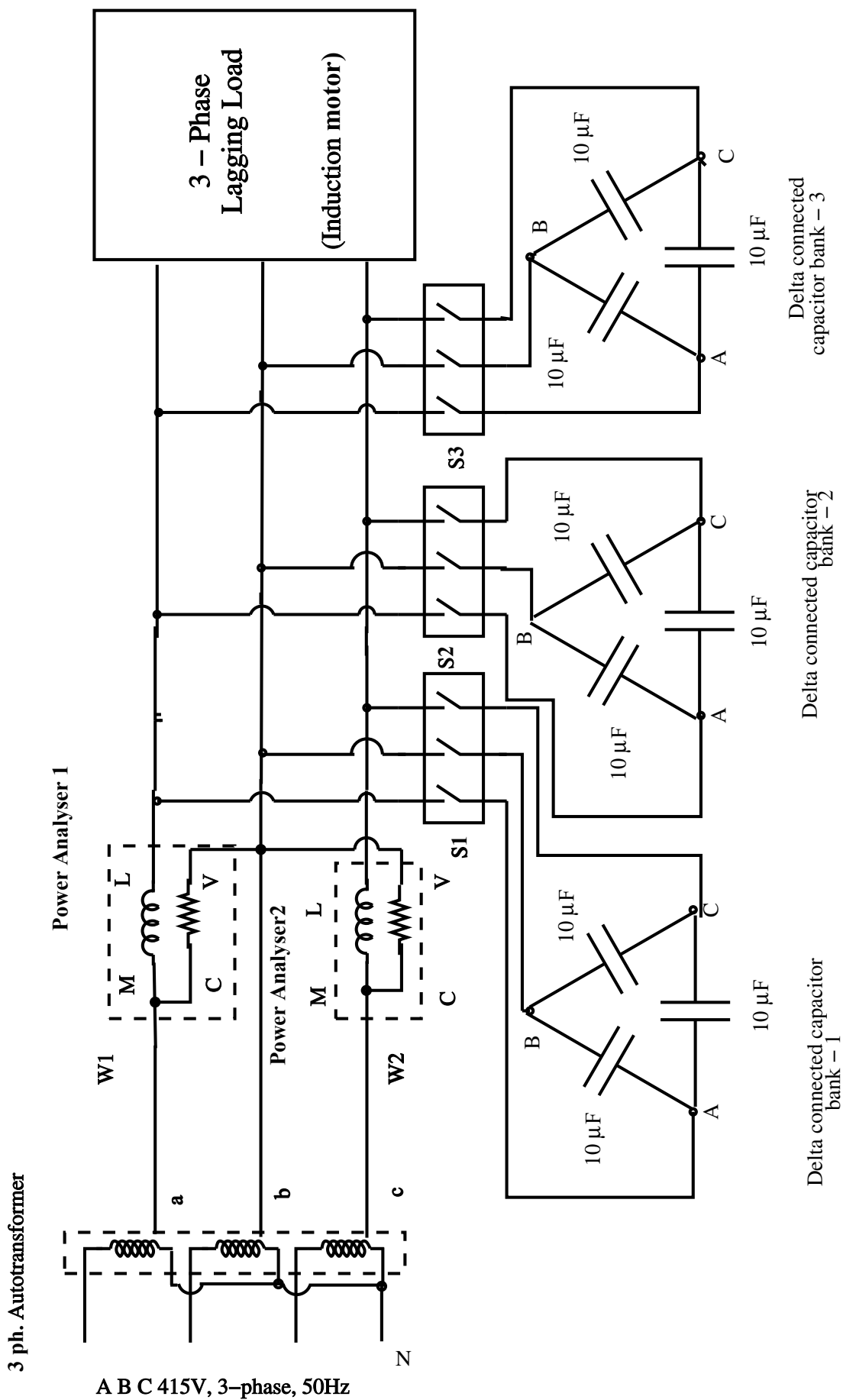
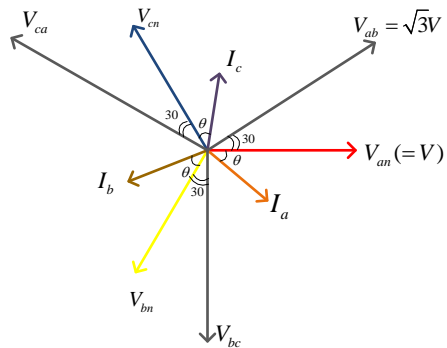
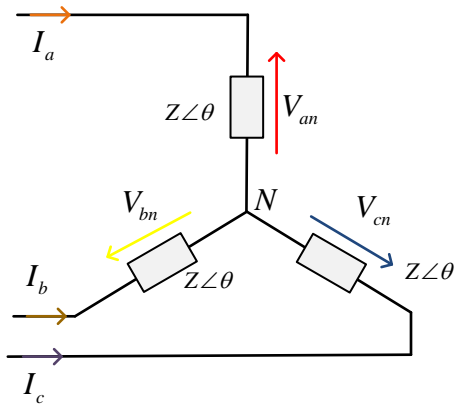
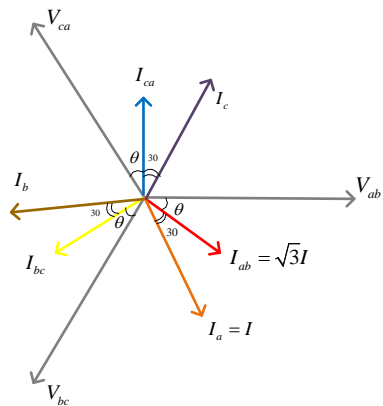
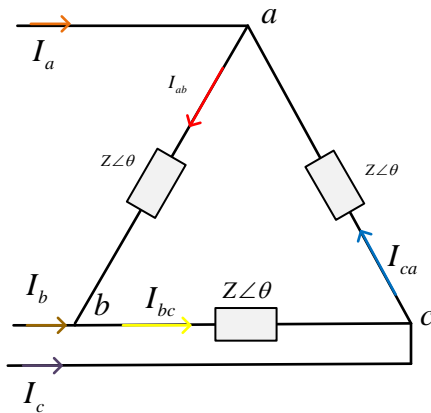


Fig.6 Circuit diagram for Power factor improvement



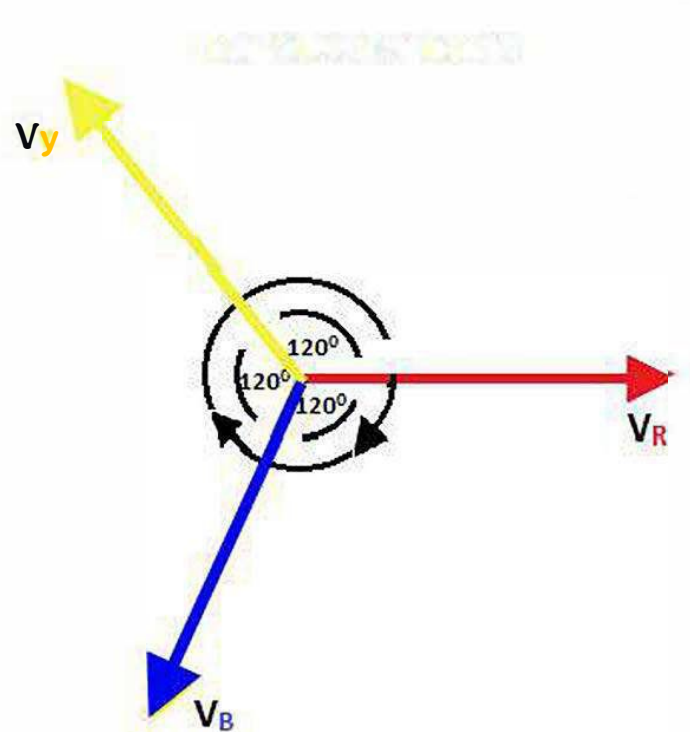
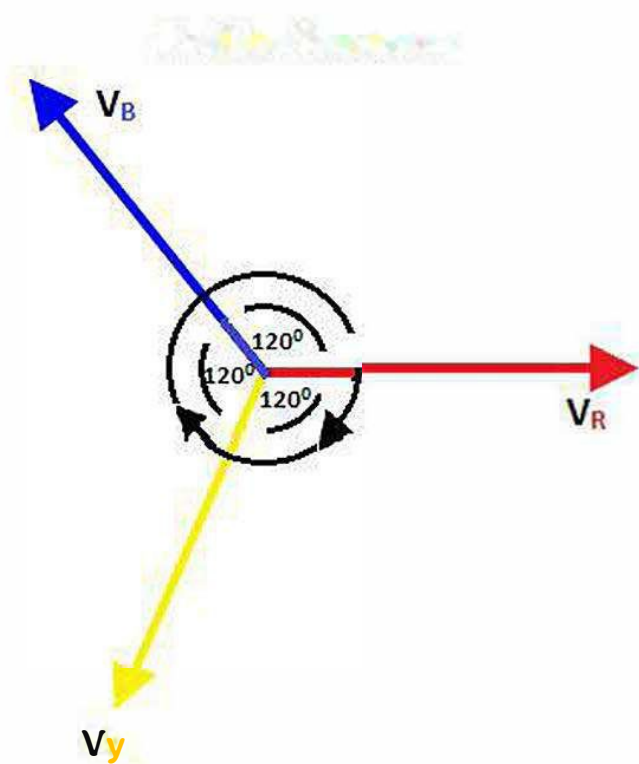
Vector diagram for 3 phase system supplying balanced *star* load



Vector diagram for 3 phase system supplying balanced *delta* load

Positive Sequence (RYB)

Negative Sequence (RBY)



Phase Sequence in a balanced three Phase System : R is considered as reference phasor