PMATH 333

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Preface

The purpose of these notes is to provide the reader with a secondary reference to the material covered in this course. It may be incomplete or contain errors.

The notes are taken down by Iris Jiang and transcribed by Sibelius Peng.

For any questions, send me an email via `https://notes.sibeliusp.com/contact/`.

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CHAPTER 1

Informal Intro

Derivative

instantaneous rate of change.

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \qquad \text{if it exists.}$$

Example.

If $f : \mathbb{R} \to \mathbb{R}$ has $f'(x) = 0$ for all $x \in \mathbb{R}$. Then f is constant.

First goal of course

Defining $\mathbb{R}$ and proving it has no gaps.

Dedekind Cut

A dedekind cut is a subset $A \subset \mathbb{Q}$ such that

1. $\varnothing \neq A \neq \mathbb{Q}$
2. If $x \in A$ and $q \in \mathbb{Q}$ with $q \leq x$, then $q \in A$
3. A has no largest element. That is if $x \in A$, then there exists $y \in A$ with $x < y$.

Lemma 1.1

Let q be a rational number with $0 < q$ and $q^2 < 2$. Then there exists some $r \in \mathbb{Q}$ with $q < r$ and $r^2 < 2$.

Proof:

Exercise. □

Theorem 1.2: Density of $\mathbb{Q}$ in $\mathbb{R}$

If $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$, then there exists $q \in \mathbb{R}$ with $\alpha < q < \beta$.

Proof:

Exercise. □

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