Sushant Prabhu | AAE632 PSF1

PSF1

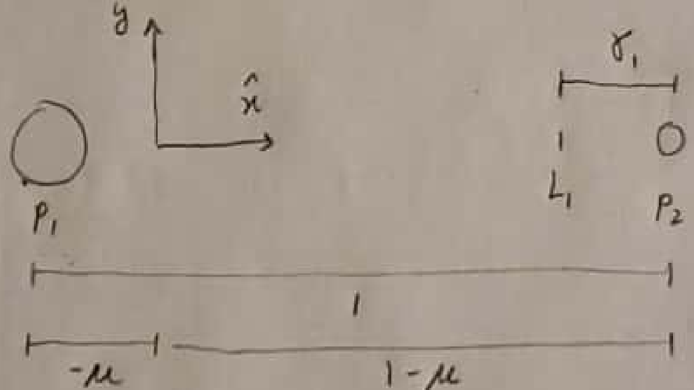
Given: Earth-moon system at L_1 point

Find: a) Eigenvalues and eigenvectors. Do the posses stable and unstable mode? Plot L_1 and eigenvectors position and direction. Are they orthogonal? Identify E^S and E^U

b) Move along $\hat{x}_1 \rightarrow \hat{\xi}$ $x_0 = 15, 60, 200$ km. Plot the path. Comment on path. Do you need ZVC?

Solution:

a) It has been a while since we derived the L_1 libration point so let us refresh our memory.



∴ We can use this polynomial to solve for:

$$f(\gamma_1) = -\gamma_1^5 + (3-\mu)\gamma_1^4 + (2\mu-3)\gamma_1^3 + \mu\gamma_1^2 - 2\mu\gamma_1 + \mu$$

Just use Horner's algorithm with Newton's method solve for γ_1

$$\left.\begin{aligned} x_{L_1} &= 1-\mu-\gamma_1 \\ y_{L_1} &= 0 \\ z_{L_1} &= 0 \end{aligned}\right\} \rightarrow \text{position of } L_1$$

continued...

Now to find equilibrium point at L_1 we need to linearize the non-linear equation about point L_1. We have already linearized 3BCRP equation before

Non-linear equations:

$$\ddot{x} - 2n\dot{y} - n^2 x = \frac{-(1-\mu)(x+\mu)}{d^3} - \frac{\mu(x-1+\mu)}{r^3}$$

$$\ddot{y} + 2n\dot{x} - n^2 y = -\frac{(1-\mu)y}{d^3} - \frac{\mu y}{r^3}$$

$$\ddot{z} = -\frac{(1-\mu)z}{d} - \frac{\mu z}{r^3}$$

$$d = [(x+\mu)^2 + y^2 + z^2]^{1/2} \qquad r = [(x+x-1)^2 + y^2 + z^2]^{1/2}$$

After an expansion about the equilibrium point:

$$\left.\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= U^*_{xx}\xi + U^*_{xy}\eta + U^*_{xz}\zeta \\ \ddot{\eta} + 2\dot{\xi} &= U^*_{yx}\xi + U^*_{yy}\eta + U^*_{yz}\zeta \\ \ddot{\zeta} &= U^*_{zx}\xi + U^*_{zy}\eta + U^*_{zz}\zeta \end{aligned}\right\} \text{Linear equations}$$

$$U^*_{xx} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)(x+\mu)^2}{d^5} + \frac{3\mu(x-1+\mu)^2}{r^5}$$

$$U^*_{yy} = 1 - \frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)y^2}{d^5} + \frac{3\mu y^2}{r^5}$$

$$U^*_{zz} = -\frac{(1-\mu)}{d^3} - \frac{\mu}{r^3} + \frac{3(1-\mu)z^2}{d^5} + \frac{3\mu z^2}{r^5}$$

$$U^*_{xy} = U^*_{yx} = \frac{3(1-\mu)(x+\mu)y}{d^5} + \frac{3\mu(x+\mu-1)y}{r^5}$$

$$U^*_{xz} = U^*_{zx} = \frac{3(1-\mu)(x+\mu)z}{d^5} + \frac{3\mu(x+\mu-1)z}{r^5}$$

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continue...

$$U^*_{zy} = U^*_{yz} = \frac{3(1-\mu)yz}{d^5} + \frac{3\mu yz}{r^5}$$

In our case $y=0$ and $z=0$ $\therefore U^*_{zy} = U^*_{yz} = U^*_{xz} = U^*_{zx} = 0$

Let us write the linear equations in a state space form and leave the z elements in despite it being a planar problem

$$\dot{\bar{x}} = [A]\,\bar{x} \qquad \bar{x} = \begin{bmatrix} \xi \\ \eta \\ \zeta \\ \dot{\xi} \\ \dot{\eta} \\ \dot{\zeta} \end{bmatrix} \qquad \dot{\bar{x}} = \begin{bmatrix} \dot{\xi} \\ \dot{\eta} \\ \dot{\zeta} \\ \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \end{bmatrix}$$

$$\therefore A = \left.\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ U^*_{xx} & U^*_{xy} & U^*_{xz} & 0 & 2 & 0 \\ U^*_{yx} & U^*_{yy} & U^*_{yz} & -2 & 0 & 0 \\ U^*_{zx} & U^*_{zy} & U^*_{zz} & 0 & 0 & 0 \end{bmatrix}\right\} \text{Find eigenvalues and eigenvectors of this matrix}$$

$\lambda_1 = -2.9321$ $\quad v_1 = [0.2932,\ 0.1349,\ 0,\ -0.8598,\ -0.3956,\ 0]^T$

$\lambda_2 = 2.9321$ $\quad v_2 = [-0.2932,\ 0.1349,\ 0,\ -0.8598,\ 0.3956,\ 0]^T$

$\lambda_3 = 2.3344i$ $\quad v_3 = [-0.1058,\ -0.3793i,\ 0,\ -0.2469i,\ 0.8854,\ 0]^T$

$\lambda_4 = -2.3344i$ $\quad v_4 = [-0.1058,\ 0.3793i,\ 0,\ 0.2469i,\ 0.8854,\ 0]^T$

$\lambda_5 = 2.2688i$ $\quad v_5 = [0,\ 0,\ 0.4033i,\ 0,\ 0,\ 0.9151]^T$

$\lambda_6 = -2.2688i$ $\quad v_6 = [0,\ 0,\ -0.4033i,\ 0,\ 0,\ 0.9151]^T$

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Continued...

The eigenvalues do posses stable and unstable characteristics. λ_1 is stable and λ_2 is unstable. Others are periodic

See Figure: F1.1 for plot of E^s and E^u

The eigenvectors are not orthogonal. Visually we can see this but $\bar{v}_1 \cdot \bar{v}_2 = 0.515 \neq 0$

b) In order to calculate $\dot{\eta}$ we need C - Jacobi constant

$$x_{L_1}^2 + \cancel{y^2}^{\,0} + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} = C_{L_1} \sim \text{Jacobi at } L_1$$

$$\therefore x_0^2 + \frac{2(1-\mu)}{d} + \frac{2\mu}{r} - v^2 = C_{L_1} \quad \rightarrow \text{here } v = \dot{\eta}$$

$$\therefore \dot{\eta} = \sqrt{C_{x_0} - C_{L_1}}$$

C_{x_0} is Jacobi Constant at $\dot{x}_0$ with $v = 0$

See Figure: F1.2, F1.3, F1.4 and F1.5. The smaller the perturbation the closer to the eigen space. Also spacecraft appears to arrive through the stable space and leave by unstable. It also does a loop near L_1 as it changes eigen space

The zoomed out versions show that the eigenvectors cross the ZVC which is not possible. Since this is a linear approximation it is only true near libration points

PSF1

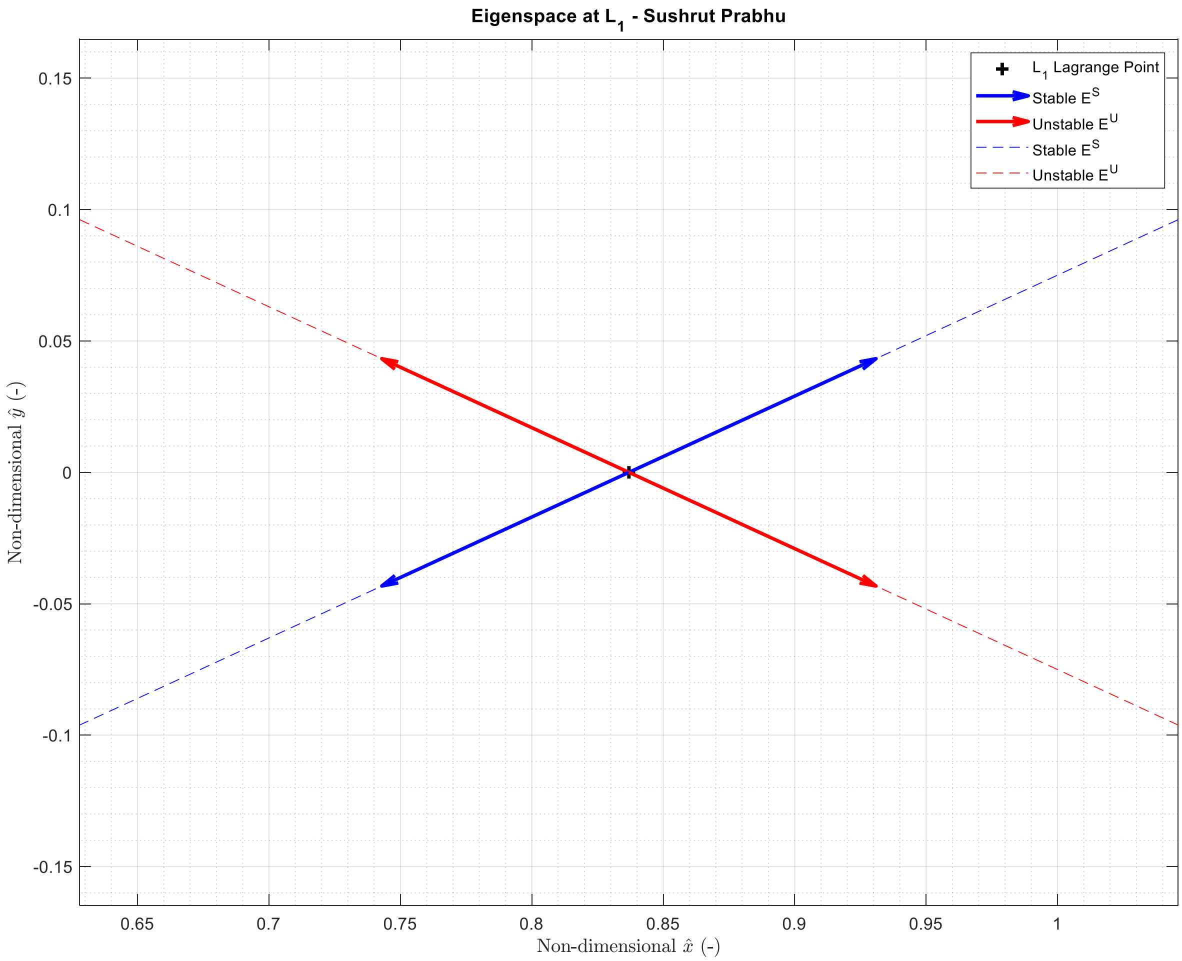

Figure F1.1: The eigenvectors of the position space.

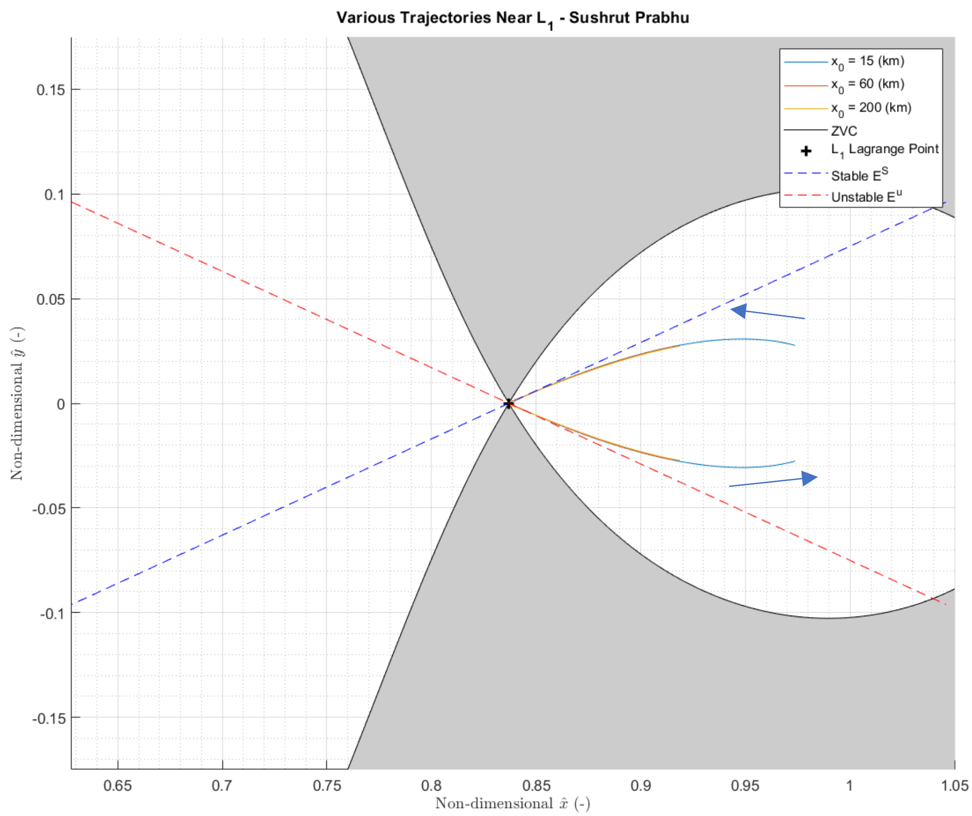

Figure F1.2: Zoomed out view of the trajectories with positive velocity.

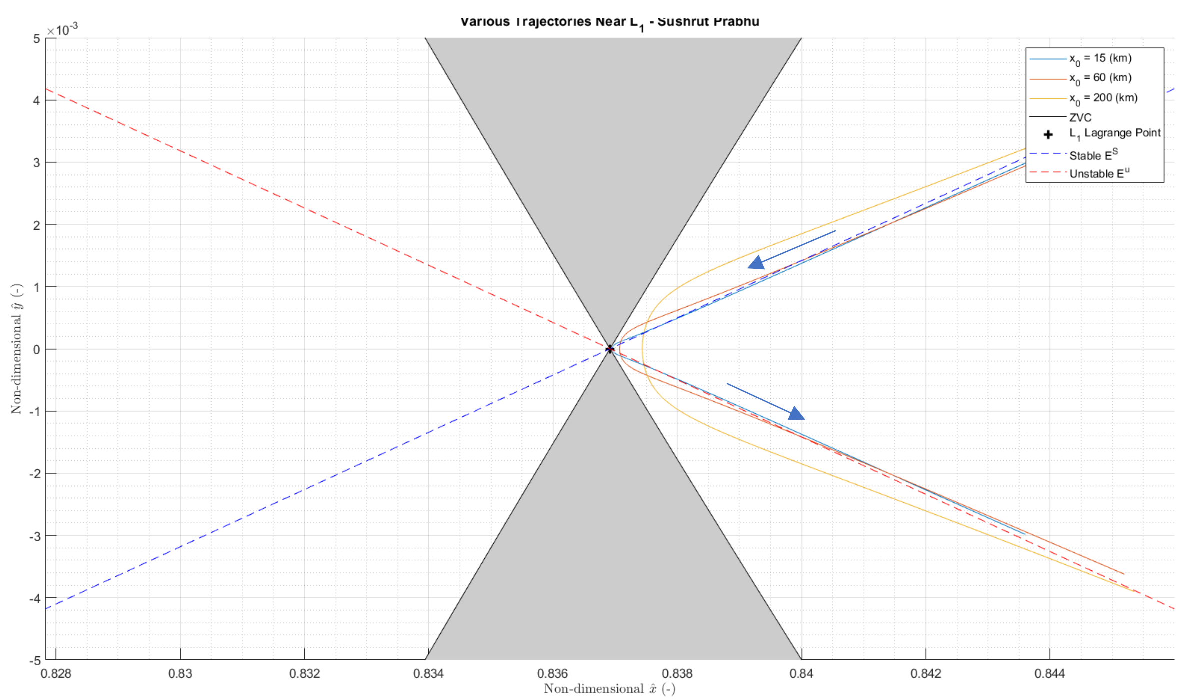

Figure F1.3: Zoomed in view of the trajectories with positive velocity.

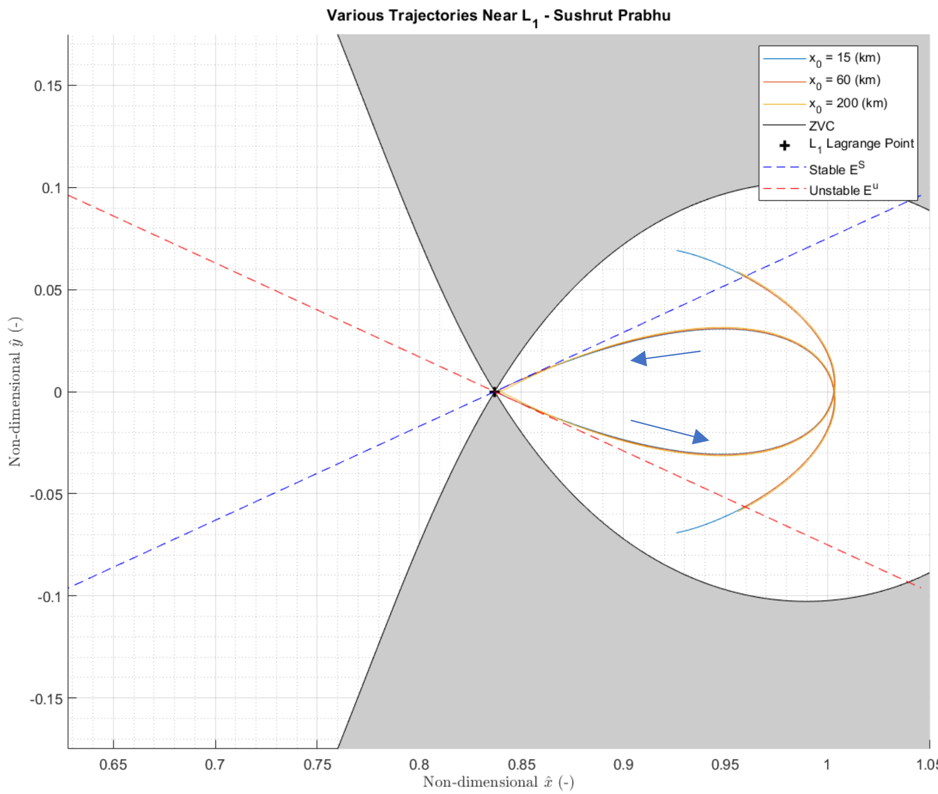

Figure F1.4: Zoomed out view of the trajectories with negative velocity.

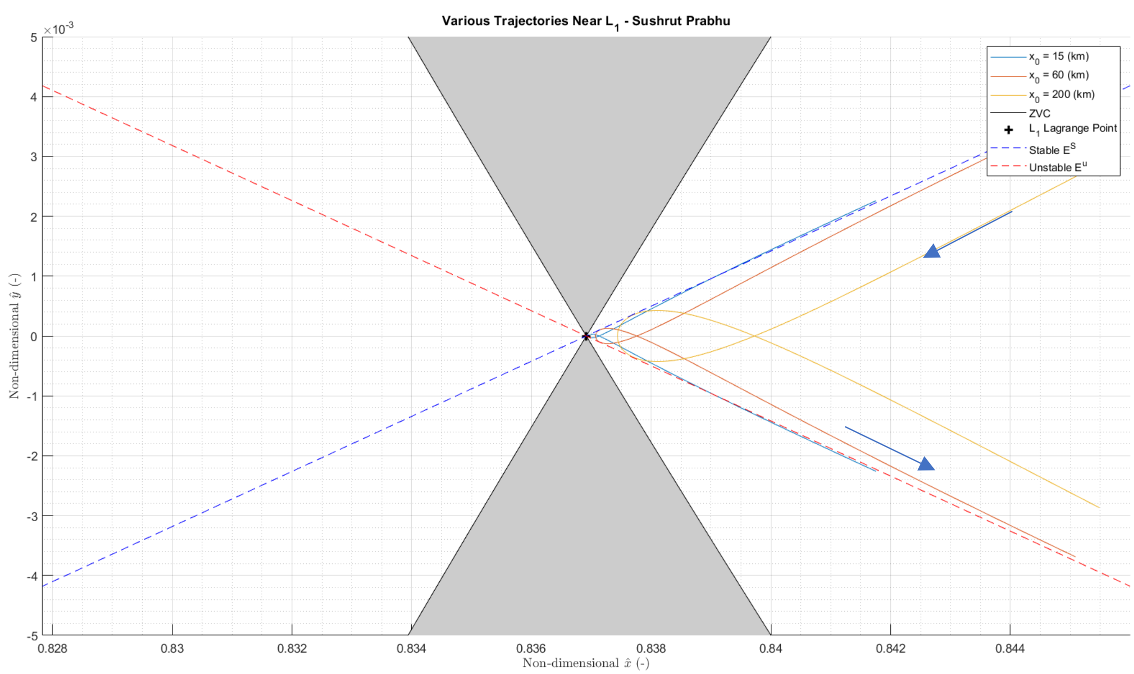

Figure F1.5: Zoomed in view of the trajectories with negative velocity.

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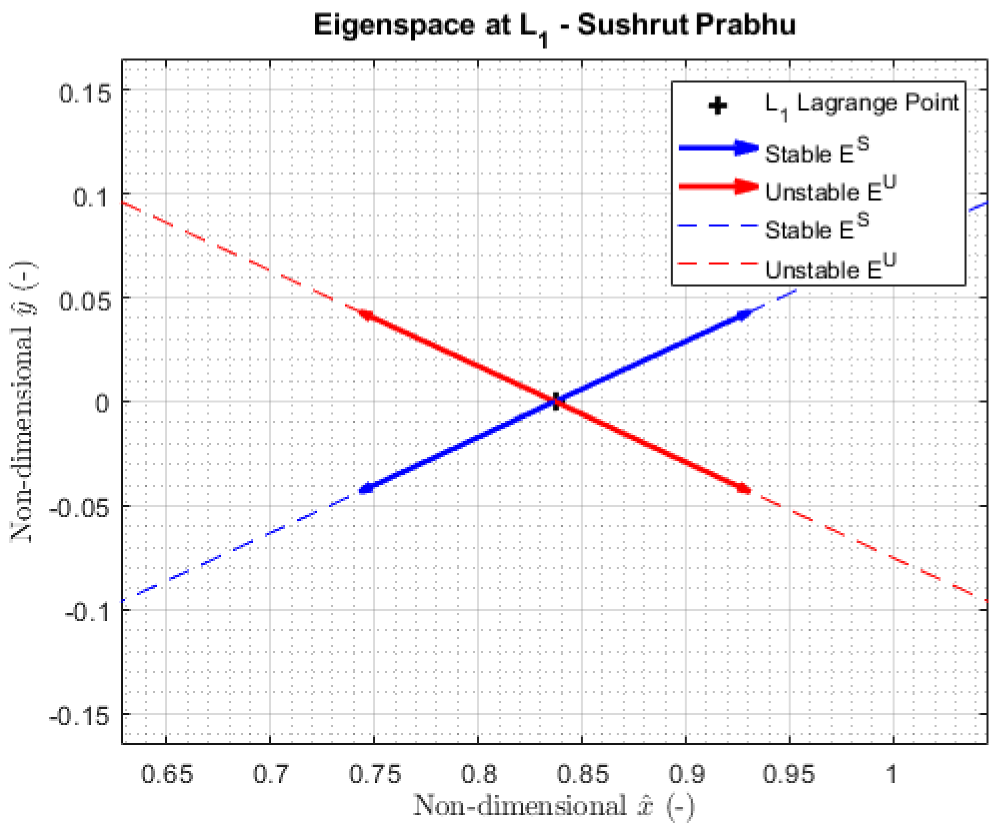
PSF1

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

Part a)

```
A = A_t(dim_vals{7,2},0,0,dim_vals{4,2});
[V,D] = eig(A);
D = diag(D);
e1 = V(1:2,1)/norm(V(1:2,1))*.23;
e2 = V(1:2,2)/norm(V(1:2,2))*.23;



figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
quiver(dim_vals{7,2},0,e1(1)*.5,e1(2)*.5,'b','LineWidth',2)
quiver(dim_vals{7,2},0,e2(1)*.5,e2(2)*.5,'r','LineWidth',2)
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
```

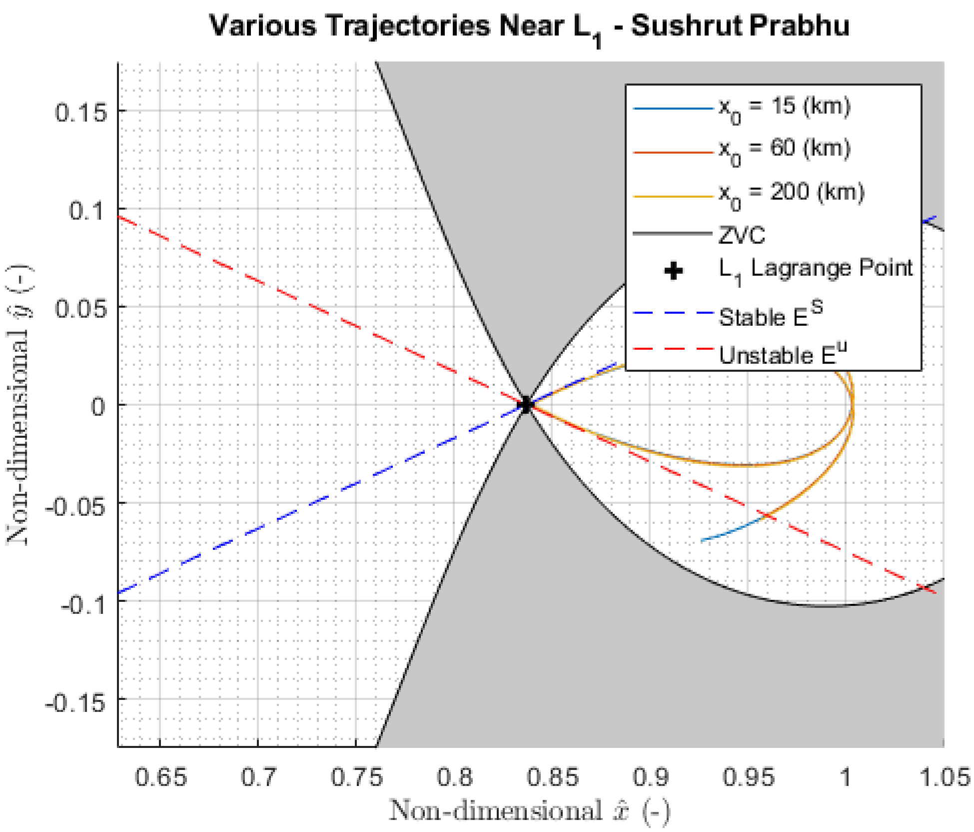
```
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
quiver(dim_vals{7,2},0,-e1(1)*.5,-e1(2)*.5,'b','LineWidth',2)
quiver(dim_vals{7,2},0,-e2(1)*.5,-e2(2)*.5,'r','LineWidth',2)
axis equal
grid on
grid minor
legend('L_1 Lagrange Point','Stable E^S','Unstable E^U','Stable
 E^S','Unstable E^U')
title('Eigenspace at L_1 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
```



Part b)

```
C_L1 = Jacobi_C(dim_vals{7,2},0,0,0,dim_vals{4,2});

x0_dim = [15, 60, 200];
x0 = x0_dim/dim_vals{2,2};
t_end = [3 2.4 2];
% t_end = [2.3 1.7 1.3]
% t_end = [1.7 1.4 1];


figure
hold on
for n = 1:length(x0)
```

```
    eta_dot = sqrt(Jacobi_C(dim_vals{7,2}+x0(n),0,0,0,dim_vals{4,2}) -
 C_L1);

    IC = [dim_vals{7,2}+x0(n) 0 0 0 eta_dot 0];

    [~,y1] = ode45(@cr3bp_df,[0 -t_end(n)],IC,options,dim_vals{4,2});
    [~,y2] = ode45(@cr3bp_df,[0 t_end(n)],IC,options,dim_vals{4,2});

    plot([flip(y1(:,1));y2(:,1)],[flip(y1(:,2));y2(:,2)])
end
[X,Y] = meshgrid(.7:0.001:1.05,-0.1750:0.001:0.1750);
% [X,Y] = meshgrid(.83:0.00001:0.84,-0.005:0.00001:0.005);
C = Jacobi_C(X,Y,0,0,dim_vals{4,2});

m = 2;
map = ones(m , 3)*.8;

colormap(map);

contourf(X,Y,-C,-[C_L1 C_L1]);
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
axis equal
grid on
grid minor
legend(['x_0 = ',num2str(x0_dim(1)), ' (km)'],
 ['x_0 = ',num2str(x0_dim(2)), ' (km)' ] ,['x_0 =
 ',num2str(x0_dim(3)), ' (km)'], 'ZVC', 'L_1 Lagrange Point', 'Stable
 E^S', 'Unstable E^u')
title('Various Trajectories Near L_1 - Sushrut Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
```



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PSF 2

Given: The L_1 liberation points for the Earth-Moon system

Find: a) Shift off equilibrium point along local stable and unstable direction. How is that accomplished?
How far should you step? Does it matter?
Propagate +ve or -ve time? Propagate +ve and -ve

b) If you propagate from L_1 does it reach L_2? Moon? Earth?

c) Overlay manifold with E3, comments?

d) Same analysis for L_2. Do you reach L_1? L_1 and L_2 plots. Similar to class?

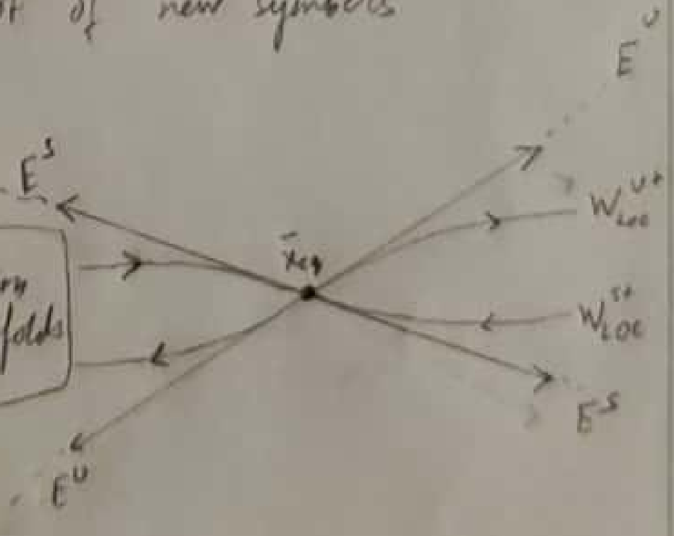
Solution:

After finding the eigenspaces we can use those to create manifolds.

Here are the steps: (Example for stable Manifold)

$\bar{x}_{U+} = \bar{x}_{eq} + d\bar{V}^{WU}$ ← Lot of new symbols

$$\bar{V}^{WU} = \frac{\bar{V}^{U}}{[x_0^2 + y_0^2 + z_0^2]^{1/2}}$$



$\boxed{\therefore\ \bar{x}_{U+} = \bar{x}_{eq} + d \cdot \bar{V}^{WU}}$ ← Same equation for all manifolds

Now the question is how do we pick d

continued...

In order to understand how to pick d, I plotted some variations of d and the effect on manifold. All of these solutions had a reasonable Jacobi.
Note: As you go closer there can be issues with integration.

Eventhough we can use same equation to find each manifold this does not mean "d" will be the same. But there is symmetry about x-axis

See Figures: F2.3 and F2.7

These show the closest approach to the L_1 point and that within the ranges chosen the "d" value doesn't matter. But also note the difference. There is a symmetry for the solution about the x-axis but the Earth side start vs Lunar side start. Evident in the trajectories in Figures: F2.1, F2.2, F2.5, and F2.6

These aspects are also evident in the relationship of "d" and time The closer you start the more time to reach and get away L_1 point.
See Figures: F2.4 and F2.8

Continued...

b) The manifold can never reach L_2. Firstly, this is evident in the Figure: F2.10. Furthermore, the Jacobi constant of the manifold also suggests that it is physically impossible to get to L_2.
The figure shows that it is possible to reach the Earth. Moreover, it also goes around the Moon.
See Figures: F2.10, F2.11, and F2.12

c) After propagating the periodic orbit for a reasonably long time we see it depart on an unstable manifold. See Figures F2.11 and F2.12

d) The L_2 orbit does reach L_1 or atleast close to it. This makes sense as Jacobi would allow it Figure F2.13

Similar to the periodic orbit at L_1 the orbit at L_2 after some time has a departurture almost parallel to the unstable manifold.
See Figures: F2.14 and F2.15

See all local manifold. Figure F2.16.
They look similar to the ones in class

PSF2

Part a)

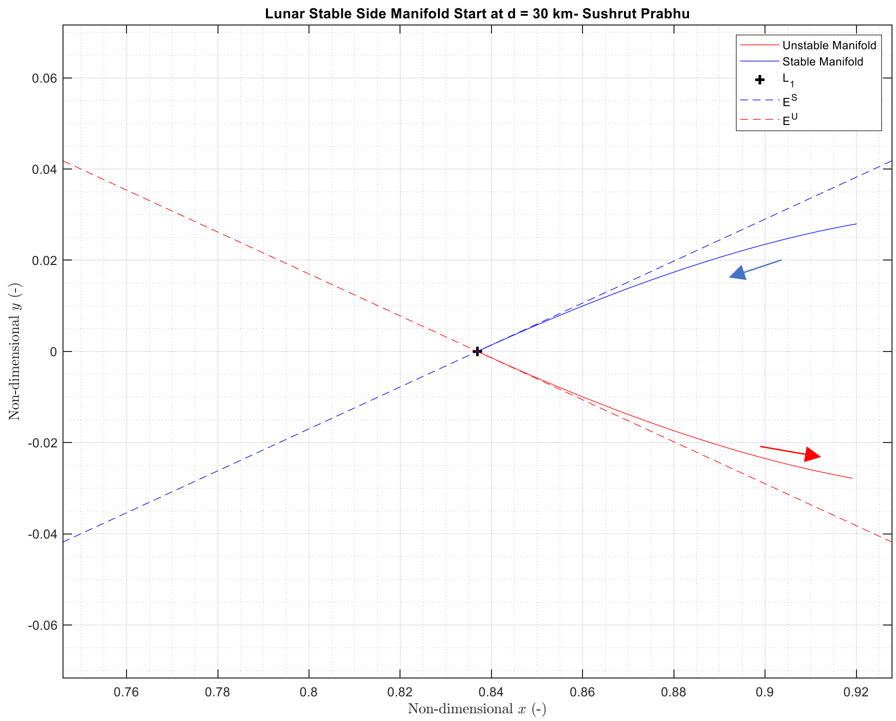


Figure F2.1: Lunar side starting point on stable eigenspace forward and backwards propagated.

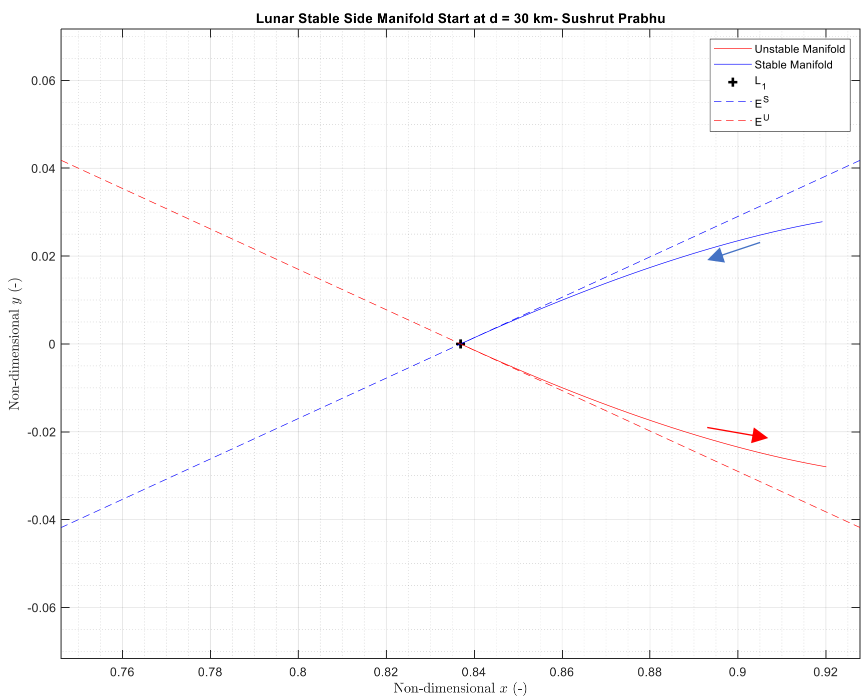


Figure F2.2: Lunar side starting point on unstable eigenspace forward and backwards propagated.

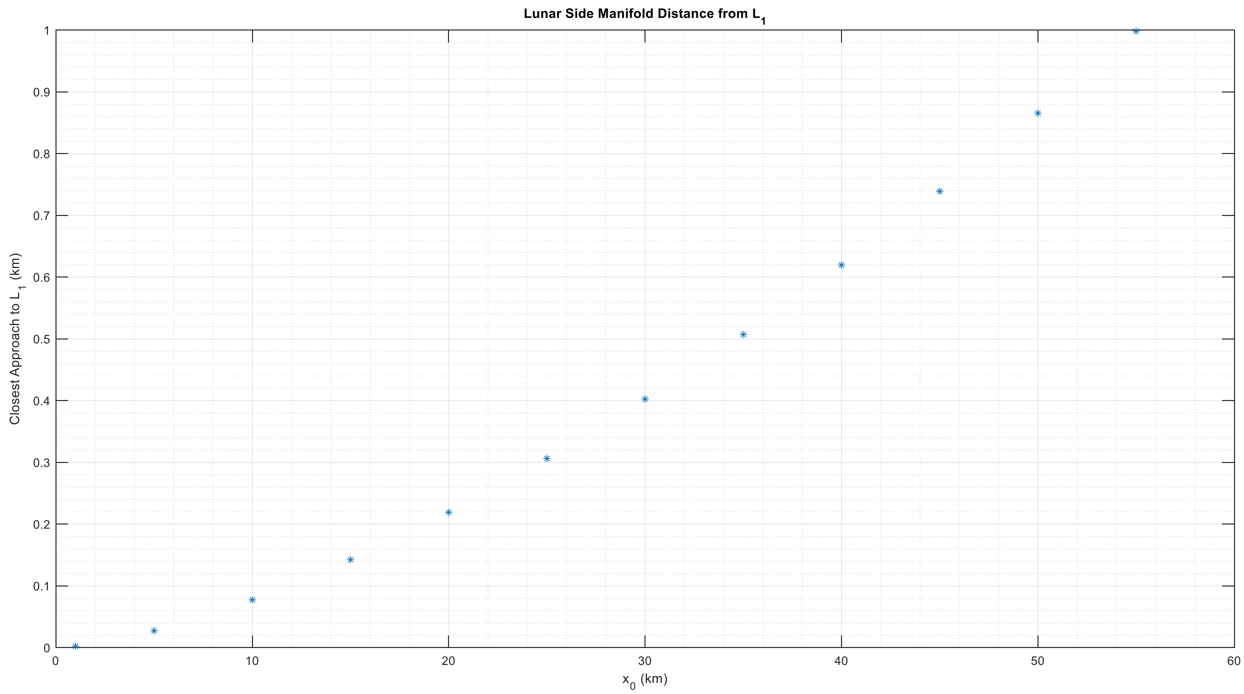


Figure F2.3: Lunar side starting point closest approach for various perturbations.

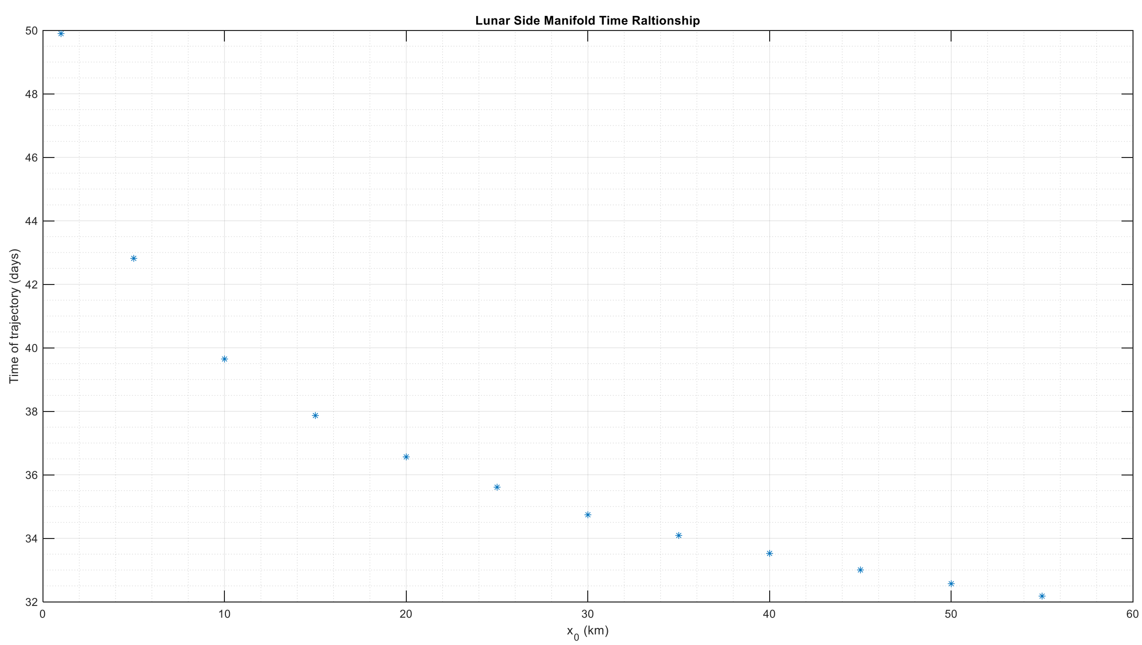


Figure F2.4: Lunar side starting point time taken to reach a similar point.

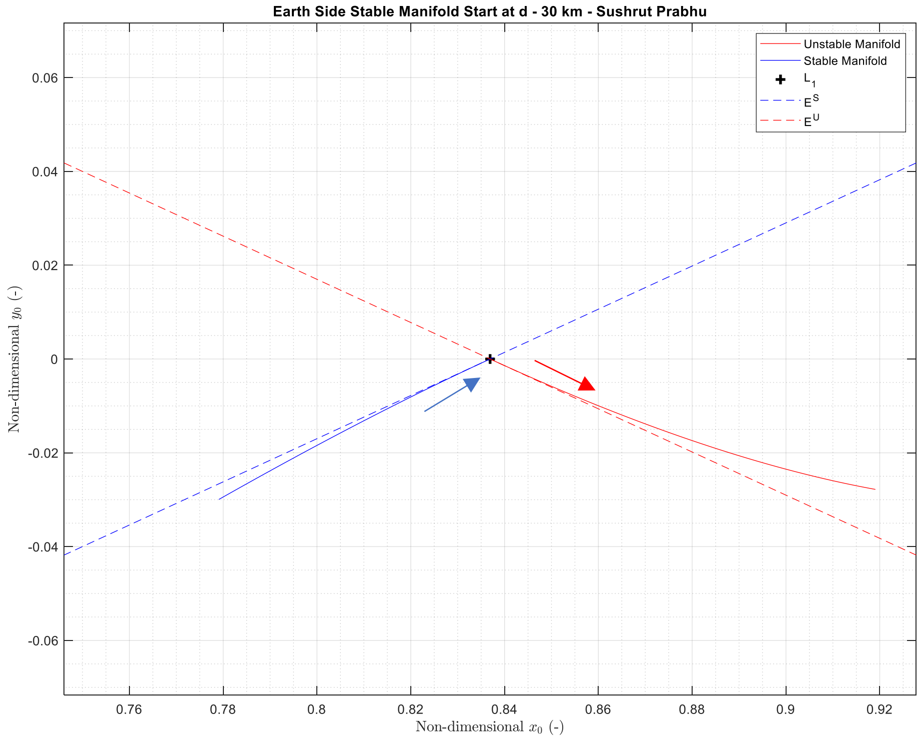


Figure F2.5: Earth side starting point on stable eigenspace forward and backwards propagated.

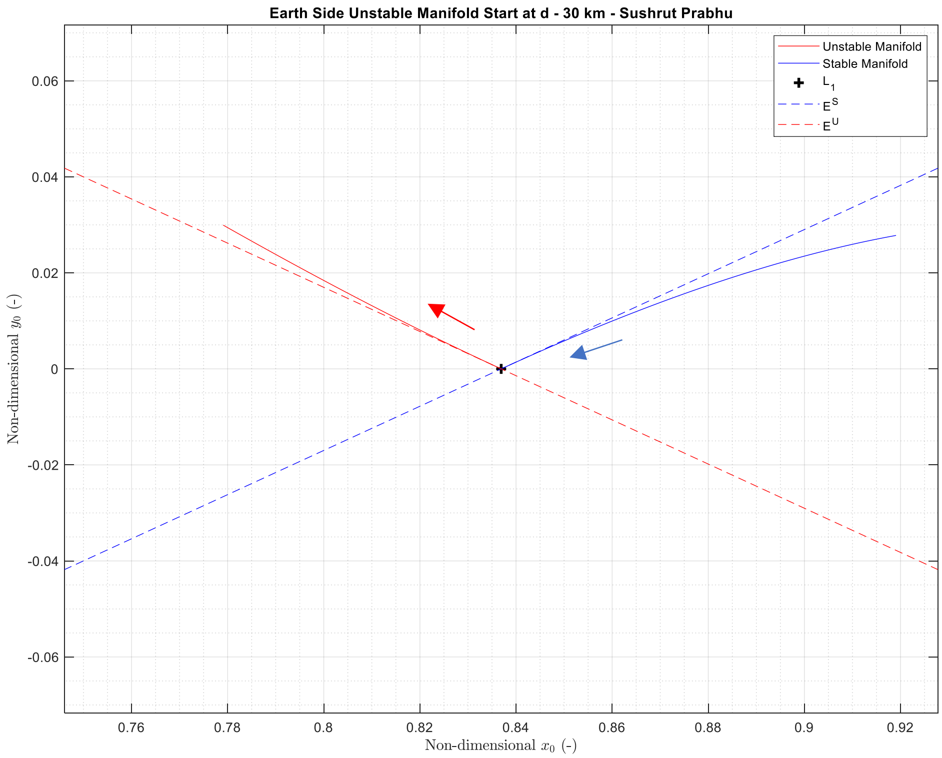


Figure F2.6: Earth side starting point on unstable eigenspace forward and backwards propagated.

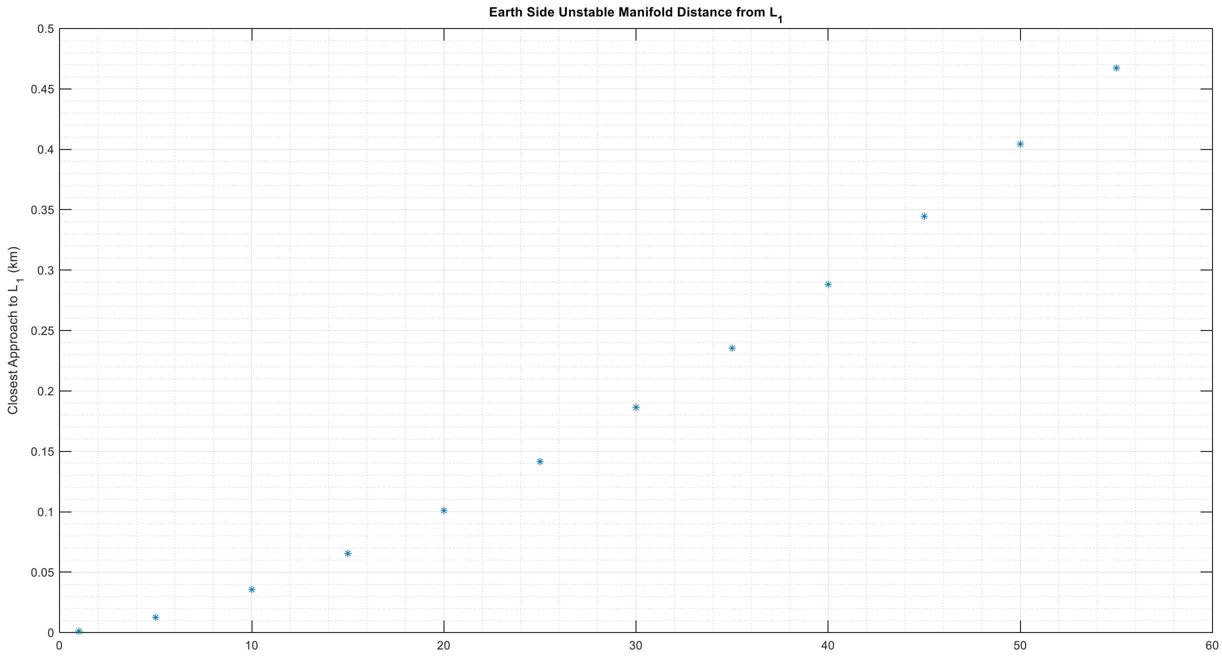


Figure F2.7: Earth side starting points closest approach for various perturbations.

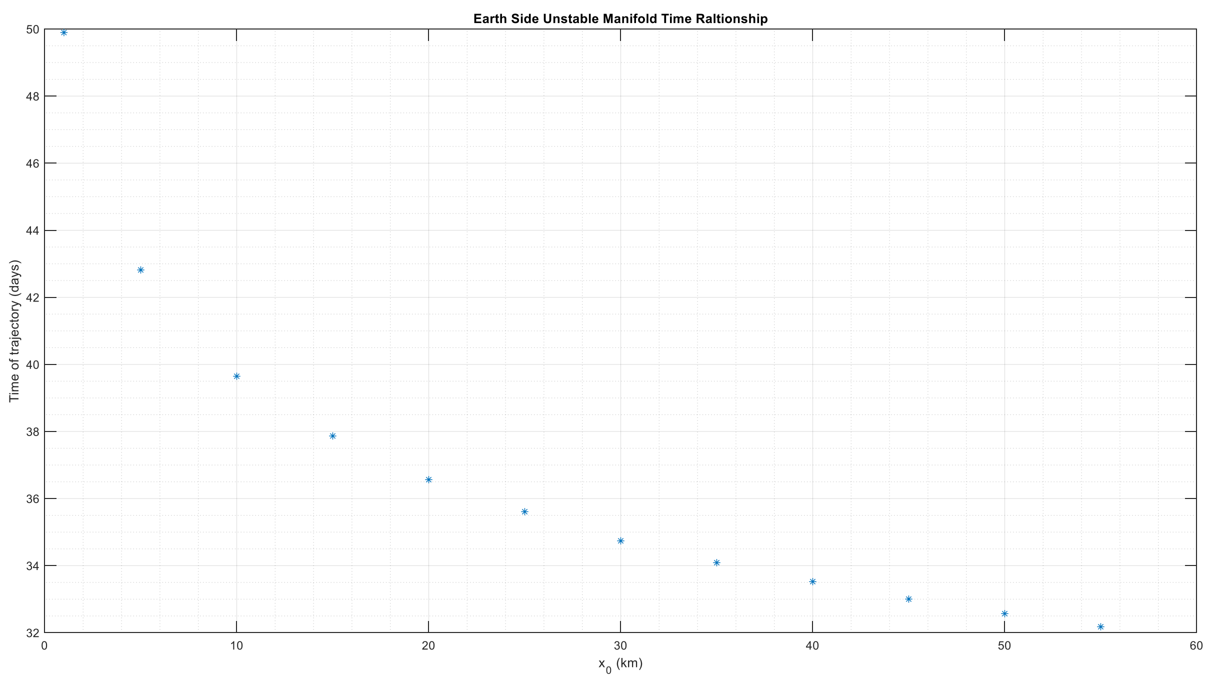


Figure F2.8: Earth side starting points time taken to reach a similar point.

Part b)

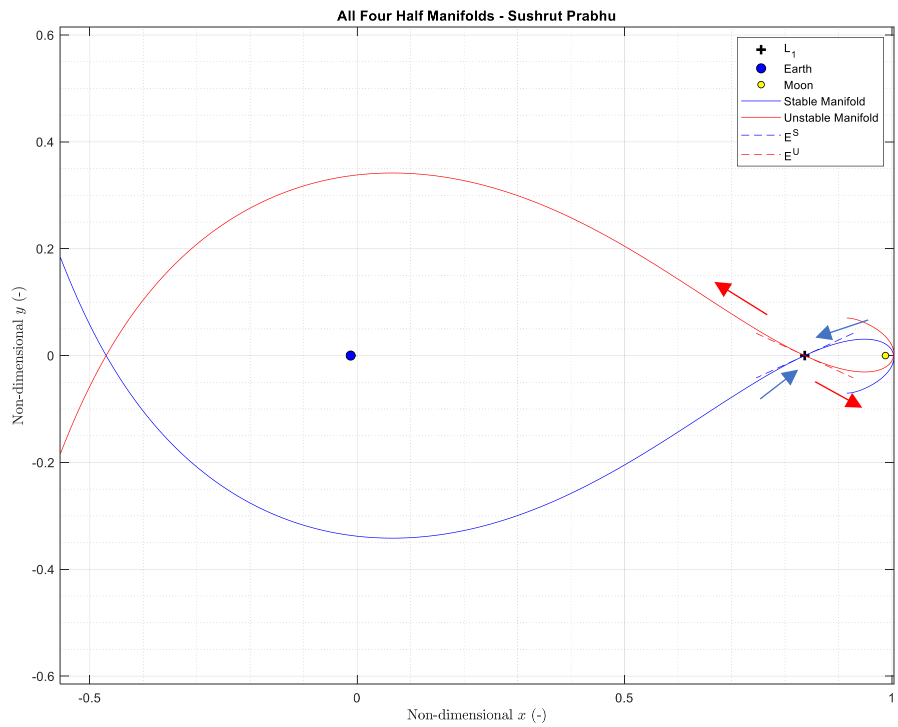


Figure F2.10: Four half manifolds at L1 point.

Part c)

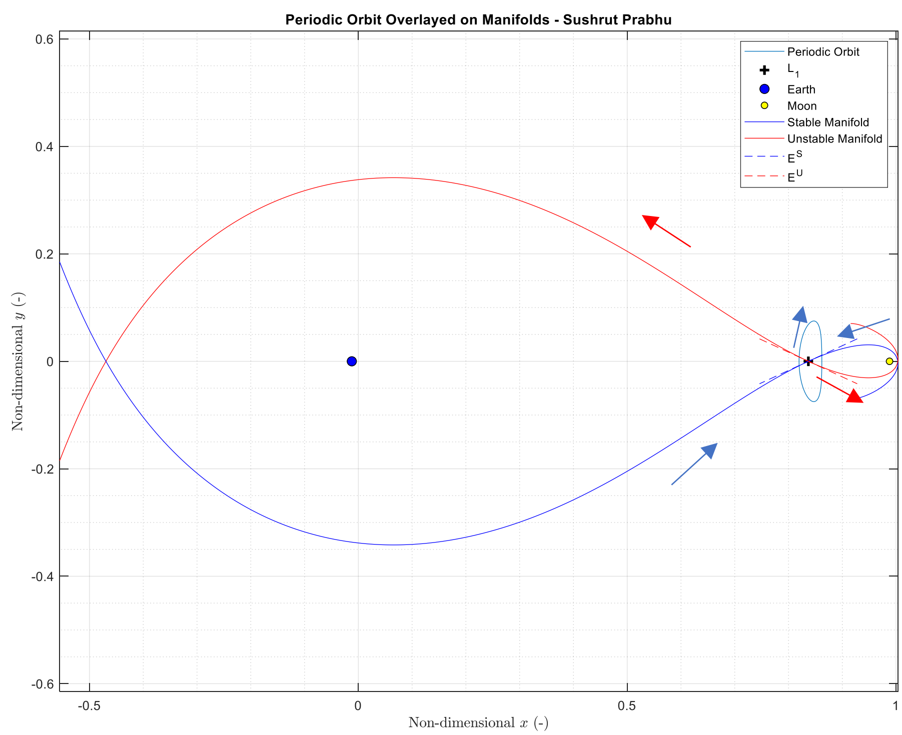


Figure F2.11: Four half manifolds at L1 point with 2 periods of a periodic orbit.

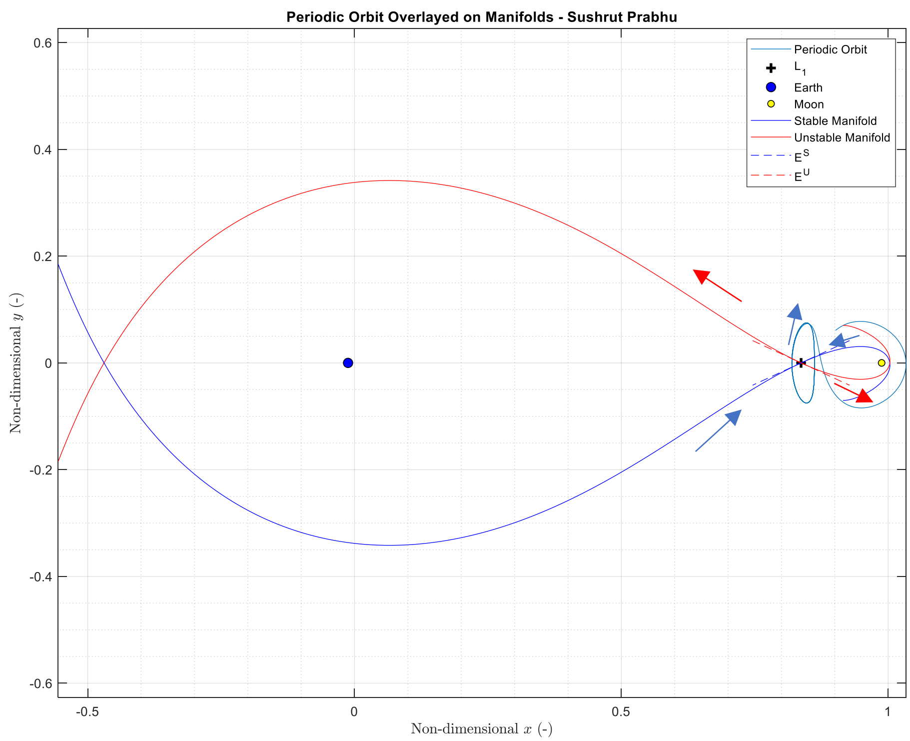


Figure F2.12: Four half manifolds at L1 point with 5 periods of a periodic orbit.

Part d)

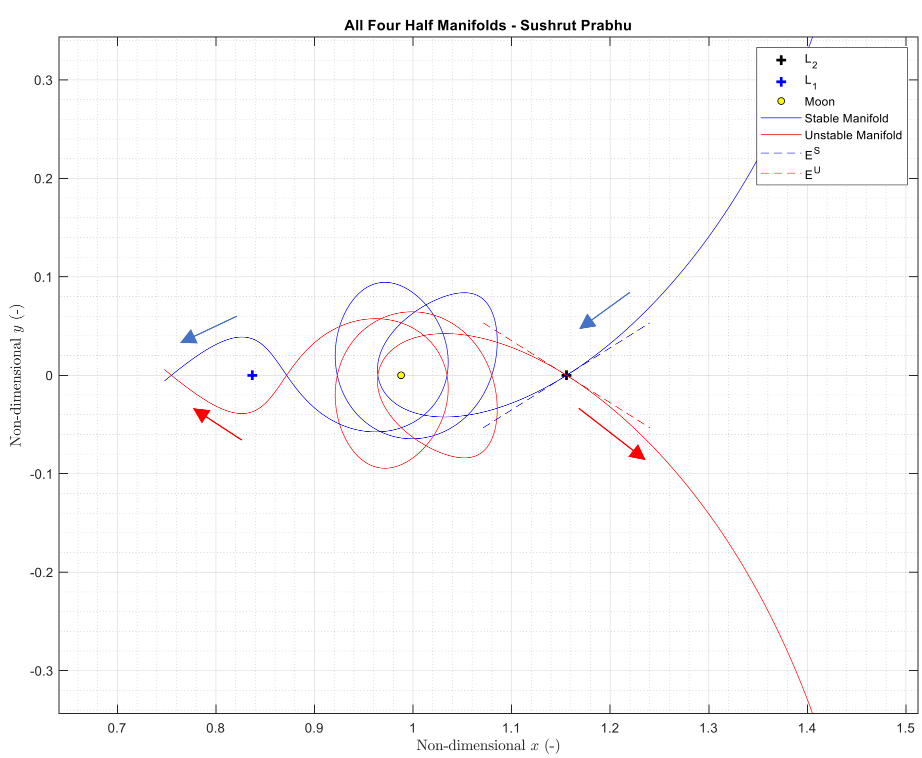


Figure F2.13: Four half manifolds at L2 point.

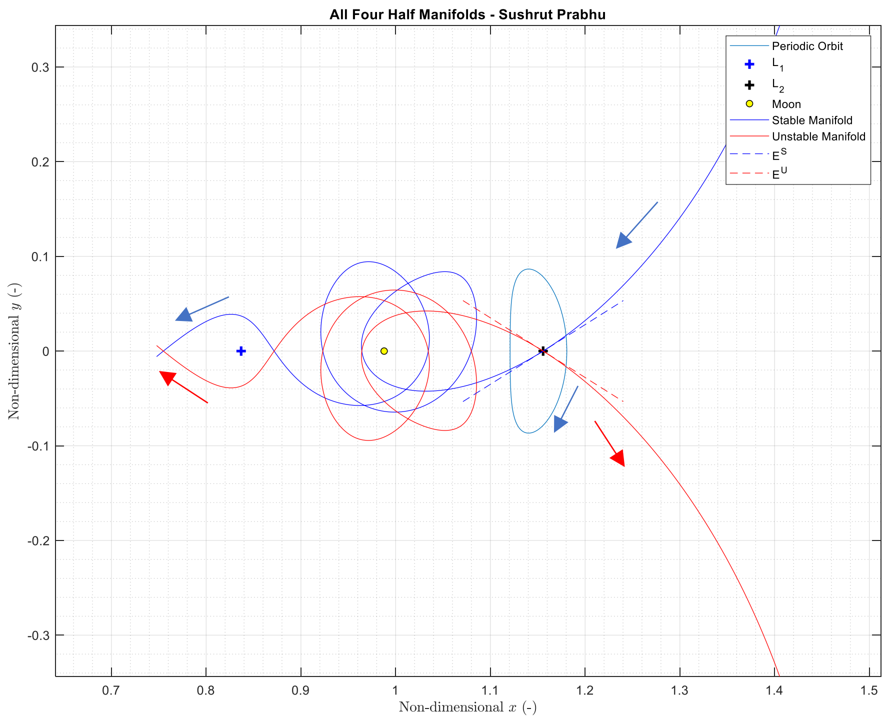


Figure F2.14: Four half manifolds at L2 point with 2 periods of a periodic orbit.

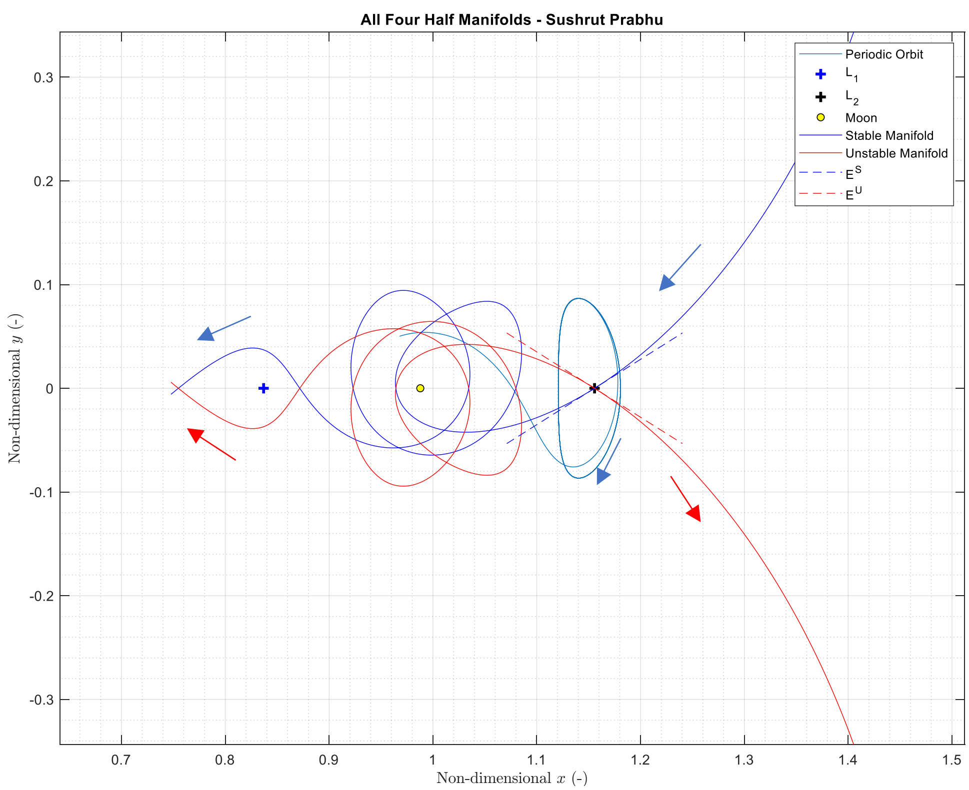


Figure F2.15: Four half manifolds at L2 point with 6 periods of a periodic orbit.

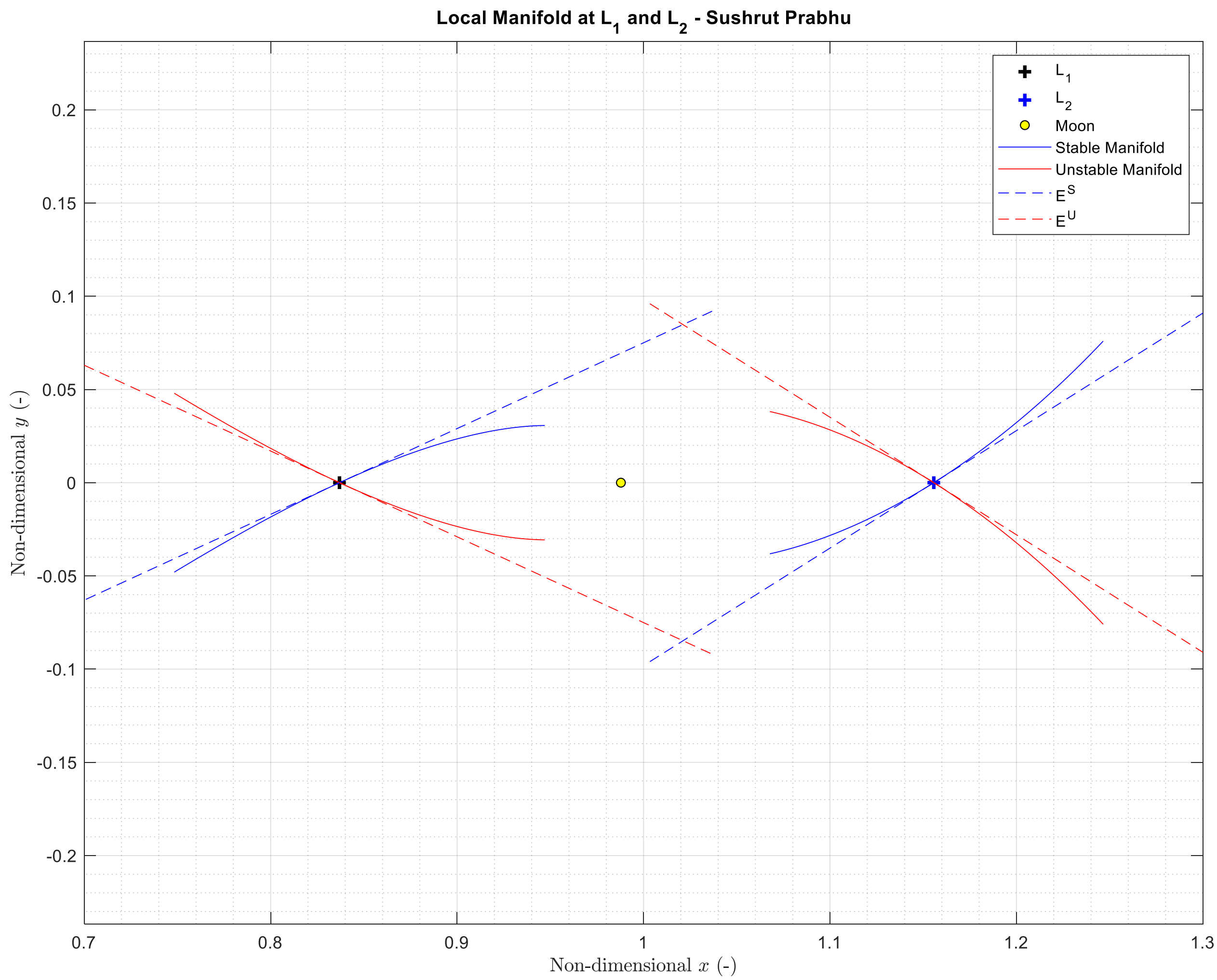


Figure F2.16: Four half manifolds at L2 point with 6 periods of a periodic orbit.

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PSF2

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};

% Lagrange Point 2
dim_vals{10,2} = abs(L2_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{11,2} = 1-dim_vals{4,2} + dim_vals{10,2};
dim_vals{12,2} = dim_vals{10,2}*dim_vals{2,2};
dim_vals{13,2} = dim_vals{11,2}*dim_vals{2,2};
```

Part a)

```
A = A_t(dim_vals{7,2},0,0,dim_vals{4,2});
[V,D] = eig(A);
D = diag(D);
```

```
e1 = V(1:2,1)/norm(V(1:2,1))*.1;
e2 = V(1:2,2)/norm(V(1:2,2))*.1;

Vs = V(1:end,1)/norm(V(1:3,1));
Vu = V(1:end,2)/norm(V(1:3,2));

d_dim = [55,50,45,40,35,30,25,20,15,10,5,1];

% Moon Side
t_end1 = [5.27, 5.33, 5.4, 5.49, 5.57, 5.67, 5.8, 5.95, 6.15, 6.42,
 6.91, 8.00];
t_end2 = [2.14, 2.17, 2.2, 2.23, 2.28, 2.33, 2.4, 2.47, 2.57, 2.71,
 2.95, 3.49];
n = 1;
for d = d_dim/dim_vals{2,2}
    x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vs';
    IC = x0;
    t_end =t_end1(n);

    [t1,y1] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

    t_end = t_end2(n);
    [t2,y2] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

    y = [flip(y1(:,1)),flip(y1(:,2)) ; y2(:,1), y2(:,2)];
    v = [flip(y1(:,4)),flip(y1(:,5)) ; y2(:,4), y2(:,5)];
    t = [flip(t1);t2];

    min_error(n) = min(vecnorm(y - [dim_vals{7,2},
 0].*ones(size(y)),2,2));

    J_L1 = Jacobi_C(dim_vals{7,2},0,0,0,dim_vals{4,2});
    Jt =
 Jacobi_C(y(:,1),y(:,2),zeros(size(y(:,1))),vecnorm(v(:,1:end),2,2),
 dim_vals{4,2});
    J_error = abs(Jt-J_L1)/abs(J_L1);

    if n == 5
        figure (1)
        plot(y1(:,1),y1(:,2),'r')
        hold on
        plot(y2(:,1),y2(:,2),'b')

%          figure
%          plot(t,J_error)
%          title(['Jacobi Bounds Check for x_0 = ' num2str(d_dim(n)) '
 km - Sushrut Prabhu'])
%          grid on
%          ylabel("Non-dimensional $$J_C$$ (-)","Interpreter", "latex")
%          xlabel("Time (-)")

    end
    n = n+1;
end
```

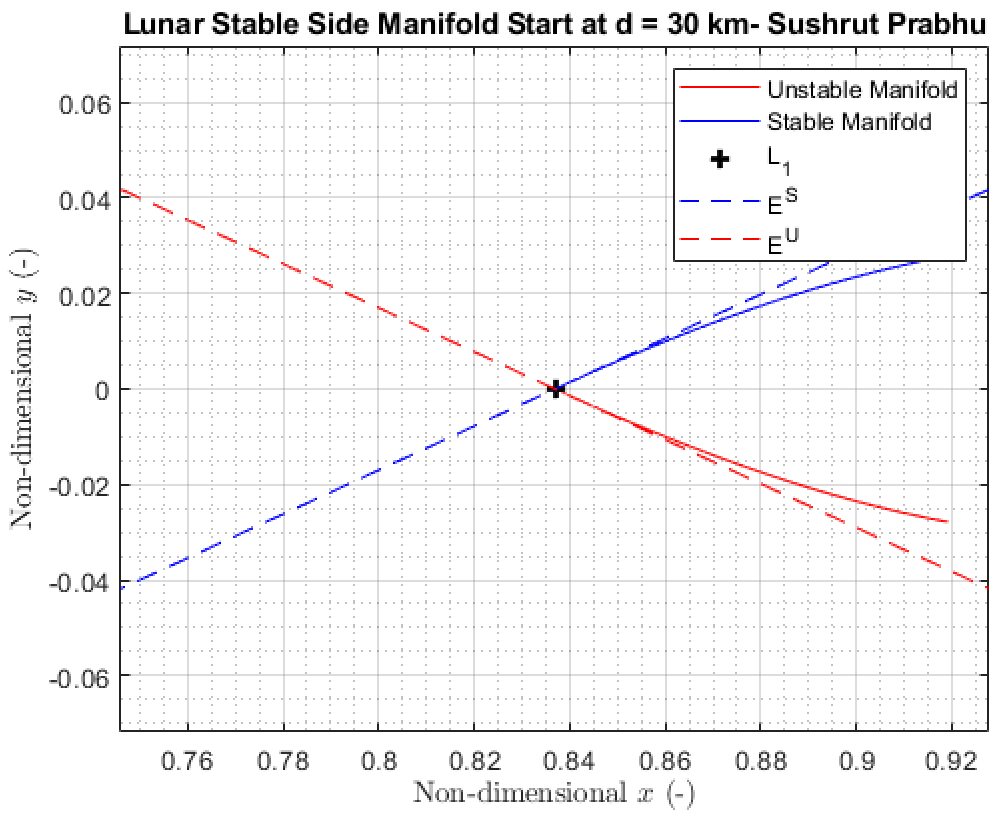
```
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
axis equal
title(['Lunar Stable Side Manifold Start at d = 30 km- Sushrut
 Prabhu'])
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
legend('Unstable Manifold', 'Stable Manifold', 'L_1', 'E^S','E^U')

figure
plot(d_dim,min_error*dim_vals{2,2},'*')
grid on
grid minor
title('Lunar Side Manifold Distance from L_1')
xlabel('x_0 (km)')
ylabel('Closest Approach to L_1 (km)')


figure
plot(d_dim,(t_end1+t_end2)*dim_vals{5,2}/3600/24,'*')
grid on
grid minor
title('Lunar Side Manifold Time Raltionship')
xlabel('x_0 (km)')
ylabel('Time of trajectory (days)')



% Moon Side
t_end2 = [5.27, 5.33, 5.4, 5.49, 5.57, 5.67, 5.8, 5.95, 6.15, 6.42,
 6.91, 8.00];
t_end1 = [2.14, 2.17, 2.2, 2.23, 2.28, 2.33, 2.4, 2.47, 2.57, 2.71,
 2.95, 3.49];
n = 1;
for d = d_dim/dim_vals{2,2}
    x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vu';
    IC = x0;
    t_end =t_end1(n);

    [t1,y1] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

    t_end = t_end2(n);
    [t2,y2] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

    y = [flip(y1(:,1)),flip(y1(:,2)) ; y2(:,1), y2(:,2)];
    v = [flip(y1(:,4)),flip(y1(:,5)) ; y2(:,4), y2(:,5)];
    t = [flip(t1);t2];

    min_error(n) = min(vecnorm(y - [dim_vals{7,2},
 0].*ones(size(y)),2,2));
```

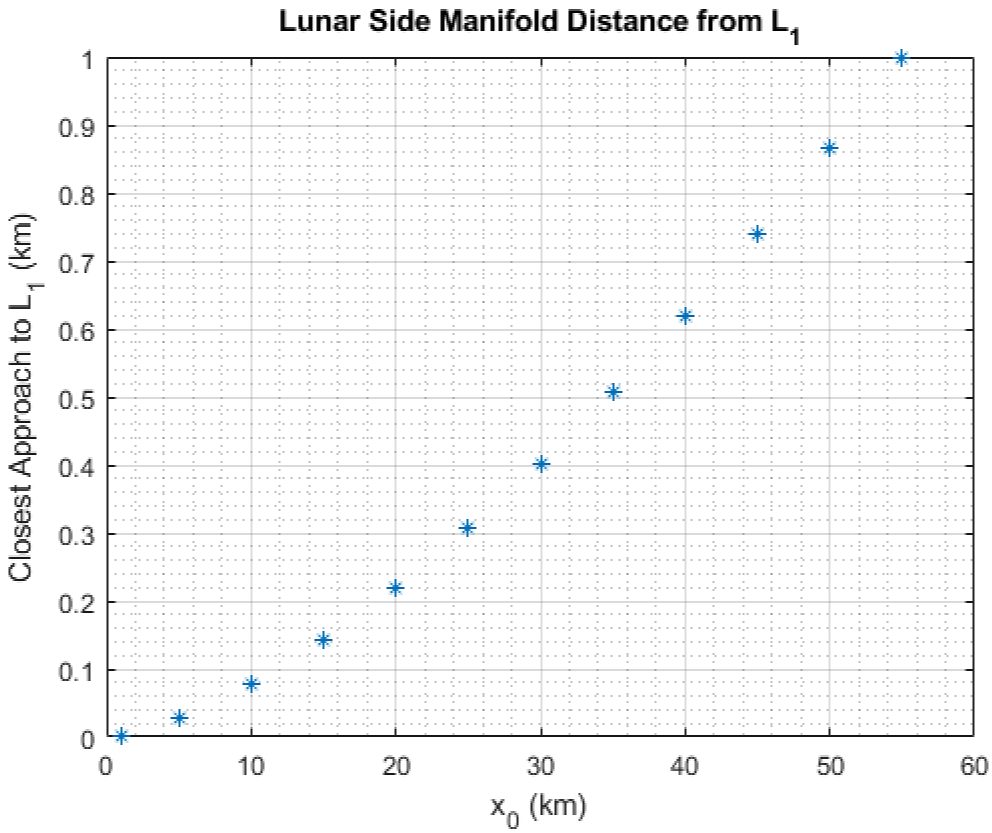
```
    J_L1 = Jacobi_C(dim_vals{7,2},0,0,0,dim_vals{4,2});
    Jt =
 Jacobi_C(y(:,1),y(:,2),zeros(size(y(:,1))),vecnorm(v(:,1:end),2,2),
 dim_vals{4,2});
    J_error = abs(Jt-J_L1)/abs(J_L1);

    if n == 5
        figure (4)
        plot(y1(:,1),y1(:,2),'r')
        hold on
        plot(y2(:,1),y2(:,2),'b')
    end
    n = n+1;
end

plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
axis equal
title(['Lunar Unstable Side Manifold Start at d = 30 km- Sushrut
 Prabhu'])
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
legend('Unstable Manifold', 'Stable Manifold', 'L_1', 'E^S','E^U')

figure
plot(d_dim,min_error*dim_vals{2,2},'*')
grid on
grid minor
title('Lunar Side Manifold Distance from L_1')
xlabel('x_0 (km)')
ylabel('Closest Approach to L_1 (km)')


figure
plot(d_dim,(t_end1+t_end2)*dim_vals{5,2}/3600/24,'*')
grid on
grid minor
title('Lunar Side Manifold Time Raltionship')
xlabel('x_0 (km)')
ylabel('Time of trajectory (days)')




% Earth Side Stable
d_dim = [55,50,45,40,35,30,25,20,15,10,5,1];
t_end1 = [5.27, 5.33, 5.4, 5.49, 5.57, 5.67, 5.8, 5.95, 6.15, 6.42,
 6.91, 8.00];
```

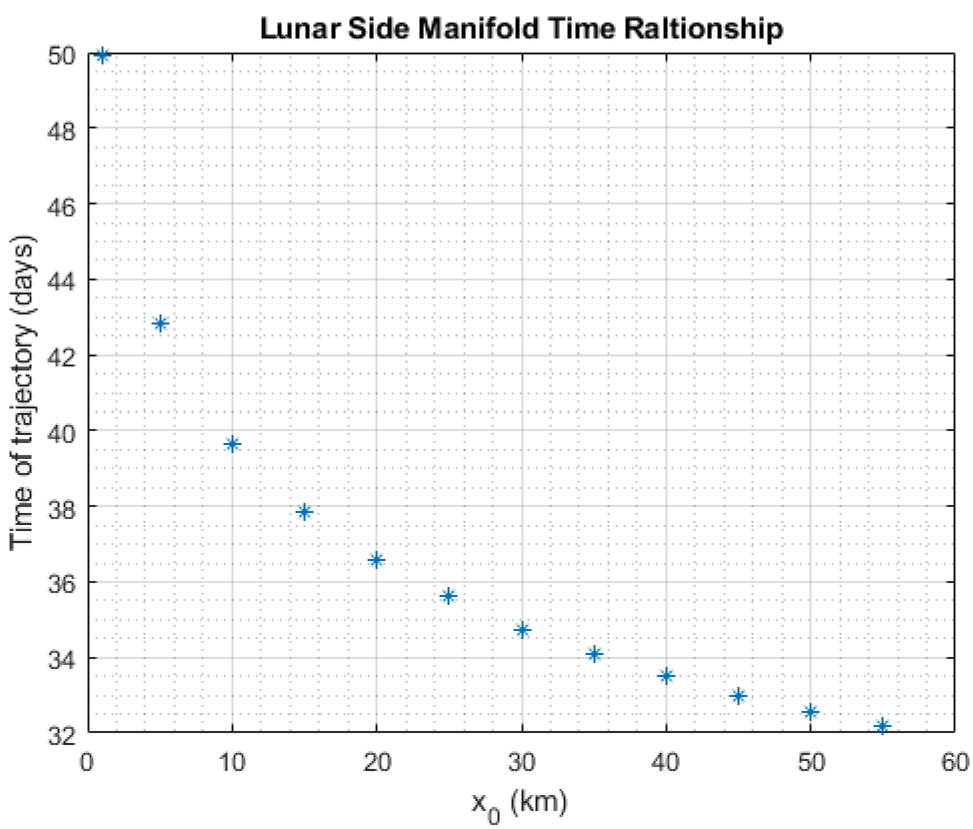
```
t_end2 = [2.14, 2.17, 2.2, 2.23, 2.28, 2.33, 2.4, 2.47, 2.57, 2.71,
 2.95, 3.49];
n = 1;
for d = d_dim/dim_vals{2,2}
    x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vs';
    IC = x0;
    t_end =t_end1(n);

    [t1,y1] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

    t_end = t_end2(n);
    [t2,y2] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

    y = [flip(y1(:,1)),flip(y1(:,2)) ; y2(:,1), y2(:,2)];
    v = [flip(y1(:,4)),flip(y1(:,5)) ; y2(:,4), y2(:,5)];
    t = [flip(t1);t2];

    min_error(n) = min(vecnorm(y - [dim_vals{7,2},
 0].*ones(size(y)),2,2));

    J_L1 = Jacobi_C(dim_vals{7,2},0,0,0,dim_vals{4,2});
    Jt =
 Jacobi_C(y(:,1),y(:,2),zeros(size(y(:,1))),vecnorm(v(:,1:end),2,2),
 dim_vals{4,2});
    J_error = abs(Jt-J_L1)/abs(J_L1);

    if n == 5
        figure (7)
        plot(y1(:,1),y1(:,2),'r')
        hold on
        plot(y2(:,1),y2(:,2),'b')
    end
    n = n+1;
end

plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
axis equal
title(['Earth Side Stable Manifold Start at d - 30 km - Sushrut
 Prabhu'])
legend('1','2','3','4')
grid on
grid minor
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y_0$$ (-)","Interpreter", "latex")
legend('Unstable Manifold', 'Stable Manifold', 'L_1', 'E^S','E^U')

figure
plot(d_dim,min_error*dim_vals{2,2},'*')
grid on
grid minor
title('Earth Side Unstable Manifold Distance from L_1')
xlabel('x_0 (km)')
```

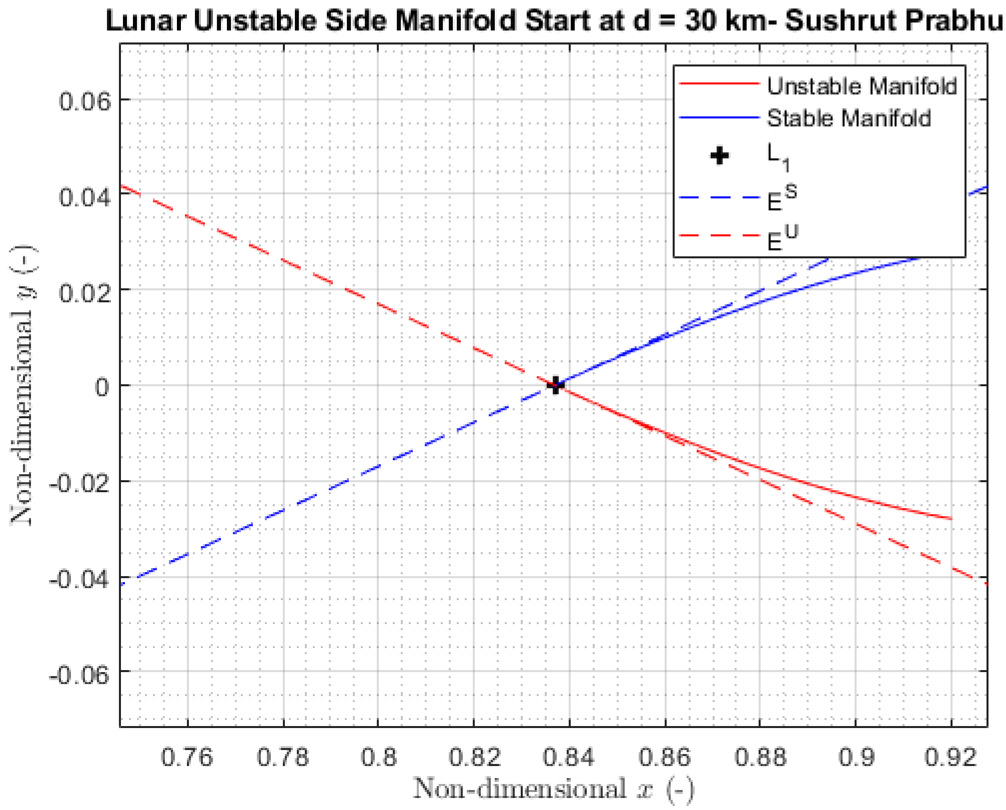
```
ylabel('Closest Approach to L_1 (km)')

figure
plot(d_dim,(t_end1+t_end2)*dim_vals{5,2}/3600/24,'*')
grid on
grid minor
title('Earth Side Unstable Manifold Time Raltionship')
xlabel('x_0 (km)')
ylabel('Time of trajectory (days)')

% Earth Side Unstable
d_dim = [55,50,45,40,35,30,25,20,15,10,5,1];
t_end2 = [5.27, 5.33, 5.4, 5.49, 5.57, 5.67, 5.8, 5.95, 6.15, 6.42,
 6.91, 8.00];
t_end1 = [2.14, 2.17, 2.2, 2.23, 2.28, 2.33, 2.4, 2.47, 2.57, 2.71,
 2.95, 3.49];
n = 1;
for d = d_dim/dim_vals{2,2}
    x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vu';
    IC = x0;
    t_end =t_end1(n);

    [t1,y1] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

    t_end = t_end2(n);
    [t2,y2] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

    y = [flip(y1(:,1)),flip(y1(:,2)) ; y2(:,1), y2(:,2)];
    v = [flip(y1(:,4)),flip(y1(:,5)) ; y2(:,4), y2(:,5)];
    t = [flip(t1);t2];

    min_error(n) = min(vecnorm(y - [dim_vals{7,2},
 0].*ones(size(y)),2,2));

    J_L1 = Jacobi_C(dim_vals{7,2},0,0,0,dim_vals{4,2});
    Jt =
 Jacobi_C(y(:,1),y(:,2),zeros(size(y(:,1))),vecnorm(v(:,1:end),2,2),
 dim_vals{4,2});
    J_error = abs(Jt-J_L1)/abs(J_L1);

    if n == 5
        figure (10)
        plot(y1(:,1),y1(:,2),'r')
        hold on
        plot(y2(:,1),y2(:,2),'b')
    end
    n = n+1;
end

plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
axis equal
```

```
title(['Earth Side Unstable Manifold Start at d - 30 km - Sushrut
 Prabhu'])
legend('1','2','3','4')
grid on
grid minor
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y_0$$ (-)","Interpreter", "latex")
legend('Unstable Manifold', 'Stable Manifold', 'L_1', 'E^S','E^U')


figure
plot(d_dim,min_error*dim_vals{2,2},'*')
grid on
grid minor
title('Earth Side Unstable Manifold Distance from L_1')
xlabel('x_0 (km)')
ylabel('Closest Approach to L_1 (km)')

figure
plot(d_dim,(t_end1+t_end2)*dim_vals{5,2}/3600/24,'*')
grid on
grid minor
title('Earth Side Unstable Manifold Time Raltionship')
xlabel('x_0 (km)')
ylabel('Time of trajectory (days)')
```

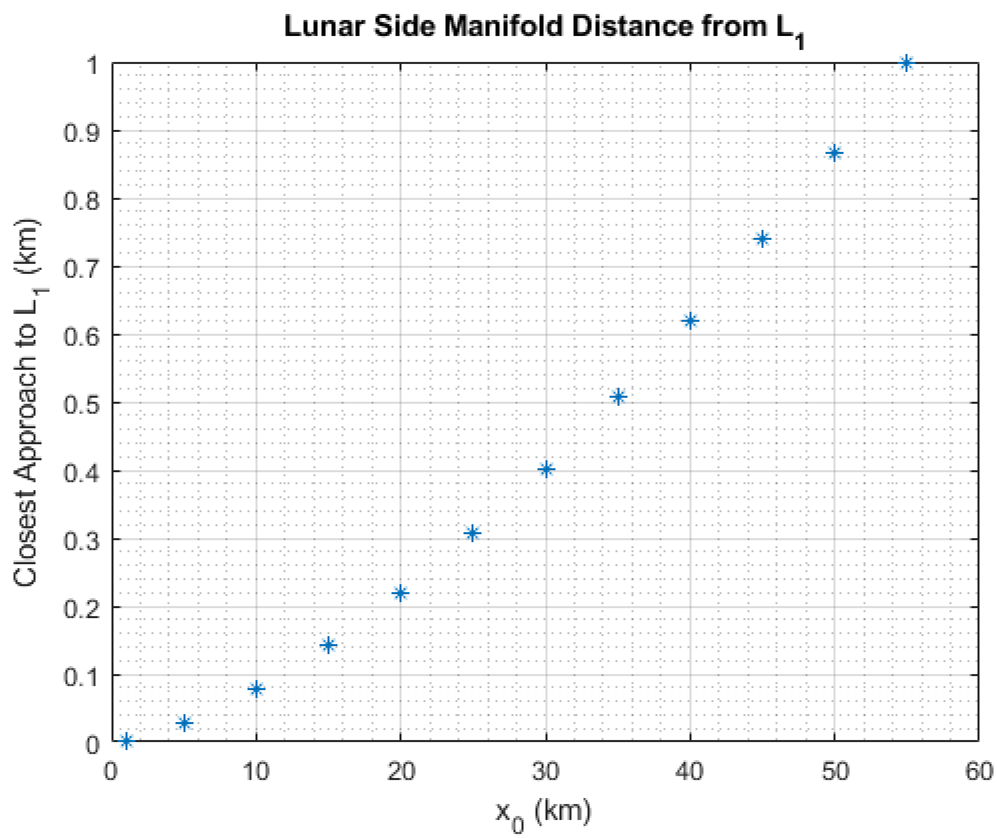



Lunar Side Manifold Distance from L_1
Closest Approach to L_1 (km)
x_0 (km)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60

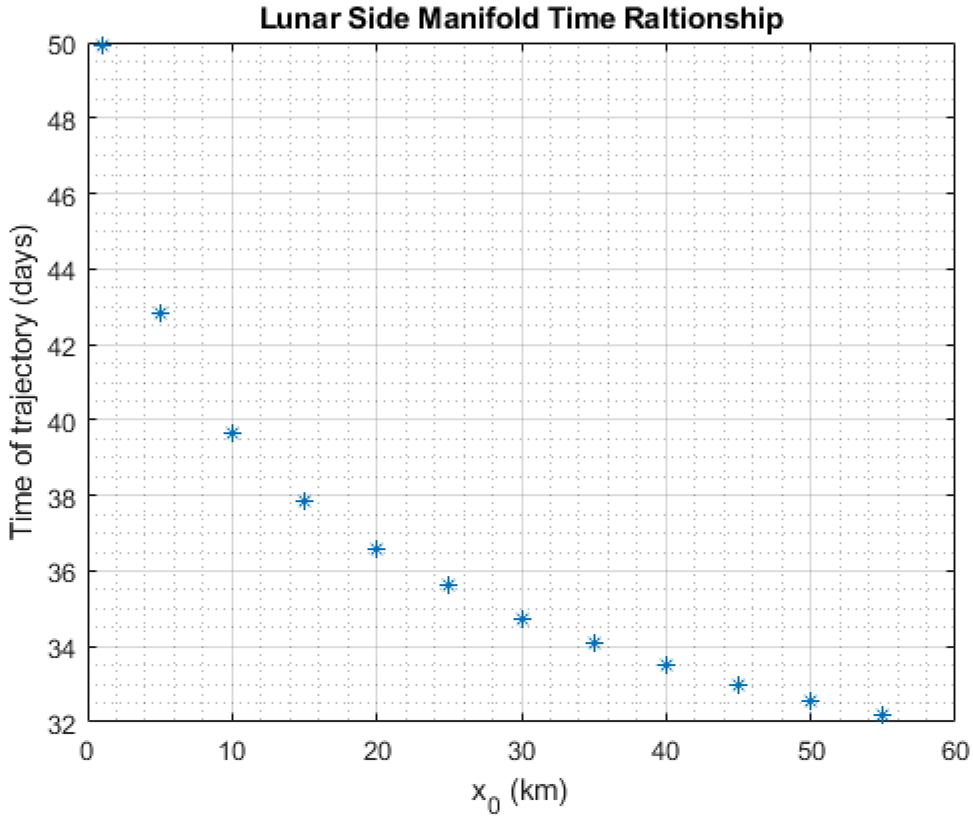

Lunar Side Manifold Time Raltionship
Time of trajectory (days)
x_0 (km)
32
34
36
38
40
42
44
46
48
50
0
10
20
30
40
50
60

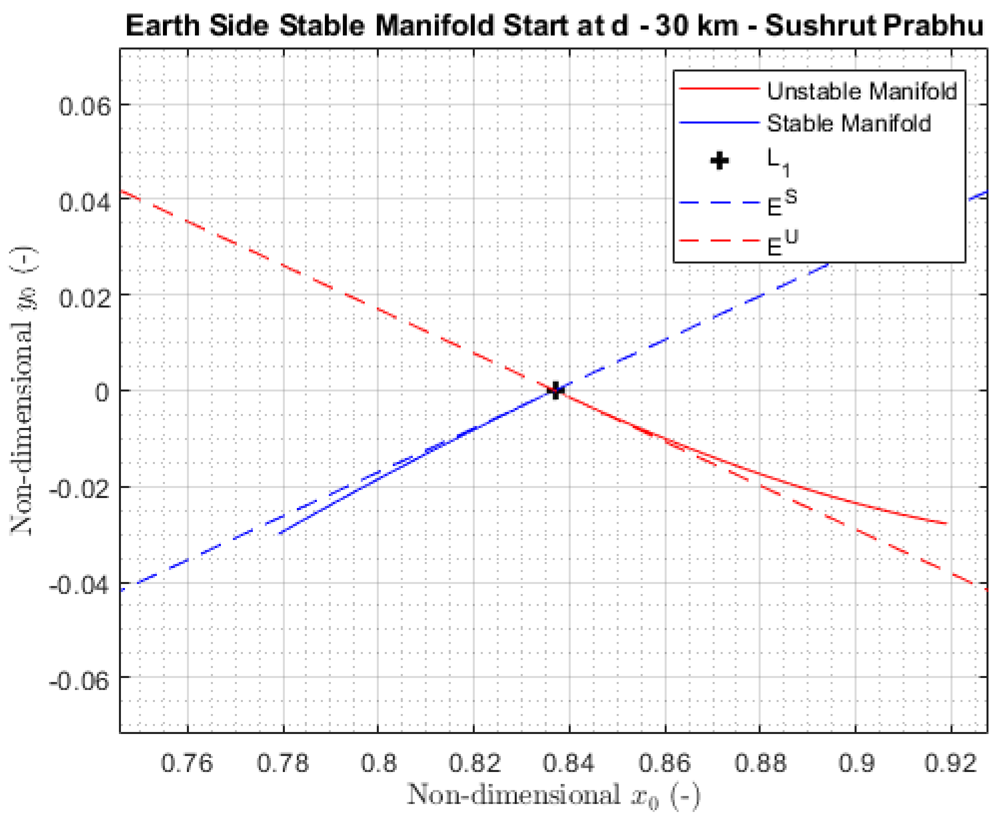


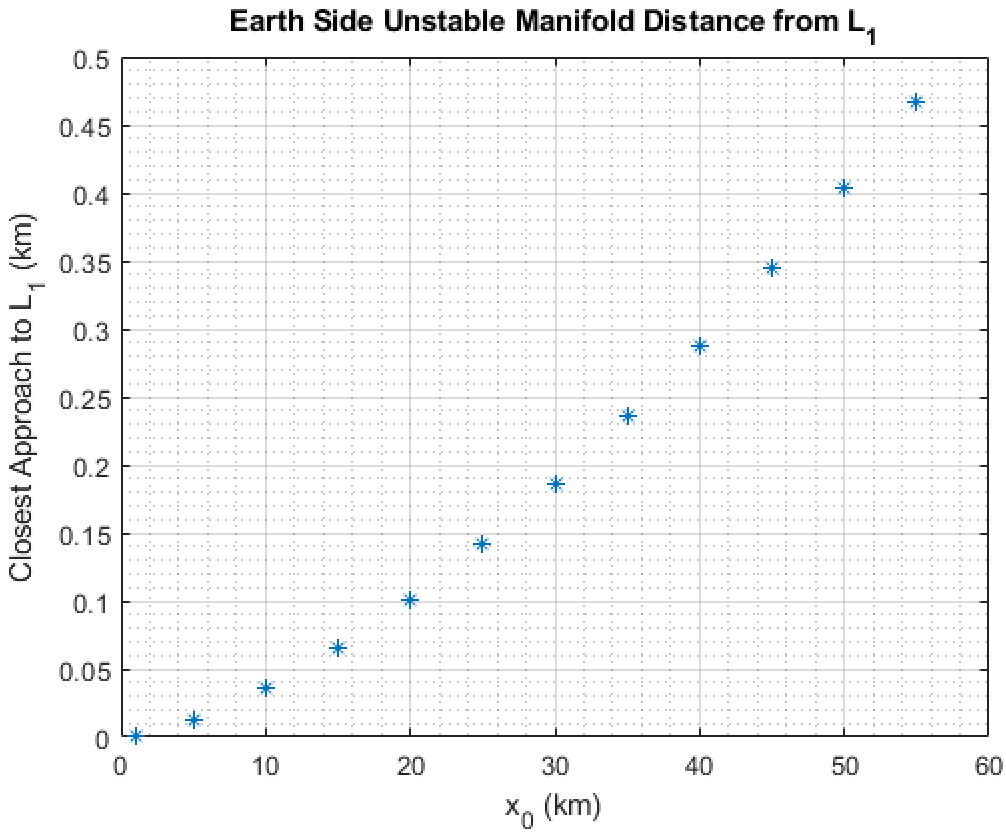
Lunar Unstable Side Manifold Start at d = 30 km- Sushrut Prabhu
Unstable Manifold
Stable Manifold
L_1
E^S
E^U
Non-dimensional y (-)
Non-dimensional x (-)

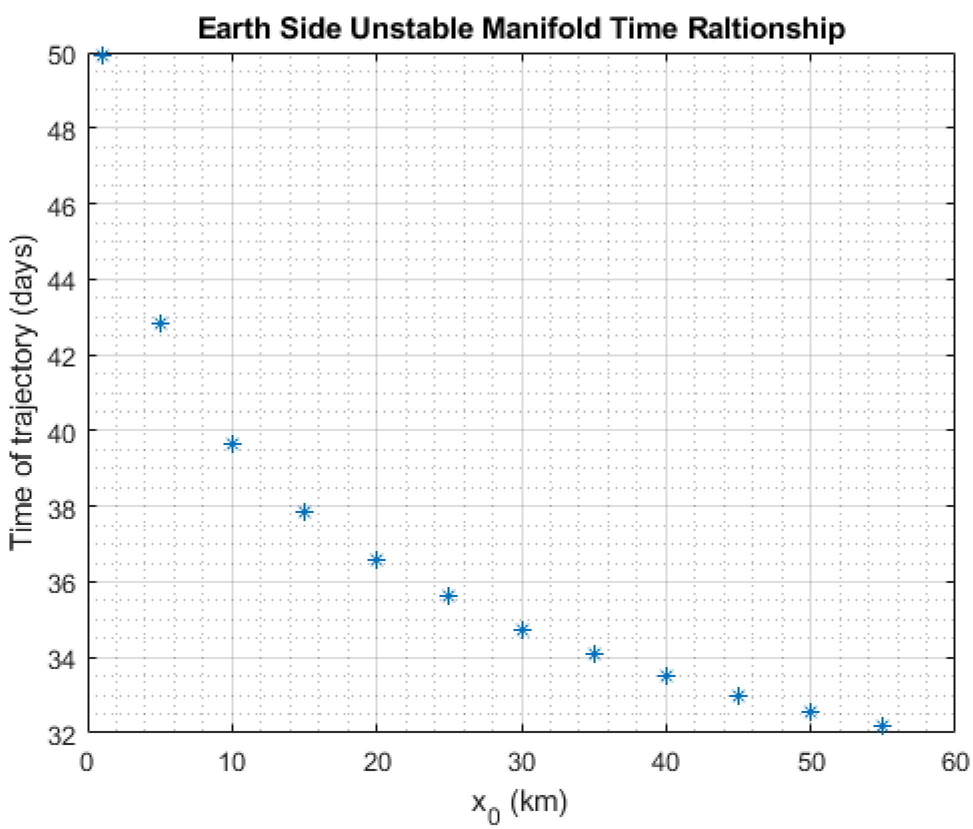


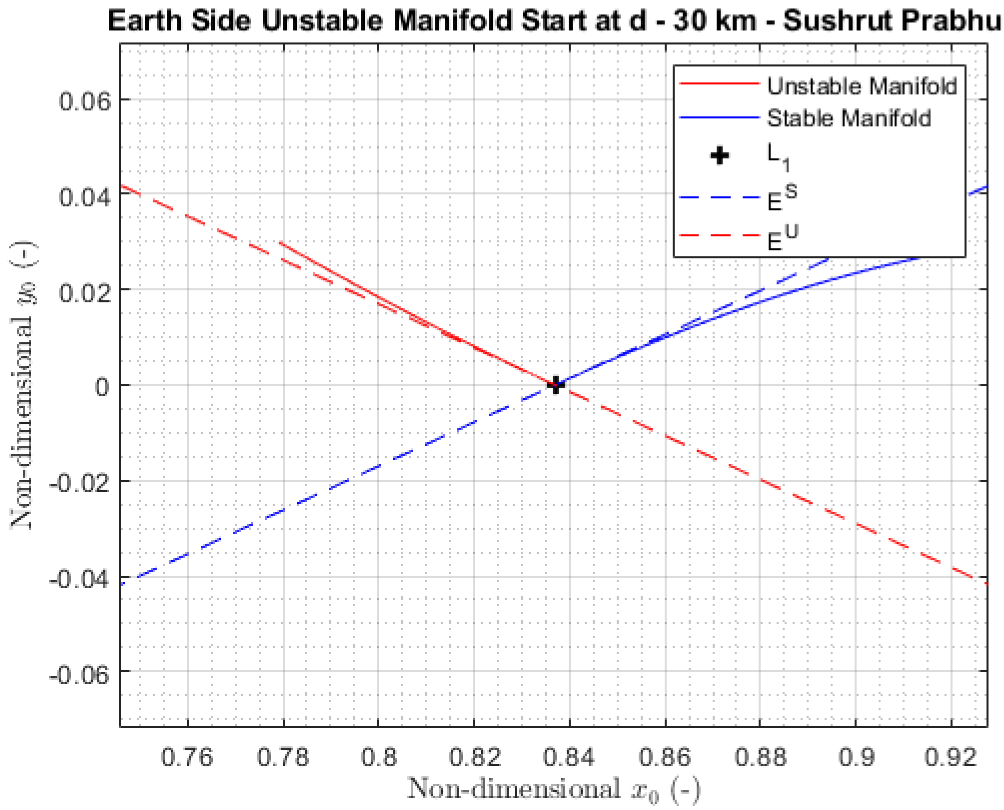
Lunar Side Manifold Distance from L_1
Closest Approach to L_1 (km)
x_0 (km)

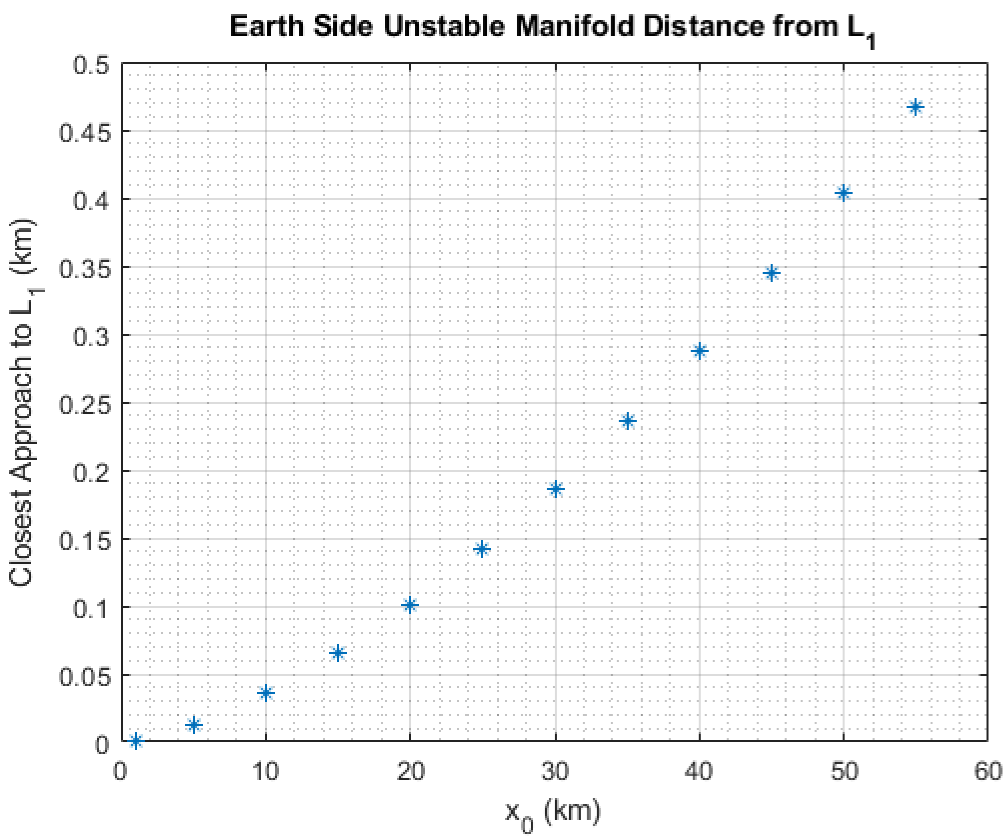

Lunar Side Manifold Time Raltionship
Time of trajectory (days)
50
48
46
44
42
40
38
36
34
32
0
10
20
30
40
50
60
x_0 (km)

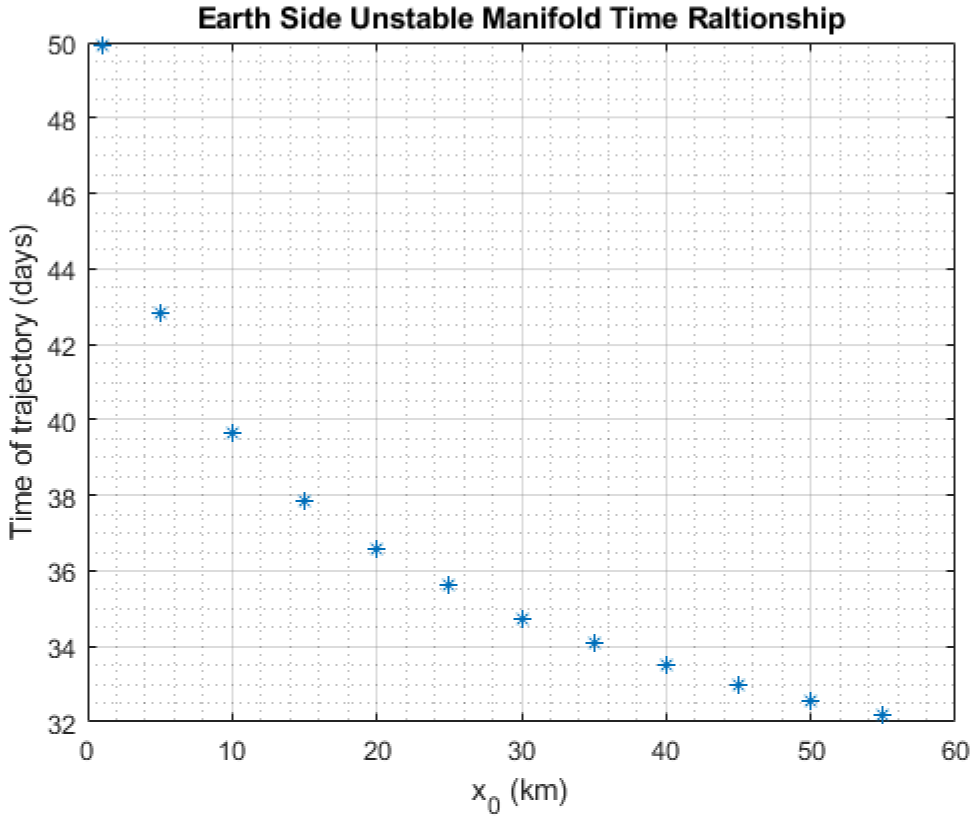

Earth Side Stable Manifold Start at d - 30 km - Sushrut Prabhu
Unstable Manifold
Stable Manifold
L_1
E^S
E^U
Non-dimensional y_0 (-)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.76
0.78
0.8
0.82
0.84
0.86
0.88
0.9
0.92
Non-dimensional x_0 (-)


Earth Side Unstable Manifold Distance from L_1
Closest Approach to L_1 (km)
x_0 (km)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
50
60


Earth Side Unstable Manifold Time Raltionship
Time of trajectory (days)
x_0 (km)
50
48
46
44
42
40
38
36
34
32
0
10
20
30
40
50
60


Earth Side Unstable Manifold Start at d - 30 km - Sushrut Prabhu
Unstable Manifold
Stable Manifold
L_1
E^S
E^U
Non-dimensional y_0 (-)
Non-dimensional x_0 (-)

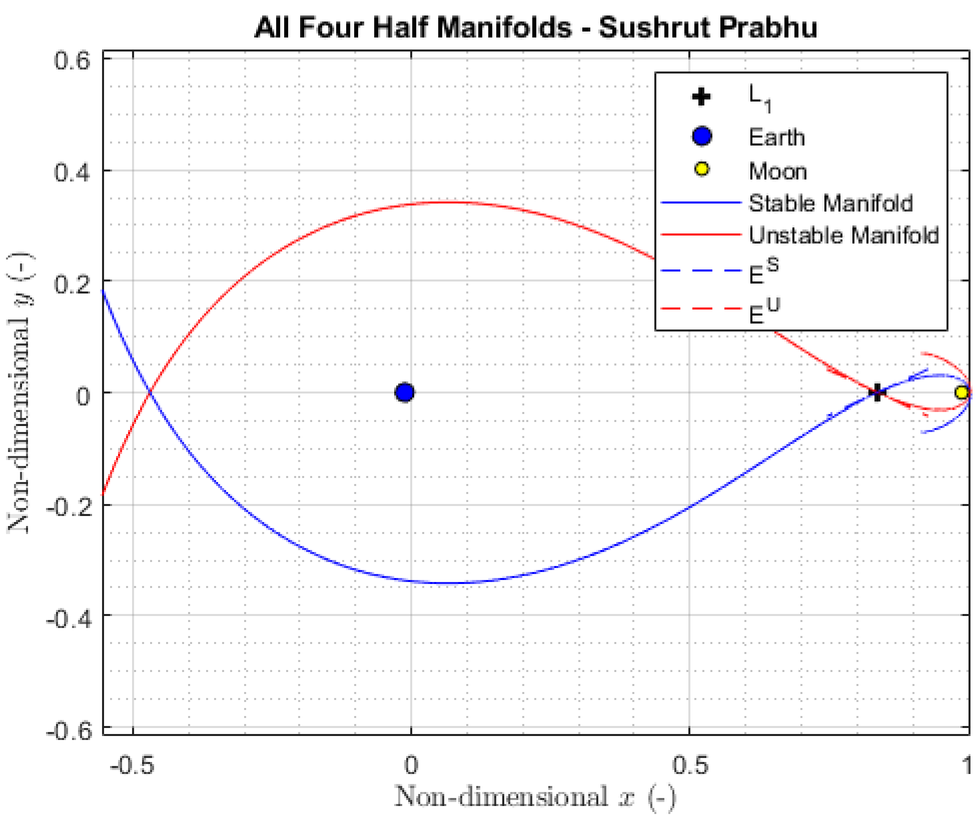

Earth Side Unstable Manifold Distance from L_1
Closest Approach to L_1 (km)
x_0 (km)



Part b)

```
d = 30/dim_vals{2,2};
t_end = 2.8;

x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vs';
JC1 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y1] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vu';
JC4 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y4] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

t_end = 4.1;
x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vu';
JC2 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y2] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vs';
JC3 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y3] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});
```

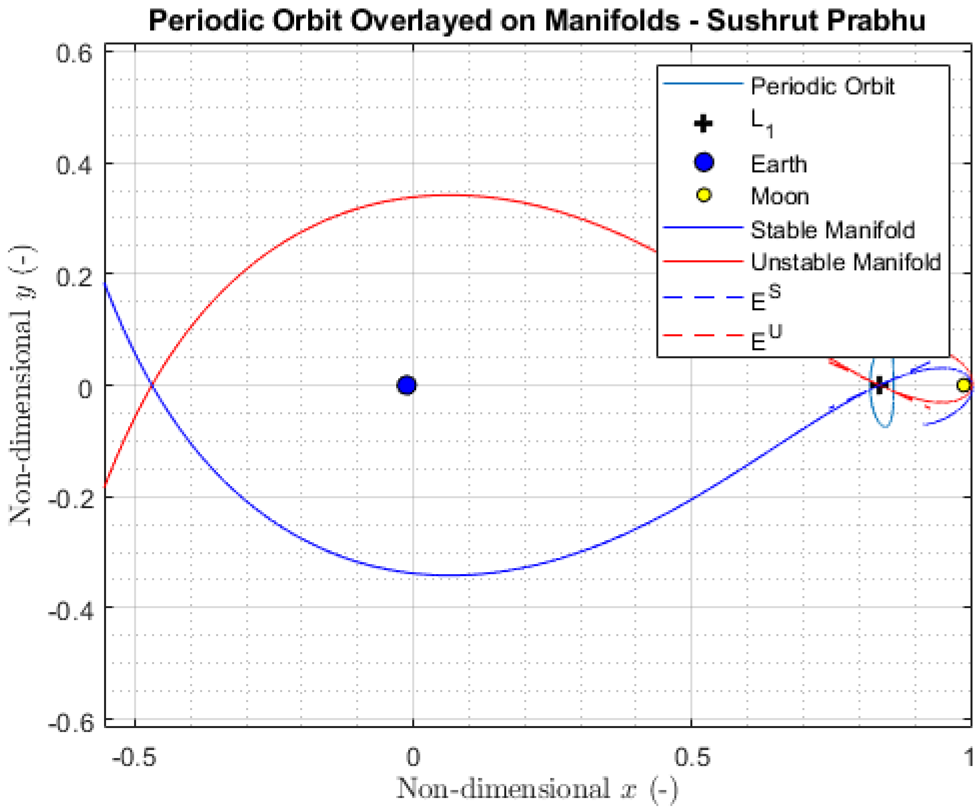
```
figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('L_1','Earth','Moon','Stable Manifold','Unstable
 Manifold','E^S','E^U')
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
title('All Four Half Manifolds - Sushrut Prabhu')
```

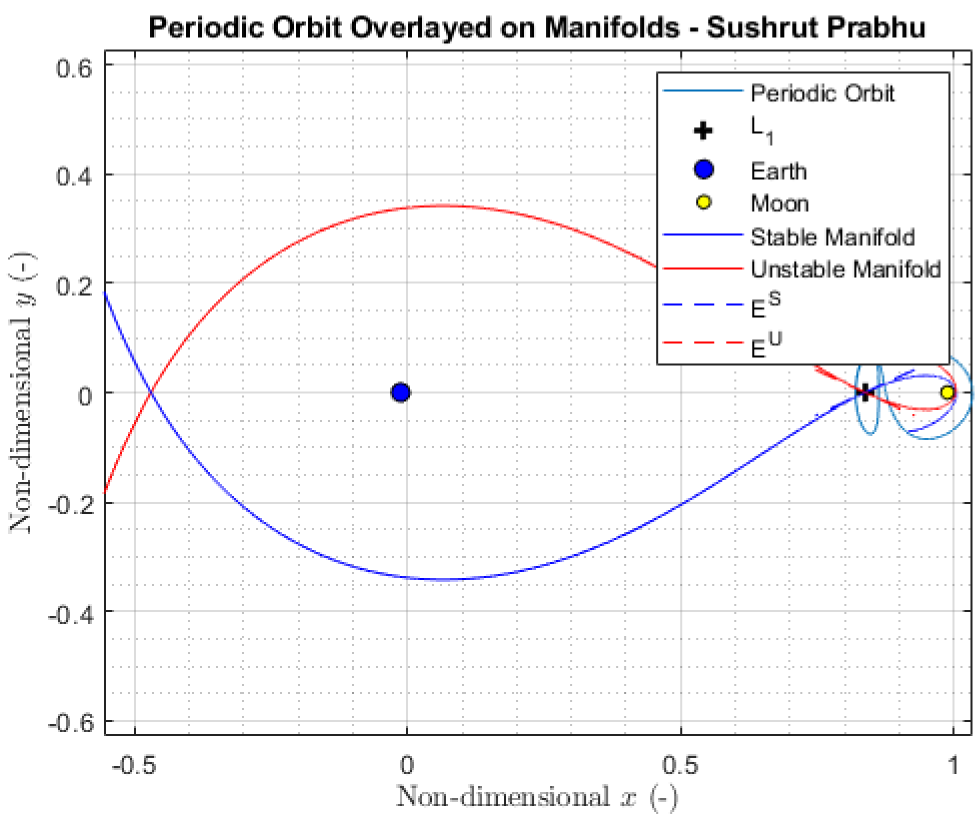


Part c)

```
y = [dim_vals{7,2}+.025, 0, 0, 0, -.1752, 0];
t_end = 2.78*.5;
```

```
[yn,t_end] = Target3d_per([0
 0],y(1:3),y(4:6),t_end,dim_vals{4,2},"planar", 10^-13,"");
[~,y]=ode45(@cr3bp_df,[0 t_end*2],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('Periodic Orbit','L_1','Earth','Moon','Stable
 Manifold','Unstable Manifold','E^S','E^U')
grid on
grid minor
title('Periodic Orbit Overlayed on Manifolds - Sushrut Prabhu')
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")


[~,y]=ode45(@cr3bp_df,[0 t_end*9],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot(-dim_vals{4,2},0,'ko','MarkerSize',7,'MarkerFaceColor','b')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{7,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{7,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('Periodic Orbit','L_1','Earth','Moon','Stable
 Manifold','Unstable Manifold','E^S','E^U')
grid on
grid minor
title('Periodic Orbit Overlayed on Manifolds - Sushrut Prabhu')
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
```


Periodic Orbit Overlayed on Manifolds - Sushrut Prabhu
Periodic Orbit
L_1
Earth
Moon
Stable Manifold
Unstable Manifold
E^S
E^U
Non-dimensional y (-)
Non-dimensional x (-)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.5
0
0.5
1

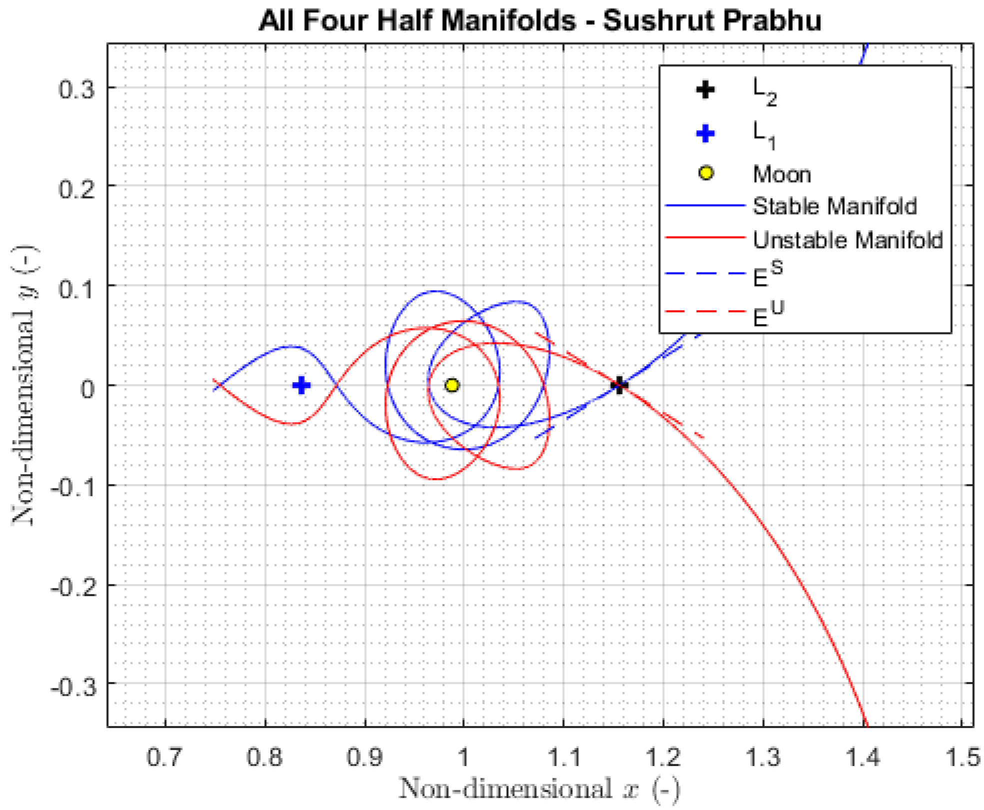

Periodic Orbit Overlayed on Manifolds - Sushrut Prabhu
Periodic Orbit
L_1
Earth
Moon
Stable Manifold
Unstable Manifold
E^S
E^U
Non-dimensional y (-)
Non-dimensional x (-)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.5
0
0.5
1

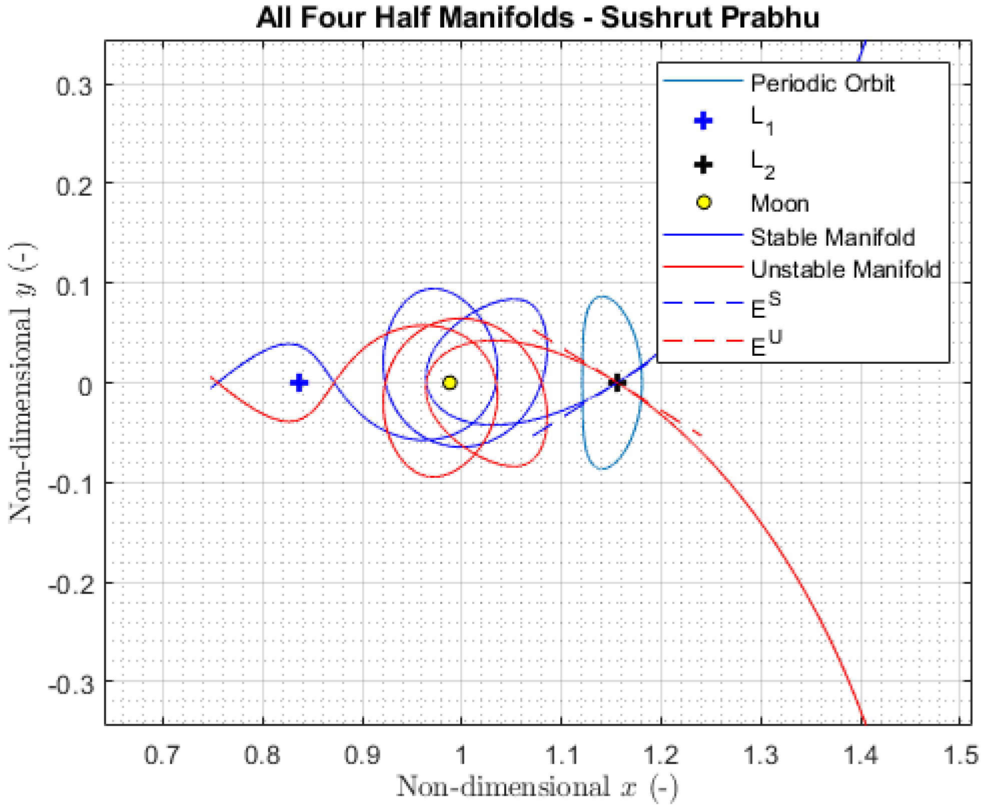
Part d)

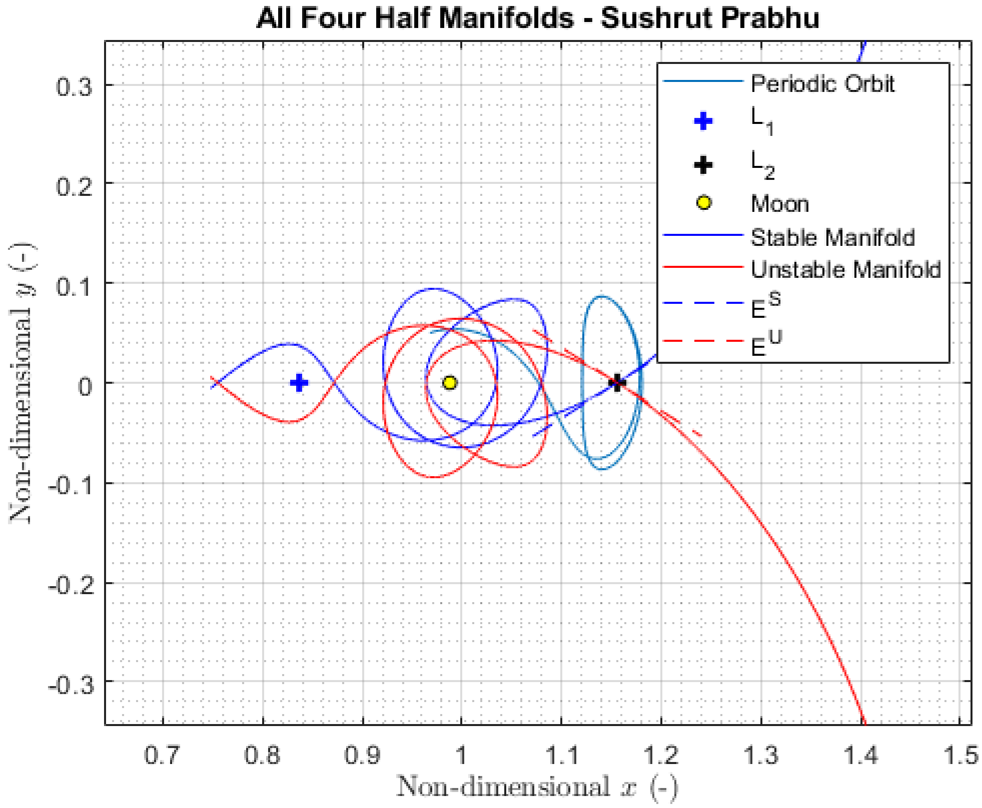
```
A = A_t(dim_vals{11,2},0,0,dim_vals{4,2});
[V,D] = eig(A);
D = diag(D);

e1 = V(1:2,2)/norm(V(1:2,2))*.1;
e2 = V(1:2,1)/norm(V(1:2,1))*.1;

Vu = V(1:end,1)/norm(V(1:3,1));
Vs = V(1:end,2)/norm(V(1:3,2));

d = 50/dim_vals{2,2};

t_end = 7.2;
x0 = [dim_vals{11,2} 0 0 0 0 0] + d*Vs';
JC1 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y1] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{11,2} 0 0 0 0 0] + d*Vu';
JC2 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y2] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

t_end = 4.1;
x0 = [dim_vals{11,2} 0 0 0 0 0] - d*Vs';
JC3 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y3] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{11,2} 0 0 0 0 0] - d*Vu';
JC4 = Jacobi_C(x0(1),x0(2),x0(3),norm(x0(4:6)),dim_vals{4,2});
IC = x0;
[~,y4] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});


figure
plot(dim_vals{11,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(dim_vals{7,2},0,'+b','LineWidth',2,'MarkerSize',7)
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{11,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{11,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('L_2','L_1','Moon','Stable Manifold','Unstable
 Manifold','E^S','E^U')
grid on
grid minor
```

```
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
title('All Four Half Manifolds - Sushrut Prabhu')


y = [dim_vals{11,2}+.025, 0, 0, 0, -.16, 0];
t_end = 3.5*.5;

[yn,t_end] = Target3d_per([0
 0],y(1:3),y(4:6),t_end,dim_vals{4,2},"planar", 10^-13,"");
[~,y]=ode45(@cr3bp_df,[0 t_end*2],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+b','LineWidth',2,'MarkerSize',7)
plot(dim_vals{11,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{11,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{11,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('Periodic Orbit','L_1','L_2','Moon','Stable Manifold','Unstable
 Manifold','E^S','E^U')
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
title('All Four Half Manifolds - Sushrut Prabhu')


[~,y]=ode45(@cr3bp_df,[0 t_end*9],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+b','LineWidth',2,'MarkerSize',7)
plot(dim_vals{11,2},0,'+k','LineWidth',2,'MarkerSize',7)
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y1(:,1),y1(:,2),'b')
plot(y2(:,1),y2(:,2),'r')
plot([e1(1),-e1(1)]+dim_vals{11,2},[e1(2),-e1(2)],'--b')
plot([e2(1),-e2(1)]+dim_vals{11,2},[e2(2),-e2(2)],'--r')
plot(y3(:,1),y3(:,2),'b')
plot(y4(:,1),y4(:,2),'r')
axis equal
legend('Periodic Orbit','L_1','L_2','Moon','Stable Manifold','Unstable
 Manifold','E^S','E^U')
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
```

```
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
title('All Four Half Manifolds - Sushrut Prabhu')
```



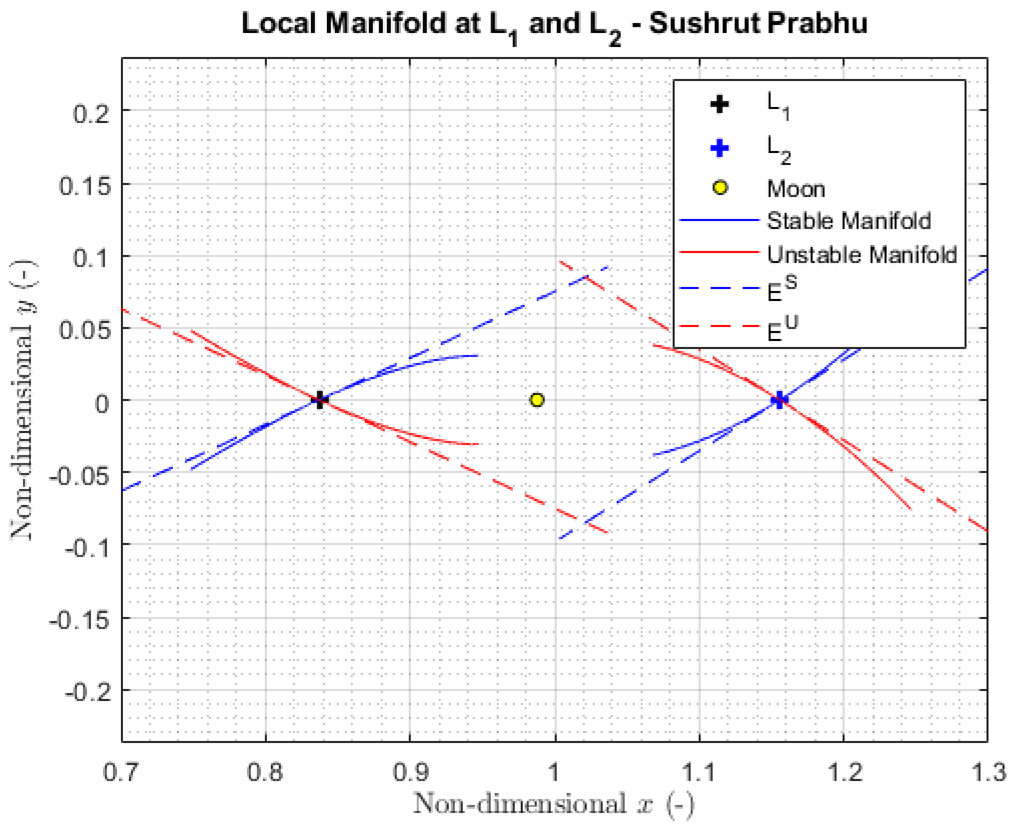

All Four Half Manifolds - Sushrut Prabhu
Periodic Orbit
L_1
L_2
Moon
Stable Manifold
Unstable Manifold
E^S
E^U
Non-dimensional y (-)
Non-dimensional x (-)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5


All Four Half Manifolds - Sushrut Prabhu
Periodic Orbit
L_1
L_2
Moon
Stable Manifold
Unstable Manifold
E^S
E^U
Non-dimensional y (-)
Non-dimensional x (-)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5

Local Manifolds

```
A = A_t(dim_vals{7,2},0,0,dim_vals{4,2});
[V,D] = eig(A);
D = diag(D);

e11 = V(1:2,1)/norm(V(1:2,1))*.22;
e12 = V(1:2,2)/norm(V(1:2,2))*.22;

Vs = V(1:end,1)/norm(V(1:3,1));
Vu = V(1:end,2)/norm(V(1:3,2));
d = 30/dim_vals{2,2};
t_end = 2.4;

x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vs';
IC = x0;
[~,y11] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vu';
IC = x0;
[~,y14] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

t_end = 2.5;
x0 = [dim_vals{7,2} 0 0 0 0 0] + d*Vu';
IC = x0;
[~,y12] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{7,2} 0 0 0 0 0] - d*Vs';
IC = x0;
[~,y13] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});


A = A_t(dim_vals{11,2},0,0,dim_vals{4,2});
[V,D] = eig(A);
D = diag(D);

e21 = V(1:2,1)/norm(V(1:2,1))*.18;
e22 = V(1:2,2)/norm(V(1:2,2))*.18;

Vu = V(1:end,1)/norm(V(1:3,1));
Vs = V(1:end,2)/norm(V(1:3,2));
d = 30/dim_vals{2,2};

t_end = 3.2;
x0 = [dim_vals{11,2} 0 0 0 0 0] + d*Vs';
IC = x0;
[~,y21] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{11,2} 0 0 0 0 0] + d*Vu';
IC = x0;
[~,y22] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

t_end = 3.5;
```

```
x0 = [dim_vals{11,2} 0 0 0 0 0] - d*Vs';
IC = x0;
[~,y23] = ode45(@cr3bp_df,[0 -t_end],IC,options,dim_vals{4,2});

x0 = [dim_vals{11,2} 0 0 0 0 0] - d*Vu';
IC = x0;
[~,y24] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});




figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(dim_vals{11,2},0,'+b','LineWidth',2,'MarkerSize',7)
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y11(:,1),y11(:,2),'b')
plot(y12(:,1),y12(:,2),'r')
plot([e11(1),-e11(1)]+dim_vals{7,2},[e11(2),-e11(2)],'--b')
plot([e12(1),-e12(1)]+dim_vals{7,2},[e12(2),-e12(2)],'--r')
plot(y13(:,1),y13(:,2),'b')
plot(y14(:,1),y14(:,2),'r')
plot(y21(:,1),y21(:,2),'b')
plot(y22(:,1),y22(:,2),'r')
plot([e21(1),-e21(1)]+dim_vals{11,2},[e21(2),-e21(2)],'--r')
plot([e22(1),-e22(1)]+dim_vals{11,2},[e22(2),-e22(2)],'--b')
plot(y23(:,1),y23(:,2),'b')
plot(y24(:,1),y24(:,2),'r')
axis equal
xlim([.7 1.3])
legend('L_1','L_2','Moon','Stable Manifold','Unstable
 Manifold','E^S','E^U')
grid on
grid minor
xlabel("Non-dimensional $$x$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$y$$ (-)","Interpreter", "latex")
title('Local Manifold at L_1 and L_2 - Sushrut Prabhu')
```



Published with MATLAB® R2018a

Sushant Prabhu | AAE632 | PSF3

PSF3

Given: Lyapunov orbit at $\xi = 0.01$ for the Earth moon system

Find: (a) For L_1 Lyapunov orbit, list eigenvalues and eigenvectors. How to check for pairs of λ_i $\bar{v}_i$? Dimension of eigenspace

(b) Fixed point at $\eta = 0$? Plot $E^u(\bar{x}^*)$ and $E^s(\bar{x}^*)$. Plot negative and positive direction. Which directions to plot? Configuration space?

(c) Plot velocity components?

(d) Eigenvectors for 20 points. Plot periodic orbit

Solution:

a) In order to get the eigenvalue λ_i and eigenvectors $\bar{v}_i$ we need the monodromy matrix

$$\therefore \phi(T,0) = G \begin{bmatrix} 0 & -I \\ I & -2\Omega \end{bmatrix} \phi^T(T/2, 0) \begin{bmatrix} -2\Omega & I \\ -I & 0 \end{bmatrix} G\, \phi(T/2, 0)$$

Monodromy Matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{See Table: F3.1 for all eigenvalues with the eigenvectors}}$

continued...

To ensure that eigenvalues are paired withe the correct eigenvector just have to check the definition:

$$A\bar{v}_i = \lambda_i \bar{v}_i \quad \boxed{\therefore \|A\bar{v}_i - \lambda_i \bar{v}_i\| = 0}$$

Note: The eigenspace we are working with is 6 dimensional but our problem is reduced to a planar one. This means we can reduce it to 4-dimension.

6) The fixed point is just the initial conditions of the periodic orbit: $\bar{r} = 0.8469\,\hat{x} + 0\hat{y} + 0\hat{z}$

$\bar{v} = 0\,\hat{x} + 0.0782\,\hat{y} + 0\,\hat{z}$

So let us start by plotting in configuration space. Moreover, the z-direction is 0 because this is a planar problem so we can plot the x-y direction See figure: 3.1

Again, we cannot plot in 6 dimensions. So we can seperate position and velocity and plot those. But since this is a planar problem we only need to plot x-z plane

continued...

c) See figure: F3.2 The eigenspace can be scaled as proffered. Because the eigenvectors only provide a direction and so the can be scaled. Just need the origin to be at the fixed point.

d) See Table F3.2 for all the eigenvectors. Note: I only write eigenvalues once because the are independent of the position on the orbit.
See both figures: F3.3 and F3.4
I just extended eigenvectors to form the tube shape.

Table F3.2 is quite long and I tried to colour code it to make it easier to understand.
Again: Eigenvalues will be the same as in Table F3.1

PSF3

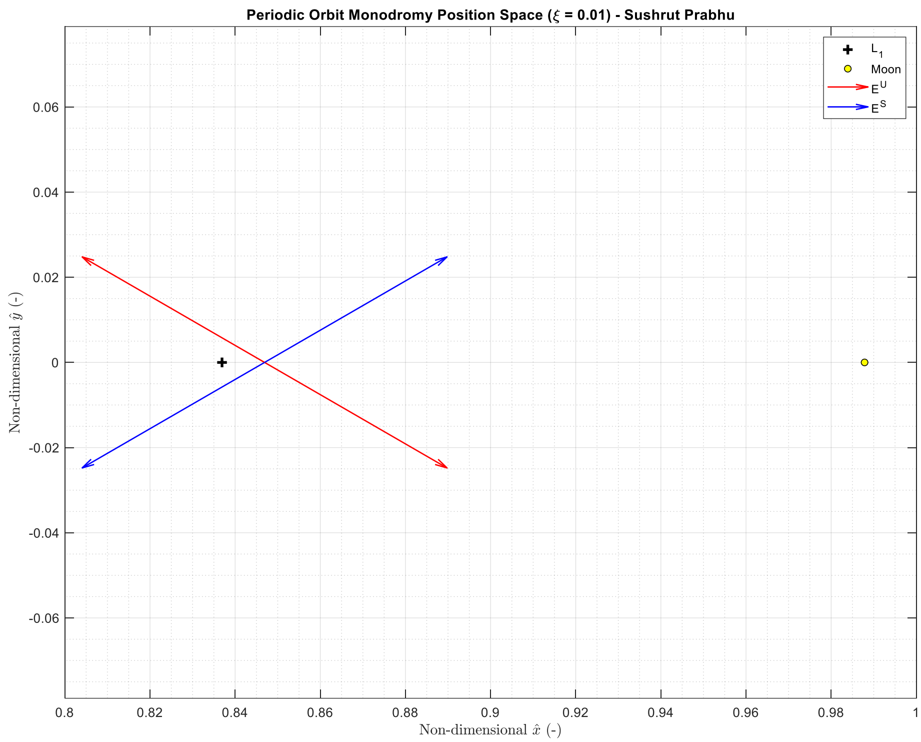


Figure F3.1: The eigenvectors in position space for a period orbit at L1.

Eigenvalues	$\overrightarrow{V_1}$	$\overrightarrow{V_2}$	$\overrightarrow{V_3}$	$\overrightarrow{V_4}$	$\overrightarrow{V_5}$	$\overrightarrow{V_6}$
2561.14	0.274958	0.274958	3.523479e-13 +1.690329e-07i	3.523479e-13 - 1.690329e-07i	9.187940e-15 - 3.958570e-15i	9.187940e-15 +3.958570e-15i
0.000390	-0.158987	0.158987	0.906723	0.906723	-2.543915e-15 - 1.924178e-14i	-2.543915e-15 +1.924178e-14i
1.000000 +4.804469e-08i	-1.054024e-25	6.913819e-19	-2.115522e-17 +1.016396e-24i	-2.115522e-17 - 1.016396e-24i	-4.996163e-16 + 0.447240i	-4.996163e-16 - 0.447240i
1.000000 -4.804469e-08i	0.849125	-0.849125	0.421727 +2.048316e-19i	0.421727 -2.048316e-19i	3.868837e-16 + 1.989120e-15i	3.868837e-16 + 1.989120e-15i
0.989859 +0.142056i	-0.422030	-0.422030	-2.404125e-12 -1.240846e-06i	-2.404125e-12 +1.240846e-06i	-7.016691e-14 +1.013059e-14i	-7.016691e-14 - 1.013059e-14i
0.989859 -0.142056i	3.279217e-25	-2.150986e-18	6.581686e-17 -3.162151e-24i	6.581686e-17 +3.162151e-24i	0.894414	0.894414

Table F3.1: Eigenvalues and eigenvectors of the Monodromy matrix.

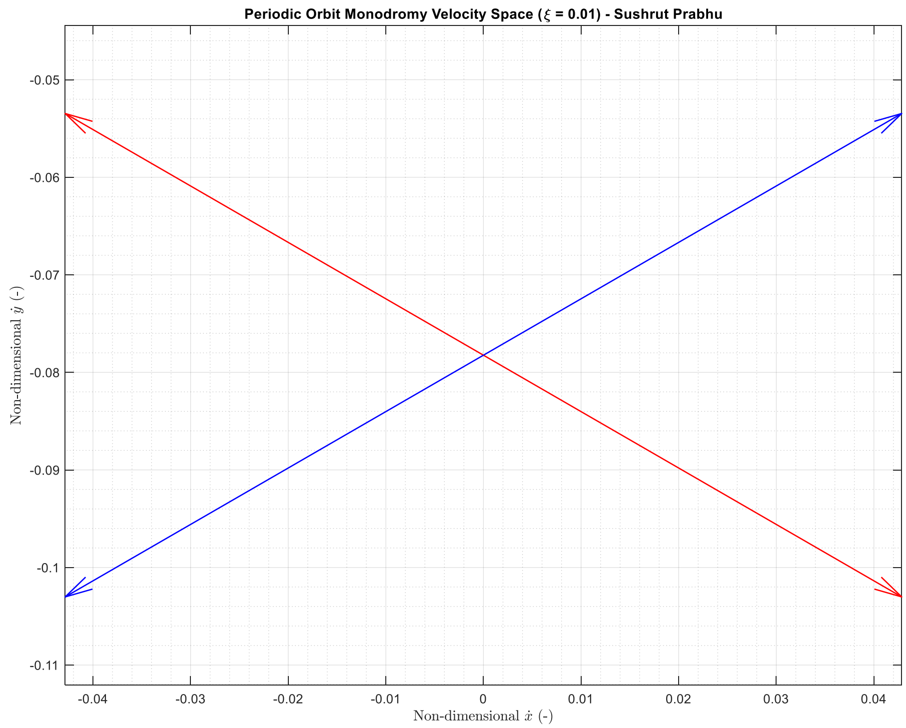


Figure F3.2: The eigenvectors in velocity space for a period orbit at L1.

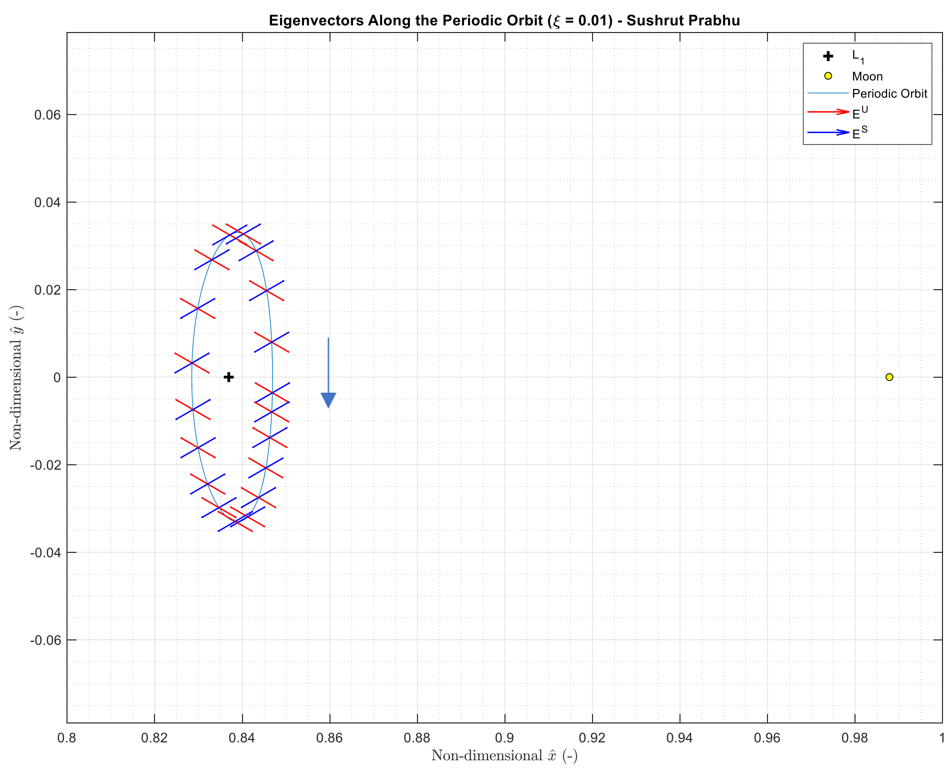


Figure F3.3: The eigenvectors in position space for a period orbit at L1.

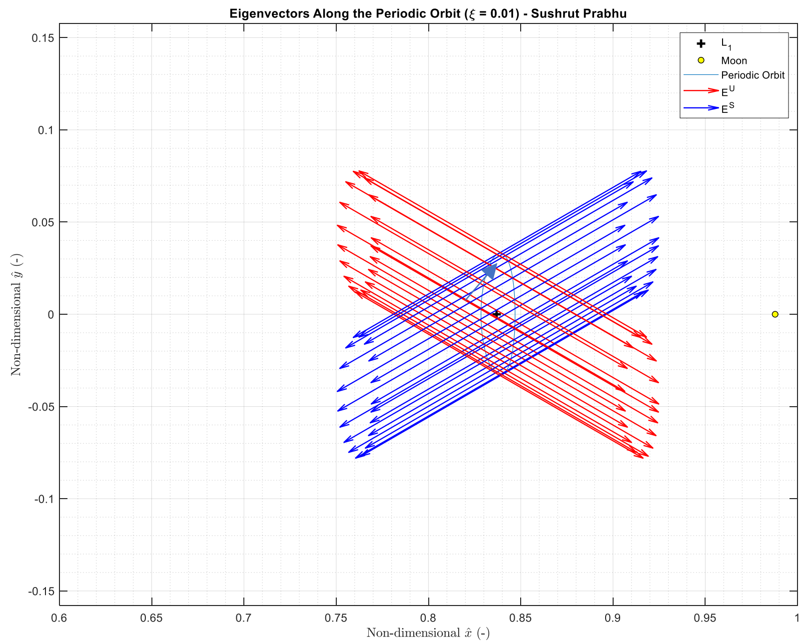


Figure F3.4: The eigenvectors in position made longer to create a tube.

Period Elapsed	$\vec{V_1}$	$\vec{V_2}$	$\vec{V_3}$	$\vec{V_4}$	$\vec{V_5}$	$\vec{V_6}$
0.0161	0.31459	0.24041	0.01838	0.01838	0	0
	-0.17826	0.14146	0.90172	0.90172	-0	-0
	0	0	-0	-0	0.038879 +0.44474i	0.038879 -0.44474i
	0.97342	-0.73991	0.42265	0.42265	-0	-0
	-0.46344	-0.3833	-0.22957 -1e-06i	-0.22957 +1e-06i	-0	-0
	0	0	0	0	0.88942 -0.11463i	0.88942 +0.11463i
0.0357	0.371	0.20414	0.04098	0.04098	0	0
	-0.20439	0.12222	0.8821	0.8821	-0	-0
	0	0	-0	-0	0.085815 +0.43494i	0.085815 -0.43494i
	1.1485	-0.62455	0.42595	0.42595	-0	-0
	-0.51812	-0.33935	-0.50528 -1e-06i	-0.50528 +1e-06i	-0	-0
	0	0	0	0	0.86986 -0.25264i	0.86986 +0.25264i

.0628	0.46523	0.16337	0.072381	0.072381	0	0
	-0.2453	0.099421	0.83181	0.83181	-0	-0
	0	0	-0	-0	0.14781 +0.40975i	0.14781 -0.40975i
	1.4362	-0.4944	0.43206	0.43206	-0	-0
	-0.60164	-0.28485	-0.86413 -1e-06i	-0.86413 +1e-06i	-0	-0
	0	0	0	0	0.81987 -0.4335i	0.81987 +0.4335i
.09884	0.62598	0.1223	0.1143	0.1143	0	0
	-0.30942	0.075001	0.72766	0.72766	-0	-0
	0	0	-0	-0	0.22229 +0.35734i	0.22229 -0.35734i
	1.9155	-0.36362	0.43442	0.43442	-0	-0
	-0.73133	-0.22301	-1.2817 -1e-06i	-1.2817 +1e-06i	-0	-0
	0	0	0	0	0.71699 -0.64694i	0.71699 +0.64694i
0.1425	0.89985	0.086071	0.16512	0.16512	0	0
	-0.40857	0.052191	0.54914	0.54914	-0	-0
	0	0	0	0	0.29801 +0.26682i	0.29801 -0.26682i
	2.7093	-0.2501	0.40893	0.40893	-0	-0
	-0.9384	-0.16133	-1.6821 -1e-06i	-1.6821 +1e-06i	-0	-0
	0	0	0	0	0.54268 -0.8564i	0.54268 +0.8564i
0.1918	1.3394	0.05866	0.2146	0.2146	0	0
	-0.55446	0.034286	0.30432	0.30432	-0	-0
	0	0	0	0	0.35508 +0.14162i	0.35508 -0.14162i
	3.9508	-0.1668	0.32102	0.32102	-0	-0
	-1.2707	-0.10979	-1.9556	-1.9556	-0	-0
	0	0	0	0	0.30747 -1.0039i	0.30747 +1.0039i
0.2439	2.024	0.039366	0.24902	0.24902	0	0

	-0.77009	0.021648	0.019294 -1e-06i	0.019294 +1e-06i	-0	-0
	0	0	0	0	0.37962 -0.005167i	0.37962 +0.005167i
	5.8548	-0.11034	0.15432	0.15432	-0	-0
	-1.8237	-0.071399	-2.0414	-2.0414	0	0
	0	0	0	0	0.038481 -1.0543i	0.038481 +1.0543i
0.2965	3.0467	0.02644	0.25555	0.25555	0	0
	-1.0897	0.013458	-0.26555	-0.26555	-0	-0
	0	0	0	0	0.366 -0.15261i	0.366 +0.15261i
	8.6874	-0.073722	-0.068908	-0.068908	-0	-0
	-2.7413	-0.045354	-1.9242	-1.9242	0	0
	0	0	0	0	-0.22662 -0.99845i	-0.22662 +0.99845i
0.3443	4.401	0.018424	0.23247	0.23247	-0	-0
	-1.5261	0.008673	-0.49884	-0.49884	-0	-0
	0	0	0	0	0.3223 -0.27405i	0.3223 +0.27405i
	12.453	-0.051421	-0.28683	-0.28683	-0	-0
	-4.1038	-0.029554	-1.6534 +1e-06i	-1.6534 -1e-06i	0	0
	0	0	0	0	-0.44294 -0.86451i	-0.44294 +0.86451i
0.3908	6.2801	0.012958	0.18391	0.18391	-0	-0
	-2.1657	0.005645	-0.68341	-0.68341	-0	-0
	0	0	0	0	0.25517 -0.37128i	0.25517 +0.37128i
	17.711	-0.036286	-0.47866	-0.47866	-0	-0
	-6.2192	-0.019245	-1.2589 +1e-06i	-1.2589 -1e-06i	0	0
	0	0	-0	-0	-0.61637 -0.67081i	-0.61637 +0.67081i
0.4425	9.3187	0.008748	0.10518	0.10518	-0	-0
	-3.2828	0.003511	-0.82188	-0.82188	-0	-0

	0	0	0	0	0.15852 -0.44681i	0.15852 +0.44681i
	26.257	-0.024588	-0.63281	-0.63281	-0	-0
	-10.066	-0.011807	-0.70204 +1e-06i	-0.70204 -1e-06i	0	0
	0	0	-0	-0	-0.75334 -0.40006i	-0.75334 +0.40006i
0.4898	13.376	0.006098	0.019143	0.019143	-0	-0
	-4.914	0.002294	-0.87543	-0.87543	-0	-0
	0	0	0	0	0.057068 -0.48041i	0.057068 +0.48041i
	37.68	-0.017172	-0.69464	-0.69464	-0	-0
	-15.81	-0.007501	-0.12662 +1e-06i	-0.12662 -1e-06i	0	0
	0	0	-0	-0	-0.81832 -0.12064i	-0.81832 +0.12064i
0.5375	19.237	0.004239	-0.069818	-0.069818	-0	-0
	-7.5009	0.001516	-0.85352	-0.85352	0	0
	0	0	0	0	-0.049386-0.47724i	-0.049386 +0.47724i
	54.123	-0.011944	-0.6692	-0.6692	0	0
	-24.972	-0.004753	0.46351 +1e-06i	0.46351 -1e-06i	0	0
	0	0	-0	-0	-0.82064 +0.16953i	-0.82064 -0.16953i
0.6064	32.495	0.002505	-0.18027	-0.18027	-0	-0
	-14.091	0.000864	-0.69277	-0.69277	0	0
	0	0	0	0	-0.19451 -0.40805i	-0.19451 +0.40805i
	91.016	-0.007063	-0.48882	-0.48882	0	0
	-48.025	-0.002491	1.2318 +1e-06i	1.2318 -1e-06i	0	0
	0	0	-0	-0	-0.7124 +0.56207i	-0.7124 -0.56207i
0.6751	54.658	0.00148	-0.24528	-0.24528	-0	-0
	-26.576	0.000518	-0.40825	-0.40825	0	0
	0	0	-0	-0	-0.30775 -0.27327i	-0.30775 +0.27327i

	152.42	-0.004199	-0.19877	-0.19877	0	0
	-90.253	-0.001355	1.7813 +1e-06i	1.7813 -1e-06i	0	0
	0	0	-0	-0	-0.48699 +0.8674i	-0.48699 -0.8674i
0.7402	89.379	0.000895	-0.25432	-0.25432	0	0
	-48.081	0.000333	-0.068308 +1e-06i	-0.068308 -1e-06i	0	0
	0	0	-0	-0	-0.3685 -0.10384i	-0.3685 +0.10384i
	249.91	-0.002575	0.09117	0.09117	0	0
	-159.77	-0.000802	2.027	2.027	0	0
	0	0	-0	-0	-0.19182 +1.0315i	-0.19182 -1.0315i
0.7846	125.33	0.000631	-0.23309	-0.23309	0	0
	-71.432	0.000251	0.17686 +1e-06i	0.17686 -1e-06i	0	0
	0	0	-0	-0	-0.37796 +0.02263i	-0.37796 -0.02263i
	353.65	-0.001844	0.25425	0.25425	0	0
	-232.04	-0.000581	2.0201	2.0201	-0	-0
	0	0	-0	-0	0.036949 +1.0562i	0.036949 -1.0562i
0.8456	200.74	0.000387	-0.1781	-0.1781	0	0
	-120.9	0.000172	0.4934	0.4934	0	0
	0	0	-0	-0	-0.34536 +0.19129i	-0.34536 -0.19129i
	579.91	-0.00116	0.39397	0.39397	0	0
	-377.22	-0.000393	1.7656 -1e-06i	1.7656 +1e-06i	-0	-0
	0	0	-0	-0	0.35582 +0.96117i	0.35582 -0.96117i
0.9090	332.5	0.00023	-0.10558	-0.10558	0	0
	-203.85	0.000115	0.75256	0.75256	0	0
	0	0	-0	-0	-0.25811 +0.33665i	-0.25811 -0.33665i
	992.43	-0.000705	0.43507	0.43507	0	0

	-601.71	-0.000274	1.2012 -1e-06i	1.2012 +1e-06i	-0	-0
	0	0	0	0	0.64798 +0.70466i	0.64798 -0.70466i
0.9667	533.62	0.000142	-0.038138	-0.038138	0	0
	-318.88	0.000078	0.88536	0.88536	0	0
	0	0	-0	-0	-0.1412 +0.42078i	-0.1412 -0.42078i
	1634	-0.000439	0.42544	0.42544	-0	-0
	-882.7	-0.0002	0.47125 -1e-06i	0.47125 +1e-06i	-0	-0
	0	0	0	0	0.83079 +0.35721i	0.83079 -0.35721i

Table F.2: The eigenvector for 20 different position.

Table of Contents

PSF3

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

Part a

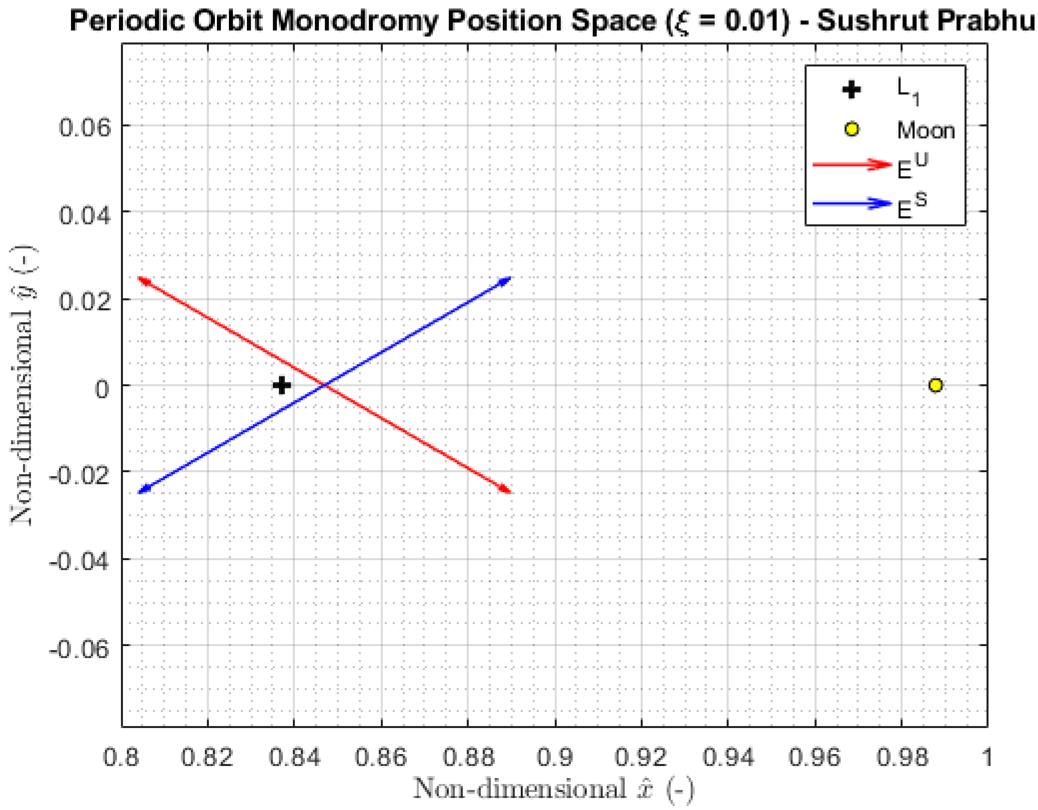
```
y = [dim_vals{7,2}+.01 0 0 0 -.082 0];
t_end = 2.711/2;

[yn,t_end] = Target3d_per([0
 0],y(1:3),y(4:6),t_end,dim_vals{4,2},"planar", 10^-13,"");

STM0 = eye(6);
STM0 = STM0(:)';
IC = [yn, STM0];
G = [1 0 0 0 0 0; 0 -1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 -1 0 0; 0 0 0 0 1
 0;0 0 0 0 0 -1];
```

```
Omega = [0 1 0; -1 0 0; 0 0 0];

[~,y]=ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

stm_12 = reshape(y(end,7:end),6,6)';
monodromy = G* [zeros(3,3), -eye(3);eye(3), -2*Omega]*stm_12' *
 [-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

[V,D] = eig(monodromy);
D = diag(D);

for n = 1:length(D)
    test = D(n)*V(:,n)-monodromy*V(:,n);
    if test <= 10^-11
        fprintf('Eigenvalue %i is paired with eigenvector %i.\n',
[n,n])
    else
        fprintf('There is mismatching')
    end
end
```

```
Eigenvalue 1 is paired with eigenvector 1.
Eigenvalue 2 is paired with eigenvector 2.
Eigenvalue 3 is paired with eigenvector 3.
Eigenvalue 4 is paired with eigenvector 4.
Eigenvalue 5 is paired with eigenvector 5.
Eigenvalue 6 is paired with eigenvector 6.
```

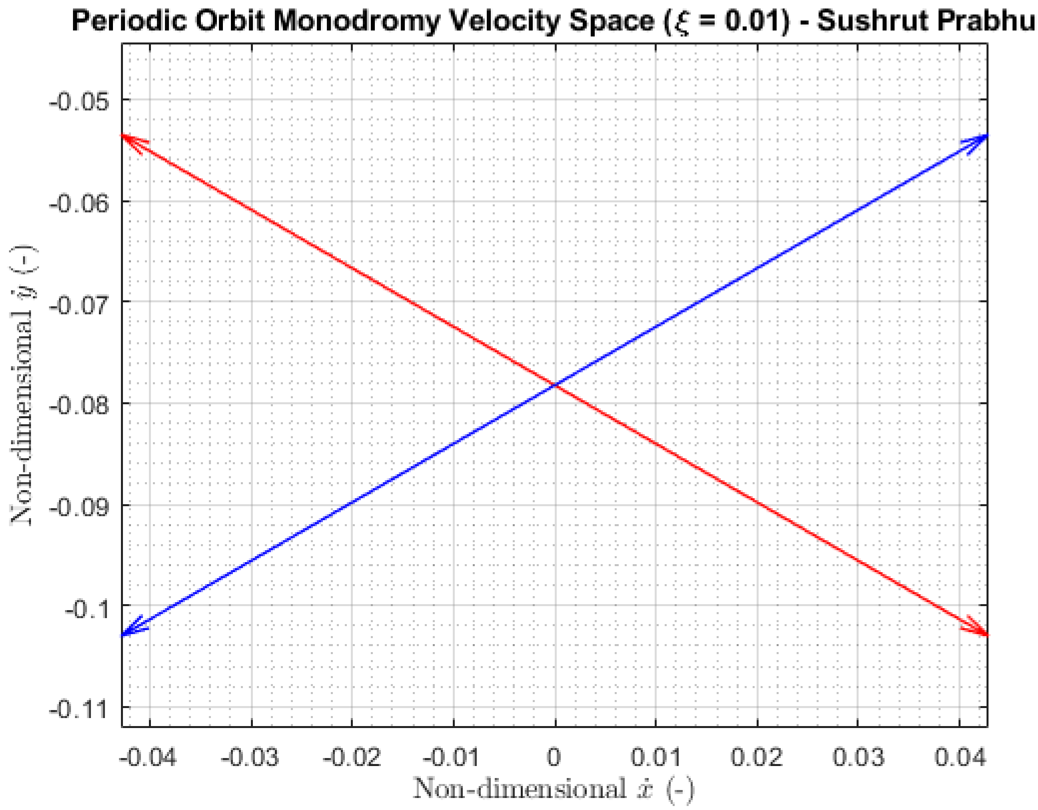
Part b

```
xi0 = 0.01;
Eu = V(1:2,1)/norm(V(1:2,1))*.055;
Es = V(1:2,2)/norm(V(1:2,1))*.055;

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
quiver(dim_vals{7,2}+xi0,0,Eu(1),Eu(2),'r','LineWidth',1)
quiver(dim_vals{7,2}+xi0,0,Es(1),Es(2),'b','LineWidth',1)
quiver(dim_vals{7,2}+xi0,0,-Eu(1),-Eu(2),'r','LineWidth',1)
quiver(dim_vals{7,2}+xi0,0,-Es(1),-Es(2),'b','LineWidth',1)
axis equal
xlim([0.8 1])
grid on
grid minor
legend('L_1','Moon','E^U','E^S')
title('Periodic Orbit Monodromy Position Space (\xi = 0.01) - Sushrut
 Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
```



Part c)

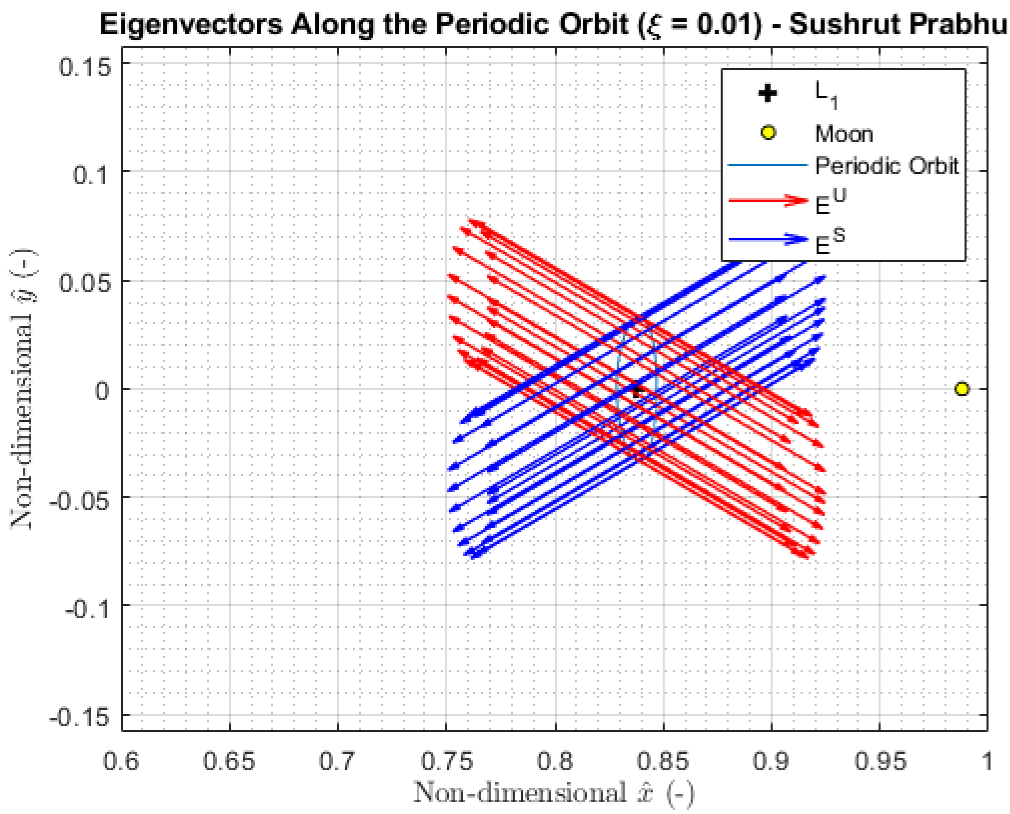
```
Ev_u = V(4:5,1)/norm(V(4:5,1));
Ev_s = V(4:5,2)/norm(V(4:5,1));

figure
quiver(0,yn(5),Eu(1),Eu(2),'r','LineWidth',1)
hold on
quiver(0,yn(5),Es(1),Es(2),'b','LineWidth',1)
quiver(0,yn(5),-Eu(1),-Eu(2),'r','LineWidth',1)
quiver(0,yn(5),-Es(1),-Es(2),'b','LineWidth',1)
axis equal
grid on
grid minor
title('Periodic Orbit Monodromy Velocity Space (\xi = 0.01) - Sushrut
 Prabhu')
xlabel("Non-dimensional $$\dot{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\dot{y}$$ (-)","Interpreter", "latex")
```



Part d)

```
count = 21;
[t,y]=ode45(@cr3bp_STM_df3d,[0 t_end*2],IC,options,dim_vals{4,2});
phi = y(:,7:end);

V_vec = [];
t_vec = [];

figure
plot(dim_vals{7,2},0,'+k','LineWidth',2,'MarkerSize',7)
hold on
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot(y(:,1),y(:,2))

for n = floor(length(y)/count):floor(length(y)/count+2):length(y)
    phi_t = reshape(phi(n,:),6,6)';
    V_t = phi_t*V;

    V_vec = [V_vec; V_t];
    t_vec = [t_vec; t(n)/t_end/2];

    Eu = V(1:2,1)/norm(V(1:2,1))*.1;
    Es = V(1:2,2)/norm(V(1:2,1))*.1;
    quiver(y(n,1),y(n,2),Eu(1),Eu(2),'r','LineWidth',1)
```

```
    quiver(y(n,1),y(n,2),Es(1),Es(2),'b','LineWidth',1)
    quiver(y(n,1),y(n,2),-Eu(1),-Eu(2),'r','LineWidth',1)
    quiver(y(n,1),y(n,2),-Es(1),-Es(2),'b','LineWidth',1)
end

V_vec = round(V_vec,6);

csvwrite('Vectors.csv',V_vec)

axis equal
grid on
grid minor
xlim([0.6 1])
title('Eigenvectors Along the Periodic Orbit (\xi = 0.01) - Sushrut
 Prabhu')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
legend('L_1','Moon','Periodic Orbit','E^U','E^S')
```



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A_t

Intermediate caluclation for STM

```
function A = A_t(x,y,z,miu)

if isnan(z)
    [Uxx,Uyy,~,Uxy,~,~]= Unn(x,y,0,miu);
    A = [zeros(2,2), eye(2,2); Uxx, Uxy, 0, 2; Uxy, Uyy, -2, 0];
else
    [Uxx,Uyy,Uzz,Uxy,Uxz,Uyz]= Unn(x,y,z,miu);
    A = [zeros(3,3), eye(3,3); Uxx, Uxy,Uxz, 0, 2,0; Uxy, Uyy, Uyz, 0,
 -2, 0; Uxz, Uyz, Uzz, 0, 0, 0];
end

end
```

```
Not enough input arguments.

Error in A_t (line 5)
if isnan(z)
```

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acc_nonlin

Non-linear acceleration for STM

```
function acc = acc_nonlin(r,v,miu)

dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
rr = sqrt((r(1)+miu-1)^2 + r(2)^2 + r(3)^2);

acc(1) = 2*v(2) + r(1) - (1-miu)*(r(1)+miu)/dd^3 - miu*(r(1)-1+miu)/
rr^3;
acc(2) = -2*v(1) + r(2) - (1-miu)*r(2)/dd^3 - miu*r(2)/rr^3;
acc(3) = -(1-miu)*r(3)/dd^3 - miu*r(3)/rr^3;
```

```
Not enough input arguments.

Error in acc_nonlin (line 5)
dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
```

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cr3bp_STM_df2d

2d STM

```
function dx = cr3bp_STM_df2d(t,x,miu)
dx = zeros(20,1);

d = sqrt((x(1)+miu)^2 + x(2)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2);
phi = reshape(x(5:end),4,4)';

dx(1) = x(3);
dx(2) = x(4);
dx(3) = 2*x(4) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(4) = -2*x(3) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;

phi_dot = [A_t(x(1),x(2),NaN,miu)*phi]';

dx(5:end) = phi_dot(:);
end
```

```
Not enough input arguments.

Error in cr3bp_STM_df2d (line 6)
d = sqrt((x(1)+miu)^2 + x(2)^2);
```

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cr3bp_STM_df3d

3d STM

```
function dx = cr3bp_STM_df3d(t,x,miu)

dx = zeros(42,1);

d = sqrt((x(1)+miu)^2 + x(2)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2);
phi = reshape(x(7:end),6,6)';

d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2 + x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = 2*x(5) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(5) = -2*x(4) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;
dx(6) = -(1-miu)*x(3)/d^3 - miu*x(3)/r^3;

phi_dot = [A_t(x(1),x(2),x(3),miu)*phi]';

dx(7:end) = phi_dot(:);
end
```

```
Not enough input arguments.

Error in cr3bp_STM_df3d (line 7)
d = sqrt((x(1)+miu)^2 + x(2)^2);
```

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Target3d_per

Periodic orbit targeter

```
function  [IC_final,t_end] = Target3d_per(r_des,r,v,t_end,miu,fix,
 tol, pl)
% Initialization
% t = 0:.001:t_end;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance
error = 2*tol;
phi_0 = eye(6);
phi_0 = phi_0(:)';
i = 1;

if pl == "plot"
    figure
    hold on
end

while error > tol
    % Non-linear propgation with phi
    IC = [r,v,phi_0];
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,miu);

    % Phi at final time
    phi_t = y(:,7:end);
    phi_tf = reshape(phi_t(end,:),6,6)';
    acc = acc_nonlin(y(end,1:3),y(end,4:6),miu);

    if fix == "planar"
        phi_main_tf = [phi_tf(2,5), y(end,5); phi_tf(4,5), acc(1)];
        err = r_des - [y(end,2), y(end,4)]';
    elseif fix == "x0"
        phi_main_tf = [phi_tf(4,3), phi_tf(4,5); phi_tf(6,3),
 phi_tf(6,5)] + 1/y(end,5)*[acc(1), acc(3)]'*[phi_tf(2,3),
 phi_tf(2,5)];
        err = r_des - [y(end,4), y(end,6)]';
    elseif fix == 2
        phi_main_tf = [phi_tf(1:2,4),y(end,3:4)'];
    elseif dt == 2
        phi_main_tf = [phi_tf(1:2,3),y(end,3:4)'];
    end

    delv1 = phi_main_tf^-1 * err;

    if fix == "planar"
        IC = [r,v+[0, delv1(1), 0]];
        t_end = t_end + delv1(2);
    elseif fix == "x0"
        IC = [r+[0,0,delv1(1)],v+[0,delv1(2),0]];
    elseif fix == 1
```

```
        IC = [r,v+[0, delv1(1), 0]];
        t_end = t_end+delv1(2);
    elseif fix == 2
        IC = [r,v+[delv1(1) 0, 0]];
        t_end = t_end+delv1(2);
    end

    [~,yn] = ode45(@cr3bp_df,[0 t_end],IC,options,miu);

    r = yn(1,1:3);
    v = yn(1,4:6);

    if fix == "planar"
        error = max(abs([yn(end,2),yn(end,4)]-r_des'));
    elseif fix == "x0"
        error = max(abs([yn(end,4),yn(end,6)]-r_des'));
    end

    i = i+1;

    if i > 500
        error = tol/2;
        fprintf("Did not Coverge")
    end
    if pl == "plot"
        if error > tol
            ite = plot(yn(:,1),yn(:,2),'-.b');
            if i > 2

 set(get(get(ite,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
            end
        else
           plot(yn(:,1),yn(:,2),'r');
        end
    end
end

IC_final = yn(1,:);
end
```

Not enough input arguments.

Error in Target3d_per (line 7)
*error = 2*tol;*

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Characteristic Elements

```
function [lstar, mstar, tstar] = charE(D1,D2,m1,m2)
G = 6.6738*10^-20;

lstar = D1+D2;
mstar = m1 + m2;

tstar = sqrt(lstar^3/G/mstar);
```

```
Not enough input arguments.

Error in charE (line 5)
lstar = D1+D2;
```

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horner_alg

```
function [alpha,beta] = horner_alg(n,a,z0)
alpha = a(1);
beta = 0;

for k = 2:n
   beta = alpha + z0*beta;
   alpha = a(k) + z0*alpha;
end


end
```

```
Not enough input arguments.

Error in horner_alg (line 3)
alpha = a(1);
```

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```
function C = Jacobi_C(x,y,z,v,miu)

d = sqrt((x+miu).^2 + y.^2 +z.^2);
r = sqrt((x-1+miu).^2 + y.^2 + z.^2);
C = x.^2 + y.^2 + 2*(1-miu)./d + 2*miu./r - v.^2;

end
```

```
Not enough input arguments.

Error in Jacobi_C (line 3)
d = sqrt((x+miu).^2 + y.^2 +z.^2);
```

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Unn

```
function [Uxx,Uyy,Uzz,Uxy,Uxz,Uyz]= Unn(x,y,z,miu)

xm = (x+miu);
xm1 = x+miu-1;

d = sqrt(xm^2 + y^2 +z^2);
r = sqrt(xm1^2 + y^2 +z^2);

Uxx = 1-(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*xm^2 / d^5 + 3*miu*xm1^2 /
 r^5;
Uyy =  1-(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*y^2 / d^5 + 3*miu*y^2 /
 r^5;
Uzz = -(1-miu)/d^3 - miu/r^3 + 3*(1-miu)*z^2 / d^5 + 3*miu*z^2 / r^5;

Uxy = 3*(1-miu)*xm*y/d^5 + 3*miu*xm1*y/r^5;
Uxz = 3*(1-miu)*xm*z/d^5 + 3*miu*xm1*z/r^5;
Uyz = 3*miu*y*z/d^5 + 3*miu*y*z/r^5;
end
```

```
Not enough input arguments.

Error in Unn (line 4)
xm = (x+miu);
```

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Solar Systems Constants

Constants

```
classdef SolarS
    properties
        % Distances from Sun/ planet to planet/ Moon
        dMercury = 57909226.542
        dVenus = 108209474.537
        dEarth = 149597870.7
        dMars = 227943816.693
        dJupiter = 778340816.693
        dSaturn = 1426666414.180
        dM_E = 384400
        dMoon = 384400+149597870.7
        dPluto = 5906440596.5288
        dTitan_S = 1221865
        dPhobos_M = 9376;
        dEuropa_J = 671100
        dOberon_U = 583500
        dTriton_N = 354759
        dCharon_P = 17536

        % Mass of Planet/Star/ Moon (G*m)
        mSun = 132712440017.99
        mVenus = 324858.5988
        mEarth = 398600.4415
        mMars = 42828.3142
        mJupiter = 126712767.8578
        mSaturn = 37940626.0611
        mUranus = 5794549.0070719
        mNeptune = 6836534.06387
        mPluto = 981.600887707;

        mMoon = 4902.8011
        mTitan = 8979.766
        mPhobos = 7.11328968e-04
        mEuropa = 3203.31978
        mOberon = 192.4249
        mTriton = 1427.8589
        mCharon = 103.2187

        % Radius of Moon/Planet
        rEarth = 6378.136;
        rMars = 3397.00;
        rMoon = 1738.10;
        rSaturn = 60268.00;
        rJupiter = 71492.00;
        % Eccentricity of Planets
        eEarth = 0.01671022
        eSaturn = 0.05386179
```

```
    end
end


ans = 

  SolarS with properties:

     dMercury: 5.7909e+07
       dVenus: 1.0821e+08
       dEarth: 1.4960e+08
        dMars: 2.2794e+08
     dJupiter: 7.7834e+08
      dSaturn: 1.4267e+09
         dM_E: 384400
        dMoon: 1.4998e+08
       dPluto: 5.9064e+09
     dTitan_S: 1221865
    dPhobos_M: 9376
    dEuropa_J: 671100
    dOberon_U: 583500
    dTriton_N: 354759
    dCharon_P: 17536
         mSun: 1.3271e+11
       mVenus: 3.2486e+05
       mEarth: 3.9860e+05
        mMars: 4.2828e+04
     mJupiter: 1.2671e+08
      mSaturn: 3.7941e+07
      mUranus: 5.7945e+06
     mNeptune: 6.8365e+06
       mPluto: 981.6009
        mMoon: 4.9028e+03
       mTitan: 8.9798e+03
      mPhobos: 7.1133e-04
      mEuropa: 3.2033e+03
      mOberon: 192.4249
      mTriton: 1.4279e+03
      mCharon: 103.2187
       rEarth: 6.3781e+03
        rMars: 3397
        rMoon: 1.7381e+03
      rSaturn: 60268
     rJupiter: 71492
       eEarth: 0.0167
      eSaturn: 0.0539
```

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Target3d

Periodic orbit targeter

```
function  [IC_final,t_end, tb] = Target3d(r_des,r,v,t_end,miu, tol,
 pl)
% Initialization
% t = 0:.001:t_end;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance
error = 2*tol;
phi_0 = eye(6);
phi_0 = phi_0(:)';
i = 1;
v0 = v;

if pl == "plot"
    figure
    plot3(r(1),r(2),r(3),'*')
    hold on
end

while error > tol
    % Non-linear propgation with phi
    IC = [r,v,phi_0];
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,miu);

    % Phi at final time
    phi_t = y(:,7:end);
    phi_tf = reshape(phi_t(end,:),6,6)';

    K = -phi_tf(1:3,4:6);
    Fx = r_des - y(end,1:3)' ;
    delv1 = K.' * (K*K.')^-1 * Fx;

    IC = [r, v - delv1'];
    [~,yn] = ode45(@cr3bp_df,[0 t_end],IC,options,miu);
    v = yn(1,4:6);

    error = max(abs(yn(end,1:3)-r_des'));

    tb{:,i} = {v-v0, norm(v-v0), Fx, norm(Fx)};
    i = i+1;

    if pl == "plot"
        if error > tol
            pl3 = plot3(yn(:,1),yn(:,2),yn(:,3),'-.b');
            if i > 2

 set(get(get(pl3,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
            end
        else
```

```
            plot3(yn(:,1),yn(:,2),yn(:,3))
        end
    end

    if i > 20
        error = tol/2;
        fprintf("Did not Coverge")
    end
end

IC_final = IC;

end
```

```
Not enough input arguments.

Error in Target3d (line 7)
error = 2*tol;
```

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