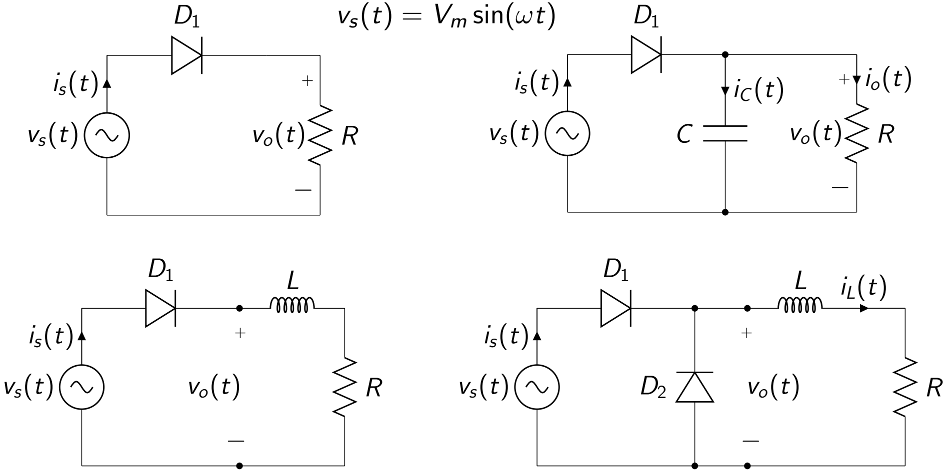
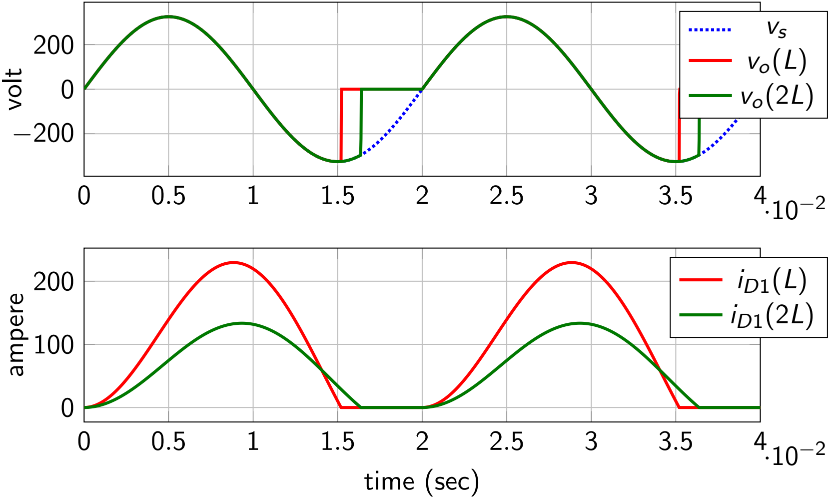
Single-phase half-wave (1-pulse) rectifiers

$$v_s(t) = V_m \sin(\omega t)$$



1-ph half-wave rectifier [(R-L load) / (L-filter and R-load)]

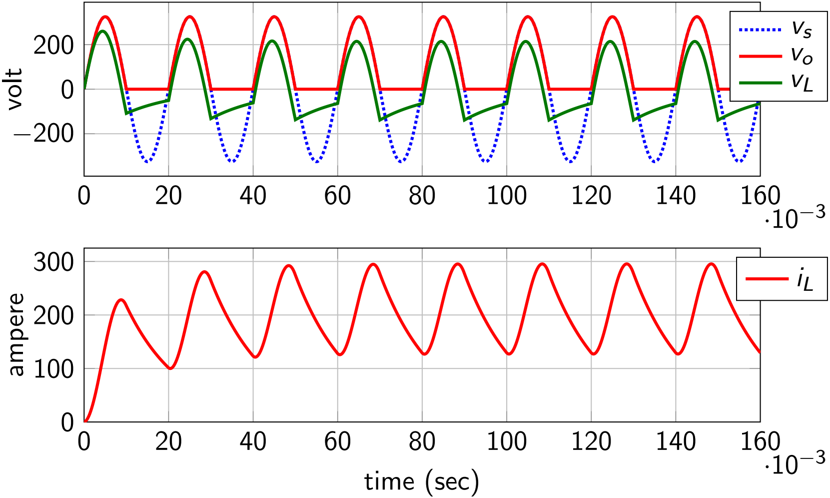
Waveforms in periodic steady state



$v_s = 230\sqrt{2}\sin(2\pi \cdot 50 \cdot t)$V, R=0.5Ω, L=6.5mH, $i_{D1} = i_s$

1-ph half-wave rectifier (Free-wheeling diode, L-filter and R-load)

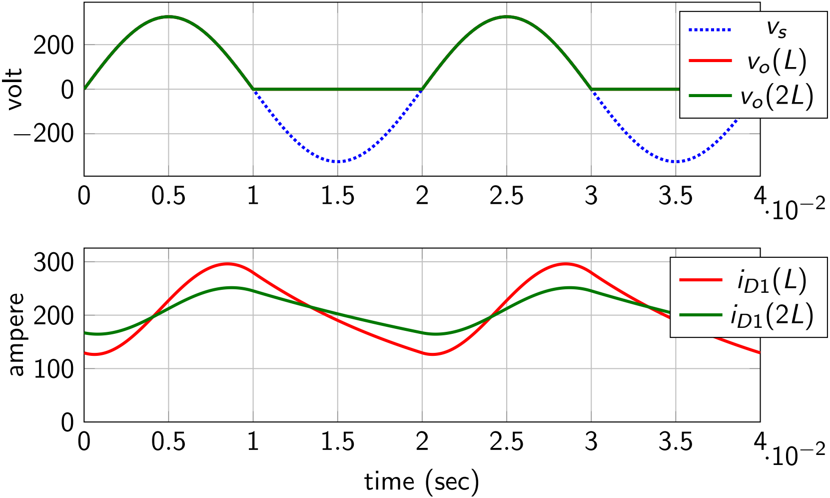
Waveforms: Start-up transient and periodic steady state



$v_s = 230\sqrt{2}\sin(2\pi \cdot 50 \cdot t)$V, R=0.5Ω, L=6.5mH, $i_{D1} = i_s$, v_L: volt across L

1-ph half-wave rectifier (Free-wheeling diode, L-filter and R-load)

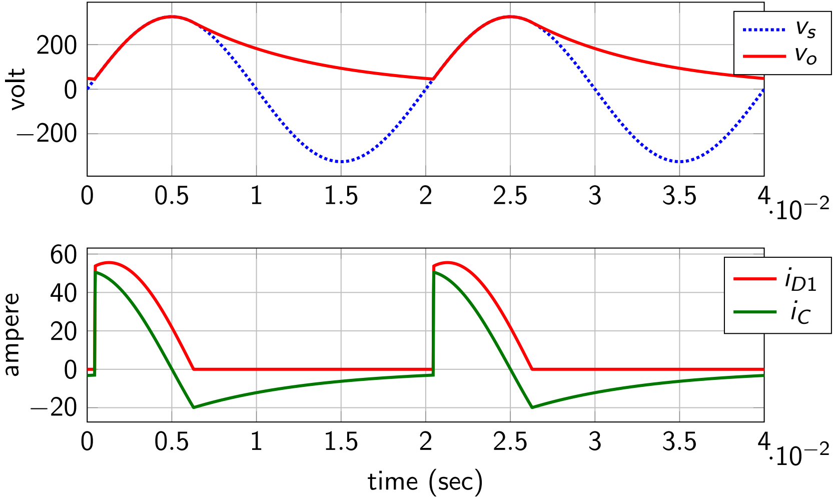
Waveforms in periodic steady state



$v_s = 230\sqrt{2}\sin(2\pi \cdot 50 \cdot t)$V, R=0.5Ω, L=6.5mH, $i_{D1} = i_s$ **Diode currents?**

1-ph half-wave rectifier (Capacitive filter and R load)

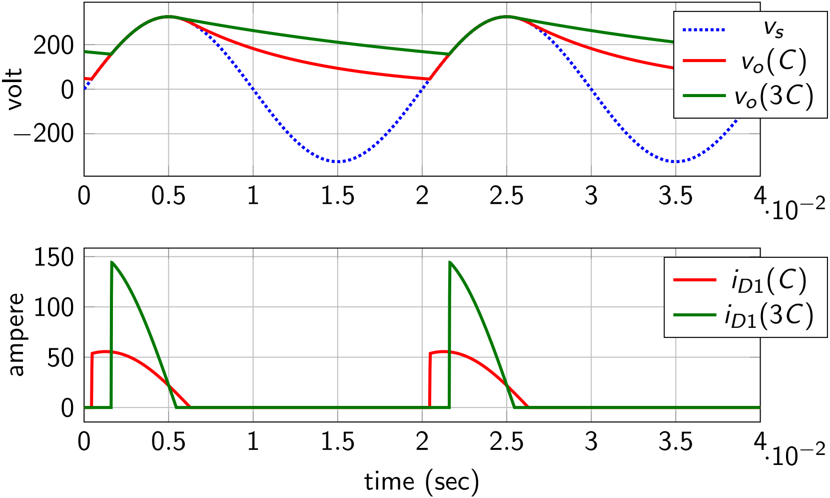
Waveforms in periodic steady state



$v_s = 230\sqrt{2}\sin(2\pi \cdot 50 \cdot t)$V, R=15Ω, C=0.5mF, $i_{D1} = i_s$

1-ph half-wave rectifier (Capacitive filter and R-load)

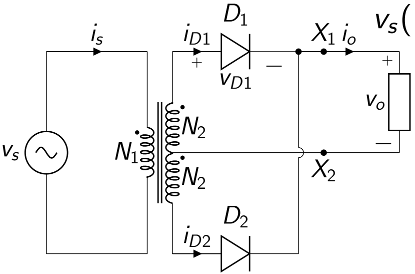
Waveforms in periodic steady state



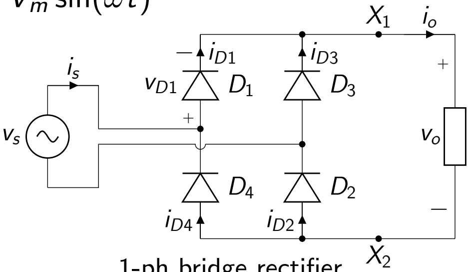
$v_s = 230\sqrt{2}\sin(2\pi \cdot 50 \cdot t)$V, R=15Ω, C=0.5mF, $i_{D1} = i_s$ **i_s vs i_o!**

Single-phase full-wave (2-pulse) rectifiers

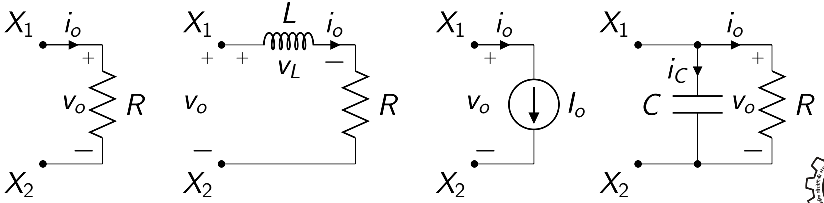
$$v_s(t) = V_m \sin(\omega t)$$



1-ph full-wave 2-diode rectifier



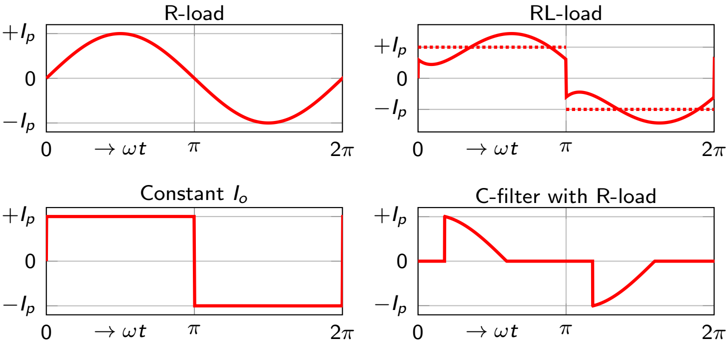
1-ph bridge rectifier



R with high L

$\omega L \gg R$

1-ph full-wave rectifier: Waveforms of input current $i_s(t)$

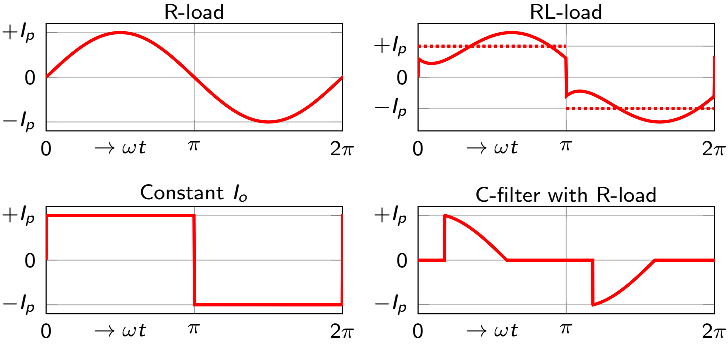


Waveforms of $i_s(t)$ are the same for full-wave bridge rectifier and full-wave 2-diode rectifier (with $N_1 = N_2$ and ideal transformer)

Load type	I_p
R-load	$\frac{V_m}{R}$
RL-load	$\frac{2V_m}{\pi R}$
Const I_o	I_o
C-filter with R-load	$\frac{V_m}{R}\sin(\omega t_n) + \omega C V_m \cos(\omega t_n)$

ωt_n is the instant at which the diodes turn-ON

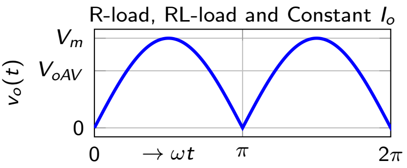
1-ph full-wave rectifier: Waveforms of input current $i_s(t)$

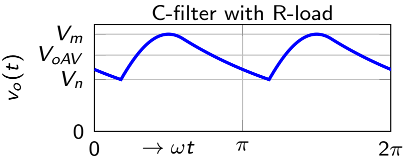


In the case of constant current (I_o) load,

- $I_{sRMS} = I_o$; Fundamental $I_{s1RMS} = \frac{4I_o}{\pi}\frac{1}{\sqrt{2}} = 0.9I_o$; nth harmonic $I_{snRMS} = \frac{I_{s1RMS}}{n}$
- $DPF = 1$ and $DF_1 = 0.9 \Rightarrow$ power factor $PF = 0.9$
- $i_s(t)$ doesn't contain even harmonics
- Total harmonic distortion $I_{sTHD} = 48.43\%$

1-ph full-wave rectifier: Waveforms of output voltage $v_o(t)$

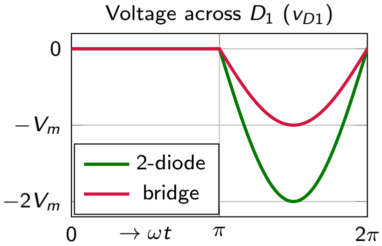




$V_n = V_m \sin(\omega t_n)$ and ωt_n is the instant at which the diodes turn-ON $\qquad K = \omega RC$

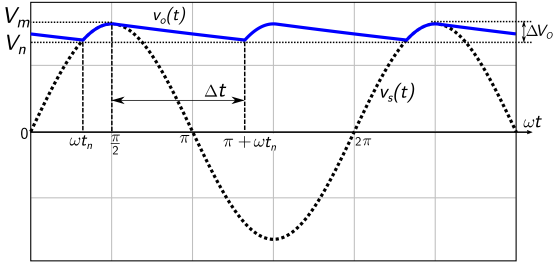
Load type	V_{oAV}
R-Load RL-Load Const I_o	$\dfrac{2V_m}{\pi}$
C-filter with R-load	$\dfrac{V_m}{\pi}\left[\cos(\omega t_n) + \sqrt{1+K^2} - \dfrac{K^2}{\sqrt{1+K^2}}\, e^{\frac{-\tan^{-1}K - \omega t_n}{K}}\right]$

1-ph full-wave rectifier: Voltage across diode



- Peak inverse voltage (PIV) across each diode in a 2-diode rectifier (with $N_1 = N_2$) is twice that in a bridge rectifier
- In practical diodes, $v_{D1} \neq 0$ when D_1 is ON. Forward voltage drop across two diodes appears between source and load in bridge rectifier, while the center-tap (2-diode) rectifier has only one diode-voltage-drop between source and load

1-ph full-wave rectifier: Design of C-filter



Specifications: (i) Input voltage and frequency (V_m and ω) (ii) average output power (P_o) (iii) peak-to-peak ripple in output voltage (ΔV_o)

Assumptions: (i) Diodes stop conducting at $\omega t = \pi/2$ and (ii) ripple in output power is negligible

Peak-to-peak ripple $\Delta V_o = V_m - V_n = V_m - [V_m \sin(\omega t_n)] \Rightarrow \omega t_n = \sin^{-1}[1 - (\Delta V_o / V_m)]$

Output power P_o is supplied only by the capacitor for the duration $\Delta t = \dfrac{(\pi/2) + \omega t_n}{\omega}$

Change in energy of the capacitor $\dfrac{1}{2} C \left(V_m^2 - V_n^2\right) = P_o \Delta t$

Connection of multiple diodes

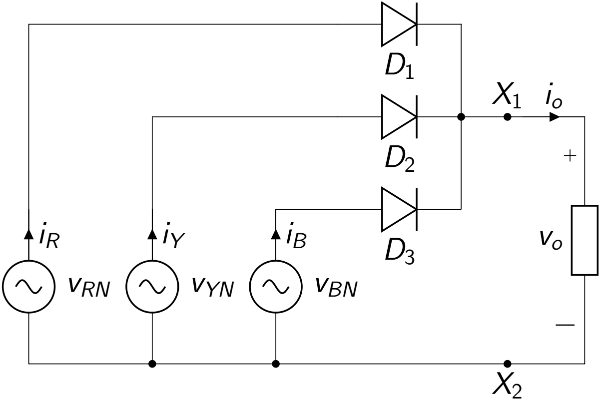
Common cathode connection

Diode with its anode at the most positive voltage conducts

Common anode connection

Diode with its cathode at the most negative voltage conducts

3-ph half-wave (3-pulse) rectifier

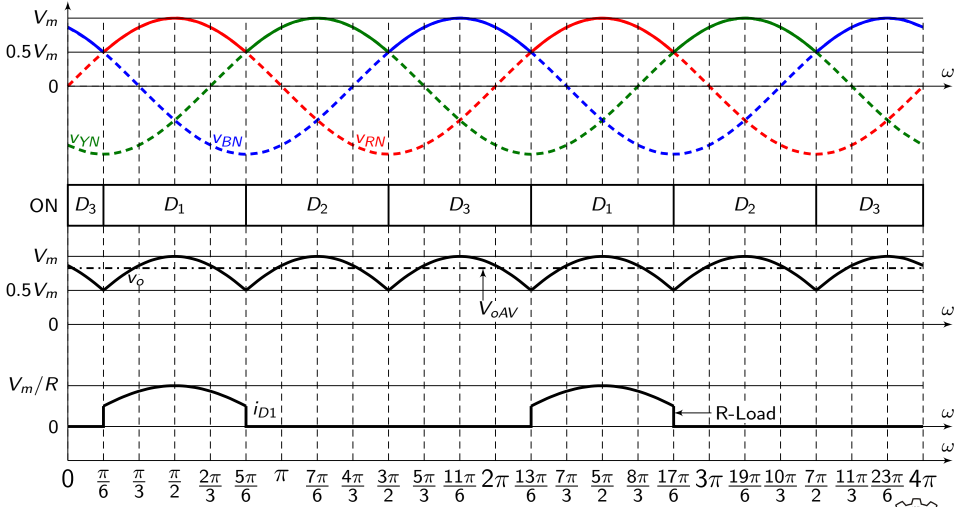


$$v_{RN} = V_m \sin(\omega t) \qquad v_{YN} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \qquad v_{BN} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$i_R = i_{D1}$ and similar definitions for $i_Y \& i_B$

3-ph half-wave (3-pulse) rectifier

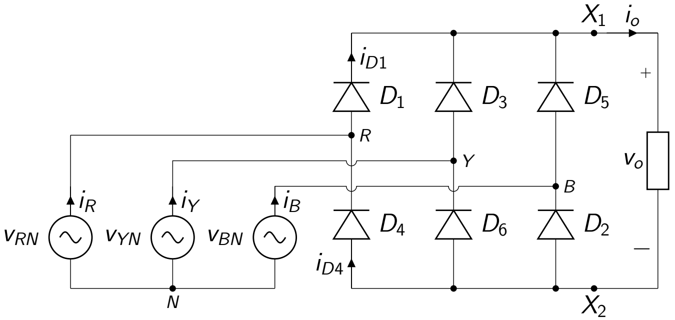
Waveforms in periodic steady state



3-ph half-wave (3-pulse) rectifier

- Average output voltage
  $$V_{oAV} = \frac{1}{(2\pi/3)} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} V_m \sin(\omega t)\ d(\omega t) = \frac{3\sqrt{3}V_m}{2\pi}$$
- Peak-to-peak ripple in output voltage
  $V_{oPP} = 0.5V_m$
- Each diode conducts for 120° (one-third of a line cycle)
- Input current $i_R(t)$ contains DC component (finite average value)

3-ph full-wave/bridge (6-pulse) rectifier



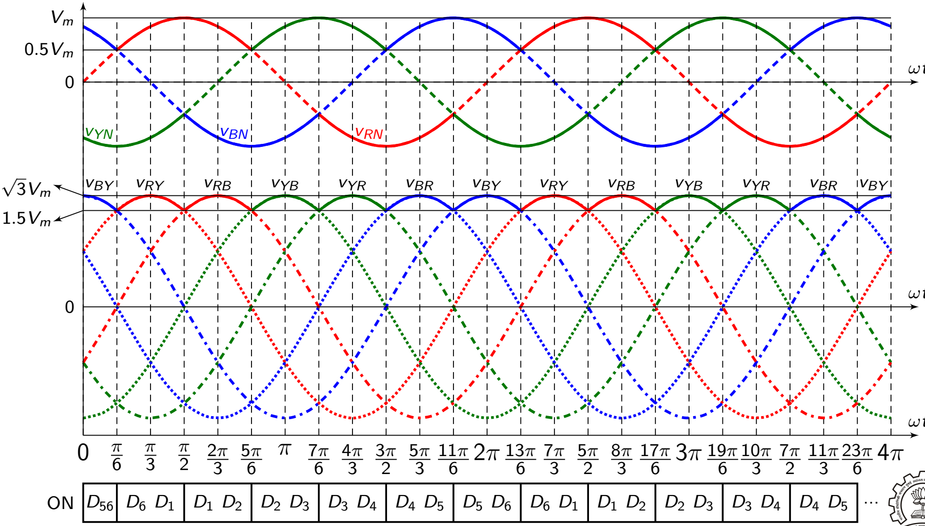
$v_{RN} = V_m \sin(\omega t) \qquad v_{YN} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \qquad v_{BN} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$

$i_R = i_{D1} - i_{D4}$ and similar definitions for $i_Y \& i_B$

PIV for each diode is $\sqrt{3}V_m$

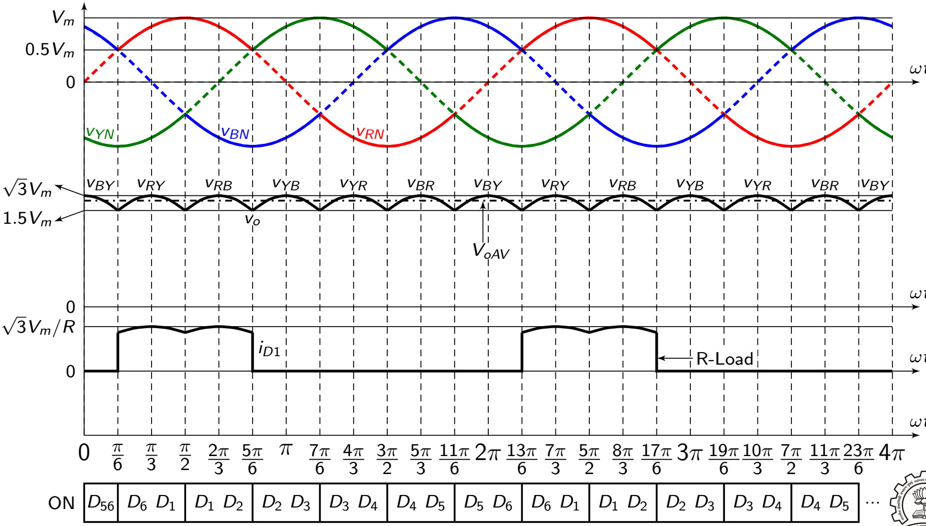
3-ph full-wave/bridge (6-pulse) rectifier (R, RL and constant I_o)

Waveforms in periodic steady state



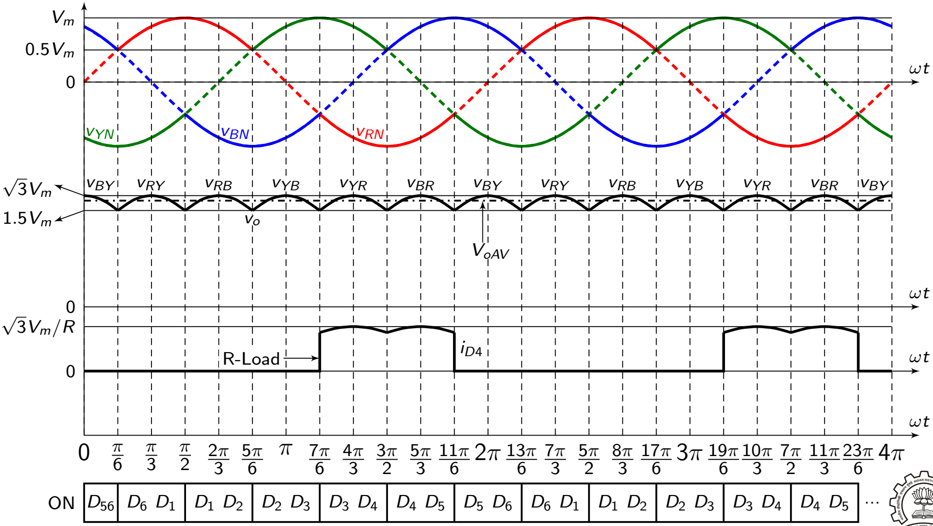
3-ph full-wave/bridge (6-pulse) rectifier (R, RL and constant I_o)

Waveforms in periodic steady state



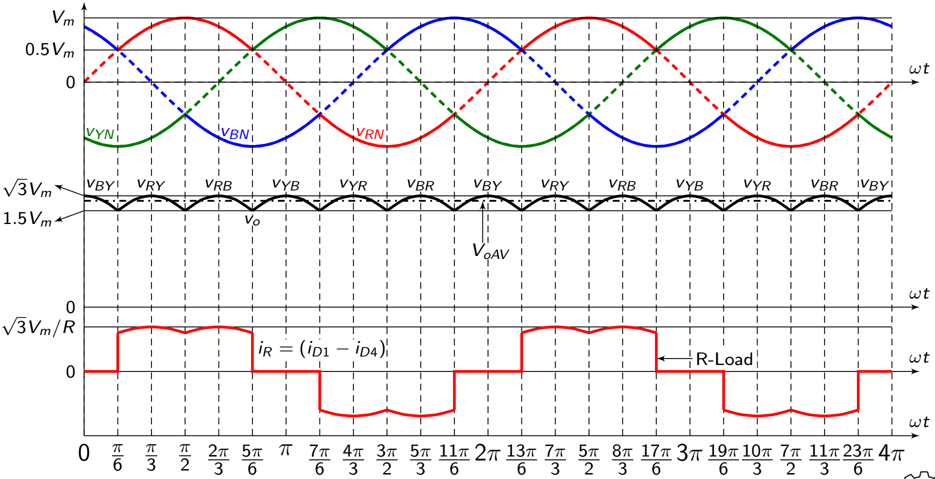
3-ph full-wave/bridge (6-pulse) rectifier (R, RL and constant I_o)

Waveforms in periodic steady state



3-ph full-wave/bridge (6-pulse) rectifier (R, RL and constant I_o)

Waveforms in periodic steady state



ON	D_{56}	D_6 D_1	D_1 D_2	D_2 D_3	D_3 D_4	D_4 D_5	D_5 D_6	D_6 D_1	D_1 D_2	D_2 D_3	D_3 D_4	D_4 D_5	···

3-ph full-wave/bridge (6-pulse) rectifier (R, RL and constant I_o)

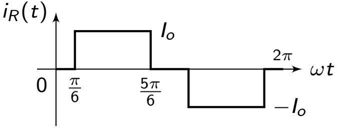
- Average output voltage

$$
\begin{aligned}
V_{oAV} &= \frac{1}{(\pi/3)} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} v_{RY}(t)\; d(\omega t) \\
&= \frac{3}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)\; d(\omega t) = \frac{3\sqrt{3} V_m}{\pi}
\end{aligned}
$$

- Maximum value of $v_o(t)$ is peak line-to-line voltage $\sqrt{3}V_m$
- Peak-to-peak ripple in output voltage
  $V_{oPP} = \sqrt{3}V_m - 1.5V_m = 0.232V_m$
- Frequency of output voltage ripple is 300Hz (6 times the line frequency)
- Each diode conducts for 120° (one-third of a line cycle), two diodes conduct together
- Input current $i_R(t)$ doesn't contain even harmonics; doesn't contain triplen harmonics

3-ph full-wave/bridge (6-pulse) rectifier (constant I_o)

In the case of constant current (I_o) load, $i_R(t)$ has the shape of a quasi-square wave



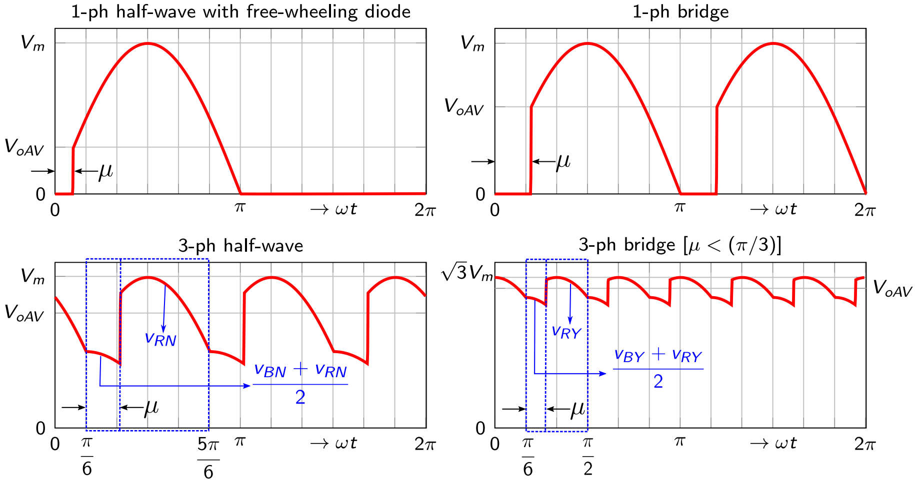
- $I_{RRMS} = I_o\sqrt{\frac{2}{3}}$; Fundamental $I_{R1RMS} = \frac{\sqrt{6}}{\pi} I_o$; nth harmonic $I_{RnRMS} = \frac{I_{R1RMS}}{n}$
- $DPF = 1$ and $DF_1 = 0.955 \Rightarrow$ power factor $PF = 0.955$
- Order of harmonics in $i_R(t)$ is 5,7,11,13,17,19,... or $6m \pm 1$, $m = 1, 2, 3, \ldots$
- Total harmonic distortion $I_{RTHD} = 31.08\%$

Effect of source inductance: Commutation (Constant load current I_o)

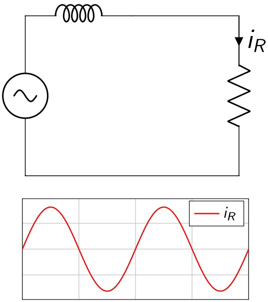
- Commutation: The process of current being transferred from one diode to the other
- Commutating inductance L_s is in series with v_s for 1-phase rectifiers
- For 3-phase rectifiers, L_s is in series with each of the sources, v_{RN}, v_{YN} and v_{BN}
- Commutation interval (in terms of angle) is denoted by μ
- Effects of commutation
  - Load regulation (Variation of output voltage with load)
  - Change in the shape of source (input) current waveform

Type of rectifier	$\cos\mu$	V_{oAV}
1-ph half-wave (free-wheeling diode)	$1 - \dfrac{\omega L_s I_o}{V_m}$	$\dfrac{V_m}{\pi}\left(1 - \dfrac{\omega L_s I_o}{2V_m}\right)$
1-ph bridge	$1 - \dfrac{2\omega L_s I_o}{V_m}$	$\dfrac{2V_m}{\pi}\left(1 - \dfrac{\omega L_s I_o}{V_m}\right)$
3-ph half-wave	$1 - \dfrac{2\omega L_s I_o}{\sqrt{3}V_m}$	$\dfrac{3\sqrt{3}V_m}{2\pi}\left(1 - \dfrac{\omega L_s I_o}{\sqrt{3}V_m}\right)$
3-ph bridge	$1 - \dfrac{2\omega L_s I_o}{\sqrt{3}V_m}$	$\dfrac{3\sqrt{3}V_m}{\pi}\left(1 - \dfrac{\omega L_s I_o}{\sqrt{3}V_m}\right)$

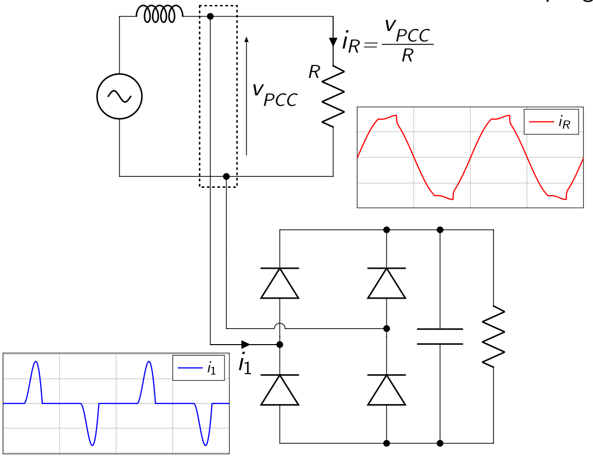
Effect of source inductance: Waveforms of output voltage $v_o(t)$



Input current of a rectifier: Effect on other loads



PCC: Point of Common Coupling

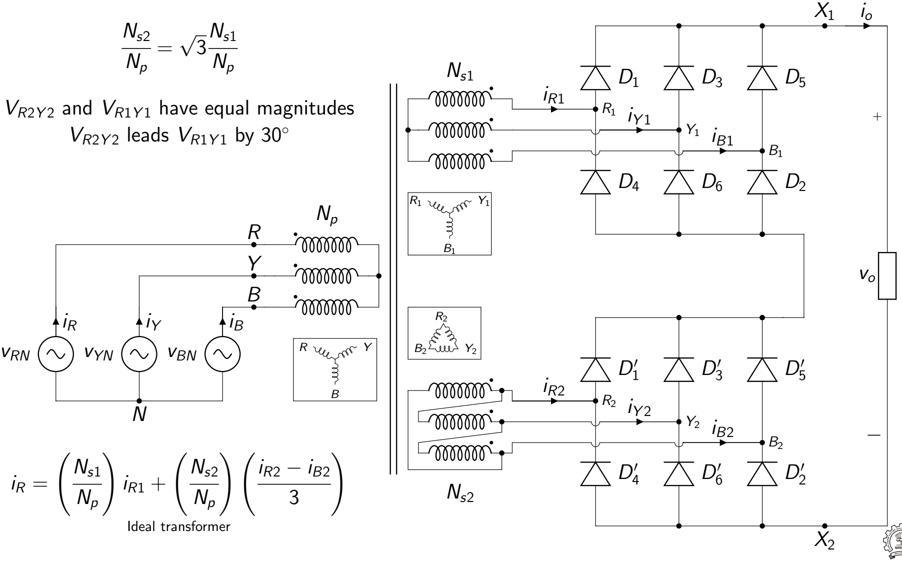


3-phase 12-pulse series-type diode rectifier

$$\frac{N_{s2}}{N_p} = \sqrt{3}\frac{N_{s1}}{N_p}$$

V_{R2Y2} and V_{R1Y1} have equal magnitudes

V_{R2Y2} leads V_{R1Y1} by 30°

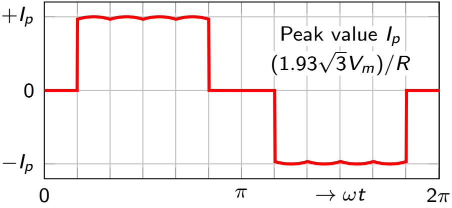


$$i_R = \left(\frac{N_{s1}}{N_p}\right) i_{R1} + \left(\frac{N_{s2}}{N_p}\right)\left(\frac{i_{R2} - i_{B2}}{3}\right)$$

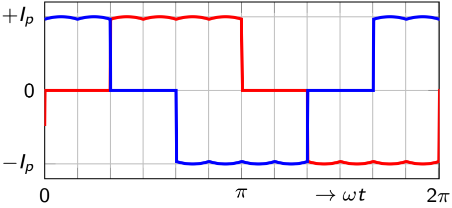
Ideal transformer

3-phase 12-pulse diode rectifier: R-load

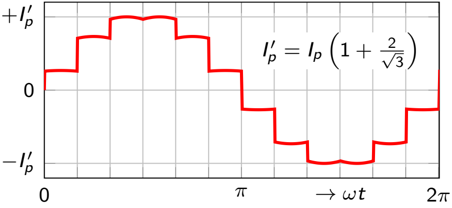
Line current in secondary 1: i_{R1}



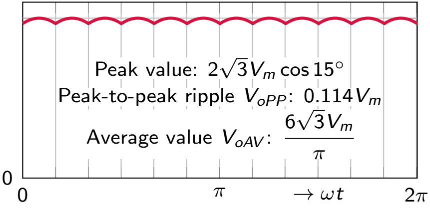
Line currents in secondary 2: i_{R2} and i_{B2}



Line current in primary: i_R



Output voltage: v_o

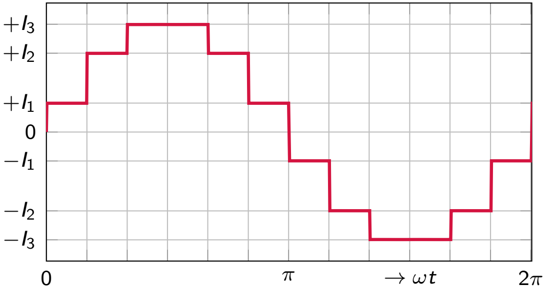


$N_{s1} = N_p$

3-phase 12-pulse diode rectifier (constant current I_o load)

$N_{s1} = N_p$

Input current: i_R



$$I_1 = \frac{I_o}{\sqrt{3}}$$

$$I_2 = I_o\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$I_3 = I_o\left(1 + \frac{2}{\sqrt{3}}\right)$$

- $I_{RRMS} = I_o\sqrt{\frac{4}{3} + \frac{2}{\sqrt{3}}}$; Fundamental $I_{R1RMS} = \frac{2\sqrt{6}}{\pi} I_o$
- $DPF = 1$ and $DF_1 = 0.9886 \Rightarrow$ power factor $PF = 0.9886$
- Order of harmonics in $i_R(t)$ is 11,13,23,25,... or $12m \pm 1$, $m = 1, 2, 3, \ldots$
- Total harmonic distortion $I_{RTHD} = 15.24\%$

Module 1: Summary

- Diode characteristics and loss calculations
- 1-pulse, 2-pulse, 3-pulse, 6-pulse and 12-pulse rectifiers
- R-load, constant current load (R with high L) and C-filter with R-load
- Effect of source inductance: commutation, load regulation
- Periodic steady-state and measures of periodic waveforms
- Fourier analysis and waveform symmetries
- Harmonic distortion at point of common coupling