Program : **B.Tech**

Subject Name: **Signals and Systems**

Subject Code: **EC-402**

Semester: **4th**

Unit-1 Introduction of Signals and Systems: Definition of signal, Classification of Signal and representation: Continuous time and discrete time, even/odd, periodic/aperiodic, random/deterministic, energy/power, one/multidimensional, some standard signals, , Basic Operations on Signals for CT/DT signal, transformation of independent & dependent variables,
Definition of system and their classification: CT/DT, linear/non-linear, variant/non-variant, causal and non-causal system state/dynamic system, interconnection of systems. System properties: linearity: additivity and homogeneity, shift-invariance, causality, stability, realizability.

Basic Definitions:
Signals : A function of one or more independent variables which contain some information is called signal.

Systems: A system is a set of elements or functional blocks that are connected together and produces an output in response to an input signal.

Classification of Signals :

I. Periodic and Non-Periodic Signals :

A signal that repeats at regular time interval is called as periodic signal. The periodicity of the signal is represented mathematically

$x(t) = x(t+T_0)$; T_0 is the period of the continuous time (CT) signal
$x(n) = x(n+N)$; N is the period of the discrete time (DT) signal

A signal that does not repeats at regular time interval is called non-periodic signal. It does not satisfy the periodicity condition.

$x(t) \neq x(t+T_0)$; for continuous time (CT) signal
$x(n) \neq x(n+N)$; for discrete time (DT) signal

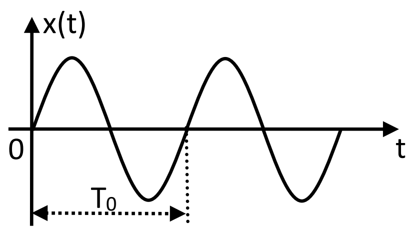


Fig. 1.1 (a) : Periodic Signal

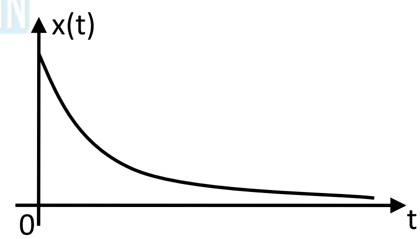


Fig. 1.1 (b) : Non-Periodic Signal

(a) The sum of two continuous-time periodic signals $x_1(t)$ and $x_2(t)$ with period T_1 and T_2 is periodic if the ratio of their respective periods T_1/T_2 is a rational number or ratio of two integers, otherwise not periodic.
(b) The fundamental period is the least common multiple (LCM) of T_1 and T_2.
(c) The sum of two discrete-time periodic sequence is always periodic.

Example: Determine whether the following signals are periodic or not? If periodic find the fundamental period.

(a) $\sin(12\pi t)$ (b) $e^{j4\pi t}$ (c) $\sin(10\pi t) + \cos(20\pi t)$

(a) Given $x(t) = \sin(12\pi t)$, Since x(t) is a sinusoidal signal it is periodic
Comparing x(t) with $\sin(\omega t)$, we get $\omega = 12\pi$ or $T = 2\pi/\omega = 2\pi / 12\pi = 1/6$ sec.

(b) Given $x(t) = e^{j4\pi t}$, Since x(t) is complex exponential signal it is periodic.
Comparing x(t) with $e^{j\omega t}$, we get $\omega = 4\pi$ or $T = 2\pi/\omega = 2\pi / 4\pi = 1/2$ sec.

(c) Given $x(t) = \sin(10\pi t) + \cos(20\pi t)$, Let $x(t) = x_1(t) + x_2(t)$, where $x_1(t) = \sin(10\pi t)$ and $x_2(t) = \cos(20\pi t)$
Comparing $x_1(t)$ and $x_2(t)$ with $\sin(\omega_1 t)$ and $\cos(\omega_2 t)$. we get $\omega_1 = 10\pi$ and $\omega_2 = 20\pi$

$\therefore \frac{T_1}{T_2} = \frac{1/5}{1/10} = 2$, Since T_1/T_2 is a rational number, the given signal is periodic and the fundamental period is T= T_1 =2T_2 = 1/5 Sec.

II. Even and Odd Signals :

A signal is said to be even signal if it is symmetrical about the amplitude axis. The even signal amplitude is not altered when the time axis is inverted.

$$x(t) = x(-t) \quad ; \quad x(n) = x(-n)$$

A signal is said to be odd signal if it is anti-symmetrical about the amplitude axis. The odd signal amplitude is inverted when the time axis is inverted.

$$x(t) = - x(-t) \quad ; \quad x(n) = - x(-n)$$

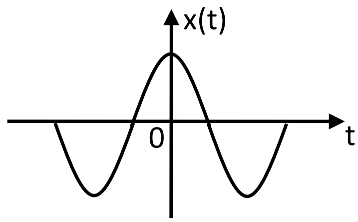


Fig. 1.2 (a) : Symmetric (Even) Signal

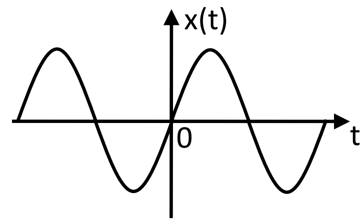


Fig. 1.2 (b) : Anti-Symmetric (Odd) Signal

If the signal does not satisfies either the condition for even signal or the condition for odd signal then it is neither an even signal nor the odd signal. However it contains both even and odd components in the signal. The even and odd components can be calculated as $x_e(t) = \frac{1}{2}[\, x(t) + x(-t)\, ; \; x_o(t) = \frac{1}{2}[\, x(t) - x(-t)$

Find even and odd components of following signals

(i) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$ (ii) $x(t) = (1 + t^3)\cos^3(10t)$ (iii) $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

(i) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

Now $x(-t) = 1 + (-t) + 3(-t)^2 + 5(-t)^3 + 9(-t)^4$

$x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$

$$\therefore x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$= \frac{1}{2}[1 + t + 3t^2 + 5t^3 + 9t^4 + 1 - t + 3t^2 - 5t^3 + 9t^4]$$

$$\boxed{\therefore x_e(t) = 1 + 3t^2 + 9t^4}$$

and $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$= \frac{1}{2}[1 + t + 3t^2 + 5t^3 + 9t^4 - 1 + t - 3t^2 + 5t^3 - 9t^4]$$

$$\boxed{\therefore x_o(t) = t + 5t^3}$$

(ii) $x(t) = (1 + t^3)\cos^3(10t)$

Now $x(t) = (1 + t^3)(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t))$

$$\therefore x(-t) = (1 + (-t)^3)(\frac{3}{4}\cos(-10t) + \frac{1}{4}\cos(-30t))$$

$$x(-t) = (1 - t^3)(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t)$$

To find the even and odd components, consider the equation

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \frac{1}{2}\{[(1 + t^3)(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t)] + [(1 - t^3)(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t))]$$

$$\boxed{\therefore x_e(t) = (\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t))}$$

and

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x_o(t) = \frac{1}{2}\{[(1 + t^3)\left(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t)\right] - [(1 - t^3)(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t))]$$

$$\boxed{\therefore x_o(t) = t^3(\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t))}$$

(iii) x(t) = cos(t) + sin(t) + sin(t) cos(t)

Now x(-t) = cos(-t) + sin(-t) + sin(-t) cos(-t)

x(-t) = cos(t) - sin(t) - sin(t) cos(t)

$$\therefore x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$= \frac{1}{2}[\cos(t) + \sin(t) + \sin(t)\cos(t) + \cos(t) - \sin(t) - \sin(t)\cos(t)]$$

$$\boxed{\therefore x_e(t) = \cos(t)}$$

and $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$= \frac{1}{2}[\cos(t) + \sin(t) + \sin(t)\cos(t) - \cos(t) + \sin(t) + \sin(t)\cos(t)]$$

$$\boxed{\therefore x_o(t) = \sin(t)\cos(t) + \cos(t)}$$

III. Energy and Power Signals :

The signal which has finite energy and zero average power is called as energy signal.

If x(t) has $0 < E < \infty$ and $P = 0$, then it is a energy signal, where E is the energy and P is the average power of signal x(t).

The signal which has finite average power and infinite energy is called as power signal.

If x(t) has $0 < P < \infty$ and $E = \infty$, then it is a power signal, where E is the energy and P is the average power of signal x(t).

If the signal does not satisfies either the condition for energy signal or the condition for power signal then it is neither an energy signal nor the power signal.

Energy of Signal:

For real CT signal, energy is calculated as

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

For complex valued CT signal, energy is calculated as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

For real DT signal, energy is calculated as

$$E = \sum_{n=-\infty}^{\infty} x(n)^2$$

For complex valued DT signal, energy is calculated as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Power of Signal:

For real CT signal, average power is calculated as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} x^2(t)dt$$

For complex valued CT signal, average power is calculated as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 dt$$

For real DT signal, average power is calculated as

$$P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-\infty}^{\infty} x(n)^2$$

For complex valued DT signal, average power is calculated as

$$P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Comparison of Energy Signal & Power Signal

Sl. No.	Energy Signal	Power Signal
1	Total normalized energy is finite and non zero	The normalized average power is finite and non zero
2	Non periodic signals are energy signals	Periodical signals are power signals
3	Power of energy signal is zero	Energy of power signal is infinite

Example : (i) Prove the following (a) The power of the energy signal is zero over infinite time (b) The energy of the power signal is infinite over infinite time

Solution :

(a) Power of the energy signal

Let x(t) be an energy signal

$$\therefore \text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \left[\lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \right]$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} [E] \qquad \text{Since } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\boxed{\therefore \quad P = \frac{1}{2\infty} [E] = 0 \text{ X } E = 0}$$

Thus, the power of the energy signal is zero over infinite time.

(b) Energy of the power signal

Let x(t) be the power signal

$$\therefore \text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Consider the limits of integration as –T to T and take limit T tends to . this will not change the meaning of above equation

$$\therefore \quad E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \left[2T \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right]$$

$$\therefore \quad E = \lim_{T\to\infty} 2T \left[\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right] = \lim_{T\to\infty} 2TP$$

$$\boxed{\therefore \quad E = \infty}$$

Thus, the energy of the power signal is infinite over infinite time.

Example : (ii) Sketch the given signal x(t) = $e^{-a|t|}$ for a > 0. Also determine whether the signal is a power signal or energy signal or neither.

Solution : The given signal is x(t) = $e^{-a|t|}$ for a > 0

It can be expressed as

$$x(t) = \begin{cases} e^{-at} & \text{for } t > 0 \\ e^{at} & \text{for } t < 0 \end{cases}$$

The sketch of this signal is shown in Fig. (a)

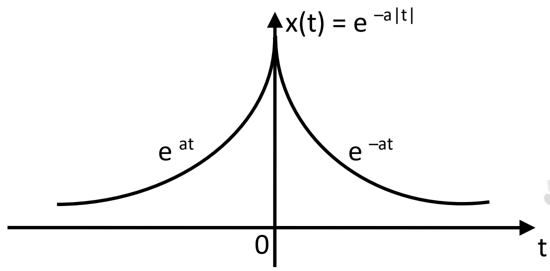


Fig.(a) : Waveforms for Example (ii)

The energy of the signal is expressed as

$$\text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} e^{-2a|t|} dt$$

$$\therefore \quad E = \int_{-\infty}^{0} e^{-2a(-t)}\, dt + \int_{0}^{\infty} e^{-2a(t)} dt$$

$$\therefore \quad E = \int_{0}^{\infty} e^{-2at}\, dt + \int_{0}^{\infty} e^{-2at} dt = 2\int_{0}^{\infty} e^{-2at} dt$$

$$\therefore \quad E = 2\left[\frac{e^{-2at}}{-2a}\right]_0^{\infty} = -\frac{1}{a}\left[e^{-\infty} - e^{0}\right] = \frac{1}{a} \text{ Joules}$$

Since energy is finite, the signal is energy signal.

Example : (iii) The signal x(t) is shown in Fig.(b). Determine whether the signal is a power signal or energy signal or neither.

Solution : The given signal x(t) can be expressed as

$$x(t) = \begin{cases} 2 & \text{for } -1 \le t \le 0 \\ 2e^{-t/2} & \text{for } \quad t > 0 \end{cases}$$

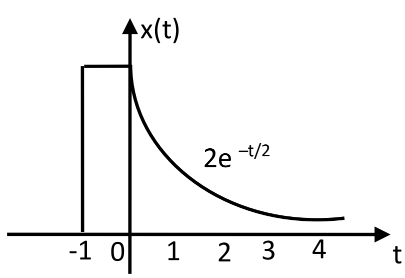


Fig.(b) : Waveforms for Example (iii)

The energy of the signal is expressed as

$$\text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\therefore \quad E = \int_{-1}^{0} 2^2\, dt + \int_{0}^{\infty} [2e^{-(t/2)}]^2 dt$$

$$\therefore \quad E = \int_{-1}^{0} 4\, dt + \int_{0}^{\infty} 4e^{-t} dt$$

$$\therefore \quad E = 4[t]_{-1}^{0} + 4\left[\frac{e^{-t}}{-1}\right]_0^{\infty} = 4 - 4\left[e^{-\infty} - e^{0}\right] = 4 + 4 = 8 \text{ Joules}$$

Since energy is finite, the signal is energy signal.

Example : (iv) The signal x(t) is shown in Fig.(c). Determine whether the signal is a power signal or energy signal or neither.

Solution : The given signal x(t) is a periodic signal with period from -1 to 1 (T=2) , can be expressed as

$$x(t) = t \quad \text{for} -1 \leq t \leq 1$$

Since the signal is periodic, it is a power signal and the average power can be calculated.

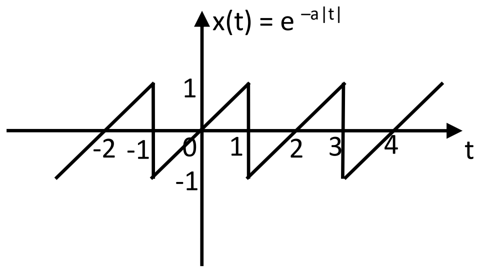


Fig.(c) : Waveforms for Example (iv)

The average power of the signal is expressed as

$$\text{Power P} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\therefore \quad p = \frac{1}{2}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{2}\int_{-1}^{1} t^2 dt$$

$$\therefore \quad P = \frac{1}{2}\left[\frac{t^3}{3}\right]_{-1}^{1} = \frac{1}{6}.2 = \frac{1}{3} \text{ Watts}$$

Since energy is finite, the signal is energy signal.

IV. Deterministic and Random Signals

A signal which has regular pattern and can be completely represented by mathematical equation at any time is call deterministic signal.

i.e., sine wave, exponential signal, square wave and triangular wave etc.

A signal which has uncertainty about its occurrence is called random signal. A random signal cannot be represented by mathematical equation.

i.e., noise is a random signal

Elementary Signals :

(i) *Unit Step Signal :* The unit step signal has a constant amplitude of unity for t ≥ 0 and zero for negative values of t.

The mathematical expression for CT unit step signal : $u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

The mathematical expression for DT unit step signal : $u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$

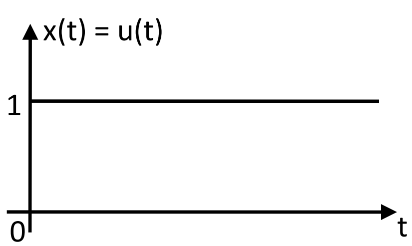


Fig. 1.3 (a) : CT unit Step Signal

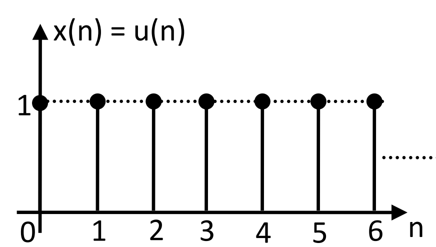
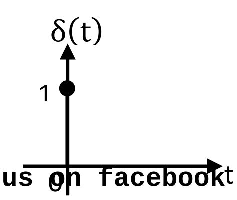
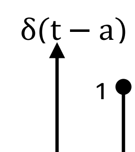


Fig. 1.3 (b) : DT unit Step Signal

(ii) Unit Impulse Signal: This signal is most widely used elementary signal in the analysis of systems. It is also called Dirac delta signal. It is defined as

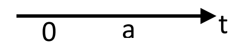
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$





and $\delta(t) = 0 \text{ for } t \neq 0\ \delta(t)$

$\delta(t)$ *i.e.,* $\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$



(iii) *Unit Ramp Signal:* The unit ramp signal r(t) is that signal which starts at t=0 and increases linearly with time and is defined as

Continuous time ramp signal $r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$ or $r(t) = t\,u(t)$

Discrete time ramp sequence $r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$ or $r(n) = n\,u(n)$

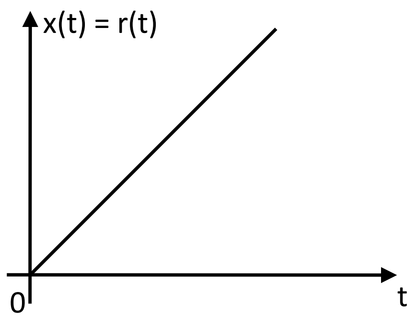


Fig. 1.4 (a) : CT Ramp Signal

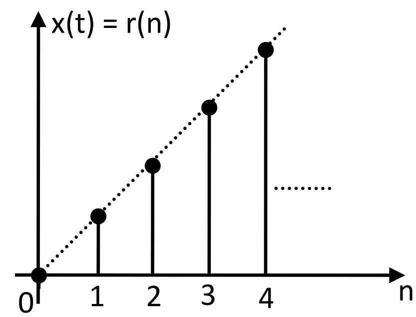
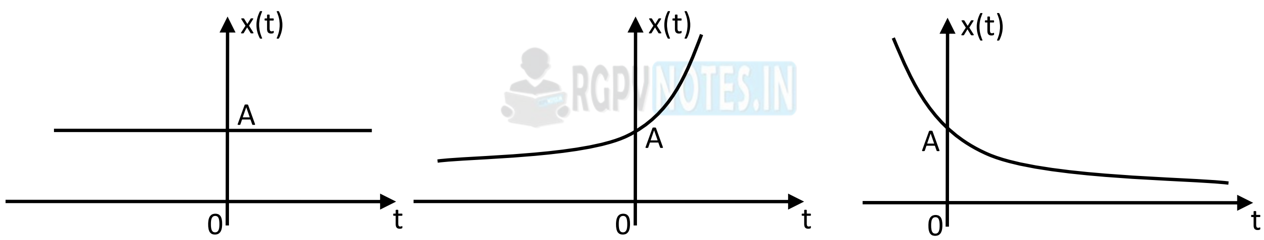


Fig. 1.4 (b) : DT Ramp Signal

(iv) *Exponential Signal :* The continuous-time real exponential signal has general form as $x(t) = A\,e^{\alpha t}$, where both A and α are real number.



$x(t) = A\,e^{\alpha t}$ for $\alpha = 0$ $x(t) = A\,e^{\alpha t}$ for $\alpha > 0$ $x(t) = A\,e^{\alpha t}$ for $\alpha < 0$

Fig. 1.5: CT Exponential Signal

(v) *Sinusoidal Signal:* The CT sinusoidal signal has general form as $x(t) = A \sin(\omega t + \phi)$

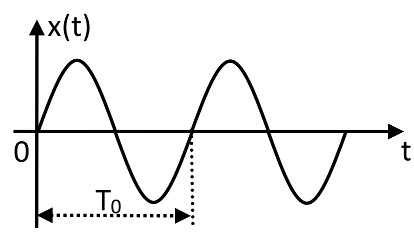


Fig. 1.6: CT Sinusoidal Signal

Relationships between the signals:

Relation between unit step and unit ramp signal:

The unit ramp signal is defined as,

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Differentiating r(t) with respect to t,

$$\frac{d\,r(t)}{dt} = \begin{cases} \frac{d\,(t)}{dt} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$\boxed{\therefore\ u(t) = \frac{d\,r(t)}{dt}}$$

The unit step signal is defined as,

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Integrating u(t) with respect to t,

$$\int u(t)\,dt = \int 1\,dt = t$$

$$\boxed{\therefore r(t) = \int u(t)\,dt}$$

Signal Operations and Properties:

Following are the operations performed on the signals;

(i) *Time Shifting (Delay / Advance):* The signal can be delayed or advanced by a constant time factor. The signal x(t) is time delayed if the time factor is having negative value. The signal x(t) is time advanced if the time factor is having positive value.
i.e., x(t) is right shifted if it is represented as x(t-2) and left shifted if it is represented as x(t+2). The Fig. shows the time shifting operations.

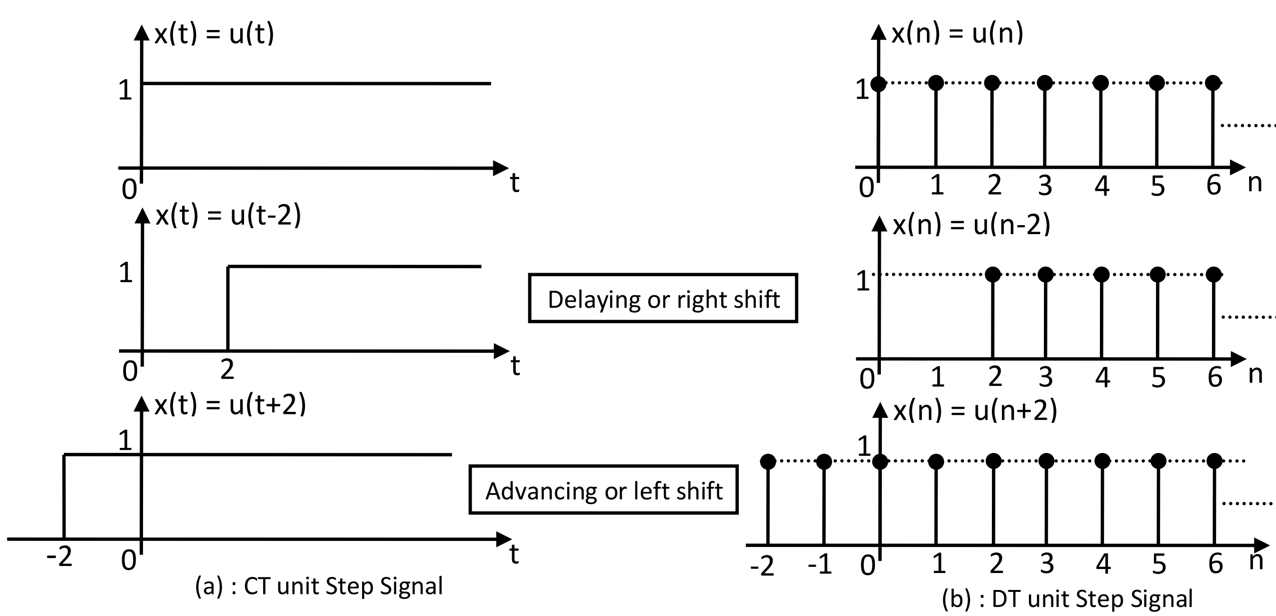
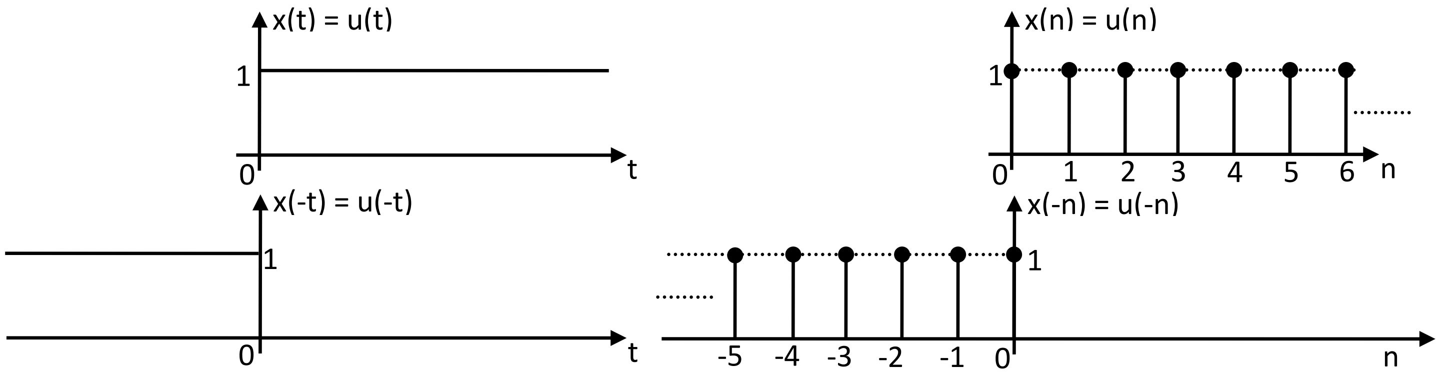


Fig. 1.8: Time Shifting Operation

(ii) *Time Folding:* Time folding is also called as time reversal of signal x(t) and is denoted by x(-t). The signal x(-t) is obtained by replacing t with –t in the given x(t).



(a) : CT unit Step Signal

(b) : DT unit Step Signal

Fig. 1.9: Time Folding Operation

(iii) *Time Scaling (Compression / Expansion):* The time scaling may be compression or expansion of time x(t). It is expressed as y(t) = x(at) where a is the scaling factor.

(iv) *Amplitude Scaling:* The amplitude scaling of the CT signal x(t) is represented as y(t) = A x(t)

(v) *Signal Addition :* The two or more CT signals can be added. The value of new signal is obtained by adding the value of each signal at every instant of time. Subtraction of one signal from other can also be performed in the similar way.

(vi) *Signal Multiplication:* The two signal can be multiplied in its continuous time domain. The value of new signal is obtained by multiplying the two signal values at every instant of time.

Example : Sketch the following signals

(a) $x(t) = 2u(t+2) - 2u(t-3)$ (b) $x(t) = u(t+4)\,u(-t+4)$ (c) $x(t) = r(-t)\,u(t+2)$

(c) $x(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$

Solution: (a) Given $x(t) = 2u(t+2) - 2u(t-3)$

- Consider the elementary signal u(t).
- The signal 2u(t+2) is obtained by shifting u(t) to the left by 2 units and multiplying by 2
- The signal -2u(t-3) is obtained by shifting u(t) to the right by 3 units and multiplying by -2
- The signal x(t) is obtained by adding 2u(t+2) and – 2u(t-3)
- The sketch of all the signals are shown in Fig. (a).

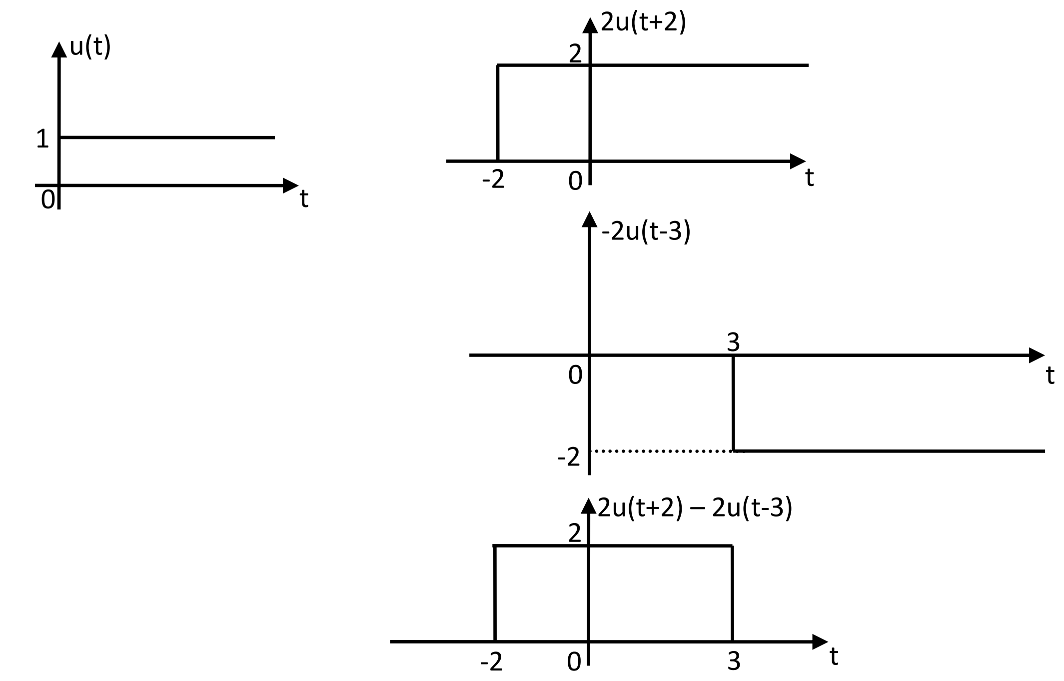


Fig.(a) : Waveforms for Example (a)

Solution: (b) Given $x(t) = u(t+4)\,u(-t+4)$

- Consider the elementary signal u(t).
- The signal u(t+4) is obtained by shifting u(t) to the left by 4 units
- The signal u(-t+4) is obtained by reversing u(t) and then shifting u(-t) to the right by 4 units
- The signal x(t) is obtained by multiplying u(t+4) and u(-t+4)
- The sketch of all the signals are shown in Fig. (b).

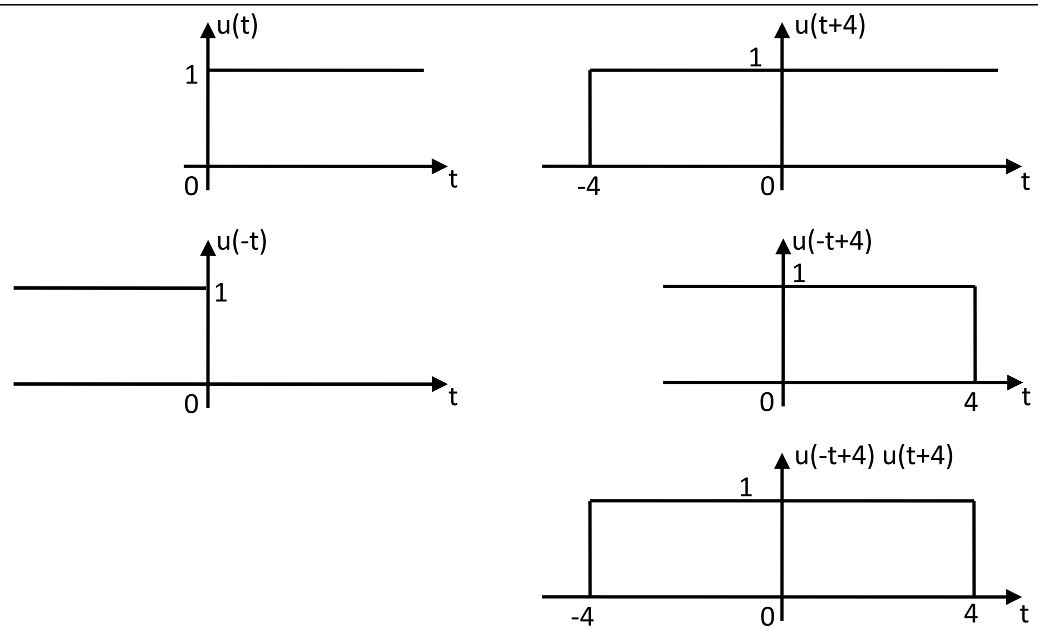


Fig.(b) : Waveforms for Example (b)

Solution: (c) Given x(t) = r(-t) u(t+2)

- Consider the elementary signals u(t) and r(t).
- The signal u(t+2) is obtained by shifting u(t) to the left by 2 units
- The signal r(-t) is obtained by time reversing r(t)
- The signal x(t) is obtained by multiplying r(-t) and u(t+2)
- The sketch of all the signals are shown in Fig. (c).

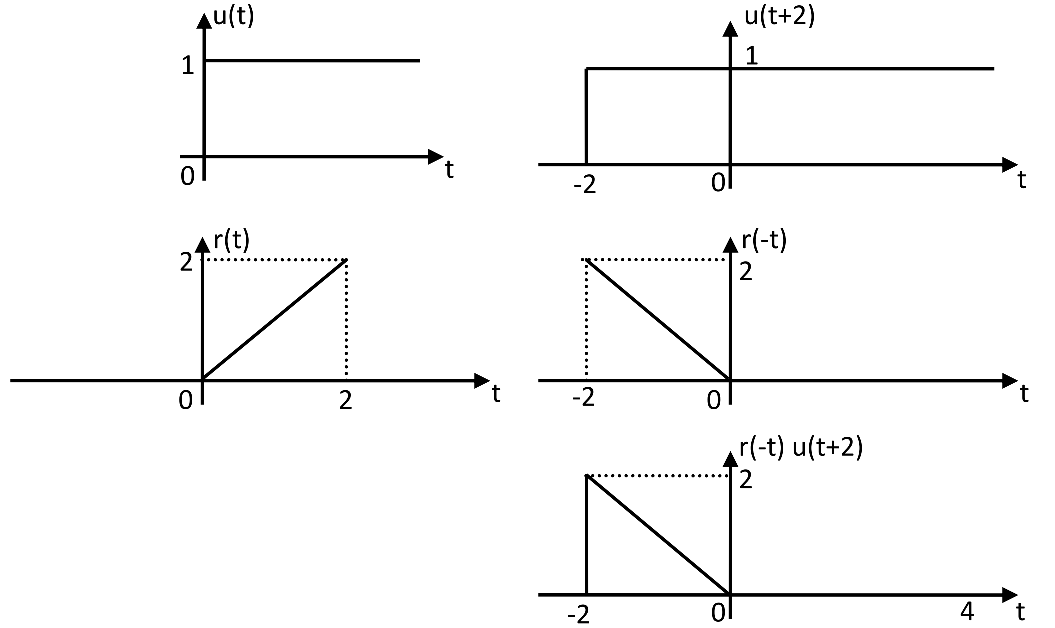


Fig.(c) : Waveforms for Example (c)

(a) Solution: (d) Given x(t) = r(t) – r(t-1) – r(t-3) + r(t-4)

- Consider the elementary signals r(t).
- The signal r(t-1) is obtained by shifting r(t) to the right by 1 unit with slope -1
- Similarly the signal r(t-3) is obtained by shifting r(t) to the right by 3 units with slope -1 and the signal r(t-4) is obtained by shifting r(t) to the right by 4 units with slope 1
- The signal x(t) is obtained by adding r(-t), -r(t-1), -r(t-3) and r(t+4)
- The sketch of all the signals are shown in Fig. (d).

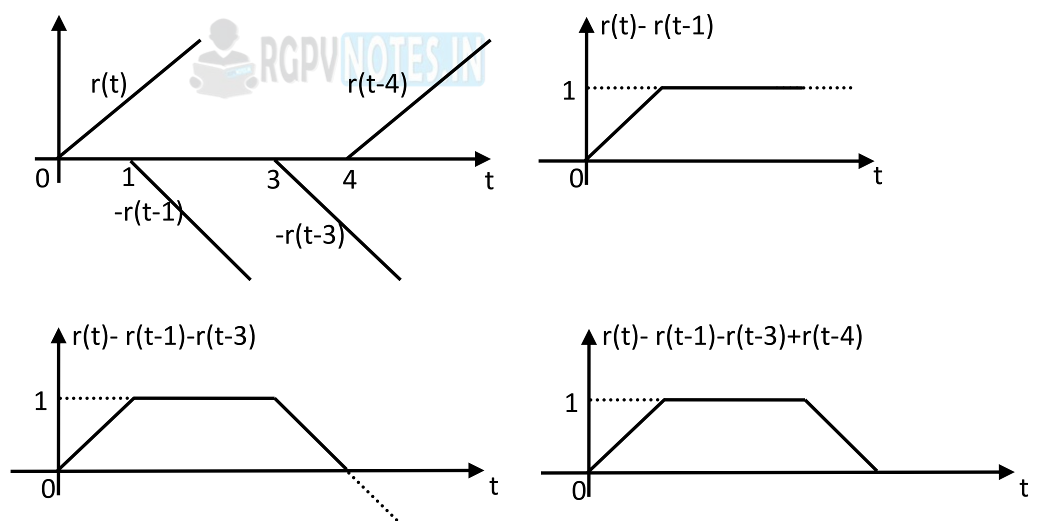


Fig.(d) : Waveforms for Example (d)

Basic System Properties :

(I) Causality (II) Time Invariant & Variant (III) Linearity (IV) Stability (V) Static & Dynamic (VI) Invertible & Non-Invertible

(I) Causal and Non-causal System : A system is said to be **causal** if its output $y(t)$ at any arbitrary time t_0 depends only on the values of its input $x(t)$ for $t \le t0$. In the causal system the output does not begin before the input signal is applied. If the independent variable represents time, a system must be causal in order to be physically realizable. Noncausal systems can sometimes be useful in practice, however, as the independent variable need not always represent time.

Example : Determine whether the following systems are causal or non-causal

(i) $y(t) = 0.2x(t) – x(t-1)$ (ii) $y(t) = 0.8x(t-1)$ (iii) $y(n) = x(n-1)$

(iv) $y(t) = x(t+1)$ (v) $y(n-2) = x(n)$ (vi) $y(n) = x(n) –x(n+1)$

Solution:

(i) Given that $y(t) = 0.2x(t) – x(t-1)$

In the above equation put t=0 then $y(0) = 0.2x(0) – x(-1)$

put t=1 then $y(1) = 0.2x(1) – x(0)$

Since the output y(t) depends on the present and the past input values of x(t), the system is **causal**

(ii) Given that $y(t) = 0.8x(t-1)$

In the above equation put t=0 then $y(0) = 0.8\, x(-1)$

put t=1 then $y(1) = 0.8\, x(0)$

Since the output y(t) depends on only the past input values of x(t), the system is **causal**

(iii) Given that $y(n) = x(n-1)$

In the above equation put n=0 then $y(0) = x(-1)$

put n=1 then $y(1) = x(0)$

Since the output y(n) depends on only the past input values of x(n), the system is **causal**

(iv) Given that $y(t) = x(t+1)$

In the above equation put t=0 then $y(0) = x(1)$

put t=1 then $y(1) = x(2)$

Since the output y(t) depends on future input values of x(t), the system is **non-causal**

(v) Given that $y(n-2) = x(n)$

In the above equation put n=0 then $y(-2) = x(0)$

put n=1 then $y(-1) = x(1)$

Since the output y(t) depends on future input values of x(t), the system is **non-causal**

(vi) Given that $y(n) = x(n) –x(n+1)$

In the above equation put n=0 then $y(0) = x(0) – x(1)$

put n=1 then $y(1) = x(1) – x(2)$

Since the output y(t) depends on the present and the future input values of x(t), the system is **non-causal**

(II) Time Invariant & Variant System: Let $y(t)$ be the response of a system to the input $x(t)$, and let t_0 be a time-shift constant. If, for any choice of $x(t)$ and t_0, the input $x(t - t_0)$ produces the output $y(t - t_0)$, the system is said to be **time invariant**. A system is time invariant, if a time shift in the input signal results in an identical time shift in the output signal.

(III) Linear and Non-Linear System: Let $y_1(t)$ and $y_2(t)$ denote the responses of a system to the inputs $x_1(t)$ and $x_2(t)$, respectively. If, for any choice of $x_1(t)$ and $x_2(t)$, the response to the input $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$, the system is said to possess the **additivity** property.

Let $y(t)$ denote the response of a system to the input $x(t)$, and let a denote a complex constant. If, for any choice of $x(t)$ and a, the response to the input $ax(t)$ is $ay(t)$, the system is said to possess the **homogeneity** property.

If a system possesses both the additivity and homogeneity properties, it is said to be **linear**. Otherwise, it is said to be **nonlinear**.

The two linearity conditions (i.e., additivity and homogeneity) can be combined into a single condition known as superposition. Let $y_1(t)$ and $y_2(t)$ denote the responses of a system to the inputs $x_1(t)$ and $x_2(t)$, respectively, and let a and b denote complex constants. If, for any choice of $x_1(t)$, $x_2(t)$, a, and b, the input $ax_1(t)+bx_2(t)$ produces the response $ay_1(t)+by_2(t)$, the system is said to possess the **superposition** property.

To show that a system is linear, we can show that it possesses both the additivity and homogeneity properties, or we can simply show that the superposition property holds.

Example : Determine whether the following systems are linear or non-linear

(i) $y(t) = t.x(t)$ (ii) $y(t) = x^2(t)$ (iii) $y(t) = ax(t) + b$

Solution:

(i) Given that $y(t) = t.x(t)$

Let $y_1(t) = tx_1(t)$ and $y_2(t) = tx_2(t)$

Now, the linear combination of the two outputs will be

$$y_3(t) = a_1 y_1(t) + a_2y_2(t) = a_1 tx_1(t) + a_2tx_2(t)$$

Also the response to the linear combination of input will be

$$y_4(t) =f[a_1x_1(t) + a_2x_2(t)] = t[a_1x_1(t) + a_2x_2(t)]$$
$$y_4(t) = a_1t\, x_1(t) + a_2t\, x_2(t)$$

Since the output $y_3(t) = y_4(t)$, the system is **linear system.**

(ii) Given that $y(t) = x^2(t)$

Let $y_1(t) = x_1^2(t)$ and $y_2(t) = x_2^2(t)$

Now, the linear combination of the two outputs will be

$$y_3(t) = a_1 y_1(t) + a_2y_2(t) = a_1 x_1^2(t) + a_2 x_2^2(t)$$

Also the response to the linear combination of input will be

$$y_4(t) =f[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$$
$$y_4(t) = a_1 x_1^2(t) + a_2 x_2^2(t) + 2a_1 a_2 x_1(t) x_2(t)$$

Since the output $y_3(t) \neq y_4(t)$, the system is **not a linear system.**

(iii) Given that $y(t) = ax(t) + b$

Let $y_1(t) = ax_1(t)+b$ and $y_2(t) = ax_2(t)+b$

Now, the linear combination of the two outputs will be

$$y_3(t) = a_1 y_1(t) + a_2y_2(t) = a_1(ax1(t)+b) + a_2 (ax_2(t)+b)$$

Also the response to the linear combination of input will be

$$y_4(t) =f[a_1x_1(t) + a_2x_2(t)] = a[a_1x_1(t) + a_2x_2(t)] + b$$

Since the output $y_3(t) \neq y_4(t)$, the system is **not a linear system.**

(IV) Memory and Memory-less System: A system is said to have **memory** if its output $y(t)$ at any arbitrary time $t0$ depends on the value of its input $x(t)$ at any time other than $t = t_0$. If a system does not have memory, it is said to be **memory less**.

(V) Invertible and Non-invertible System: A system is said to be **invertible** if its input $x(t)$ can always be uniquely determined from its output $y(t)$. From this definition, it follows that an invertible system will always produce distinct outputs from any two distinct inputs. If a system is invertible, this is most easily demonstrated by finding the inverse system. If a system is not invertible, often the easiest way to prove this is to show that two distinct inputs result in identical outputs.

(VI) Stability : The bounded-input bounded-output (BIBO) stability is most commonly defined in system analysis. A system having the input $x(t)$ and output $y(t)$ is **BIBO stable** if, a bounded input produces a bounded output.