EN304- ELECTRICAL ENERGY SYSTEMS/ POWER SYSTEMS

Department of Energy Science and Engineering

This is only reading material (not lecture slides)

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SLIDE DECK 2: AC ANALYSIS

Outline

- Single Phase AC
- Elementary Circuits
- Series Circuits
- Parallel Circuit
- Sinusoidal Steady State Analysis
- AC Power

Single Phase AC

A sinusoid is a signal that has the form of the sine or cosine function

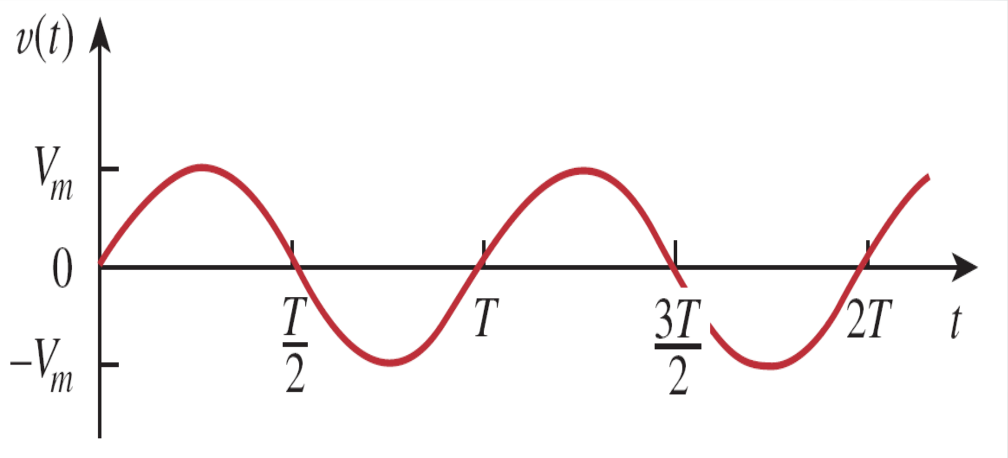
$$v(t) = V_m Sin(wt)$$

Where,

V_m = The amplitude of the sinusoid

ω = The angular frequency in radians/s

ωt = The argument of the sinusoid



- T is called the period of the sinusoid

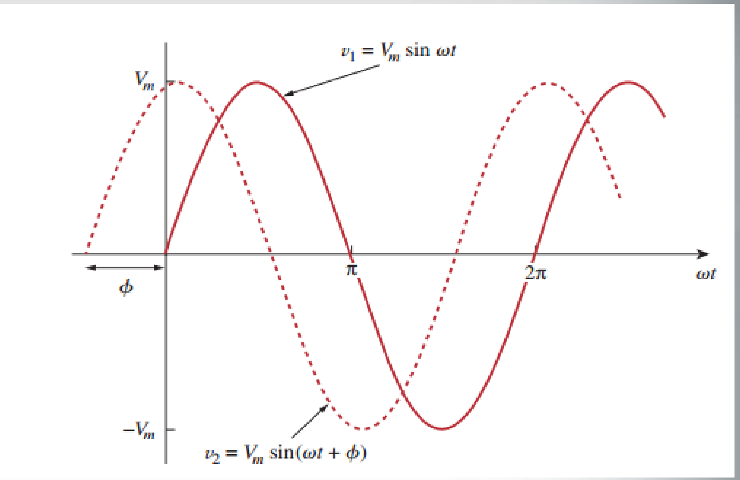
$$T = \frac{2\prod}{\omega}$$

The reciprocal of T is the number of cycles per second, known as the cyclic frequency (f) of the sinusoid

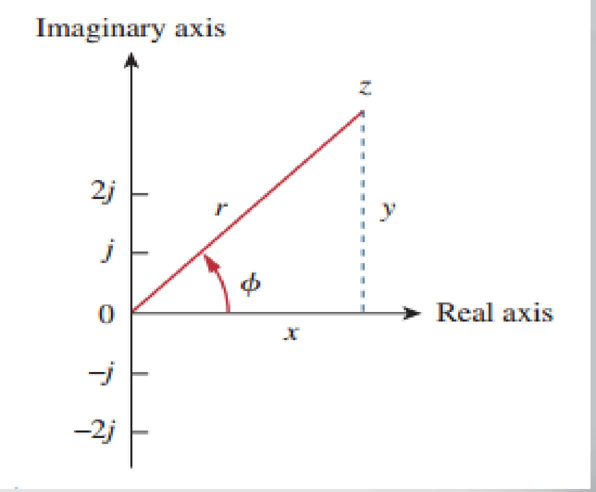
$$\omega = 2\prod f$$

Single Phase AC Phasors

- Φ is called the phase of the sinusoid
- A phasor is a complex number that represents the **amplitude** and **phase** of a sinusoid.
- A complex number z can be represented by **rectangular** form: $\mathrm{z} = \mathrm{x} + \mathrm{jy}$ where $\mathrm{j} = \sqrt{-1}$,
- or it can be represented by **polar** form: : $\mathrm{z} = \mathrm{r}\angle\theta$. r is the amplitude and θ is the phase shift.
- Exponential form: $\mathrm{z} = \mathrm{r}e^{j\theta}$ $\quad r = \sqrt{x^2 + y^2}\,, \quad \theta = \tan^{-1}\frac{y}{x}$

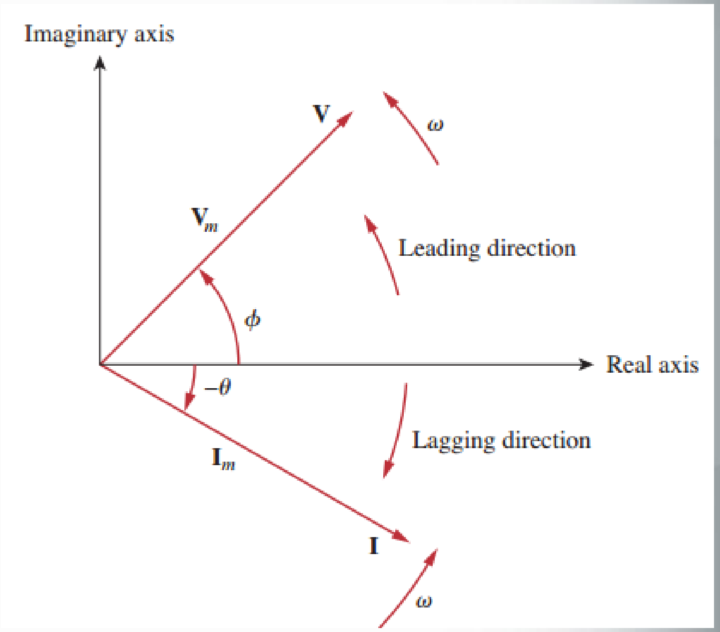


Time representation	Phasor representation
$V_m \sin(\omega t)$	$V_m\angle 0$
$V_m \sin(\omega t + \theta)$	$V_m\angle -\theta$
$V_m \sin(\omega t - \theta)$	$V_m\angle\theta$



Single phase AC: Phasors

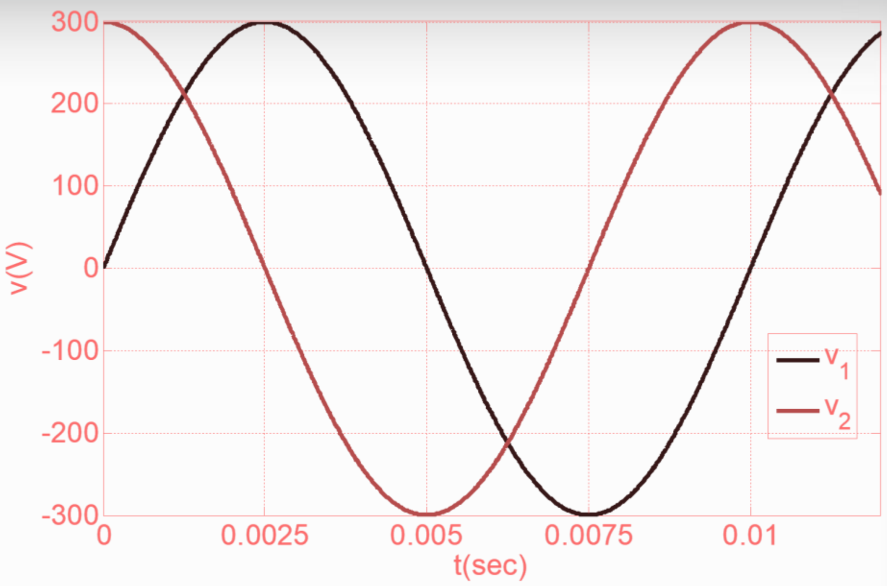
- Operations on complex numbers
  - Addition $z_1 + z_2 = (x_1 + x_2) + \mathrm{j}(y_1 + y_2)$
  - Subtraction $z_1 - z_2 = (x_1 - x_2) + \mathrm{j}(y_1 - y_2)$
  - Multiplication $z_1 * z_2 = (r_1 * r_2)\angle(\theta_1 + \theta_2)$
  - Division $z_1/z_2 = (r_1/r_2)\angle(\theta_1 - \theta_2)$
  - Reciprocal $1/z = (1/r)\angle(-\theta)$
  - Square root $\sqrt{z} = (\sqrt{r})\angle(\theta/2)$
  - Complex conjugate $z^* = r\angle(-\theta) = \mathrm{x} - \mathrm{jy}$
- Phasors rotates in anti-clockwise
- Let $\mathrm{V} = V_m\angle\phi$ and $\mathrm{I} = \mathrm{I}_m\angle - \theta$.
  Note the leading and lagging directions.



Note that phasors rotate in anti-clockwise

Single phase AC: exercise

- Write the sinusoid of the signals in previous slide in three forms (rectangular, polar and exponential).



- Find the result in both rectangular and polar forms, for the following, using complex quantities:

  a) $\dfrac{5-j12}{15\angle 53.1°}$ b) $(5-j12)+15\angle -53.1°$

  b) $\dfrac{2\angle 30°-4\angle 210°}{5\angle 450°}$ c) $\left(5\angle 0° + \dfrac{1}{3\sqrt{2}\angle -45°}\right)\times 2\angle 210°$

- Use phasor technique to evaluate the expression and then find the numerical value at t = 10 ms

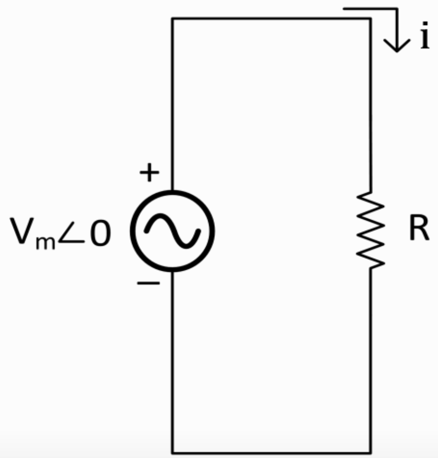
$$i(t) = 150\cos(100t-45°) + 500\sin(100t) + \frac{d}{dt}[\cos(100t-30°)]$$

Elementary circuits

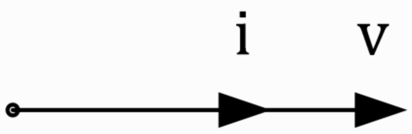
1. **Purely resistive circuit**
   - Voltage and current are in phase.
   - The instantaneous value of the current though the resistance is given by Ohm law,

$$i = \frac{v}{R} = \frac{V_m \sin(\omega t)}{R} = \frac{V_m}{R}\sin(\omega t) = \mathrm{I}_m \sin(\omega t)$$

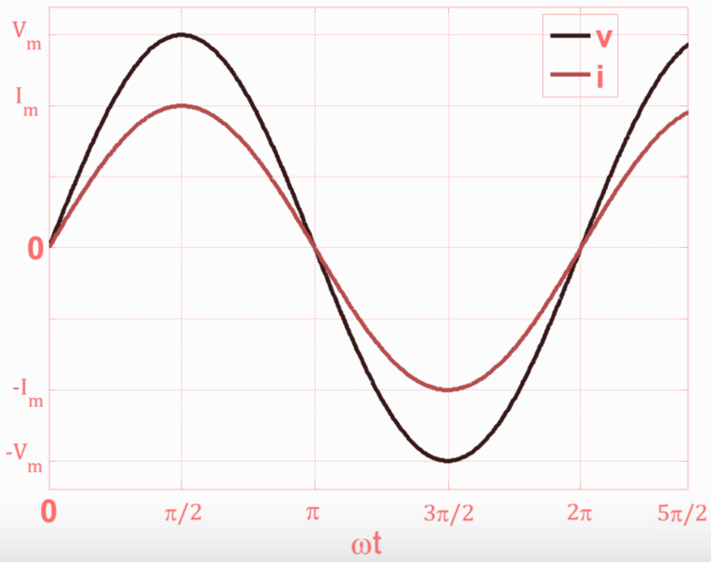
Circuit diagram



Phasor diagram



Waveforms



Elementary circuits

2. Purely inductive circuit

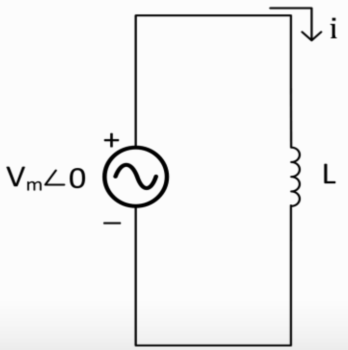
- The current lags the voltage by $90°$.
- The voltage across the inductor is given by $v = L\frac{di}{dt} = V_m \sin(\omega t)$

$$\therefore \; di = \frac{V_m}{\omega L}\sin(\omega t)dt$$

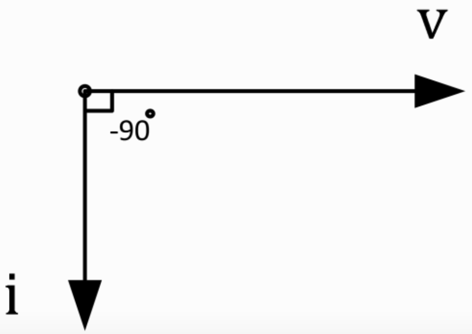
By integrating both sides

$$i = -\frac{V_m}{\omega L}\cos(\omega t) = \frac{V_m}{\omega L}\sin(\omega t - 90°) = \mathrm{I}_m \sin(\omega t - 90°)$$

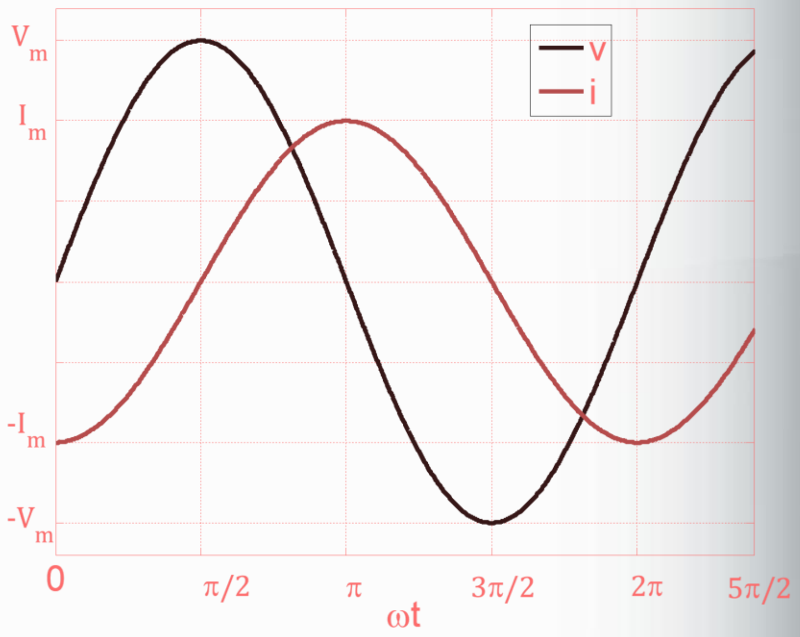
Circuit diagram



Phasor diagram



Waveforms



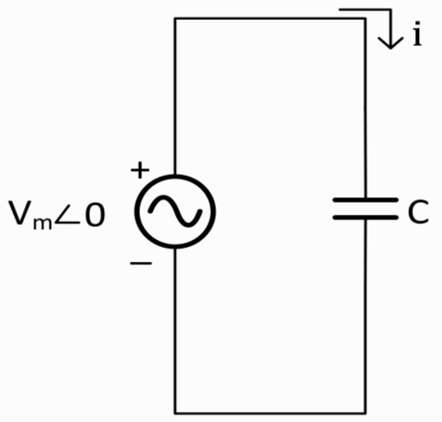
Elementary circuits

3. Purely capacitive circuit

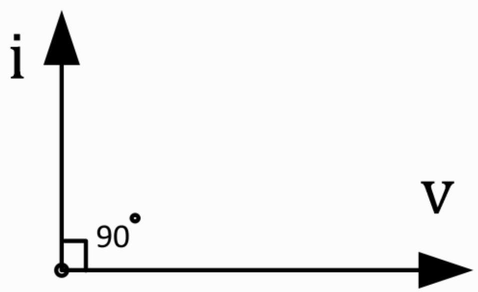
- The current leads the voltage by 90°.
- The current through the capacitor is given by $i = C\frac{dv}{dt}$

$$\therefore\ i = C\frac{d}{dt}\left(V_m \sin(\omega t)\right) = V_m\omega C\cos(\omega t) = V_m\omega C\sin(\omega t + 90°) = \mathrm{I}_m\sin(\omega t + 90°)$$

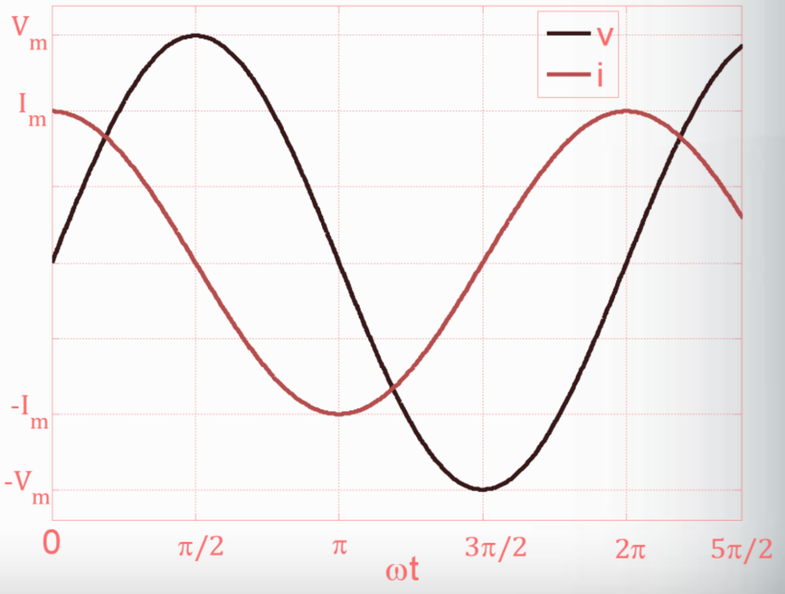
Circuit diagram



Phasor diagram



Waveforms



Elementary circuits

4. Ohm's law in AC circuits

- $$\frac{v}{i} = R, \qquad \frac{v}{i} = j\omega\mathrm{L} \quad \text{and} \quad \frac{v}{i} = \frac{1}{j\omega\mathrm{C}}$$

  In general, **impedance** $\boldsymbol{Z}$ represents the opposition that the circuit exhibits to the flow of sinusoidal current, where $\frac{v}{i} = Z$ and it is measured in Ω.
- The admittance (Y): is the reciprocal of impedance. It is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. It is measured in Siemens (S).

  $$\frac{i}{v} = Y = G + jB$$

  where G is called conductance and B is known as susceptance.
- Summary

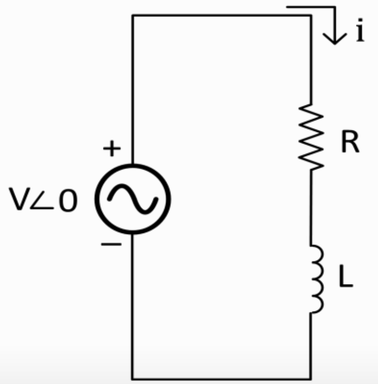
Element	*Impedance*	*Admittance*
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega\mathrm{L}$	$Y = \frac{1}{j\omega\mathrm{L}}$
C	$Z = \frac{1}{j\omega\mathrm{C}}$	$Y = j\omega\mathrm{C}$

Series circuit: Inductive circuit (RL circuit)

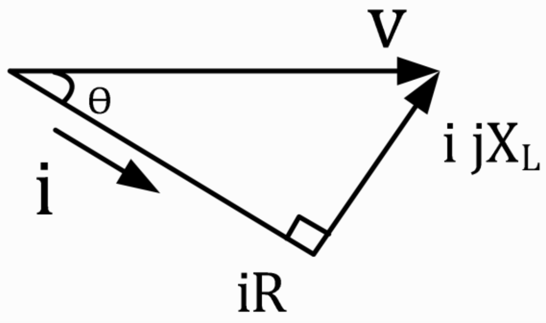
- The current lags the voltage by $\theta = tan^{-1}(\frac{X_L}{R})$.
- The circuit impedance is given by $Z\angle\theta = R + jX_L, \;\; where\, X_L = \omega L = 2\pi f L$
- The current can be calculated by

$$i = \frac{V\angle 0}{Z\angle\theta} = \frac{V + j0}{R + jX_L} = |i|\angle - \theta$$

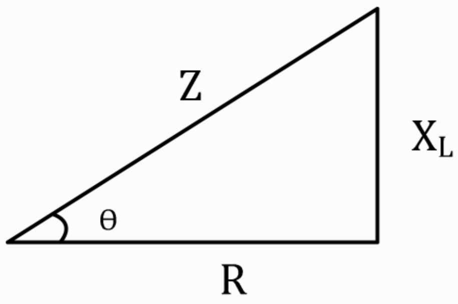
Circuit diagram



Phasor diagram



Impedance triangle

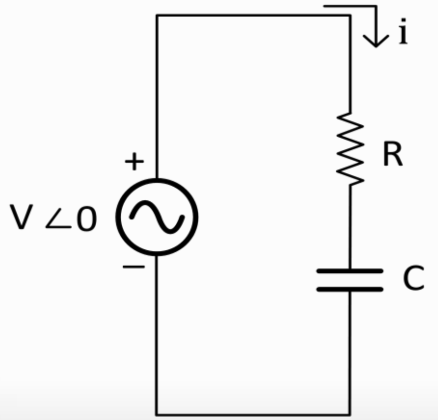


Series circuit: Capacitive circuit (RC circuit)

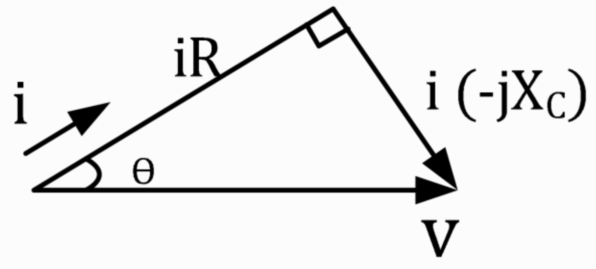
- The current leads the voltage by $\theta = \tan^{-1}\left(-\frac{X_C}{R}\right)$
- The circuit impedance is given by $Z\angle -\theta = R - jX_c$, $where\, X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
- The current can be calculated by

$$i = \frac{V\angle 0}{Z\angle -\theta} = \frac{V + j0}{R - jX_C} = |i|\angle\theta$$

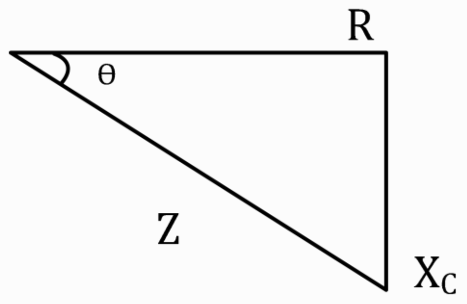
Circuit diagram



Phasor diagram



Impedance triangle



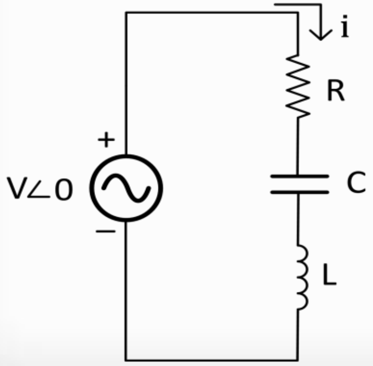
Series circuit: RLC

- The circuit impedance is given by $Z\angle\pm\theta = R + j(X_L - X_c)$
- The current can be calculated by

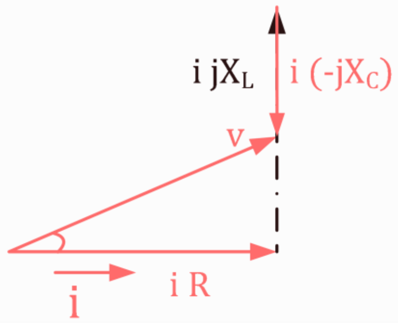
$$i = \frac{V\angle 0}{Z} = \frac{V + j0}{R + j(X_L - X_c)}$$

- Three cases can be obtained:1) $X_L > X_c$ → The current lags the voltage
  2) $X_L < X_c$ → The current leads the voltage
  3) $X_L = X_c$ → The current and the voltage are in phase (resonance)

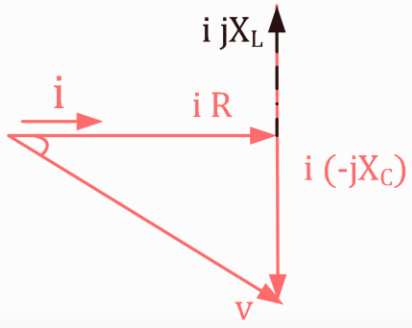
Circuit diagram



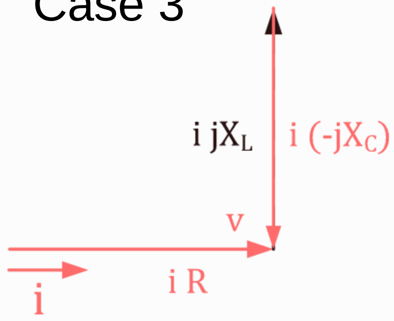
Case 1



Case 2



Case 3



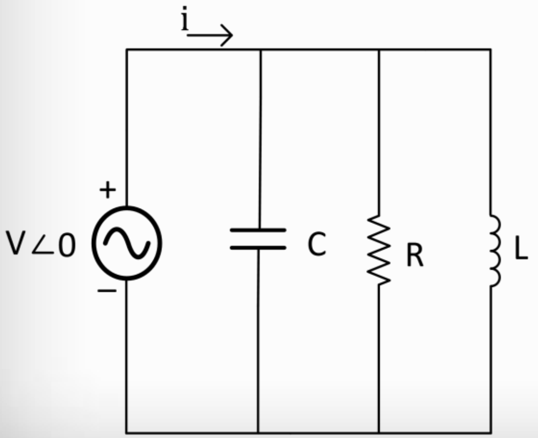
Parallel circuit: RLC

- The circuit admittance is given by $Y = Y_R + Y_L + Y_C = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$
- The current can be calculated by

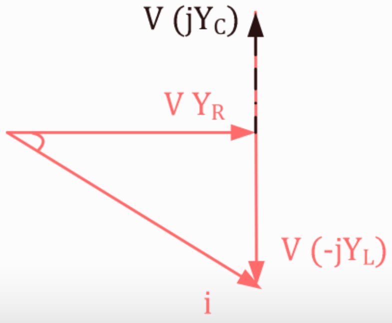
$$i = (V\angle 0)Y \ = V\left[\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right] \quad , |i| = |V|\sqrt{\frac{1}{R^2+\left(\omega C - \frac{1}{\omega L}\right)^2}}$$

- Three cases can be obtained: 1) $X_L < X_c$ $\rightarrow$ The current lags the voltage
  2) $X_L > X_c$ $\rightarrow$ The current leads the voltage
  3) $X_L = X_c$ $\rightarrow$ The current and the voltage are in phase (resonance)

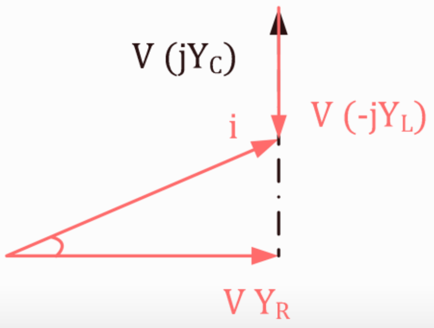
Circuit diagram



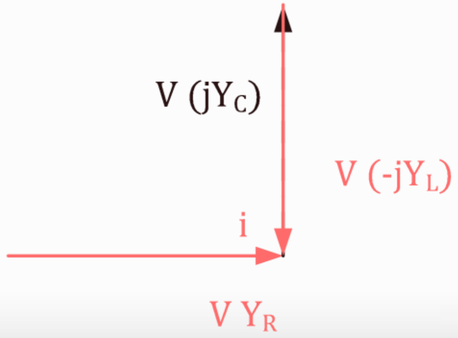
Case 1



Case 2



Case 3



Sinusoidal Steady State Analysis

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc)
3. transform the resulting phasor to the time domain.

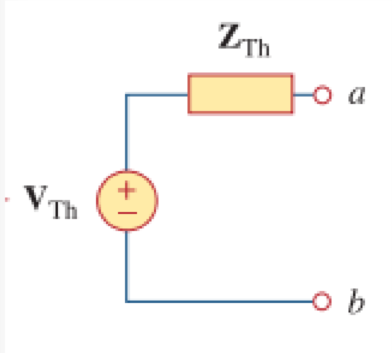
- Nodal Analysis – Since KCL is valid for phasors so we can analyze ac circuits by nodal analysis.
- Mesh Analysis- Since KVL is valid for phasors so we can analyze ac circuits by nodal analysis.
- Superposition – Since ac circuits are linear so superposition is applied in similar way as it is applied to dc circuits
- Source Transformation – In frequency domain it involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance .

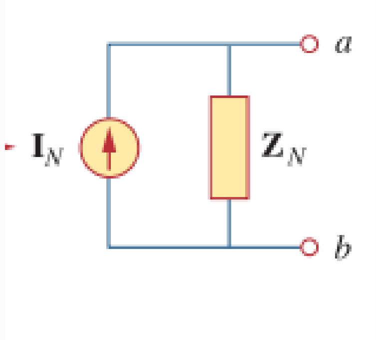
$$V_s = Z_s I_s \Leftrightarrow I_s = \frac{V_s}{Z_s}$$

Sinusoidal Steady State Analysis

- Thevenin and Norton Equivalent Circuits

Thevenin's and Norton's theorems are applied to ac circuits in the same way as they are to dc circuits. The only difference is that we have to deal with impedance rather than resistance.



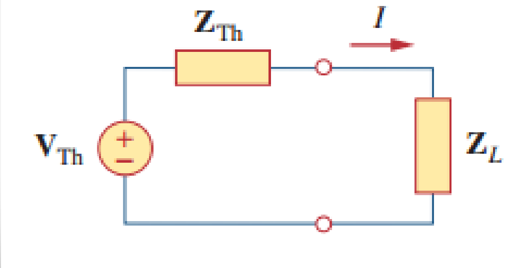


$$V_{Th} = Z_N I_N \,, Z_{Th} = Z_N$$

Fig. Thevenin equivalent

Fig. Norton equivalent

- Maximum Power transfer theorem



For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{TH}

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*$$

AC Power

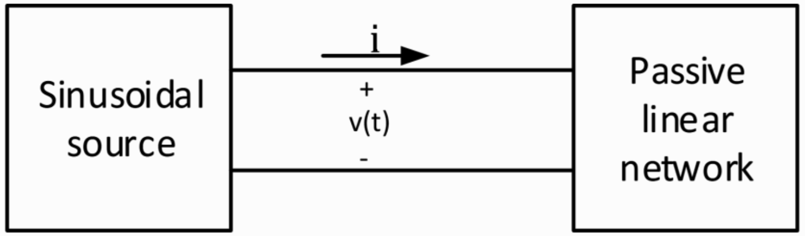
- The **instantaneous power** (in watts): is the power at any instant of time. $p(t) = v(t)\ i(t)$

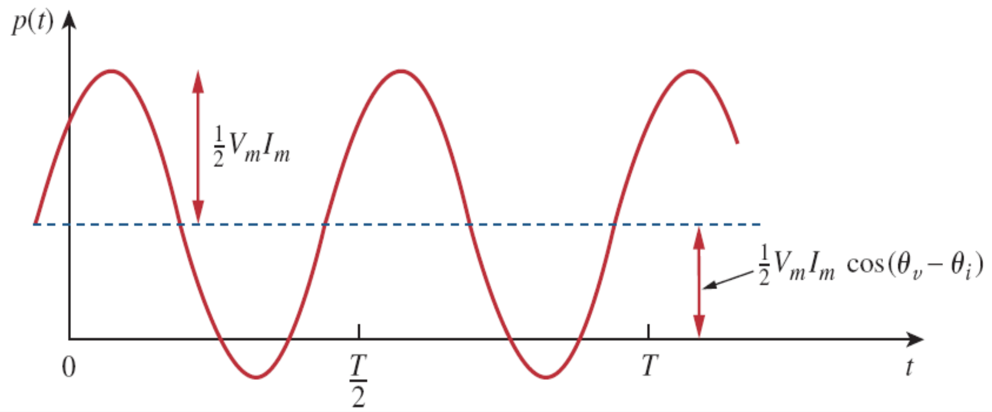
consider $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$\Longrightarrow \quad p(t) = \underbrace{V_m I_m \cos(\theta_v - \theta_i)} + \underbrace{\tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$

- Time independent.
- Constant
- Depends on the phase difference between voltage and current

- Time dependent.
- Sinusoidal function with twice frequency of voltage or current.





- When $p(t)$ is positive, power is absorbed by the circuit.
- When $p(t)$ is negative, power is absorbed by the source.

AC Power

- The **average power** (in watts): is the average of the instantaneous power over one period.

$$\boldsymbol{P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{2}V_m} I_m \cos(\theta_v - \theta_i)$$

  - It is time independent.
  - For R load $\theta_v - \theta_i = 0$, hence $\boldsymbol{P} = \frac{1}{2}V_m I_m$, i.e. R load consume/absorb power at all times.
  - For L or C Loads $\theta_v - \theta_i = \pm 90$, hence $\boldsymbol{P = 0}$, i.e. L and C loads consume zero average power.

- **Complex power** $\boldsymbol{S}$ (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is **real power** $\boldsymbol{P}$ and its **imaginary part** is reactive power $\boldsymbol{Q}$.
- ***Effective (or RMS) value:*** The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

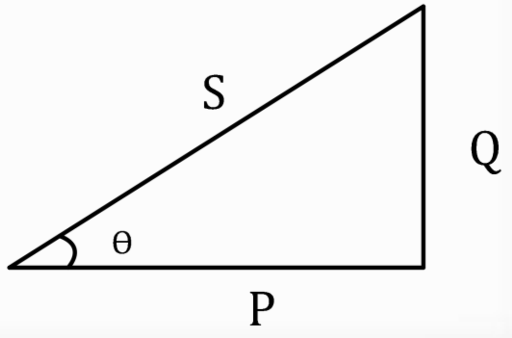
AC Power

$$V_{rms} = V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}, \quad I_{rms} = I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

- For sinusoid $i(t) = I_m \cos(\omega t)$: $\quad I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \frac{I_m}{\sqrt{2}}$
- Similarly for sinusoid $v(t) = V_m \cos(\omega t)$: $\quad V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \frac{V_m}{\sqrt{2}}$
- The complex power can be expressed by,

$$S = \frac{1}{2}VI^* = V_{rms}\,I_{rms}^* = V_{rms}I_{rms}\angle(\theta_v - \theta_i) = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*}$$

- Where P and Q are the real and imaginary parts of complex power S
- $P = Re(S) = I_{rms}^2 R \qquad\qquad Q = Im(S) = I_{rms}^2 X$



AC Power

Note: 1. Q = 0 for resistive loads (unity pf)
2. Q < 0 for capacitive loads (leading pf)
3. Q > 0 for inductive loads (lagging pf)

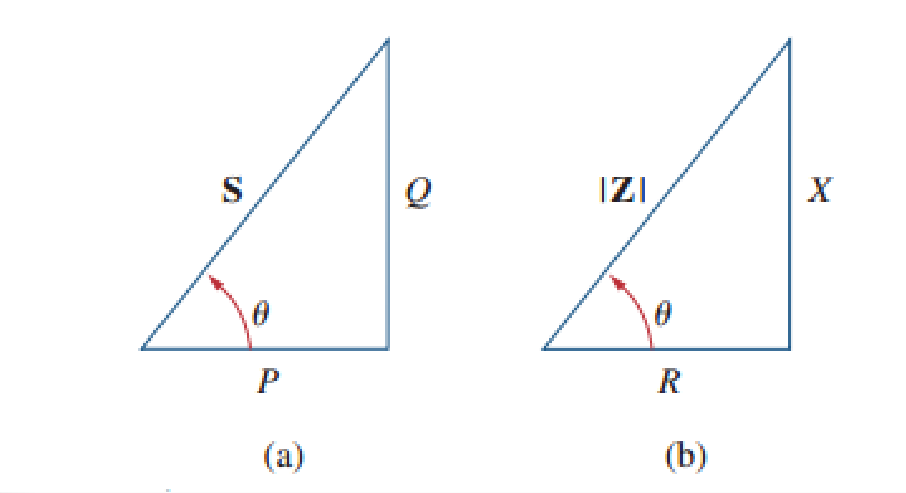


Fig.(a) Power triangle, (b) impedance triangle.