# EN304- ELECTRICAL ENERGY SYSTEMS

Department of Energy Science and Engineering

This is only reading material (not lecture slides)

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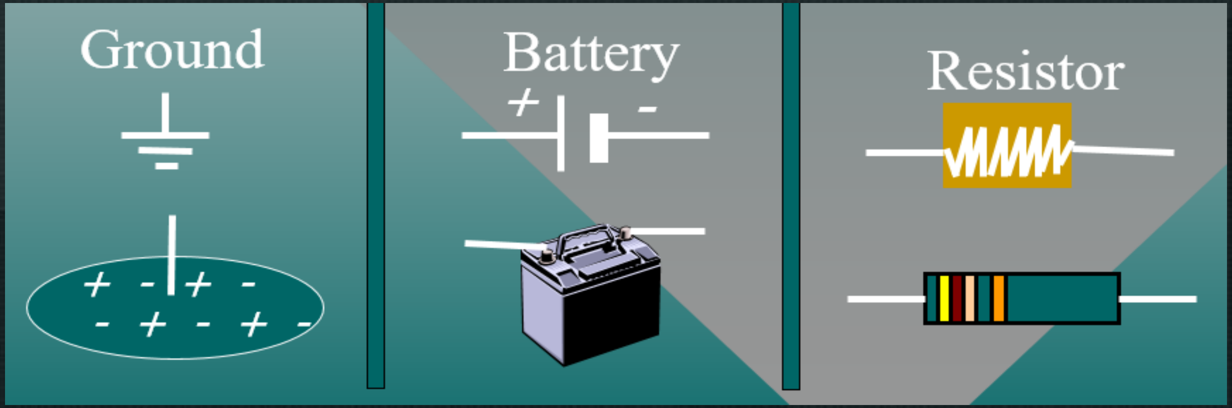
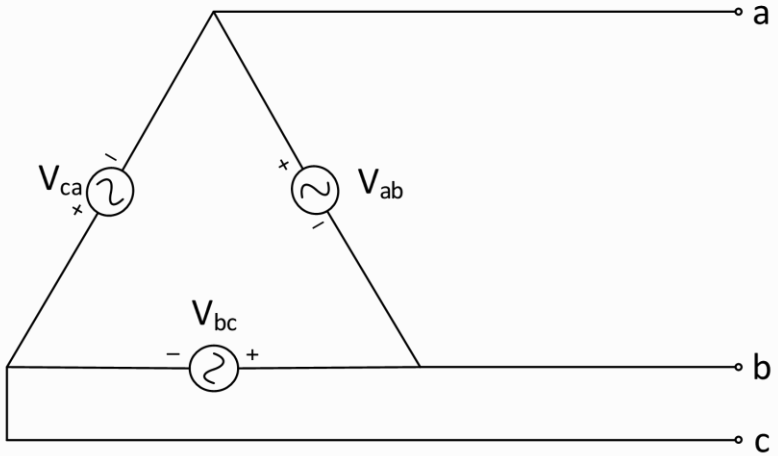
## SLIDE DECK 1: THREE PHASE CIRCUITS

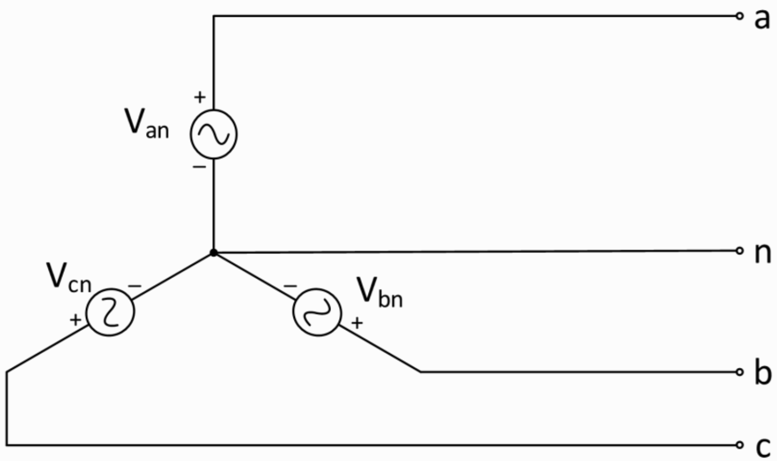


# Concept of three phase system

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected or delta-connected as shown in the figure.




Delta-connected source




wye-connected source

Where $v_{an}$, $v_{bn}$ and $v_{cn}$ are respectively the voltage differences between the phases a, b, c and neutral n. and they are called phase voltages.

# Concept of three phase system (wye connected source)

- Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.
- Note that for balanced phase voltages the following equations are valid:

$$v_{an} + v_{bn} + v_{cn} = 0$$
$$|v_{an}| = |v_{bn}| = |v_{cn}|$$

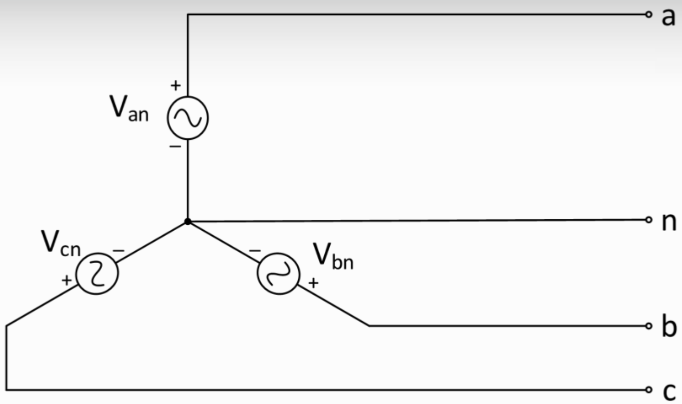
- For balanced phase voltage, assuming

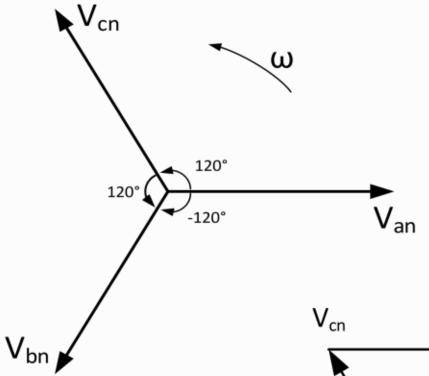
$|V_{an}| = |V_{bn}| = |V_{cn}| = V_P$, we can write the following phasors of phase voltages,

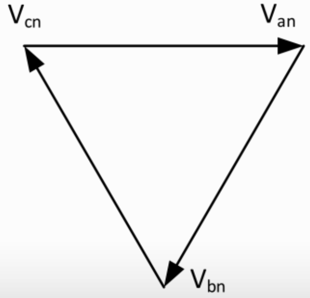
$$V_{an} = V_P\angle 0°$$
$$V_{bn} = V_P\angle -120°$$
$$V_{cn} = V_P\angle -240° = V_P\angle +120°$$

See the phasor diagram and notice that the positive direction is counter clockwise direction.

- Notice that the $120°$ phase difference between the phases is the reason of $v_{an} + v_{bn} + v_{cn} = 0$. (prove it mathematically)



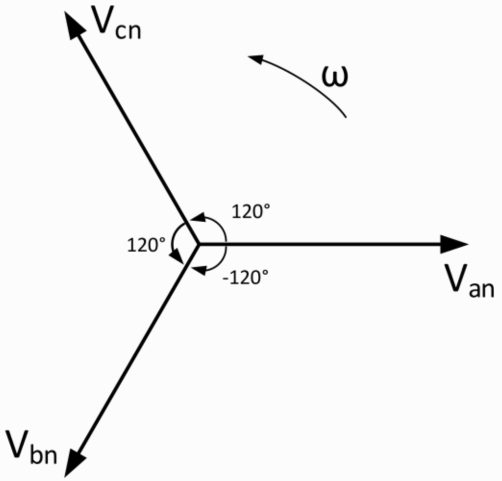



# Phase sequence in three phase system

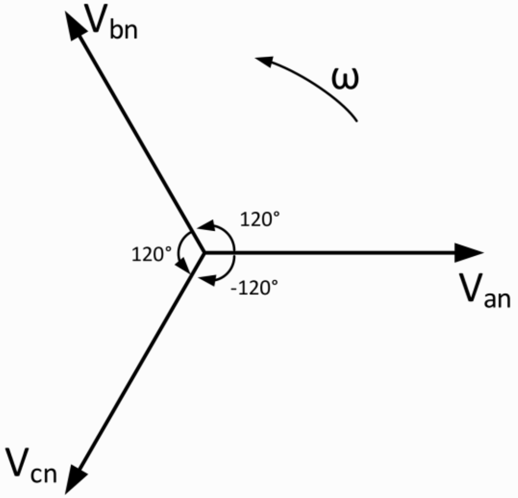
- The phase sequence is the time order in which the voltages pass through their respective maximum values.
- The phase sequence is determined by the order in which the phasors pass through a fixed point in the phasor diagram. abc sequence is called positive sequence and acb sequence is called negative sequence. See the following phasors, mathematical representations and waveforms for both sequences.

**Positive sequence**




$$V_{an} = V_P\angle 0°$$
$$V_{bn} = V_P\angle -120°$$
$$V_{cn} = V_P\angle -240° = V_P\angle +120°$$

**Negative sequence**



$$V_{an} = V_P\angle 0°$$
$$V_{cn} = V_P\angle -120°$$
$$V_{bn} = V_P\angle -240° = V_P\angle +120°$$

Concept of $3\phi$ system

Y-connected source

Phase sequence

Line & phase voltages in Y-connected sys.

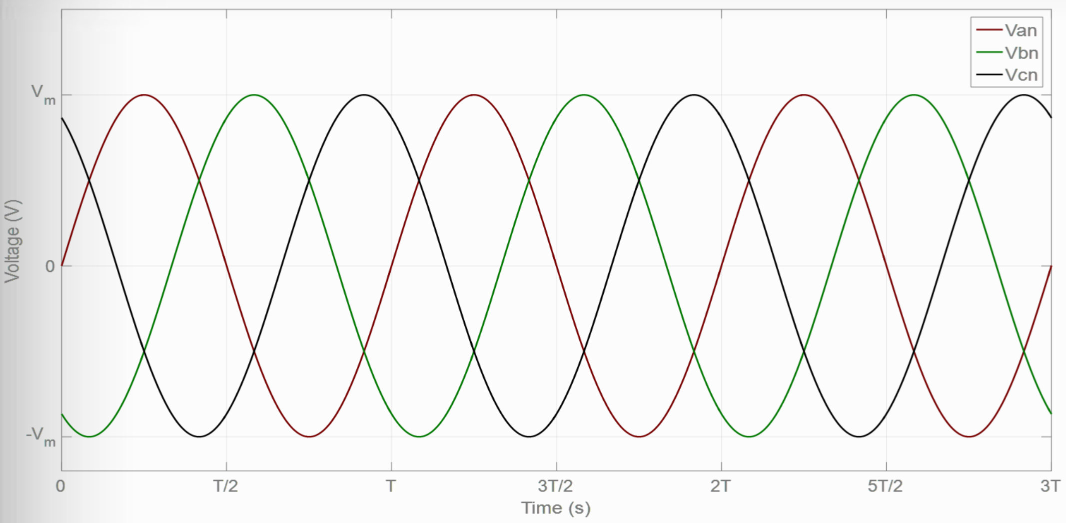
Line & phase currents in Δ-connected sys.

Power in $3\phi$ system

Why $3\phi$ system?

# Phase sequence in three phase system

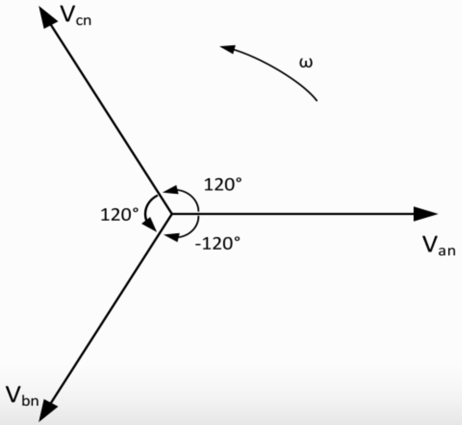
## Positive sequence



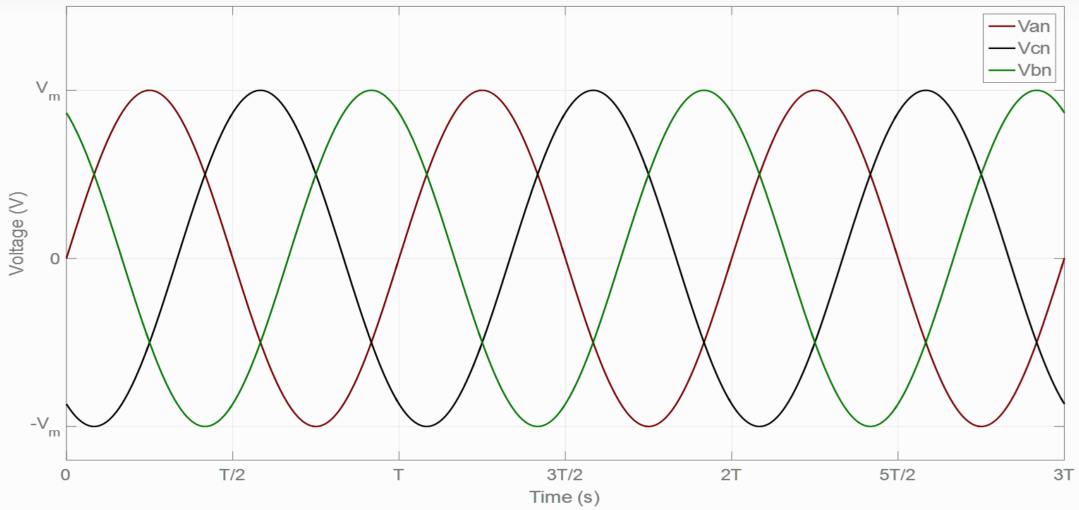

$$v_{an}(t) = V_{an}\sin(\omega t)$$
$$v_{bn}(t) = V_{bn}\sin(\omega t - 120°)$$
$$v_{cn}(t) = V_{cn}\sin(\omega t - 240°)$$

OR

$$v_{an}(t) = V_{an}\sin(\omega t)$$
$$v_{bn}(t) = V_{bn}\sin(\omega t + 240°)$$
$$v_{cn}(t) = V_{cn}\sin(\omega t + 120°)$$



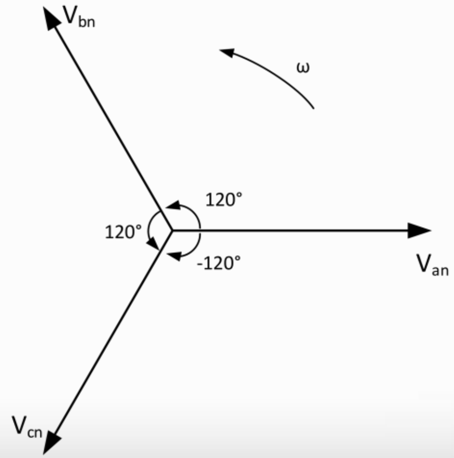

## Negative sequence




$$v_{an}(t) = V_{an}\sin(\omega t)$$
$$v_{bn}(t) = V_{bn}\sin(\omega t + 120°)$$
$$v_{cn}(t) = V_{cn}\sin(\omega t + 240°)$$

OR

$$v_{an}(t) = V_{an}\sin(\omega t)$$
$$v_{bn}(t) = V_{bn}\sin(\omega t - 240°)$$
$$v_{cn}(t) = V_{cn}\sin(\omega t - 120°)$$




- Concept of $3\phi$ system
- Y-connected source
- Phase sequence
- Line & phase voltages in Y-connected sys.
- Line & phase currents in Δ-connected sys.
- Power in $3\phi$ system
- Why $3\phi$ system?

# Line voltage and phase voltage in balanced system (wye connected source)

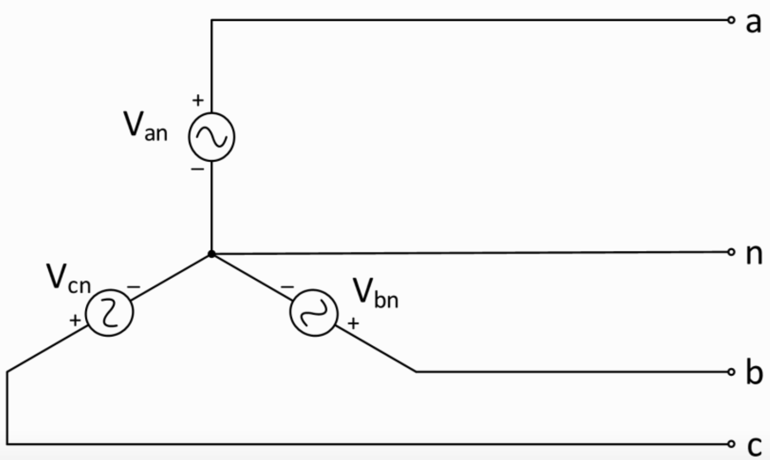
- The voltage difference between any two lines is called line voltage ($V_{ab}, V_{bc}, V_{ca}$). These line voltages are related to phase voltages.

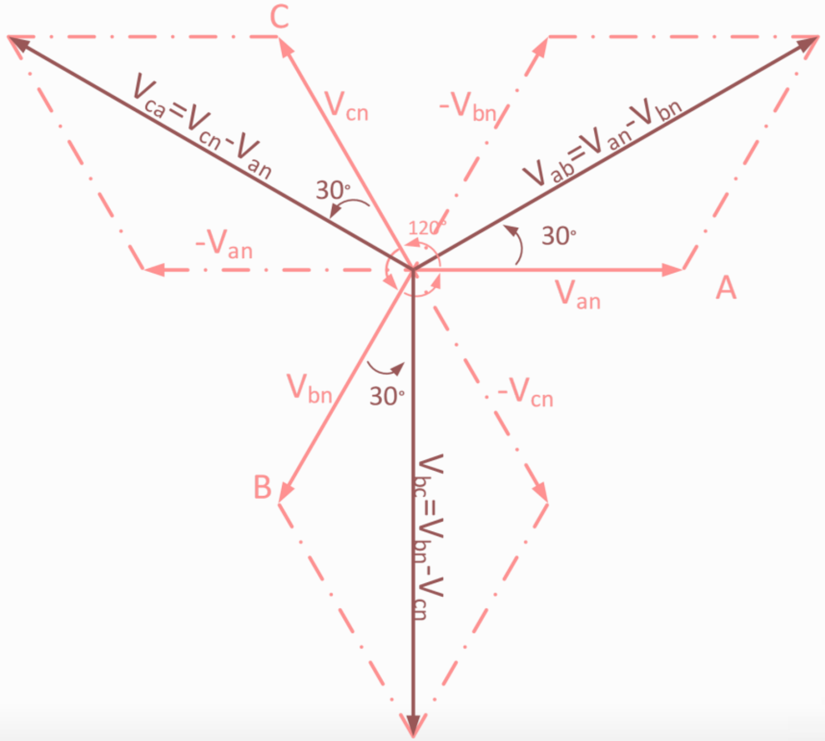
$$V_{ab} = V_{an} - V_{bn} = V_P\angle 0° - V_P\angle -120° = \sqrt{3}V_P\angle 30°$$

Similarly,

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_P\angle -90°$$
$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_P\angle -210°$$

See the phasor diagram of phase and line voltage.



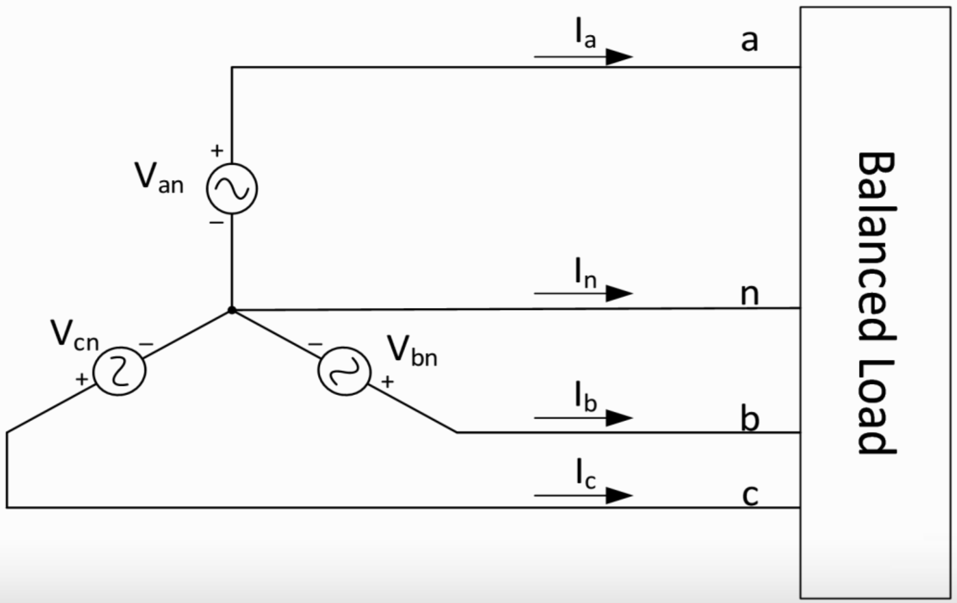



# Line voltage and phase voltage in balanced system (wye connected source)

- Notice that in balanced voltage system: $|V_{ab}| = |V_{bc}| = |V_{ca}| = |V_L| = \sqrt{3}\,|V_P|$
$$|V_{an}| = |V_{bn}| = |V_{cn}| = |V_P|$$
where $V_L$ is the line voltage.

- When a balanced wye-connected source is loaded by balanced load, the phase currents are balanced as well. If $I_a$ is taken as reference, then
$$I_b = I_a\angle -120°$$
$$I_c = I_a\angle -240°$$

The neutral current is calculated by applying KCL,
$$I_n = I_a + I_b + I_c = 0$$
Hence, when a balanced wye-connected source is loaded by balanced load, the neutral current is zero
And the neutral voltage is zero as well.
- Note that in wye-connected systems line currents
And phase currents are equal.

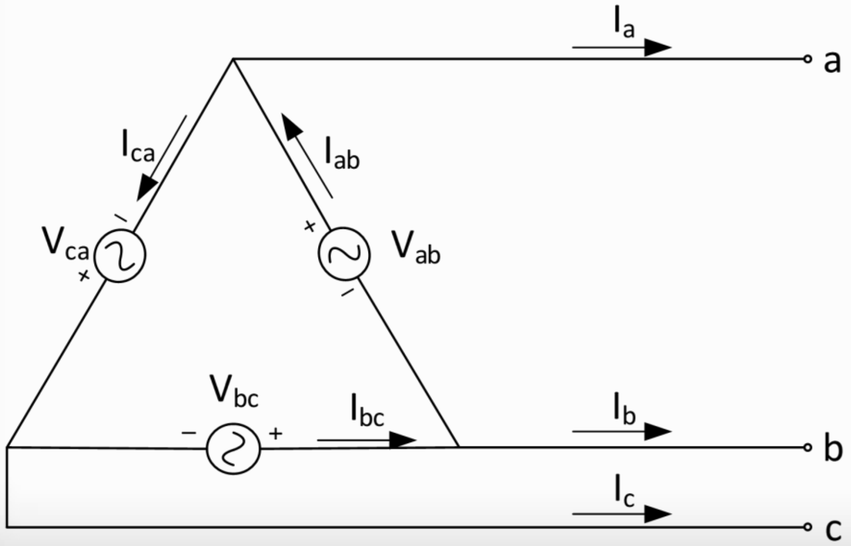


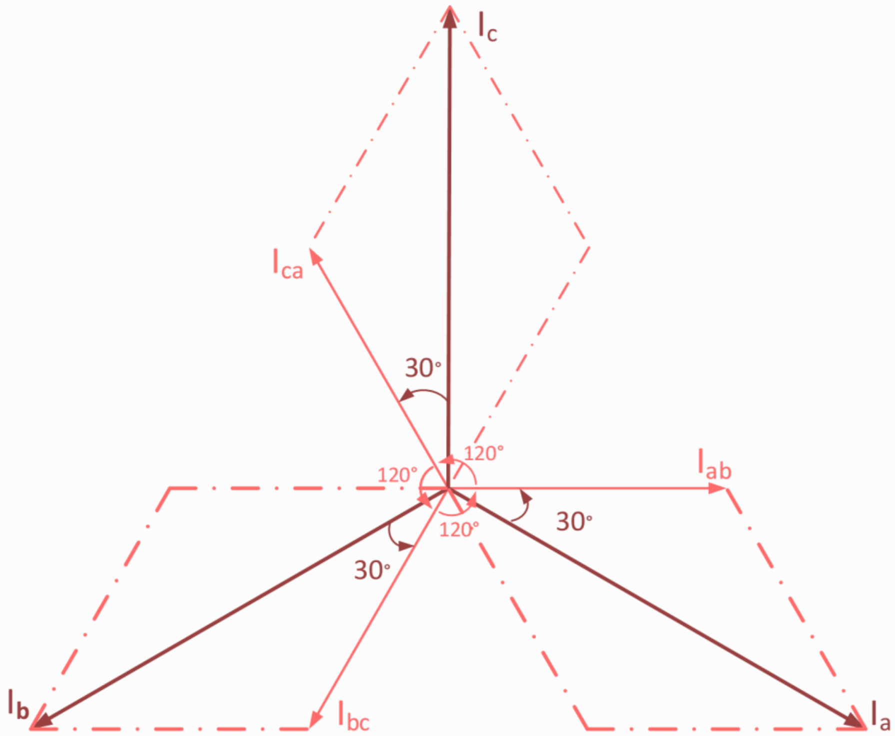
# Three phase balanced system (delta connected source)

- In delta connected system, neutral does not exist.
- $I_{ab}$, $I_{bc}$ and $I_{ca}$ are the phase currents. $I_a$, $I_b$ and $I_c$ are the line currents. Assuming that source and the load are balanced, the phase currents can be written as follows,

$$I_{ab} = I_P\angle 0°$$
$$I_{bc} = I_P\angle -120°$$
$$I_{ca} = I_P\angle -240°$$

Where $I_P$ is the phase current.




- Concept of $3\phi$ system
- Y-connected source
- Phase sequence
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- Power in $3\phi$ system
- Why $3\phi$ system?

# Three phase balanced system (delta connected source)

- Applying KCL at the nodes a, b and c we get,

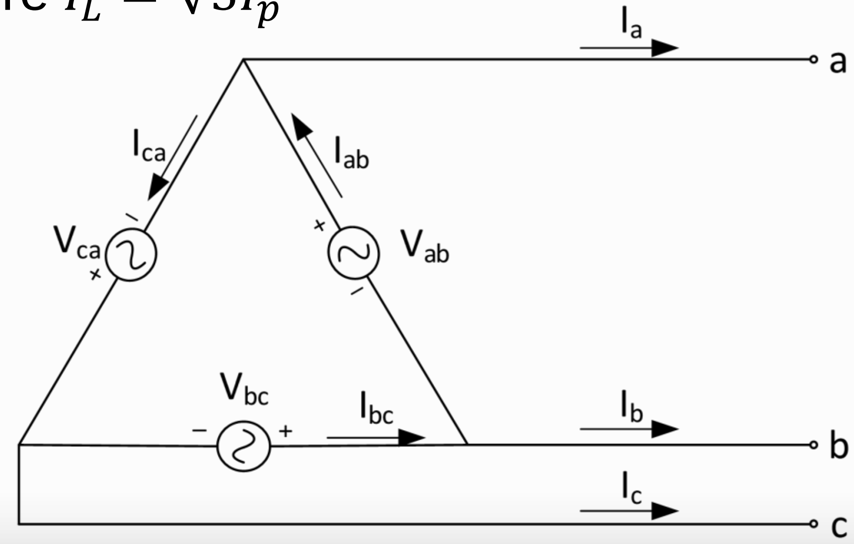
$$I_a = I_{ab} - I_{ca} = I_P\angle 0° - I_P\angle -240° = \sqrt{3}I_P\angle -30°$$

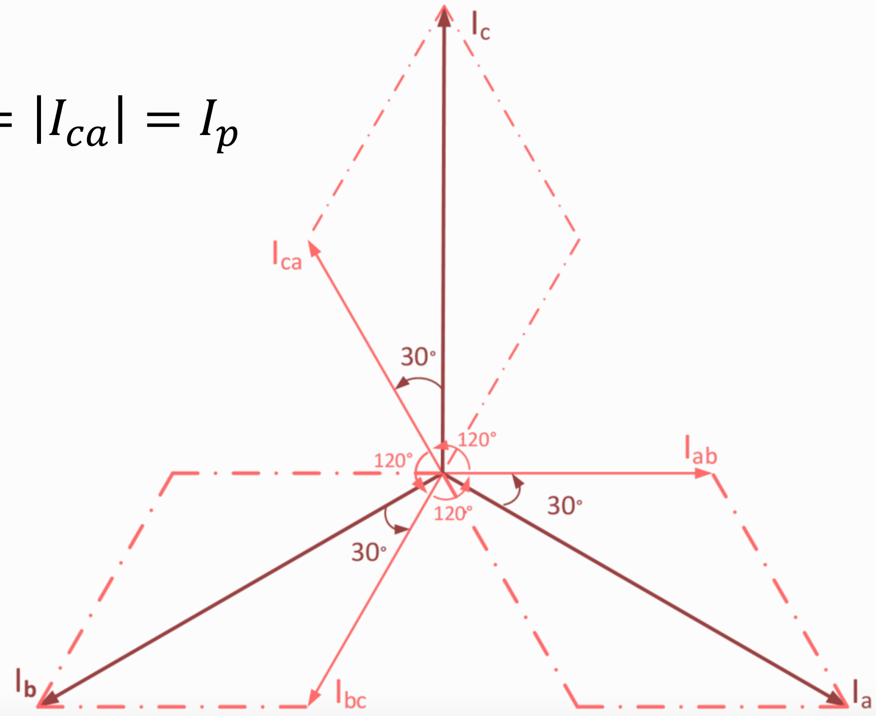
Similarly,

$$I_b = I_{bc} - I_{ab} = \sqrt{3}\,I_P\angle -150°$$
$$I_c = I_{ca} - I_{bc} = \sqrt{3}\,I_P\angle -270°$$

- Notice that $|I_a| = |I_b| = |I_c| = I_L$ and $|I_{ab}| = |I_{bc}| = |I_{ca}| = I_p$

where $I_L = \sqrt{3}I_p$



# Power in balanced three phase system

- For a Y-connected load, the phase voltages are given by,

$$v_{an}(t) = \sqrt{2}v_P \sin(\omega t)$$
$$v_{bn}(t) = \sqrt{2}v_P \sin(\omega t - 120°)$$
$$v_{cn}(t) = \sqrt{2}v_P \sin(\omega t - 240°)$$

- If the load impedance is $Z_Y = Z\angle\theta$, then the phase currents lag their corresponding voltages by $\theta$ as follow,

$$i_a(t) = I \sin(\omega t - \theta)$$
$$i_b(t) = I \sin(\omega t - \theta - 120°)$$
$$i_c(t) = I \sin(\omega t - \theta - 240°)$$

Where $I = \frac{\sqrt{2}v_P}{Z}$

# Power in balanced three phase system

- The total instantaneous power in the load is the sum of the instantaneous powers in the three phases,

$$P = P_a + P_b + P_c = v_{an} i_a + v_{bn} i_b + v_{cn} i_c \Longrightarrow \boxed{P = 3 v_P i_P \cos(\theta) = \sqrt{3}\, v_L i_L \cos(\theta) \quad (*)}$$

- Similarly, reactive power and complex power can be calculated as follows,

$$\boxed{Q = 3 v_P i_P \sin(\theta) = \sqrt{3}\, v_L i_L \sin(\theta)} \quad \boxed{S = 3 v_P i_P^*}$$

- Note that these equations apply for both Y-connected and Delta-connected loads.