EN 304- ELECTRICAL ENERGY SYSTEMS

Department of Energy Science and Engineering

This is only reading material (not lecture slides)

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SLIDE DECK 5: TRANSFORMER

Transformer- Definition

- A transformer is a static machine used for transforming power through electromagnetic induction from one circuit at a given voltage level to another circuit at different or same voltage level, however without changing frequency.

- It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core.

- Transformers have no moving parts, rugged and durable in construction, thus requiring less maintenances. Therefore, they have a very high efficiency—as high as 99%.

- There is no electrical connection between the primary and secondary. Hence, the ac power is transferred from primary to secondary through magnetic flux

Terminology

- V_1= Applied ac voltage
- V_2= Voltage across the load
- E_1= Primary induced emf
- E_2= Secondary induced emf
- N_1= No of turns on the primary
- N_2= No of turns on the secondary
- I_1= current flowing in the primary
- I_2= current flowing in the secondary

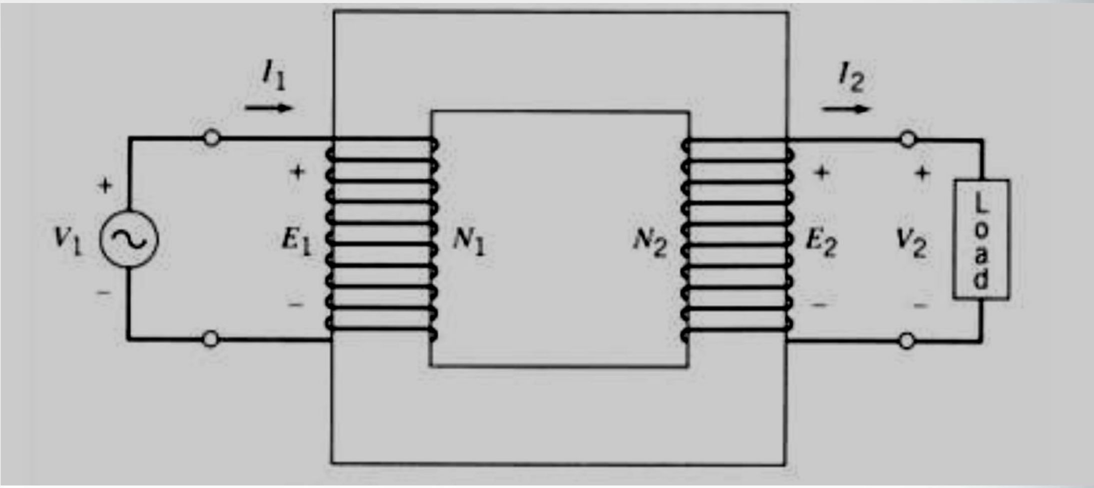


Fig 1: Two winding transformer

Transformer: Principle of operation

- Transformers are devices that transfer energy from one circuit to another by means of a common magnetic field. Except auto-transformers, there is no direct electrical connection from one circuit to the other.
- Transformers work on the principle of electromagnetic induction, where mutual induction results in emf induced in the secondary winding.
- The ability of iron or steel to carry magnetic flux is much greater than air. Modern electrical steels have permeabilities in the order of 1500 compared with 1.0 for air. This means that the ability of a steel core to carry magnetic flux is 1500 times that of air.
- Ideally the flux generated by the first coil will be transferred through the transformer core, this flux induces an emf in the second coil and the voltage appears on the second winding terminals. Increasing the flux in the core requires increasing either the first coil voltage or number of turns. Increasing the second coil voltage for the same flux can be done by increasing the coil number of turns of the second coil.

Transformer: Principle of operation

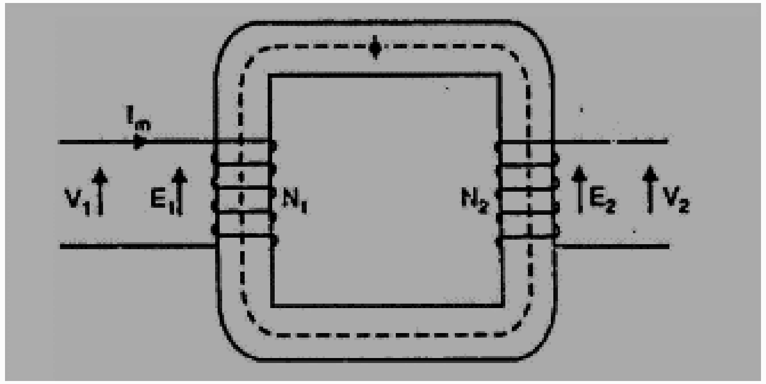
- There is no electrical connection between the primary and secondary. Hence, the a.c. power is transferred from primary to secondary through magnetic flux.
- Where, V_1 = Applied a.c voltage, V_2 = voltage across the load
  E_1 = primary induced emf, E_2 = Secondary induced emf
  N_1 = No of turns on the primary, N_2 = No of turns on the secondary
  I_1 = current flowing in the primary, I_2 = current flowing in the secondary
- If $V_2 > V_1$, it is called a step up-transformer. On the other hand, if $V_2 < V_1$, it is called a step-down transformer.
- Working: It works on the principle of Faraday's laws of electromagnetic induction

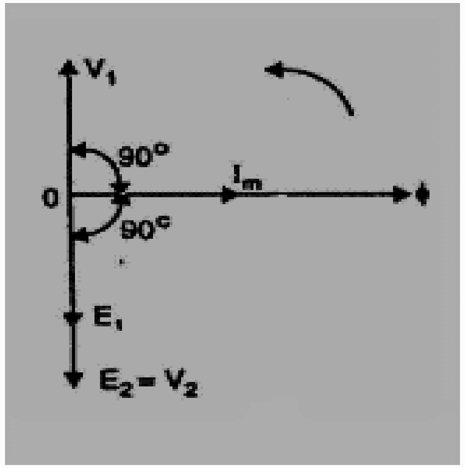
$$E_1 = -N_1 \frac{d\phi}{dt}, \qquad E_2 = -N_2 \frac{d\phi}{dt} \quad \Longrightarrow \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Ideal Transformer on No-Load:

An ideal transformer is one that has

1. **Zero leakage flux:** Fluxes produced by the primary and secondary currents are confined within the core
2. **The windings have no resistance:** Induced voltages equal to applied voltages
3. **The core has infinite permeability:** Reluctance (opposition to flux) of the core is zero; Negligible current is required to establish magnetic flux
4. **Loss-less magnetic core:** No hysteresis or eddy current losses





Note: V_1 - supply voltage ; I_m- magnetising current; E_1 - self induced emf ; E_2 - mutually induced emf

EMF Equation of a Transformer

The sinusoidal flux produced by the primary is $\emptyset = \emptyset_m$ Sin ωt

The instantaneous emf induced in the primary is, e_1= -$N_1 \frac{d\emptyset}{dt}$ = -$N_1 \frac{d}{dt}(\emptyset_m$ Sin $\omega t)$ =-$\omega\emptyset_m N_1$cos ωt

$$e_1 = 2\pi f \emptyset_m N_1 \sin(\omega t - 90^0)$$

maximum value of induced emf, $E_{m1} = 2\pi f \emptyset_m N_1$

The rms value is given by , $E_1 = \frac{E_{m1}}{\sqrt{2}}$ = 4.44 $f N_1 \emptyset_m$; Similarly $E_2 = \frac{E_{m2}}{\sqrt{2}}$ = 4.44 $f N_2 \emptyset_m$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$

volt-amperes input to the primary is equal to the output volt-amperes i.e. $V_1 I_1 = V_2 I_2$

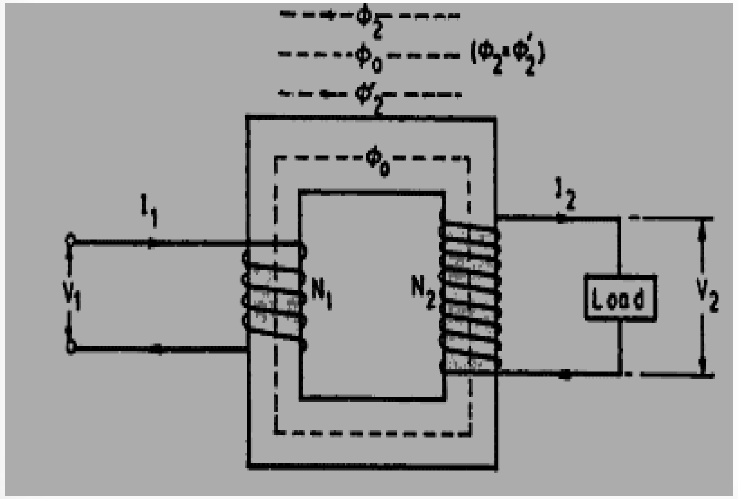
Voltage Transformation Ratio (K): $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$ $\Longrightarrow$ $\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

Ideal Transformer on Load:

- The secondary current I_2 sets up an mmf N_2I_2 which produces a flux in the opposite direction to the flux ∅ originally set up in the primary by the magnetizing current.
- power factor on the primary side is equal to the power factor on the secondary side

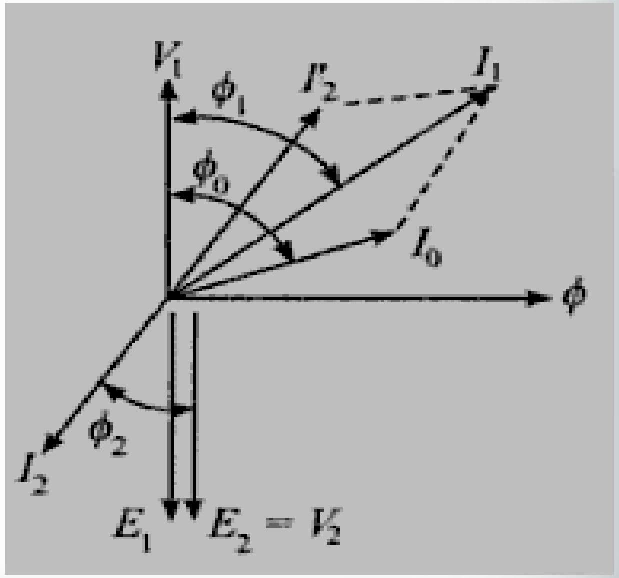
$$\emptyset_1 = \emptyset_2 \quad \Longrightarrow \quad \cos\emptyset_1 = \cos\emptyset_2$$

- Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e., $V_1 I_1 \cos\emptyset_1 = V_2 I_2 \cos\emptyset_2$



$$N_1 I_1 = N_2 I_2$$

$$I_1 = \frac{N_2}{N_1} I_2 = kI_2$$



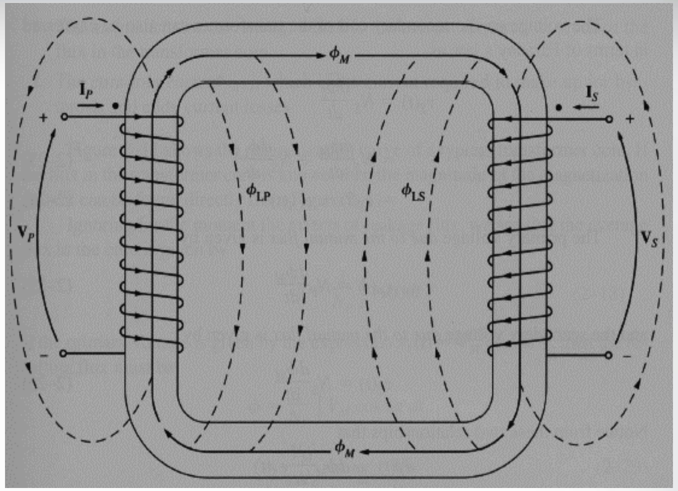
Practical Transformer on No-Load:

A practical transformer differs from the ideal transformer in many respects. The practical transformer has

(i) iron losses

(ii) winding resistances and

(iii) magnetic leakage, giving rise to leakage reactances.



Leakage flux: flux that links only one of the transformer windings but not the other one

Mutual flux: flux that remains in the core and links both the windings

ϕ_p: total average primary flux

ϕ_M : flux linking both primary and secondary windings

ϕ_{LP}: primary leakage flux

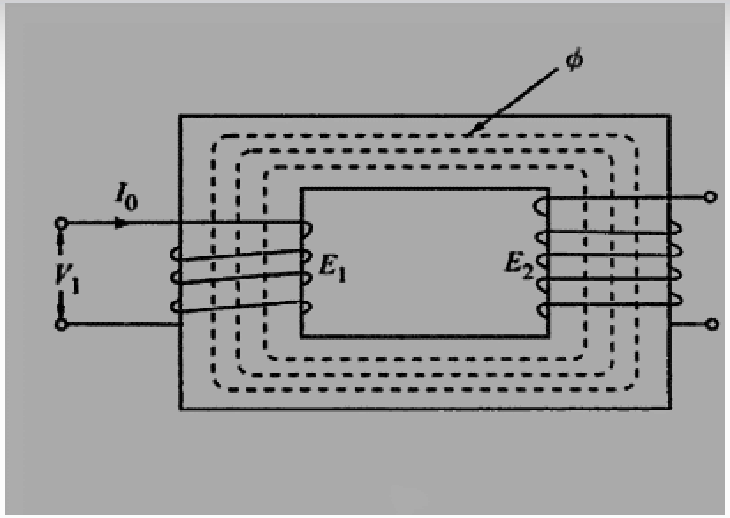
ϕ_S: total average secondary flux

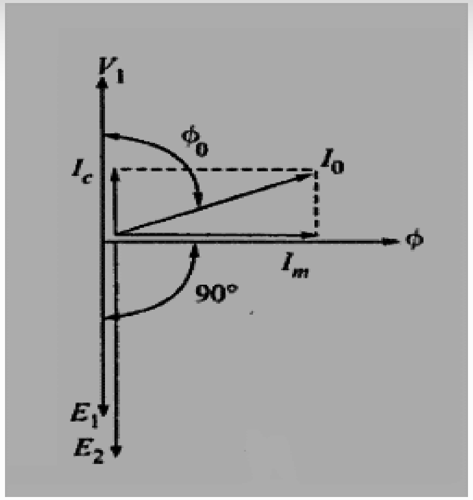
ϕ_{LS}: secondary leakage flux

$$\varphi_P = \varphi_M + \varphi_{LP}$$

$$\varphi_S = \varphi_M + \varphi_{LS}$$

Practical Transformer on No-Load:





When an ac power source is connected to a transformer, a current flows in its primary circuit, even when the secondary circuit is open circuited. This current is the current required to produce flux in the ferromagnetic core and is called *excitation current*. It consists of two components:

1. The *magnetization current* I_m, which is the current required to produce the flux in the transformer core
2. The *core-loss current* I_c, which is the current required to make up for hysteresis and eddy current losses

Practical Transformer on No-Load:

- The no-load primary current I_0 can be resolved into two rectangular components:

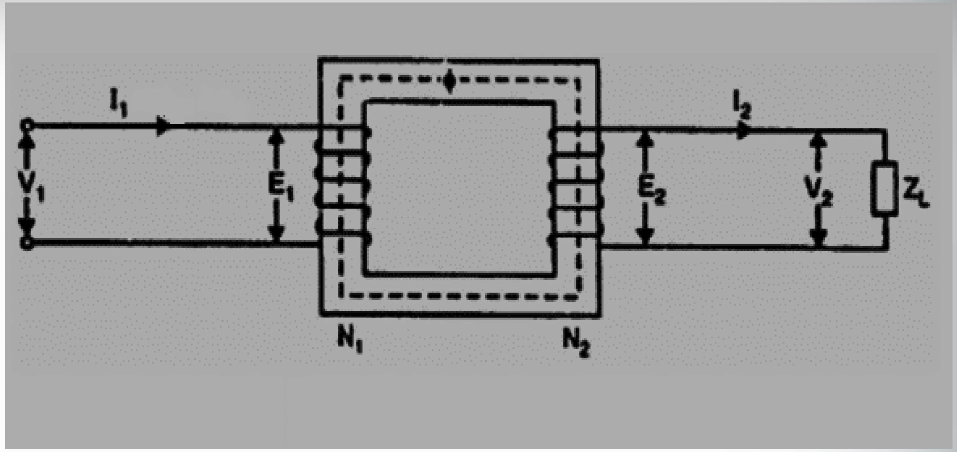
  $I_c = I_0 \text{Cos}\, \emptyset_0$ and $I_m = I_0 \text{Sin}\, \emptyset_0$

  $I_0 = \sqrt{{I_c}^2 + {I_m}^2}$ and no load pf = $\text{Cos}\, \emptyset_0 = \frac{I_c}{I_0}$

- It is to be known that no load primary copper loss (i.e. ${I_0}^2$R) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e., No load input power, W_0 = Iron loss

Practical Transformer on Load:

(1) No winding resistance and leakage flux -



The total primary current I_1 must meet two requirements:

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

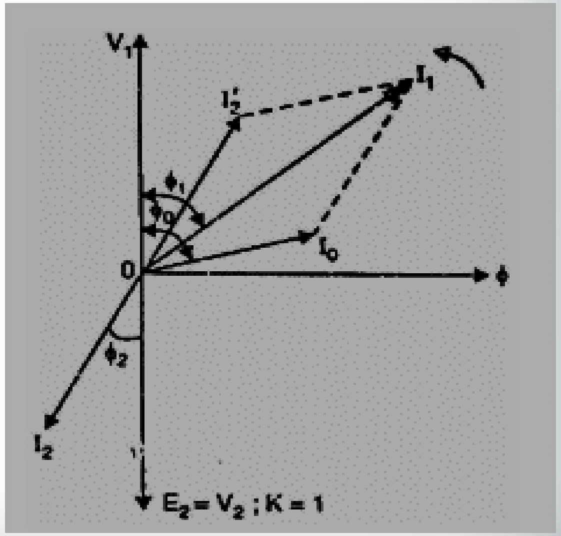
(b) It must supply a current I_0 to counteract the demagnetizing effect of secondary current I_2.

The magnitude of I'_2 will be such that: $$N_1 I'_2 = N_2 I_2$$

$$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

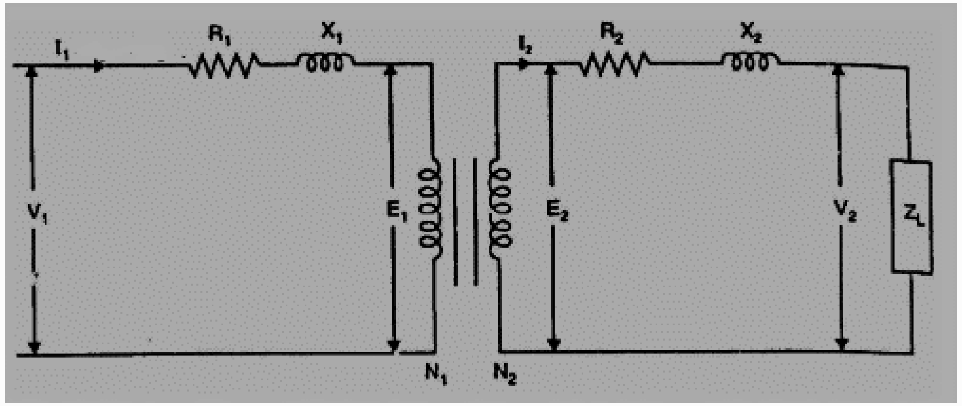
The total primary current I_1 is the phasor sum of I'_2 and I_0 ; $$I_1 = I_2' + I_0$$

$$I_2' = -K\, I_2$$



Practical Transformer on Load:

(2) Transformer with resistance and leakage reactance:

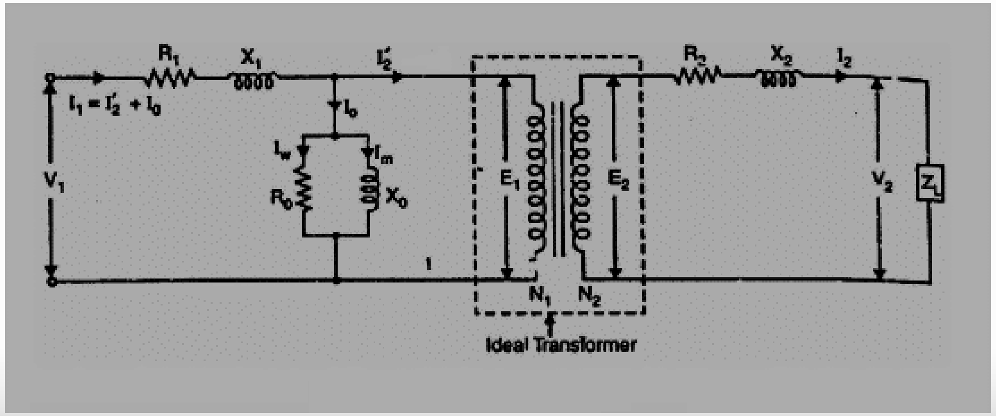


The total primary current I_1 is the phasor sum of I'_2 and I_0 ;

$$I_1 = I'_2 + I_0 \qquad I_1 = I_0 + (-KI_2)$$

$$I'_2 = -KI_2$$

(a) Exact Equivalent Circuit of a Loaded Transformer



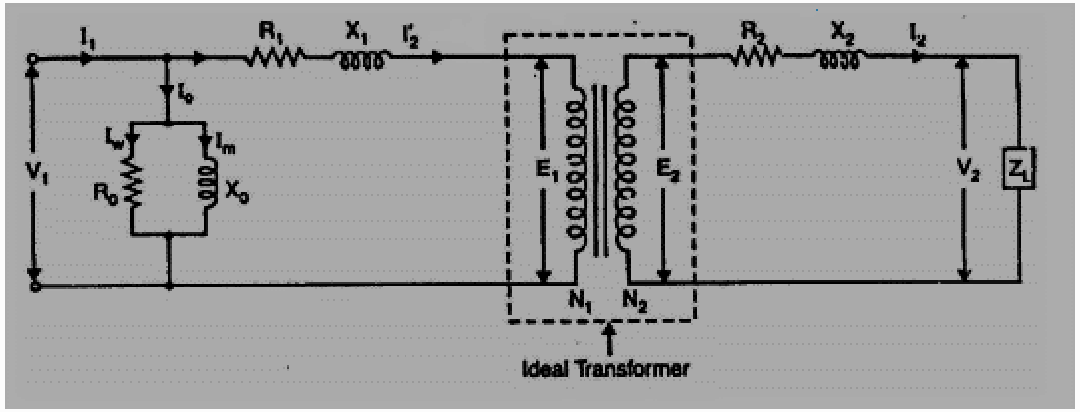
$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_2'}{I_2}$$

$$V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2 Z_2$$

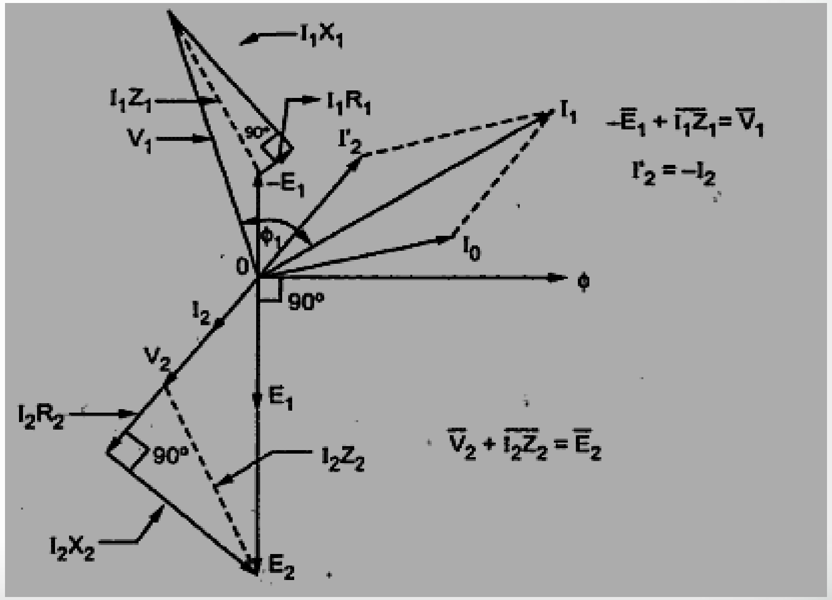
$$V_1 = -E_1 + I_1(R_1 + jX_1) = -E_1 + I_1 Z_1$$

Practical Transformer on Load contd..

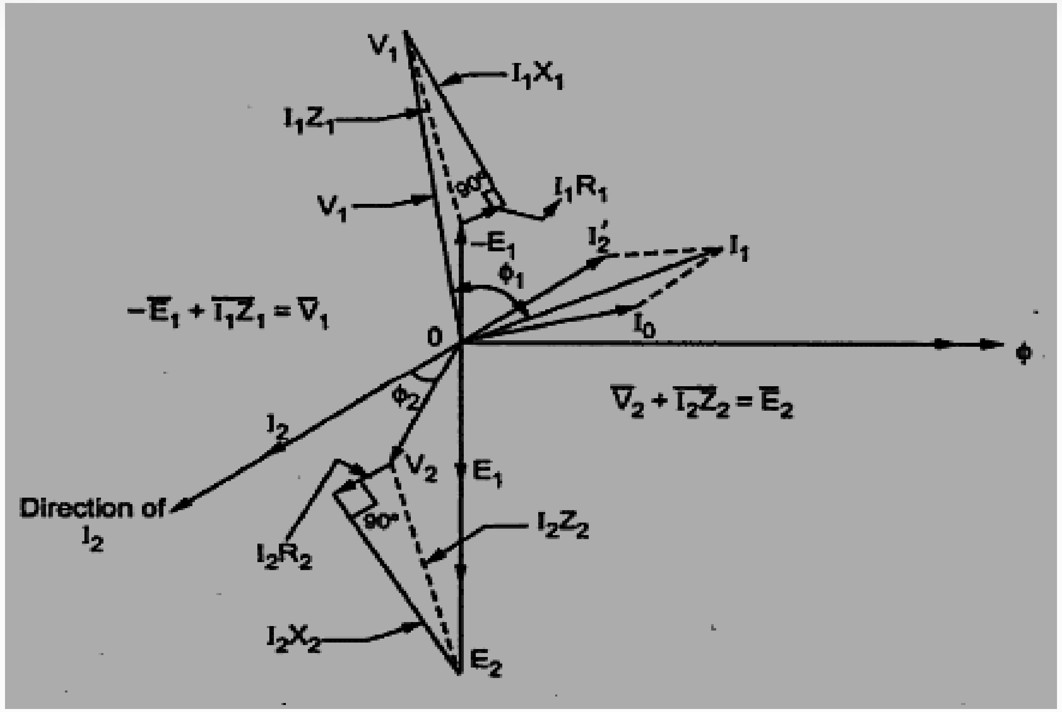
(b) Approximate Equivalent Circuit of a Loaded Transformer



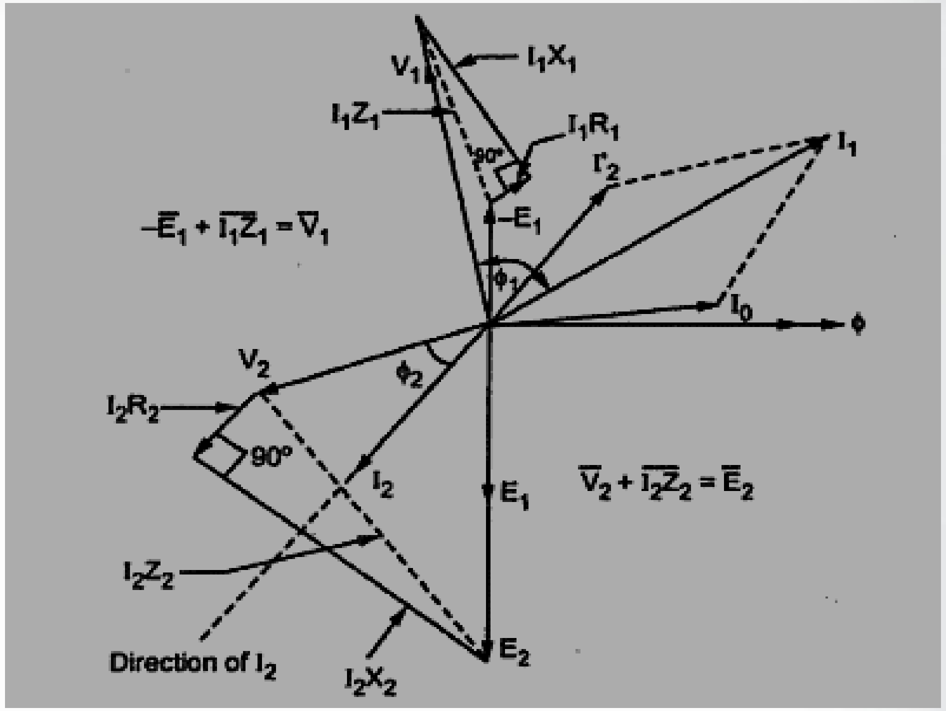
Phasor diagram for unity power factor load



Phasor diagrams for transformers on load



lagging power factor load



leading power factor load

Equivalent Circuit of Transformer

The deviations from the ideal transformer are modelled in the equivalent circuit as follows:

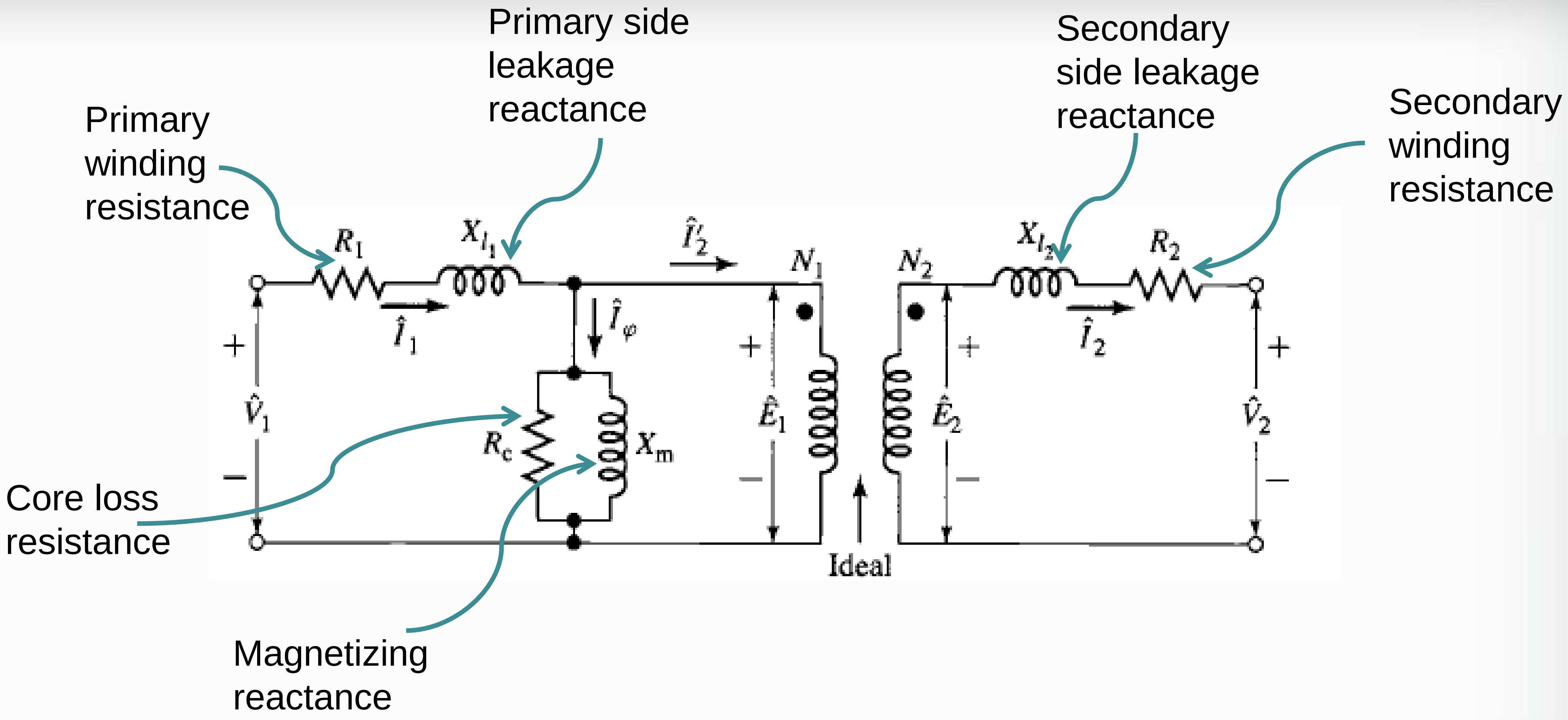
i. Leakage flux : Leakage flux is modelled as a leakage inductor in series, on both primary and secondary sides ⇒ X_{l1}, X_{l2}

ii. Winding resistance: Winding resistance of primary and secondary are both represented by resistors in series, on both primary and secondary sides ⇒ R_1, R_2

iii. Finite permeability(finite magnetizing current): A part of the input current goes as magnetizing current which is accounted by means of a shunt branch connected magnetizing inductor ⇒ X_m

iv. Core losses: Core losses are constant. It varies only with E_1^2 or $\Phi_m^2 f^2$. It is represented by a shunt branch resistor ⇒ R_c

Equivalent Circuit of Transformer



Equivalent Circuit of Transformer

- **Referring impedance on secondary side to primary side**:

We know that Z_2 on the secondary side is given by : $Z_2 = \dfrac{V_2}{I_2}$

Now, Z_2 can be written as follows: $Z_2 = \dfrac{(N_2 / N_1)}{(N_1 / N_2)} \dfrac{V_1}{I_1}$

The impedance measured across primary winding is $Z_2' = \dfrac{V_1}{I_1} = \left(\dfrac{N_1}{N_2}\right)^2 Z_2$

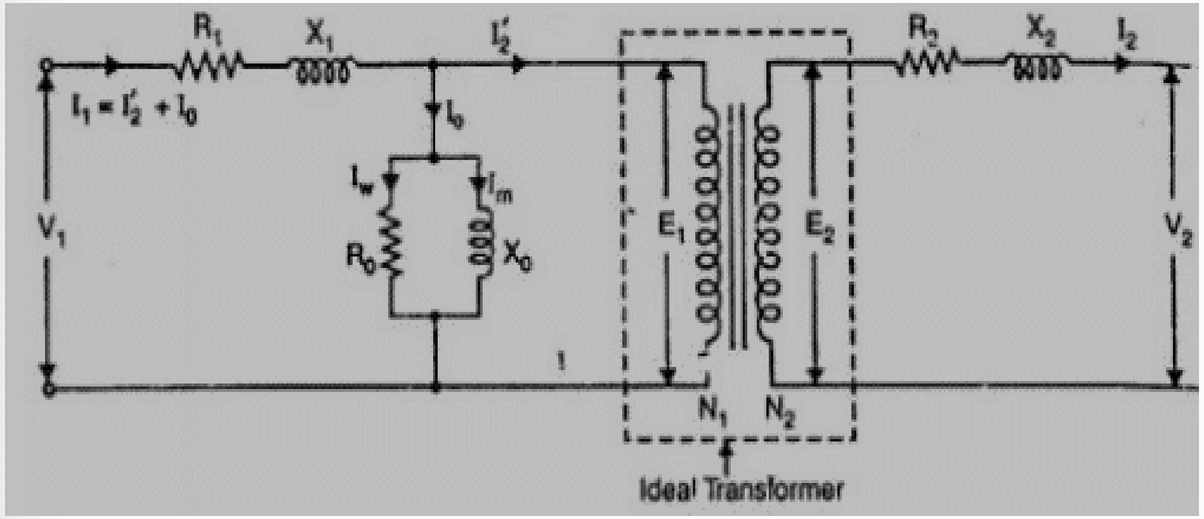
Transformer ratio is given by: $K = \dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{I_1}{I_2}$

Therefore, the impedance on secondary side referred to primary is given by $Z_2' = \dfrac{1}{K^2} Z_1$

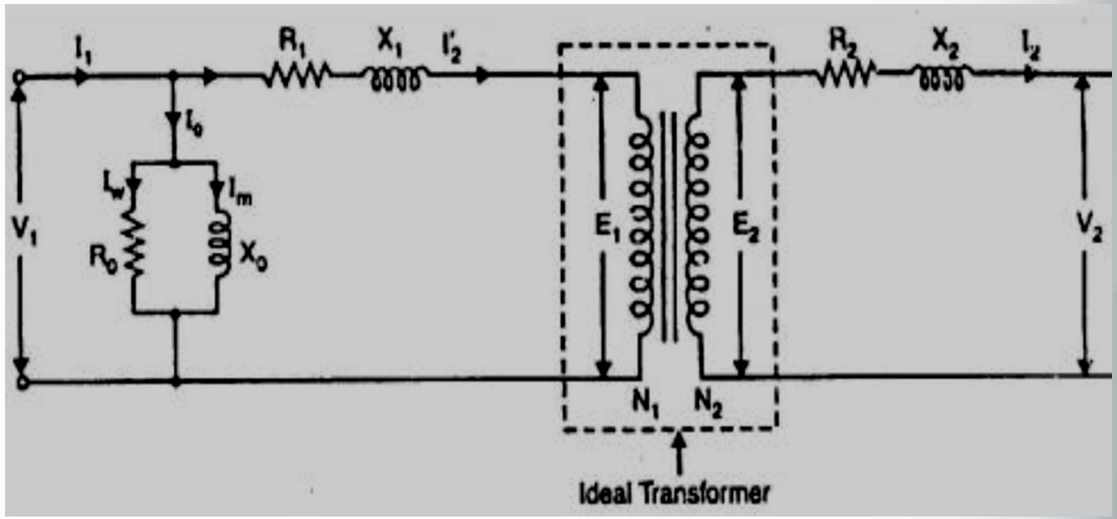
Similarly, when primary impedance is referred to secondary side, we get $Z_1' = K^2 Z_2$

Exact and Approximate Equivalent circuits

Exact Equivalent circuit:

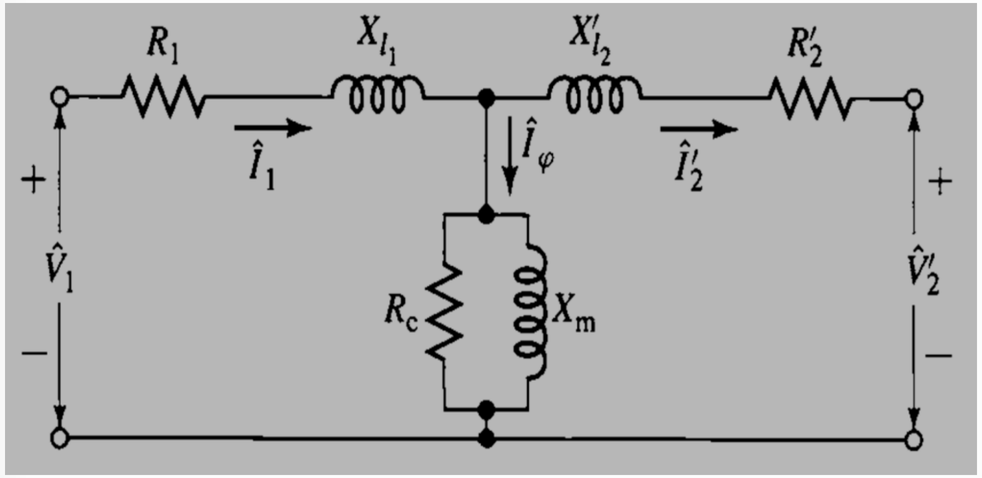


Approximate Equivalent circuit:



Exact and Approximate Equivalent circuits referred to primary side

Exact Equivalent circuit referred to primary side:



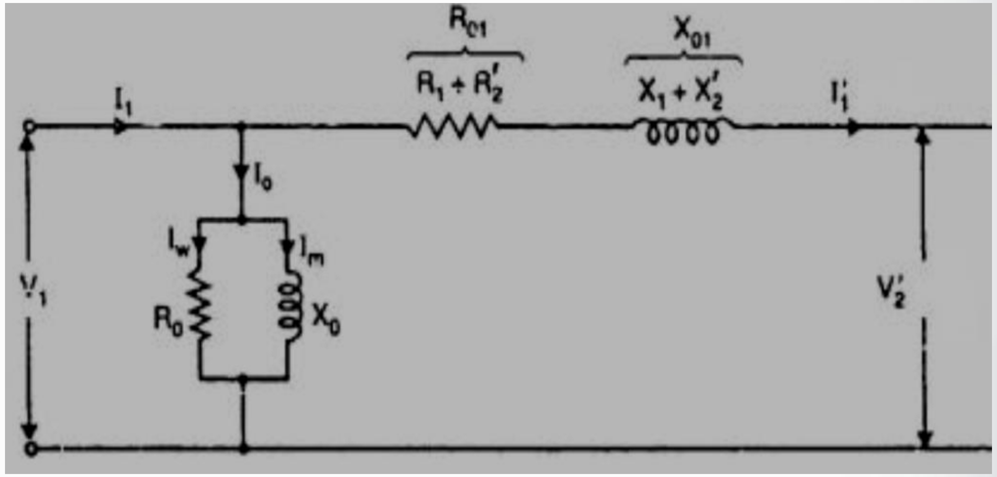
$$R_2' = R_2 / K^2$$

$$X_2' = X_2 / K^2$$

$$V_2' = V_2 / K$$

$$I_2' = KI_2$$

Approximate Equivalent circuit referred to primary side:



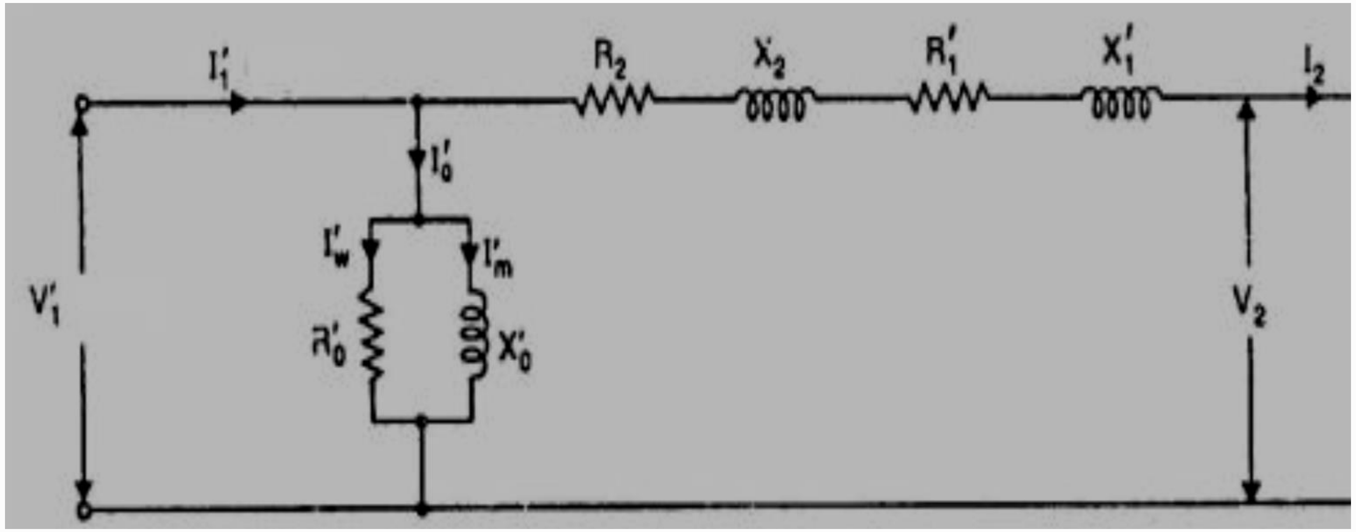
$$R_2' = R_2 / K^2$$

$$X_2' = X_2 / K^2$$

$$V_2' = V_2 / K$$

$$I_2' = KI_2$$

Approximate Equivalent circuit referred to secondary side



$$R_1' = R_1 K^2 \qquad V_1' = V_1 K$$

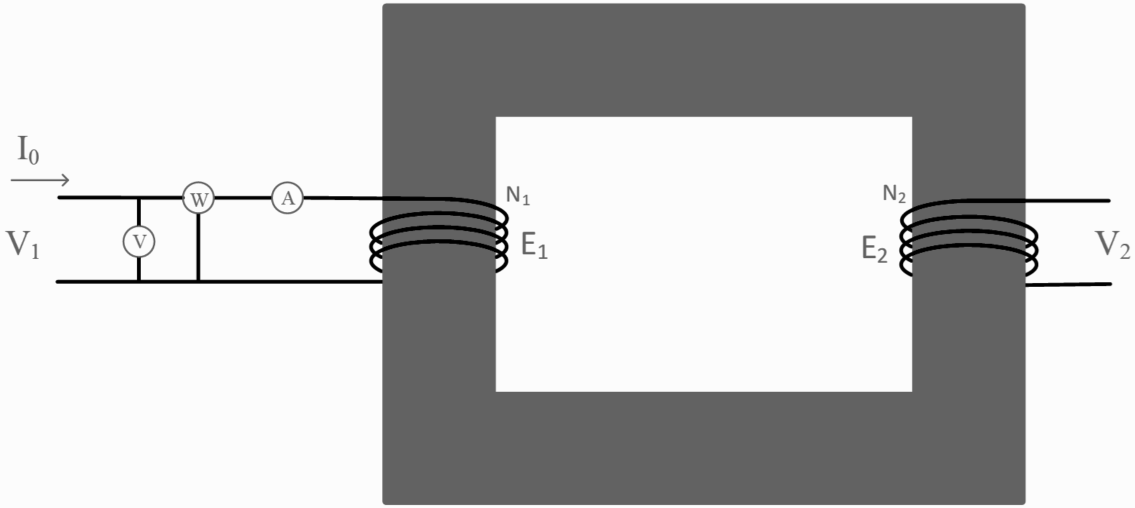
$$X_1' = X_1 K^2 \qquad I_1' = I_1 / K$$

Transformer Tests

- The performance of a transformer can be calculated on the basis of equivalent circuit
- The four main parameters of equivalent circuit are:
  - R_{01} as referred to primary (or secondary R_{02})
  - the equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02})
  - Magnetising susceptance B_0 (or reactance X_0)
  - core loss conductance G_0 (or resistance R_0)
- The above constants can be determined by two tests
  - Open circuit test (O.C test / No load test)
  - Short circuit test (S.C test/Impedance test)
- These tests are economical and convenient
  - these tests furnish the result without actually loading the transformer

Open-circuit Test

In Open Circuit Test the transformer's **secondary winding is open-circuited**, and its **primary winding is connected to a full-rated line voltage**.



$$\text{Core loss} = W_{oc} = V_0 I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{W_{oc}}{V_0 I_0}$$

$$\mathrm{I_c}\, or\ \mathrm{I_w} = I_0 \cos\phi_0$$

$$\mathrm{I_m}\, or\ \mathrm{I}_\mu = I_0 \sin\phi_0 = \sqrt{I_0^2 - I_w^2}$$

$$\mathrm{I_0 = V_0 Y_0}; \quad \therefore \mathrm{Y_o} = \frac{I_0}{V_0}$$

$$\mathrm{R_o} = \frac{\mathrm{V_o}}{\mathrm{I_w}}$$

$$\mathrm{X_o} = \frac{\mathrm{V_o}}{\mathrm{Im}}$$

- To find

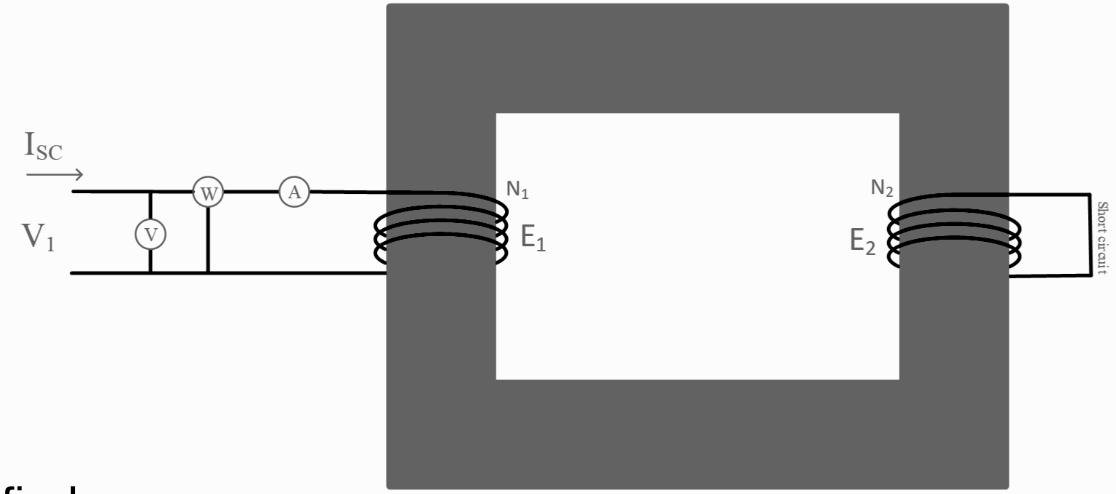
  (i) No load loss or core loss

  (ii) No load current I_o which is helpful in finding G_o(or R_o) and B_o (or X_o)

Short-circuit Test

In Short Circuit Test the **secondary terminals are short circuited**, and the **primary terminals are connected to a fairly low-voltage source.**

The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. The input voltage, current and power is measured.



Full load cu loss = $W_{sc} = I_{sc}^2 R_{01}$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$\therefore X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

- To find

  (i) Full load copper loss – to pre determine the efficiency

  (ii) Z_{01} or Z_{02}; X_{01} or X_{02}; R_{01} or R_{02} - to predetermine the voltage regulation

Transformer Voltage Regulation

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it.

Voltage regulation is defined as the change in secondary terminal voltage when transformer state changes from no load to full load or vice versa, and the change in terminal voltage is expressed as percentage change with respect to the terminal voltage at the initial state of the transformer (no load voltage/full load voltage).

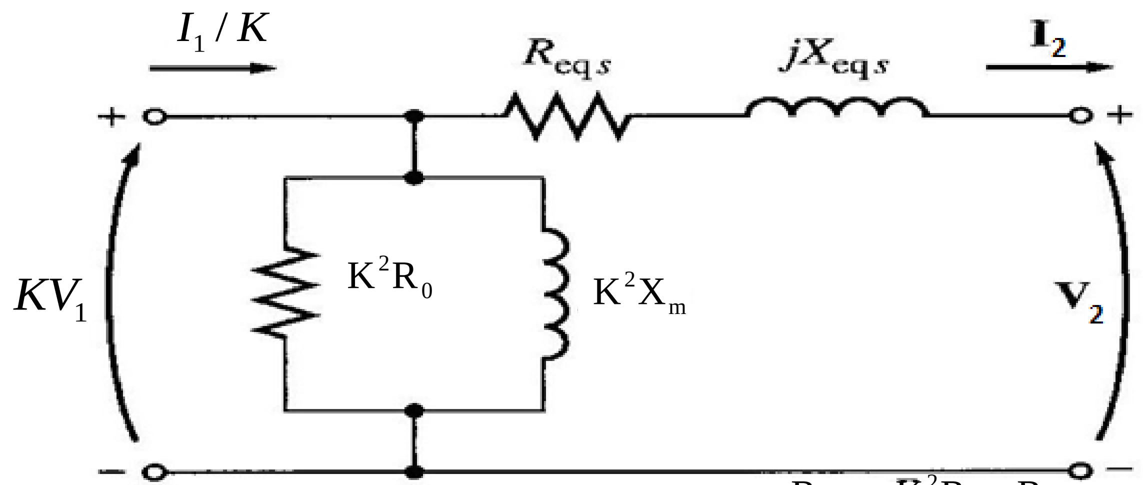
$$\%\,\mathrm{Voltage\,Regulation} = \frac{V_2[\mathrm{no-load}] - V_2[\mathrm{full-load}]}{V_2[\mathrm{no-load}]} \times 100$$

$$\approx \frac{V_1[\mathrm{no-load}] - V_1[\mathrm{full-load}]}{V_1[\mathrm{no-load}]} \times 100$$

Referred to the primary side

Transformer Voltage Regulation

To determine the voltage regulation of a transformer, it is necessary to understand the voltage drops within it. Below is approximate equivalent circuit referred to secondary side



$$R_{eqs} = K^2R_1 + R_2$$

$$X_{eqs} = K^2X_1 + X_2$$

Transformer Voltage Regulation

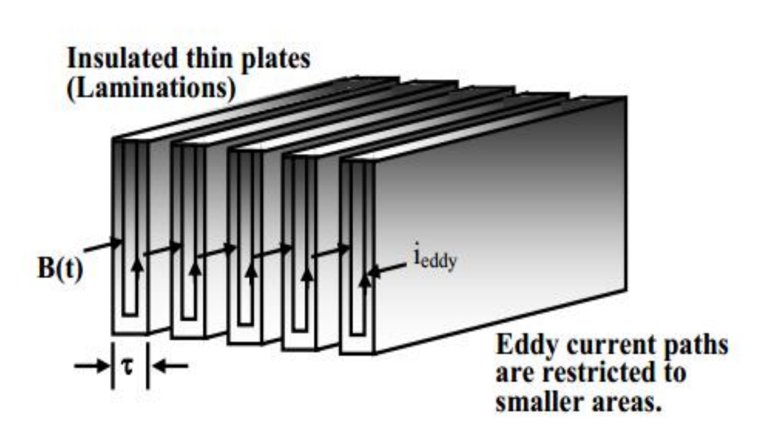
- Ignoring the excitation of the branch (since the current flow through the branch is considered to be small), more consideration is given to the series impedances (R_{eq} +jX_{eq}).
- ***Voltage Regulation depends on magnitude of the series impedance and the phase angle of the current flowing through the transformer.***
- Phasor diagrams will determine the effects of these factors on the voltage regulation. A phasor diagram consist of current and voltage vectors.
- Assume that the reference phasor is the secondary voltage, V_2. Therefore the reference phasor will have 0 degrees in terms of angle.

Based upon the equivalent circuit, apply Kirchoff Voltage Law

$$KV_1 = V_2 + R_{eq} I_2 + jX_{eq} I_2$$

Losses in a Transformer

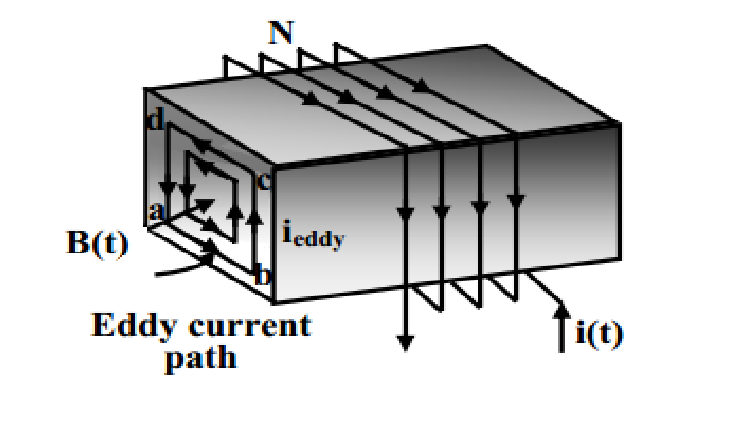
- The power losses in a transformer are of two types:
  1. Core losses/Iron losses /Constant losses
  2. Copper losses/Variable losses



These losses appear in the form of heat and result in (i) increase in temperature and (ii) drop in efficiency

(a) Core or Iron losses (P_i):

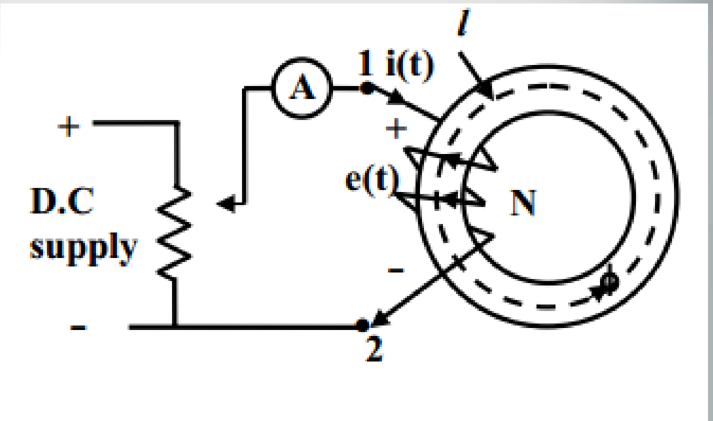
These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux.



Eddy current losses: Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.

Losses in a Transformer contd..

Hysteresis losses: Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are a complex, nonlinear function of the voltage applied to the transformer.



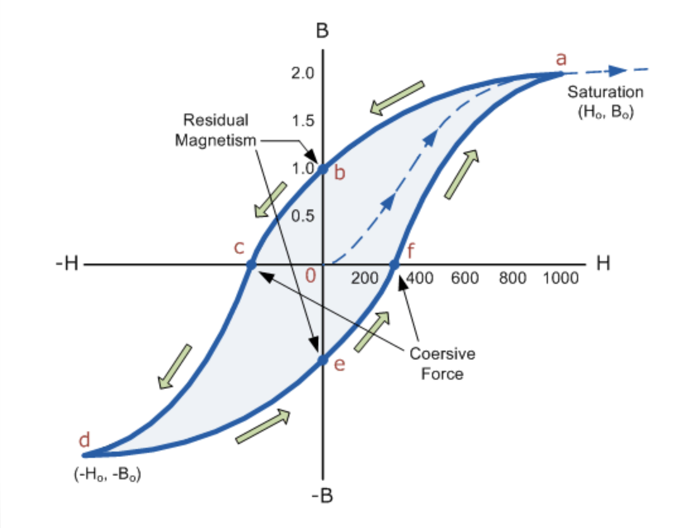
(a) Iron or Core losses

$$P_i = \text{Hysteresis loss} + \text{Eddy current loss} = \text{Constant losses}$$

- (b) Copper (I^2R) losses

Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.

$$\text{Total Cu losses, } P_c = I_1{}^2R_1 + I_2{}^2R_2 = I_1{}^2R_{01} + I_2{}^2R_{02}$$



Transformer Efficiency

$$\eta = \frac{\text{Power Output}}{\text{Power Input}}$$

$$= \frac{\text{Power Input} - \text{Losses}}{\text{Power Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Power Input}}$$

$$= 1 - \frac{P_{cu} + P_{\text{core loss}}}{P_{cu} + P_{\text{core loss}} + V_2 I_2 \cos\theta}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{V_2 I_2 \cos\theta}{P_{cu} + P_{core} + V_2 I_2 \cos\theta} \text{x}100\%$$

Usually the efficiency for a power transformer is between 0.9 to 0.99. The higher the rating of a transformer, the greater is its efficiency.

Three Phase Transformers

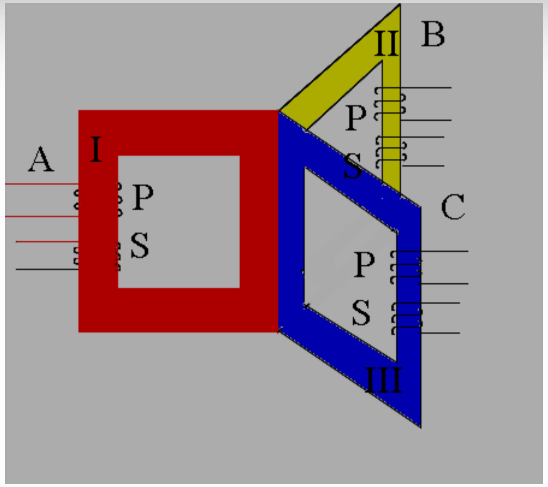
- Three phase transformer can be formed by two ways:

  (a) Three identical single-phase transformers may be connected so that the three windings of one voltage (say primary) are connected in Δ (or Y) and other three windings of the other voltage (secondary) are connected in Y (or Δ) to form a three phase transformer of different connections Δ-Y, Y-Δ, Y-Y and Δ-Δ

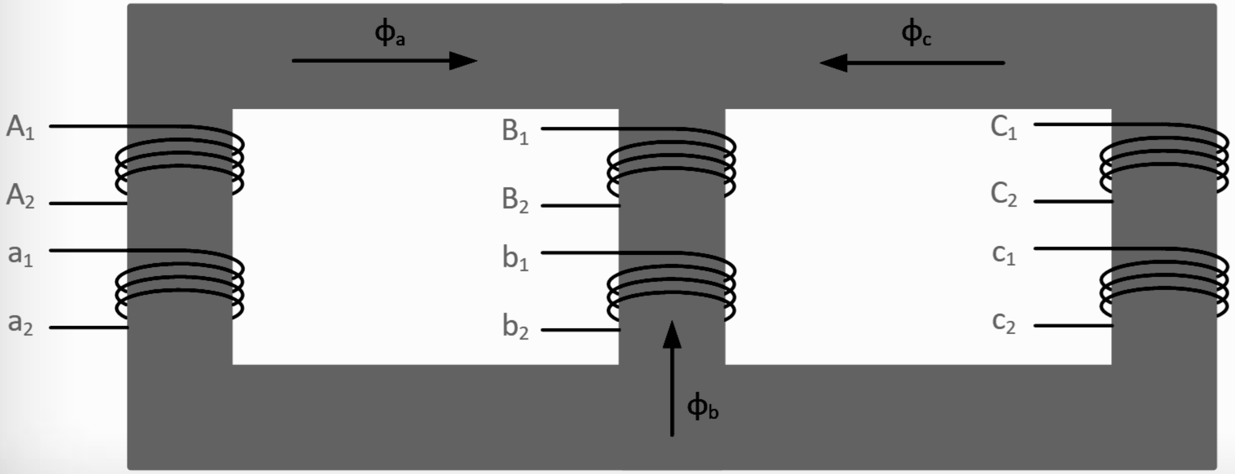
  (b) All three phases are wound on the same core, as a single unit

Three phase bank	Single unit
Reliable. If one phase is faulty , we can repair it very easily	Less reliable. Entire unit needs to be repaired if one phase is faulty
If one phase is fault, it can be operated with the other two in open-delta connection	Can't supply power to load using two phases
Cost is more	Less cost compared to 3-phase bank, due to less iron requirement because of common core

Three phase transformers



3ϕ transformer comprised of three single phase units is called transformer bank



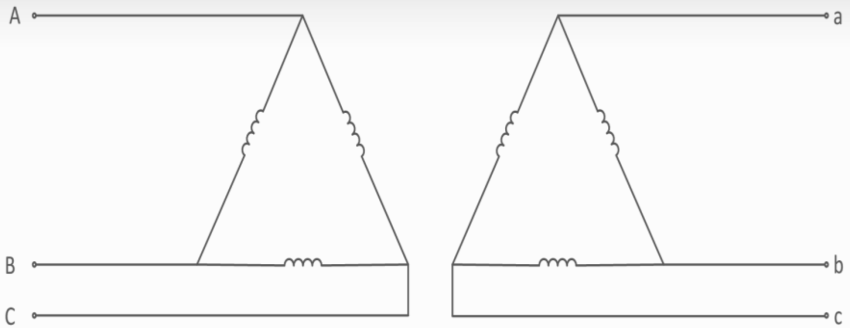
Usually 3ϕ transformers are constructed so all windings share a common core, i.e., as a single unit

Three Phase Transformer Connections

- There are 4 different ways to connect 3ϕ transformers

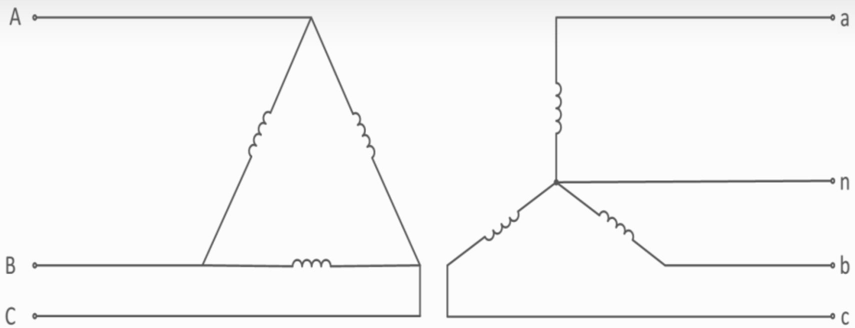
Connections	Line voltages	Phase displacement between primary and secondary
Δ-Δ	$V_1 : V_1/ K$	0^0
Y-Y	$V_1 : V_1/ K$	0^0
Δ-Y	$V_1 : (\sqrt{3} * V_1)/ K$	30^0
Y-Δ	$V_1 : (V_1/ \sqrt{3})*K$	-30^0

Three Phase Transformer Connections



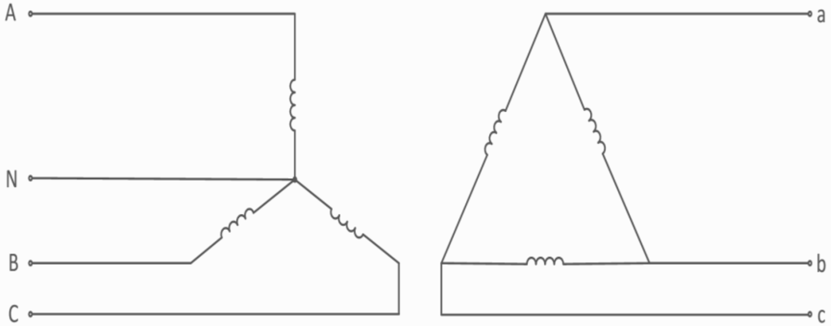
Δ/Δ connection

Phase shift = $0°$



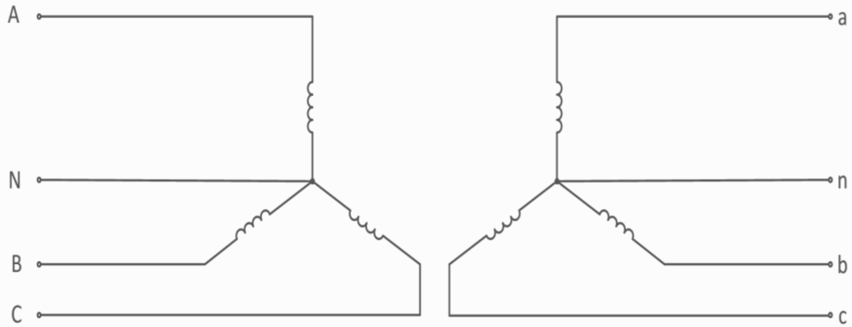
Δ/Y connection

Phase shift = $30°$



Y/Δ connection

Phase shift = $-30°$

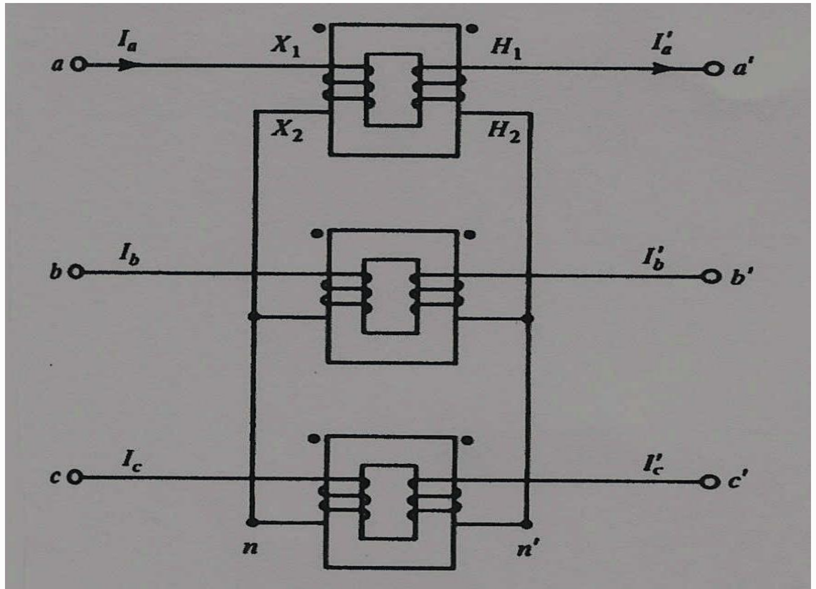


Y/Y connection

Phase shift = $0°$

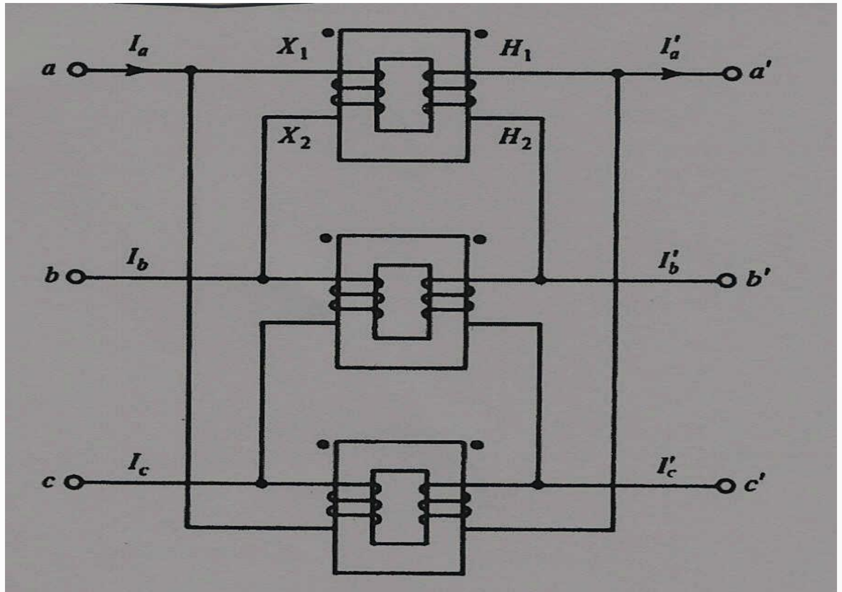
Three Phase Transformer Connections

Y-Y connection



Y-Y Connection

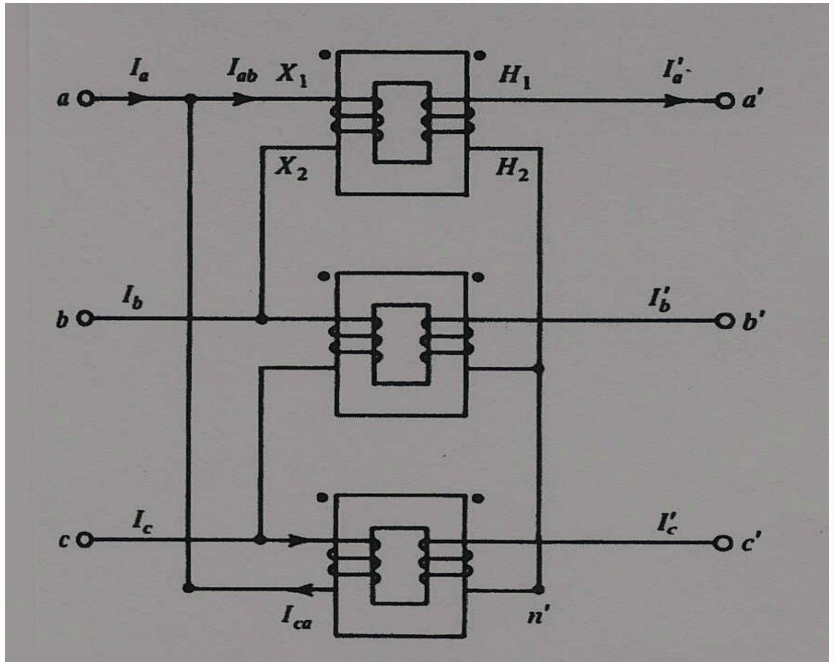
Δ-Δ connection



Δ-Δ Connection

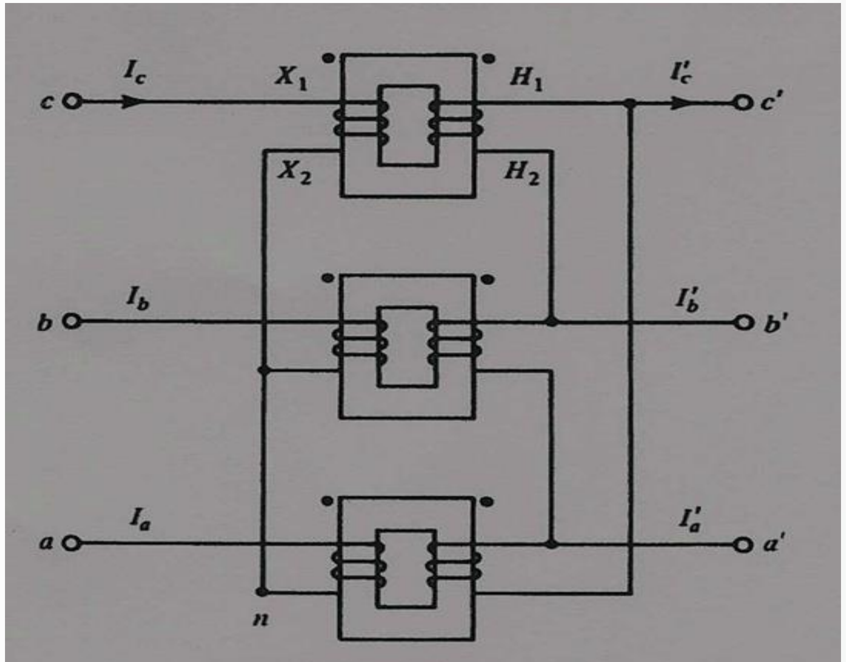
Three Phase Transformer Connections

Δ-Y connection



Δ-Y Connection

Y-Δ connection



Y-Δ Connection

Detailed 3-phase model