Measures of periodic waveforms in power electronic circuits

For a periodic waveform $f(t)$ with period T (sec) [i. e. $f(t_0) = f(t_0 + T)$, for any t_0],

$$
\begin{aligned}
\text{Peak value (Absolute peak) } F_{PK} &: \quad Max\left(|f(t)|\right) \\
\text{Average value } F_{AV} &: \quad \frac{1}{T}\int_0^T f(t)\ dt \\
\text{Root mean square (RMS) value } F_{RMS} &: \quad \sqrt{\frac{1}{T}\int_0^T f(t)^2\ dt} \\
\text{Form factor } F_{FF} &: \quad \frac{F_{RMS}}{F_{AV}} \\
\text{Ripple factor } F_{RF} &: \quad \frac{\widetilde{F}_{RMS}}{F_{AV}} = \sqrt{\frac{F_{RMS}^2 - F_{AV}^2}{F_{AV}^2}} = \sqrt{F_{FF}^2 - 1} \\
\text{Peak-to-peak value } F_{PP} &: \quad Max(f(t)) - Min(f(t)) \\
\text{Crest factor } F_{CF} &: \quad \frac{F_{PK}}{F_{RMS}} \\
\text{RMS value of the fundamental component } F_{1RMS} &: \quad \sqrt{\frac{a_1^2 + b_1^2}{2}} \\
\text{RMS value of the } n^{th} \text{ harmonic component } F_{nRMS} &: \quad \sqrt{\frac{a_n^2 + b_n^2}{2}}
\end{aligned}
$$

a_n and b_n are the coefficients in Fourier series expansion of $f(t)$. The Fourier series expansion and the coefficients are given below.

$$
\begin{aligned}
f(t) &= F_{AV} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t), \qquad \omega = \frac{2\pi}{T} \qquad (1) \\
a_n &= \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\ dt, \qquad n = 1, 2, 3, ... \\
b_n &= \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\ dt, \qquad n = 1, 2, 3, ...
\end{aligned}
$$

(F_{AV} in Eqn. (1) is written as $\dfrac{a_0}{2}$ in many textbooks.)

Substituting the expansion of $f(t)$ from Eqn. (1) in the definition of RMS value F_{RMS} and simplifying, it can be shown that

$$
\begin{aligned}
F_{RMS} &= \sqrt{F_{AV}^2 + \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2}} \\
\text{we define } \widetilde{F}_{RMS} &= \sqrt{\sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2}} = \sqrt{F_{RMS}^2 - F_{AV}^2} \\
\text{we further define } \widetilde{F}_{hRMS} &= \sqrt{\sum_{n=2}^{\infty} \frac{a_n^2 + b_n^2}{2}} = \sqrt{\widetilde{F}_{RMS}^2 - F_{1RMS}^2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Distortion factor } F_{DF1} &: \frac{F_{1RMS}}{\widetilde{F}_{RMS}} \\
\text{Total harmonic distortion } F_{THD} &: \frac{\widetilde{F}_{hRMS}}{F_{1RMS}} = \sqrt{\frac{\widetilde{F}_{RMS}^2 - F_{1RMS}^2}{F_{1RMS}^2}} = \sqrt{\frac{1}{F_{DF1}^2} - 1}
\end{aligned}
$$

$$
\text{Power factor PF} : \frac{\text{Average power}}{\text{Apparent power}}
$$

In case of non-sinusoidal current drawn from a sinusoidal voltage source,

$$
\text{PF} : \frac{V_{1RMS} I_{1RMS} \cos\phi_1}{V_{1RMS} I_{RMS}} = I_{DF1} \cos\phi_1
$$

Here, average value of the non-sinusoidal current is assumed to be zero ($I_{AV} = 0$). Thus, $I_{RMS} = \widetilde{I}_{RMS}$.

ϕ_1 is the phase angle between voltage and the fundamental component of the non-sinusoidal current.

$\cos\phi_1$ is commonly referred to as displacement power factor (DPF).
Thus, power factor = (distortion factor of current x displacement power factor)
or PF = (DF x DPF)