Symbolic reduction of block diagrams and signal flow graphs

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***Abstract*—In this paper a very easy symbolic method for reduction of block diagrams or signal flow graphs with Matlab is presented. The method can be applied to every kind of diagram and it is possible to obtain in one stage all transfer functions for a given input. The method is validated with some examples, including examples from some well-known textbooks in control.**

***Index Terms*—Block diagrams, signal flow graphs, symbolic computation, Matlab.**

I. Introduction

THIS short paper shows the possibilities to apply the Symbolic Math Toolbox of Matlab to reduce in a easy way a block diagram or signal-flow graph.

The methods for reducing block diagrams or signal-flow graphs are well known and include Mason rule [1], block manipulation [2], and equations for signals [3][4][5][6][7]. Here the third case is explored, which is the obvious method for reduction, but it is not described enough clear in engineering references. The solution proposed here improves the solution given in [6][7].

II. Description of the Method

The method of signal algebra is very clear for the example shown in Fig. 1. The relations between signals are expressed in three equations:

$$b = a - d \quad c = Gb \quad d = Hc$$

The solution of equations is the transfer function of the block diagram:

$$c = G(a-d) = G(a-Hc) = Ga - GHc$$
$$(1+GH)c = Ga$$
$$\frac{c}{a} = \frac{G}{1+GH}$$

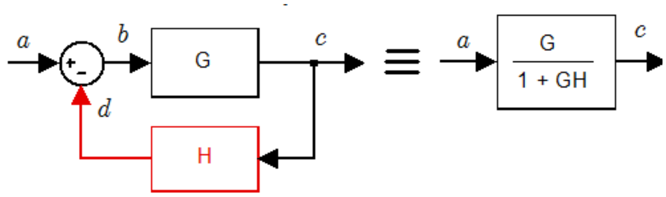

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Fig. 1. Block diagram with blocks in a loop

III. Example

This example shows how to give and solve the equations of a more complicated block diagram like Fig. 2.

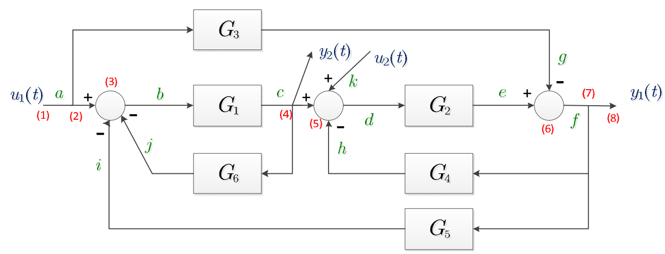

Fig. 2. Block diagram of example

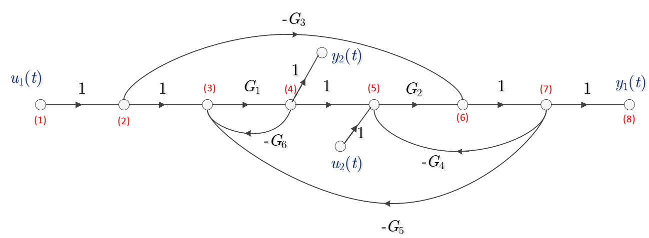

Fig. 3. Equivalent signal flow graph of example

The set of equations for signals is:

$$\begin{array}{lll} b = a - i - j & e = G_2 d & h = G_4 f \\ c = G_1 b & f = e - g & i = G_5 f \\ d = k + c - h & g = G_3 a & j = G_6 c \end{array}$$

This system of equations can be expressed in the next matrix, where the last column corresponds to input signal:

$$\begin{array}{cc} & \begin{array}{cccccccccc} b & c & d & e & f & g & h & i & j & a \end{array} \\ \mathbf{M} = & \left[\begin{array}{ccccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ -G_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -G_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -G_3 \\ 0 & 0 & 0 & 0 & -G_4 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -G_5 & 0 & 0 & 1 & 0 & 0 \\ 0 & -G_6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

The Matlab code for solution of this system of equations is (it may be better to define a matrix M with zeros and introduce every non-zero component explicitly like M(m,n)=H):

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syms G1 G2 G3 G4 G5 G6
M=[1 0 0 0 0 0 0 1 1 -1;-G1 1 0 0 0 0 0 0 0 0;0 -1 1 0 0 0 1 0 0 0;0 0 -G2 1 0 0 0 0 0 0;0 0 0 -1 1 1 0 0 0 0;0 0 0 0 0 1 0 0 0 -G3;0 0 0 0 -G4 0 1 0 0 0;0 0 0 0 -G5 0 0 1 0 0;0 -G6 0 0 0  0 0 0 1 0];
N=rref(M);
Sol = -N(5,10)
```

The solution is given at final of this page. The fifth row (variable f) is the solution of problem (the other rows give the solution for other relation between any signal and first input):

$$\frac{Y_1(s)}{U_1(s)} = \frac{G_1G_2 - G_3 - G_1G_3G_6}{1 + G_2G_4 + G_1G_6 + G_1G_2G_5 + G_1G_2G_4G_6}$$

IV. Conclusion

This short paper shows the solution of the problem of reduction of block diagrams or signal flow graphs using the Symbolic Math Toolbox of Matlab. The code is very simple and can be applied to any case. It is presented only one example, but several examples of textbooks like [2] were tested without problems.

References

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[ 1, 0, 0, 0, 0, 0, 0, 0, 0,                    -(G2*G4 + G3*G5 + 1)/(G2*G4 + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 1, 0, 0, 0, 0, 0, 0, 0,              -(G1 + G1*G2*G4 + G1*G3*G5)/(G2*G4 + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 0, 1, 0, 0, 0, 0, 0, 0,      -(G1 + G3*G4 + G1*G3*G5 + G1*G3*G4*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 0, 0, 1, 0, 0, 0, 0, 0, -(G1*G2 + G2*G3*G4 + G1*G2*G3*G5 + G1*G2*G3*G4*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6
+ 1)]
[ 0, 0, 0, 0, 1, 0, 0, 0, 0,                  (G3 - G1*G2 + G1*G3*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 0, 0, 0, 0, 1, 0, 0, 0,                                                                            -G3]
[ 0, 0, 0, 0, 0, 0, 1, 0, 0,           (G3*G4 - G1*G2*G4 + G1*G3*G4*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 0, 0, 0, 0, 0, 0, 1, 0,           (G3*G5 - G1*G2*G5 + G1*G3*G5*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
[ 0, 0, 0, 0, 0, 0, 0, 0, 1,       -(G1*G6 + G1*G2*G4*G6  + G1*G3*G5*G6)/(G2*G4  + G1*G6 + G1*G2*G5 + G1*G2*G4*G6 + 1)]
```