Theory Assignment - 4.

Q 10

a) Given

Mean of dataset $D = \{x_1, x_2, x_3 .. x_n\}$ is arbitrarily selected as the first element of the dataset

eq.

$$D = \{ x_1 , x_2 , x_3 ... x_n \}$$

$$Mean = x_1.$$

Bais of mean ~~can~~ is given by absolute expectation of a difference of from actual mean of dataset D.

$$Bias_{mean} = E[mean - mean\text{-}original]$$

$\downarrow m$ $\hookrightarrow \mu$

$$= E[m - \mu]$$

$$= E[m] - \mu.$$

$$\text{bias}_{\text{mean}} = \frac{1}{n}\sum_{i}^{n} m_i - \mu.$$

where n is no of datapoints

in D.

If for different samples, value of $E[m]$

tends to original mean μ.

ie samples $\rightarrow \infty$.

$\therefore$ $\text{Bias}_{\text{mean}} = 0$.

$\therefore$ Given Technique of selecting mean is unbiased.

b) Though the way of selecting mean comes out to be asymptotically unbiased. But one always needs to check if the metric provided is having high variance wrt to the original mean.

using Jackknife ~~re~~ Variance Estimation

$$\sigma^2_{mean} = \left(\frac{n-1}{n}\right) \sum_{i=1}^{n} (x_i - \mu)^2$$

can be inferenced that the variance tends to 0. ~~when~~ wrt to original data as $n \to \infty$. thus there can be various other techniques for the case.

Ans 4.

Given

$$P(x|\theta) = \prod_{i=1}^{d} \theta_i^{x_i} (1-\theta_i)^{1-x_i}$$

where $\theta = (\theta_1 \dots \theta_d)^t$
↳ unknown parameter vector

θ_i = probablity of $x_i = 1$

To Prove

$$MLE(\theta) = \hat{\theta} = \frac{1}{n}\sum_{K=1}^{n} x_K$$

Proof

Given dataset D, need to max $P(D|\theta)$.

$$p(D|\hat{\theta}) = \prod_{K=1}^{n} \prod_{i=1}^{d} \theta_i^{x_{Ki}} (1-\theta_i)^{(1-x_{Ki})}.$$

$$\Rightarrow \log(P(D|\hat{\theta})) = \sum_{K}^{n} \sum_{i}^{d} \left(x_{Ki} \ln\theta_i + (1-x_{Ki}) \log(1-\theta_i)\right)$$

differentiating wrt θ_i

$$\frac{\partial}{\partial\theta_i}(\log P(D|\theta)) = \sum_{K=1}^{n} \sum_{i=1}^{d} \left(\frac{d}{d\theta_i} x_{Ki} \ln\theta_i + \frac{d}{\partial\theta_i}((1-x_{Ki})(\ln(1-\theta_i)\right.$$

$$= \sum_{k=1}^{n} \left(\frac{x_{ki}}{\theta_i} - \frac{(1-x_{ki})}{(1-\theta_i)} \right). \quad \text{——— (1)}$$

Equating (1) to 0.

$$\Rightarrow \sum_{k=1}^{n} \left(\frac{x_{ki}}{\theta_i} - \frac{(1-x_{ki})}{(1-\theta_i)} \right) = 0.$$

$$\Rightarrow \sum_{k=1}^{n} x_{ki} - x_{ki}\theta_i - \theta_i + x_{ki}\theta_i) = 0.$$

$$\Rightarrow \sum_{k=1}^{n} x_{ki} = \sum_{k=1}^{n} \theta_i . n.$$

$$\Rightarrow \quad \theta_i = \frac{1}{n} \sum_{k=1}^{n} x_{ki}$$

$$\Rightarrow \quad \theta = \frac{1}{n} \sum_{1}^{n} \vec{x_k}$$

Proved