Method for Design of Analog Group Delay Equalizers

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***Abstract*— This paper provides an effective method for design of the analog allpass filter for equalization of the group delay frequency response. This method is based on combination of the Differential Evolution algorithm and modified Remez second algorithm. We are able to propose equalizers with equiripple group delay response in the passband of the lowpass filters. The procedure works automatically without an input estimation. The method is illustrated on numerical examples.**

I. INTRODUCTION

The genetic and evolutionary algorithms were found out as a powerful tool in design and optimization of electrical circuits and systems. These algorithms are time-consuming, but, on the other hand, computationally robust and lead to the optimum results of the complicated design problems solution. Excellent results have been obtained e.g. in a transfer function approximation problem solution in many cases of analog or digital filters design [1], [2].

Here we have proposed efficient design method for analog group delay equalizers. The design procedure of an analog filter with requirements for magnitude and group delay frequency response can be divided into two parts. Design of the analog filter by using standard method is the first well-established part and design of the additional analog allpass filter is the second part. It is applied in connection with the main filter for equalization of the group delay frequency response. The magnitude response and the group delay are optimized individually in cascade-coupled filter blocks. At this point, the optimization can be effectively done by adjusting the parameters of each section.

In general, an approximation of the equalizer transfer function represents an exigent mathematical problem. The commonly used design methods based on a usage of numerical methods for nonlinear equation system solving, need to estimate an initial approximation well to ensure convergence of the computed task. To avoid this, we have put together two procedures. At first, we estimate the analog allpass filter transfer function using method based on Differential Evolution (DE) algorithm. Subsequently, the estimation is used for nonlinear equation system solving. The equation system is solved using modified Remez algorithm. The algorithm works with the characteristic function of allpass filters. This procedure allows to design filters with equiripple group delay in the passband.

Note that evolutionary algorithms are powerful global minimization algorithms which simulate an evolutional process in the nature, see [3].

II. CHARACTERISTIC EQUATION OF THE ALLPASS FILTERS

Transfer function of the allpass filters has a form

$$A(s) = \frac{a(-s)}{a(s)} \tag{1}$$

where $a(s)$ is Hurwitz's polynomial. A phase characteristics results from the expression

$$A(\jmath\omega) = \frac{a(-\jmath\omega)}{a(\jmath\omega)} = e^{-\jmath\varphi(\omega)}\,, \tag{2}$$

where $\varphi(\omega)$ means the phase characteristic.

Approximation of the group delay can be constructed from the characteristic equation of the allpass filter

$$A(s) = \frac{a(-s)}{a(s)} = \frac{1 - K\phi(s)}{1 + K\phi(s)} = e^{-\varphi(s)}\,, \tag{3}$$

where $\phi(s)$ is the characteristic function of the allpass filter. It is a rational function of the complex frequency $s = \sigma + \jmath\omega$. Characteristic function has properties of the imittance function, because zeros and poles of $\phi(s)$ are located only at the imaginary axis $(s = \jmath\omega)$

$$\phi(s) = s\prod_{i=1}^{n} \frac{s^2 + \omega_{0i}^2}{s^2 + \omega_{\infty i}^2}\,. \tag{4}$$

It means, the group delay approximation is calculated by applying only one variable ω. Therefore it is the same principle like the magnitude response approximation. Thus the method is much more simple concerning an applied mathematical methods against the methods presented for example in [6], [7]. Group delay is described by the following formula

$$\tau_e(\omega) = -\frac{d\varphi(s)}{ds}\bigg|_{s=\jmath\omega}\,. \tag{5}$$

III. SOLUTION OF THE APPROXIMATION PROBLEM

Group delay characteristic of the corrected filter is marked $\tau_f(\omega)$ and group delay characteristic of the allpass filter is $\tau_e(\omega)$. Requirement of group delay approximation is the accomplishment of equation

$$\tau(\omega) = \tau_f(\omega) + \tau_e(\omega) = \tau_v(\omega) \pm \delta \tag{6}$$

A. First Part: Estimation of the Group Delay Equalizer

1) The Differential Evolution Algorithm: The DE algorithm belongs to the evolutionary algorithms group. The mentioned algorithm was developed by K. Price and for the first time was presented in 1994. In the conference First International Contest of Evolutionary Computation (1stICEO) held in Nagoya in May 1996, this algorithm turned out to be the best evolution type of algorithm for the real-valued functions solving.

The DE algorithm is a parallel direct search method which uses floating-point number representation to find continuous parameters.

This algorithm is well-known and popular for technical applications solving. Its principle can be found in [8], [1]. Several variants of DE exist. But, we have found out that the variant denoted like *DE/best/1/bin* is the most efficient version for the solved task. Therefore, we have applied this version in practical examples of analog allpass filters design.

2) The Analog Allpass Filter Estimation Method: Fundamental of the presented procedure is computation of the zeros, poles and constant K of the allpass filter characteristic function (4) so that computed allpass filter has a group delay frequency response, which equalizes the group delay frequency response of the lowpass filter. We find minimum of the $\Delta\tau(\omega)$ using the proposed DE algorithm. The mathematical formulation of the objective function is

$$F(\underline{x}) = \Delta\tau(\omega) + P_1 + P_2 = \max\Big(\tau(\omega)\Big) - \min\Big(\tau(\omega)\Big) + P_1 + P_2 \tag{7}$$

where vector $\underline{x}$ is composed of zeros, poles and constant K of the allpass filter characteristic function. P_1 and P_2 are the penalty functions, which can be computed by

$$P_1 = \sum_{i=1}^{N} \begin{cases} 20000 - 100x_i & \text{if } x_i < 0 \\ 20000 + 100x_i & \text{if } x_i > 10 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where N labels number of the unknown searched variables and x_i are elements of the vector $\underline{x}$. Because of the fact, that DE algorithm does not hold the range of the unknown searched variables, we have to use the penalty function. The penalty function P_1 is included into the objective function to ensure a prescribed interval of values of the analog allpass filter characteristic function (zeros, poles and constant K) .

$$P_2 = \sum_{i=1}^{N-1} \begin{cases} 0 & \text{if } x_i < x_{i+1} \\ 20000 + 100x_i & \text{otherwise} \end{cases} \tag{9}$$

The penalty function P_2 ensures conditions for physical implementation of the all-pass filter. The main property of the characteristic function $\phi(s)$ is that its poles and zeros have to alternate on the imaginary axis.

Here we have used the penalty function published in [1]

The vector $\underline{x}_{opt}$, for which the objective function $F(\underline{x})$ has minimum, is the wanted solution of the analog allpass filter characteristic function estimation problem.

B. Second Part: Method for Final Equiripple Approximation

The approximation algorithm seeks zeros and poles of the characteristic function $\phi(s)$, so that the group delay characteristic must be equiripple at the prescribed frequency interval. Thus, parameters of the allpass filter characteristic function obtained by the previous estimation algorithm are used as input values to the following algorithm.

The iterative procedure is carried out in the following steps :

1) Set the iteration index $\nu = 1$.
2) Initial set of zeros $\omega_{0i}^{(1)}$, poles $\omega_{\infty j}^{(1)}$ and constant $K^{(1)}$ of the characteristic function $\phi(s)$ is found using previous genetic algorithm, where $j = 1, 2, \ldots, n \oplus 2$, $i = 1, 2, \ldots, (n-1) \oplus 2$ [1] and n is the order of transfer function of the allpass filter. Superscript means the step of the iteration.
3) Find the positions of minima ω_k^{min} and maxima ω_l^{max} of the group delay curve from equation
$$\frac{d}{d\omega}\Big(\tau_e(\omega) + \tau_f(\omega)\Big) = \frac{d}{d\omega}\tau(\omega) = 0\,, \tag{10}$$
Frequencies of minima and maxima are regularly changing.
4) Create set of nonlinear equations in a form
$$\begin{aligned} \tau_e(0) + \tau_f(0) &= \tau_v \pm \delta \\ \tau_e(\omega_1^{min}) + \tau_f(\omega_1^{min}) &= \tau_v \mp \delta \\ \tau_e(\omega_1^{max}) + \tau_f(\omega_1^{max}) &= \tau_v \pm \delta \\ \tau_e(\omega_2^{min}) + \tau_f(\omega_2^{min}) &= \tau_v \mp \delta \\ \tau_e(\omega_2^{max}) + \tau_f(\omega_2^{max}) &= \tau_v \pm \delta \\ &\vdots \\ \tau_e(\omega_k^{min}) + \tau_f(\omega_k^{min}) &= \tau_v \mp \delta \\ \tau_e(\omega_l^{max}) + \tau_f(\omega_l^{max}) &= \tau_v \pm \delta \end{aligned} \tag{11}$$
and $k + l = n - 1$.
5) This system of nonlinear equations is solved by the numerical gradient method. New positions of zeros $\omega_{0i}^{(\nu)}$, poles $\omega_{\infty j}^{(\nu)}$ and constant $K^{(\nu)}$ are solutions of the nonlinear system.
6) Set $\nu = \nu + 1$ and go to step 2 until the condition in the equation (6) will be performed.

[1] Symbol $\oplus$ means the *modulo* operation

IV. EXAMPLES

The usability of this method to the allpass filter design will be presented on the solution of the following example.

It is required to equalize the group delay frequency response of the analog lowpass filter designed using a development system, for instance on web page [5].

The input requirement is to design a lowpass filter, which meet requirements to magnitude frequency response. These requirements to the magnitude frequency response can be accomplished using analog filter with 5th-order transfer function proposed by applying the Cauer approximation.

The transfer function of the filter is defined

$$H(s) = \frac{\sum_{i=0}^{4} b_i s^i}{\sum_{j=0}^{5} a_j s^j}, \tag{12}$$

where coefficients are arranged in table I.

TABLE I
THE COEFFICIENTS OF THE DESIGNED LOWPASS FILTER.

$b_0 = 0.253189$	$a_0 = 0.253189$
$b_1 = 0$	$a_1 = 0.885141$
$b_2 = 0.150948$	$a_2 = 1.407205$
$b_3 = 0$	$a_3 = 2.013651$
$b_4 = 0.019196$	$a_4 = 1.159668$
	$a_5 = 1$

The group delay frequency response of this filter is shown in Fig.1.

Group delay error[2] is $\Delta\tau_f = 10.91$ s in passband and we want to equalize group delay frequency response to receive lower group delay error and also to achieve equiripple form of the group delay frequency response.

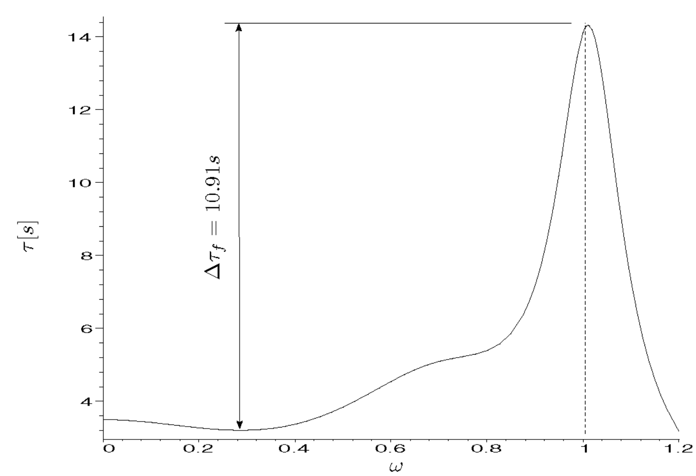

Fig. 1. The group delay frequency response of the proposed filter

We have decided for illustration of described method efficiency to use allpass filters of the order $n = 4, 5, 6$ defined by terms (3), (4).

[2]difference between maximum and minimum of the group delay in the pass-band

A. An Estimation of the Characteristic Function Parameters

As mentioned above for this part, we will compute the zeros, poles and constant K of the allpass filter characteristic functions (4) to be obtained minima of the objective functions (7). The minima are searched using DE algorithm.

The optimization task stated above was solved using the initial settings of the DE algorithm: $NP = 150$, $CR = 0.9$, $F = 0.9$. As we have indicated, the DE algorithm does not hold range of the unknown searched variables. Thus the algorithm allows to search the unknown variables values out of the initial range. We have used range of the zeros and poles $\omega_{0i}, \omega_{\infty i} \in (0, 1)$, range of the constant $K \in (0, 2)$. The group delay frequency responses were sampled at 1024 equidistant points in the passband.

We have computed these values for each allpass filter. The results are summarized in Tab.II.

TABLE II
THE ESTIMATED PARAMETERS OF THE ALLPASS CHARACTERISTIC FUNCTIONS

$n = 4$	$n = 5$	$n = 6$
$\omega_{\infty 1} = 0.3035$	$\omega_{\infty 1} = 0.2258$	$\omega_{\infty 1} = 0.1846$
$\omega_{01} = 0.6015$	$\omega_{01} = 0.4334$	$\omega_{01} = 0.3726$
$\omega_{\infty 2} = 0.8859$	$\omega_{\infty 2} = 0.6779$	$\omega_{\infty 2} = 0.5546$
$K = 0.9825$	$\omega_{02} = 0.9323$	$\omega_{02} = 0.7635$
	$K = 1.098$	$\omega_{\infty 3} = 1.018$
		$K = 1.0577$

The appropriate group delay of the total connections (lowpass and allpass filters), calculated from the mentioned values, is shown in Fig.2.

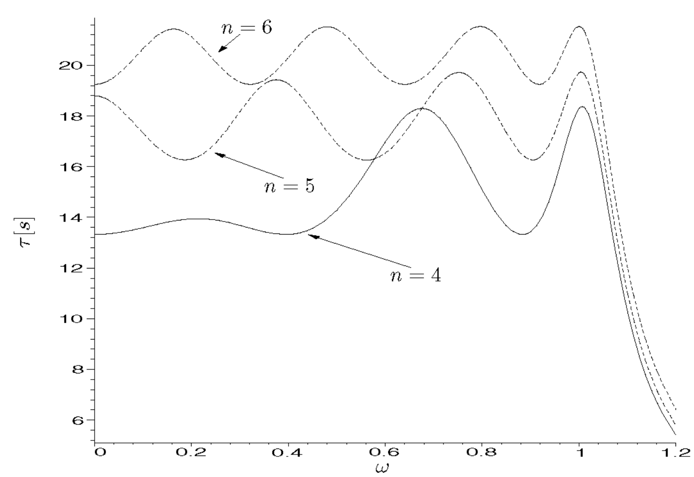

Fig. 2. The group delay frequency responses of the total connections achieved by the estimation procedure

B. Final Solution of the Group Delay Approximation

Next part of the presented algorithm solves the minimax approximation in the Chebyshev sense using modified Remez algorithm. This algorithm needs good initial estimation of the characteristic function variables to be ensured its convergence. Calculated values written in Tab.II have been used for initial variables estimation. We have found out these resultant values given in Tab.III, after performing of 20 iteration of the modified Remez algorithm.

TABLE III

THE FINAL PARAMETERS OF THE ALLPASS FILTER CHARACTERISTIC FUNCTIONS

$n = 4$	$n = 5$	$n = 6$
$\omega_{\infty 1} = 0.2588$	$\omega_{\infty 1} = 0.2206$	$\omega_{\infty 1} = 0.1842$
$\omega_{01} = 0.5434$	$\omega_{01} = 0.4277$	$\omega_{01} = 0.3718$
$\omega_{\infty 2} = 0.8152$	$\omega_{\infty 2} = 0.6716$	$\omega_{\infty 2} = 0.5538$
$K = 0.7212$	$\omega_{02} = 0.924$	$\omega_{02} = 0.7628$
	$K = 1.1351$	$\omega_{\infty 3} = 1.0166$
		$K = 1.0526$

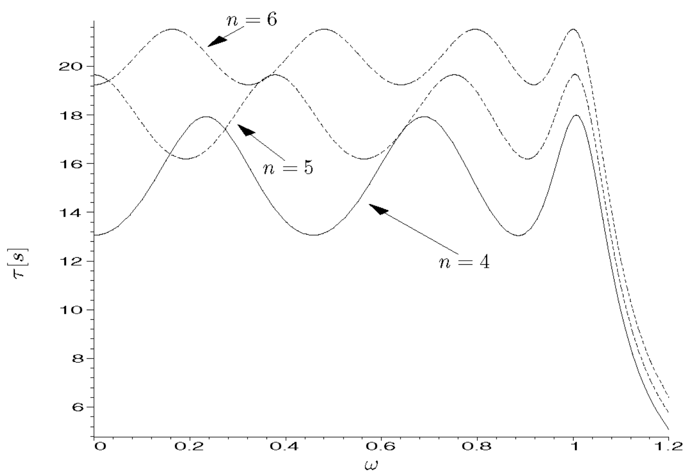

Fig. 3. The final group delay frequency response of the total connections achieved by the whole approximation procedure

TABLE IV

FINAL VALUES OF THE GROUP DELAY ERRORS

n	$\Delta\tau$
4	4.8792
5	3.4581
6	2.2852

Transfer functions of the allpass filters $A(s)$ are given by terms (3), (4). Group delay errors in the passband of the equalized filter with applied allpass filters are shown in Tab.IV. The resultant group delay of the total connection (lowpass and allpass filters), calculated from the mentioned values, is shown in Fig.3.

V. CONCLUSIONS

A usage of combination of DE algorithm and modified Remez algorithm is new unconventional design method of the group delay equalizers. You can see from the results that we achieved very small value of the group delay error. Moreover, the total group delay frequency responses have equiripple form in the pass band. It is very good property of the described method for equalizing of analog filters or another delay lines. The whole optimization algorithm is very fast. This method is able to solve this class of the tasks effectively.

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