PMATH 333

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Preface

The notes are taken down by Iris Jiang and transcribed by Sibelius Peng.

CHAPTER 1

Informal Intro

Derivative

instantaneous rate of change.

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \qquad \text{if it exists.}$$

Example.

If $f : \mathbb{R} \to \mathbb{R}$ has $f'(x) = 0$ for all $x \in \mathbb{R}$. Then f is constant.

First goal of course

Defining $\mathbb{R}$ and proving it has no gaps.

Dedekind Cut

A dedekind cut is a subset $A \subset \mathbb{Q}$ such that

1. $\varnothing \neq A \neq \mathbb{Q}$
2. If $x \in A$ and $q \in \mathbb{Q}$ with $q \leq x$, then $q \in A$
3. A has no largest element. That is if $x \in A$, then there exists $y \in A$ with $x < y$.

Lemma 1.1

Let q be a rational number with $0 < q$ and $q^2 < 2$. Then there exists some $r \in \mathbb{Q}$ with $q < r$ and $r^2 < 2$.

Proof:

Exercise. □

Theorem 1.2: Density of $\mathbb{Q}$ in $\mathbb{R}$

If $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$, then there exists $q \in \mathbb{R}$ with $\alpha < q < \beta$.

Proof:

Exercise. □

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