Sushrut Prabhu | AAE 632 PSC1

PSC1

Given: Earth-Moon System in a 3BCRP, assume position and velocity: $\bar{r} = -0.270\hat{x} - 0.420\hat{y}$
$\bar{v} = 0.300\hat{x} - 1.000\hat{y}$

Find: (a) Units or $\bar{r}$ and $\bar{v}$

(b) Since you know the location of L_i, what are Jacobi constants at L_i?

(c) What value of C corresponds to the given trajectory?
Sketch general shape of 0 velocity curve. with given IC.

(d) Numerically integrate D.E. for given IC. Plot in x-y plane.
(i) What ODE solver are you using? Why? Tolerances? Why!
(ii) 4 revolutions of large primary. How long is the non-dim time interval? Dimensional? What behaviour is apparent

(iii) Plot trajectory until it moves into Lunar Vicinity.
Is trajectory consistent with (c)? How long? Dimensional? Nondim?
Does ZVC emerge?

(iv) If you simulate longer does it ever leave Earth-Moon?

(v) Check Jacobi constant? Does it change? How many digits?
Compare constant with tolerance. Do you recommend tolerance?

Solution:

a) $\bar{r}$ and $\bar{v}$ seem to be dimensionless Because the standard units of distance is Km and the given $\bar{r}$ would be at the center of Earth

b) Note we know the distance to the libration points and the velocity at this point is $\bar{0}$. We have derived this equation in the notes

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Continued...

$d = [(x+\mu)^2 + y^2 + z^2]^{1/2}$ $r = [(x+\mu-1)^2 + y^2 + z^2]^{1/2}$

$$x^2 + y^2 + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} = C \qquad z=0,\ y=0$$

$$L_i:\ x_i^2 + \frac{2(1-\mu)}{x_i+\mu} + \frac{2\mu}{x-1+\mu} = C$$

Note that we know what x_i's are for L_1, L_2, L_3

$\therefore$

	x_i (-)	C (-)
L_1	0.8369151?	3.1883411
L_2	1.1556822	3.1721604
L_3	-1.005062	3.012147

I did L_4 and L_5 later so find on next page

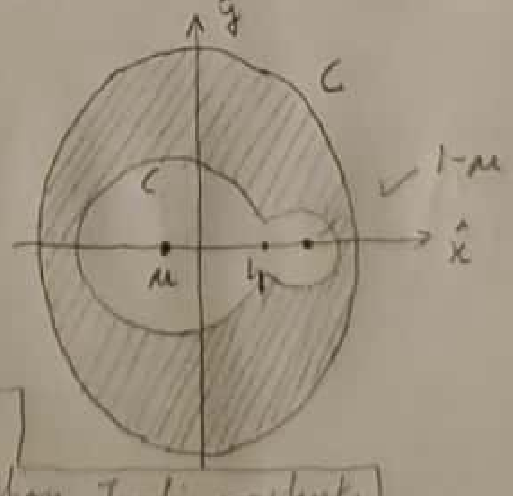
c) $C = x^2 + y^2 + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} - v^2$

$d = [(x+\mu)^2 + y^2 + z^2]^{1/2}$ $r = [(x+\mu-1)^2 + y^2 + z^2]^{1/2}$

$x = -0.27$ $y = -0.42$

$v = |\bar{v}| = |-0.3\hat{x} - 1\hat{y}| = ((-0.3)^2 + (-1)^2)^{1/2} = 1.04403$

$\therefore \boxed{C = 3.18647}$



Note: $C_1 > C > C_2$

See the plot in figure: C1.1

You can Meshgrid of X and Y.
Then do an element wise evaluation Jacobi constant.
Then plot the contour at desired contour

Continued:

d) i) $\boxed{\text{I use ode45. I tested various other Ode's but they were slower and C deviated faster. See figures C1.8 and C1.9}}$

$\boxed{\text{For the tolerance I found that the best was at relTol} \Rightarrow 10^{-13} \quad \text{absTol} \Rightarrow 10^{-15}\text{. This must be due to numerical capabilities of Matlab. See figures: C1.5, C1.6, and C1.7}}$

(ii) $\boxed{\text{See Figure C1.2 for plot of 4 revolution}}$ → Equation in notes C

$t_{end} = \boxed{18 \text{ (non-dimensional)}}$

$t_{end} = 18 \times t^* = \boxed{6.753425 \times 10^6\,s = 78 \text{ day}}$

$\boxed{\text{Motion doesn't look Keplerian but is orbiting the Earth at this point}}$

(iii) $\boxed{\text{See Figure C1.3 for plot when spacecraft has gone to lunar vicinity. Consistent with previous sketch.}}$

$\boxed{t_{end} = 800 \text{ (non-dimensional)}}$

$t_{end} = 800 \times t^* = \boxed{3.00125 \times 10^8\,s = 9.517 \text{ years}}$

$\boxed{\text{The ZVC does start to emerge. It is difficult to see the ZVC near the Moon and near } L_1}$

(iv) $\boxed{\text{See Figure C1.4 for plot of simulation for a long time}}$

$\boxed{\text{It does not leave the vicinity atleast for this time}}$

(v) Absolute error $= \dfrac{|C_t - C|}{|C|}$ → C_t is C at given time

$\boxed{\text{See Figure 5 for absolute error of Jacobi constant}}$

$\boxed{\text{It does increase over time but it is relatively small at } 10^{-11}\text{. See recomended tolerance at top of page.}}$

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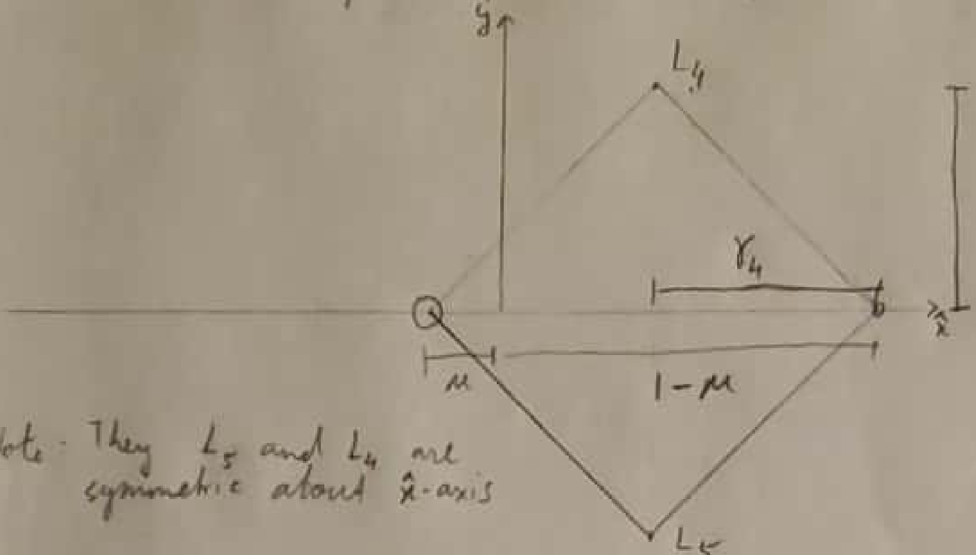
Continued part 6 Lagrange points.

6) We need to find L_4 and L_5 first.



Note: They L_5 and L_4 are symmetric about $\hat{x}$-axis

$\bar{\nabla} U^* = 0 \qquad \dot{z} = 0$

$$\therefore \frac{\partial U}{\partial x} = \frac{-(1-\mu)(x+\mu)}{d^3} - \frac{\mu(x-1+\mu)}{r^3} + n^2 x = 0 \qquad (1)$$

$$\frac{\partial U}{\partial y} = \frac{-(1-\mu)y}{d^3} - \frac{\mu y}{r^3} + n^2 y = 0 \qquad (2)$$

$$d = [(x+\mu)^2 + y^2 + \cancel{z^2}]^{1/2} \qquad r = [(x-1+\mu)^2 + y^2 + \cancel{z^2}]^{1/2}$$

We have 2 equations and 2 unknowns (1) and (2)

$$\therefore F(x) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} = \begin{bmatrix} \frac{-(1-\mu)(x+\mu)}{d^3} - \frac{\mu(x-1+\mu)}{r^3} + x \\ \frac{-(1-\mu)y}{d^3} - \frac{\mu y}{r^3} + y \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \qquad H = -J^{-1} F(X)$$

$$\therefore \boxed{X_{n+1} = X_n + H_n = X_n - J_n^{-1} F_n(x)}$$

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Continued...

$$\frac{\partial f_1}{\partial x} = -\frac{(-2(x+\mu)^2+y^2)(1-\mu)}{d^5} - \frac{\mu(y^2-2(x+m-1)^2)}{r^5} + 1$$

$$\frac{\partial f_1}{\partial y} = \frac{3y(1-\mu)(x+\mu)}{d^5} + \frac{3\mu y(x-1+m)}{r^5}$$

$$\frac{\partial f_2}{\partial x} = \frac{3y(x+\mu)(1-\mu)}{d^5} + \frac{3\mu y(x-1+\mu)}{r^5} = \frac{\partial f_1}{\partial y}$$

$$\frac{\partial f_2}{\partial y} - \frac{((x+\mu)^2-2y^2)(1-\mu)}{d^5} - \frac{\mu((x-1+\mu)^2-2y^2)}{r^5} + 1$$

Now we iterate unit we get solution:

	x_i (-)	y_i (-)	C (-)
L_4	0.48784944	0.8660254	2.98799705
L_5	0.48784944	-0.8660254	2.98799705

The position of in $\hat{x}$ direction makes sense because P_1, P_2, and L_4/L_5 form an equilateral triangle $\therefore$ $x_4 = \frac{1}{2} - \mu$ => That is what we got.

PSC1

Part c)

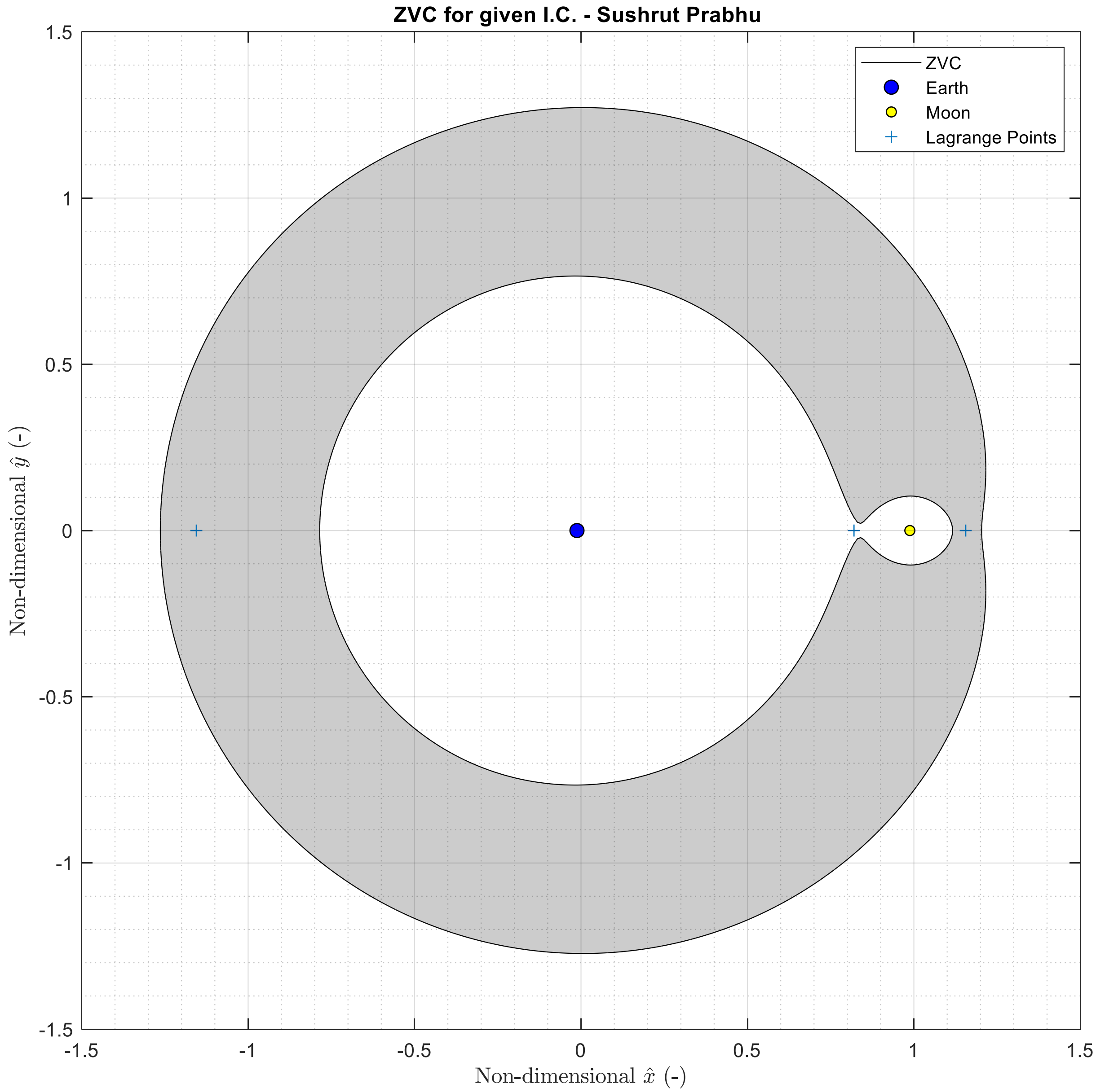


Figure C1.1: ZVC curve sketch plotted for more accuracy depiction.

Part d i)

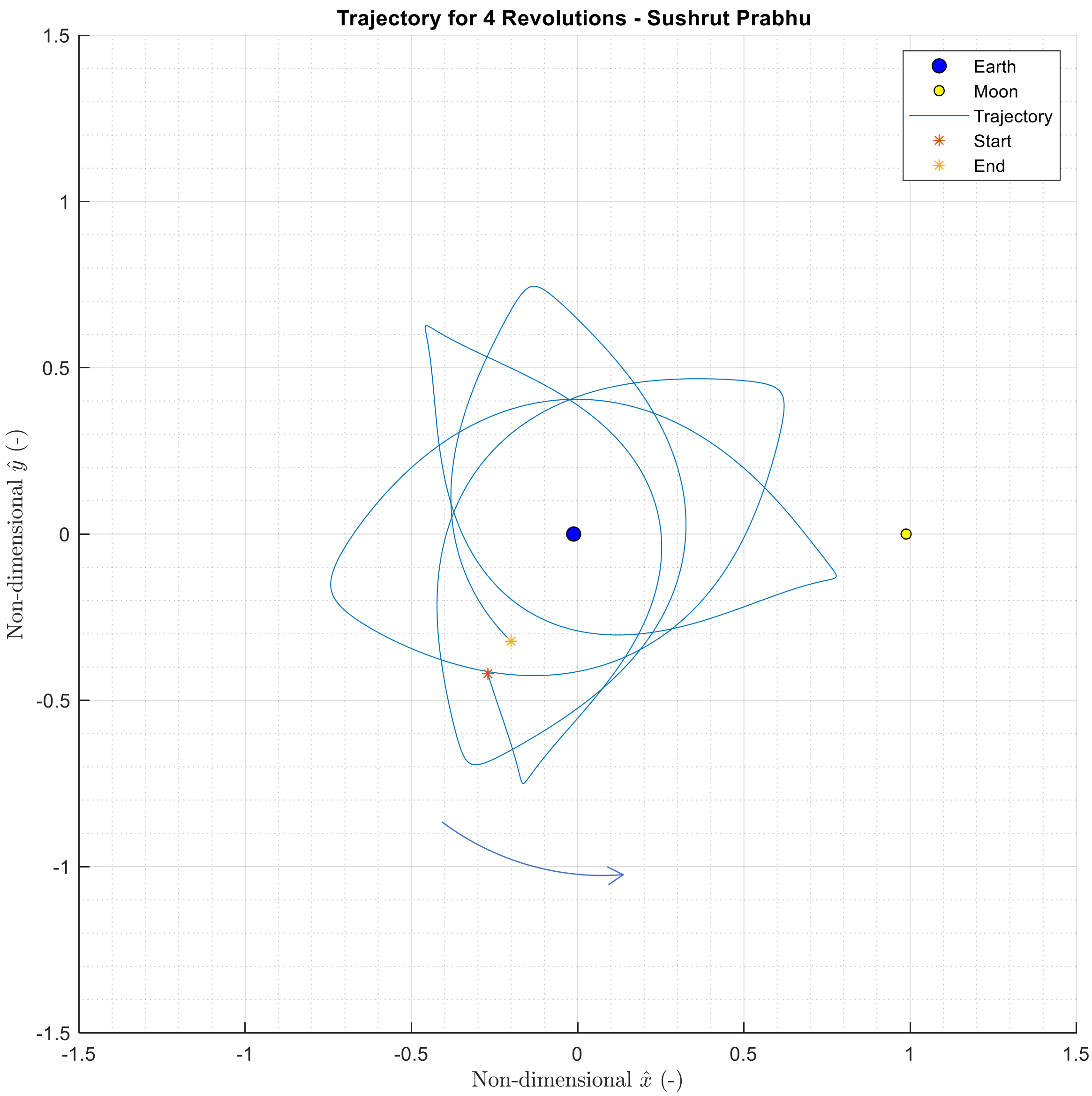


Figure C1.2: Four revolutions around the primary.

Part d ii)

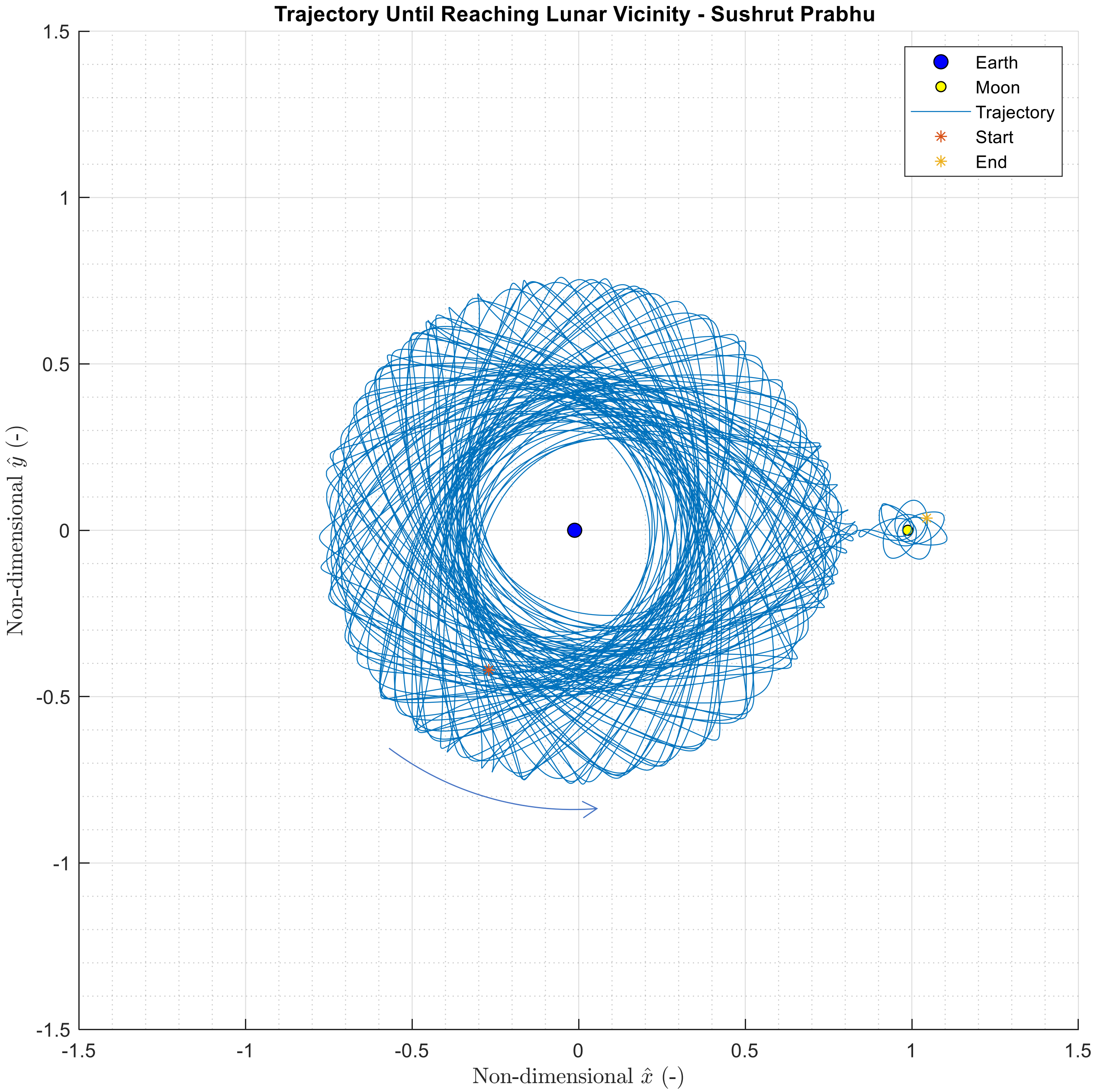


Figure C1.3: Simulation until reaching Lunar vicinity.

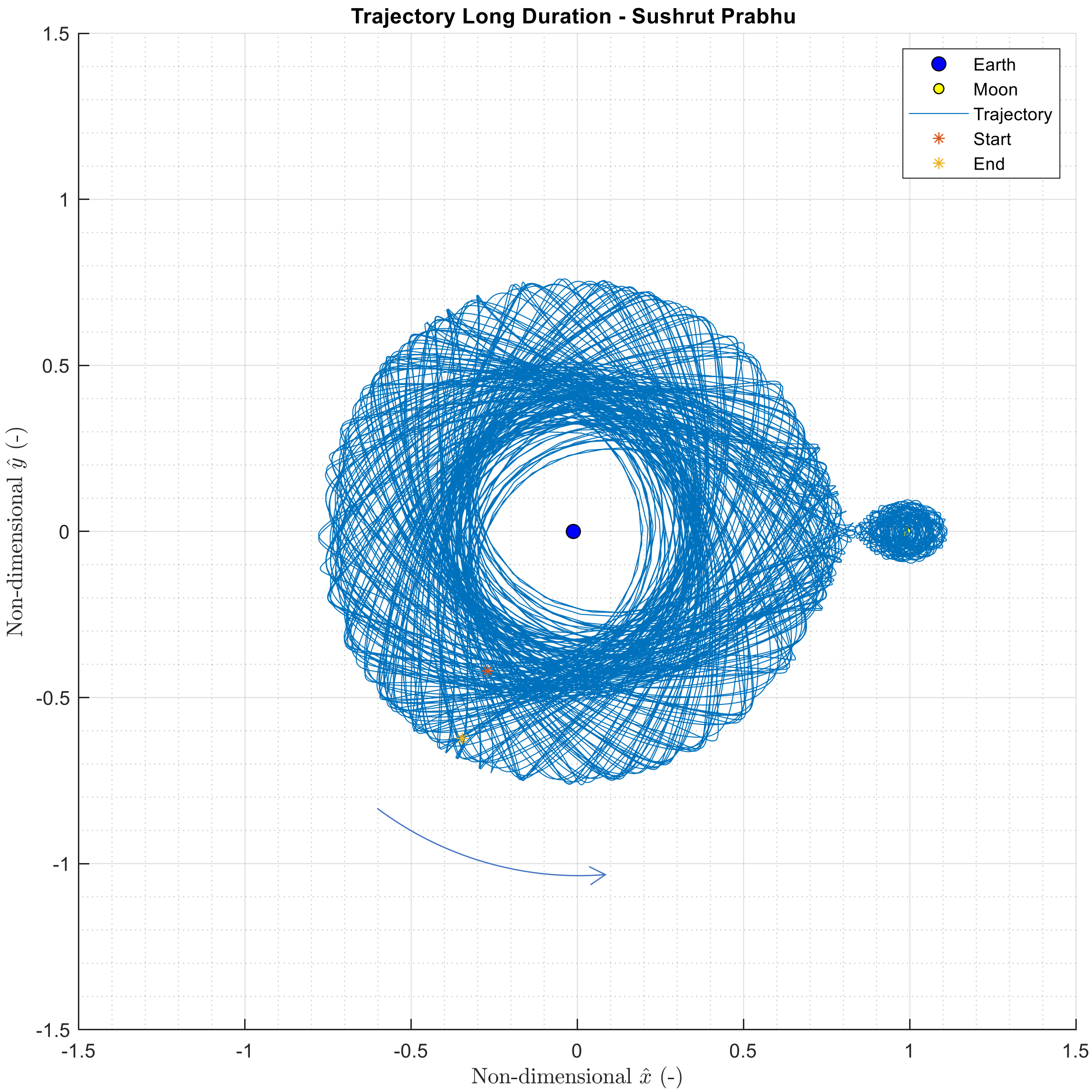


Figure C1.4: Long simulation to see general shape of ZZVC appear

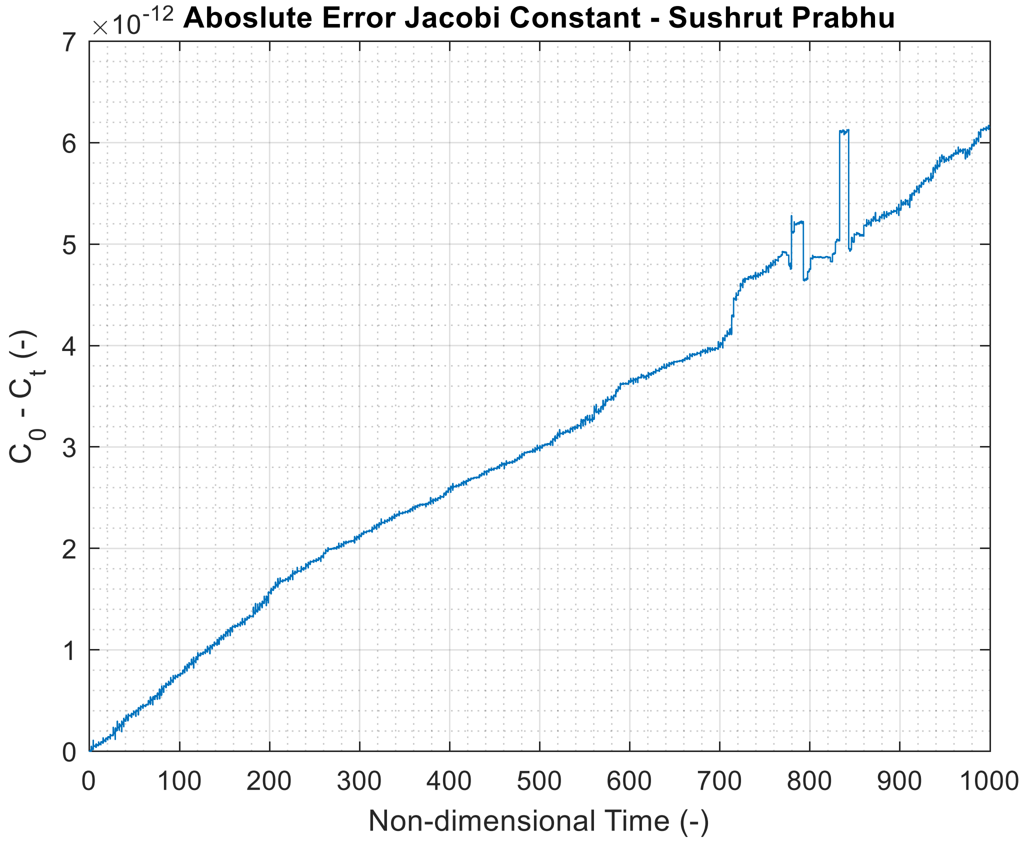


Figure C1.5: Best tolerance possible with ODE45 at (Rel: 10^-13; Abs: 10^-15)

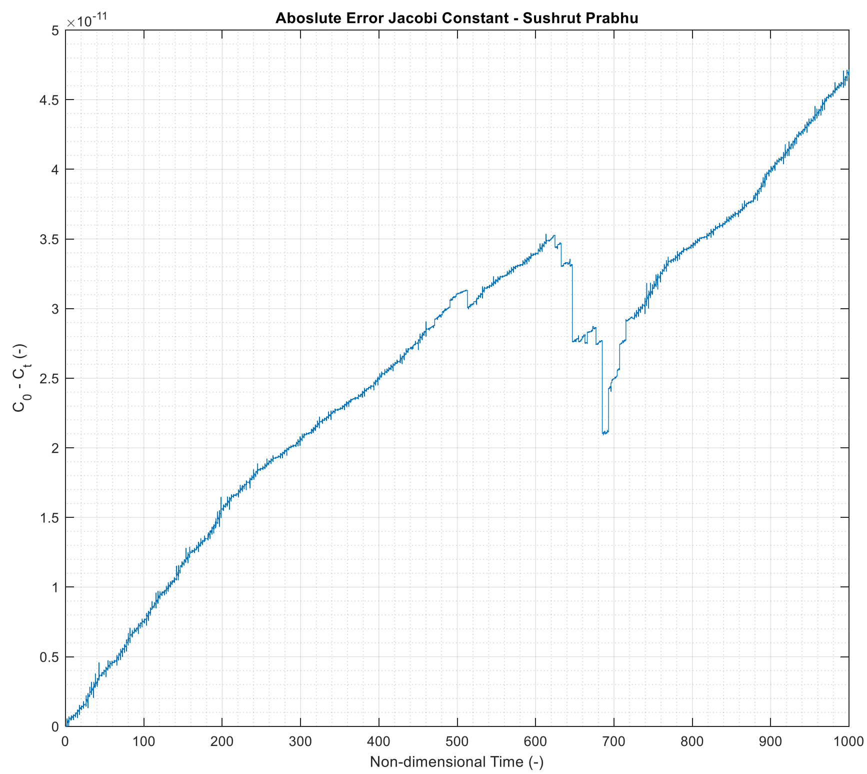


Figure C1.6: Test of tolerance possible with ODE45 at (Rel: 10^-12; Abs: 10^-15)

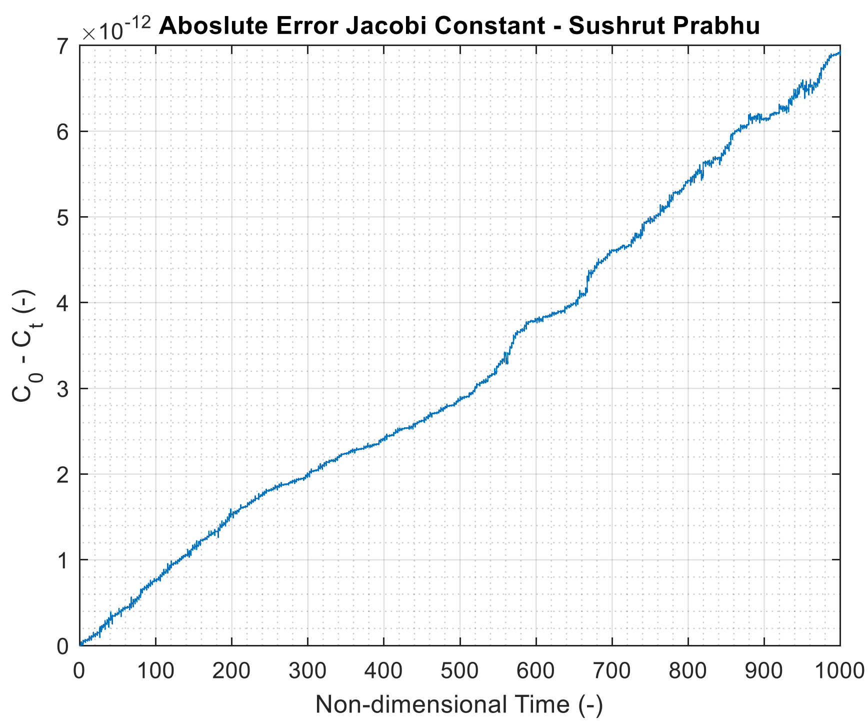


Figure C1.7: Test of tolerance possible with ODE45 at (Rel: 10^-13; Abs: 10^-17)

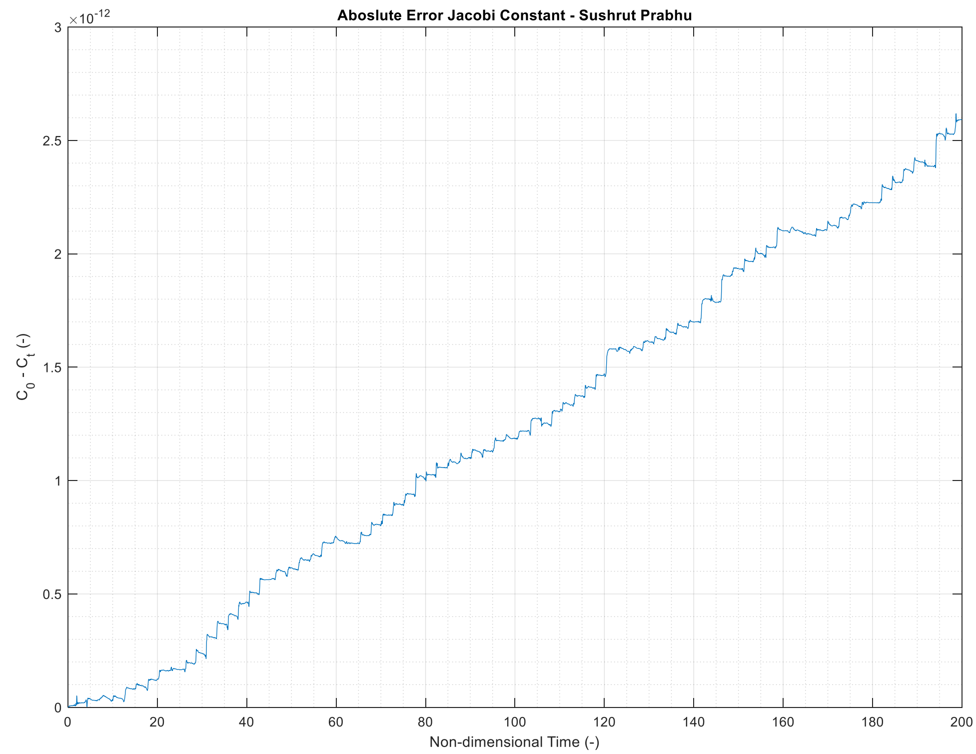


Figure C1.8: Test of tolerance possible with ODE23 at (Rel: 10^-13; Abs: 10^-15)

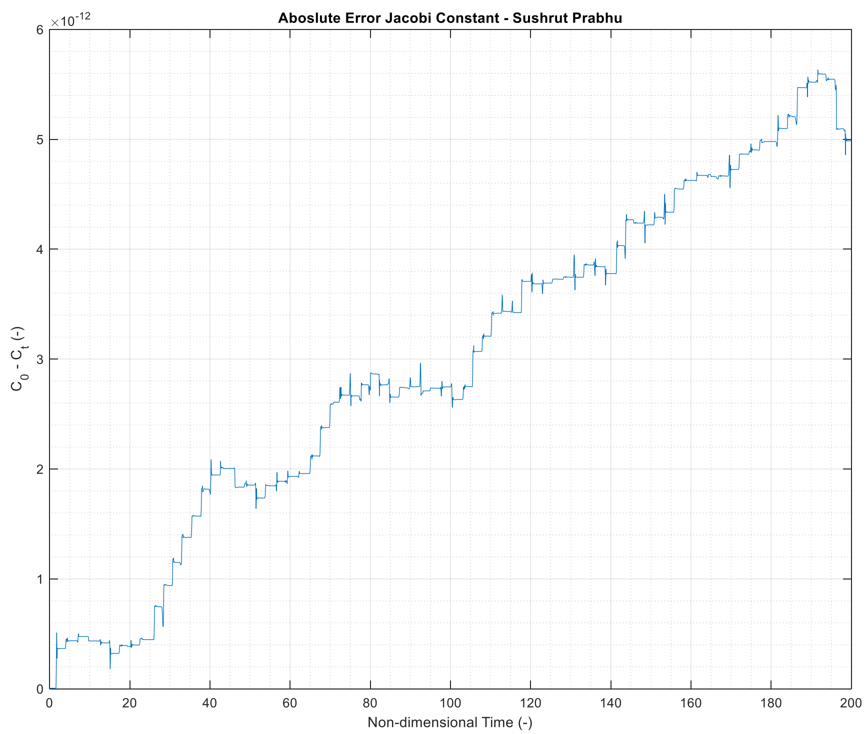


Figure C1.9: Test of tolerance possible with ODE113 at (Rel: 10^-13; Abs: 10^-15)

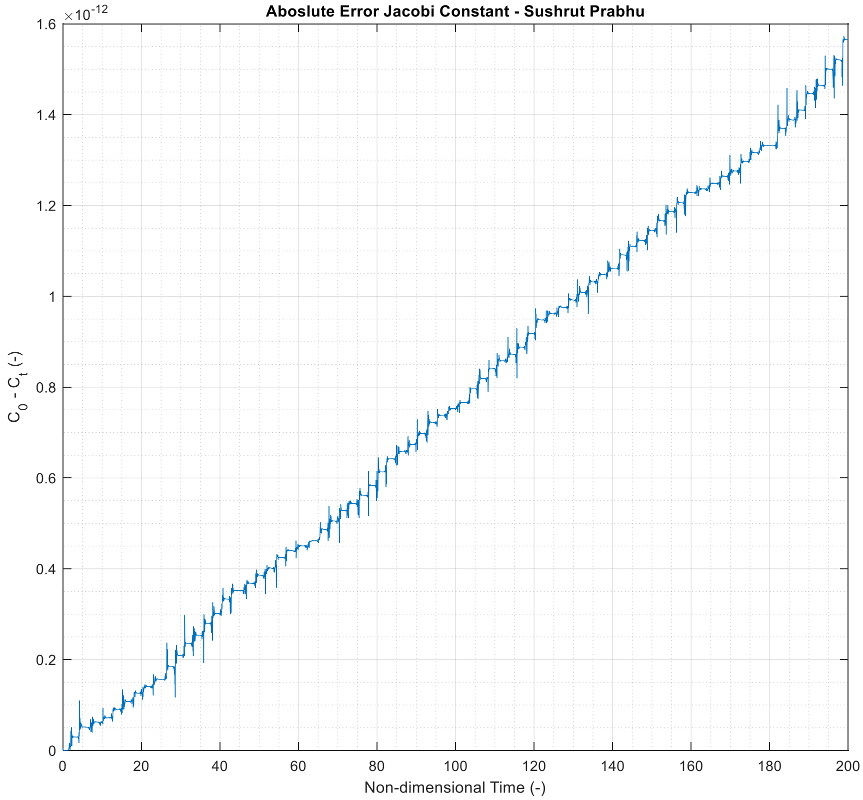


Figure C1.9: Test of tolerance possible with ODE23 at (Rel: 10^-13; Abs: 10^-15)

Table of Contents

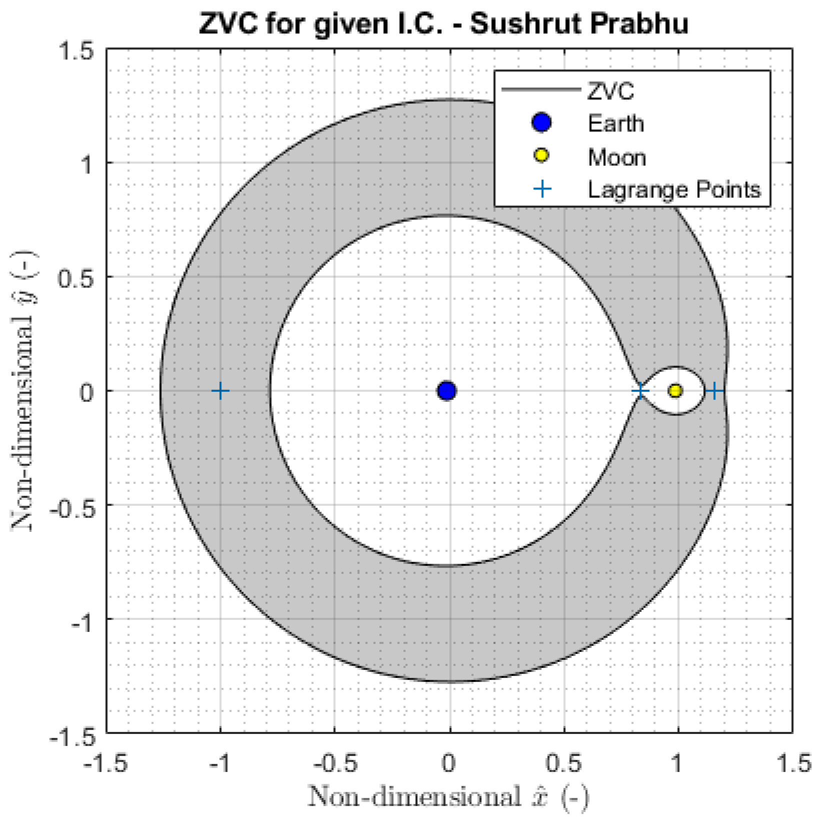
PSC1

```
clear
close all
clc
```

PSC1

```
SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' ,'gamma_2 (-)','L_2 (-)', 'gamma_2
 (km)','L_2 (km)', 'gamma_1 (-)', 'L_1 (-)', 'gamma_1 (km)', 'L_1
 (km)','gamma_3 (-)', 'L_3 (-)', 'gamma_3 (km)', 'L_3 (km)',  'L4
 (-)', 'L4 (km)', 'L5 (-)', 'L5 (km)', 'C_L1 (-)', 'C_L2 (-)', 'C_L3
 (-)', 'C_L4 (-)', 'C_L5 (-)', 't*'};
G = 6.6738*10^-20;
r = [-.27 -.42 0];
v = [.3 -1 0];

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% Solution
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{end,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon

dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;

% Lagrange Point 2
dim_vals{5,2} = abs(L2_NRmethod(dim_vals{4,2}*1.1,dim_vals{4,2},
 10^-8));
dim_vals{6,2} = 1-dim_vals{4,2} + dim_vals{5,2};
dim_vals{7,2} = dim_vals{5,2}*dim_vals{2,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};

% Lagrange Point 1
dim_vals{9,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{10,2} = 1-dim_vals{4,2} - dim_vals{9,2};
dim_vals{11,2} = dim_vals{9,2}*dim_vals{2,2};
dim_vals{12,2} = dim_vals{10,2}*dim_vals{2,2};
```

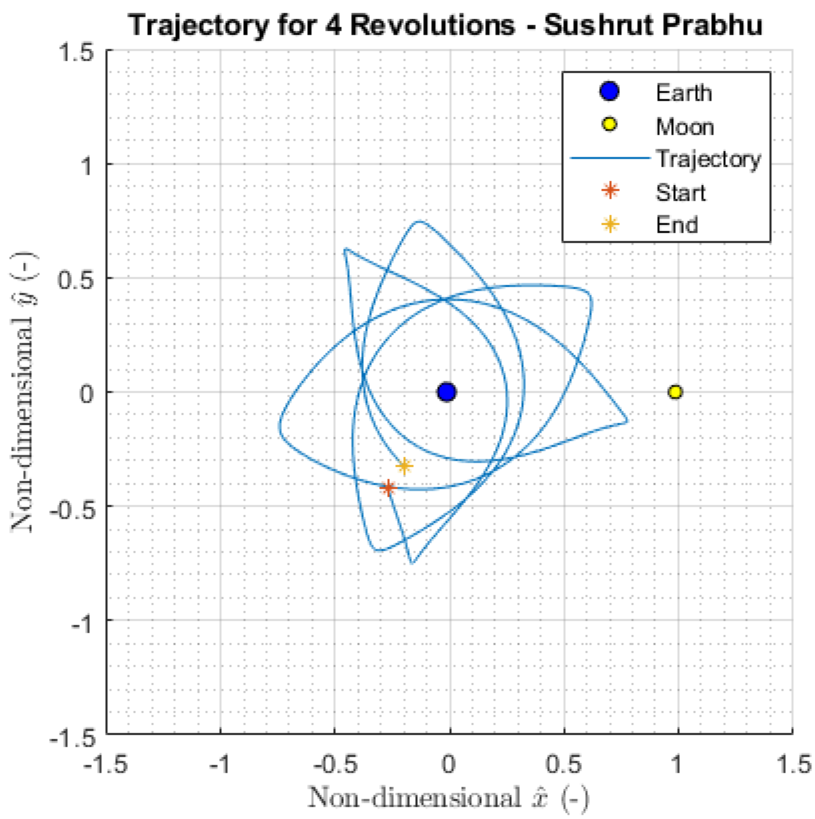
```
% Lagrange Point 3
dim_vals{13,2} = abs(L3_NRmethod(-.9,dim_vals{4,2}, 10^-8));
dim_vals{14,2} = -dim_vals{4,2} - dim_vals{13,2};
dim_vals{15,2} = dim_vals{13,2}*dim_vals{2,2};
dim_vals{16,2} = dim_vals{14,2}*dim_vals{2,2};

% L4 and L5
dim_vals{17,2} = L45_NRm(.55,.5,dim_vals{4,2},10^-7);
dim_vals{18,2} = dim_vals{17,2}*dim_vals{2,2};
dim_vals{19,2} = [dim_vals{17,2}(1);-dim_vals{17,2}(2)];
dim_vals{20,2} = dim_vals{19,2}*dim_vals{2,2};

% Jacobi
dim_vals{21,2} = Jacobi_C(dim_vals{10,2},0,0,0,dim_vals{4,2});
dim_vals{22,2} = Jacobi_C(dim_vals{6,2},0,0,0,dim_vals{4,2});
dim_vals{23,2} = Jacobi_C(dim_vals{14,2},0,0,0,dim_vals{4,2});
dim_vals{24,2} = Jacobi_C(dim_vals{17,2}(1),dim_vals{17,2}
(2),0,0,dim_vals{4,2});
dim_vals{25,2} = Jacobi_C(dim_vals{19,2}(1),dim_vals{19,2}
(2),0,0,dim_vals{4,2});


C0 = Jacobi_C(r(1),r(2),r(3),norm(v),dim_vals{4,2});

[X,Y] = meshgrid(-1.5:0.01:1.5);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

m = 2;
map = ones(m , 3)*.8;

colormap(map);
contourf(X,Y,-C,-[C0 C0]);
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
grid on
grid minor
axis square
title('ZVC for given I.C. - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')
```



Part (ii)

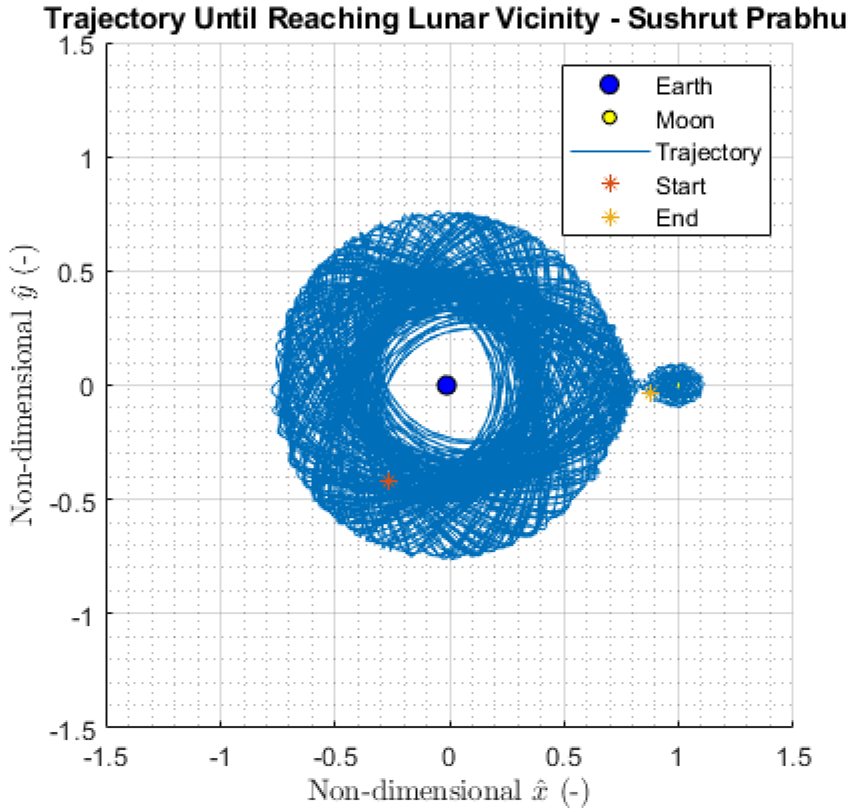
```
IC = [r,v];
t1 = 0:.005:18;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t1,IC,options,dim_vals{4,2});

figure
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
grid on
grid minor
axis square
title('Trajectory for 4 Revolutions - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Earth', 'Moon')
plot(y(:,1),y(:,2))
plot(y(1,1),y(1,2),'*')
plot(y(end,1),y(end,2),'*')
legend('Earth', 'Moon', 'Trajectory', 'Start', 'End')
xlim([-1.5 1.5])
```

```
ylim([-1.5 1.5])
```



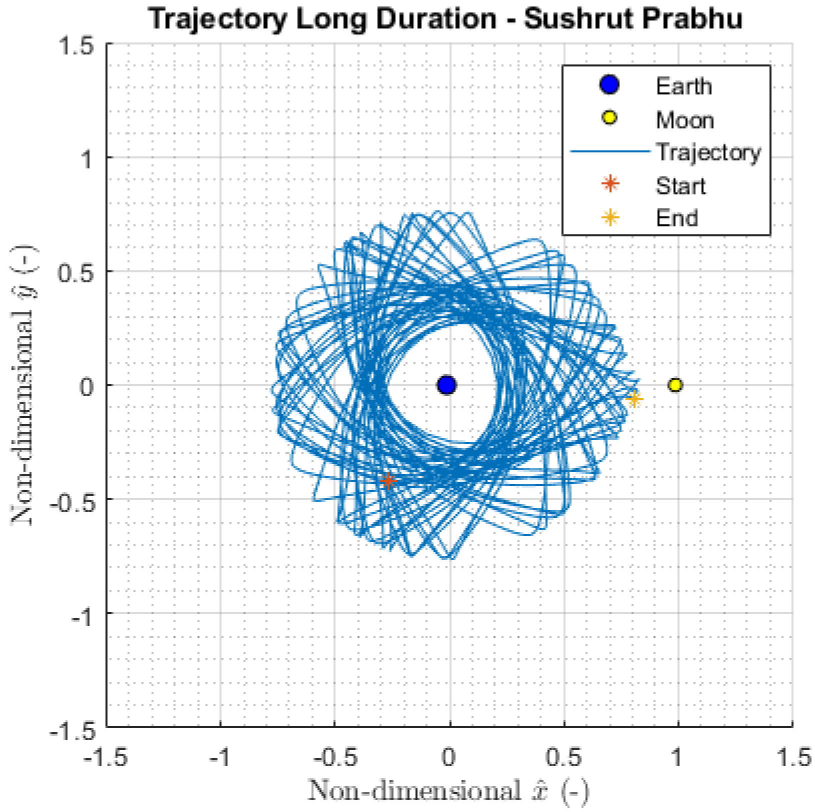
Part (iii)

```
t2 = 0:.05:800;

[~,y] = ode45(@cr3bp_df,t2,IC,options,dim_vals{4,2});

figure
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
grid on
grid minor
axis square
title('Trajectory Until Reaching Lunar Vicinity - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
plot(y(:,1),y(:,2))
plot(y(1,1),y(1,2),'*')
plot(y(end,1),y(end,2),'*')
legend('Earth', 'Moon', 'Trajectory', 'Start', 'End')
xlim([-1.5 1.5])
ylim([-1.5 1.5])
```



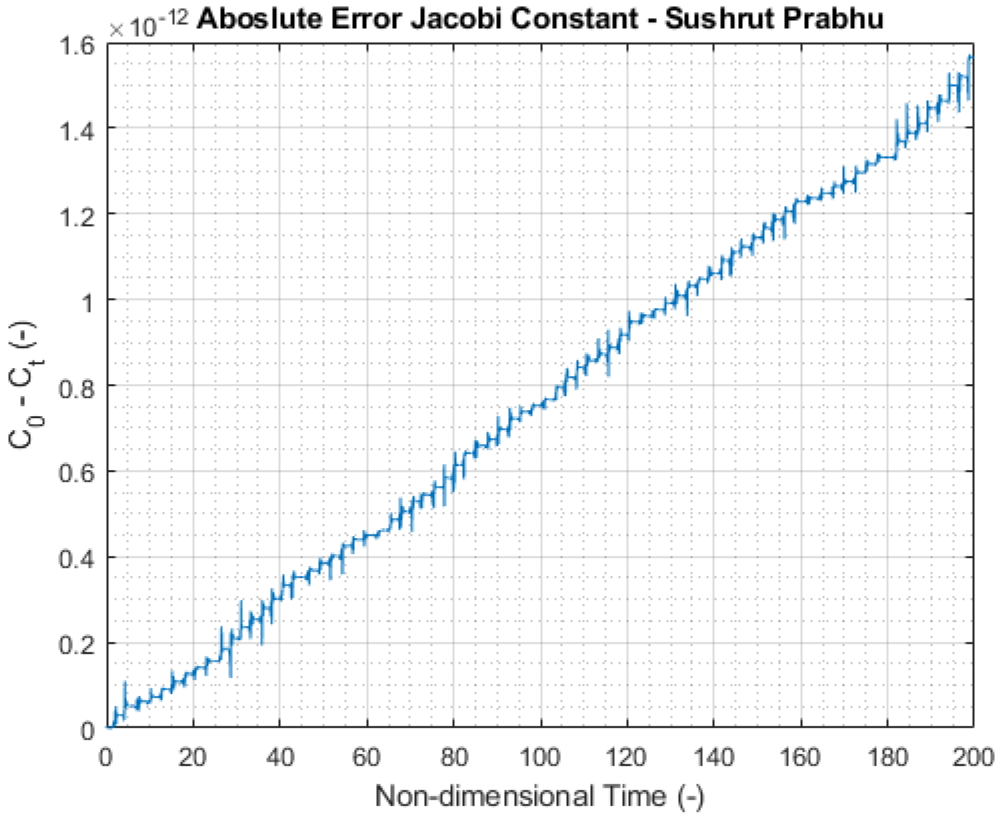
Part (iv)

```
t2 = 0:.05:200;

[t,y] = ode45(@cr3bp_df,t2,IC,options,dim_vals{4,2});

figure
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
grid on
grid minor
axis square
title('Trajectory Long Duration - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
plot(y(:,1),y(:,2))
plot(y(1,1),y(1,2),'*')
plot(y(end,1),y(end,2),'*')
legend('Earth', 'Moon', 'Trajectory', 'Start', 'End')
xlim([-1.5 1.5])
ylim([-1.5 1.5])
```



Part (v)

```
v_vec = vecnorm(y(:,4:6),2,2);
C_vec = Jacobi_C(y(:,1),y(:,2),y(:,3),v_vec,dim_vals{4,2});
delC = abs(C_vec-C0)/abs(C0);

figure
plot(t,delC)
grid on
grid minor
title('Aboslute Error Jacobi Constant - Sushrut Prabhu')
ylabel('C_0 - C_t (-)')
xlabel('Non-dimensional Time (-)')
```



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Sushant Problem | AAE632 | PSC2

PSC2

Given: The same information as PSC1

Find: a) Plot of ZVC. Do the curves correspond to the curves that emerge from simulation

b) Change value of C to get one encircling each primary and third encircling the system.

· Simulate an orbit.

c) Try a Jacobi Constant where gateway opens to L2.

· Can you find trajectory that escapes

Always indicate length of simulation, dimensional and non-dimensional.

Solution:

a) See Figure C2.1 of the ZVC this is from Matlab function

Plot was generated by creating a meshgrid of X-Y, then running meshgrid through Jacobi equation. Finally, contour plotting

b) In order to get 2 curves encircling each primary and 1 on system:

$C > C_{L_1}$ but C cannot be too big either otherwise the second primary will not have a ZVC encircling it

See Figure C2.2

The plot is for $C = 3.25$

The corresponding $v^2 = x^2 + y^2 + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} - C$

$d = [(x+\mu)^2 + y^2 + z^2]^{1/2}$ $r = [(x+\mu-1)^2 + y^2 + z^2]^{1/2}$

continued:

Let $r = [-0.27, -0.42, 0]$ this is the same as problem C1.

$|V| = 1.0131$ $\quad$ $\underline{\bar{V} = [1.0131, 0, 0]}$

I tested the ZVC for various directions of velocity

$\hat{v}_1 = [1, 0, 0]$ $\quad$ $\hat{v}_3 = [2/\sqrt{5}, 1/\sqrt{5}, 0]$

$\hat{v}_2 = [1/\sqrt{2}, 1/\sqrt{2}, 0]$ $\quad$ $\hat{v}_4 = [2/\sqrt{5}, -1/\sqrt{5}, 0]$

These 4 simulations ran for $\boxed{t = 400 \text{ (non-dim)}}$

$\boxed{\text{See Figures: C2.2, C2.3, C2.4 \& C2.5}}$ $t = t \times t^* = \boxed{4.76 \text{ years}}$

I also tested ZVC in the Lunar region

here my $r = [0.95, 0, 0]$ $\quad$ $t = 10$ (non-dim

$v = [2/\sqrt{5}, -1/\sqrt{5}, 0]$ $\quad$ $t = t \times t^* = 43.42$ days

$\boxed{\text{See figures: C2.6}}$ $\quad$ $\boxed{\text{ZVC is never violated}}$

c) To open gateway choose $C > C2$, $C = 3.1$.

Follow same steps as before. Note: not all velocity and position directions lead out of system.

I chose same position: $r = [-0.27, -0.42, 0]$

$\hat{v} = [1, 0, 0]$ $\quad$ Note $|\bar{v}| \rightarrow$ determined by C just like before

$\boxed{\text{See Figure: C2.7}}$

$t = 80$ (non-dim)

$t = t \times t^* = \boxed{0.951 \text{ years} = 347 \text{ days}}$

Subrat Pradhan | AAE632 | PSC2

PSC2 Bonus

Find: Create an algorithm to make your own ZVC curve

Solution: $\boxed{\text{Figures: C2.8, C2.9, C2.10 and C2.11}}$

I used Newton's method to get the solutions on ZVC curve. I assumed $z = 0$.

1) Symmetry along the x axis

$\therefore$ Let $$f(y) = x^2 + y^2 + \frac{2(1-\mu)}{((x+\mu)^2+y^2)^{1/2}} + \frac{2\mu}{((x+\mu-1)^2+y^2)^{1/2}} - C$$

Note $f(y) = f(-y) \rightarrow$ Cuts our work in half

2) Set up Newton Rhapson

$$f'(y) = 2y - \frac{2y(1-\mu)}{((x+\mu)^2+y^2)^{3/2}} + \frac{2\mu y}{((x+\mu-1)^2+y^2)^{3/2}}$$

$\therefore y_{n+1} = y_n - \dfrac{f(y)}{f'(y)} \rightsquigarrow$ Due to symmetry only need positive y.

3) Start with outer most curve. Note outer curve is circle so if x-range is from $-1.5 \rightarrow 1.5$ then $y_0 = 3$; approximately double the max. Discard value that are complex or do not converge.

4) The smaller curve or circles will be inside this larger curve. Redefine an x-range inside the large circle. The new $y_0 = y_{BC} \times f$: $y_{BC} \rightarrow$ Is the y-point for bigger curve. $0 < f < 1$ scaling factor for second guess ($f = 0.3$ works in most cases or use $f = 0.1$). Discard new y-point $= y_{BC}$, imaginary y-points, and non-convergent. Plot with symmetry : Note: Matlab method is faster

PSC2

Part a)

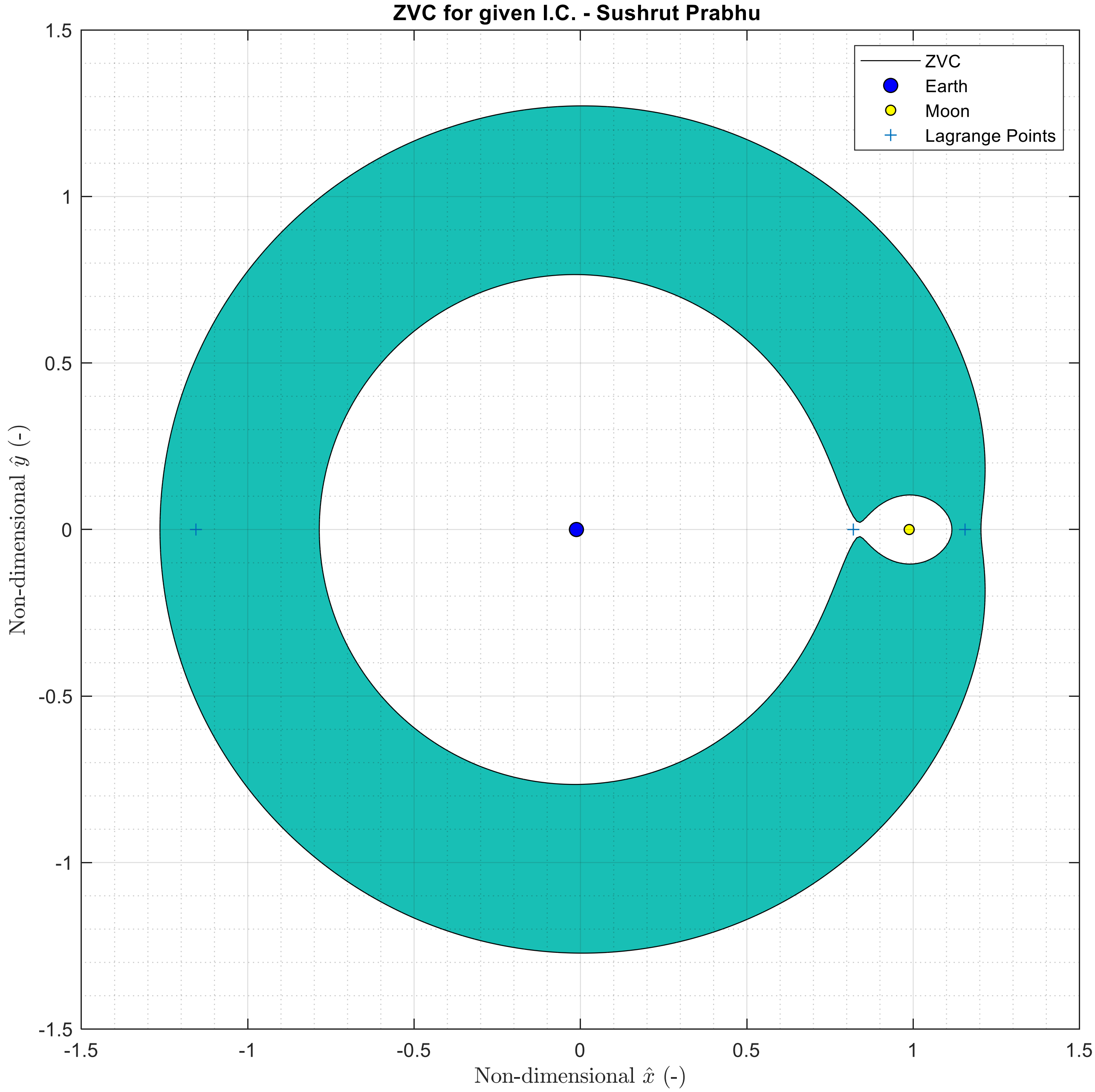


Figure C2.1: The ZVC curve for the initial conditions given in C1.

Part b)

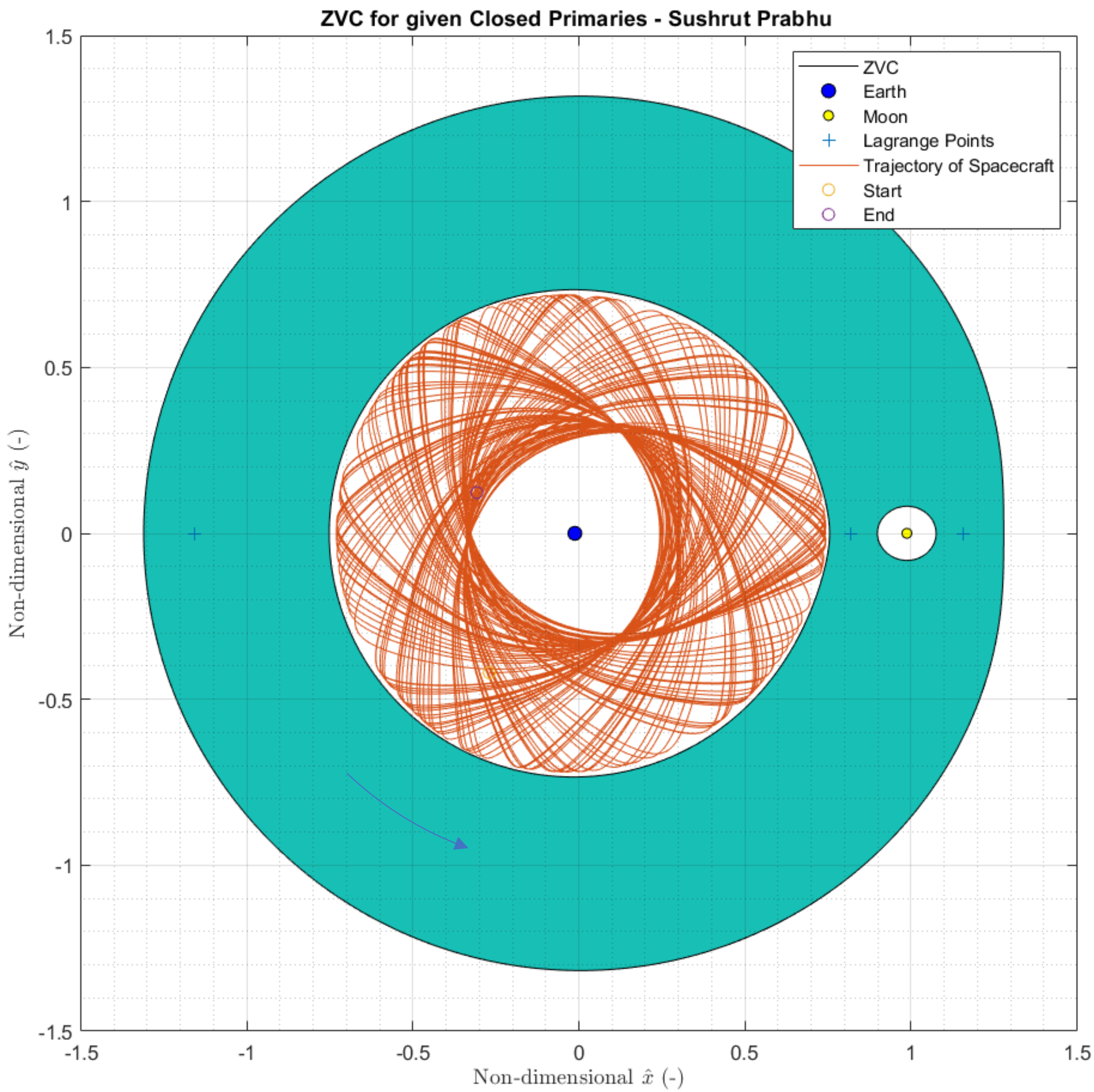


Figure C2.2: New ZVC with closed circles tested with trajectory ($\hat{v} = [1,0,0]$).

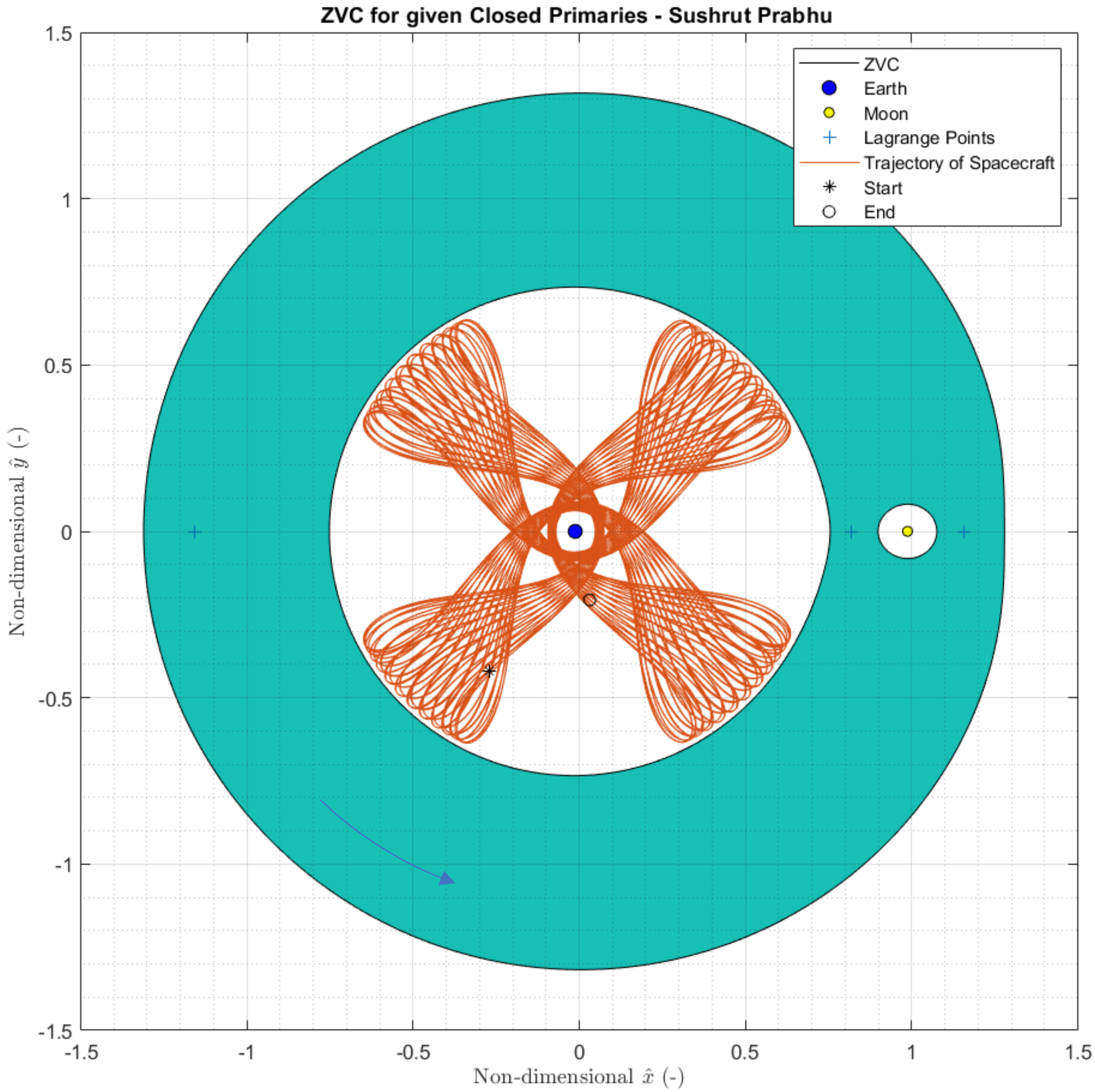


Figure C2.3: New ZVC with closed circles tested with trajectory ($\hat{v} = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]$)

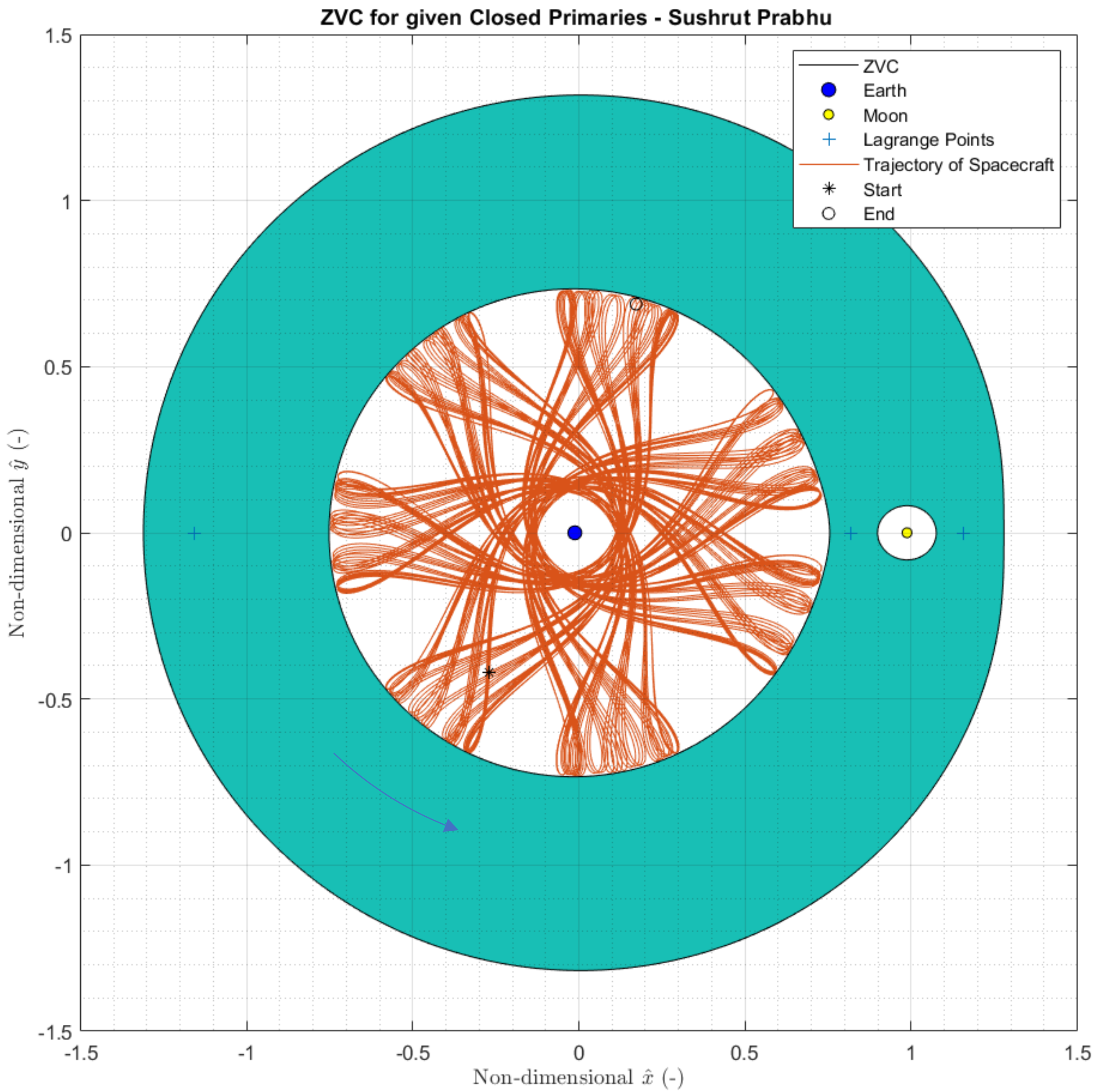


Figure C2.4: New ZVC with closed circles tested with trajectory ($\hat{v} = [\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0]$)

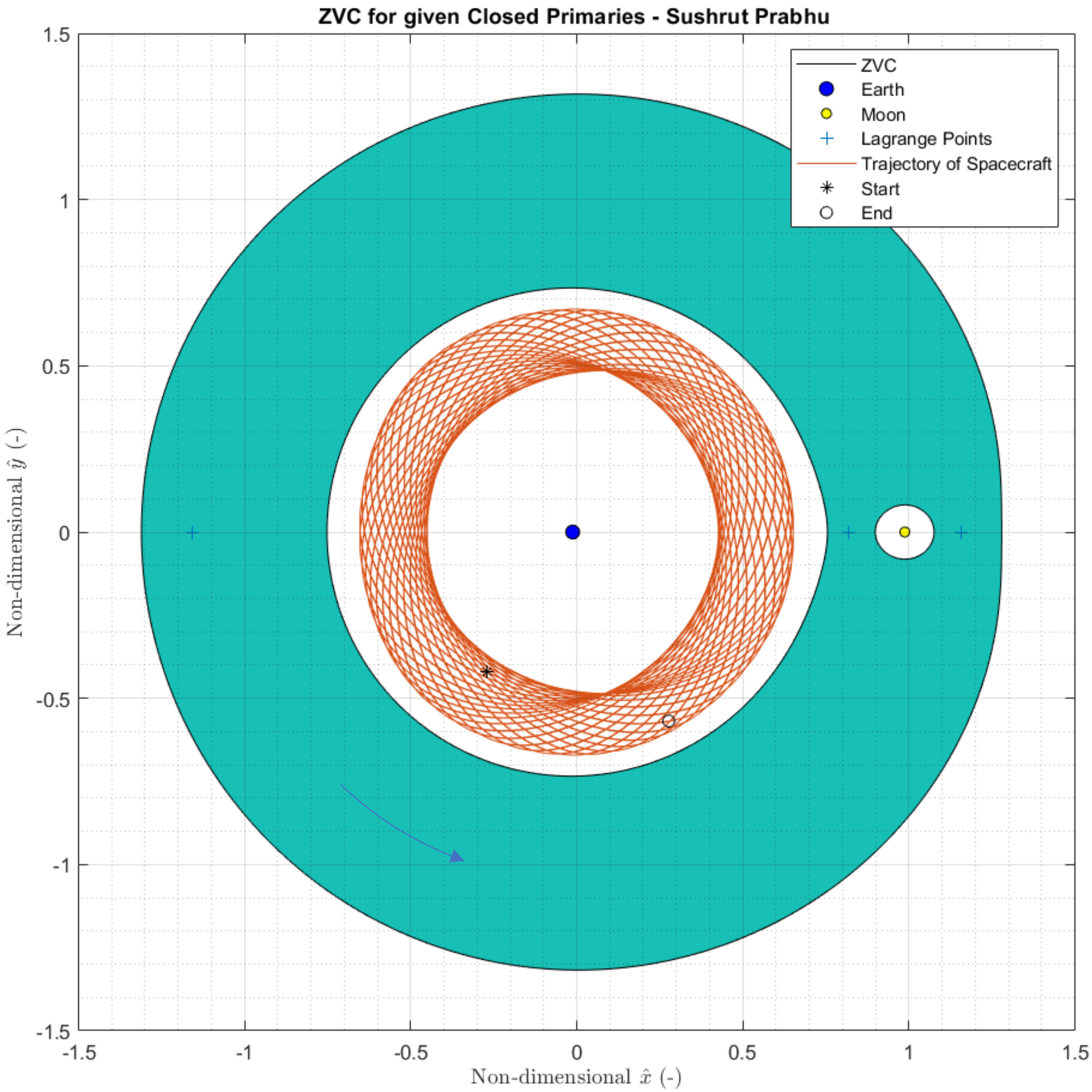


Figure C2.5: New ZVC with closed circles tested with trajectory ($\hat{v} = [\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0]$)

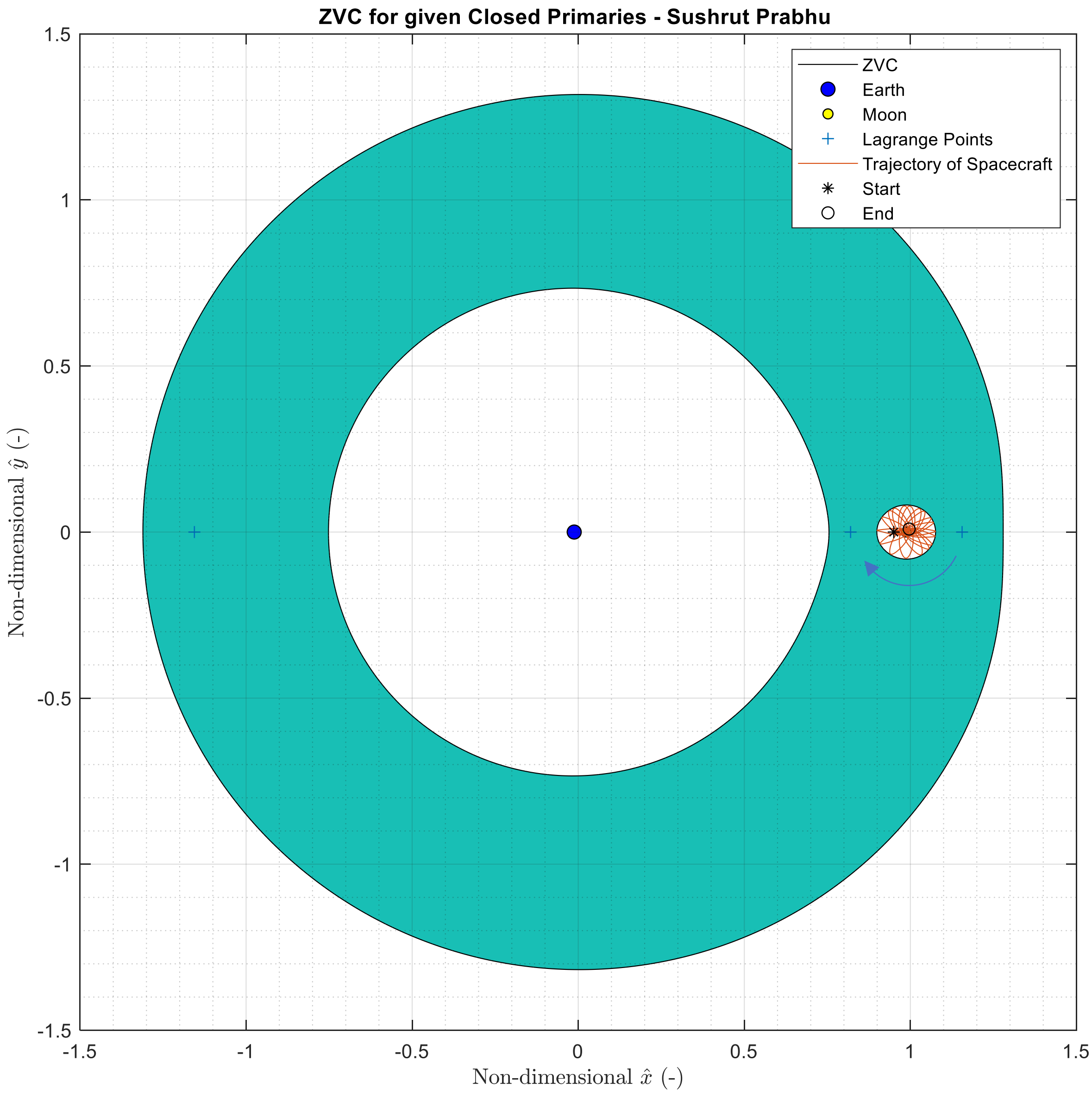


Figure C2.6: New ZVC with closed circles tested with trajectory ($\hat{v} = [\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0]$)

Part c)

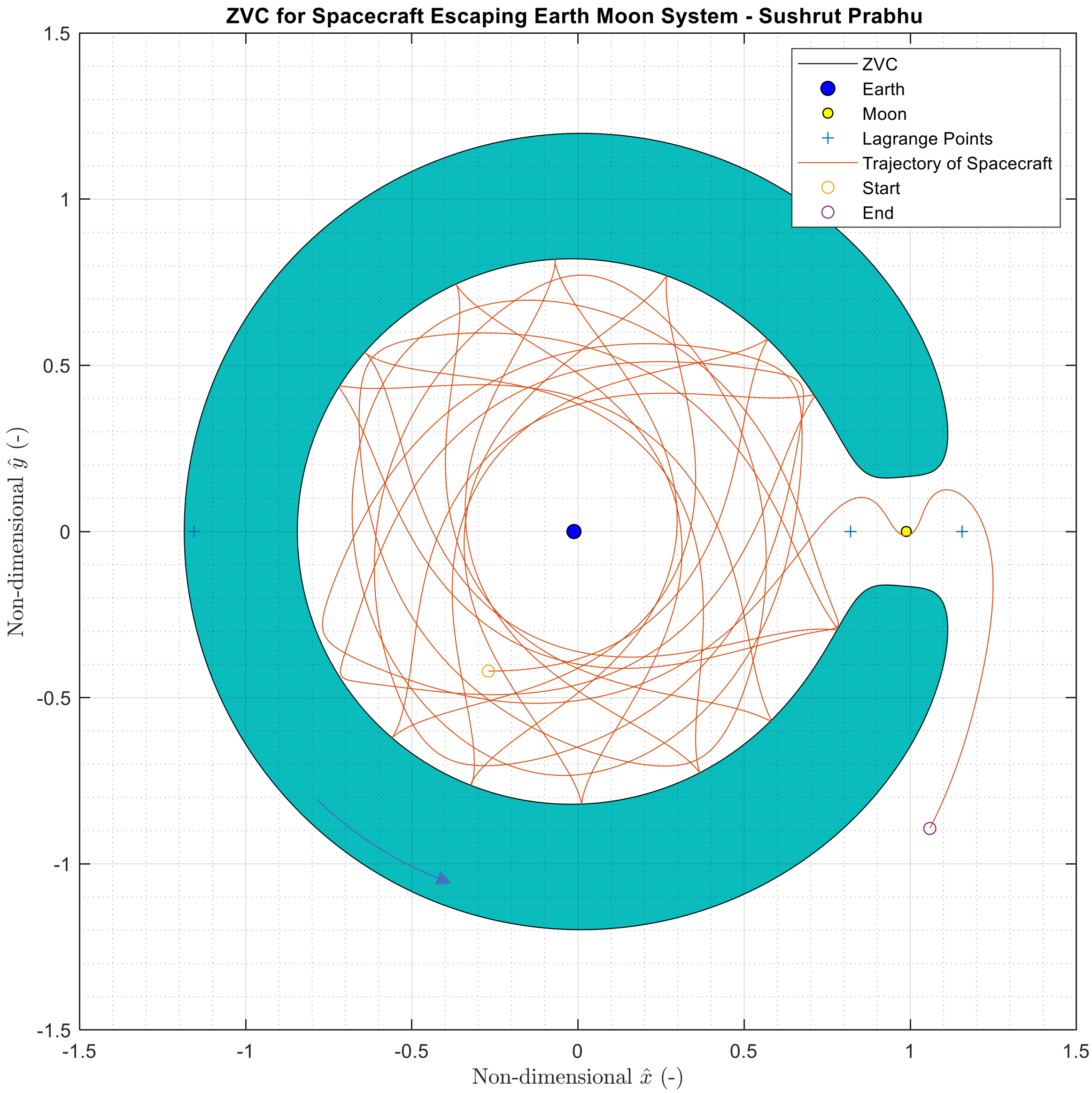


Figure C2.7: Gateway of ZVC out of the Earth-Moon System.

Problem C2 Bonus

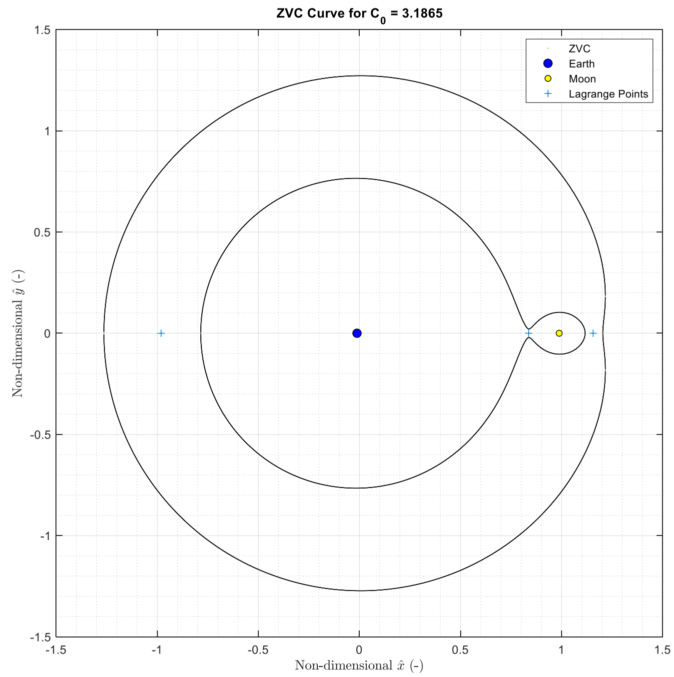


Figure C2.8: Newton Raphson Method to find ZVC same C as Figure C2.1.

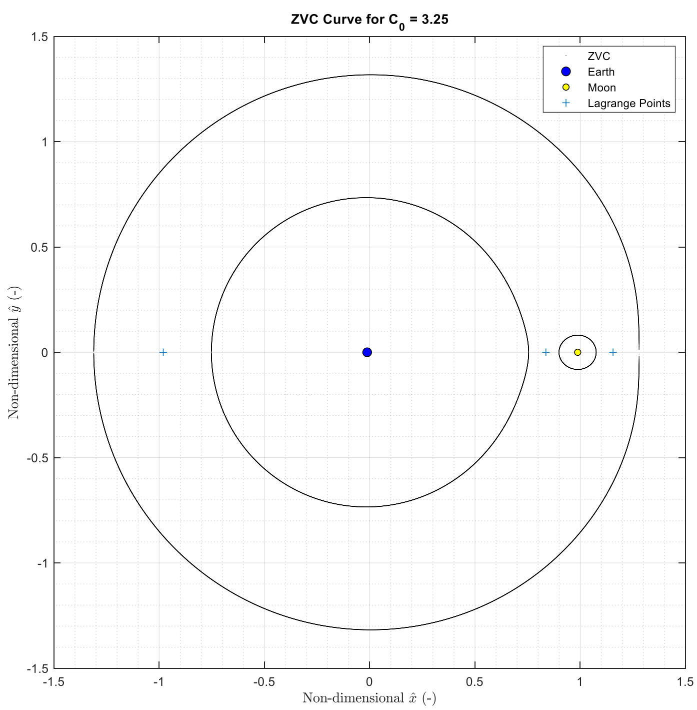


Figure C2.8: Newton Raphson Method to find ZVC same C as Figure C2.2, C2.3, C2.4, C2.5, and C2.6.

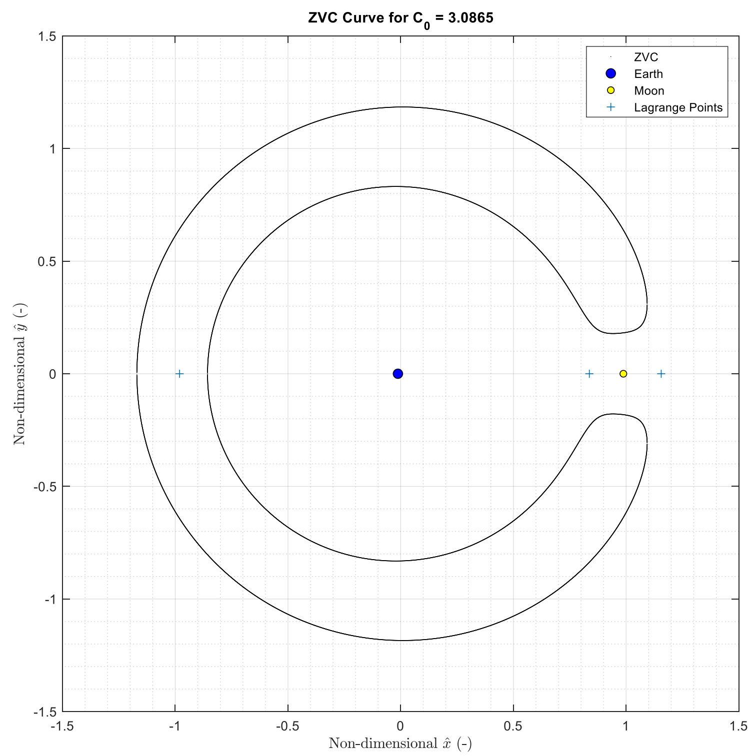


Figure C2.9: Newton Raphson Method to find ZVC same C as Figure C2.7.

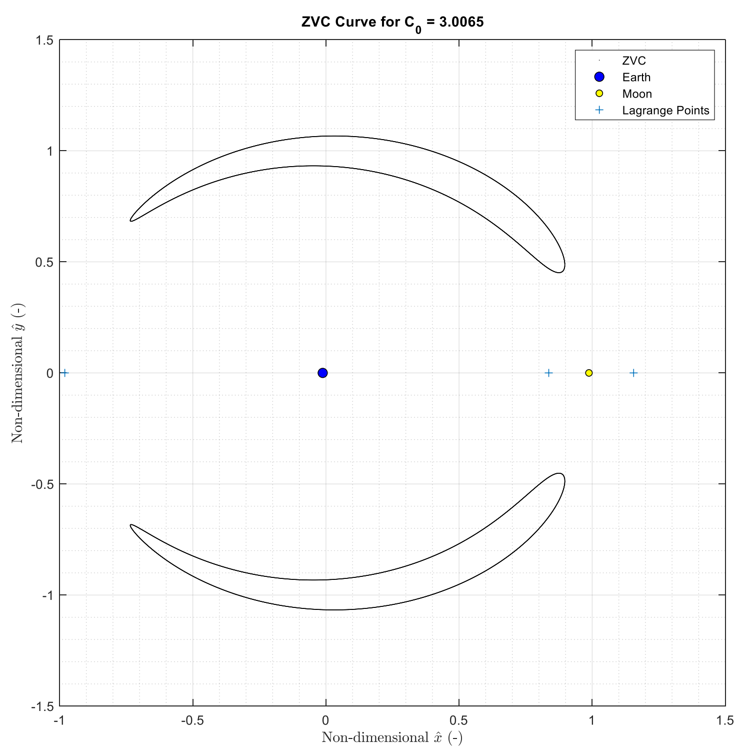


Figure C2.9: Newton Raphson Method to find ZVC

Table of Contents

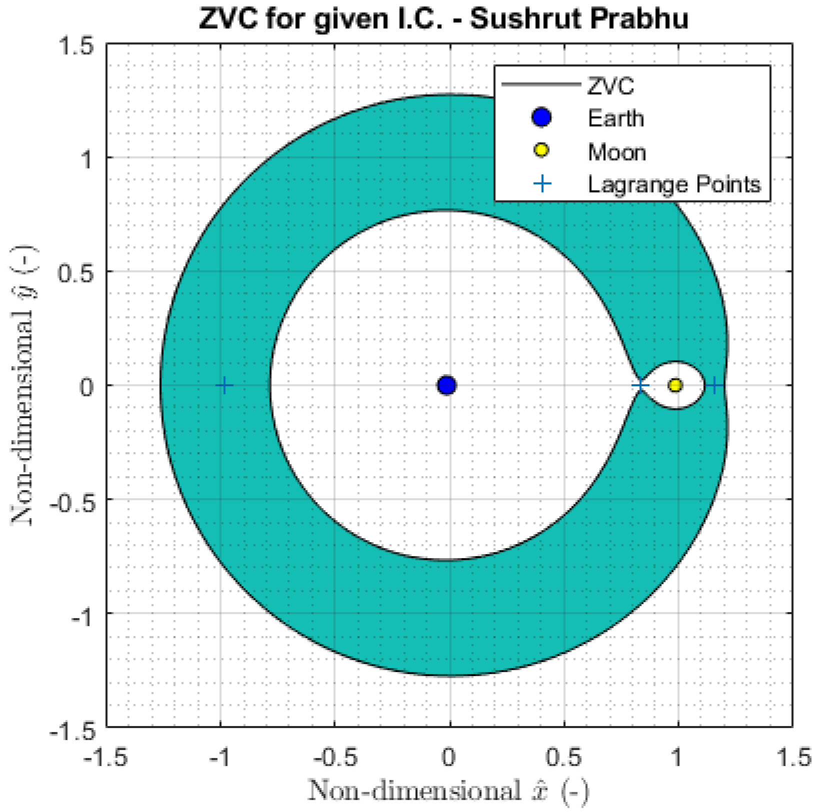
PSC2

```
clear
close all
clc
```

PSC2

```
SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' ,'gamma_2 (-)','L_2 (-)', 'gamma_2
 (km)','L_2 (km)', 'gamma_1 (-)', 'L_1 (-)', 'gamma_1 (km)', 'L_1
 (km)','gamma_3 (-)', 'L_3 (-)', 'gamma_3 (km)', 'L_3 (km)', 'C_L1
 (-)', 'C_L2 (-)', 'C_L3 (-)', 't*'};
G = 6.6738*10^-20;
r = [-.27 -.42 0];
v = [.3 -1 0];

dim_vals = num2cell(zeros(19,1));
dim_vals = [systems; param',dim_vals];

% Solution
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{20,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon

dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;

% Lagrange Point 2
dim_vals{5,2} = abs(L2_NRmethod(dim_vals{4,2}*1.1,dim_vals{4,2},
 10^-8));
dim_vals{6,2} = 1-dim_vals{4,2} + dim_vals{5,2};
dim_vals{7,2} = dim_vals{5,2}*dim_vals{2,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};

% Lagrange Point 1
dim_vals{9,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{10,2} = 1-dim_vals{4,2} - dim_vals{9,2};
dim_vals{11,2} = dim_vals{9,2}*dim_vals{2,2};
dim_vals{12,2} = dim_vals{10,2}*dim_vals{2,2};

% Lagrange Point 3
dim_vals{13,2} = abs(L3_NRmethod(-.9,dim_vals{4,2}, 10^-8));
```

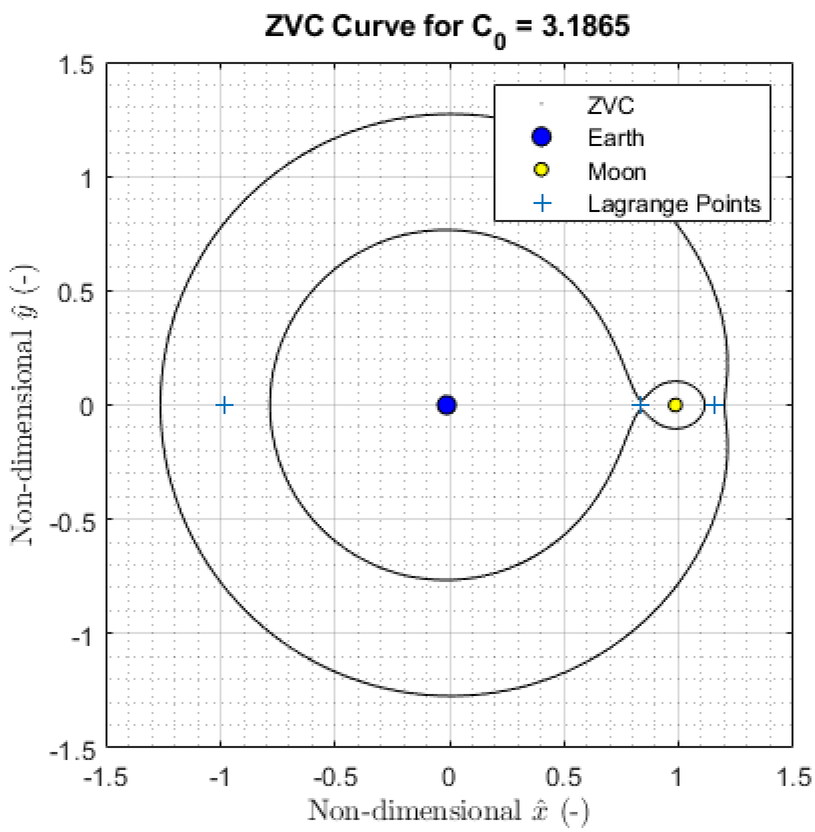
```
dim_vals{14,2} = dim_vals{4,2} - dim_vals{13,2};
dim_vals{15,2} = dim_vals{13,2}*dim_vals{2,2};
dim_vals{16,2} = dim_vals{14,2}*dim_vals{2,2};

% Jacobi
dim_vals{17,2} = Jacobi_C(dim_vals{10,2},0,0,0,dim_vals{4,2});
dim_vals{18,2} = Jacobi_C(dim_vals{6,2},0,0,0,dim_vals{4,2});
dim_vals{19,2} = Jacobi_C(dim_vals{14,2},0,0,0,dim_vals{4,2});


[X,Y] = meshgrid(-1.5:0.01:1.5);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(r(1),r(2),r(3),norm(v),dim_vals{4,2});

figure
contourf(X,Y,-C,-[C0 C0]);
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
grid on
grid minor
axis square
title('ZVC for given I.C. - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')
```

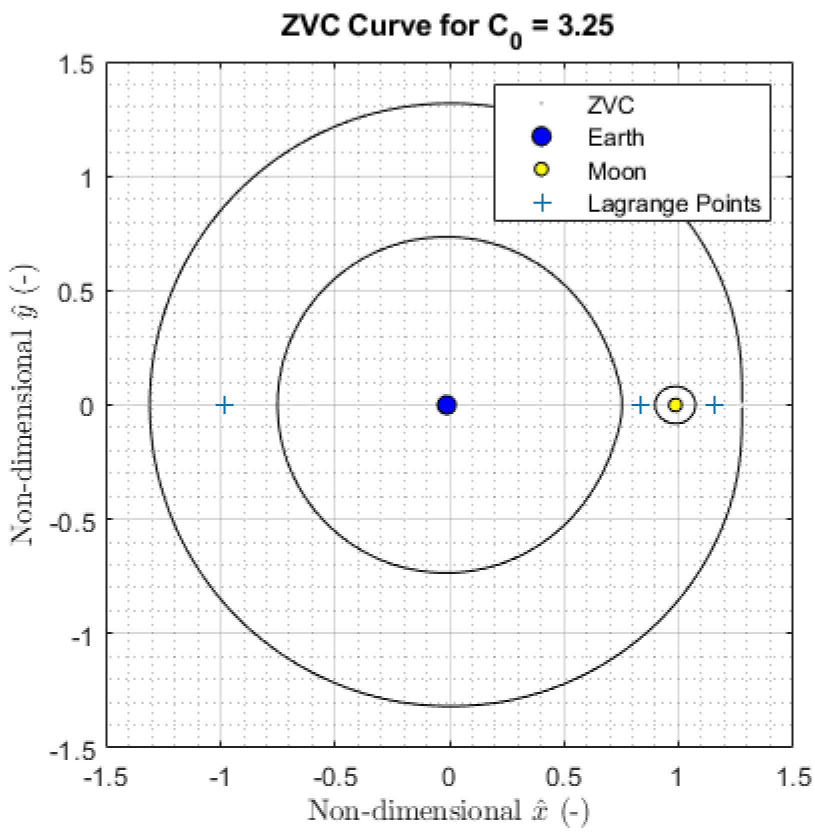


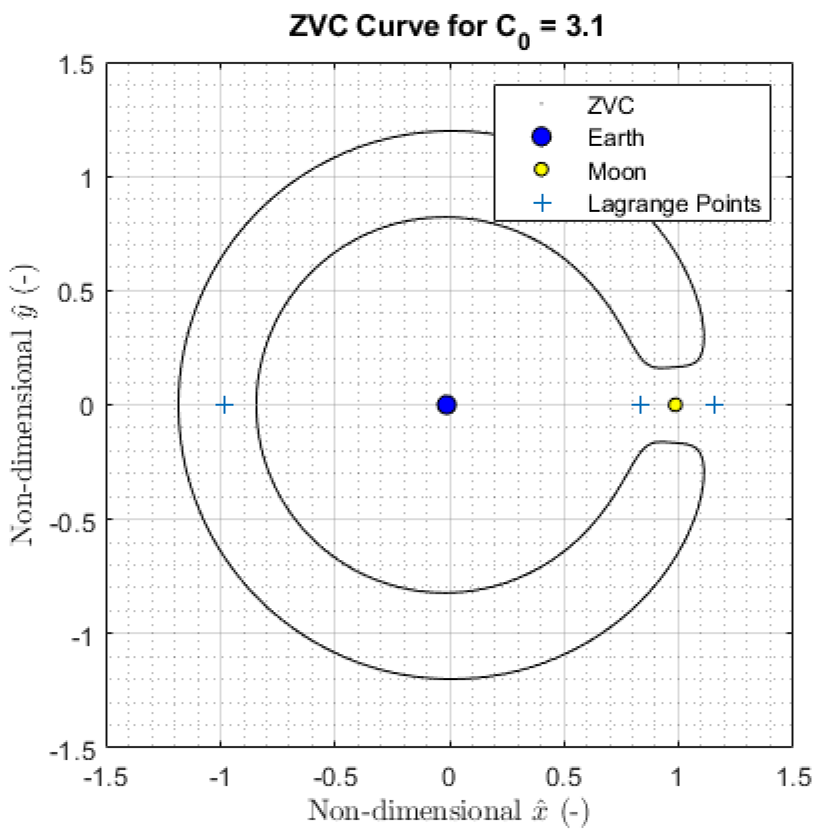
PSC2 Bonus

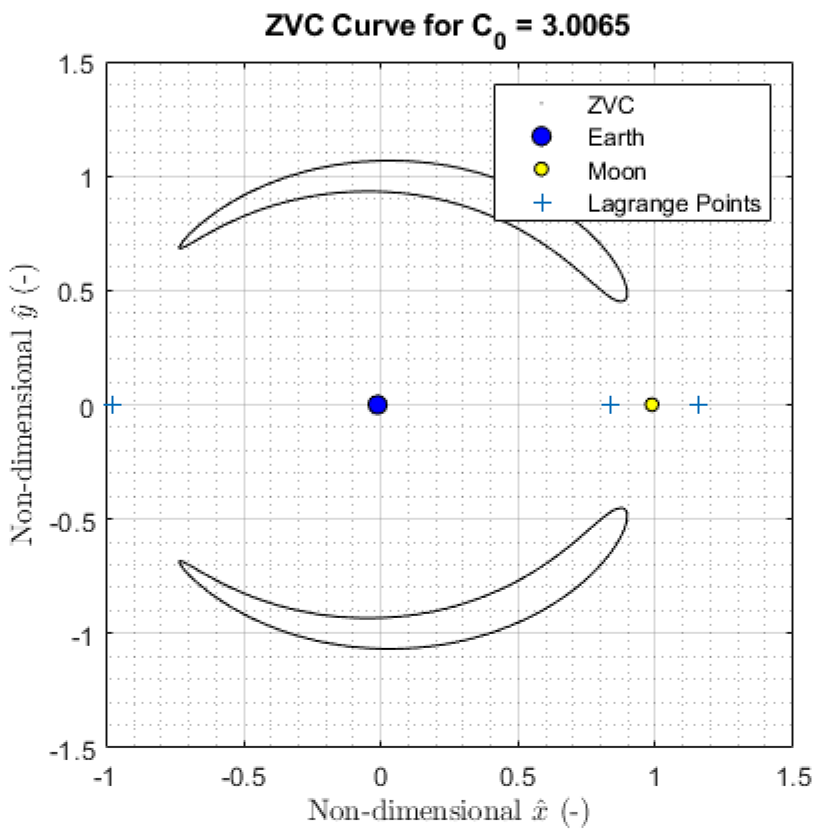
```
ZVC_bonus(-1.5:.00001:1.5,C0,dim_vals{4,2},.3)
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
axis square
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')

ZVC_bonus(-1.5:.00001:1.5,3.25,dim_vals{4,2},.1)   % Make smaller
 guess
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
axis square
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')

ZVC_bonus(-1.5:.00001:1.5,3.1,dim_vals{4,2},.3)
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
axis square
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')

ZVC_bonus(-1.5:.00001:1.5,C0-.18,dim_vals{4,2},.3)
```

```
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
axis square
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points')
```




ZVC Curve for C_0 = 3.25
ZVC
Earth
Moon
Lagrange Points
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
1.5
1
0.5
0
-0.5
-1
-1.5

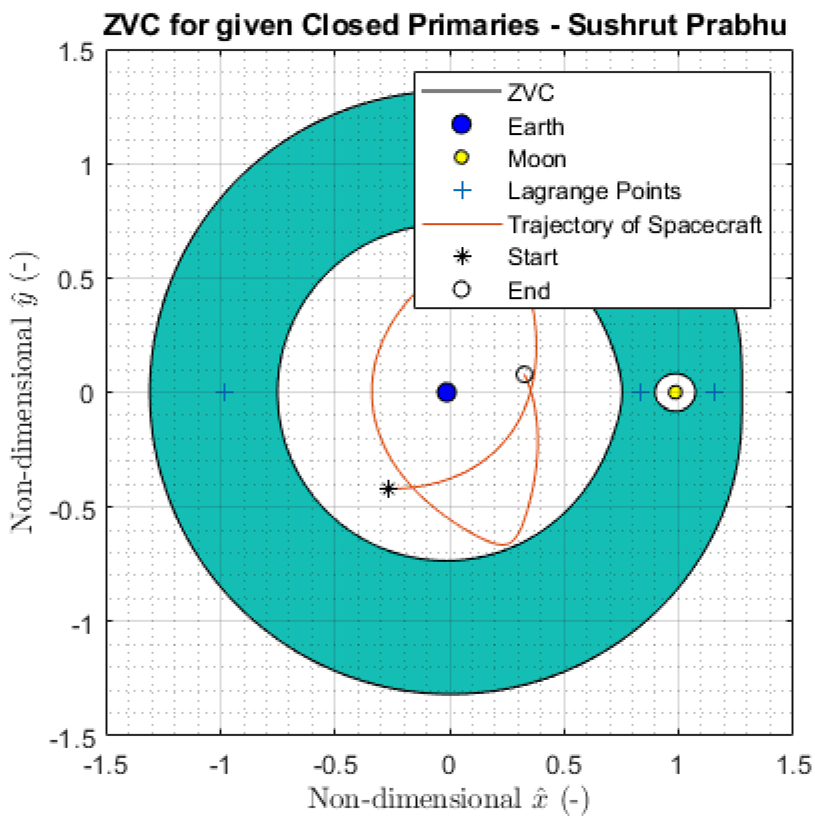

ZVC Curve for C_0 = 3.1
ZVC
Earth
Moon
Lagrange Points
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
1.5
1
0.5
0
-0.5
-1
-1.5



Part (b)

```
C02 = 3.25;
dd = sqrt((r(1)+dim_vals{4,2})^2 + r(2)^2);
rr = sqrt((r(1)+dim_vals{4,2}-1)^2+ r(2)^2);
v_mag = sqrt( r(1)^2 + r(2)^2 + 2*(1-dim_vals{4,2})/dd +
 2*dim_vals{4,2}/rr -C02);

v = v_mag*[1 0 0];

IC = [r,v];
t1 = 0:.005:5;
options=odeset('RelTol',1e-12, 'AbsTol',1e-16); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t1,IC,options,dim_vals{4,2});

figure
contourf(X,Y,-C,-[C02 C02]);
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
plot(y(:,1),y(:,2))
plot(y(1,1),y(1,2),'*k')
```

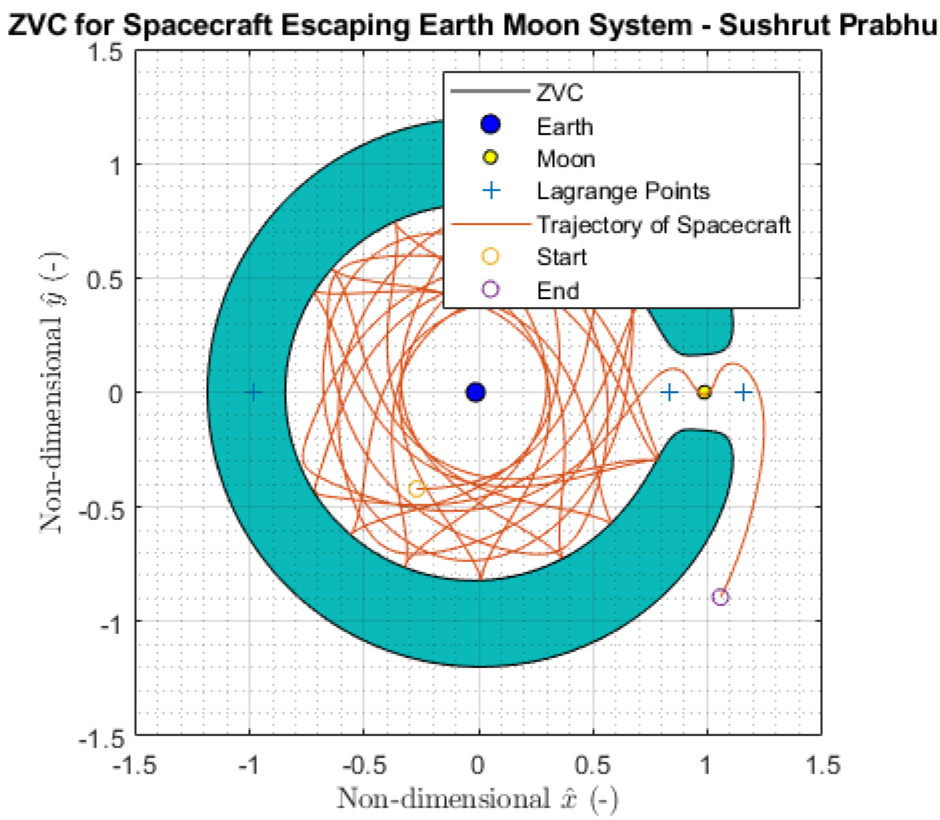
```
plot(y(end,1),y(end,2),'ok')
grid on
grid minor
axis square
title('ZVC for given Closed Primaries - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points','Trajectory of
 Spacecraft')
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points','Trajectory of
 Spacecraft','Start','End')
```



Part (c)

```
Cgate = 3.1;
v_mag = sqrt( r(1)^2 + r(2)^2 + 2*(1-dim_vals{4,2})/dd +
 2*dim_vals{4,2}/rr - Cgate);

v = v_mag*[1 0 0];

IC = [r,v];
t1 = 0:.005:80;
options=odeset('RelTol',1e-12, 'AbsTol',1e-16);        % Sets
 integration tollerance

[~,y] = ode45(@cr3bp_df,t1,IC,options,dim_vals{4,2});
```

```
figure
contourf(X,Y,-C,-[Cgate Cgate]);
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot([dim_vals{6,2},dim_vals{10,2},dim_vals{14,2}],[0,0,0],'+')
plot(y(:,1),y(:,2))
plot(y(1,1),y(1,2),'o')
plot(y(end,1),y(end,2),'o')
grid on
grid minor
axis square
title('ZVC for Spacecraft Escaping Earth Moon System - Sushrut Prabhu
 ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('ZVC', 'Earth', 'Moon', 'Lagrange Points','Trajectory of
 Spacecraft','Start','End')
```



Published with MATLAB® R2018a

Sushrut Prabhu | AAE632 PS63

PS63

Given: The linear variational equations governing ξ and η for the colinear liberation point posses 2 real and 2 imaginary roots $\lambda_2 = -\lambda_1$ $\lambda_4 = -\lambda_3$

Find a) Earth-Moon system solve linear eq and plot linear orbit about L_1 $\xi_0 = 0.01$ $\eta_0 = 0$.

ξ_0 in dimensional?

b) Max distance to L_1 in km? Period in days?

c) Find corresponding value of C and plot ZVC in x-y?

d) Use IC in non-linear algorithm. Compare linear and exact. What if ξ_0 is smaller/larger. How large can ξ_0 be?

e) Repeat similar analysis but for real roots. Is departure hyperbolic? How fast, time constant?

Solution:

The characteristic equation for colinear points is

$$\lambda^4 + (4 - U^*_{xx} - U^*_{yy})\lambda^2 + U^*_{xx} U^*_{yy} = 0 \qquad ①$$

$$U^*_{xx} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)(x+\mu)^2}{d^5} + \frac{3\mu(x-1+\mu)^2}{r^5}$$

$$U^*_{yy} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)y^2}{d^5} + \frac{3\mu y^2}{r^5} \qquad y = 0 \text{ colinear}$$

$$r = [(x+\mu)^2 + y^2 + z^2]^{1/2} \qquad d = [(x+\mu-1)^2 + y^2 + z^2]^{1/2}$$

Rewrite ①

$$\Lambda^2 + 2\beta_1\Lambda - \beta_2^2 = 0$$

$$\beta_1 = 2 - \frac{U^*_{xx} + U^*_{yy}}{2} \qquad \beta_2^2 = -U^*_{xx}U^*_{yy} > 0 \qquad \lambda = \pm\sqrt{\Lambda}$$

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Continued...

So $\Lambda_1 = -\beta_1 + (\beta_1^2 + \beta_2^2)^{1/2} \rightarrow$ Real $\quad \lambda_{1,2} = \pm\sqrt{\Lambda_1}$

$\Lambda_2 = -\beta_1 - (\beta_1^2 + \beta_2^2)^{1/2} \rightarrow$ Imag $\quad \lambda_{3,4} = \pm\sqrt{\Lambda_2}$

Solution to CE:

x direction $\quad$ y-direction

$$\xi = \sum_{i=1}^{4} A_i e^{\lambda_i t} \qquad \eta = \sum_{i=1}^{4} B_i e^{\lambda_i t}$$

$$\ddot{\xi} - 2\dot{\eta} = U^*_{xx}\,\xi + U^*_{xy}\,\eta$$
$$\ddot{\eta} + 2\dot{\xi} = U^*_{yx}\,\xi + U^*_{yy}\,\eta$$

$$\therefore \lambda_i^2 A_i e^{\lambda_i t} - 2\lambda_i B_i e^{\lambda_i t} = U^*_{xx} A_i e^{\lambda_i t} + U^*_{xy} B_i e^{\lambda_i t}$$
$$\lambda_i^2 B_i e^{\lambda_i t} + 2\lambda_i A_i e^{\lambda_i t} = U^*_{yx} A_i e^{\lambda_i t} + U_{yy} e^{\lambda_i t}$$

$$\therefore (\lambda_i^2 - U^*_{xx})A_i - (2\lambda_i + U^*_{xy})B_i = 0$$
$$(\lambda_i^2 - U^*_{yy})B_i + (2\lambda_i + U^*_{yx})A_i = 0$$

$$\left(\frac{\lambda_i^2 - U_{xx}}{2\lambda_i}\right)A_i = B_i \longrightarrow B_i = \alpha_i A_i$$

$$\therefore \xi = \boxed{A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}} + A_3 e^{\lambda_3 t} + A_4 e^{\lambda_4 t}$$
$$\eta = \boxed{\alpha_1 A_1 e^{\lambda_1 t} + \alpha_2 A_2 e^{\lambda_2 t}} + \alpha_3 A_3 e^{\lambda_3 t} + \alpha_4 A_4 e^{\lambda_4 t}$$

Note we want oscillatory motion so we do not want effect from λ_1 and λ_2 $\rightarrow \therefore A_1 = A_2 = 0$

$$\therefore \xi = A_3 e^{\lambda_3 t} + A_4 e^{\lambda_4 t}$$
$$\eta = \alpha_3 A_3 e^{\lambda_3 t} + \alpha_4 A_4 e^{\lambda_4 t}$$

Continued...

$$\therefore \xi = \xi_0 \cos(s(t-t_0)) + \frac{\eta_0}{\beta_3} \sin(s(t-t_0))$$

$$\eta = \eta_0 \cos(s(t-t_0)) - \beta_3 \xi_0 \sin(s(t-t_0))$$

$$s = \left[\beta_1 + (\beta_1^2 + \beta_2^2)^{1/2}\right]^{1/2}$$

$$\alpha_3 = i\beta_3 \qquad \beta_3 = \frac{s^2 + U_{xx}^*}{2s}$$

$$\dot{\xi}_0 = \frac{\cancel{\eta_0}^{\,0} s}{\beta_3} = 0 \qquad \dot{\eta}_0 = -\beta_3 \xi_0 s$$

$\boxed{\text{See Figure: C3.1 and C3.2 the plot of linear trajectory}}$

$$\mathcal{E}_0 = \mathcal{E}_0 \times l^* = \boxed{3844 \text{ km}}$$

b) $$d_{max} = \sqrt{\xi^2 + \eta^2} = 0.036$$

$$\therefore d_{max} = 0.036 \times l^* = \boxed{1.3787 \times 10^4 \text{ km}}$$

$$P_{ND} = \frac{2\pi}{s} = 2.6776$$

$$P_D = \mathbb{P} \times t^* = 1.0046 \times 10^6 \text{ s} = \boxed{11.6274 \text{ days}}$$

c) $\boxed{\text{See Figure: C3.1 for the plot of ZVC}}$

$$C = x^2 + y^2 + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} - v^2$$

$$v^2 = \dot{\xi}_0^2 + \dot{\eta}_0^2$$

$$d = [(x+\mu)^2 + y^2 + z^2]^{1/2} \qquad r = [(x+\mu-1)^2 + y^2 + z^2]^{1/2}$$

$$x = L_1 + \mathcal{E}_0 \qquad y = \eta_0 = 0$$

$$\therefore \boxed{C = 3.1825} \qquad \boxed{\text{All plots made for 1 period}}$$

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Continued...

d) You can see in the plot that it is not a good approximation. See Figure: C3.1 and C3.2
But how long is this? I plotted another plot against time See Figure: C3.3. It appears that for about 5 days.

See the plot for $\xi_0 = 0.017$, 0.005, 0.001
The smaller the ξ_0 the better the solutions. This is because the L_1 point behaves more linear as it is an equilibrium point.

It never creates a closed loop, it comes close to making a loop at $\xi_0 = 10^{-6}$ $\underline{\xi_0 = 384.4m}$ a very small distance. See Figure: C3.10, I also plotted other ξ_0 values See Figures: C3.4, C3.5, C3.6, C3.7, C3.8 & C3.9

e) Now we want to get the solution for λ_1 and λ_2 and suppress $A_3 = A_4 = 0$

$\therefore$ From the notes

$$\xi = \xi_0 \cosh(\lambda_1(t-t_0)) + \frac{\eta_0}{\alpha_1} \sinh(\lambda_1(t-t_0))$$

$$\eta = \eta_0 \cosh(\lambda_1(t-t_0)) + \xi_0 \alpha_1 \sinh(\lambda_1(t-t_0))$$

$$\lambda_1 = \left[-\beta_1 + (\beta_1^2 + \beta_2^2)\right]^{1/2}$$ Note we defined β_1 and β_2 earlier.

$$\alpha_i = \left(\frac{\lambda_i - U^*_{xx}}{2\lambda_i}\right) \quad \therefore \alpha_1 = \left(\frac{\lambda_1 - U^*_{xx}}{2\lambda_1}\right)$$

Continued...

$$\dot{\xi}_0 = \lambda_1 \frac{\xi_0}{\lambda} \sinh(\overset{\to 0}{0}) + \frac{\eta_0 \lambda_1}{\alpha_1} \cosh(\overset{\to 1}{0}) = \frac{\eta_0 \lambda_1}{\alpha_1} = 0$$

$$\dot{\eta}_0 = \lambda \eta_0 \sinh(\overset{\to 0}{0}) + \lambda \xi_0 \alpha_1 \cosh(\overset{\to 1}{0}) = \xi_0 \alpha_1 \lambda_1 = \boxed{-0.0418}$$

Note the departure is hyperbolic so I plotted the linear and exact solution for $t = 1$ (non dim) this is over 4 days $t = t \times t^* = 4.34$ days. The solution diverge quickly and hit the ZVC passing into restricted zone.

$\boxed{\text{The departure is hyperbolic See Figures C3.11 and C3.12}}$

The departure can be characterized

$$y = Ae^{kt} \rightarrow \tau = \frac{1}{k} \quad \text{so} \quad \tau = \frac{1}{\lambda}$$

$$\therefore \boxed{\tau = 2.9321 \text{ (non-dim)}}$$

τ, is time to grow by another factor of e OR time taken to reach asymptote at current rate of change in y direction. OR time taken to diverge 63% from linear solution.

PSC3

Part a/b/c/d)

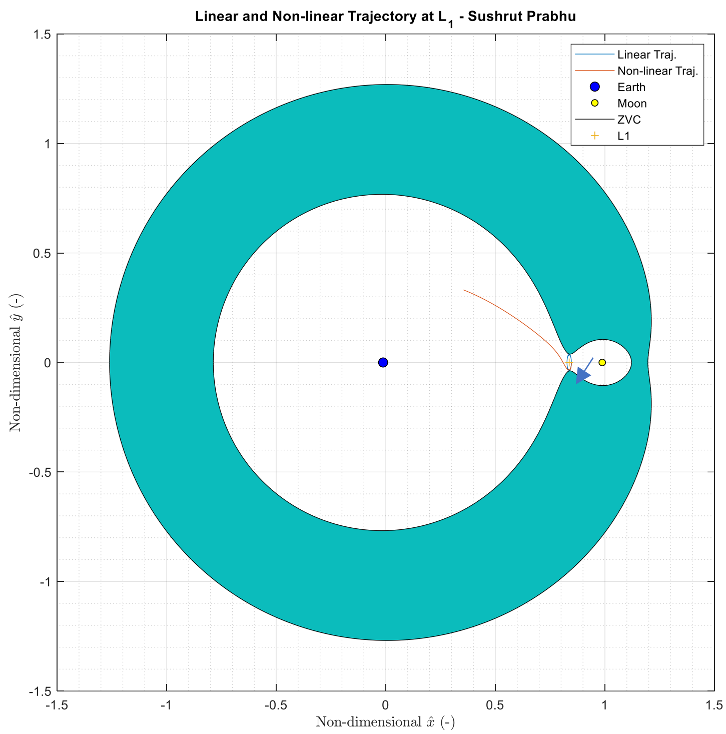


Figure C3.1: Linear and non-linear solution at L1.

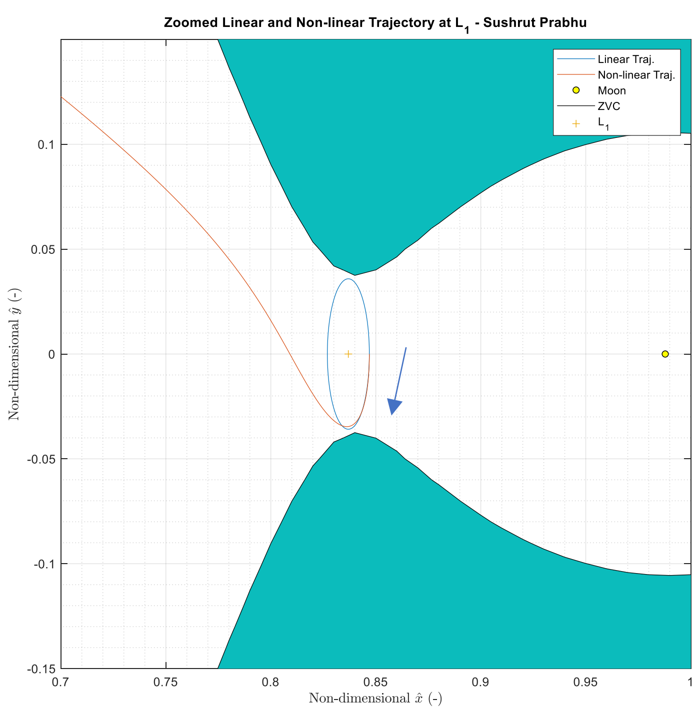


Figure C3.2: Linear and non-linear solution at L1 zoomed in.

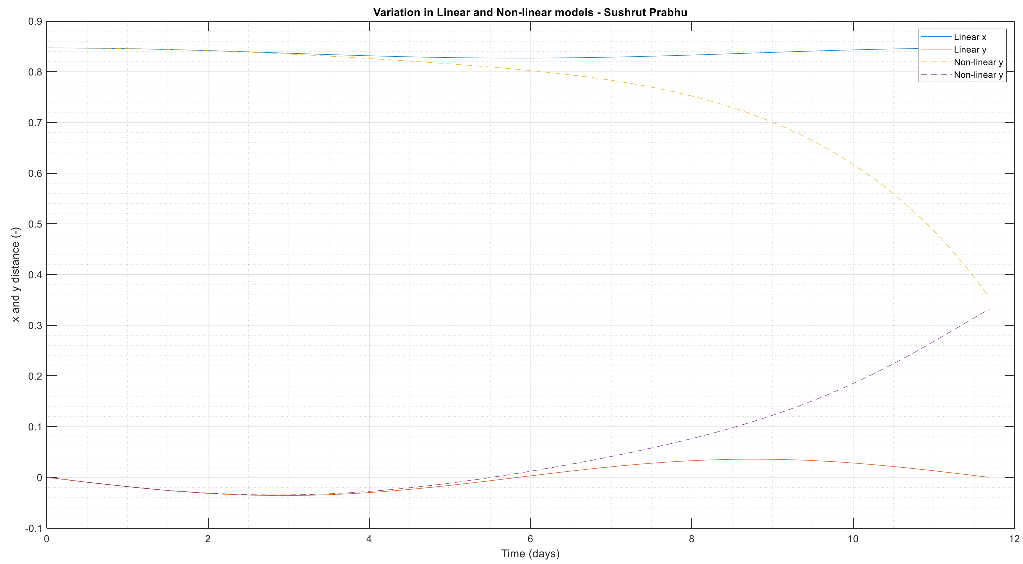


Figure C3.3: Time reference for the linear and non-linear solution at L1

Part d)

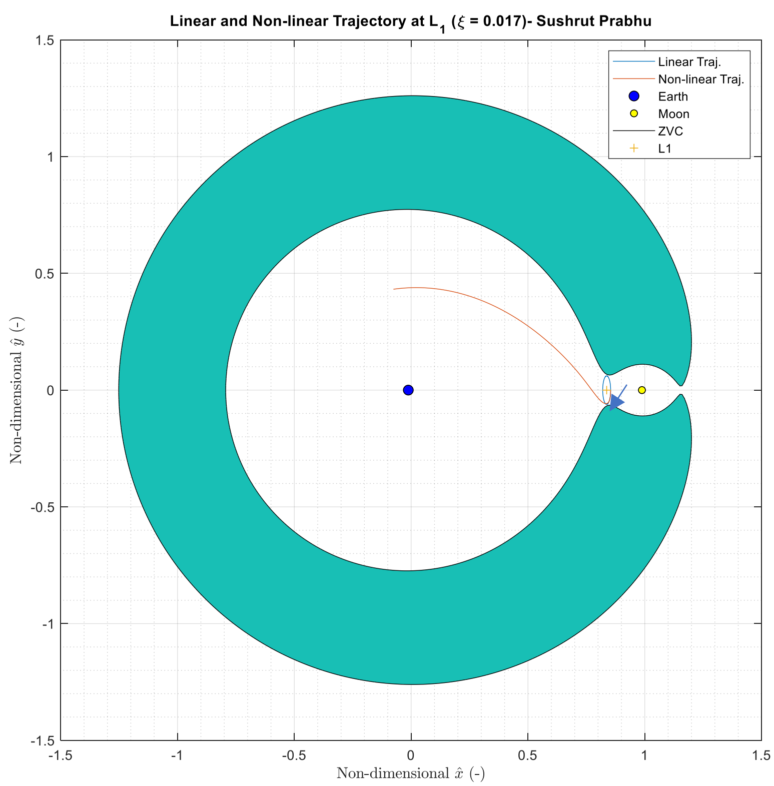


Figure C3.4: Linear and non-linear solution at L1 with a C that just opens L2.

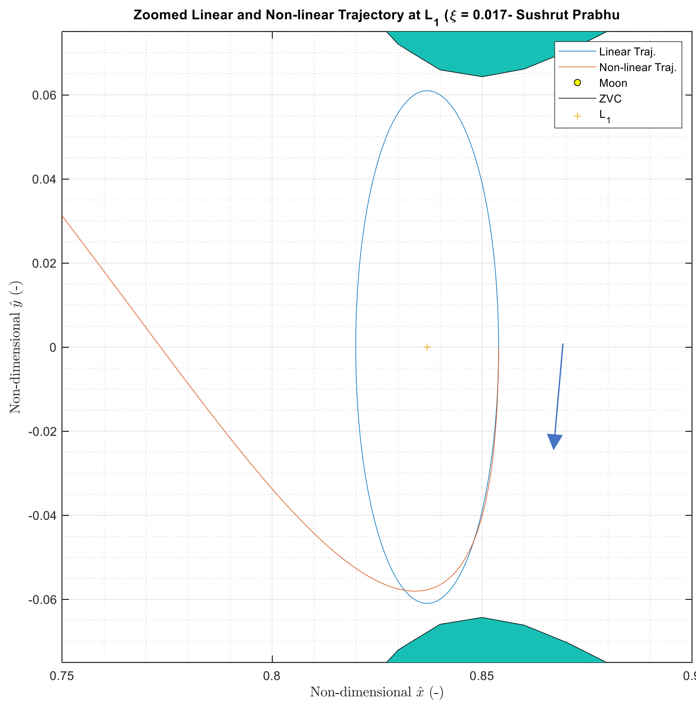


Figure C3.5: Linear and non-linear solution at L1 with a C that just opens L2

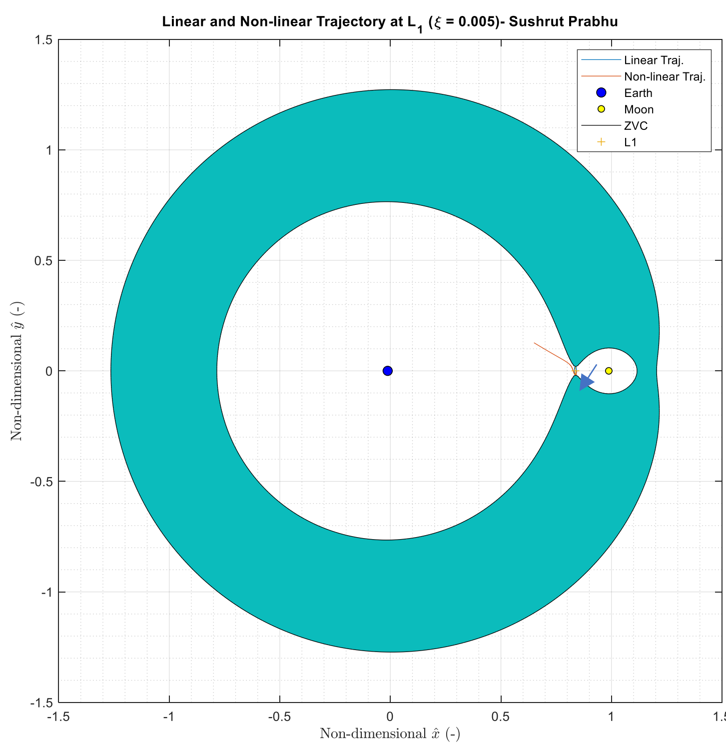


Figure C3.6: Linear and non-linear solution at L1 with $\xi_0 = 0.005$

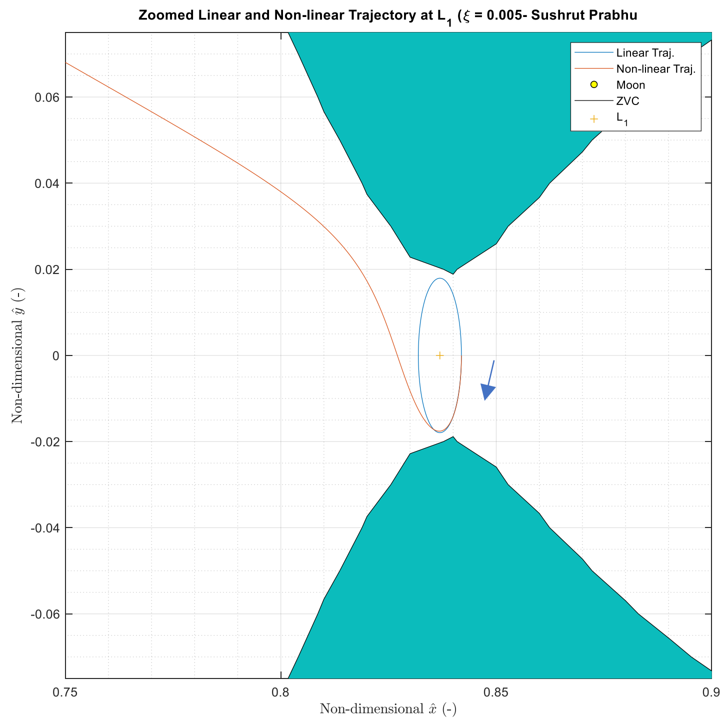


Figure C3.7: Linear and non-linear solution at L1 with $\xi_0 = 0.005$ zoomed

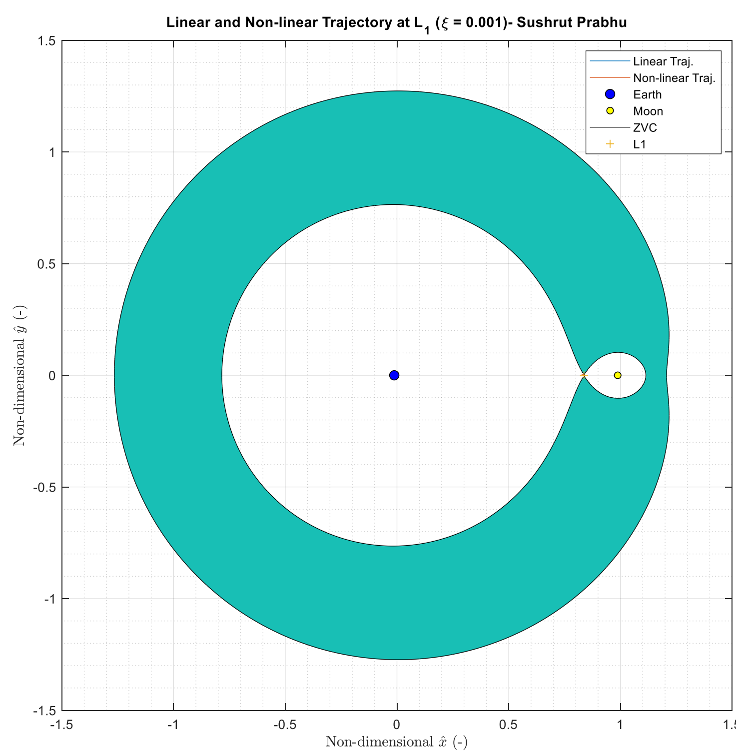


Figure C3.8: Linear and non-linear solution at L1 with $\xi_0 = 0.001$

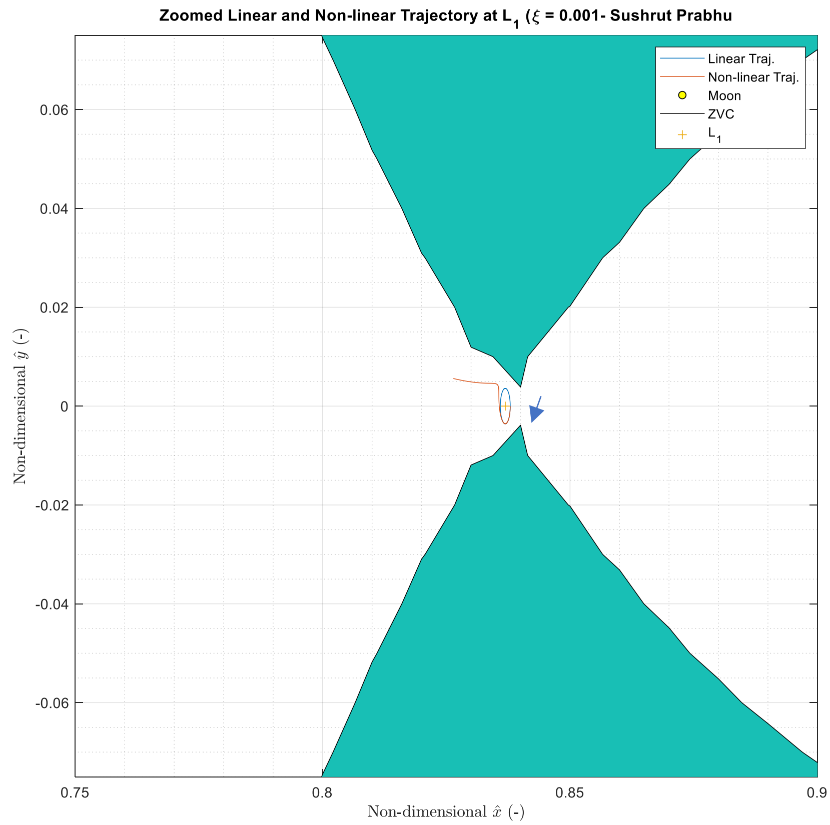


Figure C3.9: Linear and non-linear solution at L1 with $\xi_0 = 0.001$ zoomed

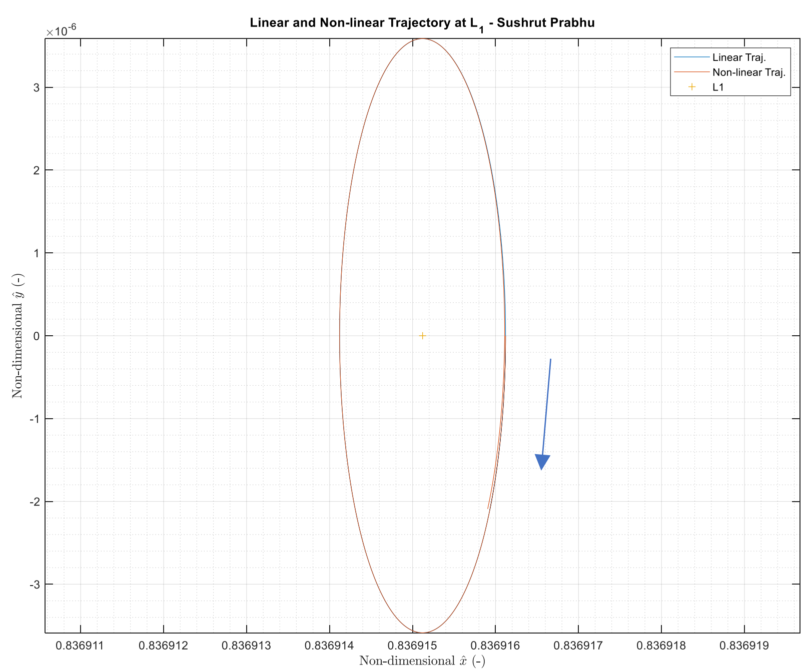


Figure C3.10: Linear and non-linear solution at L1 with $\xi_0 = 0.000001$.

Part e)

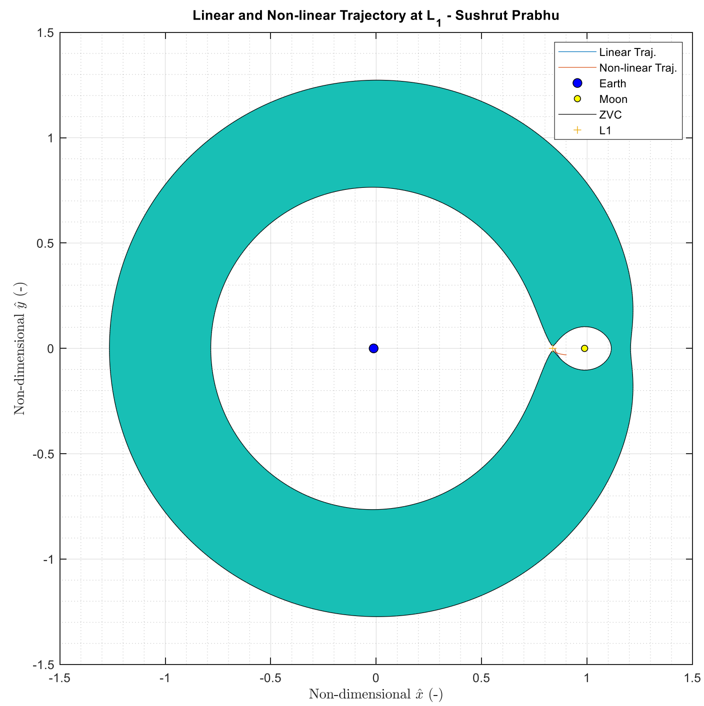


Figure C3.11: Linear and non-linear solution at L1 with hyperbolic motion.

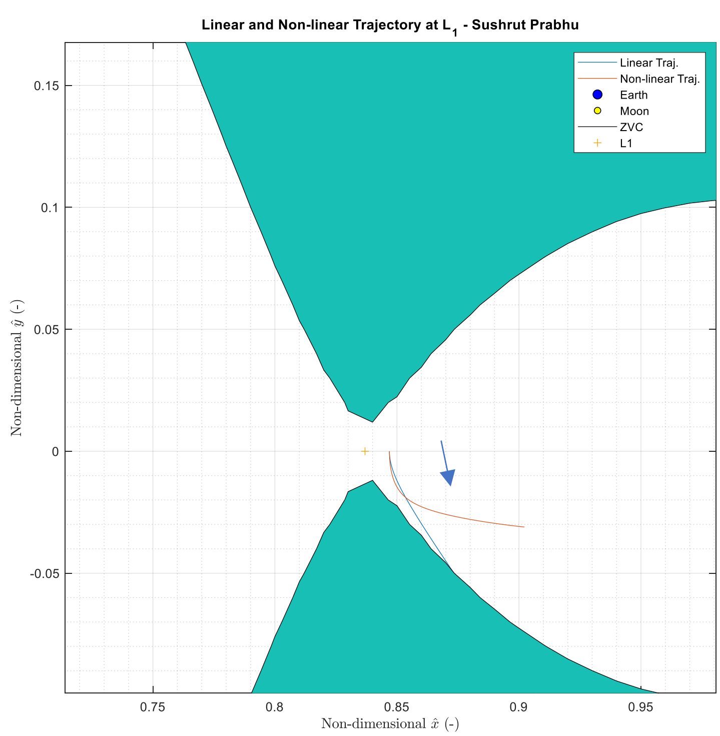


Figure C3.12: Linear and non-linear solution at L1 with hyperbolic motion.

Figure C3.13: Linear and non-linear solution at L1 with hyperbolic motion $\xi_0 = 0.001$.

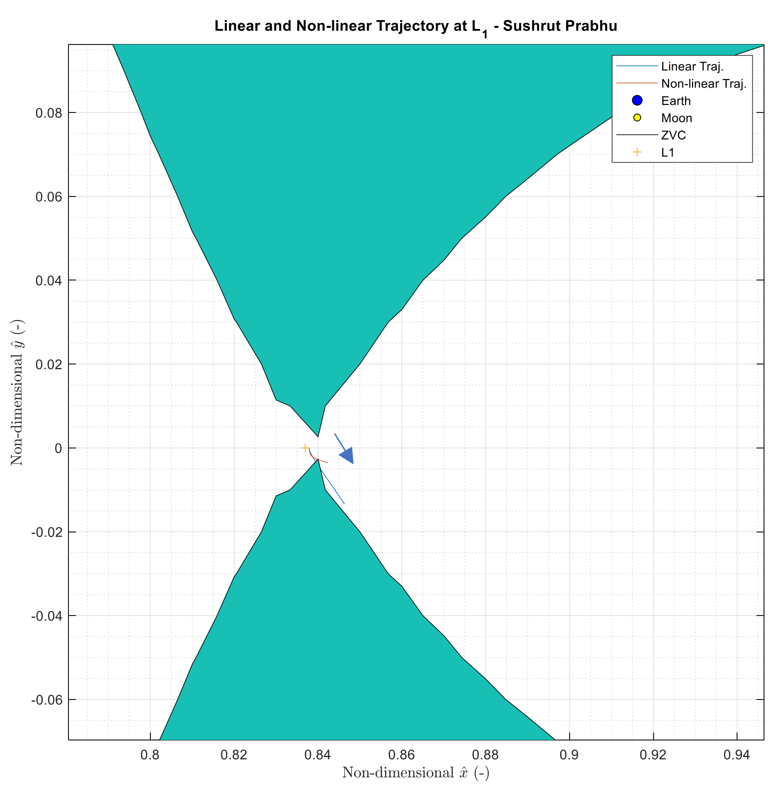


Figure C3.14: Linear and non-linear solution at L1 with hyperbolic motion $\xi_0 = 0.001$.

Table of Contents

PSC3

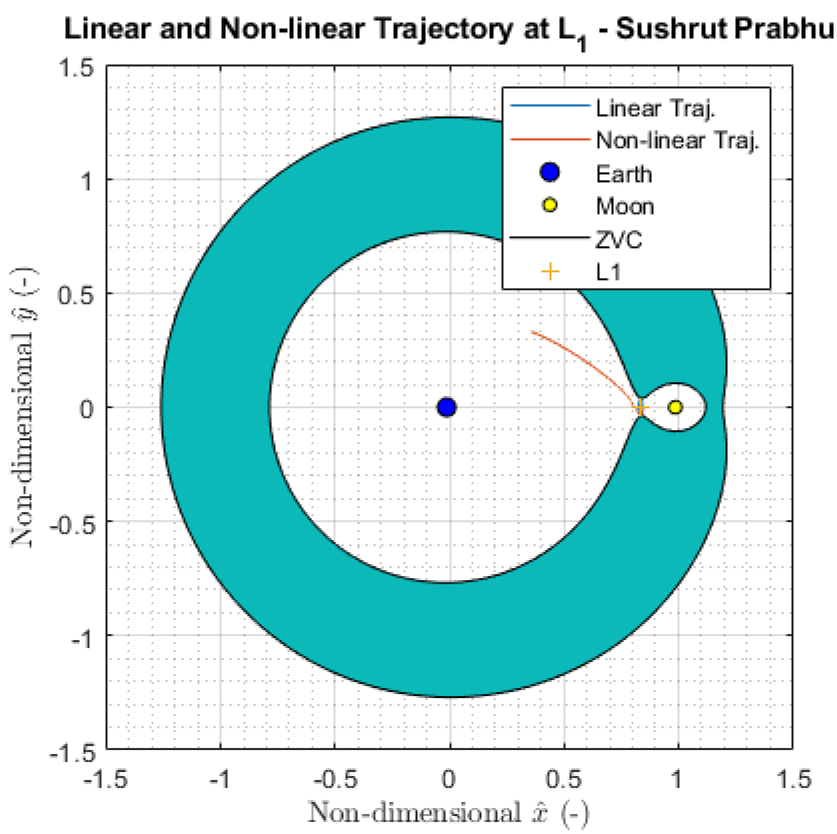
```
clear
close all
clc
```

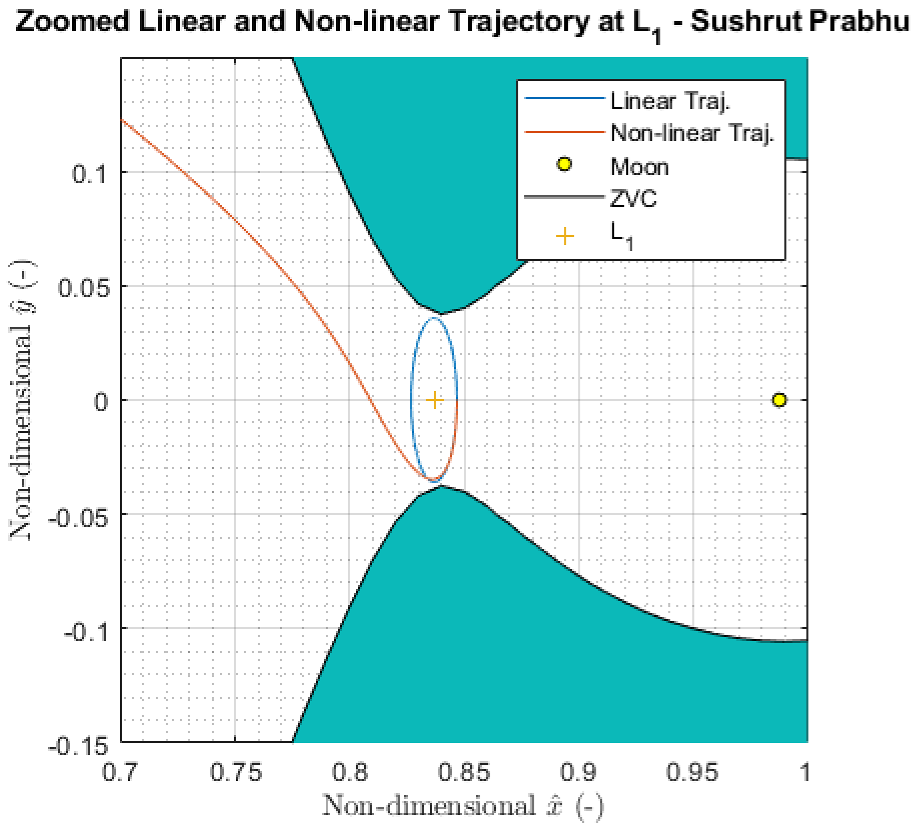
PSC3

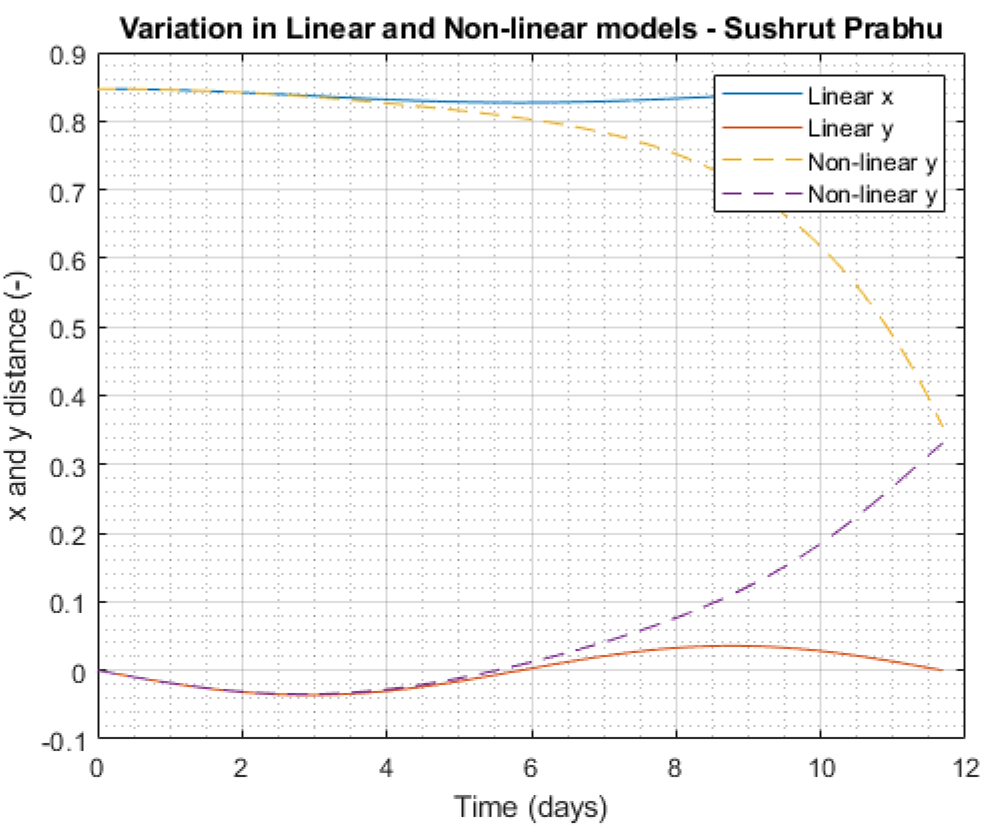
```
SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' ,'t* (s)','gamma_1 (-)', 'L_1
 (-)', 'gamma_1 (km)','L_1 (km)'};
G = 6.6738*10^-20;
xi_0 = 0.01;
eta_0 = 0;

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% Solution
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;

% Lagrange 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-10));
dim_vals{7,2} = 1-dim_vals{6,2}-dim_vals{4,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};

% dd and rr
dd = abs(dim_vals{4,2}+dim_vals{7,2});
rr = abs(dim_vals{4,2}+dim_vals{7,2}-1);

% Potentials
[Uxx,Uyy,~,~,~,~]= Unn(dim_vals{7,2},0,0,dim_vals{4,2});


beta1 = 2 - (Uxx+Uyy)/2;
beta2 = sqrt(-Uxx*Uyy);
```

```
s = sqrt(beta1 + sqrt(beta1^2 + beta2^2));
beta3 = (s^2+Uxx)/2/s;
eta_dot0 = -beta3*xi_0*s;
xi_dot0 = eta_0*s/beta3;

% Trajectory
t = 0:.001:2*pi/s;
t = t';

[xi, eta] = colinlgrange(xi_0,eta_0,s,beta3,t);
xx = xi + dim_vals{7,2};

% Max distance
max_d = max(sqrt(xi.^2 + eta.^2))*dim_vals{2,2};

% Period
Per = 2*pi/s * dim_vals{5,2};
Per_d = Per/3600/24;

% ZVC
[X,Y] = meshgrid(-1.5:0.01:1.5);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(xx(1),eta(1),0,norm([xi_dot0,eta_dot0]),dim_vals{4,2});
r = [xx(1), 0, 0];
v = [xi_dot0, eta_dot0, 0];

IC = [r,v];
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

figure
plot(xx,eta)
hold on
plot(y(:,1),y(:,2))
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
contourf(X,Y,-C,-[C0 C0]);
plot(dim_vals{7,2},0,'+')
grid on
grid minor
title('Linear and Non-linear Trajectory at L_1 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear Traj.', 'Non-linear
 Traj.', 'Earth', 'Moon', 'ZVC', 'L1')
axis equal

figure
plot(xx,eta)
hold on
```

```
plot(y(:,1),y(:,2))
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
contourf(X,Y,-C,-[C0 C0]);
plot(dim_vals{7,2},0,'+')
grid on
grid minor
title('Zoomed Linear and Non-linear Trajectory at L_1 - Sushrut
 Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear Traj.', 'Non-linear Traj.', 'Moon', 'ZVC','L_1')
xlim([0.7 1])
ylim([-0.15 .15])
axis square

figure
plot(t*dim_vals{5,2}/3600/24,[xx,eta],'-')
hold on
plot(t*dim_vals{5,2}/3600/24,[y(:,1),y(:,2)],'--')
legend('Linear x','Linear y','Non-linear y','Non-linear y')
xlabel('Time (days)')
ylabel('x and y distance (-)')
grid on
grid minor
title('Variation in Linear and Non-linear models - Sushrut Prabhu')
```



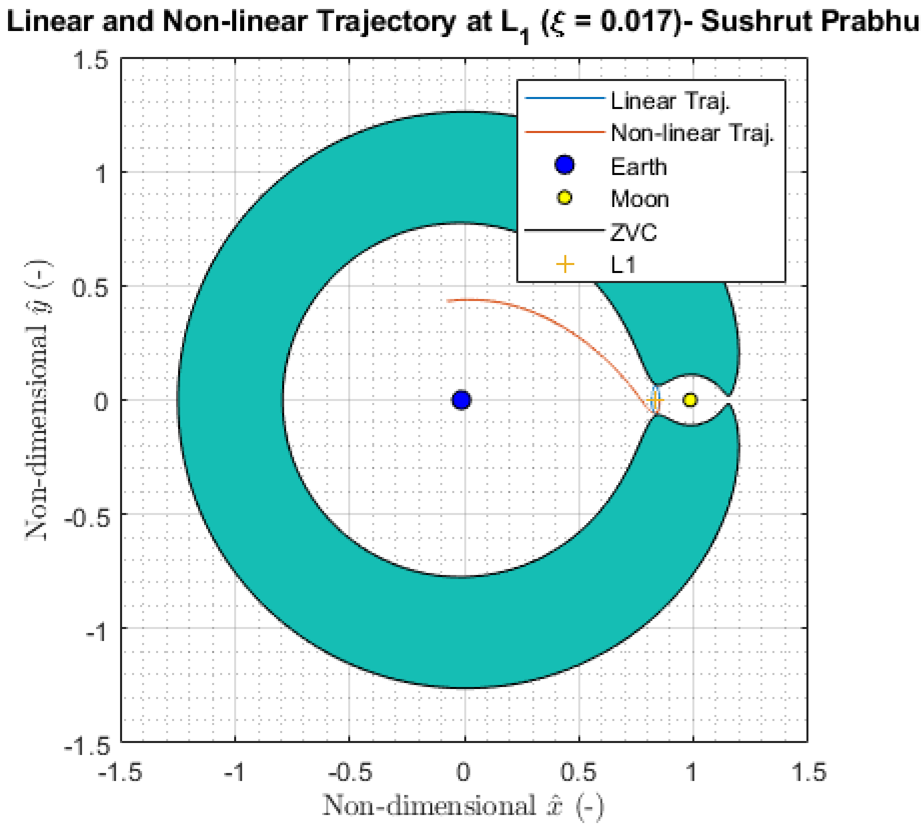

Zoomed Linear and Non-linear Trajectory at L1 - Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Moon
ZVC
L1
0.1
0.05
0
-0.05
-0.1
-0.15
0.7
0.75
0.8
0.85
0.9
0.95
1
Non-dimensional ŷ (-)
Non-dimensional x̂ (-)


Variation in Linear and Non-linear models - Sushrut Prabhu
Linear x
Linear y
Non-linear y
Non-linear y
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
2
4
6
8
10
12
x and y distance (-)
Time (days)

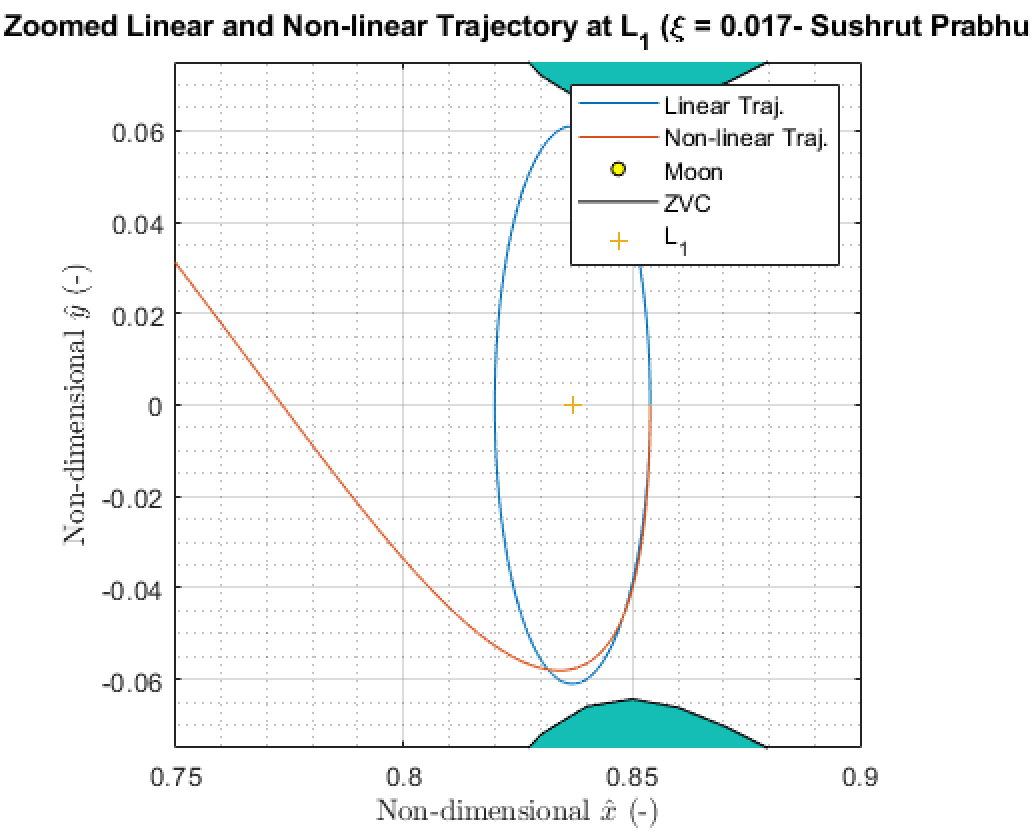
Part d

```
for xi_0 = [0.017 0.005 0.001]
    % dd and rr
    dd = abs(dim_vals{4,2}+dim_vals{7,2});
    rr = abs(dim_vals{4,2}+dim_vals{7,2}-1);

    % Potentials
    Uxx = 1 - (1-dim_vals{4,2})/dd^3 - dim_vals{4,2}/rr^3 +
 3*(1-dim_vals{4,2})*(dim_vals{4,2}+dim_vals{7,2})^2 / dd^5 +
 3*dim_vals{4,2}*(dim_vals{4,2}+dim_vals{7,2}-1)^2 / rr^5;
    Uyy = 1- (1-dim_vals{4,2})/dd^3 - dim_vals{4,2}/rr^3;

    beta1 = 2 - (Uxx+Uyy)/2;
    beta2 = sqrt(-Uxx*Uyy);

    s = sqrt(beta1 + sqrt(beta1^2 + beta2^2));
    beta3 = (s^2+Uxx)/2/s;
    eta_dot0 = -beta3*xi_0*s;
    xi_dot0 = eta_0*s/beta3;

    % Trajectory
    t = 0:.001:2*pi/s;
    t = t';

    [xi, eta] = colinlgrange(xi_0,eta_0,s,beta3,t);
    xx = xi + dim_vals{7,2};

    % Max distance
    max_d = max(sqrt(xi.^2 + eta.^2))*dim_vals{2,2};

    % Period
    Per = 2*pi/s * dim_vals{5,2};
    Per_d = Per/3600/24;

    % ZVC
    [X,Y] = meshgrid(-1.5:0.01:1.5);
    C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

    C0 =
 Jacobi_C(xx(1),eta(1),0,norm([xi_dot0,eta_dot0]),dim_vals{4,2});
    r = [xx(1), 0, 0];
    v = [xi_dot0, eta_dot0, 0];

    IC = [r,v];
    options=odeset('RelTol',1e-12, 'AbsTol',1e-16); % Sets integration
 tollerance

    [~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

    figure
    plot(xx,eta)
    hold on
```

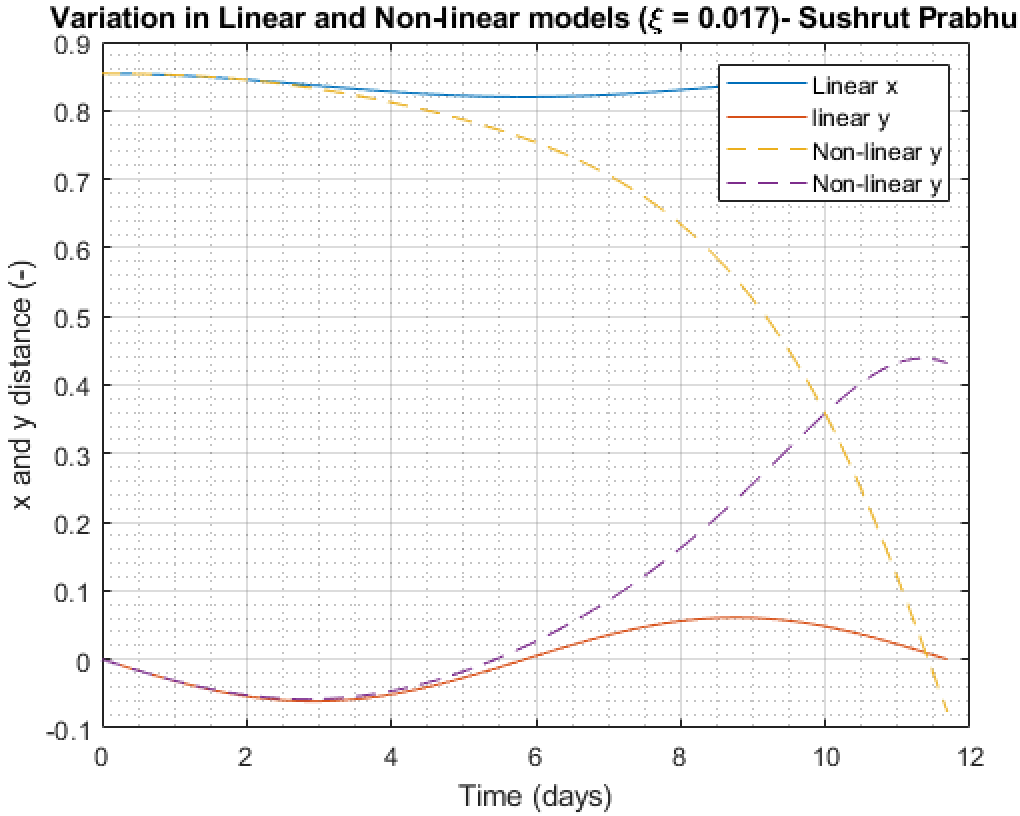
```
    plot(y(:,1),y(:,2))
    plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
    plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
    contourf(X,Y,-C,-[C0 C0]);
    plot(dim_vals{7,2},0,'+')
    grid on
    grid minor
    title(['Linear and Non-linear Trajectory at L_1 (\xi = '
 num2str(xi_0) ')- Sushrut Prabhu'])
    xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
    ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
    legend('Linear Traj.', 'Non-linear
 Traj.', 'Earth', 'Moon', 'ZVC', 'L1')
    axis equal

    figure
    plot(xx,eta)
    hold on
    plot(y(:,1),y(:,2))
    plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
    contourf(X,Y,-C,-[C0 C0]);
    plot(dim_vals{7,2},0,'+')
    grid on
    grid minor
    title(['Zoomed Linear and Non-linear Trajectory at L_1 (\xi = '
 num2str(xi_0) '- Sushrut Prabhu'])
    xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
    ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
    legend('Linear Traj.', 'Non-linear Traj.', 'Moon', 'ZVC','L_1')
    xlim([0.75 .9])
    ylim([-0.075 .075])
    axis square

    figure
    plot(t*dim_vals{5,2}/3600/24,[xx,eta],'-')
    hold on
    plot(t*dim_vals{5,2}/3600/24,[y(:,1),y(:,2)],'--')
    legend('Linear x','linear y','Non-linear y','Non-linear y')
    xlabel('Time (days)')
    ylabel('x and y distance (-)')
    grid on
    grid minor
    title(['Variation in Linear and Non-linear models (\xi = '
 num2str(xi_0) ')- Sushrut Prabhu'])
end
```



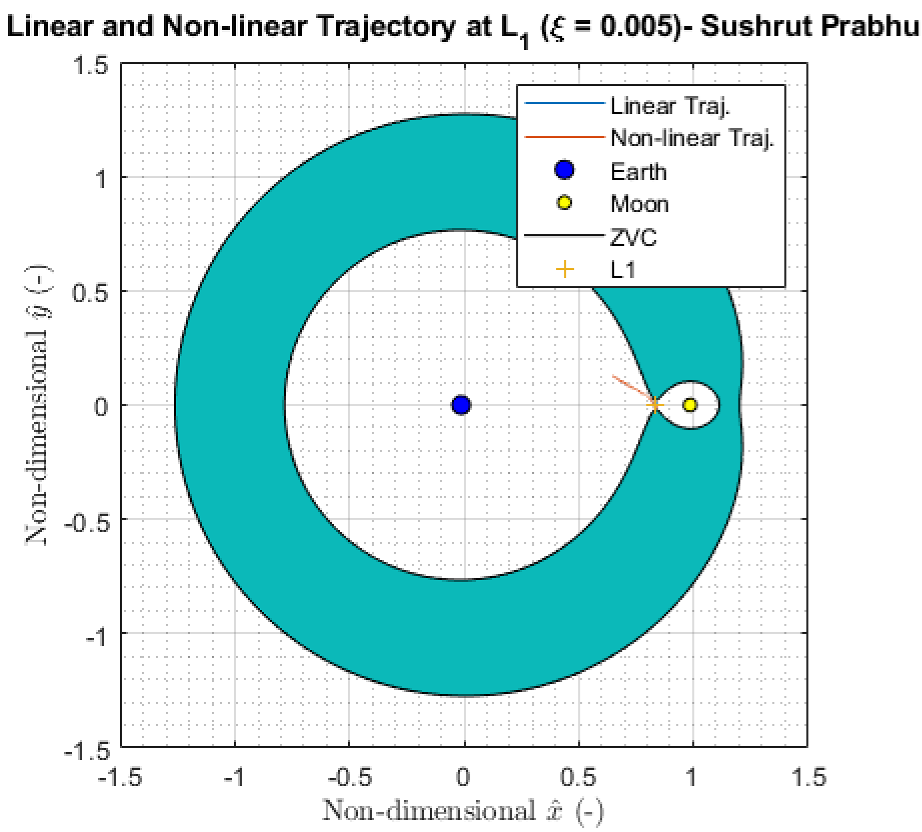
Linear and Non-linear Trajectory at L1 (ξ = 0.017)- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Earth
Moon
ZVC
L1
Non-dimensional ŷ (-)
Non-dimensional x̂ (-)



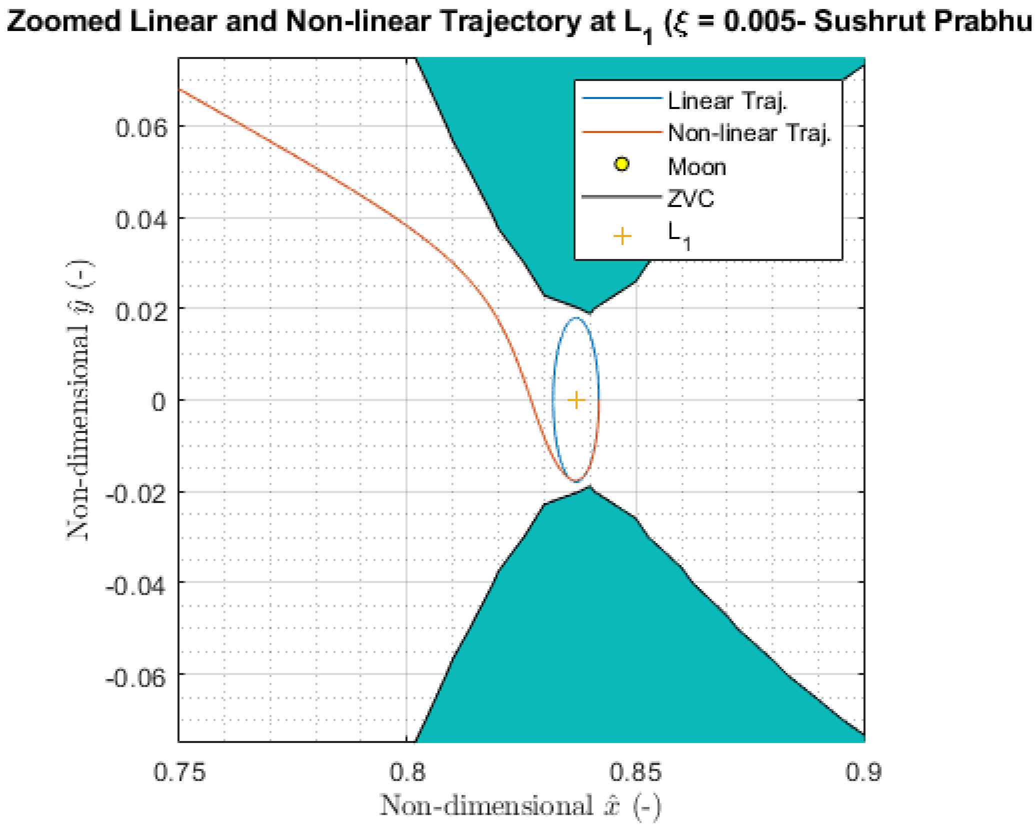
Zoomed Linear and Non-linear Trajectory at L1 (ξ = 0.017- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Moon
ZVC
L1
Non-dimensional ŷ (-)
Non-dimensional x̂ (-)



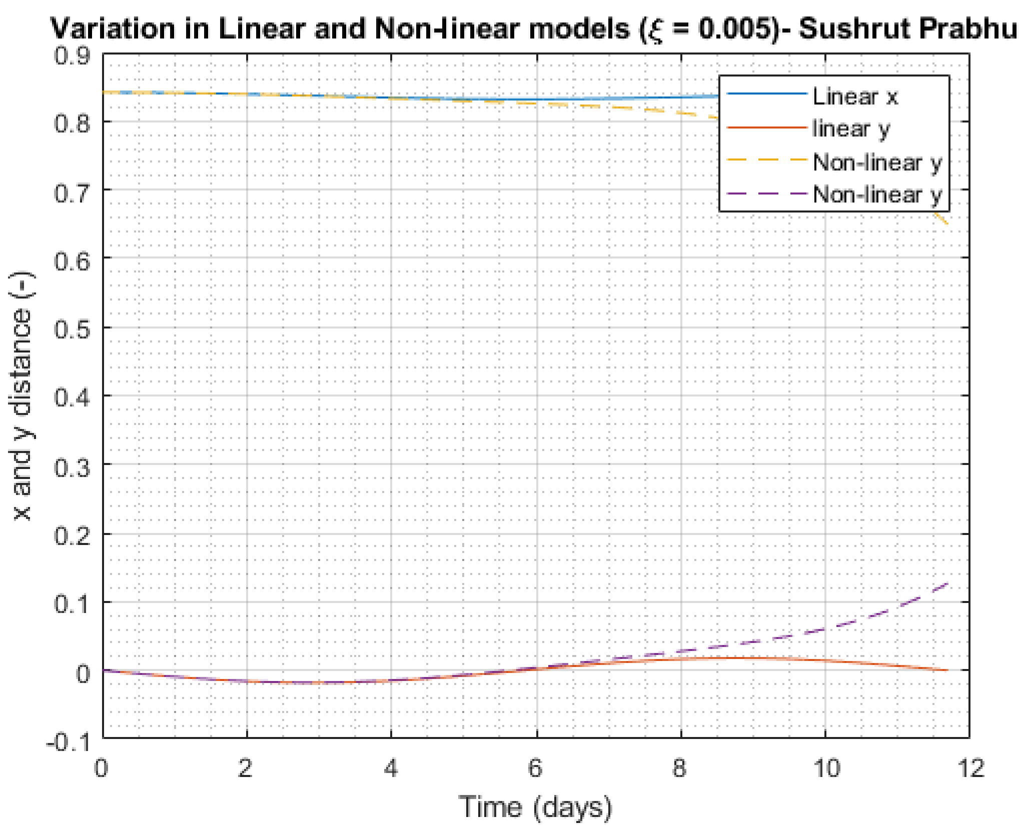
Variation in Linear and Non-linear models (ξ = 0.017)- Sushrut Prabhu
Linear x
linear y
Non-linear y
Non-linear y
x and y distance (-)
Time (days)



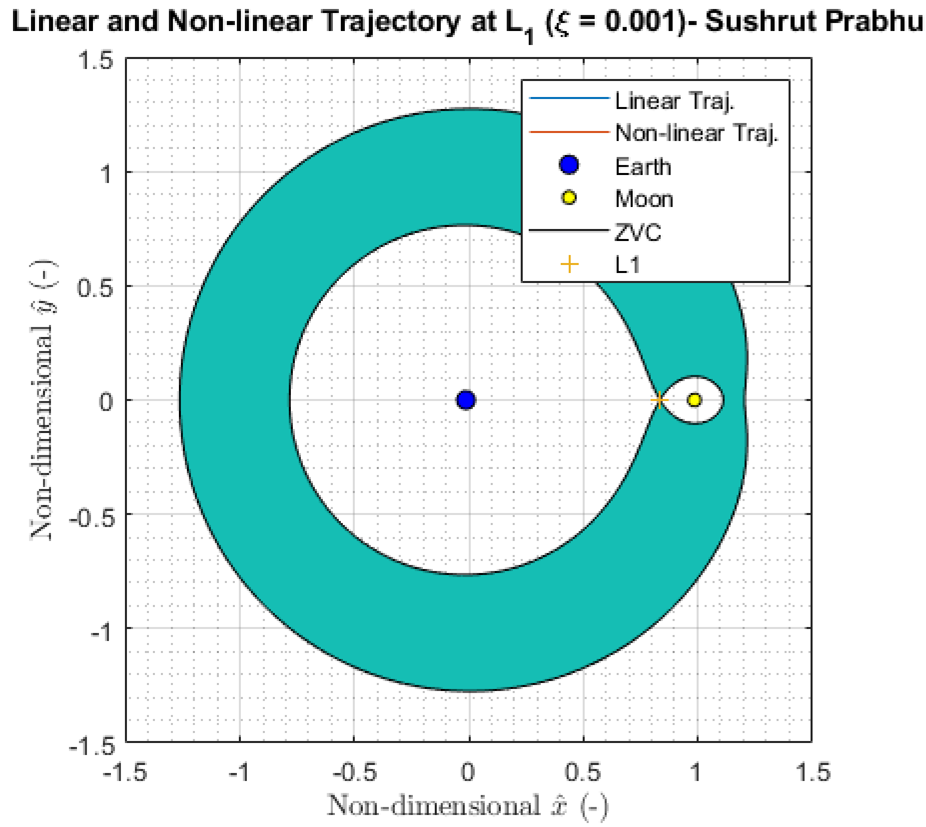
Linear and Non-linear Trajectory at L_1 (ξ = 0.005)- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Earth
Moon
ZVC
L1
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)

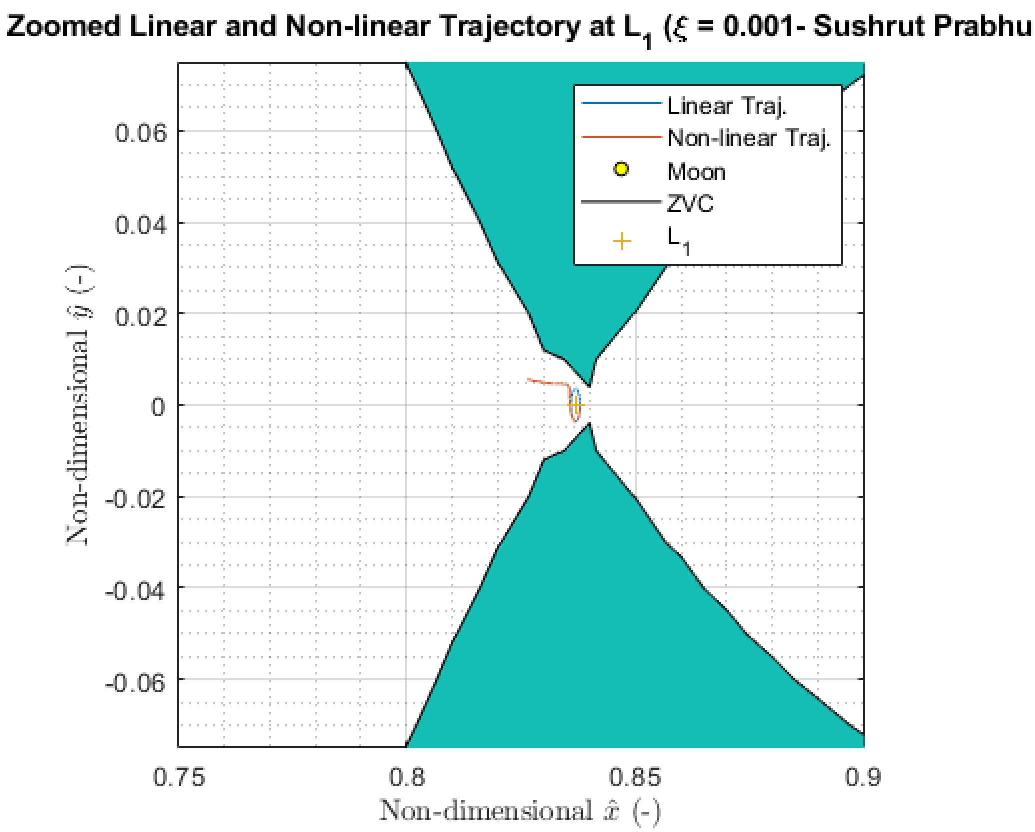


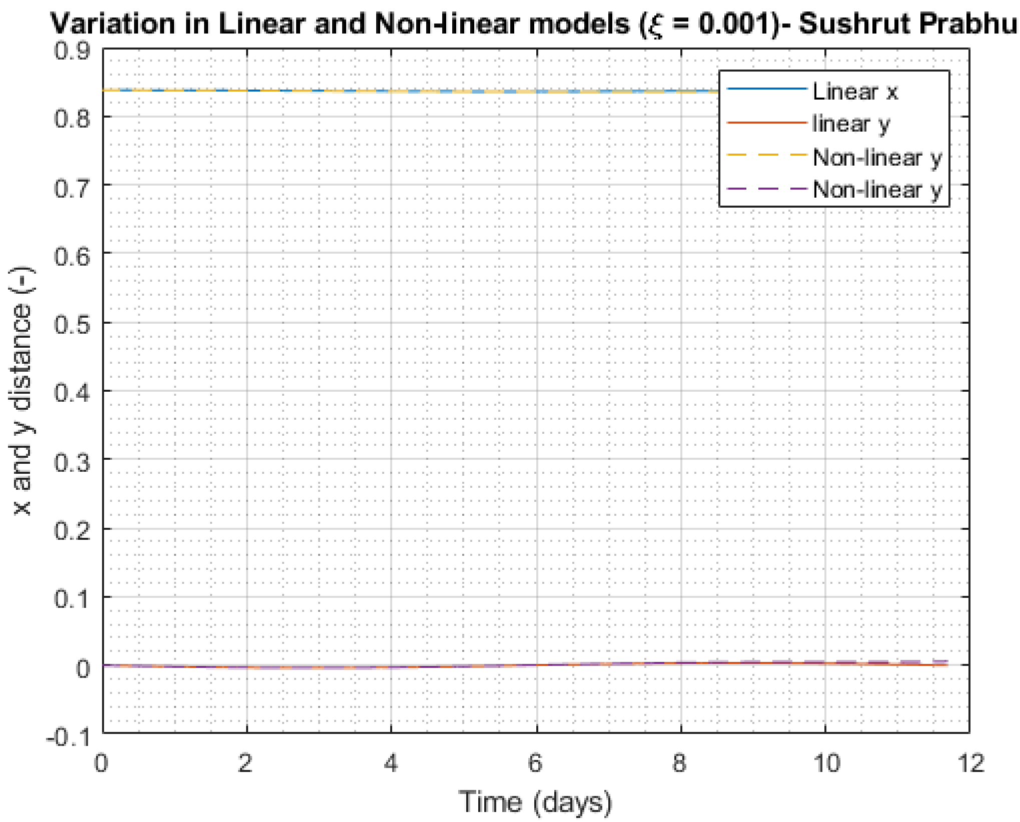
Zoomed Linear and Non-linear Trajectory at L_1 (ξ = 0.005- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Moon
ZVC
L_1
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Non-dimensional $\hat{y}$ (-)
0.75
0.8
0.85
0.9
Non-dimensional $\hat{x}$ (-)



Variation in Linear and Non-linear models (ξ = 0.005)- Sushrut Prabhu
Linear x
linear y
Non-linear y
Non-linear y
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
x and y distance (-)
0
2
4
6
8
10
12
Time (days)


Linear and Non-linear Trajectory at L_1 (ξ = 0.001)- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Earth
Moon
ZVC
L1
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
1.5
1
0.5
0
-0.5
-1
-1.5

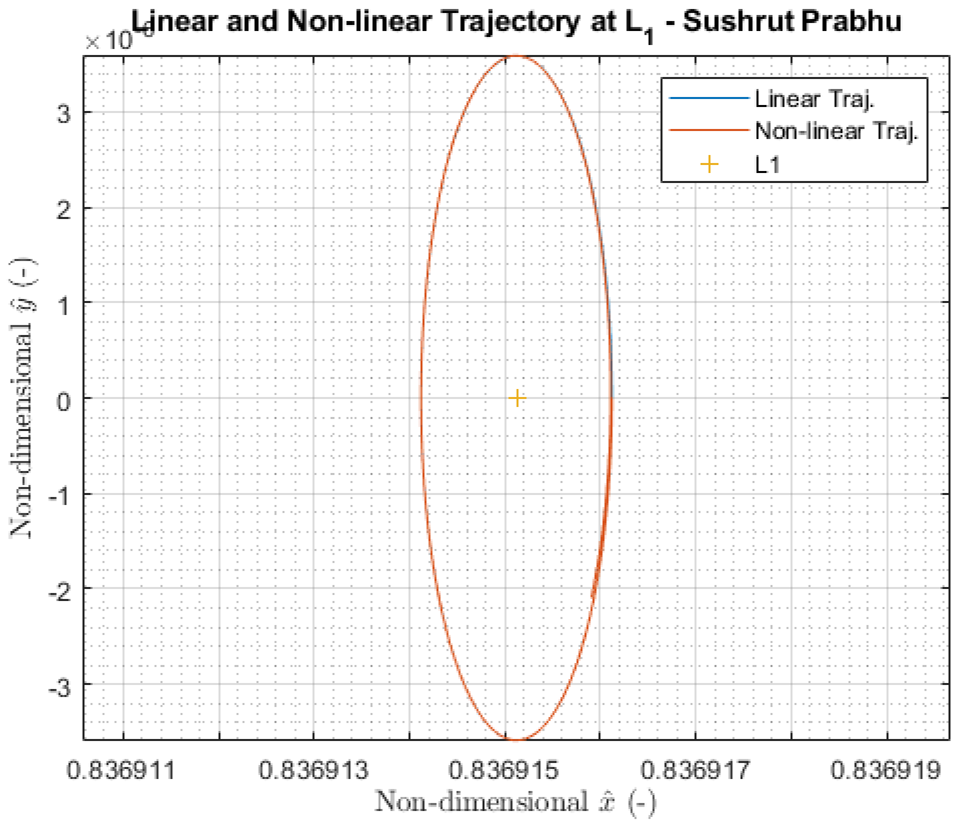

Zoomed Linear and Non-linear Trajectory at L_1 (ξ = 0.001- Sushrut Prabhu
Linear Traj.
Non-linear Traj.
Moon
ZVC
L_1
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.75
0.8
0.85
0.9



d smallest

```
xi_0 = 0.000001;

% dd and rr
dd = abs(dim_vals{4,2}+dim_vals{7,2});
rr = abs(dim_vals{4,2}+dim_vals{7,2}-1);

% Potentials
Uxx = 1 - (1-dim_vals{4,2})/dd^3 - dim_vals{4,2}/rr^3 +
 3*(1-dim_vals{4,2})*(dim_vals{4,2}+dim_vals{7,2})^2 / dd^5 +
 3*dim_vals{4,2}*(dim_vals{4,2}+dim_vals{7,2}-1)^2 / rr^5;
Uyy = 1- (1-dim_vals{4,2})/dd^3 - dim_vals{4,2}/rr^3;

beta1 = 2 - (Uxx+Uyy)/2;
beta2 = sqrt(-Uxx*Uyy);

s = sqrt(beta1 + sqrt(beta1^2 + beta2^2));
beta3 = (s^2+Uxx)/2/s;
eta_dot0 = -beta3*xi_0*s;
xi_dot0 = eta_0*s/beta3;

% Trajectory
t = 0:.001:2*pi/s*1.1;
t = t';
```

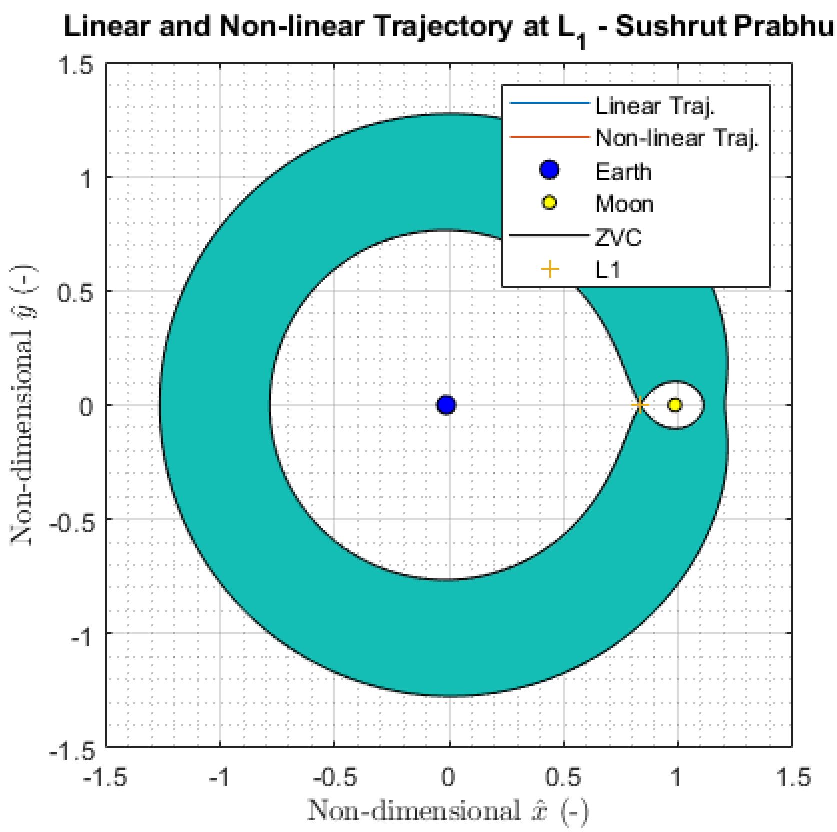
```
[xi, eta] = colinlgrange(xi_0,eta_0,s,beta3,t);
xx = xi + dim_vals{7,2};

% Max distance
max_d = max(sqrt(xi.^2 + eta.^2))*dim_vals{2,2};

% Period
Per = 2*pi/s * dim_vals{5,2};
Per_d = Per/3600/24;

% ZVC
[X,Y] = meshgrid(-1.5:0.01:1.5);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(xx(1),eta(1),0,norm([xi_dot0,eta_dot0]),dim_vals{4,2});
r = [xx(1), 0, 0];
v = [xi_dot0, eta_dot0, 0];

IC = [r,v];
options=odeset('RelTol',1e-12, 'AbsTol',1e-16); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

figure
plot(xx,eta)
hold on
plot(y(:,1),y(:,2))
plot(dim_vals{7,2},0,'+')
grid on
grid minor
title('Linear and Non-linear Trajectory at L_1 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear Traj.', 'Non-linear Traj.', 'L1')
axis equal
```



Part (e)

```
% Trajectory
t = 0:.001:1;
t = t';
xi_0 = 0.001;

lam1 = sqrt(-beta1+sqrt(beta1^2 +beta2^2));
alpha1 = (lam1-Uxx)/2/lam1;
xi_dot0 = eta_0/alpha1*lam1;
eta_dot0 = xi_0*alpha1*lam1;

xi = xi_0*cosh(lam1*t);
eta = xi_0*alpha1*sinh(lam1*t);

xx = xi + dim_vals{7,2};

r = [xx(1), 0, 0];
v = [xi_dot0, eta_dot0, 0];

IC = [r,v];
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});
```

```
% ZVC
[X,Y] = meshgrid(-1.5:0.01:1.5);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(xx(1),eta(1),0,norm([xi_dot0,eta_dot0]),dim_vals{4,2});

figure
plot(xx,eta)
hold on
plot(y(:,1),y(:,2))
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
contourf(X,Y,-C,-[C0 C0]);
plot(dim_vals{7,2},0,'+')
grid on
grid minor
title('Linear and Non-linear Trajectory at L_1 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear Traj.', 'Non-linear
 Traj.', 'Earth', 'Moon', 'ZVC', 'L1')
axis equal
```



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Sushrut Prabhu | AAE632 | PSC4

PSC4

Given: The Earth-moon system, with the LE ξ and η

Find a) Linear variational equation relative to L_4. Find roots s_1 and s_2 and general solution.

Assume you eliminate terms corresponding to long period or short period. IC $\xi = 0.01$ $\eta = 0$. What is $\dot{\xi}_0$ and $\dot{\eta}_0$?

IC for dimensional? Solve and plot linear. Period in dimensional?

Significance to long/short period?

b) Select IC for short and long frequency. Plot orbit. Are frequency commensurate? C and ZVC?

c) Use IC for non-linear and compare linear? How exact?

Compare ξ_0 by changing it.

Optional: Pluto-Charon system? What roots? Sim linear and non-lin.

Solution:

a)
$$\ddot{\xi} - 2\dot{\eta} = U^*_{xx}\xi + U^*_{xy}\eta$$
$$\ddot{\eta} + 2\dot{\xi} = U^*_{yx}\xi + U^*_{yy}\eta$$
$$\ddot{\zeta} = U^*_{zz}\zeta$$

Decouples

$$\therefore \lambda^4 + (4 - U^*_{xx} - U^*_{yy})\lambda^2 + (U^*_{xx}U^*_{yy} - U^{*2}_{xy}) = 0$$
$$\therefore \lambda^4 + \lambda^2 + \frac{27}{4}\mu(1-\mu) = 0$$
$$\Rightarrow \Lambda^2 + \Lambda + \frac{27}{4}\mu(1-\mu) = 0$$
$$\Lambda = \frac{1}{2}\left(-1 \pm [1 - 27\mu(1-\mu)]^{1/2}\right)$$

$\therefore \Lambda_1 = -0.0889$ $\quad\Lambda_2 = -0.9111$

$\lambda_{1,2} = \pm 0.2982 j$ $\quad\lambda_{3,4} = \pm 0.9545 j$

Continued...

$\therefore \boxed{s_1 = 0.298208 \qquad s_2 = 0.954501}$

$$\xi = \alpha_1 \cos s_1 t + \alpha_2 \sin s_1 t + \alpha_3 \cos s_2 t + \alpha_4 \sin s_2 t$$
$$\eta = \beta_1 \cos s_1 t + \beta_2 \sin s_1 t + \beta_3 \cos s_2 t + \beta_4 \sin s_2 t$$

Let us choose the long period

Therefore $\alpha_3 = \alpha_3 = \beta_3 = \beta_4 = 0$

$$\therefore \xi = \alpha_1 \cos s_1 t + \alpha_2 \sin s_1 t$$
$$\eta = \beta_1 \cos s_1 t + \beta_2 \sin s_1 t$$

So
$$\xi_0 = \alpha_1 \cancel{\cos(s_1 0)}^{1} + \alpha_2 \cancel{\sin(s_1 0)}^{0} = \alpha_1$$
$$\eta_0 = \beta_1 \cancel{\cos(s_1 0)}^{1} + \beta_2 \cancel{\sin(s_1 0)}^{0} = \beta_1$$

$$\dot{\xi}_0 = -\alpha_1 s_1 \cancel{\sin(s_1 0)}^{0} + \alpha_2 s_1 \cancel{\cos(s_1 0)}^{1} = \alpha_2 s_1$$
$$\dot{\eta}_0 = -\beta_1 s_1 \cancel{\sin(s_1 0)}^{0} + \beta_2 s_1 \cancel{\cos(s_1 0)}^{1} = \beta_2 s_1$$

$$\ddot{\xi}_0 = -\alpha_1 s_1^2 \cancel{\cos(s_1 0)}^{1} - \alpha_2 s_1^2 \cancel{\sin(s_1 0)}^{0} = -\alpha_1 s_1^2$$
$$\ddot{\eta}_0 = -\beta_1 s_1^2 \cancel{\cos(s_1 0)}^{1} - \beta_2 s_1^2 \cancel{\sin(s_1 0)}^{0} = -\beta_1 s_1^2$$

$\therefore$ We can use 1^{st} equation on previous page

$$\therefore -\alpha_1 s_1^2 - 2(\beta_2 s_1) = U^*_{xx} \alpha_1 + U^*_{xy} \beta_1$$
$$-\beta_1 s_1^2 + 2(\alpha_2 s_1) = U^*_{yy} \beta_1 + U^*_{yx} \alpha_1$$

$$\therefore \beta_2 = -\left(\frac{\alpha_1 (s_1^2 + U^*_{xx}) + U^*_{xy} \beta_1}{2 s_1}\right) = -\left(\frac{\xi_0 (s_1^2 + U^*_{xx}) + \eta_0 U^*_{xy}}{2 s_1}\right)$$

Continued...

$$\alpha_2 = \frac{\beta_1(U^*_{yy}+s_1^2) + U^*_{yx}\alpha_1}{2s_1} = \frac{\eta_0(U^*_{yy}+s_1^2) + U^*_{yx}\xi_0}{2s_1}$$

Summarize

$$\alpha_1 = \xi_0 \qquad \beta_1 = \eta_0 \qquad \dot{\xi} = \alpha_2 s_1 \qquad \dot{\eta} = \beta_2 s_1$$

$$\alpha_2 = \frac{\eta_0(s_1^2+U^*_{yy}) + U^*_{yx}\xi_0}{2s_1}$$

$$\beta_2 = \frac{-\xi_0(s_1^2+U^*_{xx}) - U^*_{xy}\eta_0}{2s_1}$$

$$U^*_{xx} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)(x+\mu)^2}{d^5} + \frac{3\mu(x-1+\mu)^2}{r^5}$$

$$U^*_{yy} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)y^2}{d^5} + \frac{3\mu y^2}{r^5}$$

$$U^*_{xy} = U^*_{yx} = \frac{3(1-\mu)(x+\mu)y}{d^5} + \frac{3\mu(x-1+y)y}{r^5}$$

$$\xi_0 = \xi_0 \times l^* = 3844 \text{ km}$$

$$\eta_0 = \eta_0 \times l^* = 0 \text{ km}$$

$$\dot{\xi}_0 = \dot{\xi}_0 \times \frac{l^*}{t^*} = -0.0218 \text{ km/s}$$

$$\dot{\eta}_0 = \dot{\eta}_0 \times \frac{l^*}{t^*} = -0.0362 \text{ km/s}$$

$$\mathbb{P} = \frac{2\pi}{s} = 21.0698 \text{ (non-dim)}$$

$$\mathbb{P}_{days} = \frac{\mathbb{P} \times t^*}{3600 \times 24} = \boxed{91.4952 \text{ days}}$$

See Figure: C4.1 and C4.2

Continued:

b) Recall what we had derived:

$$\xi = \alpha_1 \cos s_1 t + \alpha_2 \sin s_1 t + \alpha_3 \cos s_2 t + \alpha_4 \sin s_2 t$$
$$\eta = \beta_1 \cos s_1 t + \beta_2 \sin s_1 t + \beta_3 \cos s_2 t + \beta_4 \sin s_2 t$$

More over the differential equation was:

$$\ddot{\xi} - 2\dot{\eta} = U_{xx}\xi + U_{xy}^{*}\eta$$
$$\ddot{\eta} + 2\dot{\xi} = U_{yx}^{*}\xi + U_{yy}\eta$$

Attempt at solving for α's and β's ahead

With ξ_0, η_0, $\dot{\xi}_0$, and $\dot{\eta}$ we can solve the differential equation. But we cannot solve for α's and β's as there are 6 equations and 4 unknown. The 4 equations give above and 2 equation are the derivative of ξ and η. This means that α and β are not independent. The task does not ask us to evaluate a certain way so I evaluated the differential equations.

The frequencies are not commensurate $\mathbb{P}_L = \frac{2\pi}{s_1}$ $\mathbb{P}_S = \frac{2\pi}{s_2}$

	$\mathbb{P}_L$	$\mathbb{P}_S$
non dim	21.0698	6.5827
dim (days)	91.495	28.585

$\mathbb{P}_S \rightarrow$ about same as P_{moon} around Earth

See Figure: C4.3

$\xi_0 = 0.01$, $\eta_0 = 0$, $\dot{\xi}_0 = 0.001$, $\dot{\eta}_0 = 0.001$ $C = 2.981$

c) As ξ_0 increases the linear solution is a worse approximation for 1 frequency and both frequencies. See Figures: C4.4 C4.5 C4.6, C4.7 and C4.8

Continued.

b) Solving for α's and β's when both frequencies are excited. Due to lack of equations let us reformulate equations. Start with basic solution to differential equation with 4 roots.

$$\xi = \sum_{i=1}^{4} A_i e^{\lambda_i t} \quad ① \qquad \eta = \sum_{i=1}^{4} B_i e^{\lambda_i t} \quad ②$$

$$\left.\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= U^*_{xx}\,\xi + U^*_{xy}\,\eta \\ \ddot{\eta} + 2\dot{\xi} &= U^*_{yx}\,\xi + U^*_{yy}\,\eta \end{aligned}\right\} ③$$

Substitute ① and ② into ③ differentiate when necessary

$$\lambda_i^2 A_i e^{\lambda_i t} - 2\lambda_i B_i e^{\lambda_i t} = U^*_{xx} A_i e^{\lambda_i t} + U^*_{xy} B_i e^{\lambda_i t}$$
$$\lambda_i^2 B_i e^{\lambda_i t} + 2\lambda_i A_i e^{\lambda_i t} = U^*_{yx} A_i e^{\lambda_i t} + U^*_{yy} B_i e^{\lambda_i t}$$

$$\therefore (\lambda_i^2 - U^*_{xx})A_i - (2\lambda + U^*_{xy})B_i = 0$$
$$(\lambda_i^2 - U^*_{yy})B_i + (2\lambda + U^*_{yx})A_i = 0$$

$$\therefore \left(\frac{\lambda_i^2 - U^*_{xx}}{2\lambda_i + U^*_{xy}}\right)A_i = B_i \rightarrow \alpha_i A_i = B_i$$

different delivation for colinear

We have now reduced a variable by finding dependency.
Note $\alpha_2 = -\alpha_1$, $\alpha_4 = -\alpha_3$

So: $\xi_0 = \sum_{i=1}^{4} A_i e^{\lambda_i t_0}$, $\eta_0 = \sum_{i=1}^{4} \alpha_i A_i e^{\lambda_i t_0}$

$$\dot{\xi}_0 = \sum_{i=1}^{4} \lambda_i A_i e^{\lambda_i t_0} \qquad \dot{\eta}_0 = \sum_{i=1}^{4} \lambda_i \alpha_i A_i e^{\lambda_i t_0}$$

continued...

On the bottom of previous page we have 4 equations and 4 unknowns. So let us solve for A_i's.
This is tough, but recall some relations which will simplify the solution: $\lambda_2 = -\lambda_1$, $\lambda_4 = -\lambda_3$, $\alpha_2 = -\alpha_1$, $\alpha_4 = -\alpha_3$

See Mathematica Code for working

$$A_1 = e^{-t_0\lambda_1}\frac{\left(-\alpha_1(\dot{\eta}_0 + \eta_0\lambda_1)\lambda_3 + \alpha_3(\dot{\eta}_0\lambda_1 - \eta_0\lambda_1\lambda_3 + \alpha_1(\lambda_1+\lambda_3)\dot{\xi}_0)\right)}{2\alpha_1\lambda_1(\alpha_3\lambda_1 - \alpha_1\lambda_3)}$$

$$A_2 = e^{-t_0\lambda_1}\frac{\left(\alpha_1(-\dot{\eta}_0 + \eta_0\lambda_1)\lambda_3 + \alpha_3(\dot{\eta}_0\lambda_1 - \eta_0\lambda_1\lambda_3 + \alpha_1(-\lambda_1+\lambda_3)\dot{\xi}_0)\right)}{2\alpha_1\lambda_1(\alpha_3\lambda_1 - \alpha_1\lambda_3)}$$

$$A_3 = e^{-t_0\lambda_3}\frac{\left(\alpha_3(\dot{\eta}_0\lambda_1 - \eta_0\lambda_1\lambda_3 - \alpha_1\lambda_1^2\xi_0 + \alpha_1\lambda_3\dot{\xi}_0) + \alpha_1(-\eta_0\lambda_1^2 - \dot{\eta}_0\lambda_3 + \alpha_1\lambda_1(\lambda_3\xi_0 + \dot{\xi}_0))\right)}{2\alpha_1\lambda_1(-\alpha_3\lambda_1 + \alpha_1\lambda_3)}$$

$$A_4 = e^{-t_0\lambda_3}\frac{\left(\alpha_1(\eta_0\lambda_1^2 - \dot{\eta}_0\lambda_3 + \alpha_1\lambda_1\lambda_3\xi_0 - \alpha_1\lambda_1\dot{\xi}_0) + \alpha_3(\dot{\eta}_0\lambda_1 - \eta_0\lambda_1\lambda_3 - \alpha_1\lambda_1^2\xi_0 + \alpha_1\lambda_3\dot{\xi}_0)\right)}{2\alpha_1\lambda_1(-\alpha_3\lambda_1 + \alpha_1\lambda_3)}$$

Now we have a general solution to the A_i's therefore B_i's because we know α_i's

Next let us equate A_i's and B_i's to the general marginally stable linear form.

Continued...   Note: λ_i are complex

For notation purpose since I used α, I will now use "a" and "b" so that there is no confusion

$\therefore \xi = a_1 \cos s_1 t + a_2 \sin s_1 t + a_3 \cos s_2 t + a_4 \sin s_2 t$

$\eta = b_1 \cos s_1 t + b_2 \sin s_1 t + b_3 \cos s_2 t + b_4 \sin s_2 t$

Now let us find relation between $A_i \to a_i$ and $B_i \to b_i$

$$\therefore \xi = \sum_{i=1}^{4} A_i e^{\lambda_i t} = A_1(\cos s_1 t + j\sin s_1 t) + A_2(\cos s_1 t - j\sin s_1 t)$$
$$+ A_3(\cos s_2 t + j\sin s_2 t) + A_4(\cos s_2 t - j\sin s_2 t)$$
$$= a_1 \cos s_1 t + a_2 \sin s_1 t + a_3 \cos s_2 t + a_4 \sin s_2 t$$

$$\therefore \boxed{\begin{array}{ll} a_1 = A_1 + A_2 & a_2 = A_1 - A_2 \\ a_3 = A_3 + A_4 & a_4 = A_3 - A_4 \end{array}}$$

We can do the same with the $B_i \to b_i$ and get a similar relationship:

$$\boxed{\begin{array}{ll} b_1 = B_1 + B_2 & b_2 = B_1 - B_2 \\ b_3 = B_3 + B_4 & b_4 = B_3 - B_4 \end{array}}$$

Summary

1) We can find A_i's using ξ_0, η_0, $\dot{\xi}_0$, and $\dot{\eta}_0$

2) Then using α_i, convert A_i's to B_i's

3) Finally, use A_i's and B_i's to get a_i's and b_i's

Sushrut Prabhu | AAE632 PS4

PS4 Bonus

Given: Pluto-Charon System

Find: a) Roots of System

b) Plot linear and non-linear solution

Solution:

I assume you want this for L_4/L_5 points

$$\ddot{\xi} - 2\dot{\eta} = U^*_{xx}\xi + U^*_{xy}\eta$$

$$\ddot{\eta} + 2\dot{\xi} = U^*_{yx}\xi + U_{yy}\eta$$

$$\downarrow$$

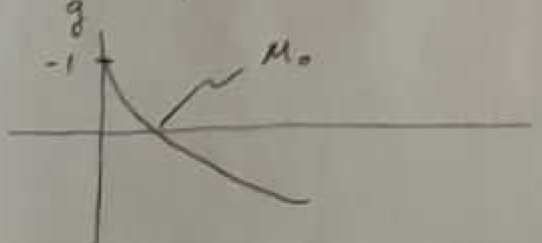
$$\lambda^4 + \lambda^2 + \frac{27}{4}\mu(1-\mu) = 0$$

$$\therefore \Lambda^2 + \Lambda + \frac{27}{4}\mu(1-\mu) = 0$$

$$\therefore \Lambda = \frac{1}{2}\left(-1 \pm \sqrt{g}\right) \qquad g = 1 - 27\mu(1-\mu)$$

$$\lambda = \pm\sqrt{\Lambda}$$

Note μ of Pluto Charon $= 0.0951 > \mu_0$



So the roots will contain real and imaginary parts

$$\therefore \boxed{\begin{array}{ll} \lambda_1 = 0.3622 + 0.7945j & \lambda_2 = -0.3622 - 0.7945j \\ \lambda_3 = 0.3622 - 0.7945j & \lambda_4 = -0.3622 + 0.745j \end{array}}$$

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Continued...

General solution to the equation is:

$$\xi = \sum_{i=1}^{4} A_i e^{\lambda_i t} = \sum_{i=1}^{4} A_i e^{\alpha_i t}(\cos\beta_i t + j\sin\beta_i t)$$

$$\eta = \sum_{i=1}^{4} B_i e^{\lambda_i t} = \sum_{i=1}^{4} B_i e^{\alpha_i t}(\cos\beta_i t + j\sin\beta_i t)$$

Note I redefined $\lambda_i \to \alpha + \beta i$

We can simplifty the equation to

$$\xi = e^{\alpha t}(a_1\cos\beta t + a_2\sin\beta t) + e^{-\alpha t}(a_3\cos\beta t + a_4\sin\beta t)$$

$$\eta = \underbrace{e^{\alpha t}(b_1\cos\beta t + b_2\sin\beta t)}_{\text{Spiral out}} + \underbrace{e^{-\alpha t}(b_3\cos\beta t + b_4\sin\beta t)}_{\text{Spiral in}}$$

Note: that the a's and b's are dependent on each other

Let us first test spiral in, so we set $a_1 = a_2 = b_1 = b_2 = 0$

$$\therefore \xi = e^{-\alpha t}(a_3\cos\beta t + a_4\sin\beta t)$$
$$\eta = e^{-\alpha t}(b_3\cos\beta t + b_4\sin\beta t)$$

Let $\xi_0 = 0.01$ and $\eta_0 = 0$, lets us find a_3 and a_4 to solve the solution

Continued

$$\xi_0 = a_3 \qquad \eta_0 = b_3 \quad \Big\} \ ①$$

$$\dot{\xi} = -\alpha e^{-\alpha t}(a_3 \cos\beta t + a_4 \sin\beta t)$$
$$+ e^{-\alpha t}(-a_3\beta \sin\beta t + a_4\beta\cos\beta t)$$

$$\dot{\eta} = -\alpha e^{-\alpha t}(b_3 \cos\beta t + b_4 \sin\beta t)$$
$$+ e^{-\alpha t}(-b_3\beta\sin\beta t + b_4\beta\cos\beta t$$

$$\therefore \left.\begin{aligned}\dot{\xi}_0 &= -a_3\alpha + a_4\beta \\ \dot{\eta}_0 &= -b_3\alpha + b_4\beta\end{aligned}\right\} ②$$

$$\ddot{\xi} = \alpha^2 e^{-\alpha t}(a_3\cos\beta t + a_4 \sin\beta t)$$
$$-2\alpha e^{-\alpha t}(-a_3\beta\sin\beta t + a_4\beta\cos\beta t)$$
$$+ e^{-\alpha t}(-a_3\beta^2\cos\beta t - a_4\beta^2\sin\beta t)$$

$$\ddot{\eta} = \alpha^2 e^{-\alpha t}(b_3\cos\beta t + b_4\sin\beta t)$$
$$-2\alpha e^{-\alpha t}(-b_3\beta\sin\beta t + b_4\beta\cos\beta t)$$
$$+ e^{-\alpha t}(-b_3\beta^2\cos\beta t - b_4\beta^2\sin\beta t)$$

$$\therefore \left.\begin{aligned}\ddot{\xi}_0 &= \alpha^2 a_3 - 2\alpha a_4\beta - a_3\beta^2 \\ \ddot{\eta}_0 &= \alpha^2 b_3 - 2\alpha b_4\beta - b_3\beta^2\end{aligned}\right\} ③$$

$\therefore$ Sub ①, ②, ③ into differential eq.

$$\alpha^2 a_3 - 2\alpha a_4\beta - a_3\beta^2 - 2(-b_3\alpha + b_4\beta) = U_{xx}a_3 + U_{xy}b_3$$

and

$$\alpha^2 b_3 - 2\alpha b_4\beta - b_3\beta^2 + 2(-a_3\alpha + a_4\beta) = U_{yy}b_3 + U_{xy}a_3$$

Continued ...:

2 equations 2 unknowns in the last part of previous pages:

$$\therefore \underline{a_4}(-2\alpha\beta) + \underline{b_4}(-2\beta) = a_3(U_{xx} + \beta^2 - \alpha^2) + b_3(U_{xy} - 2\alpha) \quad ①$$

$$\therefore \underline{b_4}(-2\alpha\beta) + \underline{a_4}(2\beta) = b_3(U_{yy} + \beta^2 - \alpha^2) + a_3(U_{xy} + 2\alpha) \quad ②$$

I underlined the unknowns, we set a_4 in terms of a_4 and solve for b_4

$$\therefore a_4 = \frac{a_3(U_{xx} + \beta^2 - \alpha^2) + b_3(U_{xy} - 2\alpha) + 2b_4\beta}{-2\alpha\beta}$$

$$\therefore b_4(-2\alpha\beta) + \left[\frac{a_3(U_{xx} + \beta^2 - \alpha^2) + b_3(U_{xy} - 2\alpha) + 2b_4\beta}{-\cancel{2}\alpha\cancel{\beta}}\right]\cancel{2\beta}$$
$$= b_3(U_{yy} + \beta^2 - \alpha^2) + a_3(U_{xy} + 2\alpha)$$

$$\therefore -2\alpha\beta b_4 + \frac{2\beta b_4}{\alpha} = \left[\frac{a_3(U_{xx} + \beta^2 - \alpha^2) + b_3(U_{xy} - 2\alpha)}{\alpha}\right]$$
$$+ b_3(U_{yy} + \beta^2 - \alpha^2) + a_3(U_{xy} + 2\alpha)$$

$$\therefore b_4(2\alpha^2\beta + 2\beta) = a_3(\alpha^2 - U_{xx} - \beta^2) + b_3(2\alpha - U_{xy})$$
$$+ \alpha\left[b_3(\alpha^2 - U_{yy} - \beta^2) - a_3(U_{xy} + 2\alpha)\right]$$

$$\therefore b_4 = \frac{\left\{a_3(\alpha^2 - U_{xx} - \beta^2) + b_3(2\alpha - U_{xy}) + \alpha\left[b_3(\alpha^2 - U_{yy} - \beta^2) - a_3(U_{xy} + 2\alpha)\right]\right\}}{(2\alpha^2\beta + 2\beta)}$$

See Figure: C4.9, C4.10, and C4.11 for different ξ_0. Note there is a positive real part so it is an unstable point. My $t = 15$ (non-dim) relatively short ~ 12 days

PS C4

Part a)

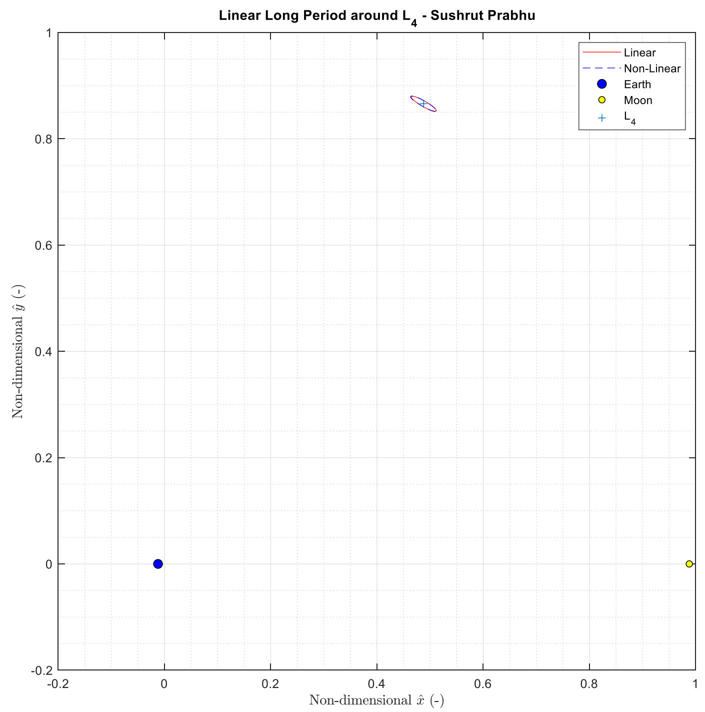


Figure C4.1: Long period orbit around L4.

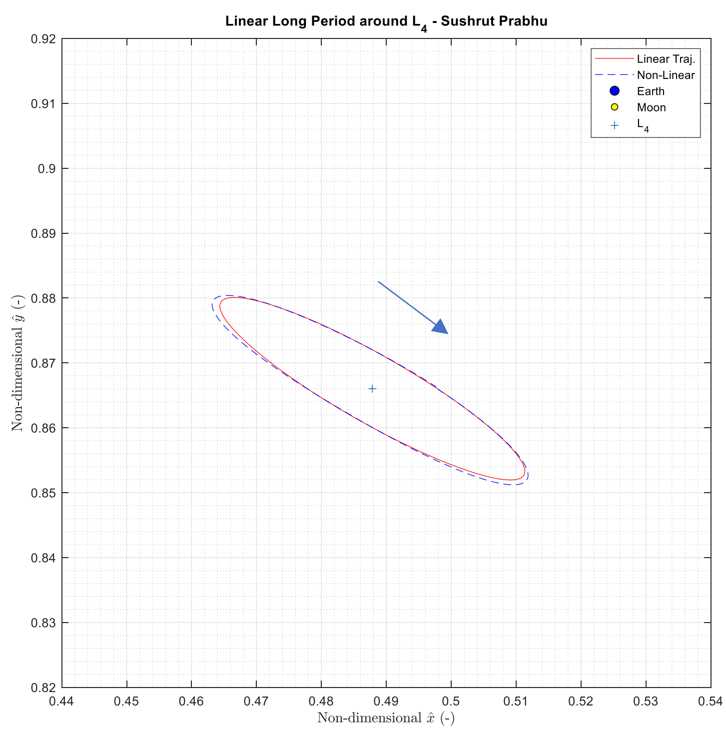


Figure C4.2: Long period orbit around L4.

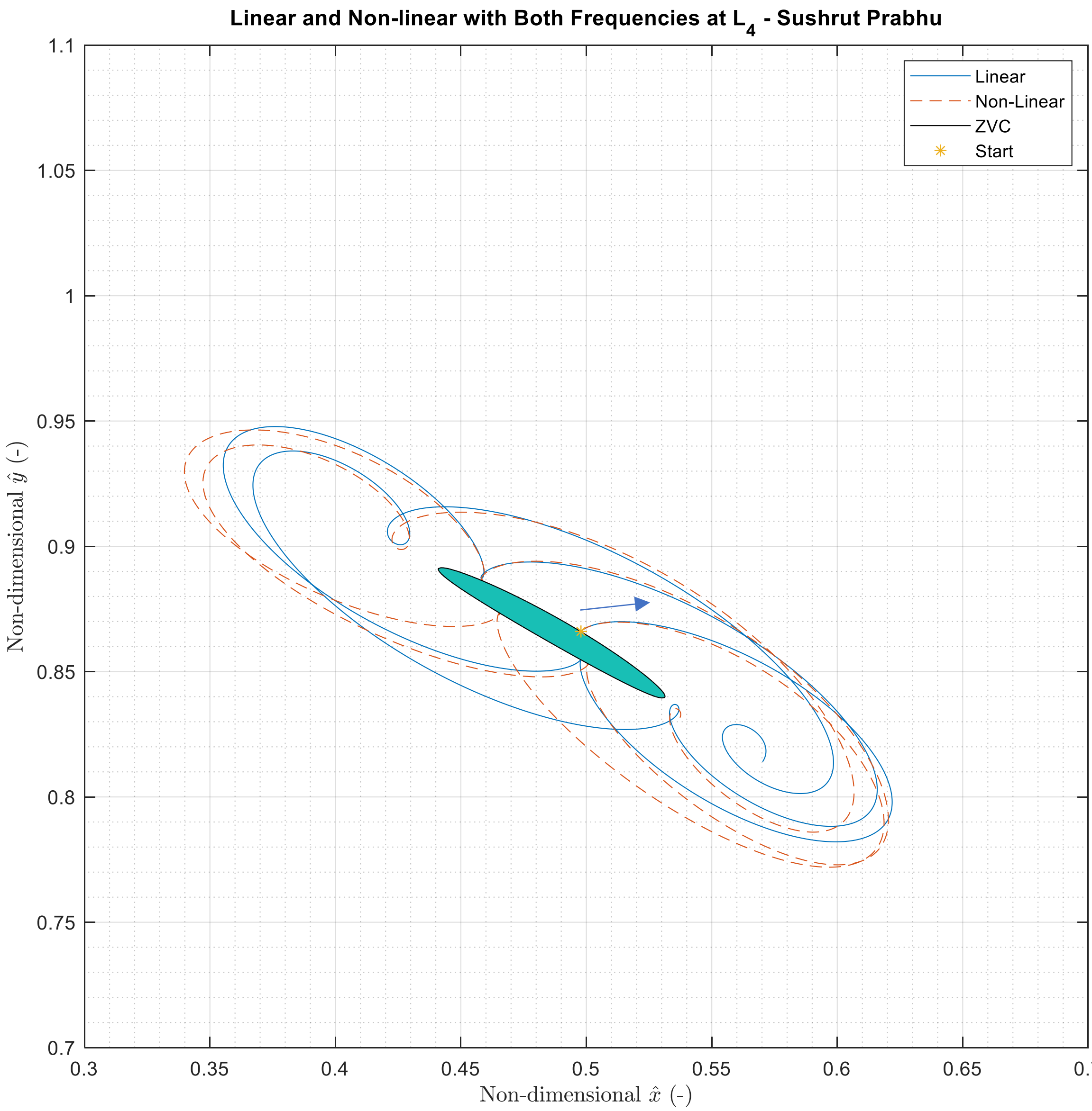


Figure C4.3: Long period and short period orbit around L4.

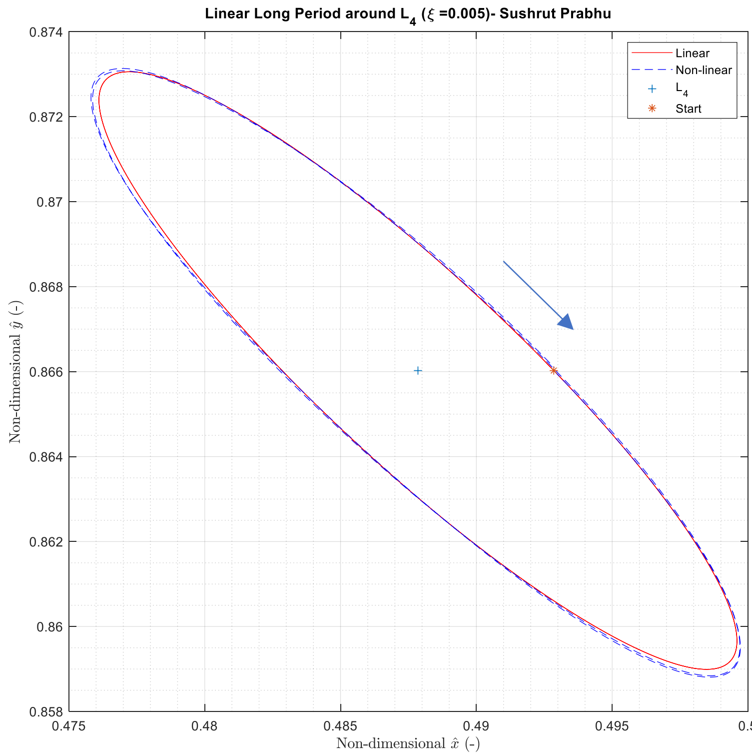


Figure C4.4: Long period orbit around L4.

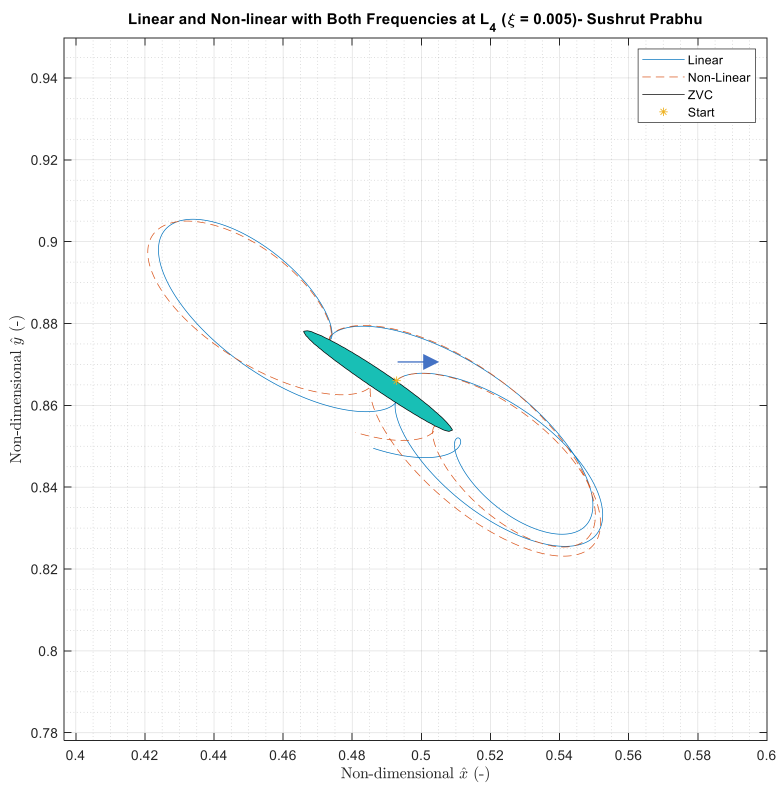


Figure C4.5: Long period and short period orbit around L4.

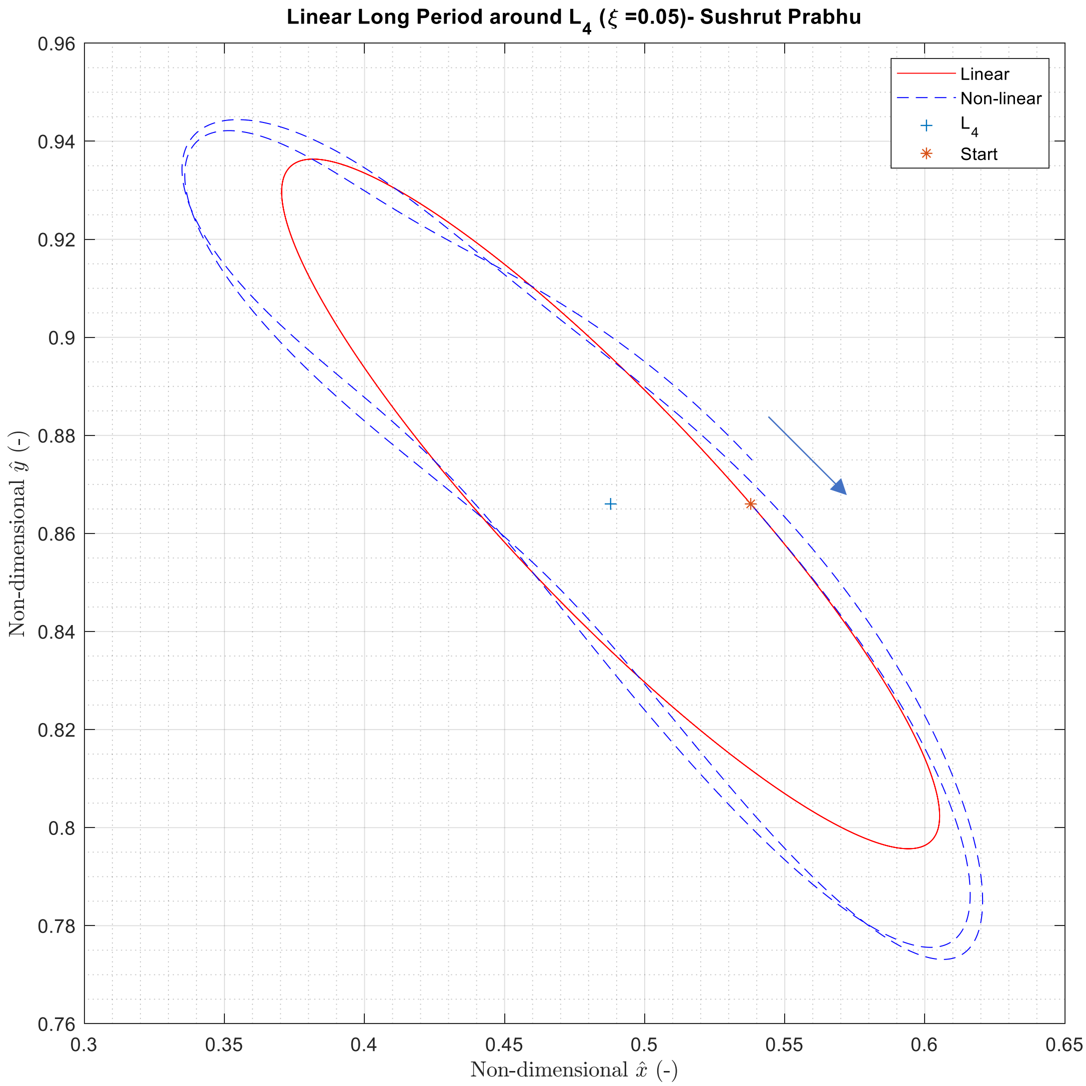


Figure C4.6: Long period orbit around L4.

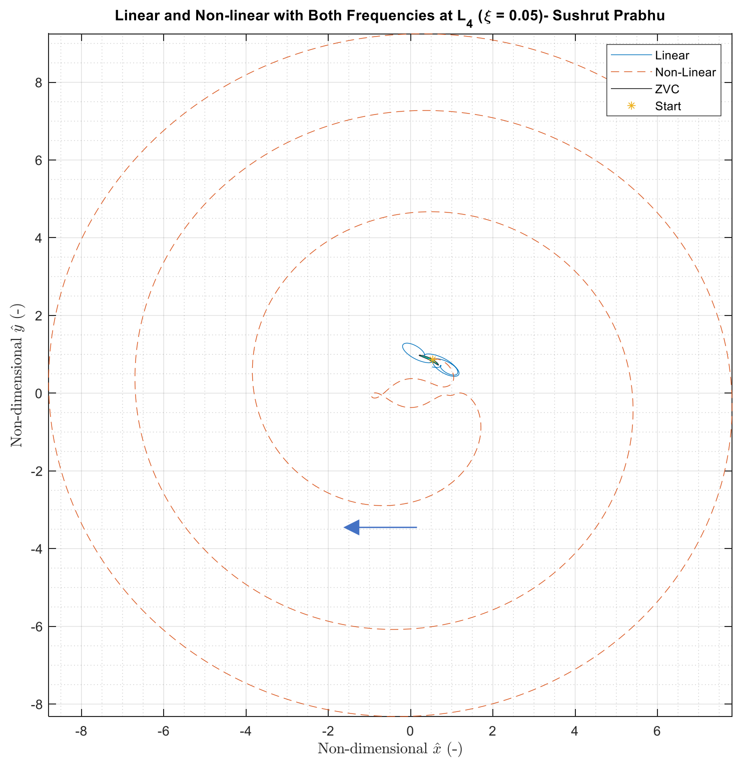


Figure C4.7: Long period and short period orbit around L4.

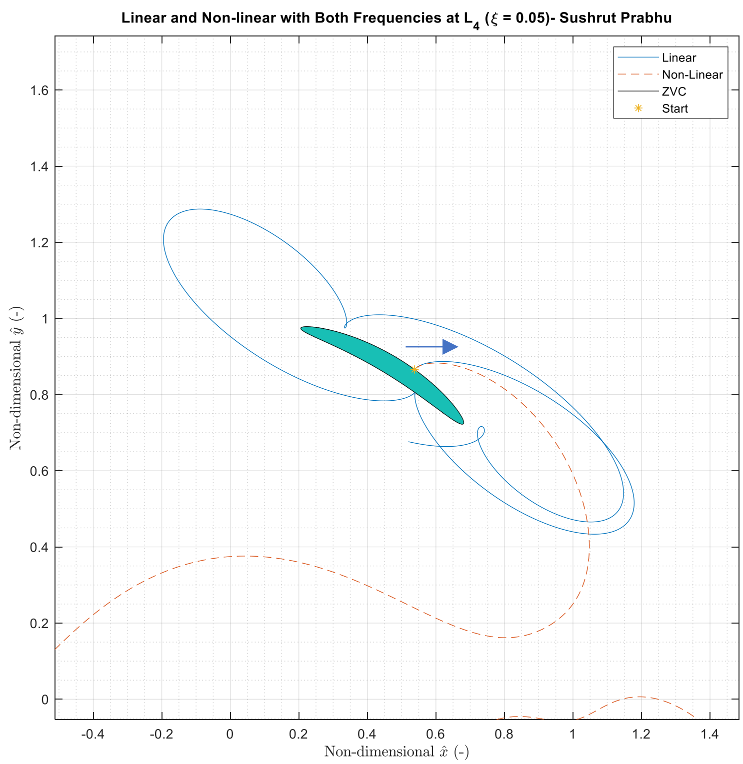


Figure C4.8: Long period and short period orbit around L4 zoomed.

PC4 Bonus

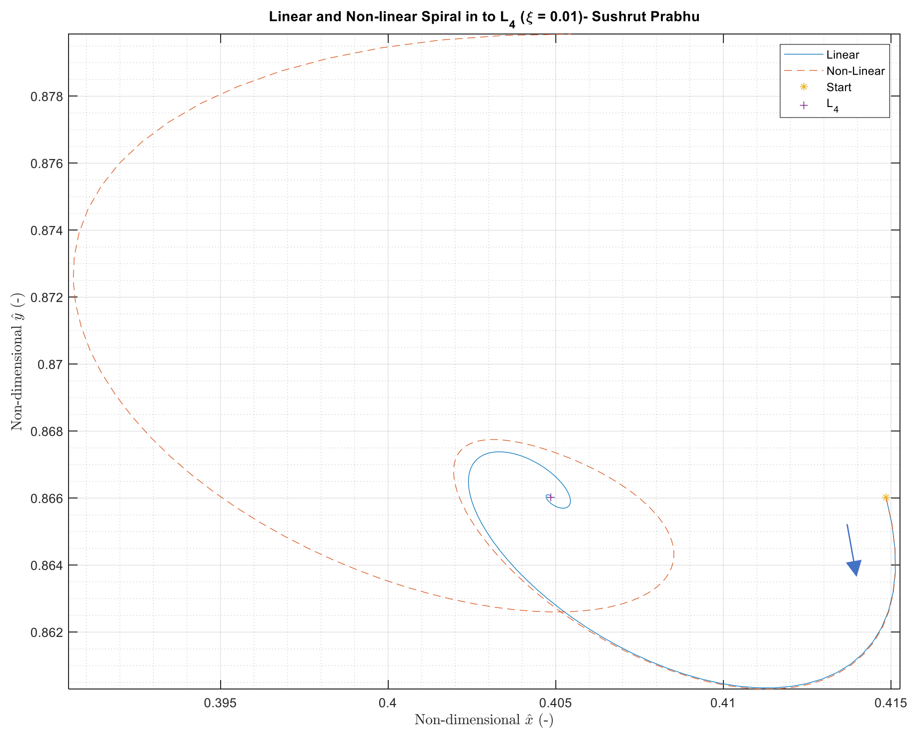


Figure C4.9: Spiral into the L4 point for Pluto Charon system.

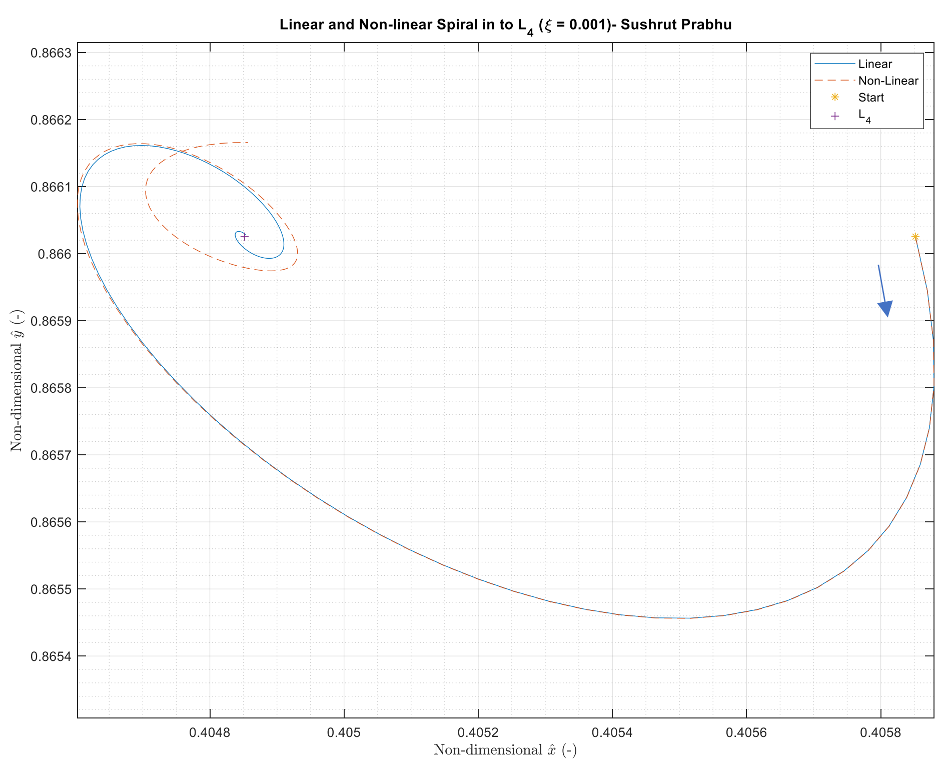
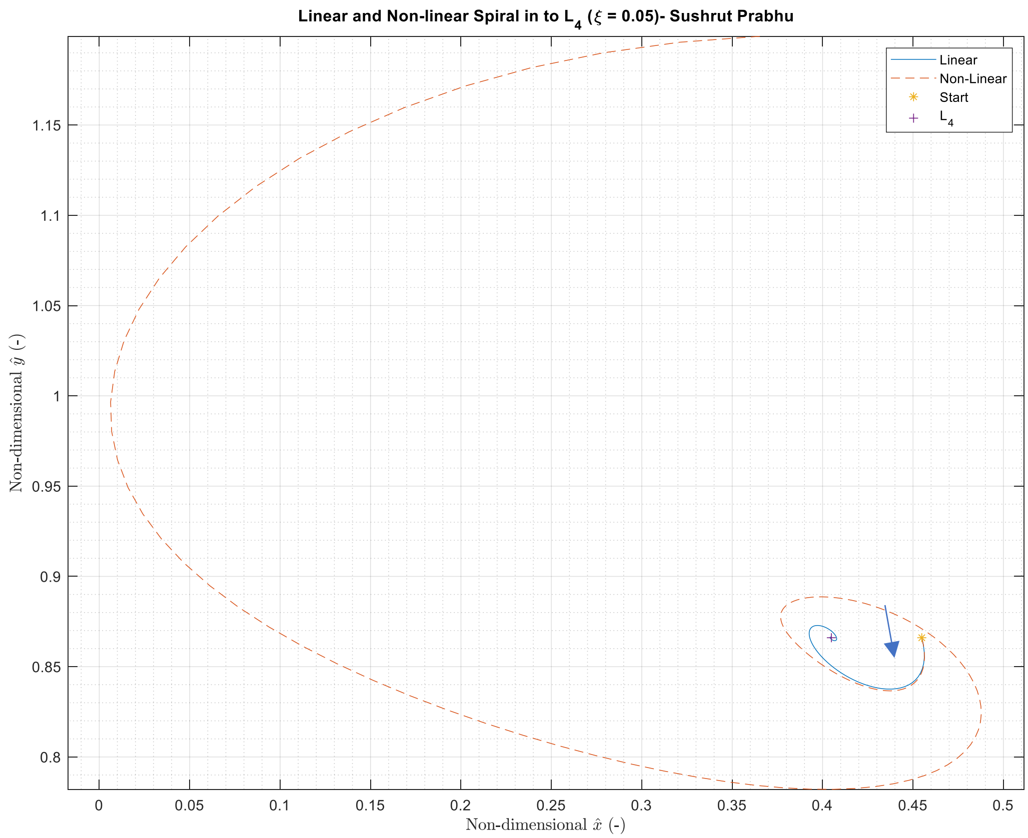


Figure C4.10: Spiral into the L4 point for Pluto Charon system with smaller perturbation.



Spiral into the L4 point for Pluto Charon System with larger perturbation.

Table of Contents

PSC4

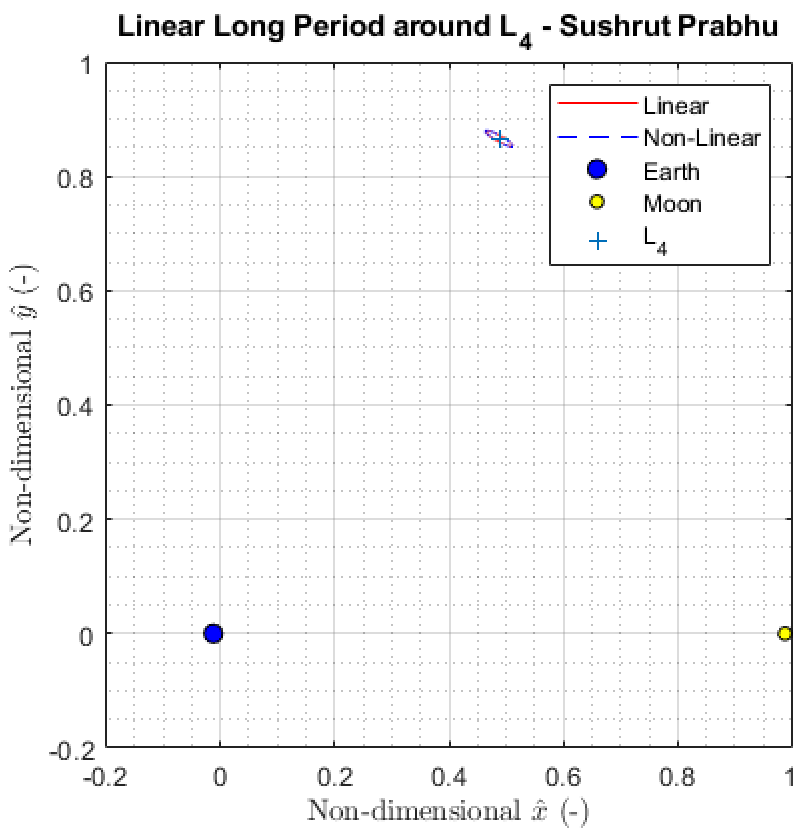
```
clear
close all
clc
```

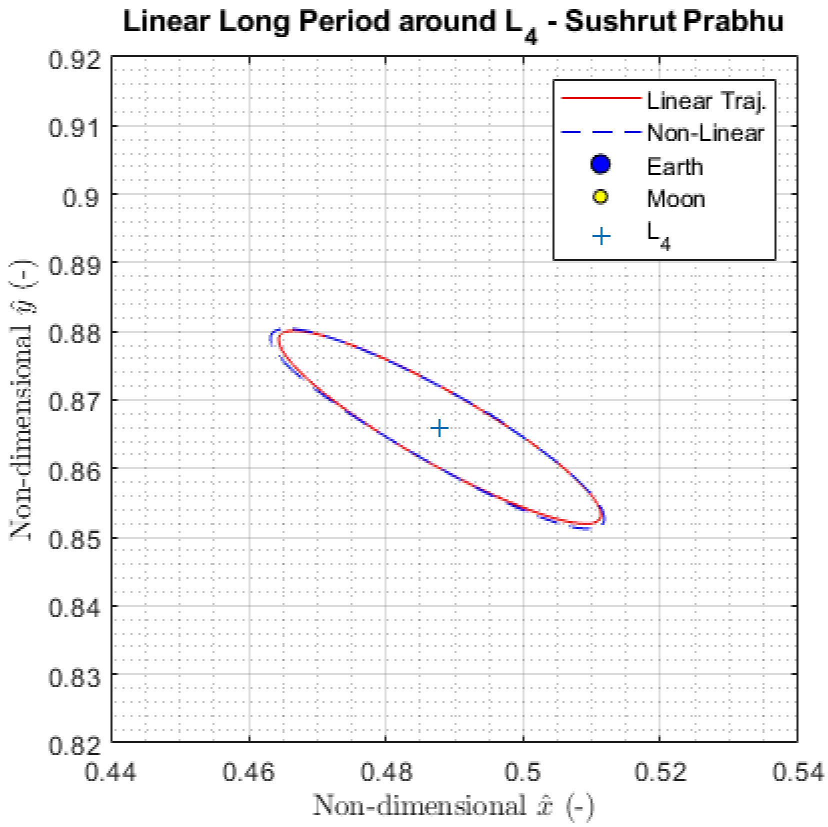
PSC4

```
SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' ,'t* (s)', 'L_4 (-)','L_4 (km)'};
G = 6.6738*10^-20;
xi_0 = 0.01;
eta_0 = 0;

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% Solution
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;

% Lagrange 4
dim_vals{6,2} = L45_NRm(.55,.5,dim_vals{4,2},10^-7);
dim_vals{7,2} = dim_vals{6,2}*dim_vals{2,2};

%
g = 1-27*dim_vals{4,2}*(1-dim_vals{4,2});

Lam1 = 0.5*(-1 + sqrt(g));
Lam2 = 0.5*(-1 - sqrt(g));

lam12 = sqrt(Lam1);
lam34 = sqrt(Lam2);

s1 = imag(lam12);
s2 = imag(lam34);

Per = 2*pi/s1;
Per_d = Per/3600/24*dim_vals{5,2};
```

```
[Uxx,Uyy,~,Uxy,~,~]= Unn(dim_vals{6,2}(1),dim_vals{6,2}
(2),0,dim_vals{4,2});
alpha2 = (eta_0*(s1^2 + Uyy) + Uxy*xi_0)/2/s1;
beta2 = (-xi_0*(s1^2 + Uxx) - Uxy*eta_0)/2/s1;
xidot_0 = alpha2*s1;
etadot_0 = beta2*s1;

xidot_0_dim = xidot_0 * dim_vals{2,2}/dim_vals{5,2};
etadot_0_dim = etadot_0 * dim_vals{2,2}/dim_vals{5,2};


t = 0:0.01:Per;
t = t';
[xi,eta]=L45_lin(t,s1,s2,xi_0,alpha2,0,0,eta_0,beta2,0,0);
xl = xi + dim_vals{6,2}(1);
yl = eta + dim_vals{6,2}(2);

% Non linear
r = [xl(1), yl(1), 0];
v = [xidot_0, etadot_0, 0];
IC = [r,v];
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});


figure
plot(xl,yl,'r')
hold on
plot(y(:,1),y(:,2),'--b')
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(dim_vals{6,2}(1),dim_vals{6,2}(2),'+')
ylim([-.2 1])
title('Linear Long Period around L_4 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear','Non-Linear',  'Earth', 'Moon', 'L_4')
grid on
grid minor
axis square

figure
plot(xl,yl,'r')
hold on
plot(y(:,1),y(:,2),'--b')
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(dim_vals{6,2}(1),dim_vals{6,2}(2),'+')
title('Linear Long Period around L_4 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear Traj.', 'Non-Linear', 'Earth', 'Moon', 'L_4')
```

```
xlim([.44 .54])
ylim([.82 .92])
grid on
grid minor
axis square
```

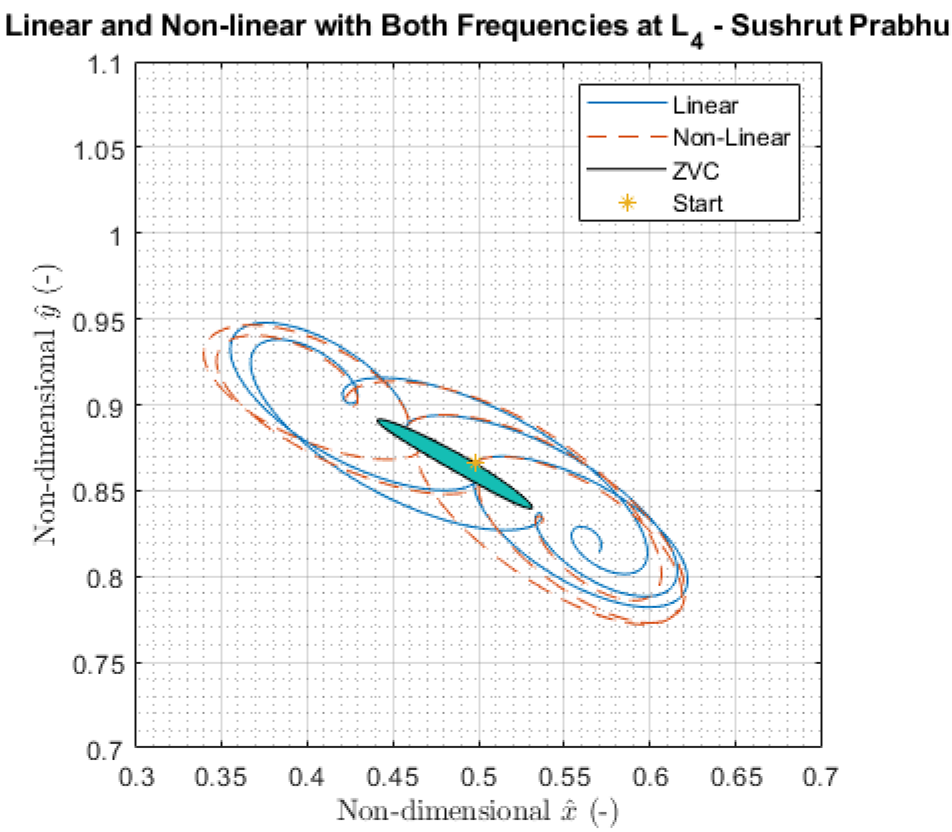




Part b

```
xidot_0 = .001;
etadot_0 = .001;

% Ode COnditions
t = 0:0.01:50;
t = t';
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

% Linear
% [Ai, Bi] = AiBi_IC(lam12,-lam12,lam34,-
lam34,Uxx,Uyy,xi_0,eta_0,xidot_0,etadot_0,Uxy);
%
%
% a = [Ai(1)+Ai(2),(Ai(1)-Ai(2))*i,Ai(3)+Ai(4),(Ai(3)-Ai(4))*i];
% b = [Bi(1)+Bi(2),(Bi(1)-Bi(2))*i,Bi(3)+Bi(4),(Bi(3)-Bi(4))*i];
%
% G1 = (s1^2 + Uxx)/(4*s1+Uxy^2);
% G2 = (s2^2 + Uxx)/(4*s2+Uxy^2);
%
% b1 = G1 * (2*a(2)*s1 - Uxy*a(1));
% b2 = -G1 * (2*a(1)*s1 + Uxy*a(2));
% b3 = G2 * (2*a(4)*s2 - Uxy*a(3));
```

```
% b4 = -G2 * (2*a(3)*s2 + Uxy*a(4));

% [xi,eta]= L45_lin(t,s1,s2,a(1),a(2),a(3),a(4),b1,b2,b3,b4);
IC = [xi_0,eta_0,xidot_0,etadot_0];
[~,xieta] = ode45(@lin45ode,t,IC,options,Uxx,Uyy,Uxy);

xi = xieta(:,1);
eta = xieta(:,2);
xl = xi + dim_vals{6,2}(1);
yl = eta + dim_vals{6,2}(2);

% Non Linear
r = [xl(1), yl(1), 0];
v = [xidot_0, etadot_0, 0];
IC = [r,v];
[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

% Jacobi
[X,Y] = meshgrid(0.3:0.001:1.2);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(r(1),r(2),r(3),norm(v),dim_vals{4,2});

figure
plot(xl,yl)
hold on
plot(y(:,1),y(:,2),'--')
contourf(X,Y,-C,-[C0 C0]);
plot(xl(1),yl(1),'*')
legend('Linear', 'Non-Linear', 'ZVC', 'Start')
ylim([.7 1.1])
xlim([.3 .7])
axis square
grid on
grid minor
title('Linear and Non-linear with Both Frequencies at L_4 - Sushrut
 Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
```

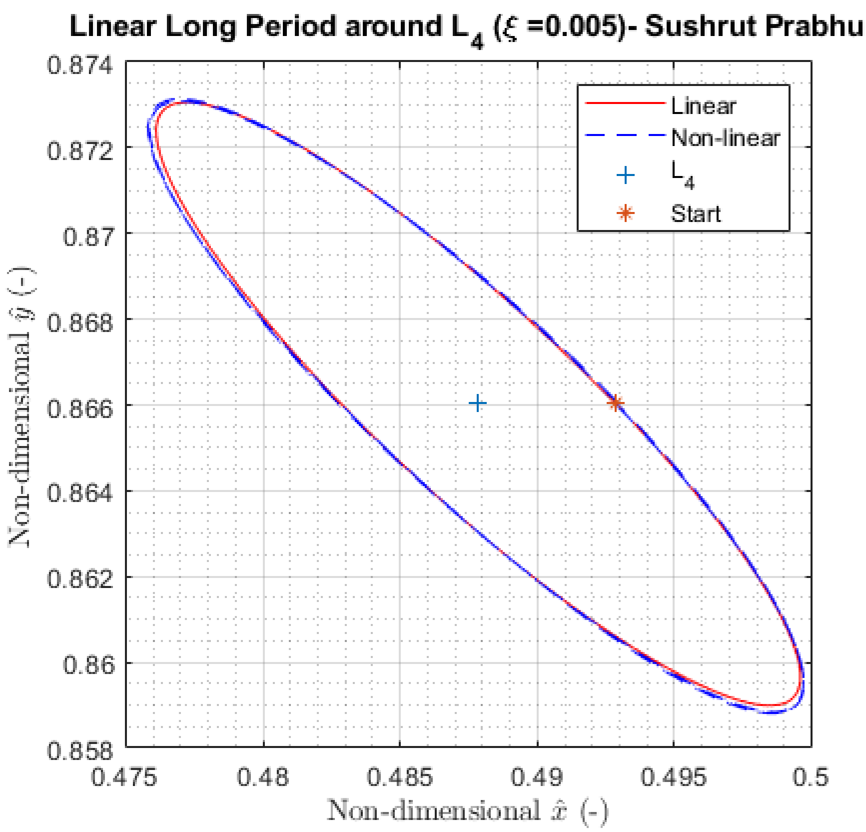


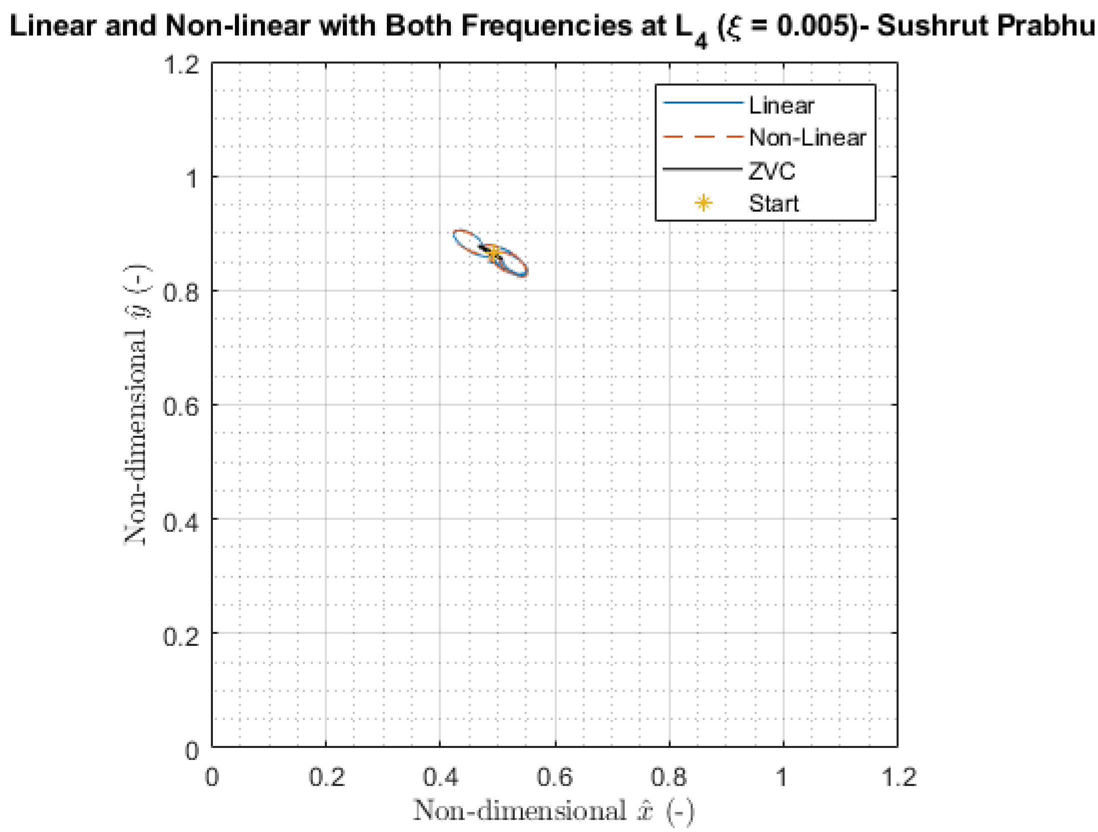
Part c

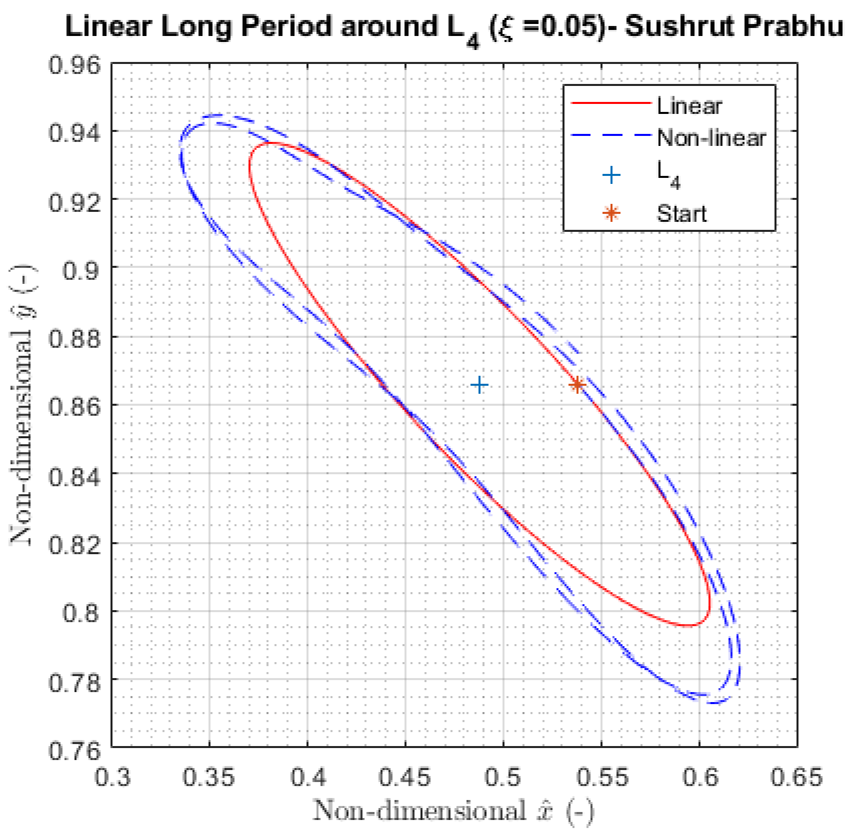
```
for xi_0 = [0.005 .05]
    % Long Period
    alpha2 = (eta_0*(s1^2 + Uyy) + Uxy*xi_0)/2/s1;
    beta2 = (-xi_0*(s1^2 + Uxx) - Uxy*eta_0)/2/s1;
    xidot_0 = alpha2*s1;
    etadot_0 = beta2*s1;

    xidot_0_dim = xidot_0 * dim_vals{2,2}/dim_vals{5,2};
    etadot_0_dim = etadot_0 * dim_vals{2,2}/dim_vals{5,2};

    t = 0:0.01:Per*2;
    t = t';
    [xi,eta]=L45_lin(t,s1,s2,xi_0,alpha2,0,0,eta_0,beta2,0,0);
    xl = xi + dim_vals{6,2}(1);
    yl = eta + dim_vals{6,2}(2);

    % Non linear
    r = [xl(1), yl(1), 0];
    v = [xidot_0, etadot_0, 0];
    IC = [r,v];
    options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance
    [~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});
```

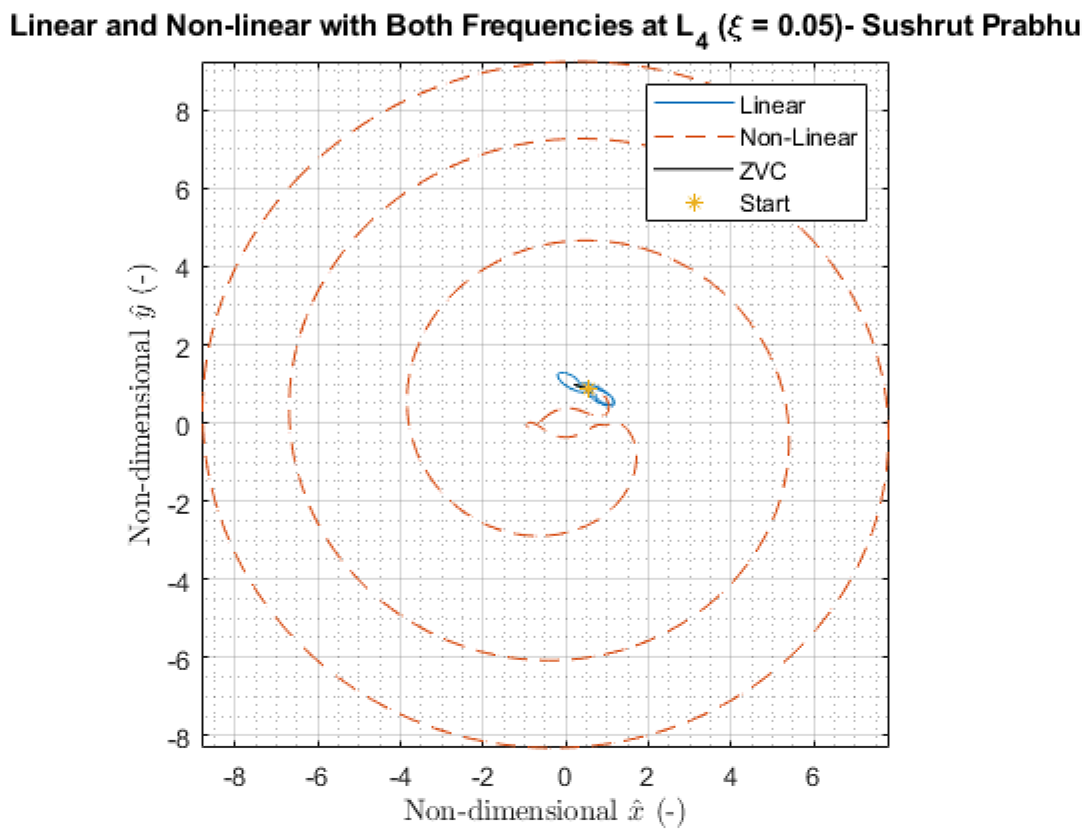
```
figure
plot(xl,yl,'r')
hold on
plot(y(:,1),y(:,2),'--b')
plot(dim_vals{6,2}(1),dim_vals{6,2}(2),'+')
plot(xl(1),yl(1),'*')
title(['Linear Long Period around L_4 (\xi =', num2str(xi_0) , ')-
Sushrut Prabhu'])
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear',  'Non-linear', 'L_4', 'Start')
grid on
grid minor
axis square


% Both frequency
xidot_0 = .001;
etadot_0 = .001;

% Ode COnditions
t = 0:0.01:(Per*1.5);
t = t';
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
tollerance

% Linear
r = [xi_0, eta_0];
v = [xidot_0, etadot_0];
IC = [r,v];
[~,xieta] = ode45(@lin45ode,t,IC,options,Uxx,Uyy,Uxy);
xl = xieta(:,1) + dim_vals{6,2}(1);
yl = xieta(:,2) + dim_vals{6,2}(2);

% Non Linear
r = [xl(1), yl(1), 0];
v = [xidot_0, etadot_0, 0];
IC = [r,v];
[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

% Jacobi
[X,Y] = meshgrid(0:0.001:1.2);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

C0 = Jacobi_C(r(1),r(2),r(3),norm(v),dim_vals{4,2});

figure
plot(xl,yl)
hold on
plot(y(:,1),y(:,2),'--')
contourf(X,Y,-C,-[C0 C0]);
plot(xl(1),yl(1),'*')
legend('Linear', 'Non-Linear', 'ZVC', 'Start')
```

```
    axis square
    grid on
    grid minor
    title(['Linear and Non-linear with Both Frequencies at L_4 (\xi =
 ' num2str(xi_0) ')- Sushrut Prabhu'])
    xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
    ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
end
```




Linear and Non-linear with Both Frequencies at L4 (ξ = 0.005)- Sushrut Prabhu
Linear
Non-Linear
ZVC
Start
Non-dimensional ŷ (-)
Non-dimensional x̂ (-)
0
0.2
0.4
0.6
0.8
1
1.2


Linear Long Period around L4 (ξ =0.05)- Sushrut Prabhu
Linear
Non-linear
L4
Start
Non-dimensional ŷ (-)
Non-dimensional x̂ (-)
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65



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PSC4

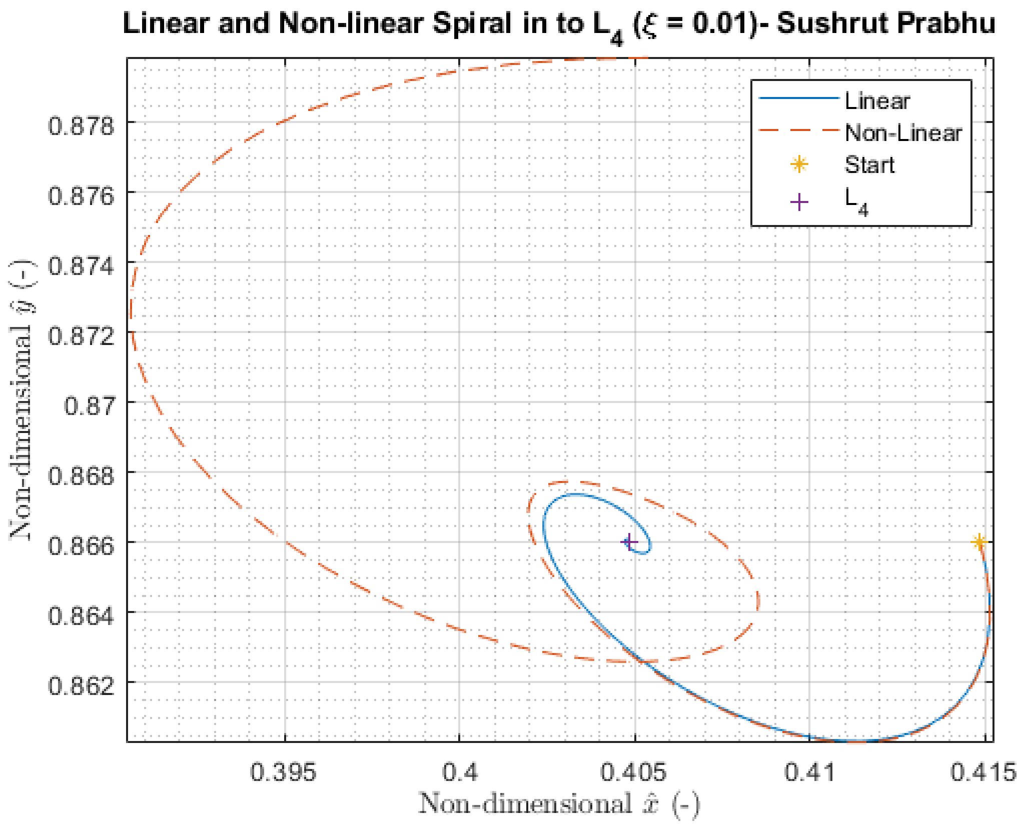
```
clear
close all
clc
```

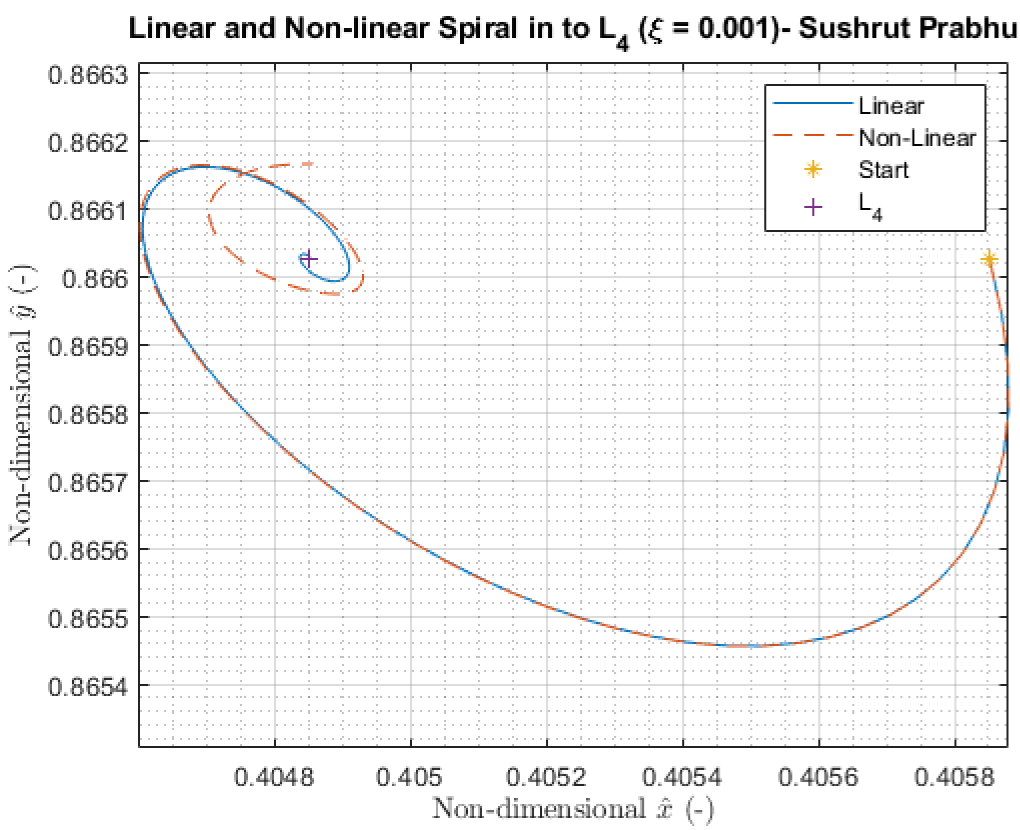
PSC4

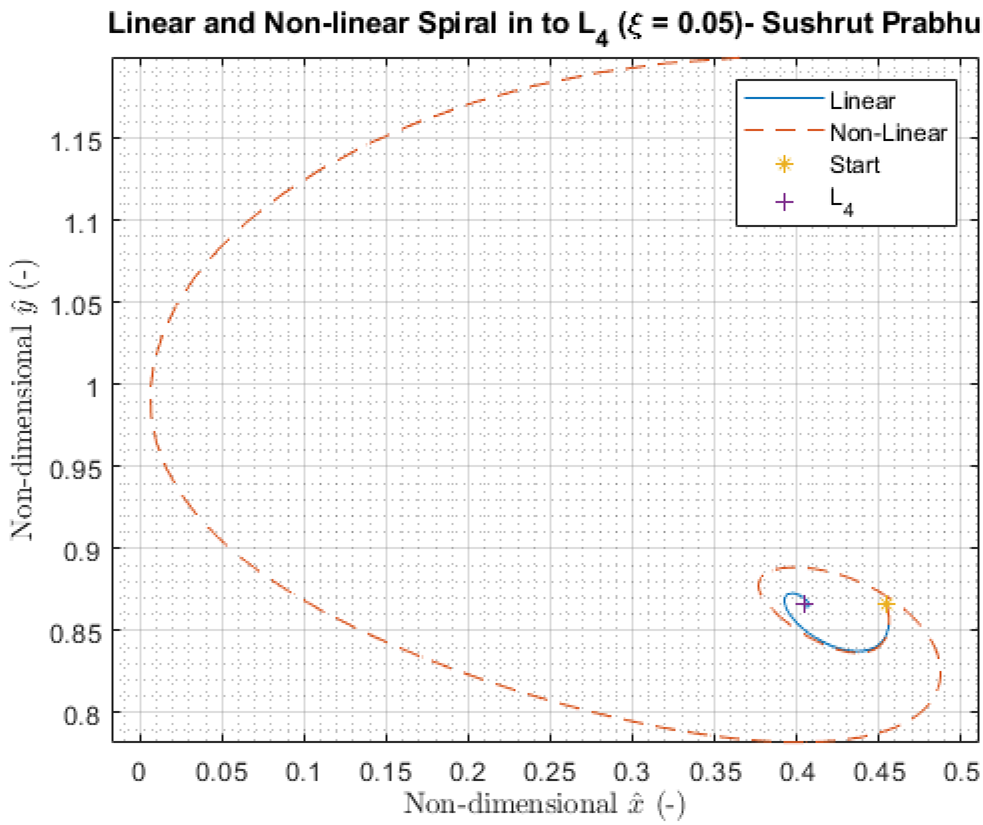
```
SS = SolarS;
systems = {'-', 'Pluto-Charon'};
param = {'l* (km)','m* (kg)','miu' ,'t* (s)', 'L_4 (-)','L_4 (km)'};
G = 6.6738*10^-20;
xi_0 = 0.01;
eta_0 = 0;


dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% Solution
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dCharon_P,0,SS.mCharon/G,SS.mPluto/G);        % Earth Moon
dim_vals{4,2} = SS.mCharon/dim_vals{3,2}/G;

% Lagrange 4
dim_vals{6,2} = L45_NRm(.55,.5,dim_vals{4,2},10^-7);
dim_vals{7,2} = dim_vals{6,2}*dim_vals{2,2};

%
g = 1-27*dim_vals{4,2}*(1-dim_vals{4,2});

Lam1 = 0.5*(-1 + sqrt(g));
Lam2 = 0.5*(-1 - sqrt(g));

lam1 = sqrt(Lam1);
lam2 = -sqrt(Lam1);
lam3 = sqrt(Lam2);
lam4 = -sqrt(lam3);

alpha = abs(real(lam1));
beta = abs(imag(lam1));

[Uxx,Uyy,~,Uxy,~,~]= Unn(dim_vals{6,2}(1),dim_vals{6,2}
(2),0,dim_vals{4,2});

a3 = xi_0;
b3 = eta_0;
b4 = ( a3*(alpha^2-Uxx-beta^2) + b3*(2*alpha - Uxy) +
 alpha*( b3*(alpha^2 - Uyy - beta^2) -a3*(Uxy+2*alpha) ))/(2*alpha^2 *
 beta + 2*beta);
a4 = (a3*(Uxx + beta^2 - alpha^2) + b3*(Uxy-2*alpha) + 2*b4*beta)/
(-2*alpha*beta);
```

```
xidot_0 = -a3*alpha + a4*beta;
etadot_0 = -b3*alpha + b4*beta;

% setup
t = 0:.1:15;
t = t';

% Linear
xi = exp(-alpha*t) .* (a3*cos(beta*t) + a4*sin(beta*t));
eta = exp(-alpha*t) .* (b3*cos(beta*t) + b4*sin(beta*t));
xl = xi + dim_vals{6,2}(1);
yl = eta + dim_vals{6,2}(2);

% Non-linear
r = [xl(1), yl(1), 0];
v = [xidot_0, etadot_0, 0];
IC = [r,v];
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

[~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

figure
plot(xl,yl)
hold on
plot(y(:,1),y(:,2),'--')
plot(xl(1),yl(1),'*')
plot(dim_vals{6,2}(1),dim_vals{6,2}(2),'+')
grid on
grid minor
title(['Linear and Non-linear Spiral in to L_4 (\xi = '
 num2str(xi_0) ')- Sushrut Prabhu'])
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('Linear', 'Non-Linear',  'Start', 'L_4')
axis equal



for xi_0 = [0.001 0.05]
    a3 = xi_0;
    b3 = eta_0;
    b4 = ( a3*(alpha^2-Uxx-beta^2) + b3*(2*alpha - Uxy) +
 alpha*( b3*(alpha^2 - Uyy - beta^2) -a3*(Uxy+2*alpha) ))/(2*alpha^2 *
 beta + 2*beta);
    a4 = (a3*(Uxx + beta^2 - alpha^2) + b3*(Uxy-2*alpha) + 2*b4*beta)/
(-2*alpha*beta);

    xidot_0 = -a3*alpha + a4*beta;
    etadot_0 = -b3*alpha + b4*beta;

    % Linear
    xi = exp(-alpha*t) .* (a3*cos(beta*t) + a4*sin(beta*t));
```

```
    eta = exp(-alpha*t) .* (b3*cos(beta*t) + b4*sin(beta*t));
    xl = xi + dim_vals{6,2}(1);
    yl = eta + dim_vals{6,2}(2);

    % Non-linear
    r = [xl(1), yl(1), 0];
    v = [xidot_0, etadot_0, 0];
    IC = [r,v];
    options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

    [~,y] = ode45(@cr3bp_df,t,IC,options,dim_vals{4,2});

    figure
    plot(xl,yl)
    hold on
    plot(y(:,1),y(:,2),'--')
    plot(xl(1),yl(1),'*')
    plot(dim_vals{6,2}(1),dim_vals{6,2}(2),'+')
    grid on
    grid minor
    title(['Linear and Non-linear Spiral in to L_4 (\xi = '
 num2str(xi_0) ')- Sushrut Prabhu'])
    xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
    ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
    legend('Linear', 'Non-Linear',  'Start', 'L_4')
    axis equal

end
```


Linear and Non-linear Spiral in to L_4 (ξ = 0.01)- Sushrut Prabhu
Linear
Non-Linear
Start
L_4
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
0.878
0.876
0.874
0.872
0.87
0.868
0.866
0.864
0.862
0.395
0.4
0.405
0.41
0.415


Linear and Non-linear Spiral in to L_4 (ξ = 0.001)- Sushrut Prabhu
Linear
Non-Linear
Start
L_4
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
0.8663
0.8662
0.8661
0.866
0.8659
0.8658
0.8657
0.8656
0.8655
0.8654
0.4048
0.405
0.4052
0.4054
0.4056
0.4058



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Finding A_is

In[16]:=
```
eq1 = ξ0 == A1 * Exp[λ1 * t] + A2 * Exp[λ2 * t] + A3 * Exp[λ3 * t] + A4 * Exp[λ4 * t];
eq2 =
  η0 == α1 * A1 * Exp[λ1 * t] + α2 * A2 * Exp[λ2 * t] + α3 * A3 * Exp[λ3 * t] + α4 * A4 * Exp[λ4 * t];
eq3 = ξ0dot == λ1 * A1 * Exp[λ1 * t] + λ2 * A2 * Exp[λ2 * t] +
    λ3 * A3 * Exp[λ3 * t] + λ4 * A4 * Exp[λ4 * t];
eq4 = η0dot == α1 * λ1 * A1 * Exp[λ1 * t] + α2 * λ2 * A2 * Exp[λ2 * t] +
    α3 * λ3 * A3 * Exp[λ3 * t] + α3 * λ4 * A4 * Exp[λ4 * t];

Solve[eq1 && eq2 && eq3 && eq4, {A1, A2, A3, A4}] /.
  {λ2 → -λ1, λ4 → -λ3, α2 → -α1, α4 → -α3} // Simplify
```

Out[20]=

$$\Big\{\Big\{A1 \to \frac{e^{-t\,\lambda 1}\left(-\alpha 1\left(\eta 0dot+\eta 0\,\lambda 1\right)\lambda 3+\alpha 3\left(\eta 0dot\,\lambda 1-\eta 0\,\lambda 1\,\lambda 3+\alpha 1\left(\lambda 1+\lambda 3\right)\xi 0dot\right)\right)}{2\,\alpha 1\,\lambda 1\left(\alpha 3\,\lambda 1-\alpha 1\,\lambda 3\right)},$$

$$A2 \to \frac{e^{t\,\lambda 1}\left(\alpha 1\left(-\eta 0dot+\eta 0\,\lambda 1\right)\lambda 3+\alpha 3\left(\eta 0dot\,\lambda 1-\eta 0\,\lambda 1\,\lambda 3+\alpha 1\left(-\lambda 1+\lambda 3\right)\xi 0dot\right)\right)}{2\,\alpha 1\,\lambda 1\left(\alpha 3\,\lambda 1-\alpha 1\,\lambda 3\right)},$$

$$A3 \to \frac{1}{2\,\alpha 1\,\lambda 1\left(-\alpha 3\,\lambda 1+\alpha 1\,\lambda 3\right)} e^{-t\,\lambda 3}\Big(\alpha 3\left(\eta 0dot\,\lambda 1-\eta 0\,\lambda 1\,\lambda 3-\alpha 1\,\lambda 1^2\,\xi 0+\alpha 1\,\lambda 3\,\xi 0dot\right)+$$
$$\alpha 1\left(-\eta 0\,\lambda 1^2-\eta 0dot\,\lambda 3+\alpha 1\,\lambda 1\left(\lambda 3\,\xi 0+\xi 0dot\right)\right)\Big),$$

$$A4 \to \frac{1}{2\,\alpha 1\,\lambda 1\left(-\alpha 3\,\lambda 1+\alpha 1\,\lambda 3\right)} e^{t\,\lambda 3}\Big(\alpha 1\left(\eta 0\,\lambda 1^2-\eta 0dot\,\lambda 3+\alpha 1\,\lambda 1\,\lambda 3\,\xi 0-\alpha 1\,\lambda 1\,\xi 0dot\right)+$$
$$\alpha 3\left(\eta 0dot\,\lambda 1-\eta 0\,\lambda 1\,\lambda 3-\alpha 1\,\lambda 1^2\,\xi 0+\alpha 1\,\lambda 3\,\xi 0dot\right)\Big)\Big\}\Big\}$$

Characteristic Elements

```
function [lstar, mstar, tstar] = charE(D1,D2,m1,m2)
G = 6.6738*10^-20;

lstar = D1+D2;
mstar = m1 + m2;

tstar = sqrt(lstar^3/G/mstar);
```

```
Not enough input arguments.

Error in charE (line 5)
lstar = D1+D2;
```

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ColinLagrange

```
function [xi,eta] = colinlgrange(xi0,eta0,s,beta3,t)

xi = xi0 * cos(s*t) + eta0/beta3 * sind(s*t);
eta = eta0 *  cos(s*t) - beta3*xi0*sin(s*t);

end
```

```
Not enough input arguments.

Error in colinlgrange (line 5)
xi = xi0 * cos(s*t) + eta0/beta3 * sind(s*t);
```

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cr3bp_df

```
function dx = cr3bp_df(t,x,miu)

dx = zeros(6,1);

d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2 + x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = 2*x(5) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(5) = -2*x(4) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;
dx(6) = -(1-miu)*x(3)/d^3 - miu*x(3)/r^3;


end
```

```
Not enough input arguments.

Error in cr3bp_df (line 6)
d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
```

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horner_alg

```
function [alpha,beta] = horner_alg(n,a,z0)
alpha = a(1);
beta = 0;

for k = 2:n
   beta = alpha + z0*beta;
   alpha = a(k) + z0*alpha;
end


end
```

```
Not enough input arguments.

Error in horner_alg (line 3)
alpha = a(1);
```

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```
function C = Jacobi_C(x,y,z,v,miu)

d = sqrt((x+miu).^2 + y.^2 +z.^2);
r = sqrt((x-1+miu).^2 + y.^2 + z.^2);
C = x.^2 + y.^2 + 2*(1-miu)./d + 2*miu./r - v.^2;

end
```

```
Not enough input arguments.

Error in Jacobi_C (line 3)
d = sqrt((x+miu).^2 + y.^2 +z.^2);
```

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Gamma1 Newton Rhapson

```
function g1_n1 = L1_NRmethod(g1_n, miu, acc)

err = acc*2;
pz = [-1,(3-miu),(2*miu-3),miu,-2*miu,miu];
while (err > acc)
    [fn,fn_p] = horner_alg(6,pz,g1_n);

    g1_n1 = g1_n - fn/fn_p;

    err = abs(g1_n1-g1_n)/abs(g1_n);
    g1_n = g1_n1;
end

end
```

```
Not enough input arguments.

Error in L1_NRmethod (line 4)
err = acc*2;
```

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Gamma2 Newton Rhapson

```
function g2_n1 = L2_NRmethod(g2_n, miu, acc)

err = acc*2;
pz = [1,(3-miu),(3-2*miu),-miu,-2*miu,-miu];
while (err > acc)
    [fn,fn_p] = horner_alg(6,pz,g2_n);

    g2_n1 = g2_n - fn/fn_p;

    err = abs(g2_n1-g2_n)/abs(g2_n);
    g2_n = g2_n1;
end

end
```

```
Not enough input arguments.

Error in L2_NRmethod (line 4)
err = acc*2;
```

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L45_lin

```
function [xi,eta]= L45_lin(t,s1,s2,a1,a2,a3,a4,b1,b2,b3,b4)

xi = a1*cos(s1*t) + a2*sin(s1*t) + a3*cos(s2*t) + a4*sin(s2*t);
eta = b1*cos(s1*t) + b2*sin(s1*t) + b3*cos(s2*t) + b4*sin(s2*t);
```

```
Not enough input arguments.

Error in L45_lin (line 4)
xi = a1*cos(s1*t) + a2*sin(s1*t) + a3*cos(s2*t) + a4*sin(s2*t);
```

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L45_NRm

```
function [FXn1]=L45_NRm(xn,yn,miu,acc)

err = 2*acc;

while err > acc
    d = sqrt((xn+miu)^2 + yn^2);
    r = sqrt((xn+miu-1)^2 + yn^2);
    df1x = -(yn^2-2*(xn+miu)^2)*(1-miu)/d^5 - miu*(yn^2-2*(xn
+miu-1)^2)/r^5 + 1;
    dfxy = 3*yn*(1-miu)*(xn+miu)/d^5 + 3*miu*yn*(xn-1-miu)/r^5;
    df2y = -(-2*yn^2 + (xn+miu)^2)*(1-miu)/d^5 - miu*(-2*yn^2 + (xn
+miu-1)^2)/r^5 + 1;

    FXn = [-(1-miu)*(xn+miu)/d^3-miu*(xn-1+miu)/r^3+xn; -(1-miu)*yn/
d^3-miu*yn/r^3+yn];

    J = [df1x, dfxy; dfxy, df2y];

    FXn1 = [xn;yn] - J^-1 * FXn;
    err = max(abs([xn;yn] - FXn1)./abs([xn;yn]));
    xn = FXn1(1); yn = FXn1(2);


end

end
```

```
Not enough input arguments.

Error in L45_NRm (line 4)
err = 2*acc;
```

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lin45ode

```
function dx = lin45ode(~,x,Uxx,Uyy,Uxy)

dx = zeros(4,1);

dx(1) = x(3);
dx(2) = x(4);
dx(3) = Uxx*x(1)+Uxy*x(2)+2*x(4);
dx(4) = Uyy*x(2)+Uxy*x(1)-2*x(3);


end
```

```
Not enough input arguments.

Error in lin45ode (line 6)
dx(1) = x(3);
```

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Solar Systems Constants

Constants

```
classdef SolarS
    properties
        % Distances from Sun/ planet to planet/ Moon
        dMercury = 57909226.542
        dVenus = 108209474.537
        dEarth = 149597870.7
        dMars = 227943816.693
        dJupiter = 778340816.693
        dSaturn = 1426666414.180
        dM_E = 384400
        dMoon = 384400+149597870.7
        dPluto = 5906440596.5288
        dTitan_S = 1221865
        dPhobos_M = 9376;
        dEuropa_J = 671100
        dOberon_U = 583500
        dTriton_N = 354759
        dCharon_P = 17536

        % Mass of Planet/Star/ Moon (G*m)
        mSun = 132712440017.99
        mVenus = 324858.5988
        mEarth = 398600.4415
        mMars = 42828.3142
        mJupiter = 126712767.8578
        mSaturn = 37940626.0611
        mUranus = 5794549.0070719
        mNeptune = 6836534.06387
        mPluto = 981.600887707;

        mMoon = 4902.8011
        mTitan = 8979.766
        mPhobos = 7.11328968e-04
        mEuropa = 3203.31978
        mOberon = 192.4249
        mTriton = 1427.8589
        mCharon = 103.2187

        % Radius of Moon/Planet
        rEarth = 6378.136;
        rMars = 3397.00;
        rMoon = 1738.10;
        rSaturn = 60268.00;
        rJupiter = 71492.00;
        % Eccentricity of Planets
        eEarth = 0.01671022
        eSaturn = 0.05386179
```

```
    end
end
```

```
ans =

  SolarS with properties:

     dMercury: 5.7909e+07
       dVenus: 1.0821e+08
       dEarth: 1.4960e+08
        dMars: 2.2794e+08
     dJupiter: 7.7834e+08
      dSaturn: 1.4267e+09
         dM_E: 384400
        dMoon: 1.4998e+08
       dPluto: 5.9064e+09
     dTitan_S: 1221865
    dPhobos_M: 9376
    dEuropa_J: 671100
    dOberon_U: 583500
    dTriton_N: 354759
    dCharon_P: 17536
         mSun: 1.3271e+11
       mVenus: 3.2486e+05
       mEarth: 3.9860e+05
        mMars: 4.2828e+04
     mJupiter: 1.2671e+08
      mSaturn: 3.7941e+07
      mUranus: 5.7945e+06
     mNeptune: 6.8365e+06
       mPluto: 981.6009
        mMoon: 4.9028e+03
       mTitan: 8.9798e+03
      mPhobos: 7.1133e-04
      mEuropa: 3.2033e+03
      mOberon: 192.4249
      mTriton: 1.4279e+03
      mCharon: 103.2187
       rEarth: 6.3781e+03
        rMars: 3397
        rMoon: 1.7381e+03
      rSaturn: 60268
     rJupiter: 71492
       eEarth: 0.0167
      eSaturn: 0.0539
```

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Unn

```
function [Uxx,Uyy,Uzz,Uxy,Uxz,Uyz]= Unn(x,y,z,miu)

xm = (x+miu);
xm1 = x+miu-1;

d = sqrt(xm^2 + y^2 +z^2);
r = sqrt(xm1^2 + y^2 +z^2);

Uxx = 1-(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*xm^2 / d^5 + 3*miu*xm1^2 /
 r^5;
Uyy =  1-(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*y^2 / d^5 + 3*miu*y^2 /
 r^5;
Uzz = -(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*z^2 / d^5 + 3*miu*z^2 / r^5;

Uxy = 3*(1-miu)*xm*y/d^5 + 3*miu*xm1*y/r^5;
Uxz = 3*(1-miu)*xm*z/d^5 + 3*miu*xm1*z/r^5;
Uyz = 3*miu*y*z/d^5 + 3*miu*y*z/r^5;
end
```

```
Not enough input arguments.

Error in Unn (line 4)
xm = (x+miu);
```

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ZVC_bonus

```
function ZVC_bonus(x1,C0,miu,guess2)

n = 1;
nn = length(x1);
nn_2 = round(nn/2);

for xn = x1
    y1(n)= ZVC_NR(3,xn,miu,C0,10^-10);
    n = n+1;
end

y1_s = max(find(isnan( y1(1:nn_2))))+1;
y1_e = nn_2+min(find(isnan( y1(nn_2:end))))-2;

x1y1 =[x1(y1_s:y1_e),fliplr(x1(y1_s:y1_e));y1(y1_s:y1_e),-
fliplr(y1(y1_s:y1_e))]';
```

Not enough input arguments.

Error in ZVC_bonus (line 5)
nn = length(x1);

Second Curve

```
x2 = x1(y1_s:y1_e);
n = 1;
for xn = x2
    y2(n)= ZVC_NR(y1(y1_s+n)*guess2,xn,miu,C0,10^-10);
    n = n+1;
end
x2y2 = [x2;y2]';

m = 1;

for n = 1:length(y2)
    if (isnan(y2(n)) || abs(y2(n) - x1y1(n,2)) < 10^-5|| abs(y2(n) +
 x1y1(n,2)) < 10^-5 )
        del(m) = n;
        m = m+1;
    end
end

x2y2(del,:)=[];
x2y2 = [x2y2;flipud(x2y2(:,1)),-flipud(x2y2(:,2))];

xy = [x1y1;x2y2];

figure
plot(xy(:,1),xy(:,2),'.k','MarkerSize',1)
hold on
```

```
% plot(x2y2(:,1),x2y2(:,2),'.k','MarkerSize',1)
axis equal
grid on
grid minor
title(['ZVC Curve for C_0 = ',num2str(C0)])
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
```

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ZVC_NR

```
function [yn] = ZVC_NR(yn,x,miu,C,acc)

err = acc*2;
n = 1;

while (err > acc)
    fn = x^2 + yn^2 + 2*(1-miu)/sqrt((x+miu)^2 + yn^2) + 2*miu/sqrt((x
+miu-1)^2 + yn^2) - C;
    fn_p = 2*yn - 2*yn*(1-miu)/sqrt((x+miu)^2 + yn^2)^3  - 2*yn*miu/
sqrt((x+miu-1)^2 + yn^2)^3;

    yn1 = yn - fn/fn_p;

    err(n) = abs(yn1-yn)/abs(yn);

    n = n+1;
    yn = yn1;

    if n > 100
        err = acc/2;
        yn = NaN(1,1);
    elseif abs(imag(yn)) > 0
        yn = 0.1;
    end
end



end
```

```
Not enough input arguments.

Error in ZVC_NR (line 4)
err = acc*2;
```

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