

Problem PSG1

Given: L<sub>1</sub> Lyapunov orbit at  $\xi = 0.01$  and  $\gamma = 0$

Find: a) Eigenvalues and eigenvectors again. I.C. for off the periodic orbit but on stable manifold.

- b) Propagate backwards in time and globalize stable manifold. Globalize both manifolds; plot in configuration space.
- c) Select other fixed point on x-axis. Compute stable manifold and plot  $y=0$  on far side of moon. How close to Moon?
- d) Produce unstable manifolds too.

Solution:

a) We have derived these orbits multiples times so I will not rederive it or its eigenvalues

$$\lambda_1 = 2.5611 \times 10^3 \quad \lambda_2 = 3.9045 \times 10^{-4} \quad \lambda_3 = \lambda_4 = 1$$

$$\lambda_5 = 0.9899 + 0.1421i \quad \lambda_6 = 0.9899 - 0.1421i$$

$$V^{ws} = \frac{V^s}{\sqrt{x^2 + y^2 + z^2}} = \begin{bmatrix} 0.8657 \\ 0.5006 \\ 0 \\ -2.6734 \\ -1.5288 \\ 0 \end{bmatrix}$$

Let  $x^* = 40 \text{ km}$

$$x^* = \begin{bmatrix} 0.847 \\ 0.0001 \\ 0 \\ -0.0003 \\ -0.0784 \\ -0 \end{bmatrix}$$

Note, that we discovered that a reasonably small perturbation was sufficient to create the manifolds.

continued...

b) See Figure: G1.1, G1.2

c) The other point on the x-axis is  $\frac{1}{2}$  a period.

The donut approach when crossing x-axis the second time is  $5.2481 \times 10^{-4}$  km

See Figure: G1.3, G1.4

These manifold shortly merge into the periodic orbit. It does about a quarter of a revolution

d) See figures: G1.5, G1.6, G1.7, G1.8, G1.9 and G1.10

The method to produce stable and unstable is similar, but you will have to propagate forward in time

# PSG1

Part b)

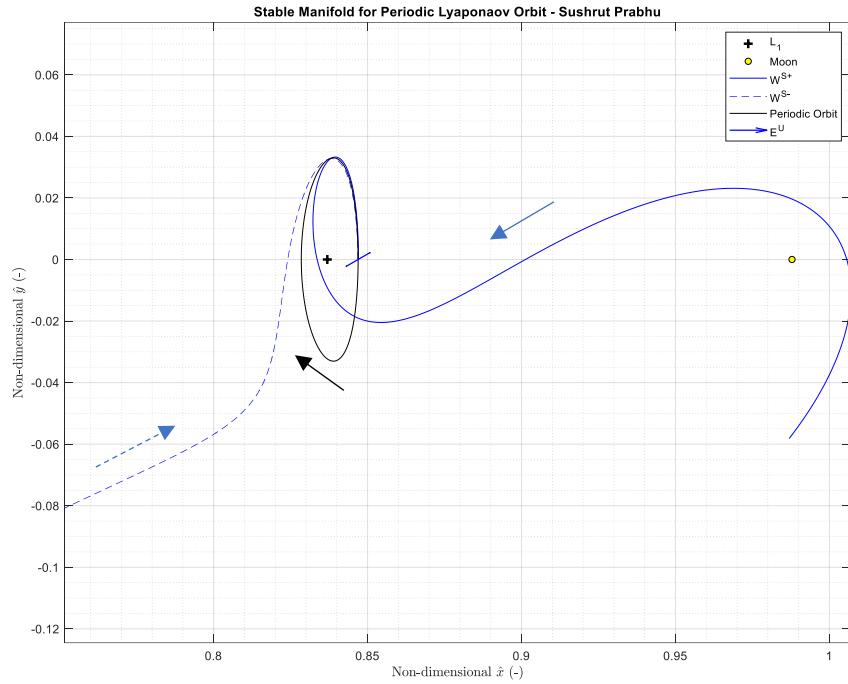


Figure G1.1: Stable manifold starting at the  $y = 0$  on the Lunar side.

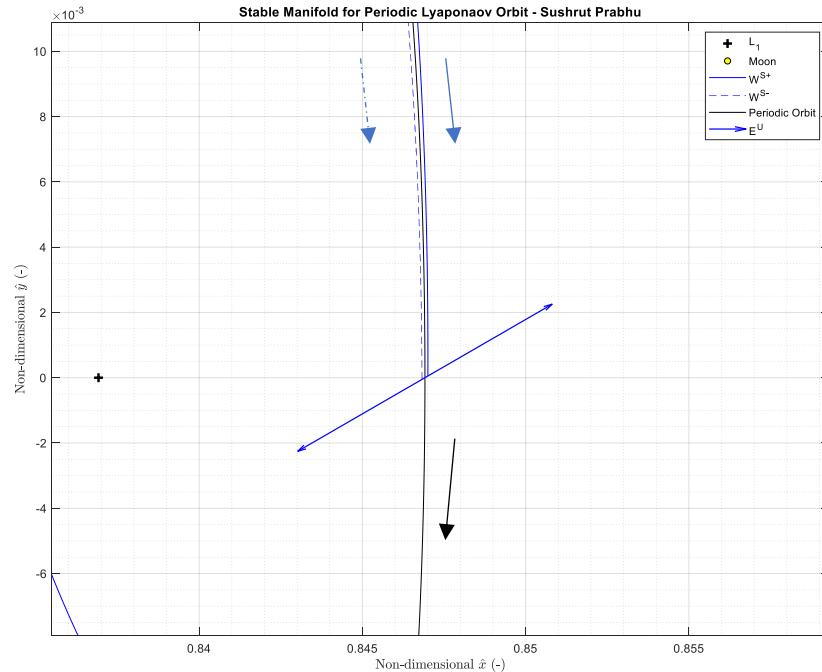


Figure G1.2: Stable manifold starting at the  $y = 0$  on the Lunar side zoomed in view.

Part c)

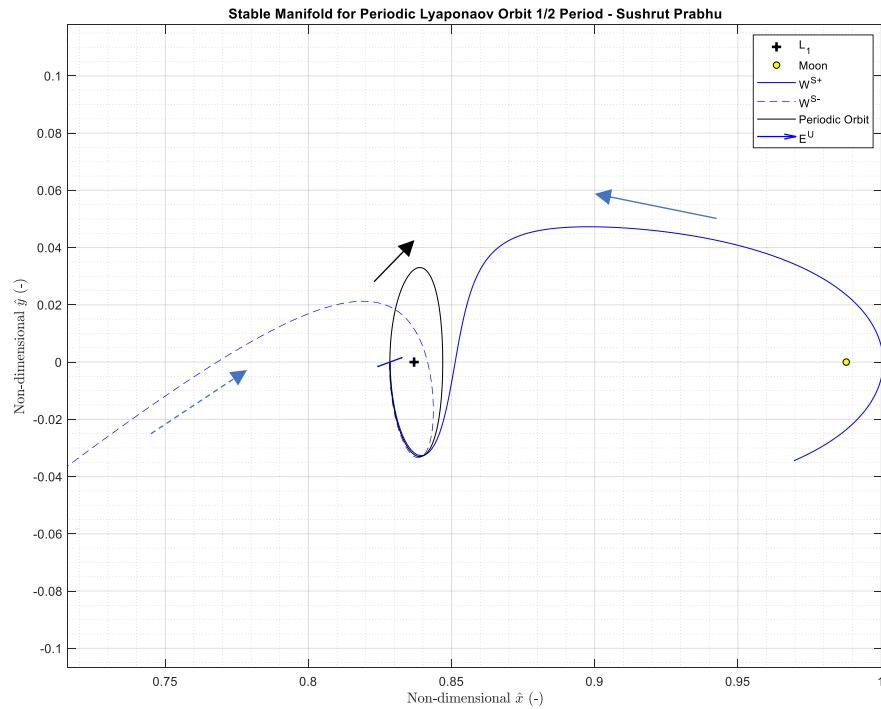


Figure G1.3: Stable manifold starting at the  $y = 0$  on the Earth side.

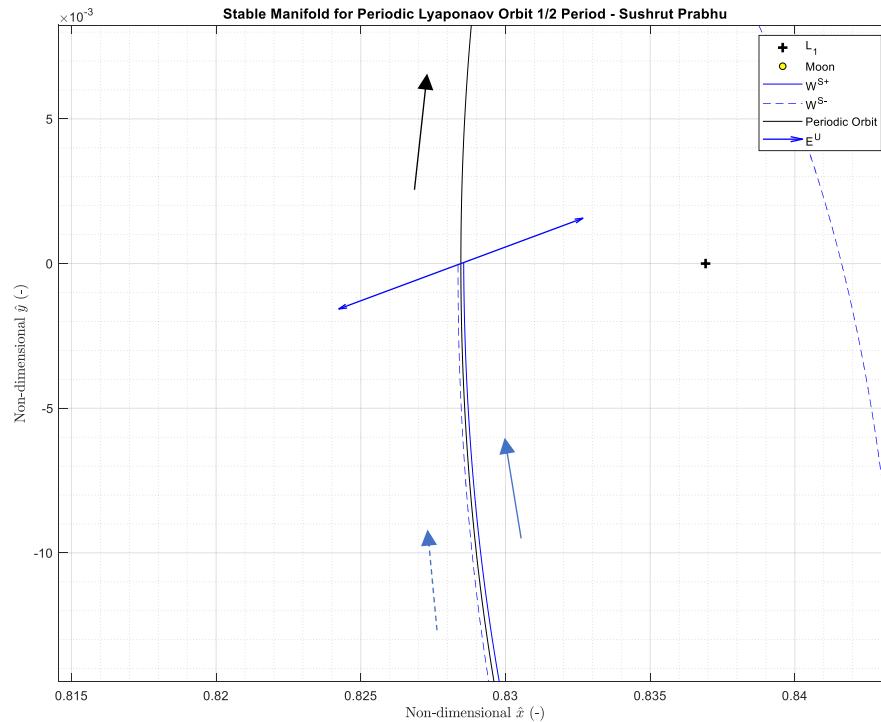


Figure G1.4: Stable manifold starting at the  $y = 0$  on the Earth side zoomed in view.

Part d)

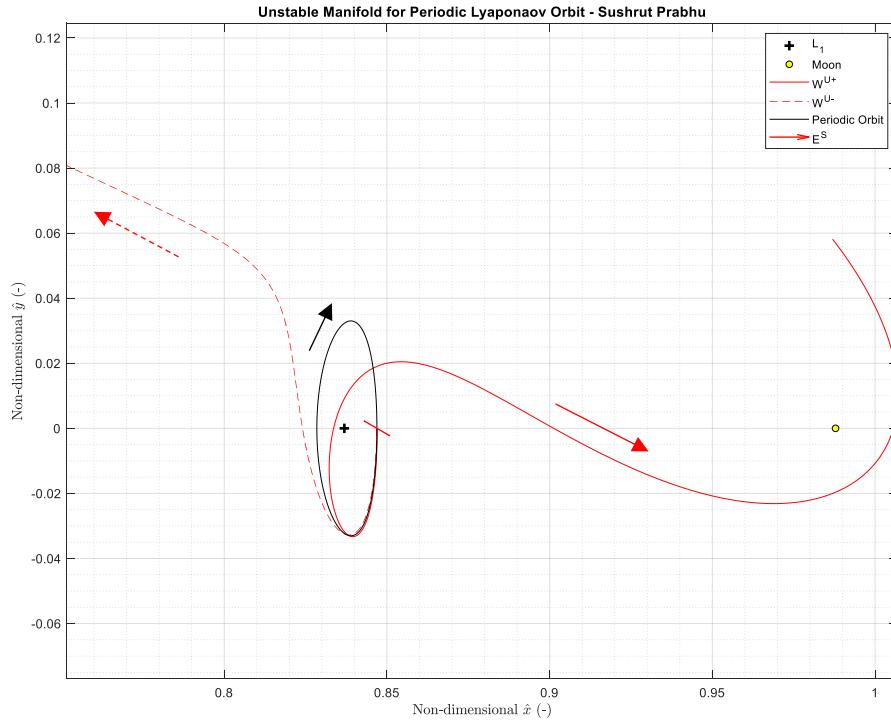


Figure G1.5: Unstable manifold starting at the  $y = 0$  on the Lunar side.

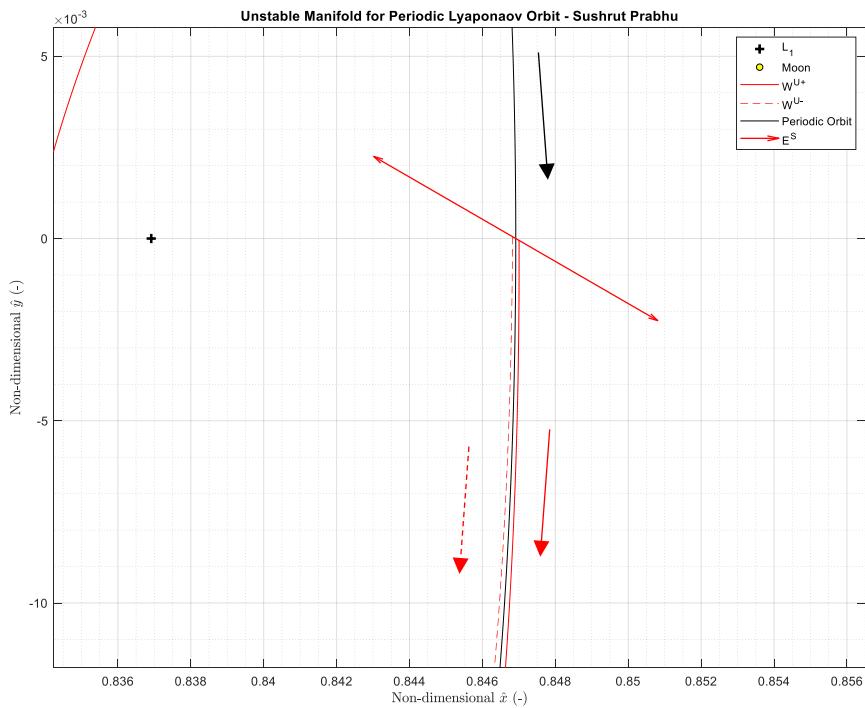


Figure G1.6: Unstable manifold starting at the  $y = 0$  on the Lunar side zoomed in view.

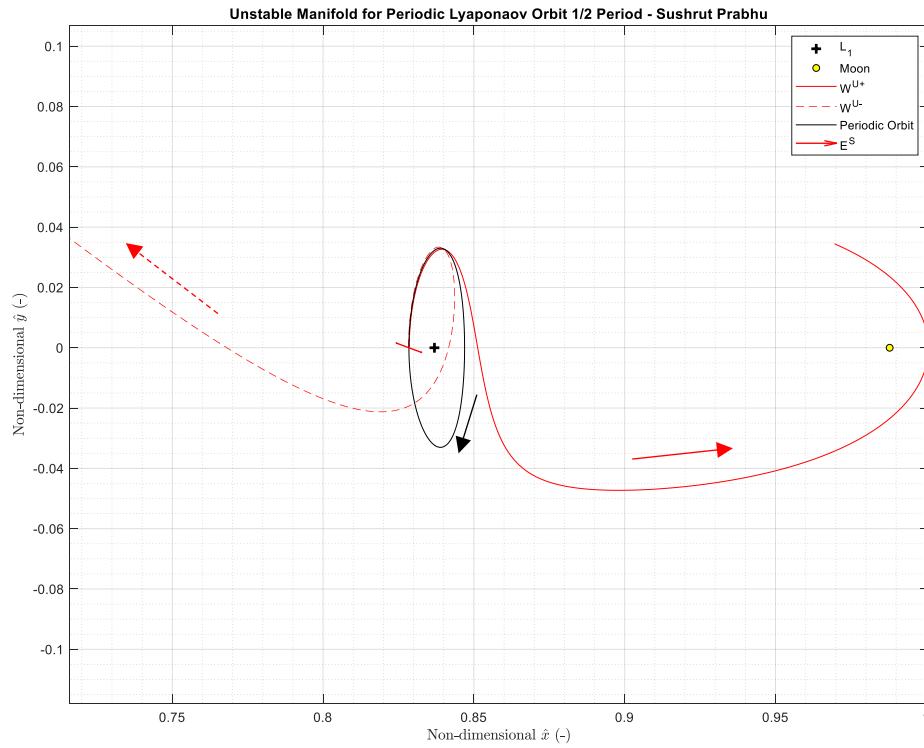


Figure G1.7: Unstable manifold starting at the  $y = 0$  on the Earth side.

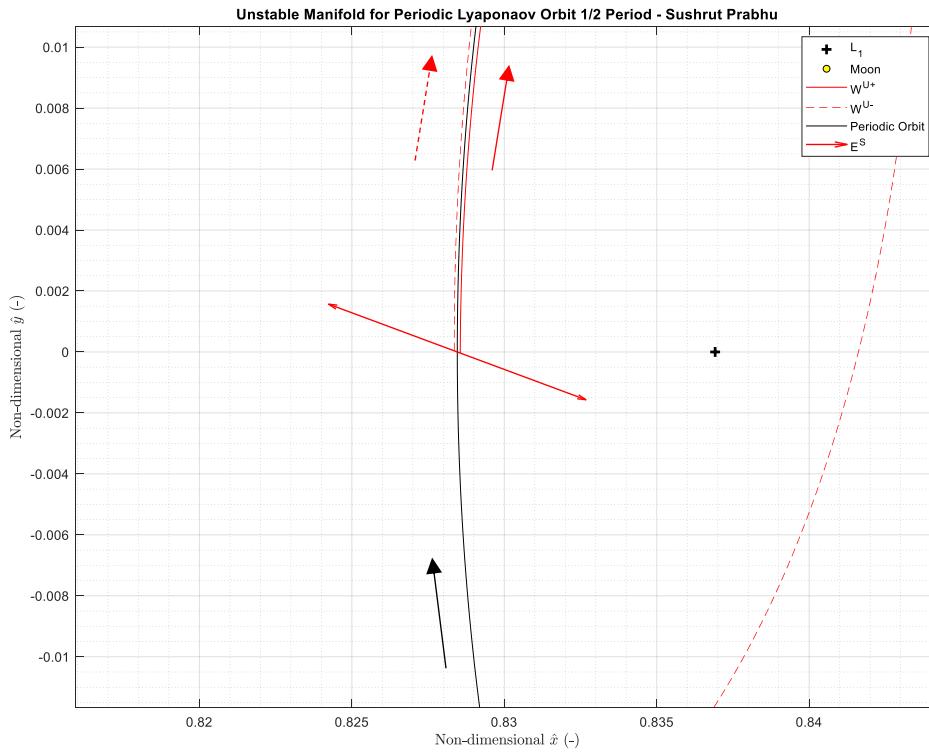


Figure G1.8: Unstable manifold starting at the  $y = 0$  on the Earth side zoomed in view.

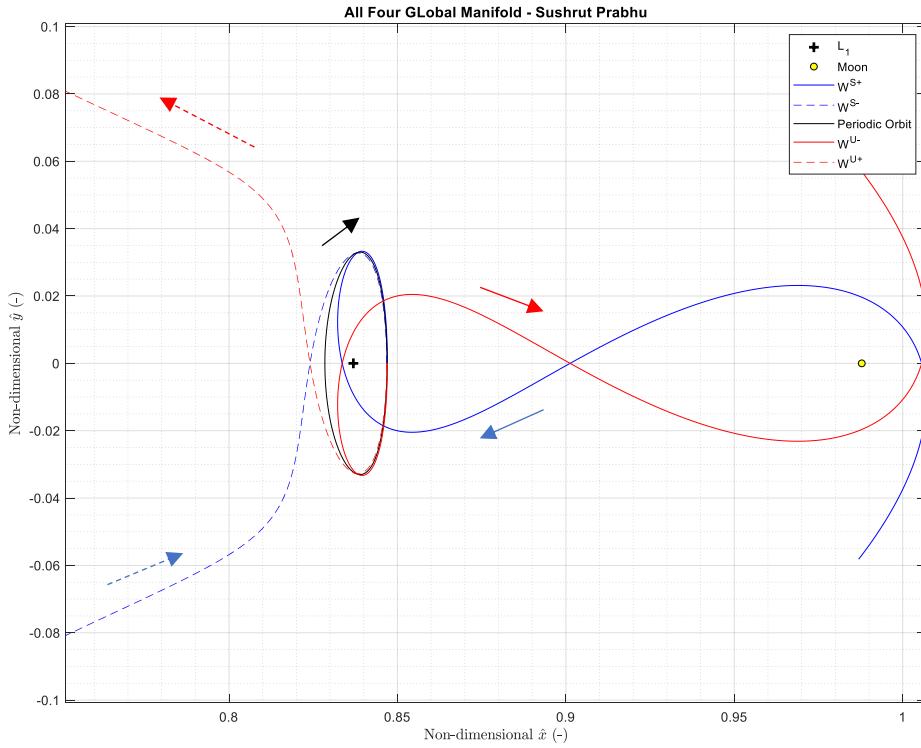


Figure G1.9: All four manifolds on the Lunar side.

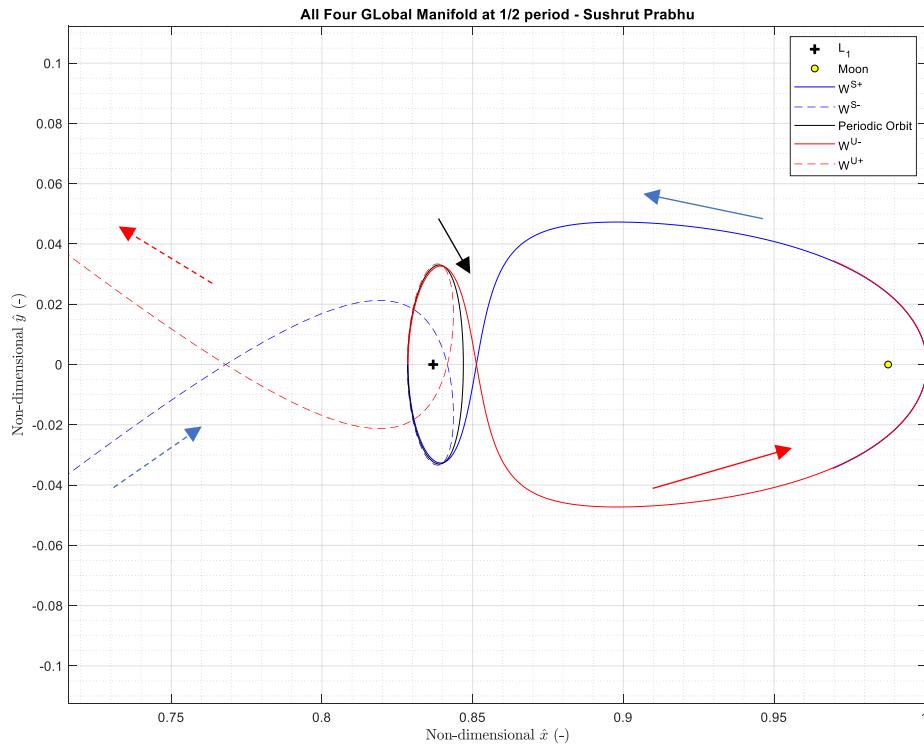


Figure G1.10: All four manifolds on the Earth side.

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## PSG1

```
clear
close all
clc

SS = SolarS;
systems = {'-' , 'Earth-Moon'};
param = {'l*' (km)', 'm*' (kg)', 'miu' , 't*' , 'gamma_1', 'L_1', 'gamma_1
(km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
tolerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G); % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G; % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}* .7, dim_vals{4,2},
10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

## Part a

```
xi = 0.01;
y = [dim_vals{7,2}+xi 0 0 0 -.082 0];
t_end = 2.711/2;

[yn,t_end] = Target3d_per([0
0],y(1:3),y(4:6),t_end,dim_vals{4,2}, "planar", 10^-13, "");

STM0 = eye(6);
STM0 = STM0(:)';
IC = [yn, STM0];
```

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```

G = [1 0 0 0 0 0; 0 -1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 -1 0 0; 0 0 0 0 1
0;0 0 0 0 -1];
Omega = [0 1 0; -1 0 0; 0 0 0];

[~,y]=ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});
[~,yfull]=ode45(@cr3bp_STM_df3d,[0 t_end*2],IC,options,dim_vals{4,2});

stm_12 = reshape(y(end,7:end),6,6)';
monodromy = G* [zeros(3,3), -eye(3);eye(3), -2*Omega]*stm_12' *
[-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

[V,D] = eig(monodromy);
D = diag(D);

V_ws = V(:,2)/norm(V(1:3,2));           % Normalized v stable

```

## Part b

```

d = 40/dim_vals{2,2};
x_star = y(1,1:6);
t_end2 = 2.5;

xs1 = x_star + d*V_ws';
[~,y1] = ode45(@cr3bp_df,[t_end2 0],xs1,options,dim_vals{4,2});

xs2 = x_star - d*V_ws';
[~,y2] = ode45(@cr3bp_df,[t_end2 0],xs2,options,dim_vals{4,2});

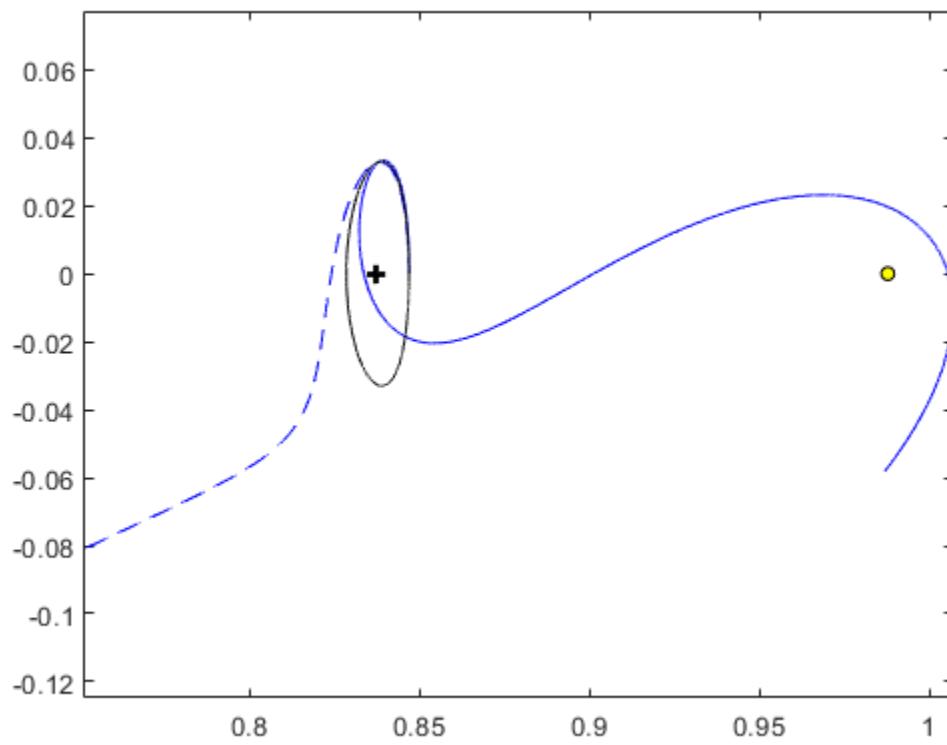
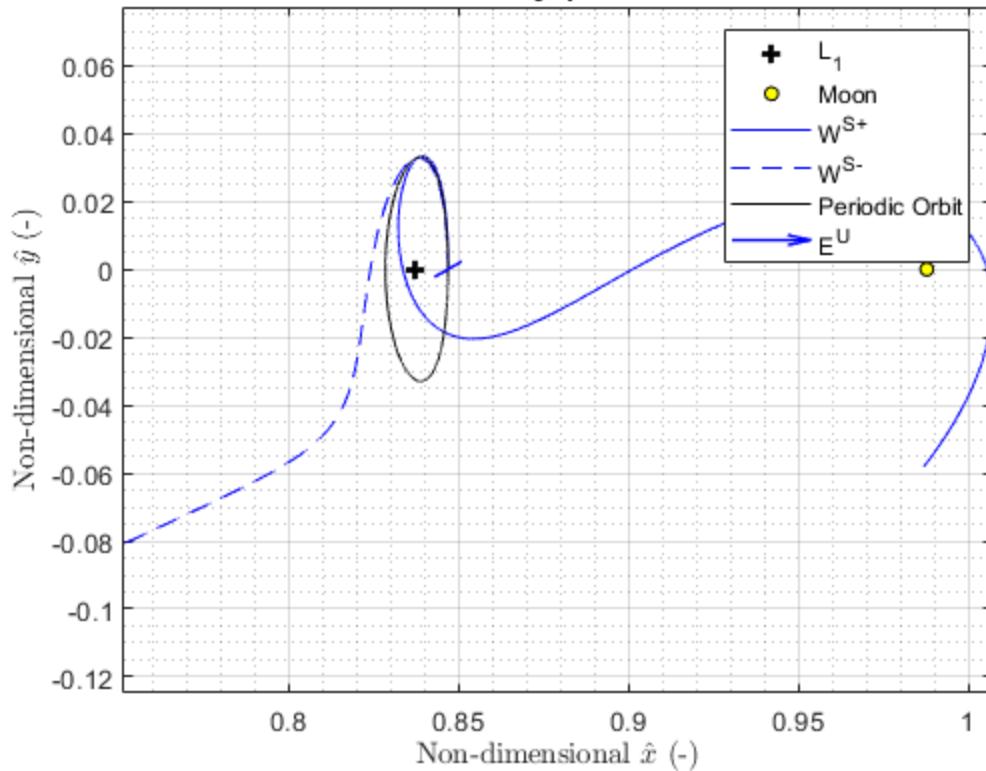
figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'--b')
plot(yfull(:,1),yfull(:,2),'k')
axis equal
quiver(y(1,1),y(1,2),V_ws(1)*.005,V_ws(2)*.005,'b','LineWidth',1)
quiver(y(1,1),y(1,2),-V_ws(1)*.005,-V_ws(2)*.005,'b','LineWidth',1)
grid on
grid minor
title('Stable Manifold for Periodic Lyaponaov Orbit - Sushrut Prabhu')
xlabel("Non-dimensional $\hat{x}$ (-)", "Interpreter", "latex")
ylabel("Non-dimensional $\hat{y}$ (-)", "Interpreter", "latex")
legend('L_1','Moon','W^S^+','W^S^-','Periodic Orbit','E^U')

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'--b')
plot(yfull(:,1),yfull(:,2),'k')
axis equal

```

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### Stable Manifold for Periodic Lyapounov Orbit - Sushrut Prabhu



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## Part c

```
phi = y(:,7:end);
phi_t = reshape(phi(end,:),6,6)';
V_t2 = phi_t*V;
t_end2 = 2.5;

V_ws = V_t2(:,2)/norm(V_t2(1:3,2));           % Normalized v stable

x_star = y(end,1:6);

xs1 = x_star + d*V_ws';
[~,y1] = ode45(@cr3bp_df,[t_end2 0],xs1,options,dim_vals{4,2});

xs2 = x_star - d*V_ws';
[~,y2] = ode45(@cr3bp_df,[t_end2 0],xs2,options,dim_vals{4,2});

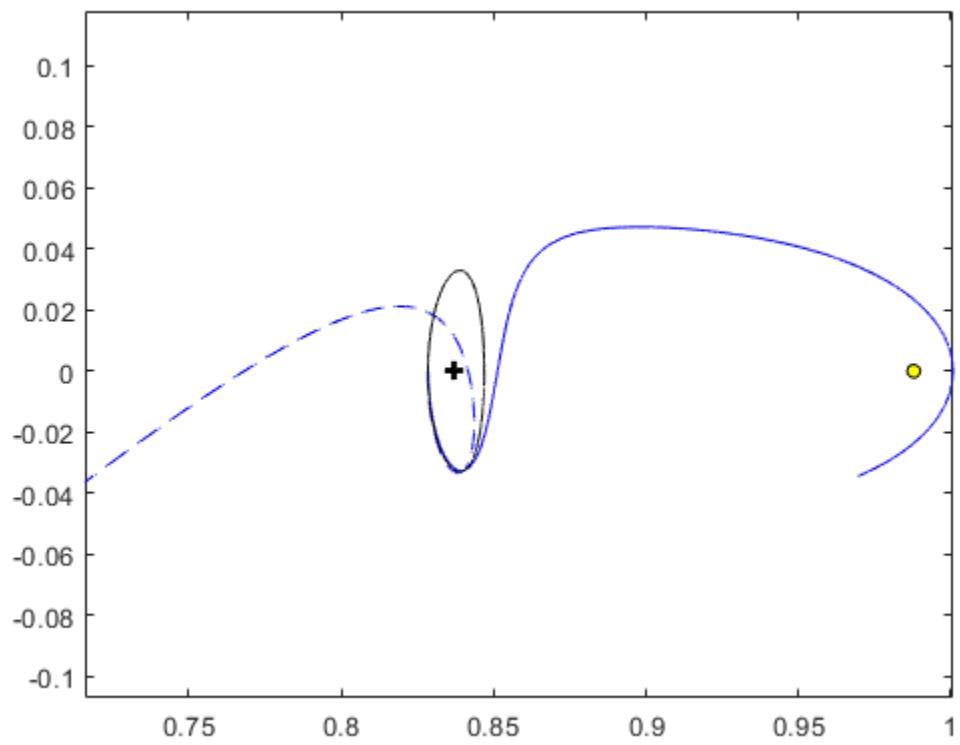
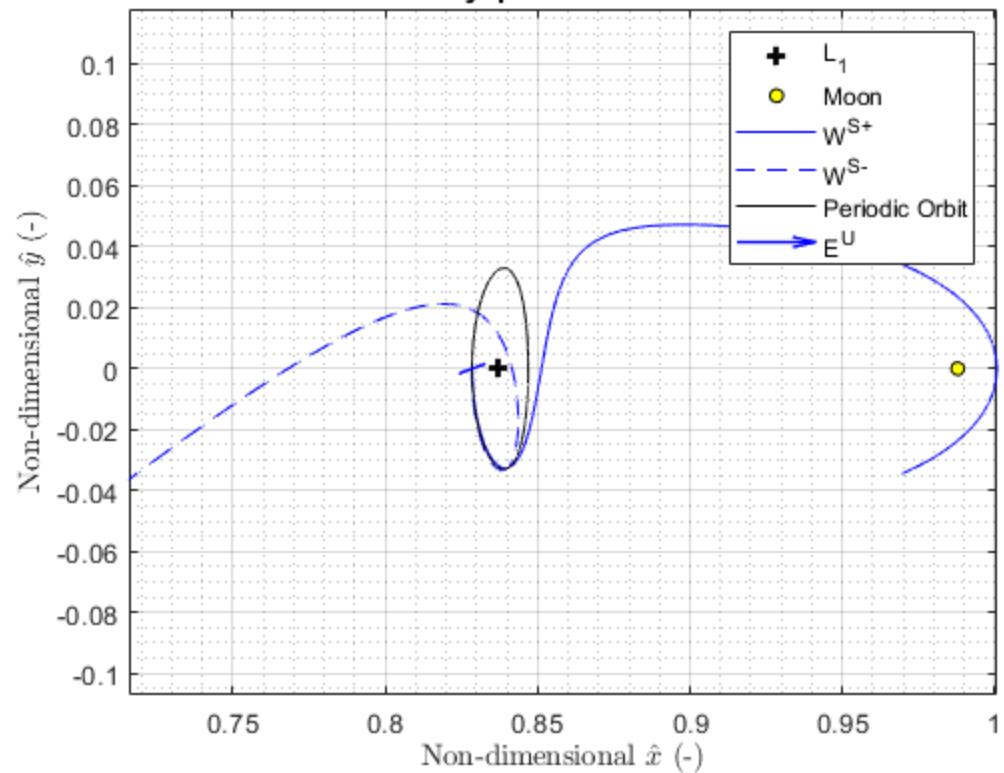
moon_dist = y1(:,1:2) - [1-dim_vals{4,2},0];
closest_app = min(vecnorm(moon_dist,1,2))*dim_vals{2,2};

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'--b')
plot(yfull(:,1),yfull(:,2),'k')
axis equal
quiver(y(end,1),y(end,2),V_ws(1)*.005,V_ws(2)*.005,'b','LineWidth',1)
quiver(y(end,1),y(end,2),-V_ws(1)*.005,-
V_ws(2)*.005,'b','LineWidth',1)
grid on
grid minor
title('Stable Manifold for Periodic Lyaponaov Orbit 1/2 Period -
Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-),"Interpreter", "latex")
legend('L_1','Moon','W^S^+','W^S^-','Periodic Orbit','E^U')

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'--b')
plot(yfull(:,1),yfull(:,2),'k')
axis equal
```

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### Stable Manifold for Periodic Lyapounov Orbit 1/2 Period - Sushrut Prabhu



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## Part d

```
V_wu = V(:,1)/norm(V(1:3,1));           % Normalized v unstable
d = 40/dim_vals{2,2};
x_star = y(1,1:6);
t_end2 = 2.5;

xu1 = x_star + d*V_wu';
[~,y1] = ode45(@cr3bp_df,[0 t_end2],xu1,options,dim_vals{4,2});

xu2 = x_star - d*V_wu';
[~,y2] = ode45(@cr3bp_df,[0 t_end2],xu2,options,dim_vals{4,2});

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'r')
plot(y2(:,1),y2(:,2),'--r')
plot(yfull(:,1),yfull(:,2),'k')
axis equal
quiver(y(1,1),y(1,2),V_wu(1)*.005,V_wu(2)*.005,'r','LineWidth',1)
quiver(y(1,1),y(1,2),-V_wu(1)*.005,-V_wu(2)*.005,'r','LineWidth',1)
grid on
grid minor
title('Unstable Manifold for Periodic Lyaponaov Orbit - Sushrut Prabhu')
xlabel("Non-dimensional $\hat{x}$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $\hat{y}$ (-)","Interpreter", "latex")
legend('L_1','Moon','W^U^+', 'W^U^-','Periodic Orbit','E^S')

figure(2)
plot(y1(:,1),y1(:,2),'r')
plot(y2(:,1),y2(:,2),'--r')
grid on
grid minor
title('All Four GLobal Manifold - Sushrut Prabhu')
xlabel("Non-dimensional $\hat{x}$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $\hat{y}$ (-)","Interpreter", "latex")
legend('L_1','Moon','W^S^+', 'W^S^-','Periodic Orbit','W^U^-','W^U^+')

V_wu = V_t2(:,1)/norm(V_t2(1:3,1));           % Normalized v stable
x_star = y(end,1:6);
t_end2 = 2.5;

xu1 = x_star + d*V_wu';
[~,y1] = ode45(@cr3bp_df,[0 t_end2],xu1,options,dim_vals{4,2});

xu2 = x_star - d*V_wu';
[~,y2] = ode45(@cr3bp_df,[0 t_end2],xu2,options,dim_vals{4,2});

figure
```

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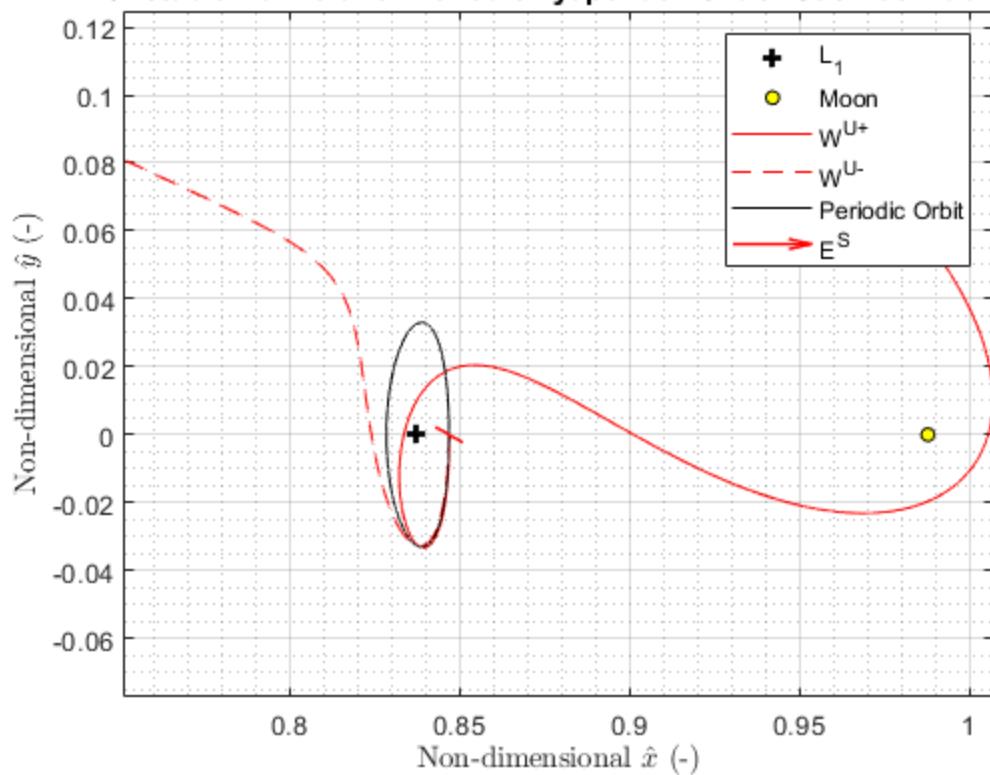
```

plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'r')
plot(y2(:,1),y2(:,2),'--r')
plot(yfull(:,1),yfull(:,2),'k')
axis equal
quiver(y(end,1),y(end,2),V_wu(1)*.005,V_wu(2)*.005,'r','LineWidth',1)
quiver(y(end,1),y(end,2),-V_wu(1)*.005,-
V_wu(2)*.005,'r','LineWidth',1)
grid on
grid minor
title('Unstable Manifold for Periodic Lyaponaov Orbit 1/2 Period -
Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-),"Interpreter", "latex")
legend('L_1','Moon','W^U^+', 'W^U^-','Periodic Orbit','E^S')

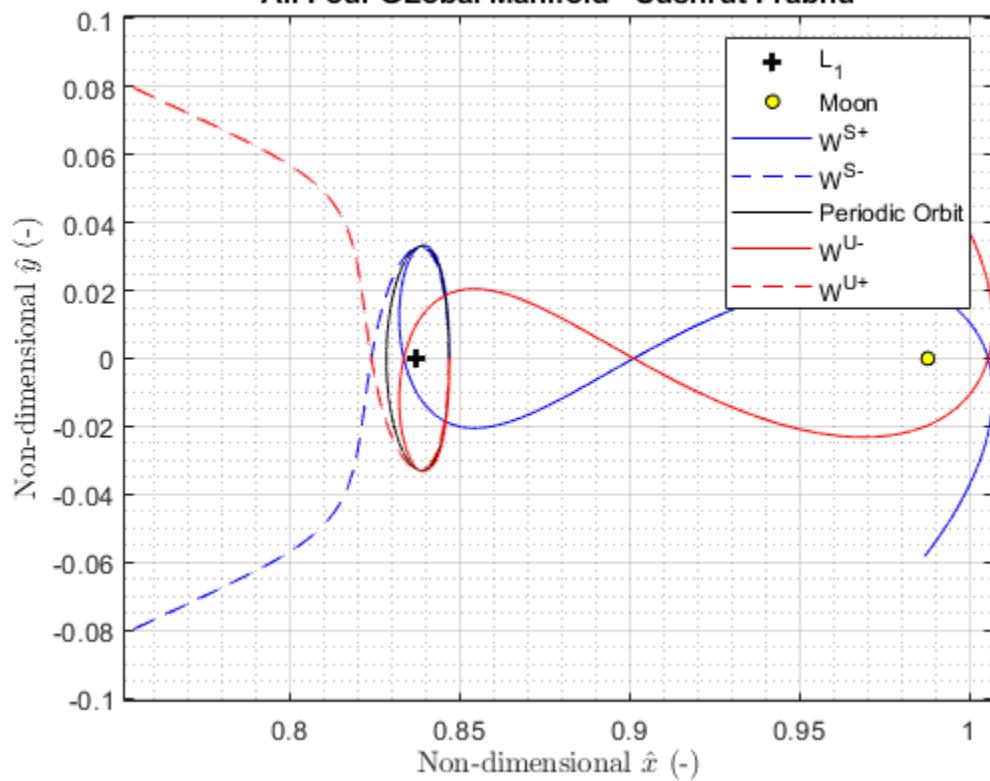
figure(4)
plot(y1(:,1),y1(:,2),'r')
plot(y2(:,1),y2(:,2),'--r')
grid on
grid minor
title('All Four GLobal Manifold at 1/2 period - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-),"Interpreter", "latex")
legend('L_1','Moon','W^S^+', 'W^S^-','Periodic Orbit','W^U^-','W^U^+')

```

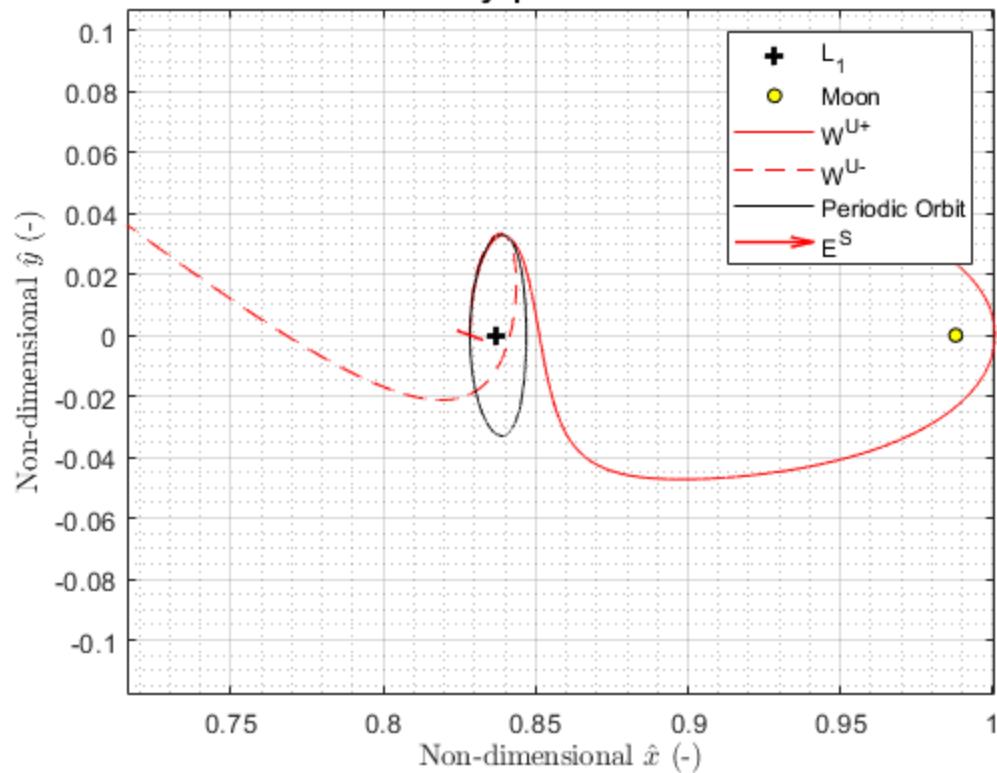
**Unstable Manifold for Periodic Lyapponov Orbit - Sushrut Prabhu**



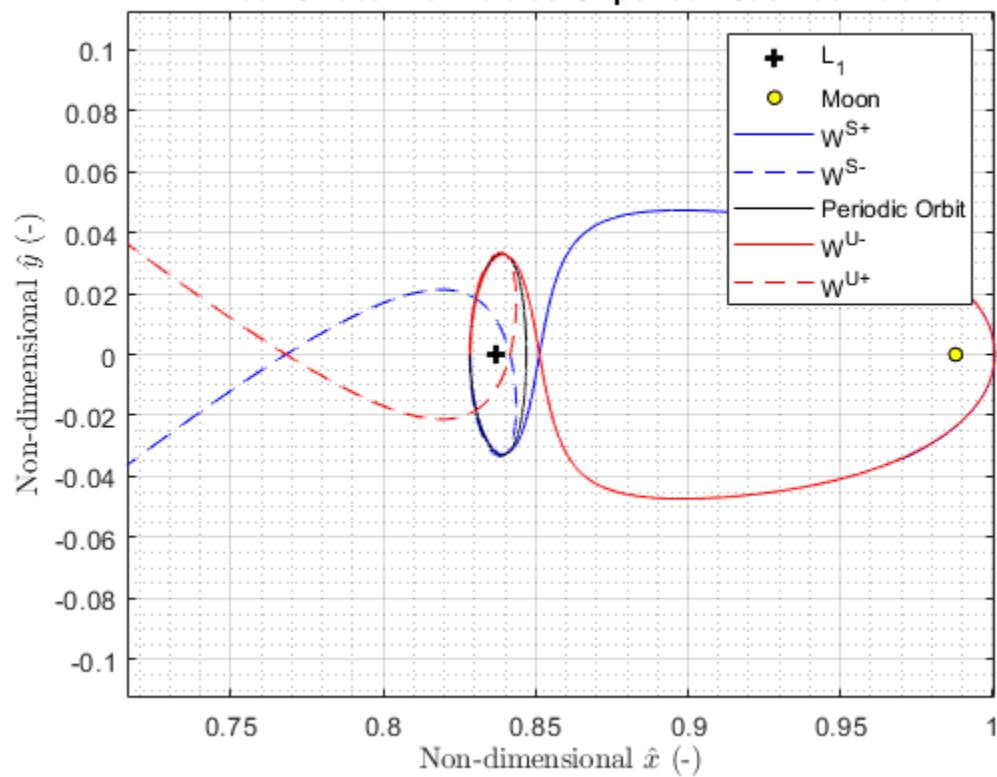
**All Four Global Manifold - Sushrut Prabhu**



**Unstable Manifold for Periodic Lyapounov Orbit 1/2 Period - Sushrut Prabhu**



**All Four Global Manifolds at 1/2 period - Sushrut Prabhu**





PSG2

Giron: Earth-Moon 3 body problem

Find: a) Plot 3D Halo. Check periodic? Correct if needed  
Are the orbits periodic?

- b) Check perpendicular crossing. Check if they meet perpendicular crossing. Use continuation to find another orbit between the two halo orbits.
- c) Lyapunov orbits, the bifurcating orbits were noted. Do these halo orbit come from this?

Solution:

- a) There are a couple of ways to check if orbit is periodic. Propagate for multiple periods and see if it is periodic. Yes, they are periodic

See Figure: G2-1, G2-2, G2-3, and G2-4

This orbit stays in that range for just under 5 periods. So this may be a propagation of numerical error

For perpendicular crossing stop the orbit after crossing the x-axis. The check velocity at  $y=0$ . If  $v_y = 0$  then it is a perpendicular crossing. These orbits cross perpendicularly

continued...

b) The target we need to build is one that aims for a 3d perpendicular crossing.

Note: we can only change  $x_0$ ,  $y_0$ , and  $t^{\text{time of flight}}$

$$\therefore \delta z_f = \frac{\partial z}{\partial x_0} \delta x_0 + \frac{\partial z}{\partial y_0} \delta y_0 + \dot{z} \delta t$$

$$\therefore \delta z_f = \frac{\partial z}{\partial x_0} \delta x_0 + \frac{\partial z}{\partial y_0} \delta y_0 + \dot{z} \delta t$$

$$\therefore \delta \dot{y}_f = \frac{\partial \dot{y}}{\partial x_0} \delta x_0 + \frac{\partial \dot{y}}{\partial y_0} \delta y_0 + \ddot{y} \delta t$$

We have 3 equations and 3 unknowns so the target should be relatively easier.

$$\therefore \begin{bmatrix} \delta x_f \\ \delta z_f \\ \delta \dot{y}_f \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{15} & \dot{x} \\ \phi_{31} & \phi_{35} & \dot{z} \\ \phi_{51} & \phi_{55} & \ddot{y} \end{bmatrix} \begin{bmatrix} \delta x_0 \\ \delta y_0 \\ \delta t \end{bmatrix}$$

No, correction was need as the target yielded the same initial conditions

We use same target to get a new halo orbit. Select  $z_0$  between the two halo orbits and then target  $x_0$ ,  $y_0$ , and  $t$ . For the initial guess we can use average values of the two orbits

See figure: b2.1, b2.2, b2.3, and b2.4

continued..

c) Bifurcations occur when eigenvalue combination change in the Monodromy matrix. So we can reduce the step size to see where this actually occurs. See figure: G2.5.

It occurs at  $x_0 = 0.8555$ . But not the initial position of the halo orbits we have is on the Earth side. The  $x_0$  position we have is on the Lunar side. So we can propagate this orbit for half a period to get  $x_0 = 0.8233$ .

Now use the targets above to target at various  $z_0$  values. Using a linear fit of the previous 2 orbits to guess the next.

See figures: G2.6, G2.7, G2.8, and G2.9

So the orbits given are a result of the bifurcation at  $x_0 = 0.8233$

## PSG2

Part b)

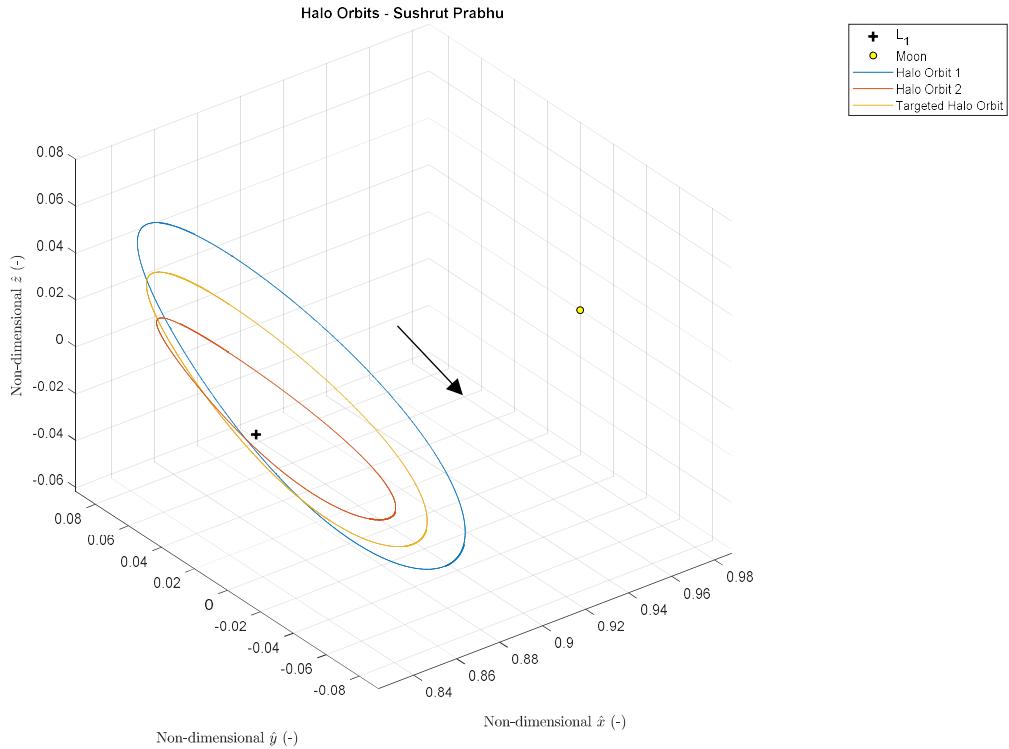


Figure G2.1: The two given halo orbits and one in between them.

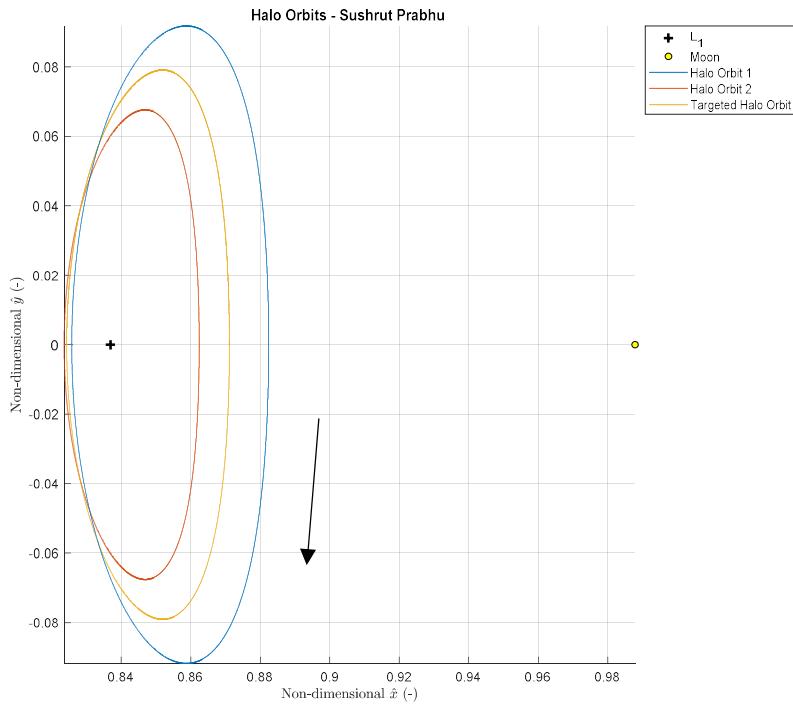


Figure G2.2: The x-y view of the two given halo orbits and one in between them.

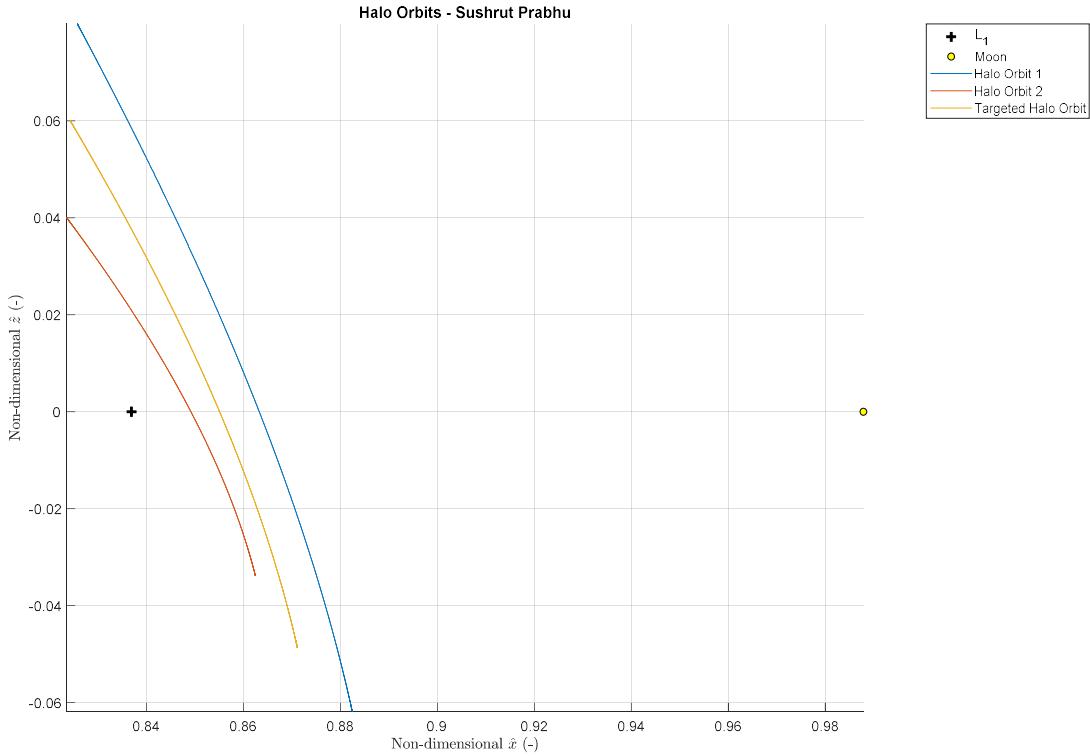


Figure G2.3: The x-z view of the two given halo orbits and one in between them.

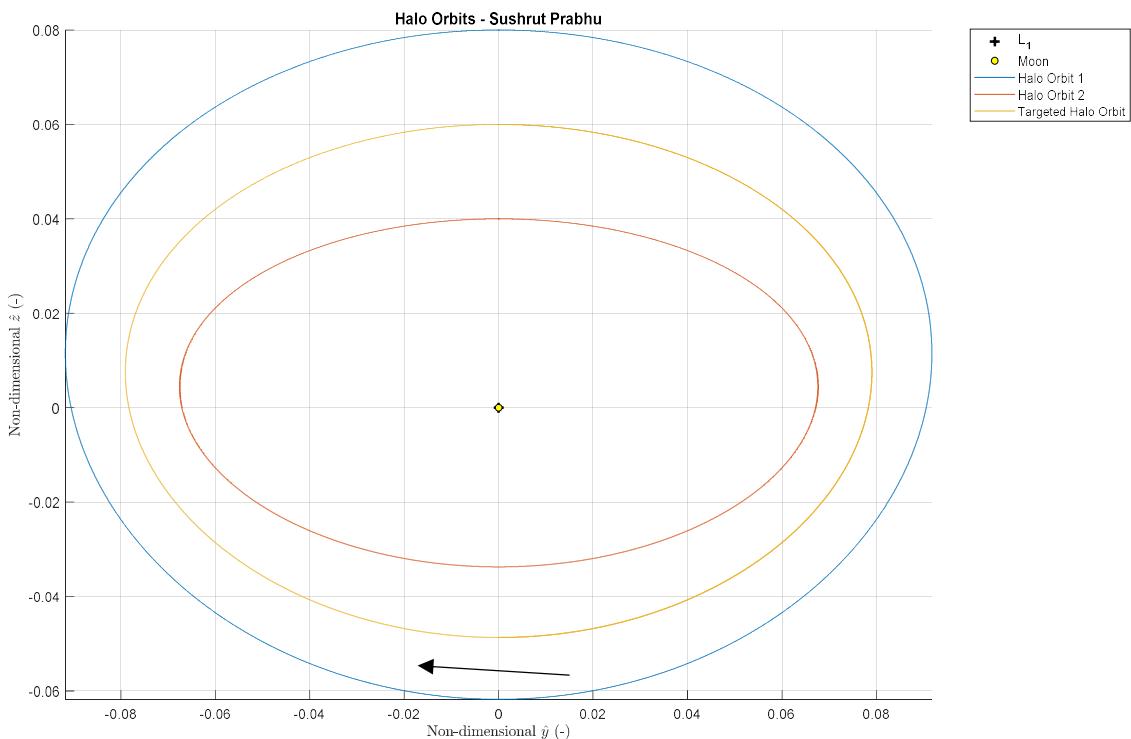


Figure G2.4: The y-z view of the two given halo orbits and one in between them

Part c)

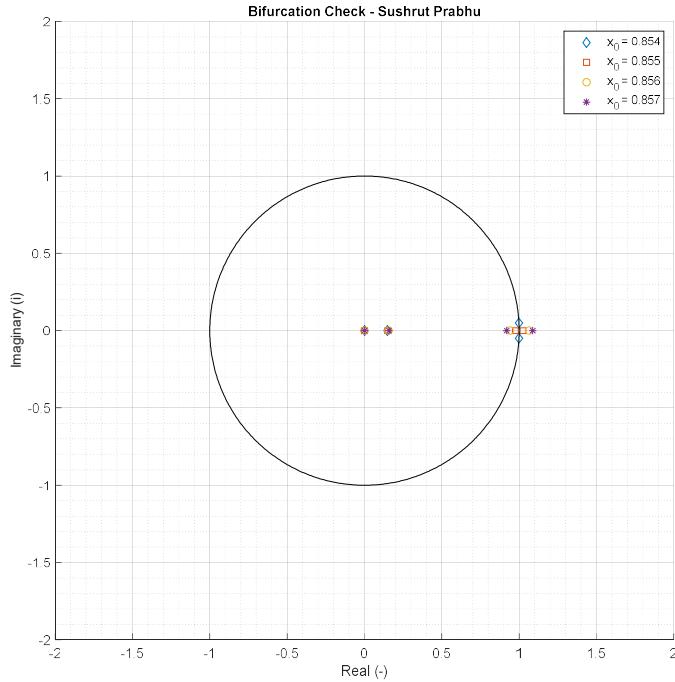


Figure G2.5: The bifurcation check with the eigenvalues of Monodromy.

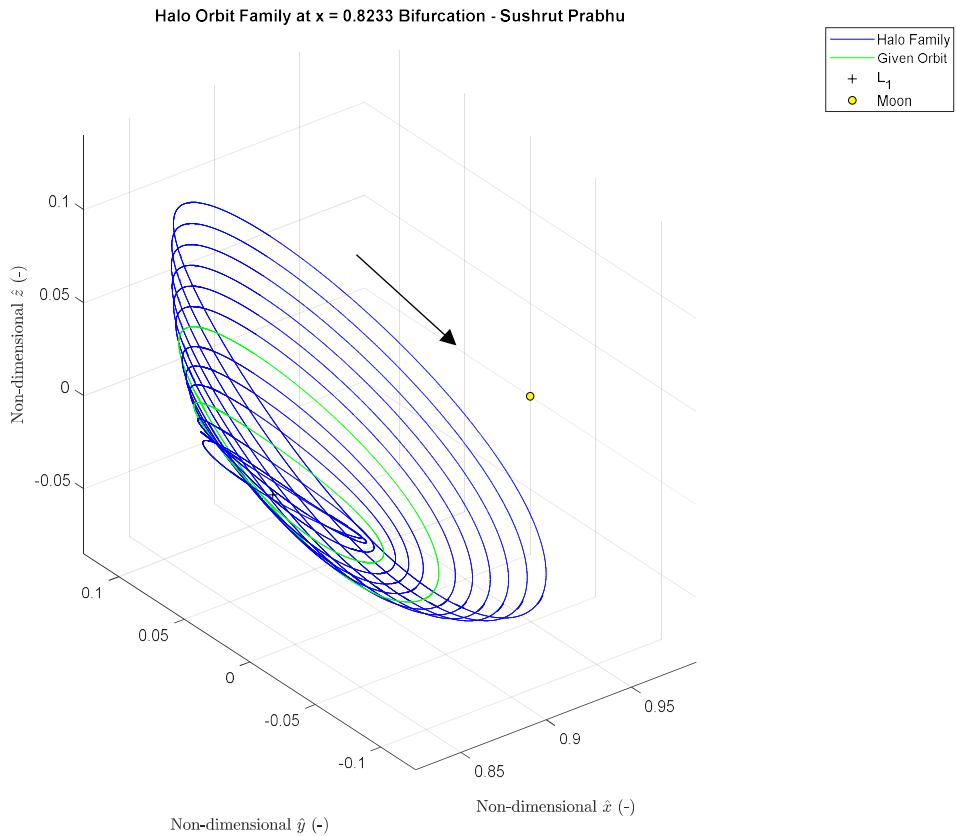


Figure G2.6: The Halo orbit family with the two given Halo orbits.

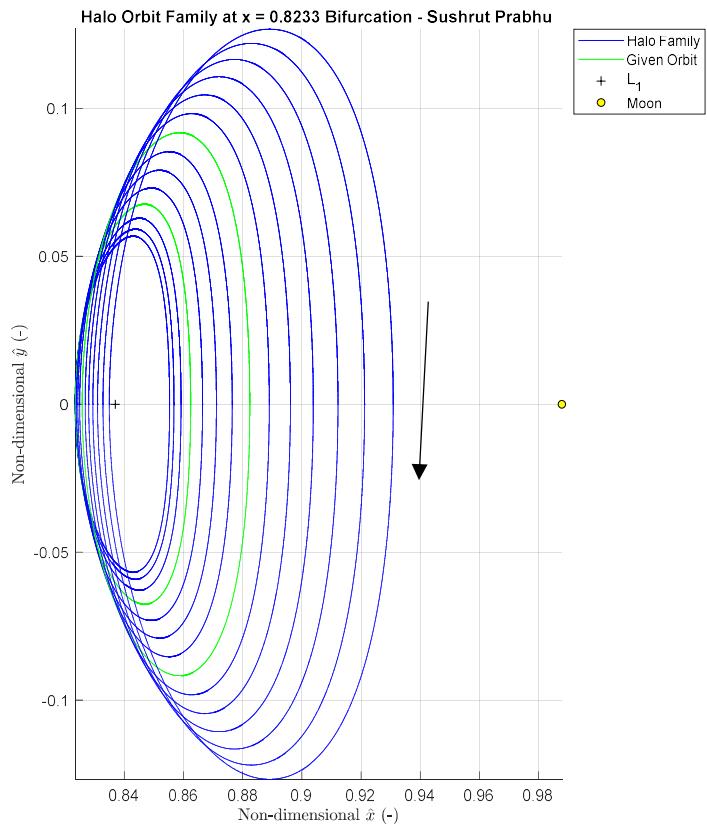


Figure G2.7: The Halo orbit family with the two given Halo orbits (x-y view).

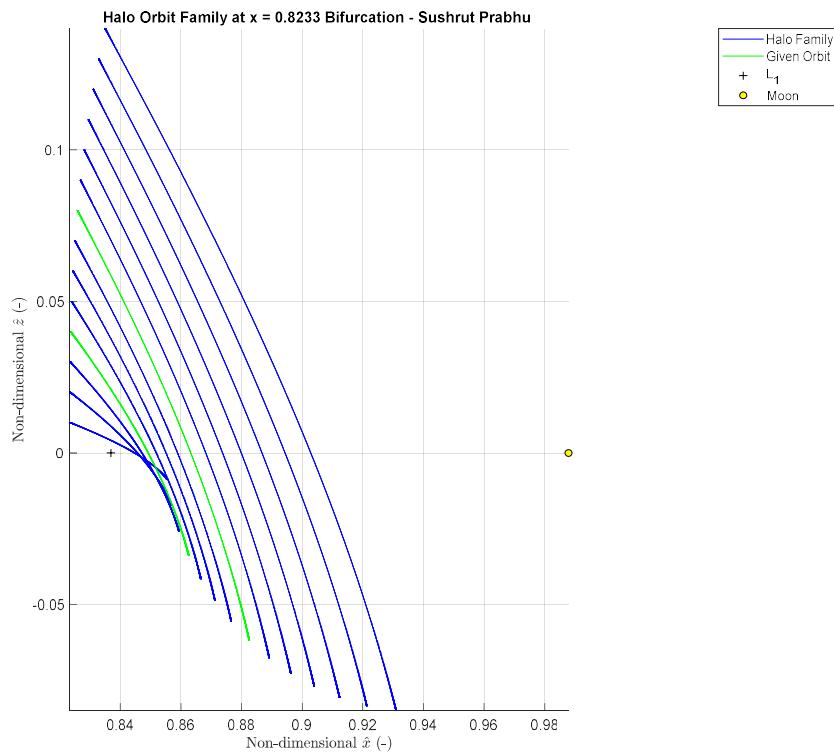


Figure G2.8: The Halo orbit family with the two given Halo orbits (x-z view).

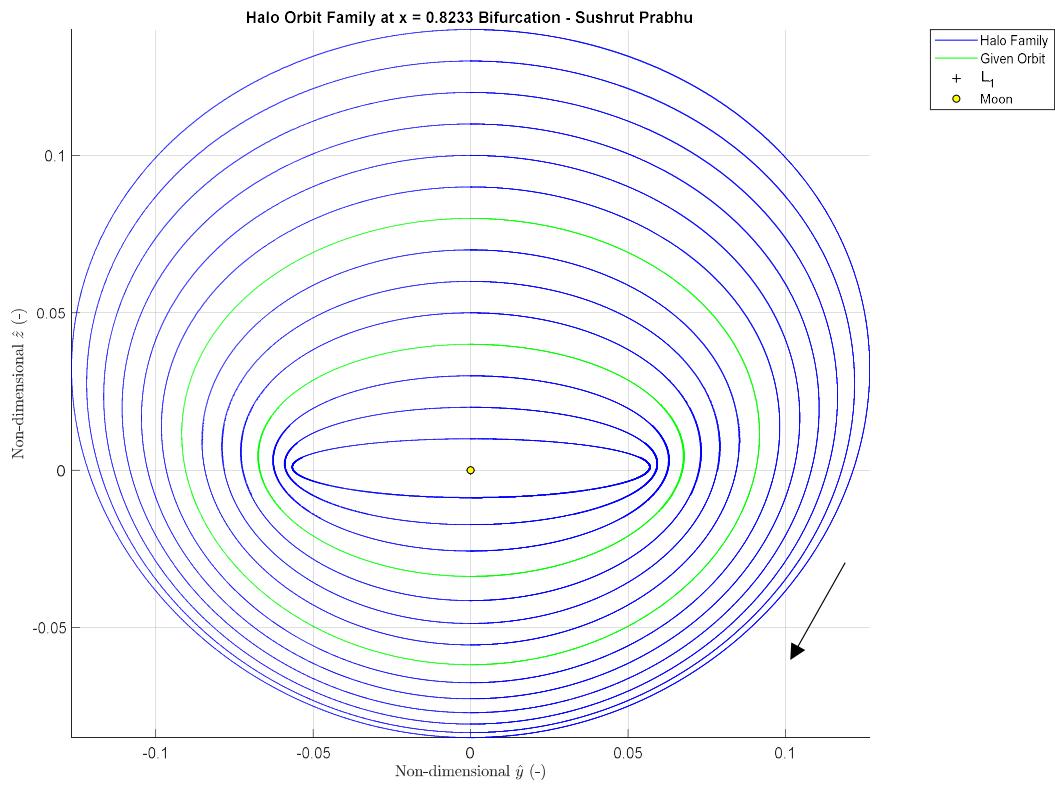


Figure G2.8: The alo orbit family with the two given Halo orbits (x-z view).

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## PSG2

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)', 'm* (kg)', 'miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
(km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
tolerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
charE(384431.4584485,0,4902.799140594719/G,398600.4480734463/G);
% Earth Moon
dim_vals{4,2} = 4902.799140594719/dim_vals{3,2}/G;    % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

## Part a)

```
IC1 = [0.82575887090385 0 0.08 0 0.19369724986446 0];
IC2 = [0.82356490862838 0 0.04 0 0.14924319723734 0];
t_end1 = 2.77648121127569;
t_end2 = 2.75330620148158;

% [IC1, t_end_half] = Target3d_per([0
0],IC1(1:3),IC1(4:6),t_end1/2,dim_vals{4,2}, "planar", 10^-13, "");
% t_end1 = t_end_half*2^4;
%
```

---

```

% [IC2, t_end_half] = Target3d_per([0
0],IC2(1:3),IC2(4:6),t_end2/2,dim_vals{4,2}, "planar", 10^-13, "");
% t_end2 = t_end_half*2*4;

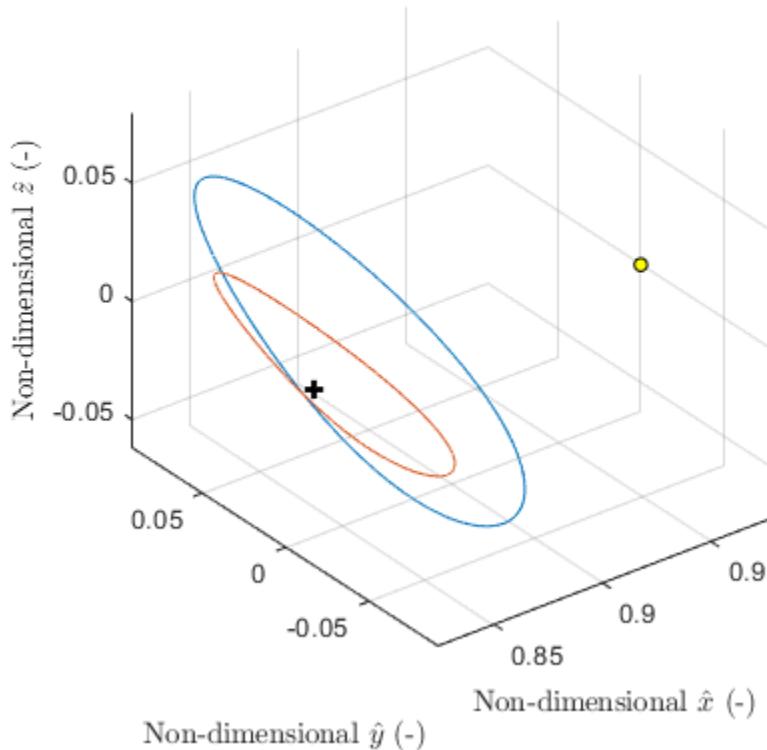
[~,y1]=ode45(@cr3bp_df,[0 t_end1],IC1,options,dim_vals{4,2});
[~,y2]=ode45(@cr3bp_df,[0 t_end2],IC2,options,dim_vals{4,2});

per1 = dot(y1(end,1:3),y2(end,1:3));
per2 = dot(y2(end,1:3),y2(end,1:3));

figure
plot3(dim_vals{7,2},0,0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot3(y1(:,1),y1(:,2),y1(:,3))
plot3(y2(:,1),y2(:,2),y2(:,3))
axis equal
grid on
xlabel("Non-dimensional  $\hat{x}$  (-)", "Interpreter", "latex")
ylabel("Non-dimensional  $\hat{y}$  (-)", "Interpreter", "latex")
zlabel("Non-dimensional  $\hat{z}$  (-)", "Interpreter", "latex")
title('Halo Orbits - Sushrut Prabhu')

```

**Halo Orbits - Sushrut Prabhu**



## Part b)

```
IC3 = (IC1+IC2)/2;
```

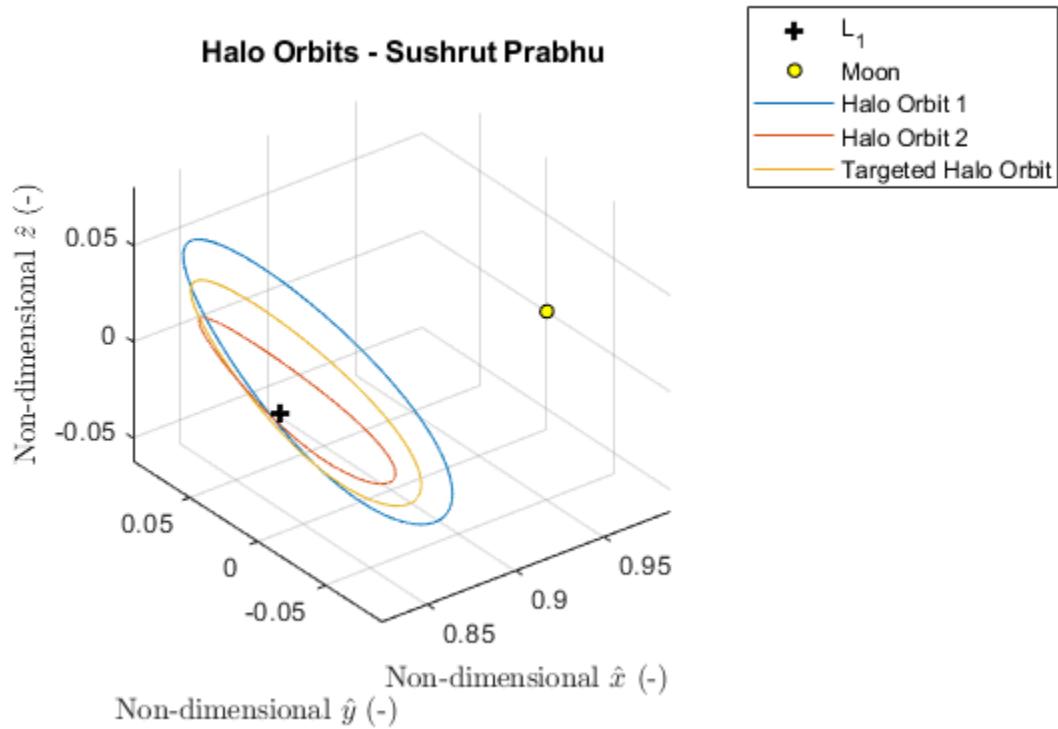
---

```

[IC3, t_end_half] = Target3d_per([0 0
0],IC3(1:3),IC3(4:6),t_end1/2,dim_vals{4,2}, "z0", 10^-13, "");
t_end3 = t_end_half*2;
[~,y3]=ode45(@cr3bp_df,[0 t_end3*1.5],IC3,options,dim_vals{4,2});

figure(1)
plot3(y3(:,1),y3(:,2),y3(:,3))
legend('L_1','Moon','Halo Orbit 1', 'Halo Orbit 2','Targeted Halo
Orbit')

```



## Part c)

```

x_unit = cos(0:.01:(2*pi));
y_unit = sin(0:.01:(2*pi));

y = [.845, 0, 0, 0, -.08, 0];

x0_step = 0.001;
k = 1;
t_end = 2.72;
G = [1 0 0 0 0 0; 0 -1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 -1 0 0; 0 0 0 0 1
0;0 0 0 0 -1];
Omega = [0 1 0; -1 0 0; 0 0 0];
IC_end = [];

```

---

---

```

while k < 15
    % Initial Guess
    IC = [y(1,1:3),y(1,4:6)];

    if k > 2
        m = (yd0_vec(k-2) - yd0_vec(k-1)) / (x0_vec(k-2) -
x0_vec(k-1));
        c = yd0_vec(k-1) - m*x0_vec(k-1);
        yg = m*(x0_step+y(1))+c;
        IC = [y(1,1:3),0,yg,0];
    end

ICm = IC + [x0_step, 0, 0, 0, 0, 0];
[t,y] = ode45(@cr3bp_df,[0 t_end],ICm,options,dim_vals{4,2});

t_end = t(find(y(:,2)>0,1));
clear t

rv_des = [0 0]';      % y = 0 and xfdot = 0
[yn,t_end] =
Target3d_per(rv_des,ICm(1:3),IC(4:6),t_end,dim_vals{4,2}, "planar",
10^-13, "");

% Final plot and solution
IC_stm = eye(6);
IC_stm = IC_stm(:)';
IC = [yn(1,1:6), IC_stm];
t_end = t_end;
[~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

% Method 2
stm_12 = reshape(y(end,7:end),6,6);
monodromy = G* [zeros(3,3), -eye(3);eye(3),
-2*Omega]*stm_12.*[-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

[V,D] = eig(monodromy);
Dvec(k,:) = diag(D)';

t_end = 2*t_end;
% [t,y] = ode45(@cr3bp_df,[0
t_end],yn(1,1:6),options,dim_vals{4,2});
%
x0_vec(k) = y(1,1);
yd0_vec(k) = y(1,5);
IC_end = [IC_end; y(end,1:6)];

if rem(k,4) == 0
    figure
    hold on
    plot(real(Dvec(k-3,:)),imag(Dvec(k-3,:)), 'd')
    plot(real(Dvec(k-2,:)),imag(Dvec(k-2,:)), 's')
    plot(real(Dvec(k-1,:)),imag(Dvec(k-1,:)), 'o')
    plot(real(Dvec(k,:)),imag(Dvec(k,:)), '*')
    plot(x_unit,y_unit, 'k')

```

---

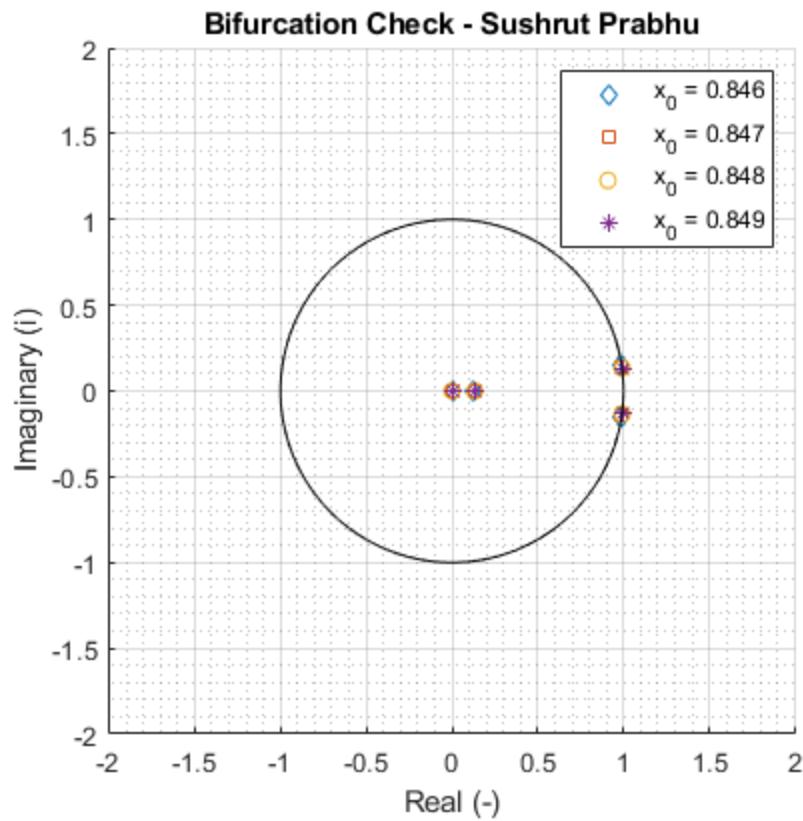
---

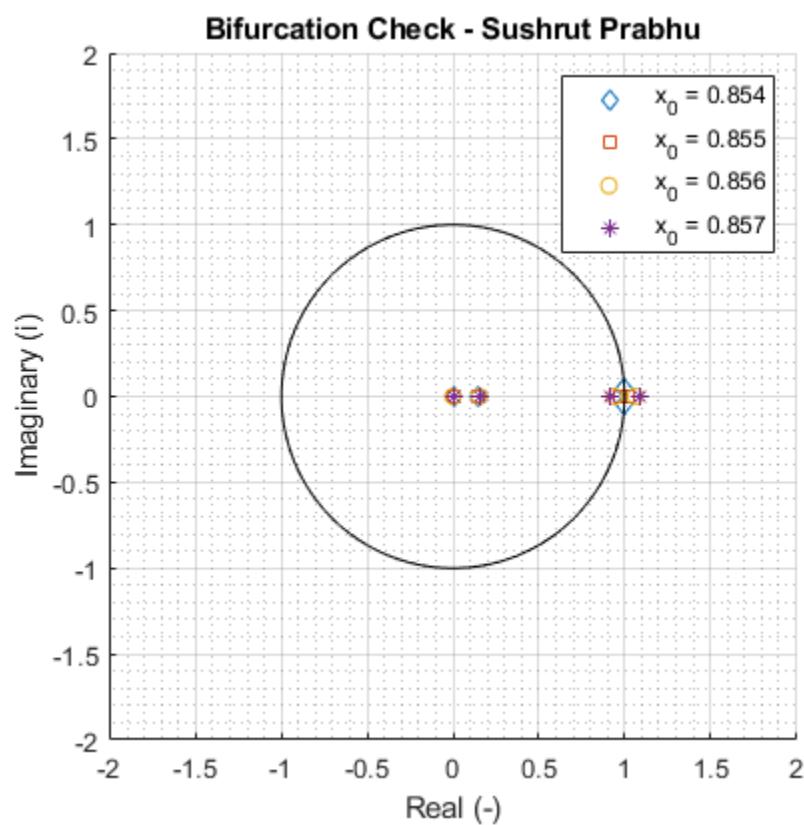
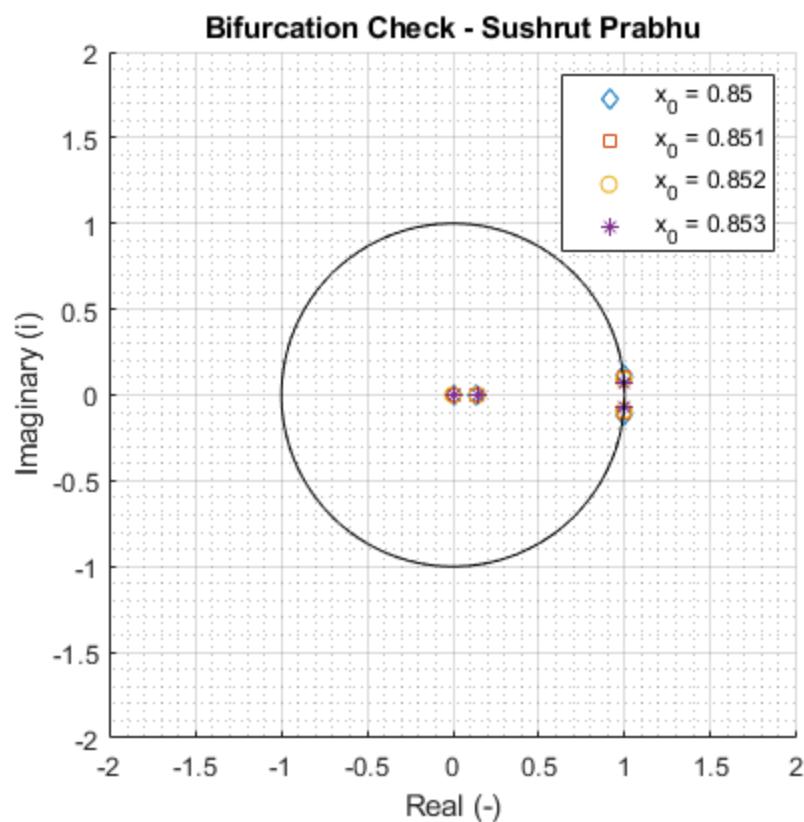
```

axis equal
grid on
grid minor
xlim([-2 2])
ylim([-2 2])
legend(['x_0 = ' num2str(x0_vec(k-3))],['x_0 = ' num2str(x0_vec(k-2))],['x_0 = ' num2str(x0_vec(k-1))],['x_0 = ' num2str(x0_vec(k))])
xlabel('Real (-)')
ylabel('Imaginary (i)')
title('Bifurcation Check - Sushrut Prabhu')
end

k = k+1;
end

```





---

# Target Halo

```
y_halo = [0.8233, 0, 0, 0, 0.127531244599002, 0];
z_step = 0.005;
k = 1;
l = 1;
IC_vec= [];

figure

while k < 30

    IC_guess = y_halo(1,:) + [0 0 z_step 0 0 0];

    [IC_halo, t_end_half] = Target3d_per([0 0
0],IC_guess(1:3),IC_guess(4:6),t_end1/2,dim_vals{4,2}, "z0",
10^-13, "");

    IC_vec = [IC_vec; IC_halo];
    t_end3 = t_end_half*2;
    [~,y_halo]=ode45(@cr3bp_df,[0
t_end3*1.5],IC_halo,options,dim_vals{4,2});

    if rem(k,2) == 0
        if IC_halo(3) == 0.04 || IC_halo(3) == 0.08
            colour = 'g';
            l = l+1;
        else
            colour = 'b';
        end
        ly_plt = plot3(y_halo(:,1),y_halo(:,2),y_halo(:,3),colour);
        hold on
    end

    if rem(k,2) == 0
        if k == 2 || k == 8

set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
        else

set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
        end
    end

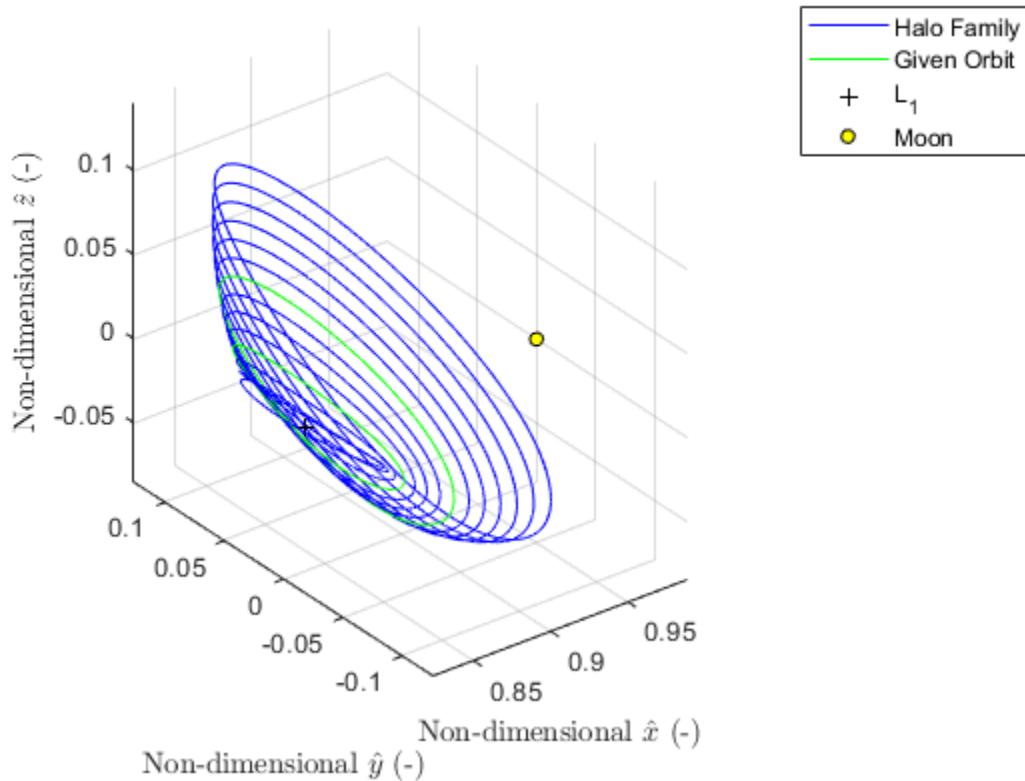
    k= k+1;
end

plot3(dim_vals{7,2},0,0,'+k')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
axis equal
grid on
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-),"Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-),"Interpreter", "latex")
```

---

```
title("Halo Orbit Family at x = 0.8233 Bifurcation - Sushrut Prabhu")
legend('Halo Family', 'Given Orbit','L_1','Moon')
```

### Halo Orbit Family at $x = 0.8233$ Bifurcation - Sushrut Prabhu



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PSG3

- Find
- a) Remeake Hénon Map  $\mu=0.5$  and  $J_0=4.5$
  - b) Using the identity periodic orbits and plot.
  - c) Reduce Jacobi constraint. How does it change?
  - i) Dusted of chaotic
  - ii) Plot quasi periodic and periodic orbit
  - d) Change  $\mu$ . what change?
  - e) Plot Lyapunov Jacobi in Earth-Moon system

Solution:

a) To create the map you just need to stop the integration when we hit the hyper plane.

The hyper plane is 2D because we will be plotting a 2D orbit. Furthermore, by constraining the elements we can understand the behaviours.

By constraining elements we reduce the problem to 2-D.  $C=C_0$  and  $y=0$   $\therefore x$  and  $z$  are free and  $y$  is constrained by  $C$

See Figure: 63.1

The quasi-Periodic Orbit is one has a periodic pattern  $x_0 = -0.609$  and  $y_0 = 2.4216$   $\omega$  From Jacobi constraint

See Figure: 63.2 and 63.3

Continued...

- b) After zooming into periodic patterns on the map we can approximate the  $x_0$  values.

$x_0 = -0.6333$  } Then use a targeter to  
 $x_0 = -0.3078$  } check periodic orbit and  
See Figure: 63.4 Jacobi.

I used a simple planar targeter that doesn't account for Jacobi constant so my Jacobi constant is accurate to  $10^{-6}$ . This is not too bad but you can build a targeter that constraints the Jacobi constant. See Figures Propagate for multiple orbits. These stay around  $P_1$  for 10-15 orbits

- c) The Jacobi value I chose was  $C = 3.8$

i) Some parts of the map become more chaotic. But there are still some patterns

ii) There is some periodic behaviour around  
 $x_0 = -0.6615$  See Figure: 63.5 and 63.7  
There is also quasi-periodic behavior around  
 $x_0 = -0.609$  See Figure: 63.5 and 63.6  
There seem to be fewer periodic behaviours in this Jacobi constant

Continued...

- d) The new value of gravitational constant  $\mu = 0.75$   
I am using the same Jacobi constant  $C = 3.8$   
The map changes a lot there are some  
new patterns that emerge and there are  
still periodic behaviours. There are some  
new solutions that emerge

See Figures: 63.8, 63.9, 63.10

You have to zoom in to notice some of these  
patterns

- e) Yes, you can see the Lyapunov orbit.  
The map looks different to all the  
previous ones. The Lyapunov orbit would be  
at  $x_0 = 0.935$ . This what is expected.  
You can see the zoomed in view too  
See Figure: 63.11 and 63.12

# PSG3

Part a)

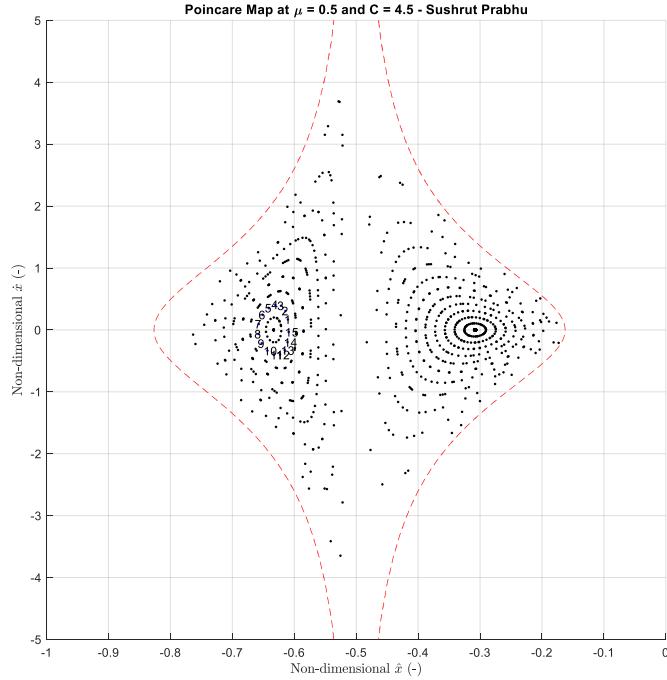


Figure G3.1: The Poincare map of the Henon problem.

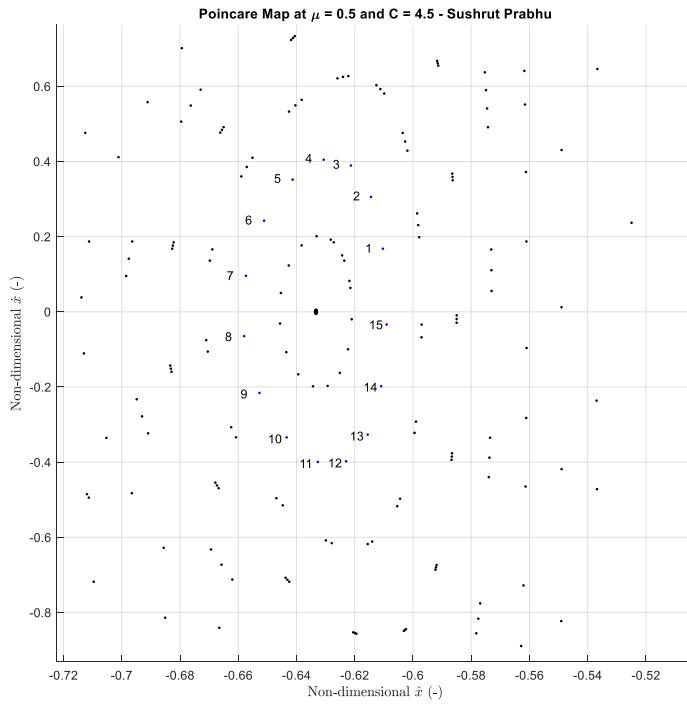


Figure G3.2: The Poincare map of the Henon problem zoomed in at the quasi periodic orbit.

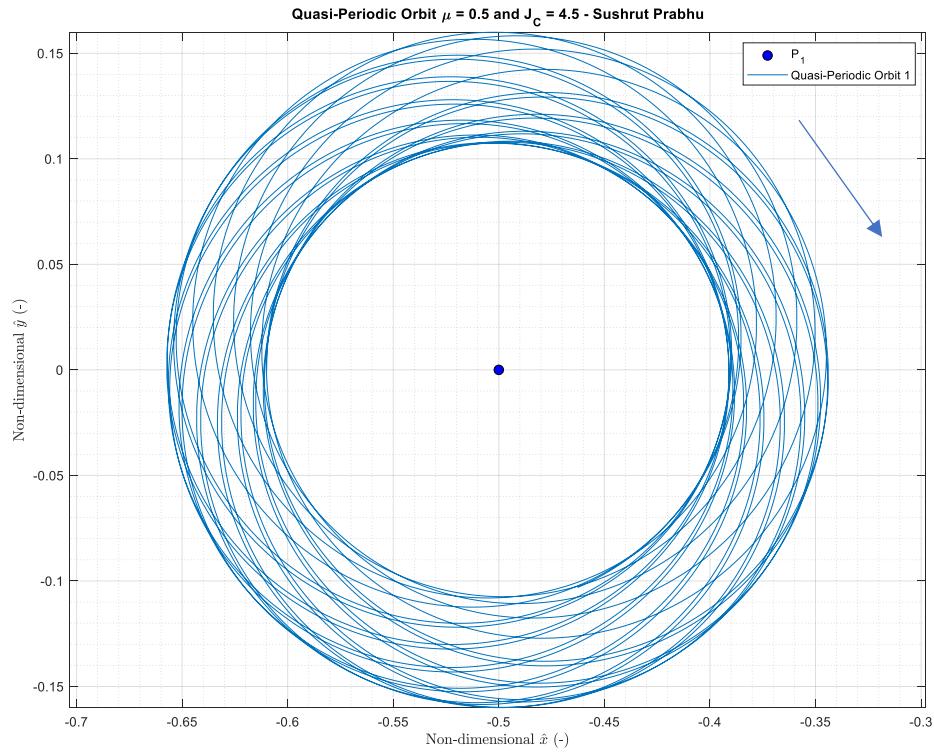


Figure G3.3: The quasi-periodic orbit from the above Poincare map.

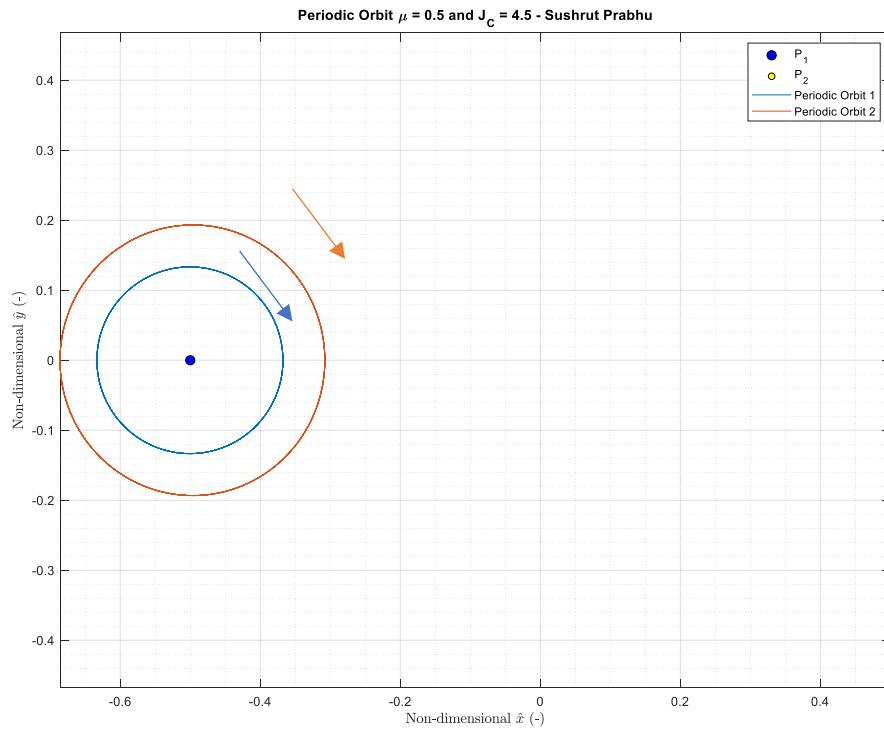


Figure G3.4: The two periodic orbits from the above Poincare map.

Part b)

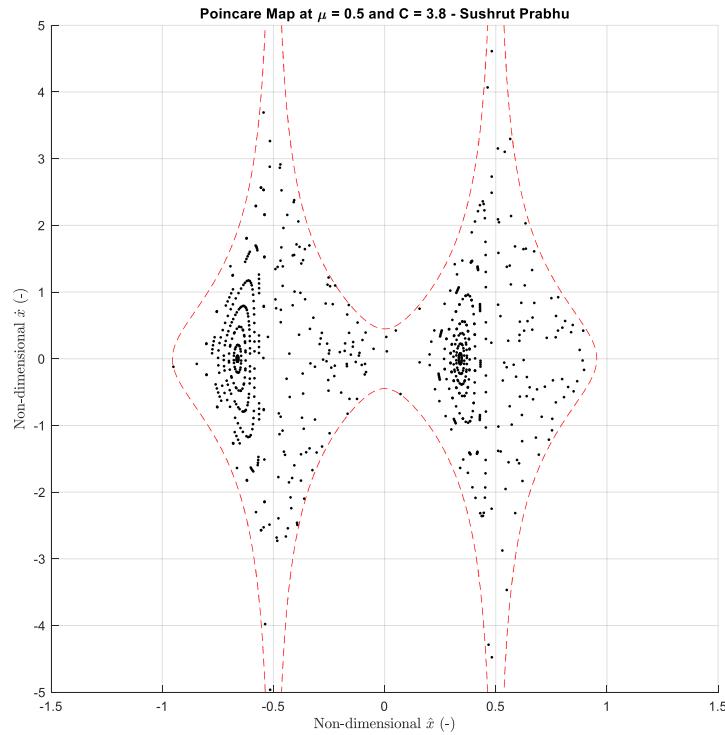


Figure G3.5: The Poincare map with different Jacobi.

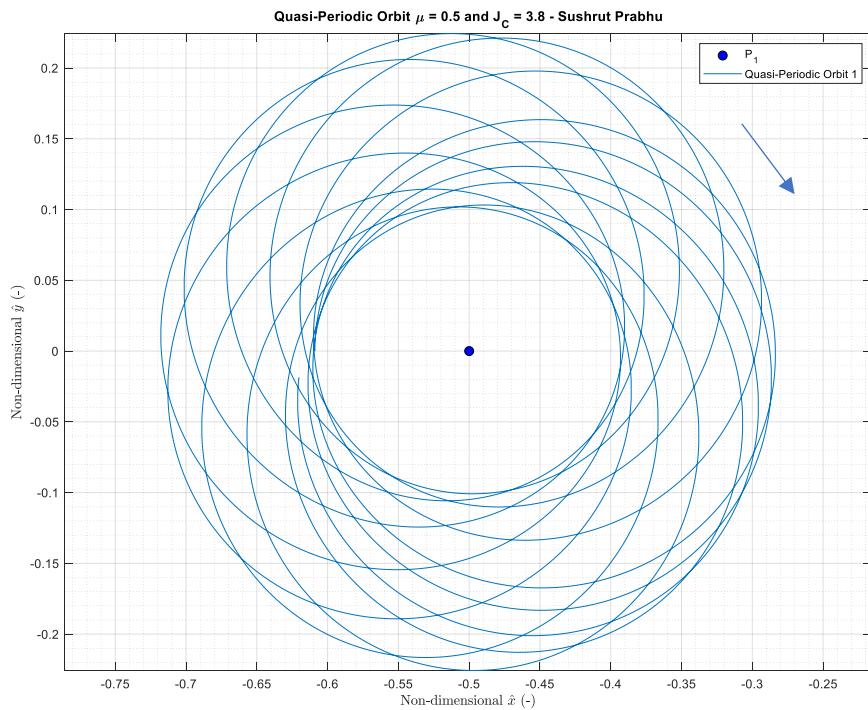


Figure G3.6: The quasi-periodic orbit from the above Poincare map.

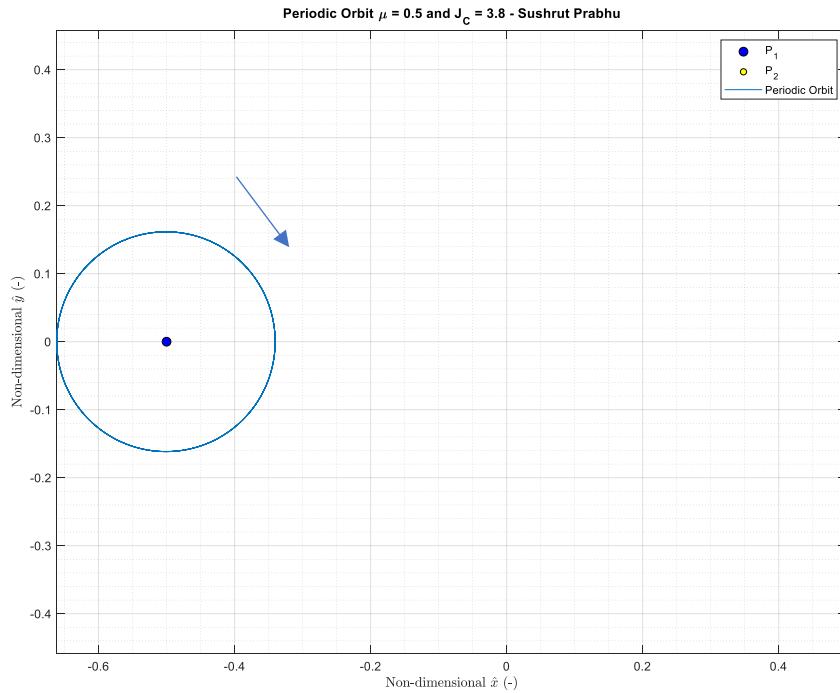


Figure G3.7: The periodic orbit on the left side from the above map.

Part c)

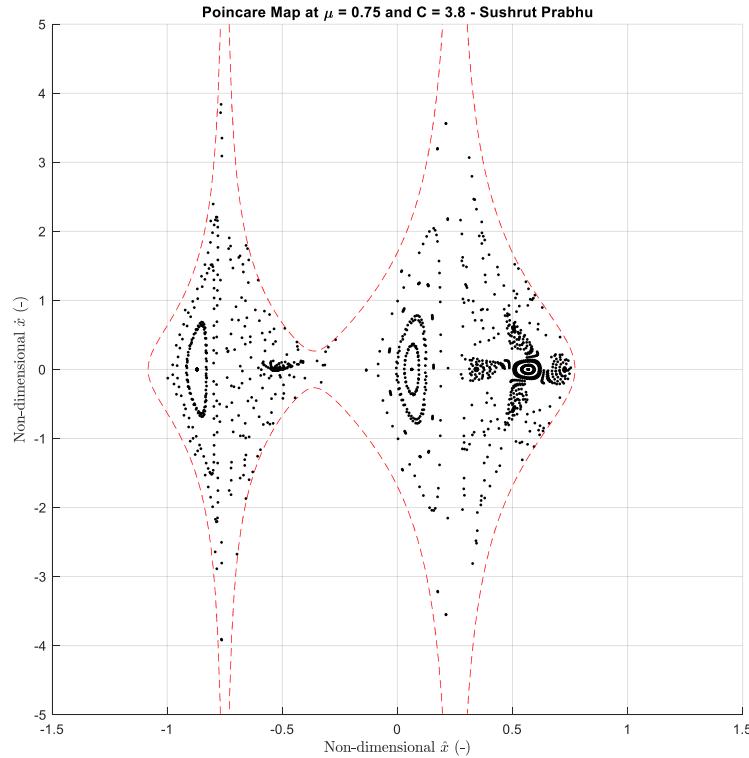


Figure G3.8: Poincare map of a different gravitational constant.

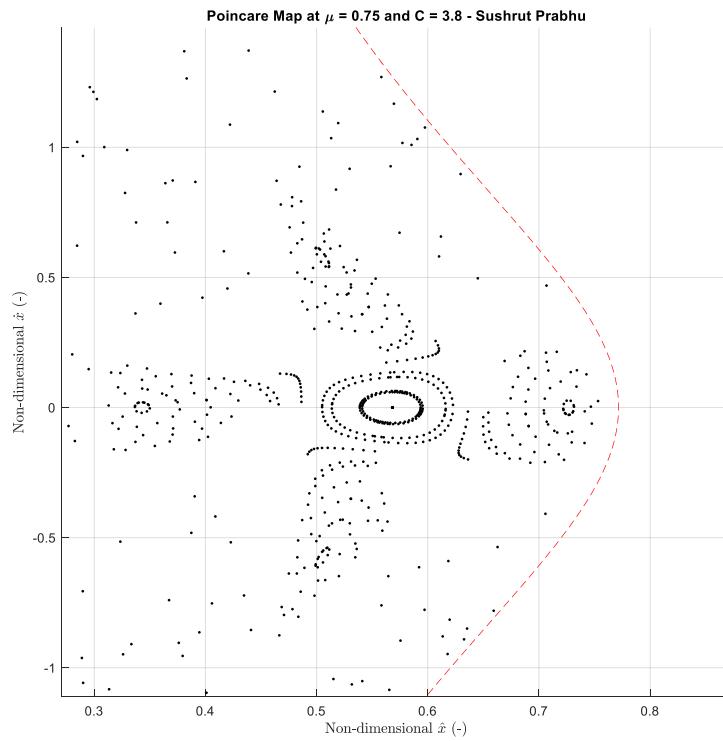


Figure G3.9: Poincare map of a different gravitational constant.

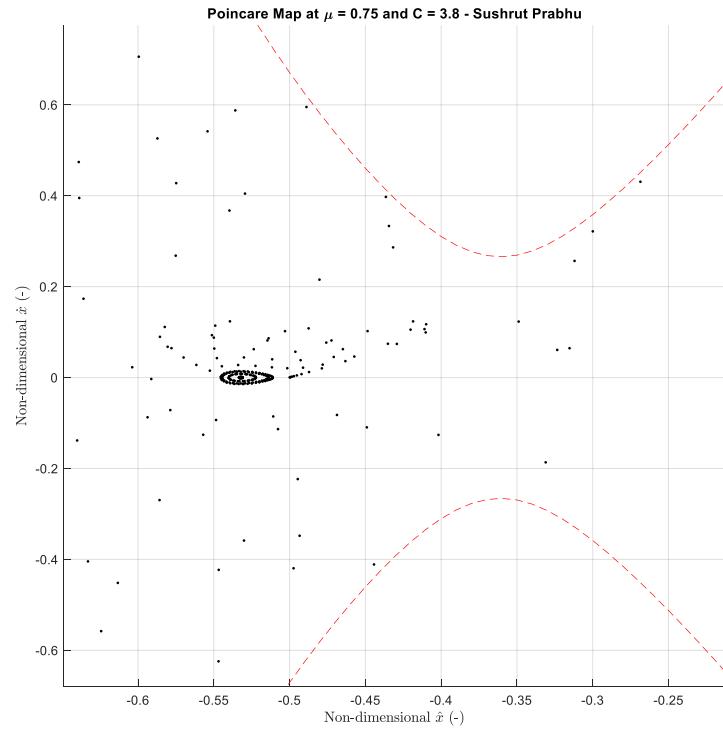


Figure G3.10: Poincare map of a different gravitational constant zoomed at periodic orbits.

Part d)

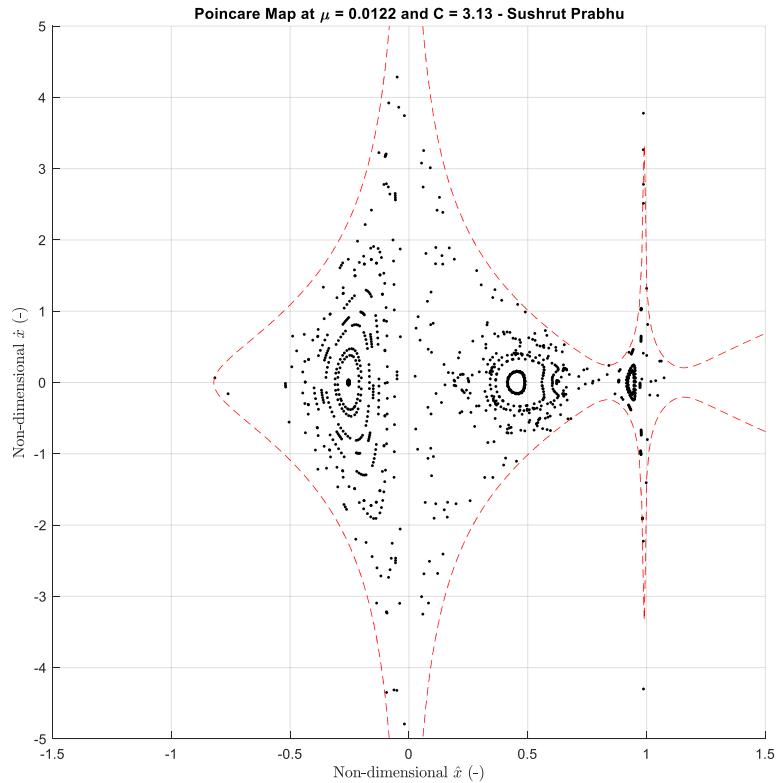


Figure G3.11: Poincare map of Earth Moon.

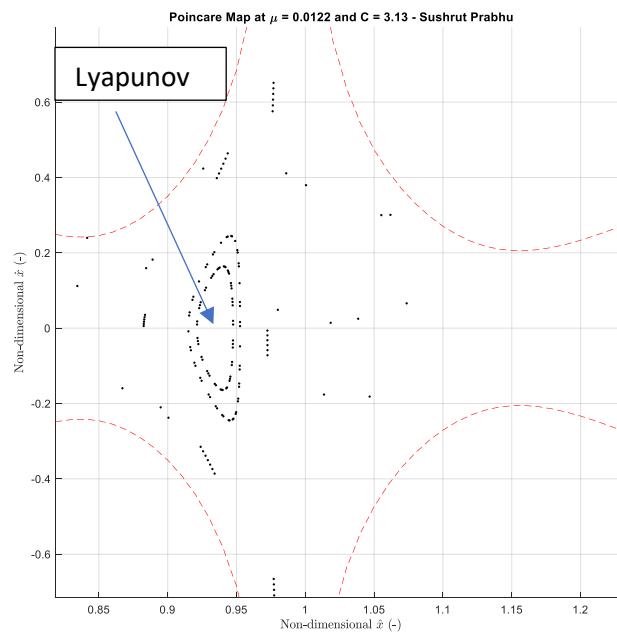


Figure G3.12: Poincare map of Earth Moon zoomed at Lyapunov.

---

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## PSE1

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)', 'm* (kg)', 'miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
(km)', 'L_1 (km)', 'gamm_2', 'L_2', 'gamma_2 (km)', 'L_2 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15,'Events',@myEvent); % Sets integration tollerance
options2=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G); % Earth Moon
% dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G; % miu
dim_vals{4,2} = .5; % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}* .7, dim_vals{4,2},
10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};

% Lagrange Point 2
dim_vals{10,2} = abs(L2_NRmethod(dim_vals{4,2}*1.1, dim_vals{4,2},
10^-8));
dim_vals{11,2} = 1 + dim_vals{10,2};
dim_vals{12,2} = dim_vals{10,2}*dim_vals{2,2};
dim_vals{13,2} = dim_vals{11,2}*dim_vals{2,2};
```

---

## Part a)

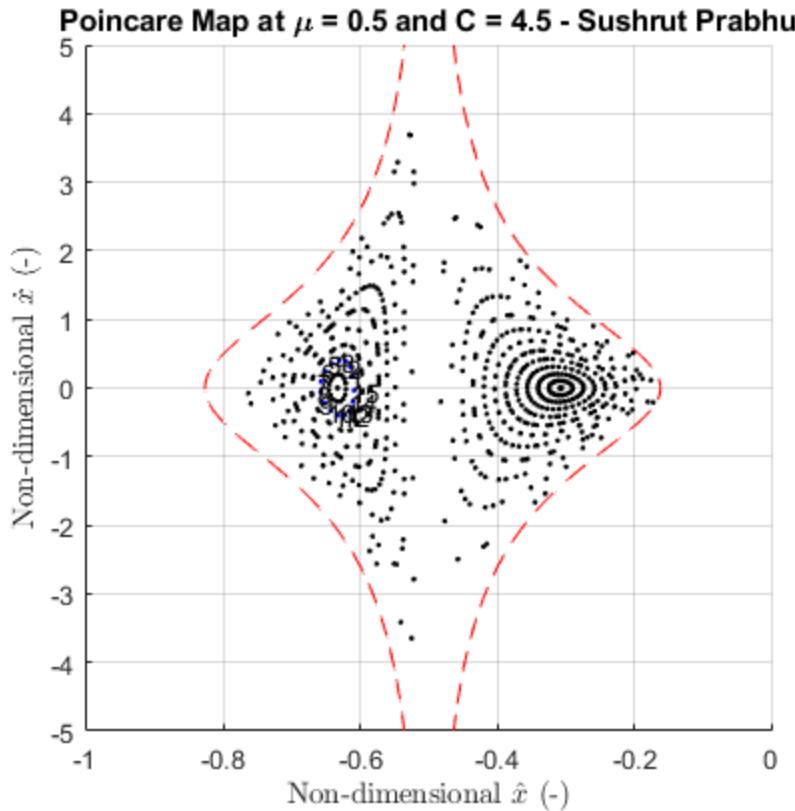
```
C0 = 4.5;
tol = 10^-13;

figure(1)
hold on
for x0 = [linspace(-.525,-.633,10), linspace(-.3095,-.483,12)]
ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2})-C0);
t_end = 10;
y = [x0 0 0 0 ydot_0 0];
x_xdot = [];
y1 = [];
k = 1;
stop = [];
error = 10^-13;
while error < 10^-10
    IC = y(end,:);
    [~,y] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});
    error = abs(C0 -
Jacobi_C(y(end,1),y(end,2),y(end,3),norm(y(end,4:6)),dim_vals{4,2}));
    if rem(k,2) == 0
        x_xdot = [x_xdot;y(end,1),y(end,4)];
    end

    k = k+1;
    if k > 100
        error = 1;
    end
end
if x0 == -.609
    plot(x_xdot(1:15,1),x_xdot(1:15,2),'.b')
    for n = 1:15
        text(x_xdot(n,1)*1.01,x_xdot(n,2)*1.01,num2str(n))
    end
else
    plot(x_xdot(:,1),x_xdot(:,2),'.k')
end

end
% JC Limit
[X,V] = meshgrid(-1:0.01:0,-5:0.01:5);
C = Jacobi_C(X,0,0,V,dim_vals{4,2});

contour(X,V,-C,-[C0 C0], '--r');
axis square
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\dot{x}$$ (-),"Interpreter", "latex")
grid on
title('Poincare Map at \mu = 0.5 and C = 4.5 - Sushrut Prabhu')
```



## Quasi

```

x0 = -0.609;
ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2})-C0);

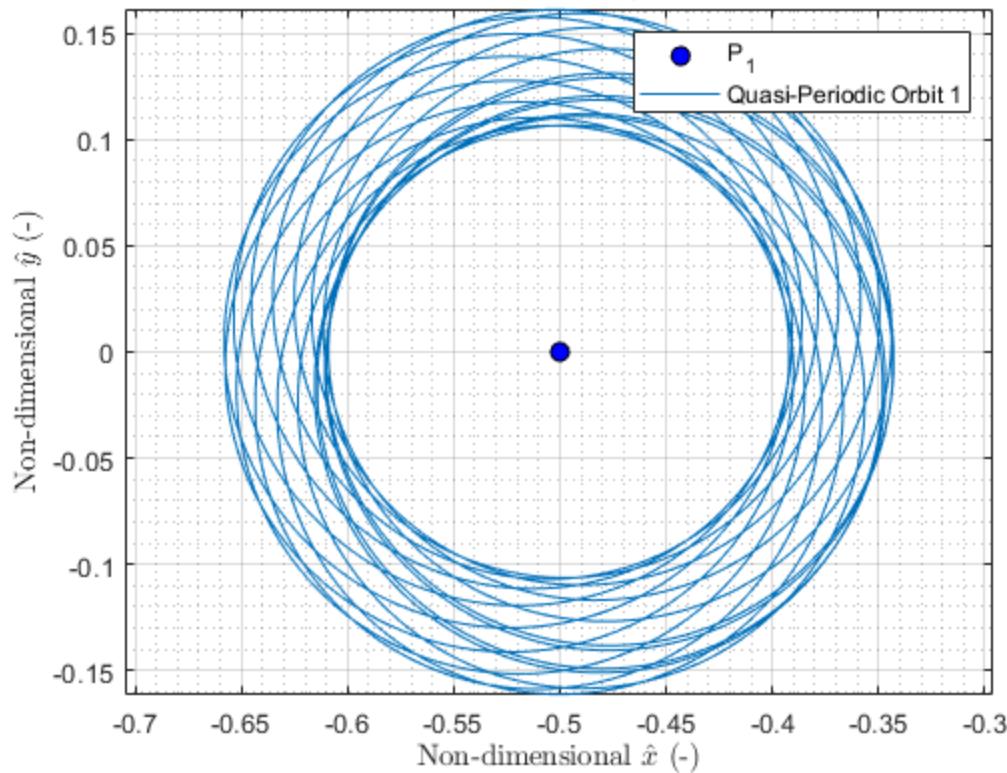
[~,yq] = ode45(@cr3bp_df,[0 7],[x0 0 0 0 ydot_0
0],options2,dim_vals{4,2});

figure
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
hold on
plot(yq(:,1),yq(:,2))
title('Quasi-Periodic Orbit \mu = 0.5 and J_C = 4.5 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\dot{\hat{x}}$$ (-)","Interpreter", "latex")
legend('P_1','Quasi-Periodic Orbit 1')
grid on
grid minor
axis equal

```

---

**Quasi-Periodic Orbit  $\mu = 0.5$  and  $J_C = 4.5$  - Sushrut Prabhu**



## Part b)

```

r_guess = -.6333192;
v_guess = sqrt(Jacobi_C(r_guess,0,0,0,dim_vals{4,2})-C0);
[IC_final,t_end] = Target3d_per([0 0],[r_guess 0 0],[0 v_guess
0],0.25,dim_vals{4,2}, "planar", 10^-10, "");

[~,y1] = ode45(@cr3bp_df,[0
t_end*10],IC_final,options2,dim_vals{4,2});

r_guess = -.3078;
v_guess = sqrt(Jacobi_C(r_guess,0,0,0,dim_vals{4,2})-C0);
[IC_final,t_end] = Target3d_per([0 0],[r_guess 0 0],[0 v_guess
0],0.25,dim_vals{4,2}, "planar", 10^-10, "");

[~,y2] = ode45(@cr3bp_df,[0
t_end*10],IC_final,options2,dim_vals{4,2});

figure
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2))
plot(y2(:,1),y2(:,2))

```

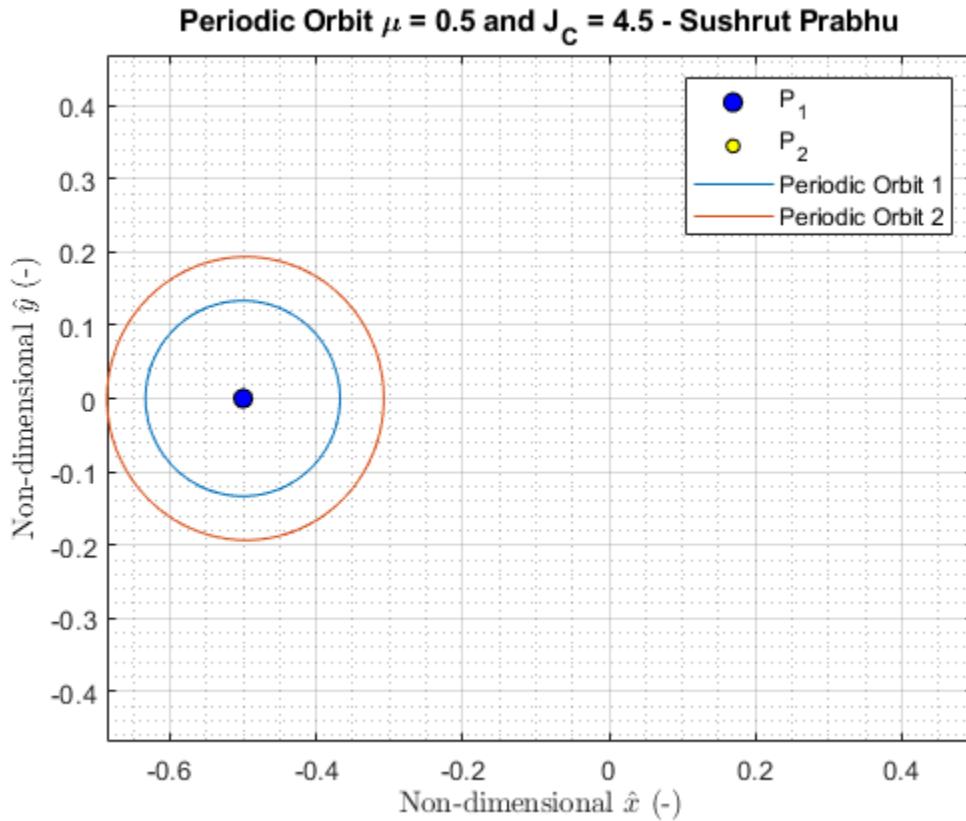
---

```

axis equal
title('Periodic Orbit \mu = 0.5 and J_C = 4.5 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-),"Interpreter", "latex")
legend('P_1','P_2','Periodic Orbit 1','Periodic Orbit 2')
grid on
grid minor

J_C0 =
Jacobi_C(y1(1,1),y1(1,2),y1(1,3),norm(y1(1,4:6)),dim_vals{4,2});
J_Cend =
Jacobi_C(y1(end,1),y1(end,2),y1(end,3),norm(y1(end,4:6)),dim_vals{4,2});

```



## Part c i)

```

C0 = 3.8;
tol = 10^-13;

figure
hold on
for x0 = [linspace(-.525,-.665,8),linspace(.3095,.48,8),
linspace(-.3095,-.48,5), linspace(.525,.633,5)]
ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2}))-C0;
t_end = 10;
y = [x0 0 0 0 ydot_0 0];
x_xdot = [];

```

---

```

y1 = [];
k = 1;
stop = [];
error = 10^-13;

while error < 10^-10
    IC = y(end,:);
    [~,y] = ode45(@cr3bp_df,[0
t_end],IC,options,dim_vals{4,2});
%
    y1 = [y1;y(:,1:2)];
    error = abs(C0 -
Jacobi_C(y(end,1),y(end,2),y(end,3),norm(y(end,4:6)),dim_vals{4,2}));
    if rem(k,2) == 0
        x_xdot = [x_xdot;y(end,1),y(end,4)];
    end

    k = k+1;
    if k > 100
        error = 1;
    end
end

plot(x_xdot(:,1),x_xdot(:,2),'.k')

%
f1 = figure;
plot(y1(:,1),y1(:,2))
%
close(f1)
end

% JC Limit
[X,V] = meshgrid(-1.5:0.01:1.5,-5:0.01:5);
C = Jacobi_C(X,0,0,V,dim_vals{4,2});

contour(X,V,-C,-[C0 C0],'--r');
axis square
xlabel("Non-dimensional $\hat{x}$ (-)", "Interpreter", "latex")
ylabel("Non-dimensional $\dot{x}$ (-)", "Interpreter", "latex")
grid on
ylim([-5,5])
title('Poincare Map at \mu = 0.5 and C = 3.8 - Sushrut Prabhu')

%
% Part c ii)
% Quasi
x0 = -0.609;
ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2}))-C0;

[~,yq] = ode45(@cr3bp_df,[0 7],[x0 0 0 0 ydot_0
0],options2,dim_vals{4,2});

figure
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
hold on

```

---

---

```

plot(yq(:,1),yq(:,2))
title('Quasi-Periodic Orbit \mu = 0.5 and J_C = 3.8 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)", "Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)", "Interpreter", "latex")
legend('P_1', 'Quasi-Periodic Orbit 1')
grid on
grid minor
axis equal

r_guess = -.661457051;
v_guess = sqrt(Jacobi_C(r_guess,0,0,0,dim_vals{4,2})-C0);
[IC_final,t_end] = Target3d_per([0 0],[r_guess 0 0],[0 v_guess
0],0.25,dim_vals{4,2}, "planar", 10^-10, "");

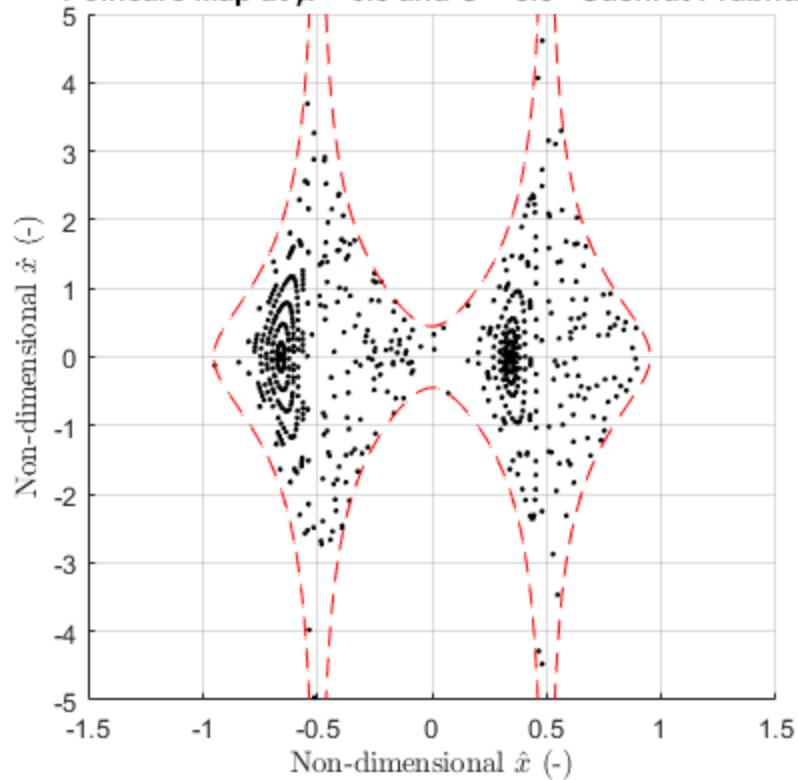
[~,yp] = ode45(@cr3bp_df,[0
t_end*10],IC_final,options2,dim_vals{4,2});

figure
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(yp(:,1),yp(:,2))
axis equal
title('Periodic Orbit \mu = 0.5 and J_C = 3.8 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)", "Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)", "Interpreter", "latex")
legend('P_1', 'P_2', 'Periodic Orbit')
grid on
grid minor

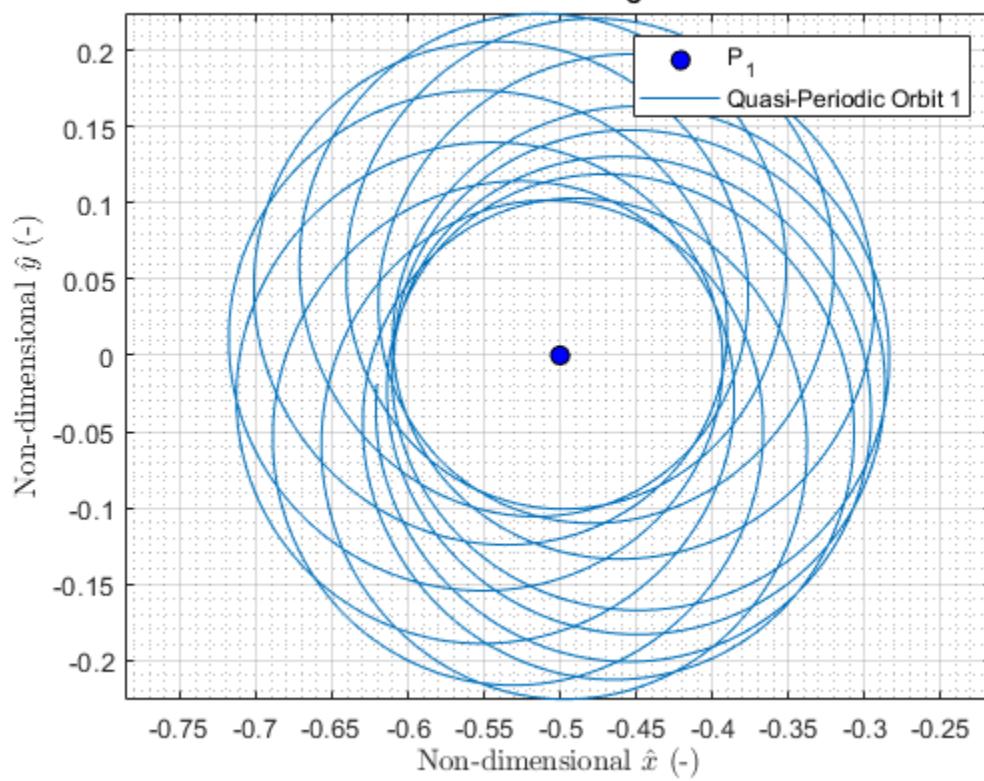
J_C0 =
Jacobi_C(yp(1,1),yp(1,2),yp(1,3),norm(yp(1,4:6)),dim_vals{4,2});
J_Cend =
Jacobi_C(yp(end,1),yp(end,2),yp(end,3),norm(yp(end,4:6)),dim_vals{4,2});

```

**Poincare Map at  $\mu = 0.5$  and  $C = 3.8$  - Sushrut Prabhu**

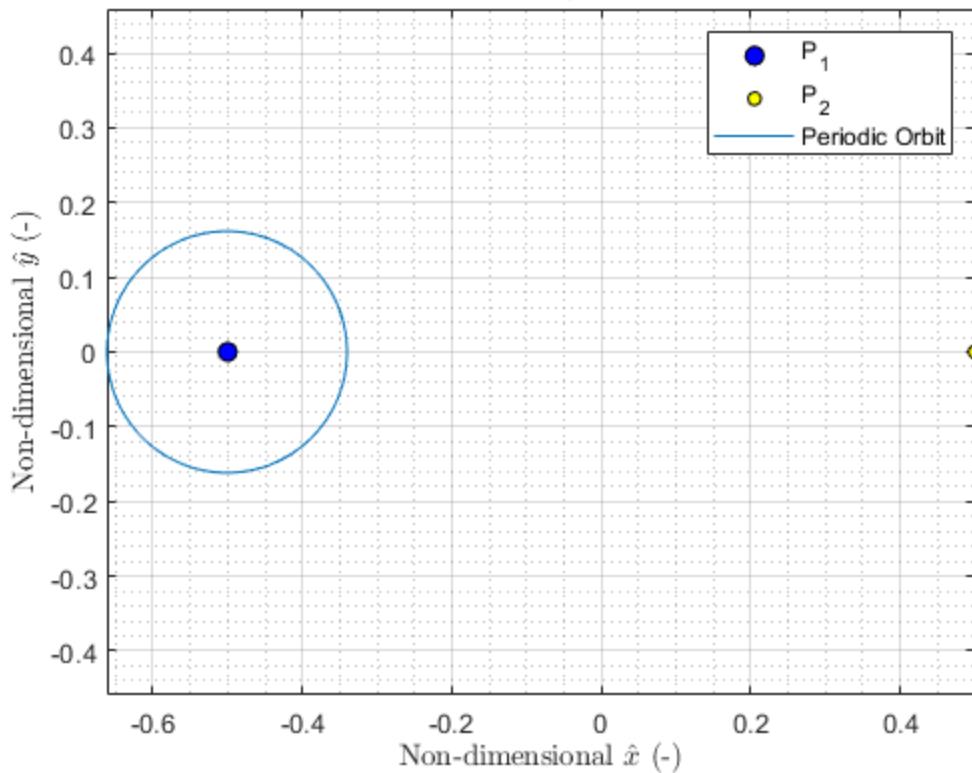


**Quasi-Periodic Orbit  $\mu = 0.5$  and  $J_C = 3.8$  - Sushrut Prabhu**



---

### Periodic Orbit $\mu = 0.5$ and $J_C = 3.8$ - Sushrut Prabhu



## Part d)

```

dim_vals{4,2} = .75;

tol = 10^-13;

figure
hold on
for x0 = [linspace(-1,-.7,8),linspace(0,.22,8), linspace(-.6,-.5,10),
linspace(.35,.65,12)]
    ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2})-C0);
    t_end = 10;
    y = [x0 0 0 0 ydot_0 0];
    x_xdot = [];
    y1 = [];
    k = 1;
    stop = [];
    error = 10^-13;

        while error < 10^-10
            IC = y(end,:);
            [~,y] = ode45(@cr3bp_df,[0
            t_end],IC,options,dim_vals{4,2});
            %
            y1 = [y1;y(:,1:2)];
    end
end

```

---

```

        error = abs(C0 -
Jacobi_C(y(end,1),y(end,2),y(end,3),norm(y(end,4:6)),dim_vals{4,2}));
        if rem(k,2) == 0
            x_xdot = [x_xdot;y(end,1),y(end,4)];
        end

        k = k+1;
        if k > 100
            error = 1;
        end
    end

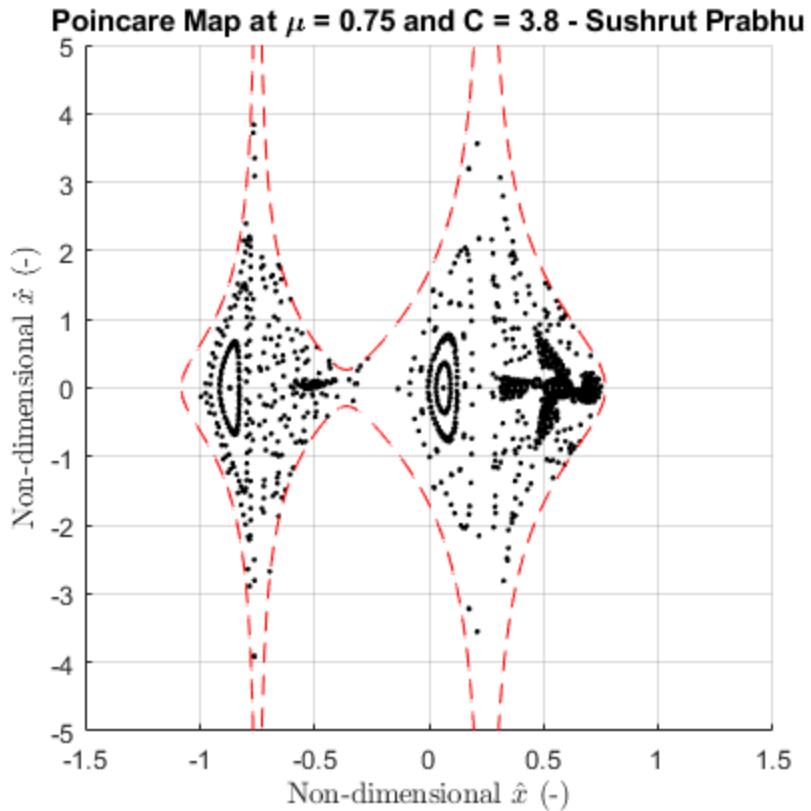
    plot(x_xdot(:,1),x_xdot(:,2),'.k')

end

% JC Limit
[X,V] = meshgrid(-1.5:0.01:1.5,-5:0.01:5);
C = Jacobi_C(X,0,0,V,dim_vals{4,2});

contour(X,V,-C,-[C0 C0], '--r');
axis square
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\dot{x}$$ (-),"Interpreter", "latex")
grid on
ylim([-5,5])
title('Poincare Map at \mu = 0.75 and C = 3.8 - Sushrut Prabhu')

```



## Part e)

```

dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;    % miu
C0 = 3.13;

tol = 10^-13;

figure
hold on
for x0 = [linspace(-.6,-.05,12),linspace(0.85,.98,5),
linspace(0.05,.85,12)]
ydot_0 = sqrt(Jacobi_C(x0,0,0,0,dim_vals{4,2})-C0);
t_end = 10;
y = [x0 0 0 0 ydot_0 0];
x_xdot = [];
y1 = [];
k = 1;
stop = [];
error = 10^-13;

while error < 10^-10
    IC = y(end,:);
    [~,y] = ode45(@cr3bp_df,[0
t_end],IC,options,dim_vals{4,2});
end

```

---

```

        error = abs(C0 -
Jacobi_C(y(end,1),y(end,2),y(end,3),norm(y(end,4:6)),dim_vals{4,2}));
        if rem(k,2) == 0
            x_xdot = [x_xdot;y(end,1),y(end,4)];
        end

        k = k+1;
        if k > 100
            error = 1;
        end
    end

plot(x_xdot(:,1),x_xdot(:,2),'.k')

end

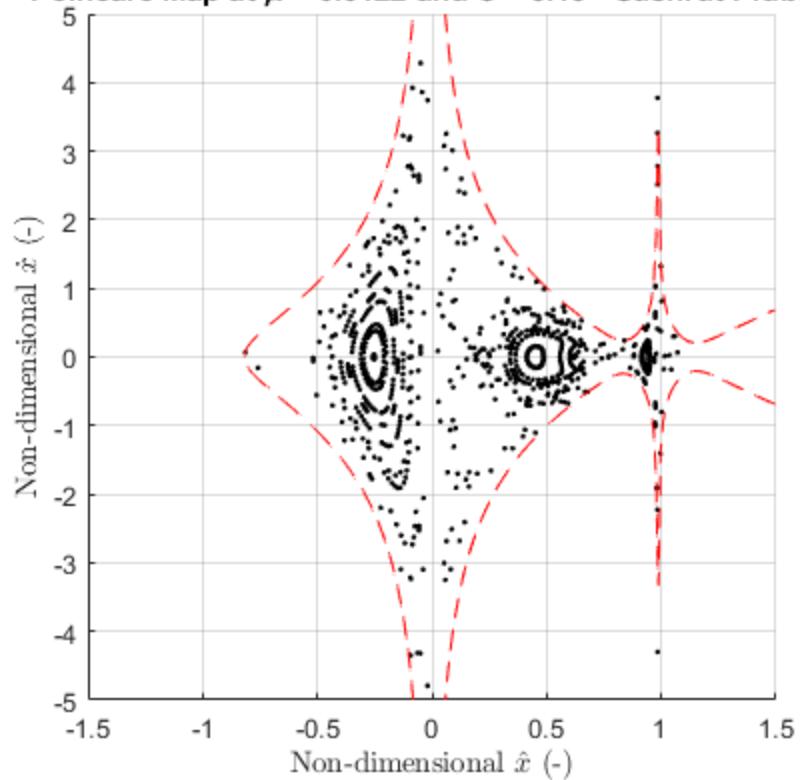
% JC Limit
[X,V] = meshgrid(-1:0.01:1.5,-5:0.01:5);
C = Jacobi_C(X,0,0,V,dim_vals{4,2});

contour(X,V,-C,-[C0 C0], '--r');
axis square
xlabel("Non-dimensional $$\hat{x}$$ (-),"Interpreter", "latex")
ylabel("Non-dimensional $$\dot{x}$$ (-),"Interpreter", "latex")
grid on
ylim([-5,5])
title('Poincare Map at \mu = 0.0122 and C = 3.13 - Sushrut Prabhu')
xlim([-1.5 1.5])

```

---

**Poincare Map at  $\mu = 0.0122$  and  $C = 3.13$  - Sushrut Prabhu**



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---

# Target3d

Periodic orbit targeter

```
function [IC_final,t_end, tb] = Target3d(r_des,r,v,t_end,miu, tol,
pl)
% Initialization
% t = 0:.001:t_end;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
tolerance
error = 2*tol;
phi_0 = eye(6);
phi_0 = phi_0(:)';
i = 1;
v0 = v;

if pl == "plot"
    figure
    plot3(r(1),r(2),r(3),'*')
    hold on
end

while error > tol
    % Non-linear propagation with phi
    IC = [r,v,phi_0];
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,miu);

    % Phi at final time
    phi_t = y(:,7:end);
    phi_tf = reshape(phi_t(end,:),6,6)';

    K = -phi_tf(1:3,4:6);
    Fx = r_des - y(end,1:3)';
    delv1 = K.' * (K*K.')^-1 * Fx;

    IC = [r, v - delv1'];
    [~,yn] = ode45(@cr3bp_df,[0 t_end],IC,options,miu);
    v = yn(1,4:6);

    error = max(abs(yn(end,1:3)-r_des'));

    tb{:,i} = {v-v0, norm(v-v0), Fx, norm(Fx)};
    i = i+1;

    if pl == "plot"
        if error > tol
            p13 = plot3(yn(:,1),yn(:,2),yn(:,3),'-.b');
            if i > 2
                set(get(get(p13,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
            end
        else
    end
```

---

```
    plot3(yn(:,1),yn(:,2),yn(:,3))
end
end

if i > 20
error = tol/2;
fprintf("Did not Coverge")
end
end

IC_final = IC;

end

Not enough input arguments.

Error in Target3d (line 7)
error = 2*tol;
```

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---

# A\_t

Intermediate calculation for STM

```
function A = A_t(x,y,z,miu)

if isnan(z)
    [Uxx,Uyy,~,Uxy,~,~] = Unn(x,y,0,miu);
    A = [zeros(2,2), eye(2,2); Uxx, Uxy, 0, 2; Uxy, Uyy, -2, 0];
else
    [Uxx,Uyy,Uzz,Uxy,Uxz,Uyz] = Unn(x,y,z,miu);
    A = [zeros(3,3), eye(3,3); Uxx, Uxy,Uxz, 0, 2,0; Uxy, Uyy, Uyz, 0,
        -2, 0; Uxz, Uyz, Uzz, 0, 0, 0];
end

end
```

*Not enough input arguments.*

```
Error in A_t (line 5)
if isnan(z)
```

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---

## acc\_nonlin

Non-linear acceleration for STM

```
function acc = acc_nonlin(r,v,miu)

dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
rr = sqrt((r(1)+miu-1)^2 + r(2)^2 + r(3)^2);

acc(1) = 2*v(2) + r(1) - (1-miu)*(r(1)+miu)/dd^3 - miu*(r(1)-1+miu)/
rr^3;
acc(2) = -2*v(1) + r(2) - (1-miu)*r(2)/dd^3 - miu*r(2)/rr^3;
acc(3) = -(1-miu)*r(3)/dd^3 - miu*r(3)/rr^3;
```

*Not enough input arguments.*

```
Error in acc_nonlin (line 5)
dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
```

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---

# cr3bp\_STM\_df2d

2d STM

```
function dx = cr3bp_STM_df2d(t,x,miu)
dx = zeros(20,1);

d = sqrt((x(1)+miu)^2 + x(2)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2);
phi = reshape(x(5:end),4,4)';

dx(1) = x(3);
dx(2) = x(4);
dx(3) = 2*x(4) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(4) = -2*x(3) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;

phi_dot = [A_t(x(1),x(2),NaN,miu)*phi]';

dx(5:end) = phi_dot(:);
end
```

*Not enough input arguments.*

*Error in cr3bp\_STM\_df2d (line 6)*  
d = sqrt((x(1)+miu)^2 + x(2)^2);

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---

# cr3bp\_STM\_df3d

3d STM

```
function dx = cr3bp_STM_df3d(t,x,miu)

dx = zeros(42,1);

d = sqrt((x(1)+miu)^2 + x(2)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2);
phi = reshape(x(7:end),6,6)';

d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2 + x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = 2*x(5) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(5) = -2*x(4) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;
dx(6) = -(1-miu)*x(3)/d^3 - miu*x(3)/r^3;

phi_dot = [A_t(x(1),x(2),x(3),miu)*phi]';

dx(7:end) = phi_dot(:);
end
```

*Not enough input arguments.*

```
Error in cr3bp_STM_df3d (line 7)
d = sqrt((x(1)+miu)^2 + x(2)^2);
```

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---

# Characteristic Elements

```
function [lstar, mstar, tstar] = charE(D1,D2,m1,m2)
G = 6.6738*10^-20;
```

```
lstar = D1+D2;
mstar = m1 + m2;

tstar = sqrt(lstar^3/G/mstar);
```

*Not enough input arguments.*

*Error in charE (line 5)*  
lstar = D1+D2;

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---

## cr3bp\_df

```
function dx = cr3bp_df(t,x,miu)

dx = zeros(6,1);

d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2 + x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = 2*x(5) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(5) = -2*x(4) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;
dx(6) = -(1-miu)*x(3)/d^3 - miu*x(3)/r^3;

end
```

*Not enough input arguments.*

*Error in cr3bp\_df (line 6)*  
d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);

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---

## horner\_alg

```
function [alpha,beta] = horner_alg(n,a,z0)
alpha = a(1);
beta = 0;

for k = 2:n
    beta = alpha + z0*beta;
    alpha = a(k) + z0*alpha;
end

end
```

*Not enough input arguments.*

*Error in horner\_alg (line 3)*  
alpha = a(1);

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---

```
function C = Jacobi_C(x,y,z,v,miu)

d = sqrt((x+miu).^2 + y.^2 + z.^2);
r = sqrt((x-1+miu).^2 + y.^2 + z.^2);
C = x.^2 + y.^2 + 2*(1-miu)./d + 2*miu./r - v.^2;

end
```

*Not enough input arguments.*

*Error in Jacobi\_C (line 3)*  
d = sqrt((x+miu).^2 + y.^2 + z.^2);

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---

# Gamma1 Newton Rhapsom

```
function g1_n1 = L1_NRmethod(g1_n, miu, acc)

err = acc*2;
pz = [-1,(3-miu),(2*miu-3),miu,-2*miu,miu];
while (err > acc)
    [fn,fn_p] = horner_alg(6,pz,g1_n);

    g1_n1 = g1_n - fn/fn_p;

    err = abs(g1_n1-g1_n)/abs(g1_n);
    g1_n = g1_n1;
end

end
```

*Not enough input arguments.*

*Error in L1\_NRmethod (line 4)*  
err = acc\*2;

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---

# Gamma2 Newton Rhapsom

```
function g2_n1 = L2_NRmethod(g2_n, miu, acc)

err = acc*2;
pz = [1,(3-miu),(3-2*miu),-miu,-2*miu,-miu];
while (err > acc)
    [fn,fn_p] = horner_alg(6,pz,g2_n);

    g2_n1 = g2_n - fn/fn_p;

    err = abs(g2_n1-g2_n)/abs(g2_n);
    g2_n = g2_n1;
end

end
```

*Not enough input arguments.*

*Error in L2\_NRmethod (line 4)*  
err = acc\*2;

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