

**Table of Laplace Transforms**

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. $e^{at}$	$\frac{1}{s-a}$
3. $t^n, n=1,2,3,\dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}, n=1,2,3,\dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n=1,2,3,\dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$ <a href="#">Heaviside Function</a>	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ <a href="#">Dirac Delta Function</a>	$e^{-cs}$
27. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	28. $u_c(t) g(t)$	$e^{-cs} \mathcal{L}\{g(t+c)\}$
29. $e^{ct} f(t)$	$F(s-c)$	30. $t^n f(t), n=1,2,3,\dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t} f(t)$	$\int_s^\infty F(u) du$	32. $\int_0^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau) g(\tau) d\tau$	$F(s) G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2 F(s) - sf'(0) - f''(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		

## Table Notes

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.
2. Recall the definition of hyperbolic functions.

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \qquad \sinh(t) = \frac{e^t - e^{-t}}{2}$$

3. Be careful when using “normal” trig function vs. hyperbolic functions. The only difference in the formulas is the “+ a<sup>2</sup>” for the “normal” trig functions becomes a “- a<sup>2</sup>” for the hyperbolic functions!
4. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1} dx$$

If  $n$  is a positive integer then,

$$\Gamma(n+1) = n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma(p+1) = p\Gamma(p)$$

$$p(p+1)(p+2)\cdots(p+n-1) = \frac{\Gamma(p+n)}{\Gamma(p)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Table 1: Table of Laplace Transforms

Number	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u_s(t)$	$\frac{1}{s}$
3	$t$	$\frac{1}{s^2}$
4	$t^n$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}$	$\frac{1}{(s+a)}$
6	$te^{-at}$	$\frac{1}{(s+a)^2}$
7	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
8	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
9	$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
10	$be^{-bt} - ae^{-at}$	$\frac{(b-a)s}{(s+a)(s+b)}$
11	$\sin at$	$\frac{a}{s^2+a^2}$
12	$\cos at$	$\frac{s}{s^2+a^2}$
13	$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
14	$e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$
15	$1 - e^{-at}(\cos bt + \frac{a}{b} \sin bt)$	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$

Table 1: Properties of Laplace Transforms

Number	Time Function	Laplace Transform	Property
1	$\alpha f_1(t) + \beta f_2(t)$	$\alpha F_1(s) + \beta F_2(s)$	Superposition
2	$f(t - T)u_s(t - T)$	$F(s)e^{-sT}; \quad T \geq 0$	Time delay
3	$f(at)$	$\frac{1}{a}F(\frac{s}{a}); \quad a > 0$	Time scaling
4	$e^{-at}f(t)$	$F(s + a)$	Shift in frequency
5	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$	First-order differentiation
6	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - f^{(1)}(0^-)$	Second-order differentiation
7	$f^n(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f^{(1)}(0) - \dots - f^{(n-1)}(0)$	n <sup>th</sup> -order differentiation
6	$\int_{0^-}^t f(\zeta)d\zeta$	$\frac{1}{s}F(s)$	Integration
7	$f(0^+)$	$\lim_{s \rightarrow \infty} sF(s)$	Post-initial value theorem
8	$\lim_{t \rightarrow \infty} f(t)$	$\lim_{s \rightarrow 0} sF(s)$	Final value theorem
9	$tf(t)$	$-\frac{dF(s)}{ds}$	Multiplication by time