The Inflationary Paradigm before and after the 2013 Data Release of the Planck Satellite

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This paper explores the influence of the results of the 2013 Planck data release on the theories of cosmic inflation. This data release introduces constraints to cosmological parameters that rule out some of the popular theories of inflation prior to 2013, but bring other theories to the forefront. In light of this, important ideas underlining both sets of theories are reviewed, and the implications of the Planck 2013 data on these theories are discussed.

I. Introduction

Despite the success of the Big Bang Theory in explaining the evolution of the early universe, there were a few problems that it could not resolve – the flatness problem, horizon problem, and magnetic monopole problem [1]. All of these problems basically boil down to the universe having expanded faster than expected, based on predicted and observed quantities, such as the critical density, size of the cosmic horizon, or density of magnetic monopoles.

While trying to solve the magnetic monopole problem, Alan Guth discovered that by the principles of General Relativity, a positive energy false vacuum would cause exponential expansion of space [2]. This regime of exponential expansion was termed inflation. Inflation was demonstrated to be able to explain the discrepancy between the expected and observed physical quantities [2].

The idea of inflation, is that very early in the universe's history, it undergoes a regime of rapid expansion, exponentially growing in an incredibly short amount of time. However, this expansion halts, and standard model Big Bang cosmology takes over from that point.

This introduced two new problems to be solved – what causes inflation to start, and what causes inflation to stop? Initial models of inflation attempting to answer these two questions provide the framework for most of the leading theories prior to 2013.

Testing inflation comes in the form of measurements of the CMB with satellites like COBE and WMAP, which constrain the initial conditions of the universe. Better measurements tightly constrain the parameters that a theory needs to predict correctly in order to be in line with observations. Theories which disagree with observations can be excluded, and eventually the right answer can be converged upon.

The latest improvements to measurements of these parameters comes from Planck's 2013 data release. These new constraints and their implications for leading theories were crucial to the advancement of the field.

Although it was expected that Planck would discover large non-Gaussian perturbations, which would require complex, multi-field models of inflation, the opposite happened. As a result, simpler models of inflation could be sufficient. This has led to interest in a class of theories known as 'cosmological attractors' [5].

Here, these cornerstones of the history of the field are explored, in order to better understand the importance of the Planck 2013 data to the advancement of the inflationary paradigm.

II. Theoretical Models of Inflation prior to 2013

The initial conception of inflation, now called 'Old Inflation', begins with a positive energy false vacuum [2]. However, the mechanism that would halt inflation in this scenario is not sufficient. This led to the theory of a scalar field rolling down a potential – an idea that is prevalent in most theories of inflation today.

A. Old Inflation

In order to resolve the flatness and horizon problem, the assumption of an adiabatically expanding universe is treated as being incorrect. Thus, it is possible for the entropy density of the universe to change. In this model, the entropy density change is as a result of supercooling as the universe undergoes a phase transition.

The mechanism which drives this supercooling is based on a negative pressure. Consider the Friedmann Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

A negative value for pressure will drive expansion. The negative pressure is said to be a result of a false vacuum, a metastable state without fields or particles, but with a large energy density. Thus, the expansion would be exponential.

$$a(t) \propto e^{\chi^t}$$

While classically stable, quantum tunneling would allow the state to decay. Tunneling decay would eventually end inflation, explaining why we don't observe it happening today. However, this tunneling would in fact lead to large inhomogeneity, which is not what is observed.

Shortly after Guth released his findings, it was found that there needed to be specific conditions for the false vacuum bubbles to radiate, known as the bubble-collision problem. Andrei Linde resolved this problem by proposing that inflation was a scalar field rolling down a potential. This was called slow-roll inflation after the slow-roll function, which describes the shape of the potential.

B. Slow-Roll Inflation

In 1981, Linde posited a solution to the 'graceful exit' problem that plagued Old Inflation by suggesting that the potential in which the scalar field sits has a hill like shape, and as the field rolls up the hill, it gradually comes to a stop. The shape of the potential was described with a slow-roll function [3].

a. Slow-Roll Function

Linde considers

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$

with $\lambda \ll 1$, an effectively flat function, but not truly flat. The universe is assumed to be in a chaotic state prior to $t \sim t_p \sim {M_p}^{-1}$. For a sufficiently small value of λ,

$$V(\phi) = \frac{1}{4}\lambda\phi^4 \lesssim {M_p}^4 \tag{1}$$

In order to constrain λ, Linde considers a locally homogenous field ϕ that expands as de Sitter space with $a(t) = a_0 e^{Ht}$, where

$$H = \left(\frac{\frac{8\pi}{3}V(\phi)}{{M_p}^2}\right)^{1/2} = \frac{\left(\frac{2\pi}{3}\lambda\right)^{1/2}\phi^2}{M_p}$$

So the equation of motion of the field is

$$\ddot{\phi} + 3H\dot{\phi} = -\lambda\phi^3$$

which implies that $\phi^2 \gg \frac{{M_p}^2}{6\pi}$. Solving for ϕ gives

$$\phi = \phi_0 e^{-\{[\sqrt{\lambda}M_p/(6\pi)^{1/2}]t\}}$$

For $\lambda \ll 1$, the time

$$\Delta t \sim \frac{(6\pi)^{1/2}}{\sqrt{\lambda} M_p}$$

during which the field decreases, is much larger than $t \sim t_p \sim (6M_p)^{-1}$. During this time the expansion is

$$a(\Delta t) = a_0 e^{(H\Delta t)} = a_0 e^{(2\pi {\phi_0}^2 / {M_p}^2)}$$

This expansion is quite large ($e^{(H\Delta t)} > e^{65}$) if

$$\phi_0 \gtrsim 3M_p$$

Applying the condition from Eq 1,

$$\lambda \lesssim 0.01$$

However, this is not the only potential possible. It is one of the simpler potentials, used as a demonstration. In fact there are many candidate potentials, including some very complex ones. There is a need to define some parameters which allow for easy comparison of various potentials, and their physical interpretations.

b. Formalized parameters

In 1994, Andrew Liddle, Paul Parsons, and John Barrow [4] formalized the slow-roll approximation, both in the case of a specified potential function (Potential Slow Roll Approximation or PSRA), and in the general case (Hubble Slow Roll Approximation or HRSA).

The PSR parameters are defined as

$$\epsilon_V(\phi) = \frac{{M_p}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$
$$\eta_V(\phi) = \frac{{M_p}^2}{8\pi}\frac{V''(\phi)}{V(\phi)}$$

These parameters can be used to describe slow-roll expansion. Additionally, certain measurable quantities, such as the spectral index of the power spectrum (discussed in Section III), can be defined in terms of these parameters. This allows for easy evaluation of a given potential with regards to whether or not it could support inflation.

The paper also discusses a better measure for inflation, the number of e-foldings of physical expansion that occur.

$$N(\phi) = \ln \frac{a_{end}}{a} = \int_{t}^{t_{end}} H dt = \int_{\phi_{end}}^{\phi} \frac{V}{V_{,\phi}} d\phi = \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon_V}}$$

To solve the horizon and flatness problems, $N(\phi) \gtrsim 60$ is required [4].

Observations of the CMB can constrain the number of e-foldings that happen between the appearance of fluctuations in the CMB, and the end of inflation.

$$N_{CMB} = \int_{\phi_{end}}^{\phi_{CMB}} \frac{d\phi}{\sqrt{2\epsilon_V}} \sim 40 \ to \ 60$$

c. Chaotic Inflation and Quantum Fluctuations

As mentioned before, the universe is said to be in a chaotic state prior to $t \sim t_p \sim {M_p}^{-1}$ [3].

Consider the simplest scalar field ϕ with mass m and potential $V = \frac{1}{2}m^2\phi^2$ [3]. The minimum of this potential is at $\phi = 0$, and if there is no expansion, the equation of motion will reduce to the simple harmonic oscillator case $\ddot{\phi} = -m^2\phi$. This is depicted in Fig 1 [3].

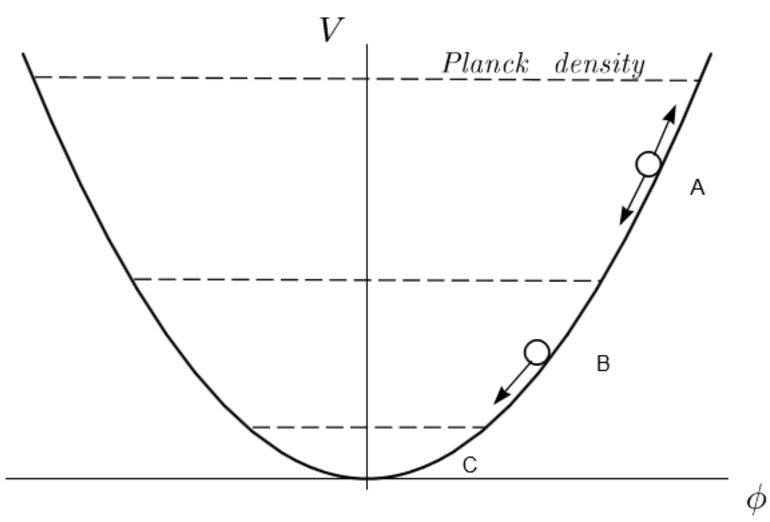

Figure 1: This plot depicts the motion of the scalar field in the potential $V = \frac{1}{2}m^2\phi^2$. It is similar to a ball rolling down a bowl filled with a viscous liquid. The regions A, B, and C are regimes for different values of ϕ. Inflation occurs in regions A ($m{M_p}^3 < V(\phi) < {M_p}^4$) and B ($m^2{M_p}^2 < V(\phi) < m{M_p}^3$). In region C, the field oscillates, leading to the production of elementary particles. [3]

But because of the expansion of the universe, there is another term that needs to be added that encapsulates this expansion. This term can be thought of like a friction that slows down a ball rolling down a hill. So the equation of motion will become

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

The Hubble parameter can be defined in terms of the scalar field, as

$$H^2 + \frac{k}{a^2} = \frac{1}{6}\left(\dot{\phi}^2 + m^2\phi^2\right) \tag{2}$$

For initially large value of ϕ, the Hubble parameter will be large to compensate. This means that the energy density of the scalar field will change very slowly, remaining almost constant. Since the energy density barely changes while the scale of the universe rises rapidly, $\ddot{\phi} \ll 3H\dot{\phi}$,

$H^2 \gg \frac{k}{a^2}$, and $\dot{\phi}^2 \ll m^2\phi^2$.

This allows for the simplification of Eq 2, by ignoring the negligible terms, to get

$$H = \frac{m\phi}{\sqrt{6}}$$

$$\dot{\phi} = -m\sqrt{\frac{2}{3}}$$

This is the regime of exponential expansion which ends once $\phi \ll M_p = 1$.

Quantum field theory suggests that what is thought of as empty space is not empty, rather it is filled with quantum fluctuations of physical fields. These fluctuations would grow with the expansion of the universe during inflation [3]. However, once the wavelength of a fluctuation grows to H^{-1}, its amplitude freezes out with a value $\delta\phi(x)$. An average amplitude for a time interval of H^{-1} is $|\delta\phi(x)| \approx \frac{H}{2\pi}$. These fluctuations are what leads to density perturbations, which later cause galaxy formation.

III. Planck Data

While theoretical constraints can be found, such as in Eq 2, observational constraints have much more power, and can shape the direction in which theorists think. Observational constraints for inflation are primarily derived from studying CMB anisotropies. WMAP provided a series of measurements, which helped define constraints incredibly well [5]. The successor to WMAP, Planck, significantly improved on the results of WMAP by providing much tighter constraints [6].

A. Observations

The primary data used to estimate the constraints on inflation, is the Planck CMB temperature likelihood [7]. This is obtained by designing a likelihood function, which propagates uncertainties. In this case, Planck opts for a hybrid approach [8], using a Gaussian approximation at high multipoles (high-ℓ) and pixel based likelihood at low multipoles (low- ℓ). The high-ℓ part is based on power spectra estimated by each detector in the frequency range of 100 – 217 GHz. The low-ℓ part is determined through a Gibbs sampling as implemented by the *Commander* Code [9], since at low-ℓ, the distribution is not Gaussian [8]. This is similar to the approach used by WMAP [10].

Combining the two, Planck is able to calculate the full power spectrum for $\ell = 2 \ to \ 2500$. This is shown in Fig 2 [7].

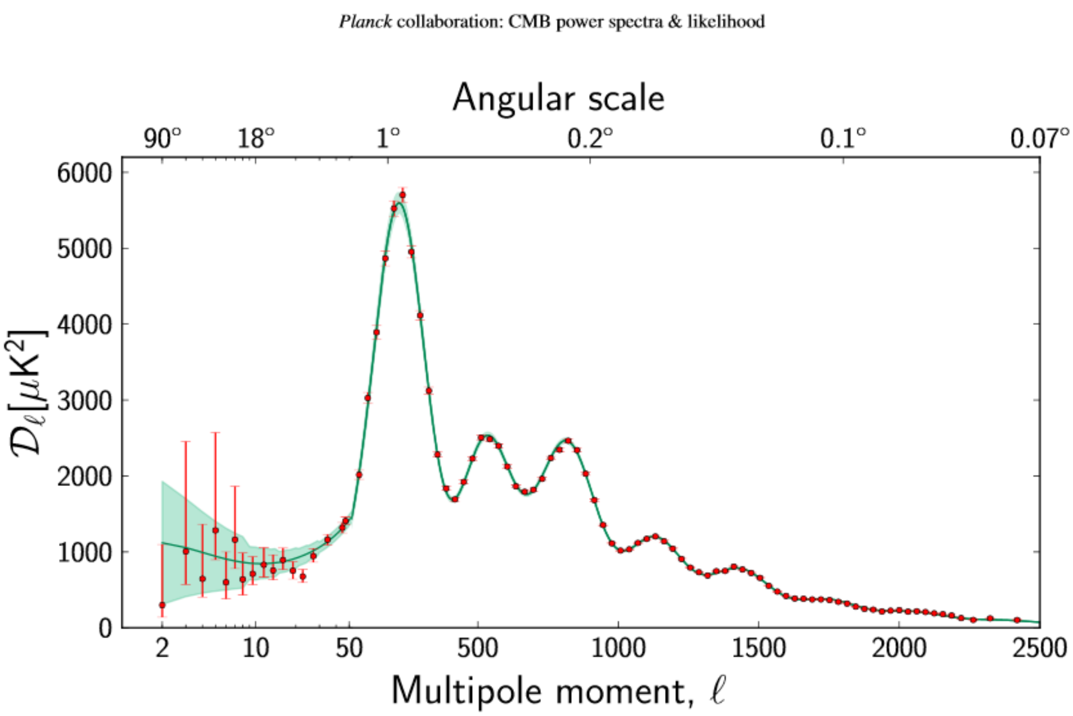

Figure 2: The 2013 Planck temperature angular power spectrum [7]. The green line is the best fit model, derived from the likelihood function. The error bars are partially due to cosmic variance, and so this contribution is depicted with the green shaded area.

While estimating the best fit model, there are certain parameters associated with the power spectrum whose values are set during the estimation process. In addition to this, the best fit model can also be used to calculate other cosmological patterns. Some of these parameters and their values with the 2013 Planck power spectrum are shown in Table 1 [7].

Table 1: A table of constraints on the basic six-parameter ΛCDM model, and derived parameters, obtained using Planck 2013 data [7].

Parameter	*Planck* Best fit	*Planck* 68% limits	*Planck*+WP Best fit	*Planck*+WP 68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14305	0.1426 ± 0.0025
Age/Gyr	13.819	13.813 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.48	1090.43 ± 0.54
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{eq}	3402	3386 ± 69	3403	3391 ± 60

B. Constraints from Planck

It is possible to describe slow-roll inflationary models using the power spectrum parameters of scalar amplitude A_s, spectral index n_s, and tensor to scalar ratio r, all defined at the pivot scale k_* [6]. It is convenient to express these quantities in terms of the PSR parameters defined in Section II.

$$A_s = \frac{V}{24\pi^2 {M_p}^4 \epsilon_V}$$

$$n_s - 1 \approx 2\eta_V - 6\epsilon_V \quad (3)$$

$$r \approx 16\epsilon_V \quad (4)$$

Planck and WMAP data combined give the following constraints to the above parameters (with $k_* = 0.002/Mpc$) [6].

$$n_s = 0.9603 \pm 0.0073 \quad (5)$$

$$r_{0.002} < 0.12 \text{ at } 95\% \text{ CL} \quad (6)$$

$$\epsilon_V < 0.008 \text{ at } 95\% \text{ CL}$$

$$\eta_V = -0.010^{+0.005}_{-0.011}$$

The constraint on r gives an upper bound on the energy scale of inflation [6].

$$V_* = \frac{3\pi^2 A_s}{2} r {M_p}^4 = (1.94 \times 10^{16} GeV)^4 \frac{r_*}{0.12} \text{ at 95\% CL}$$

Constraints for various models are depicted below, in Fig 3 [6].

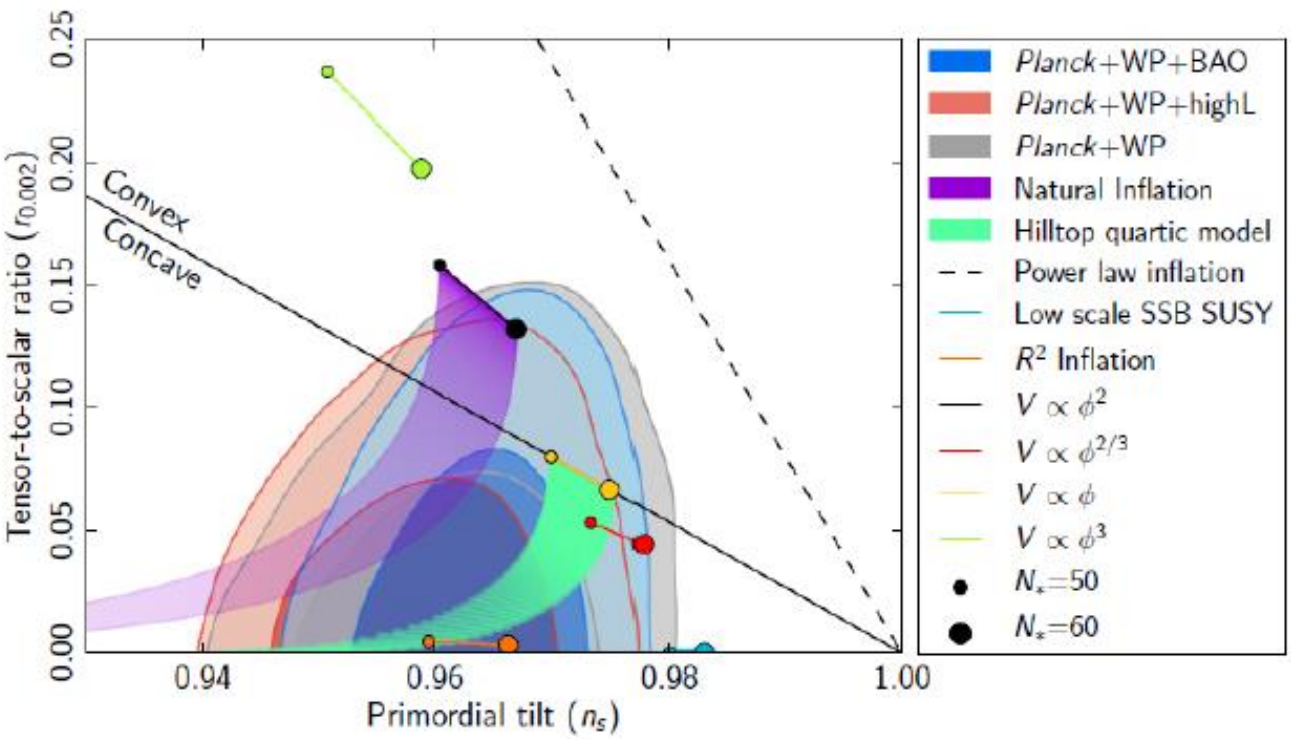

Figure 3: The confidence regions of Planck combined with various other data sets, and where popular models lie compared to the 68% and 95% CL regions [6].

B. Implications for Inflation Models

Using these constraints, it is possible to check whether or not a theory predicts the observed quantities or not. This can be done by calculating the expected values of these constraints using their PSR parameters, as in Eqs 3 and 4.

a. Power Law Potential and Chaotic Inflation

A popular class of potentials is the power law potential [6]

$$V \propto \lambda {M_p}^4 \left(\frac{\phi}{M_p}\right)^n$$

Consider the slow roll function where $n = 4$, so $V \propto \lambda\phi^4$, which is similar to the potential discussed earlier in Section II. For this potential, it is possible to calculate the following using their PSR parameters [6]

$$n_s = -24\frac{{M_p}^2}{{\phi_*}^2} + 1$$
$${\phi_*}^2 = 2{M_p}^2(4N_* + 4)$$

Thus,

$$n_s = -3\frac{1}{(N_* + 1)} + 1$$

In order to match the observation of $n_s = 0.96$, the above equation would predict $N_* = 74$, which lies outside the interval [50, 60] that is predicted. Thus this model is inconsistent with the observations. Fig 4 depicts the predicted values of the constraints [6].

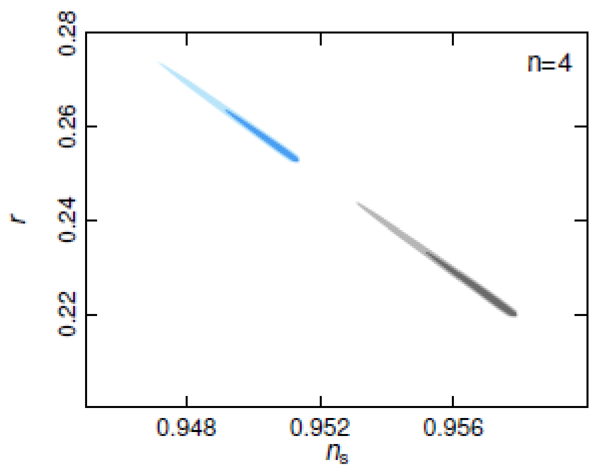

Figure 4: The predicted values of r and n_s for the restrictive (blue), and permissive (grey) entropy generation scenarios, for the potential $V \propto \lambda\phi^4$ [6].

It is clear from Fig 4, that the predicted values of r (> 0.2) and n_s (< 0.96) do not match the predictions (Eqs 5 and 6).

Now consider the quadratic potential function for chaotic inflation as suggested by Andrei Linde in 1983 [3], defined as $V = \lambda{M_p}^2\phi^2$. For this potential,

$$n_s = -8\frac{{M_p}^2}{{\phi_*}^2} + 1$$
$${\phi_*}^2 = {M_p}^2(4N_* + 2)$$

Thus,

$$n_s = -4\frac{1}{(2N_* + 1)} + 1$$

In order to match the observation of $n_s = 0.96$, the above equation would predict $N_* = 49.5$, which lies just outside the interval [50, 60] that is predicted. Thus this model is inconsistent with the observations. Fig 5 depicts the predicted values of the constraints [6]

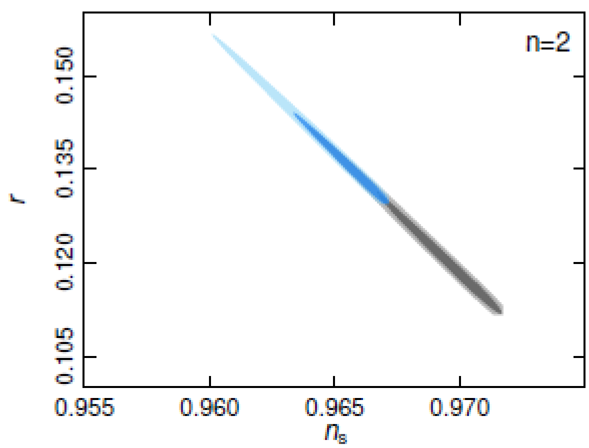

Figure 5: The predicted values of r and n_s for the restrictive (blue), and permissive (grey) entropy generation scenarios, for the potential $V \propto \lambda {M_p}^2\phi^2$ [6].

It is clear from Fig 5 that the predicted value of r (> 0.12) for the restrictive entropy generation case does not match the predictions (Eq 5). For the permissive entropy generation case, there a very small region which does seem to be within the observed parameter constraints. However, this model is outside of the 95% confidence region.

However, a linear potential defined as $V = \lambda {M_p}^3\phi$, motivated by axion monodromy [11], has $V''(\phi) = 0$, and hence $\eta_V = 0$. This model is consistent with observations. The yellow line in Fig 3, shows that this potential lies within the 95% confidence region.

b. Hilltop Models

Similar in concept to the initial ideas of inflation [12], where the field rolls from an unstable equilibrium, these models are described by

$$V = \Lambda^4\left(1 - \frac{\phi^p}{\mu^p} + \cdots\right)$$

The higher order terms are negligible during inflation, but they ensure that $V > 0$ at later times. Choosing $p = 2$ yields $V = \Lambda^4 \left(1 - \frac{\phi^2}{\mu^2}\right)$, which predicts

$$n_s = -4\frac{{M_p}^2}{\mu^2} + \frac{3r}{8} + 1$$
$$r = 32\frac{{\phi_*}^2 {M_p}^2}{\mu^4}$$

This is in agreement with observations if $\mu \geq 9M_p$ [6]. Similarly, a simple symmetry breaking model, described as $V = \Lambda^4 \left(1 - \frac{\phi^2}{\mu^2}\right)^2$ [13], is in agreement with observations if $\mu \geq 13M_p$ [6].

Also motivated by symmetry breaking, is the idea known as Natural Inflation [14, 15], described by a class of potentials of the form

$$V = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

The value of f determines whether it is a small field model or large field model. If $f \leq 1.5M_p$, it is a small field model. In this case $n_s \approx 1 - \frac{{M_p}^2}{f^2}$, which fails since $f \approx 5$ to get a value for n_s that is within constraints [6]. On the other hand, if $f \geq 1.5M_p$, $n_s \approx 1 - \frac{2}{N}$, and $r \approx \frac{8}{N}$, it is possible to approximate the $m^2\phi^2$ potential, in which case, the theory holds if $f \geq 5M_p$ [6].

c. Other Models

One popular model is known as R^2 inflation or the Starobinsky model [16]. It is described by the higher order terms of action done by gravity, including semi-classical quantum effects, formulated by

$$S = \int d^4x\sqrt{-g}\frac{{M_p}^2}{2}\left(R + \frac{R^2}{6M^2}\right)$$

This model predicts $n_s = 0.963$, and $N_* = 55$, which is fully consistent with Planck Data [6].

IV. Theories of Inflation after Planck's 2013 Data Release

One of the most important results to come out of Planck's 2013 data release [17], is

$$f_{NL}^{local} = 2.7 \pm 5.8$$

This is a measure of how Gaussian the initial conditions of the Universe were [18]. This value is quite low [17], and indicates that simpler Gaussian models can be studied preferentially over the complex models that produce non-Gaussianity locally [19].

Thus, since 2013, study of the inflationary paradigm has moved towards studying classes of potentials which match observations, and are based off of the earlier work on inflation without the non-Gaussianity. Among those classes of potentials, are 'cosmological attroctors' [19].

A. Starobinsky Model and Cosmological Attractors

As described above, the Starobinsky model arose while studying quantum effects in General Relativity [16]. For the effects to be sufficiently large, they would have to have been produced by an enormously large number of elementary particles. By choosing $M_p = 1$, we can write the Lagrangian of the effective action of the system [19].

$$L = \sqrt{-g}\left(\frac{R}{2} + \frac{R^2}{12M^2}\right)$$

Here, $M \ll M_p$ and is some mass scale. However, classically, this model is equivalent to a canonical gravity plus some scalar field, and thus by applying the transformation

$\left(1 + \frac{\phi}{3M^2}\right) g_{\mu\nu} \to g_{\mu\nu}$, and field redefinition $\varphi = \sqrt{\frac{3}{2}} \ln\left(1 + \frac{\phi}{3M^2}\right)$, the equivalent Lagrangian becomes [20]

$$L = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\varphi}\right)^2\right]$$

This Lagrangian's potential with units $M = 1$ is shown below in Fig 6 [17].

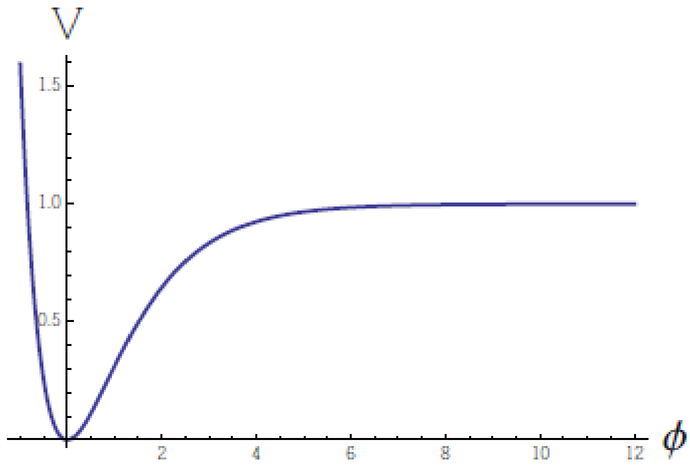

Figure 6: The potential of the Lagrangian following the transformation and field redefinition [17]. This potential is reminiscent of the simpler one in Fig 1, but more complex.

It is possible to represent this model's predictions for n_s and r in terms of the number of e-foldings, and with a large limit on e-foldings, the result is

$$n_s = 1 - \frac{2}{N} \tag{7}$$

$$r = \frac{12}{N^2} \tag{8}$$

As discussed above, for about 60 e-foldings, this model's predictions are within the 99.7% confidence region from the Planck2013 and WMAP9 data releases .

The class of potentials with similar predictions as Eqs 7 and 8, have been termed 'cosmological attractors' [17]. There are a multitude of cosmological attractors, based on various gauges for the transformation, and the coupling with supergravity, as well as superconformal theory. These models are all consistent with the predictions of Planck2013, and attempt to describe inflation as an instability in a conformon field. As one includes more physics, the potential depicted above gets more complex, with multiple local extrema. An illustration of a more complex potential is shown below in Fig 7 [17].

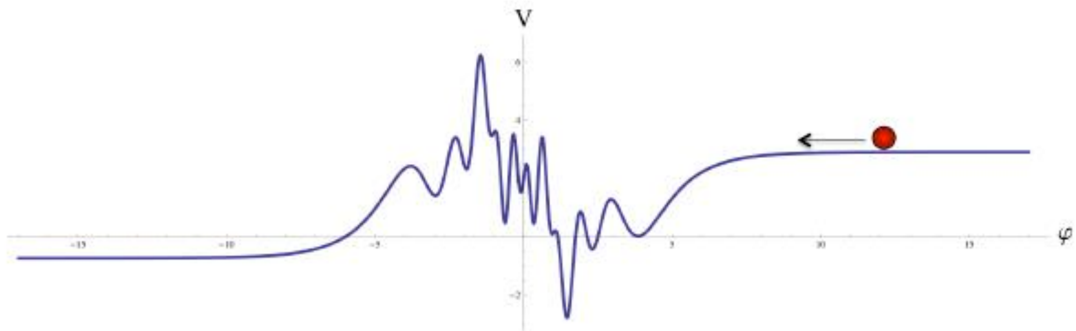

Figure 7: A more complex cosmological attractor, known as a 'superconformal attractor' due to the inclusion of superconformal theory in the physics of the potential [17].

V. Conclusions

The inflationary paradigm has greatly influenced the field of astrophysics, by fundamentally changing the landscape of study. While there is no single theory that presents itself as the clear answer, the groundwork that has been laid in the last 30 years has been instrumental.

Planck's 2013 data release has been a large factor in helping understand the nature of the universe's evolution. The constraints have eliminated a number of theories, narrowing in on the potential solutions to the quandary that is cosmic inflation. However, there are still some issues with anomalies at low monopoles [5, 21]. Looking forward, Planck released its 2015 data in February [22], in which it lists the parameters shown in Table 2 [22].

Table 2: ΛCDM model parameters within the 68% confidence region for various combinations of Planck 2015 data [22].

Planck Collaboration: Constraints on inflation

Parameter	TT+lowP	TT+lowP+lensing	TT+lowP+BAO	TT,TE,EE+lowP
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02226 ± 0.00020	0.02225 ± 0.00016
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1190 ± 0.0013	0.1198 ± 0.0015
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04095 ± 0.00041	1.04077 ± 0.00032
τ	0.078 ± 0.019	0.066 ± 0.016	0.080 ± 0.017	0.079 ± 0.017
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.093 ± 0.034	3.094 ± 0.034
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9673 ± 0.0045	0.9645 ± 0.0049
H_0	67.31 ± 0.96	67.81 ± 0.92	67.63 ± 0.57	67.27 ± 0.66
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3104 ± 0.0076	0.3156 ± 0.0091

One value that stands out, is the increased value for n_s as compared to Planck 2013, which is attributed to improvements in data processing and likelihood [22], which could influence the theories that are currently disfavored by 2013 Planck data. This increased value allows a potential like $V = \lambda {M_p}^2\phi^2$ discussed in Section III to have $N_* \approx 57$, which could have brought it back in to the 95% confidence region (however, there are other measures that still place this particular potential outside the 95% confidence region [22]).

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