Sushant Prabhu | AAE632 PS E1

Problem E1

Given: 03 and 04 where $\xi = 0.01$ and $\eta = 0$ from L_1 of the Earth-Moon System.

Find: ai) Compute atleast 20 more orbits. What is a good strategy for $\dot{y}_0$ guess? Step size?

aii) Update $\dot{y}_0$ as a function of x_0. Can you continue this strategy for initial guess of $\dot{y}_0$?

aiii) Update period and JC plots. How does it change?

aiv) Eigenvalues of STM. Three reciprocal pair? What is reciprocal of complex eigenvalue?

Solution:

ai) In the previous homework there was a linear relationship so we could use the previous 2 points to get a linear relationship to predict next $\dot{y}$

$$\dot{y}_i = \frac{\dot{y}_{i-2} - \dot{y}_{i-1}}{x_{i-2} - x_{i-1}} x_i - x_{i-1} + \dot{y}_{i-1} \qquad 3 \le i \le 20$$

The largest step size $\Rightarrow \boxed{\begin{array}{l} 0.0028 \text{ (non-dim)} \\ \Rightarrow 1.0763 \times 10^3 \text{ km} \end{array}}$

The targeter I used is as follows, we will utilize the reflective theorem

$r_f = [x_f, 0, z_f]$ even though this is a planar problem lets keep z terms in.

$v_f = [0, \dot{y}_f, \dot{z}_f]$ They will just be zero

Continued...

The variational equation is as follows:

$$\delta y_f = \frac{\partial y}{\partial \dot{y}_0}\delta \dot{y}_0 + \overset{\to 0}{\frac{\partial y}{\partial \dot{z}_0}\delta \dot{z}_0} + \dot{y}\,\delta t$$

$$\delta \dot{x}_f = \frac{\partial \dot{x}}{\partial \dot{y}_0}\delta \dot{y}_0 + \overset{\to 0}{\frac{\partial \dot{x}}{\partial \dot{z}_0}\delta \dot{z}_0} + \ddot{x}\,\delta t$$

$$\overset{\bar{rV}_{des}}{\begin{bmatrix} \delta y_f \\ \delta \dot{x}_f \end{bmatrix}} = \overset{\to \phi_{mod}}{\underset{\hookrightarrow \text{Using a } 6\times 6 \text{ STM}}{\begin{bmatrix} \phi_{25} & \dot{y} \\ \phi_{45} & \ddot{x} \end{bmatrix}}} \underset{\to \bar{rV}_{IC}}{\begin{bmatrix} \delta \dot{y}_0 \\ \delta t \end{bmatrix}}$$

$\dot{y} \to$ obtained from propagation

$\ddot{x} \to$ use the values of propogation and non-linear equation to get acceleration

$\therefore \; \dot{\bar{rV}}_{IC} = \phi_{mod}^{-1}\, \bar{rV}_{des} \to$ Targeter

21 ξ_0 positions:

3.5096×10^5 km, 3.5203×10^5 km, 3.5311×10^5 km

3.5419×10^5 km, 3.5526×10^5 km, 3.5634×10^5 km

3.5742×10^5 km, 3.5849×10^5 km, 3.5957×10^5 km

3.6064×10^5 km, 3.6172×10^5 km, 3.6280×10^5 km

3.6387×10^5 km, 3.6495×10^5 km, 3.6603×10^5 km

3.6710×10^5 km, 3.6818×10^5 km, 3.6925×10^5 km

3.7033×10^5 km, 3.7141×10^5 km, 3.7248×10^5 km

This was the max Δx_0 possible because higher ones give matrix inversion errors

continued...

aii) $\boxed{\text{See figure: E1.1 for plot of new Lyaponov orbits}}$

Again, the largest Δx_0 is 0.0028 (non-dim) or 1.0763×10^3 km, because with the strategy I have As we get closer to the moon, the relationship is non-linear and so the guesses worsen. There are errors with matrix inversion due to poor first initial guess. This works for 23 iterations

aiii) $\boxed{\text{See Figures: E1.2 and E1.3 for plots of JC and } \mathbb{P}}$

The Jacobi constant decreases almost linearly and then after about 0.96 and the it starts to curve down faster.

The $\mathbb{P}$ against x_0 position grows like a convex upward curve. There doesn't seem to be a linear part like $\dot{y}_0$ or JC

aiv) For the eigenvalues check the Table: E1.1

For complex numbers → All complex eigenvalues in pair

$\lambda_1 = a + bi$ and $\lambda_2 = \bar{\lambda}_1 = a - bi$

$\therefore \lambda_3 = \frac{1}{a+bi} \frac{(a-bi)}{(a-bi)} = \frac{a-bi}{a^2+b^2}$ $\therefore \lambda_4 = \bar{\lambda}_3 = \frac{a+bi}{a^2+b^2}$

Note: Iff $a^2+b^2=1$ then $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_4$

Continued...

To calculate the monodromy we need the ½ period STM $\phi(T/2, 0)$

$$\therefore G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

, Period

$$\phi(T, 0) = G \begin{bmatrix} 0 & -I \\ I & -2\Omega \end{bmatrix} \phi^T(T/2, 0) \begin{bmatrix} -2\Omega & I \\ -I & 0 \end{bmatrix} G\,\phi(T/2, 0)$$

A good check is that there is a reciprocal pair.

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PSE1 Optional

Given: Earth-Moon system, at point L_2

Find: Lyapunov orbits in the vicinity of L_2 starting with $\xi = 0.01$ $\eta = 0$

Solution: We can get a linear solution for $\xi = 0.01$ but it is a very poor model.

There are a couple of strategies to combat this start with a smaller ξ, mayb $\xi = 0.001$.
Or for the initial $\dot{y}_0$ corresponding to $\xi_0 = 0.01$ force it to go to cross x-axis increase $\dot{y}_0$.
This is what I did. A factor of 1.2 does the job.

Now you have a starting point you can use the same strategy. For L_2 the largest step size was 0.006 which is greater than the one used for L_1.

See figure: E1.5.

The P, $\dot{y}_0$, and J_c relationship with x_0 are far more non-linear

See figures E1.6, E1.7, E1.8

PSE1

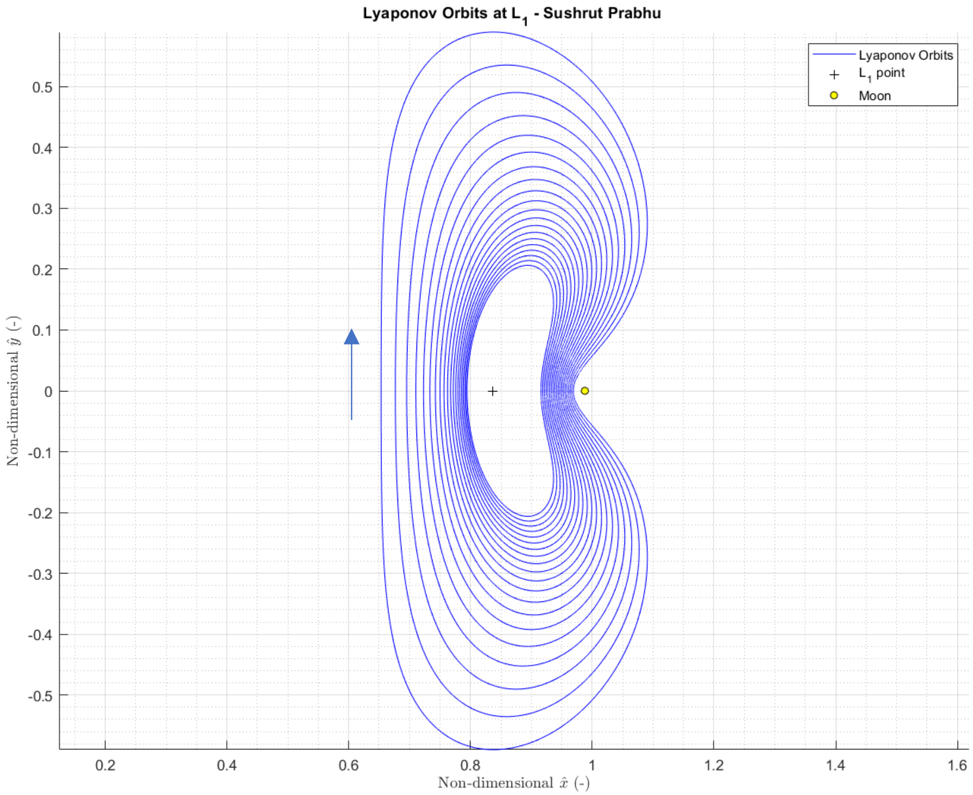


Figure E1.1: Lyapunov orbits at L1.

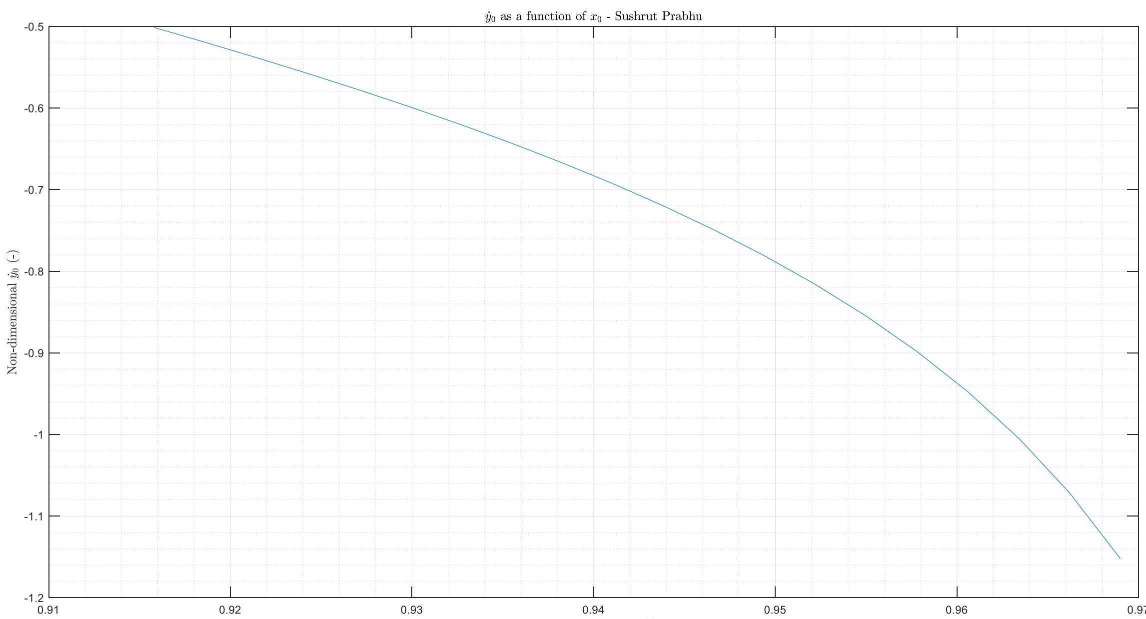


Figure E1.2: Lyapunov orbits relationship of initial x position and initial y velocity at L1.

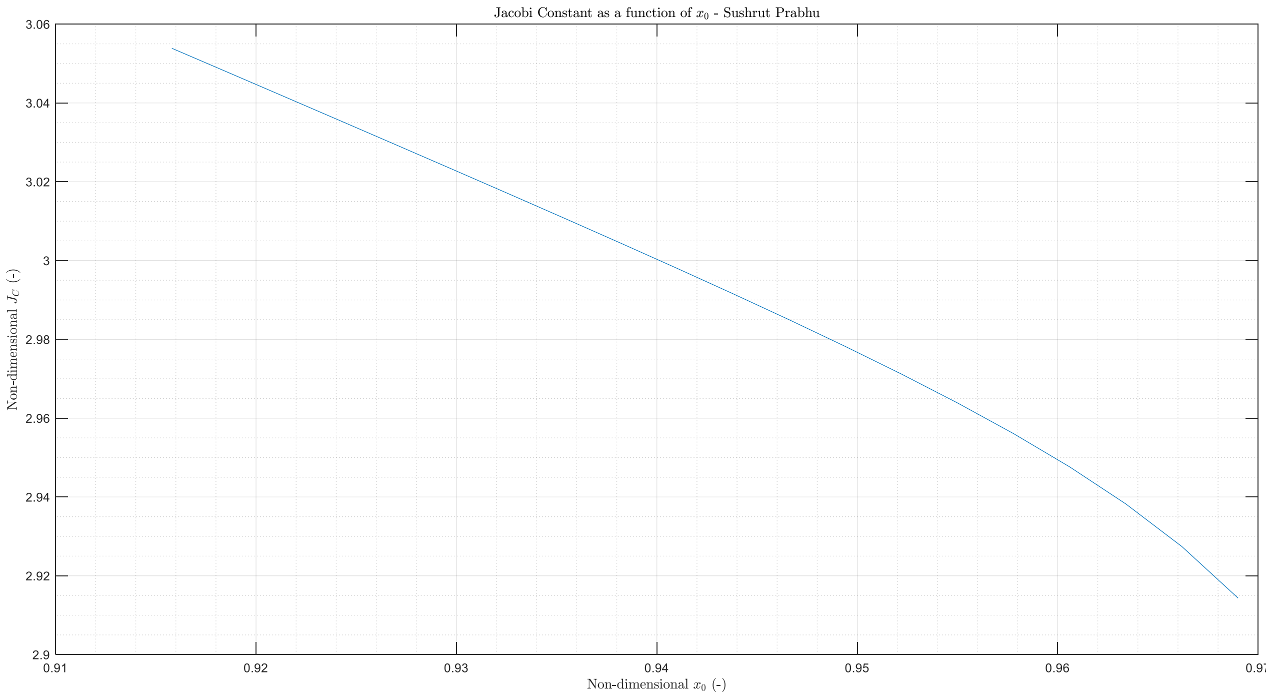


Figure E1.3: Lyapunov orbits relationship of initial x position and Jacobi constant at L1.

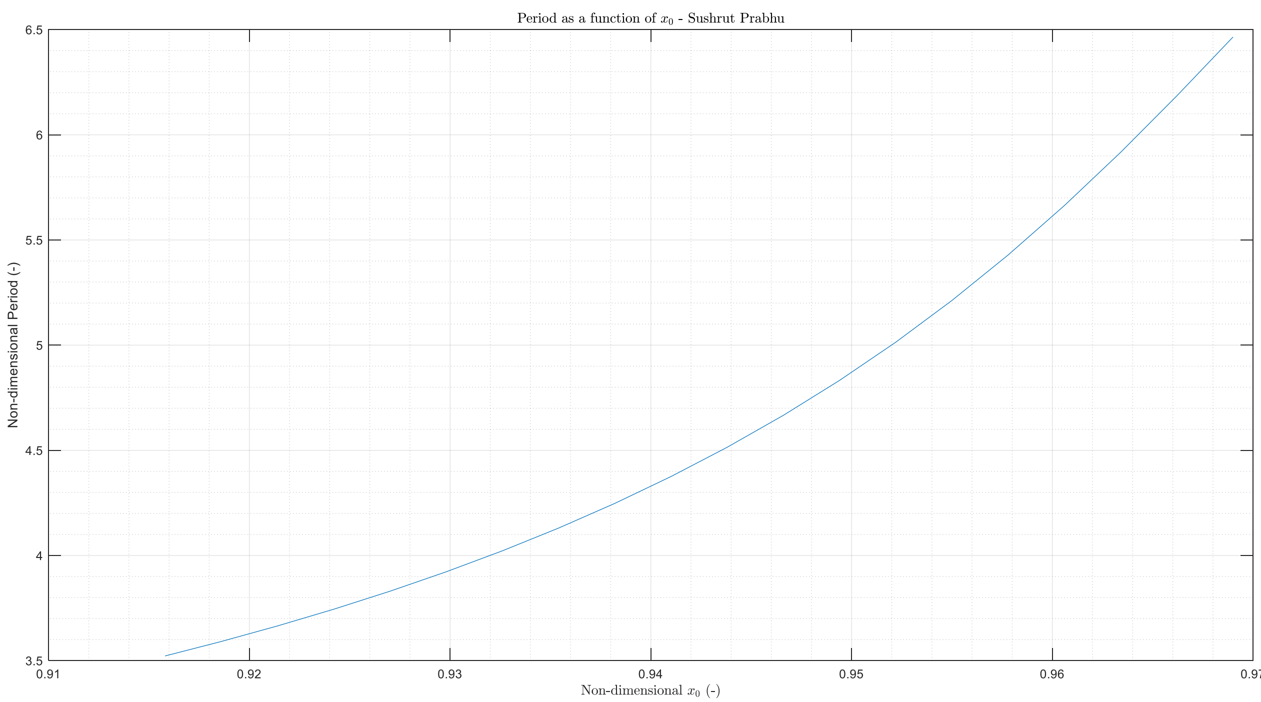


Figure E1.4: Lyapunov orbits relationship of initial x position and period at L1.

$\hat{x}$ **(non-dim)**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\lambda_3}$	$\boldsymbol{\lambda_4}$	$\boldsymbol{\lambda_5}$	$\boldsymbol{\lambda_6}$
0.9158	632.77	0.001580	1.000001	0.999999	1.586640	0.630262
0.9186	583.03	0.00171	1.000000 -5.806790e-07i	1.000000 +5.806790e-07i	1.561026	0.640604
0.9214	536.12	0.001865	1.0000000 -6.444433e-07i	1.0000000 +6.444433e-07i	1.518890	0.658375
0.9242	491.94	0.00203	1.000000 -1.548160e-06i	1.000000 +1.548160e-06i	1.453468	0.688010
0.9270	450.40	0.002220	1.000000	1.000000	1.351388	0.739980
0.9298	411.39	0.002431	0.999999	1.000001	1.164859	0.858473
0.9326	374.81	0.999999	1.000001	0.002668	0.966447 -0.256865i	0.966447 +0.256865i
0.9354	340.57	1.000003	0.999997	0.002936	0.907254 -0.420582i	0.907254 +0.420582i
0.9382	308.59	1.000002	0.999998	0.003241	0.830666 -0.556770i	0.830666 +0.556770i
0.941	278.78	1.000000 -7.361641e-07i	1.000000 +7.361641e-07i	0.003587	0.732493 -0.680774i	0.732493 +0.680774i
0.9438	251.11	1.000000 - 2.305310e-06i	1.000000 +2.305310e-06i	0.003982	0.607662 -0.794195i	0.607662 +0.794195i
0.9466	225.52	1.000001	1.000000	0.004434	0.450143 -0.892956i	0.450143 +0.892956i
0.9494	202.02	1.000004	0.999996	0.004950	0.252957 -0.967478i	0.252957 +0.967478i
0.9522	180.65	1.000002	0.999998	0.005535	0.008361 -0.999965i	0.008361 +0.999965i
0.9550	161.49	1.000007	0.999993	0.006192	-0.291688 -0.956514i	-0.291687 +0.956514i
0.9578	144.68	1.000000 -7.082430e-07i	1.000000 +7.082430e-07i	0.006912	-0.654694 -0.755894i	-0.654694 +0.755894i
0.9606	130.44	1.000004	0.999996	0.007666	-1.511146	-0.661749
0.9634	119.12	1.000004	0.999996	0.008395	-2.826209	-0.353830
0.9662	111.20	1.000000 -7.953021e-06i	1.000000 +7.953021e-06i	0.008992	-4.088580	-0.244584
0.9690	107.47	1.000000 -1.171634e-05i	1.000000 +1.171634e-05i	0.009304	-5.460904	-0.183120

Table E1.1: The eigenvalues of the monodromy matrix.

PSE1 optional

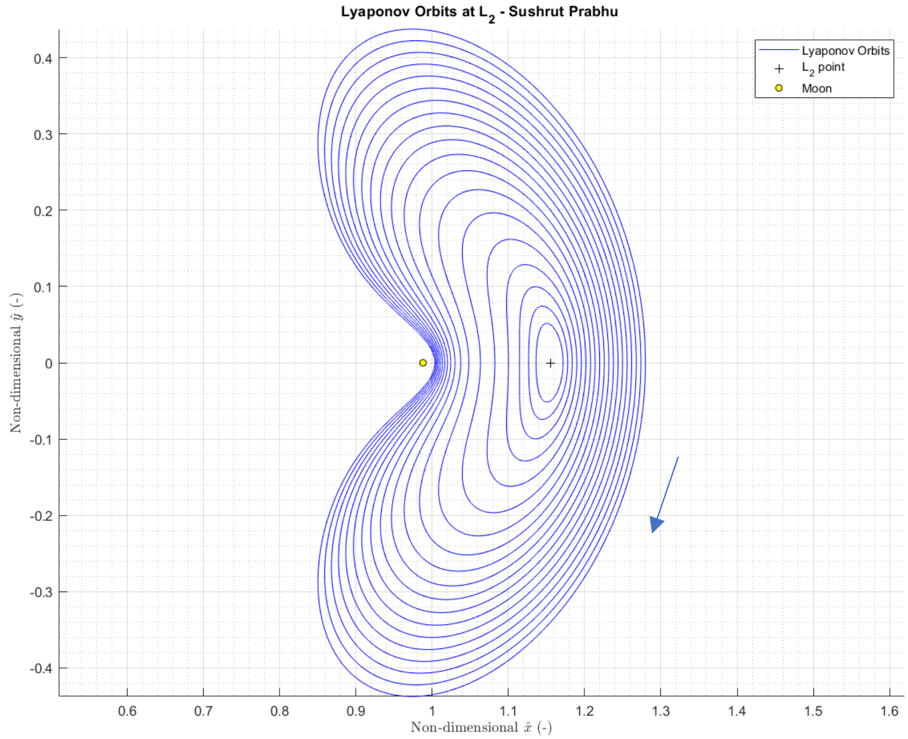


Figure E1.5: Lyapunov orbits around L2 Lagrange point.

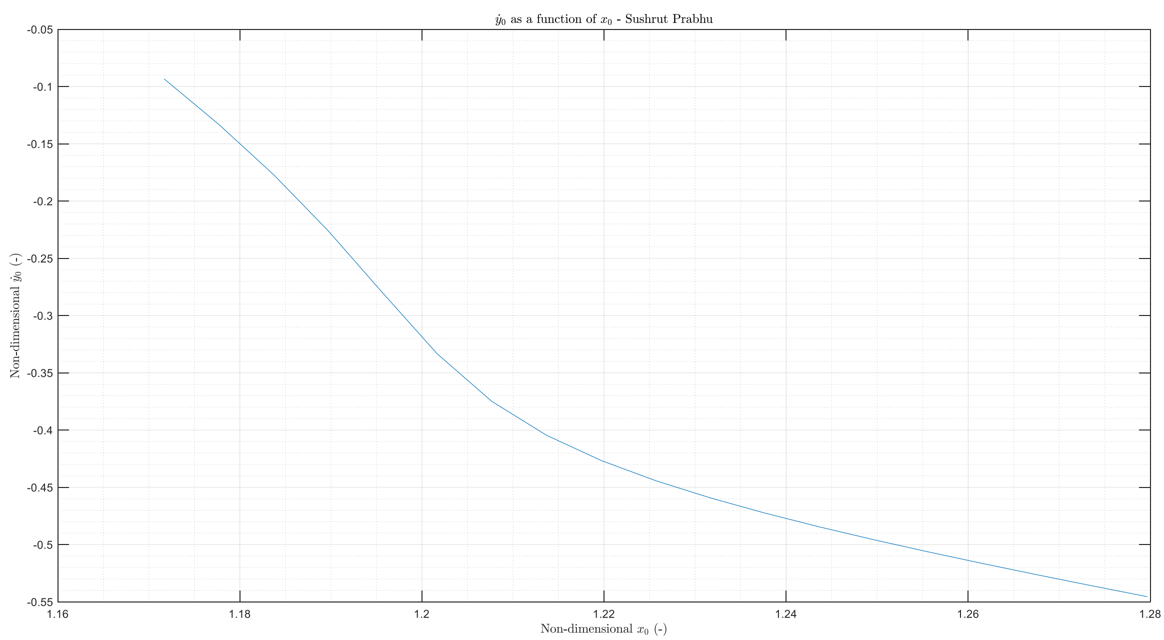


Figure E1.6: Lyapunov orbits relationship of initial x position and initial y velocity at L2.

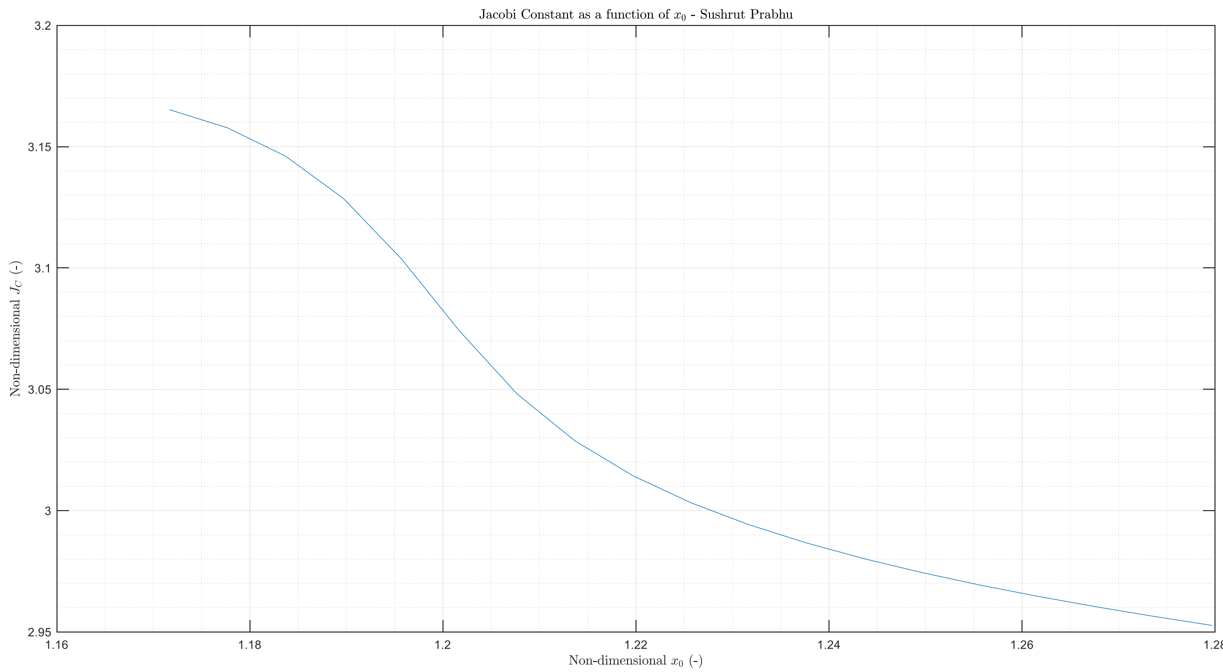


Figure E1.7: Lyapunov orbits relationship of initial x position and Jacobi constant at L1.

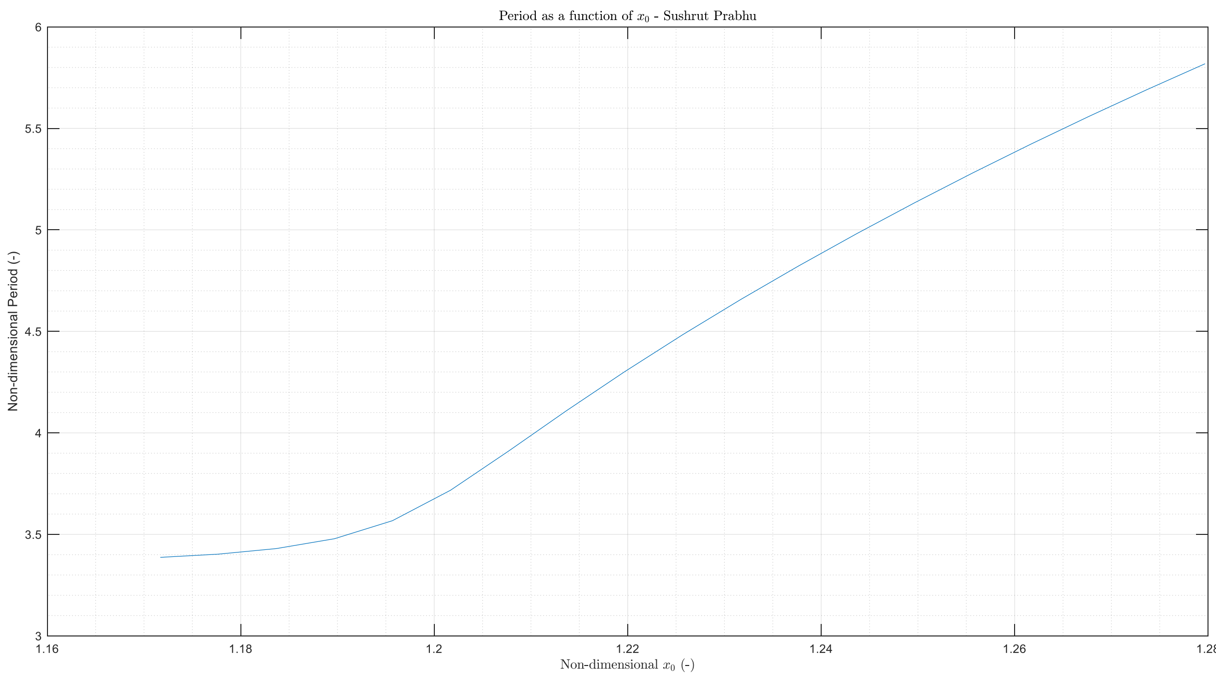


Figure E1.8: Lyapunov orbits relationship of initial x position and period at L1.

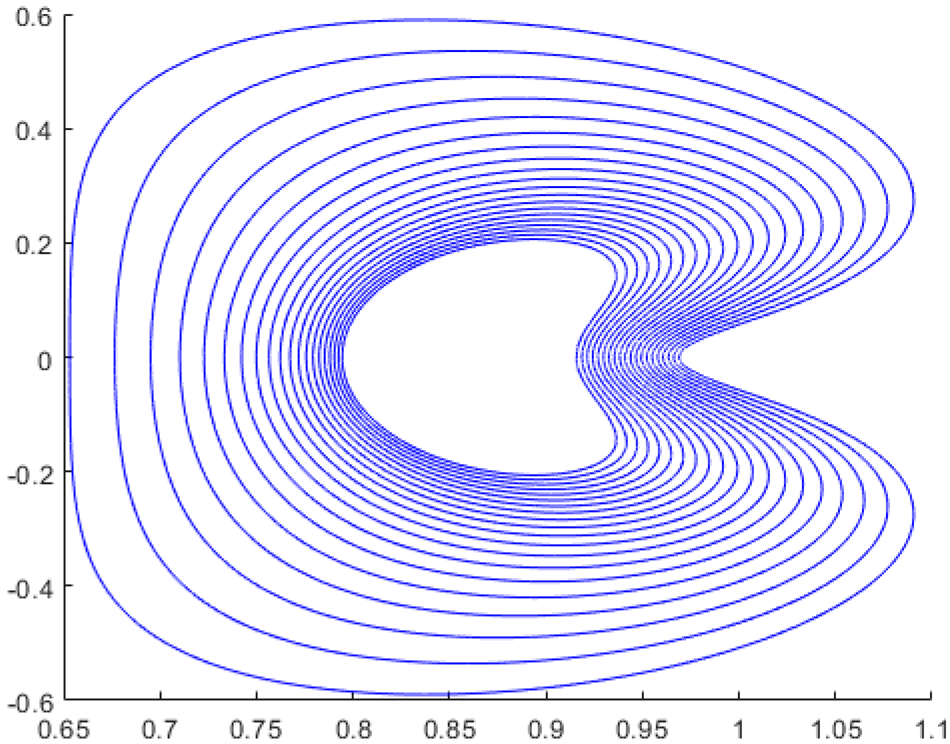
PSE1

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

Part a)

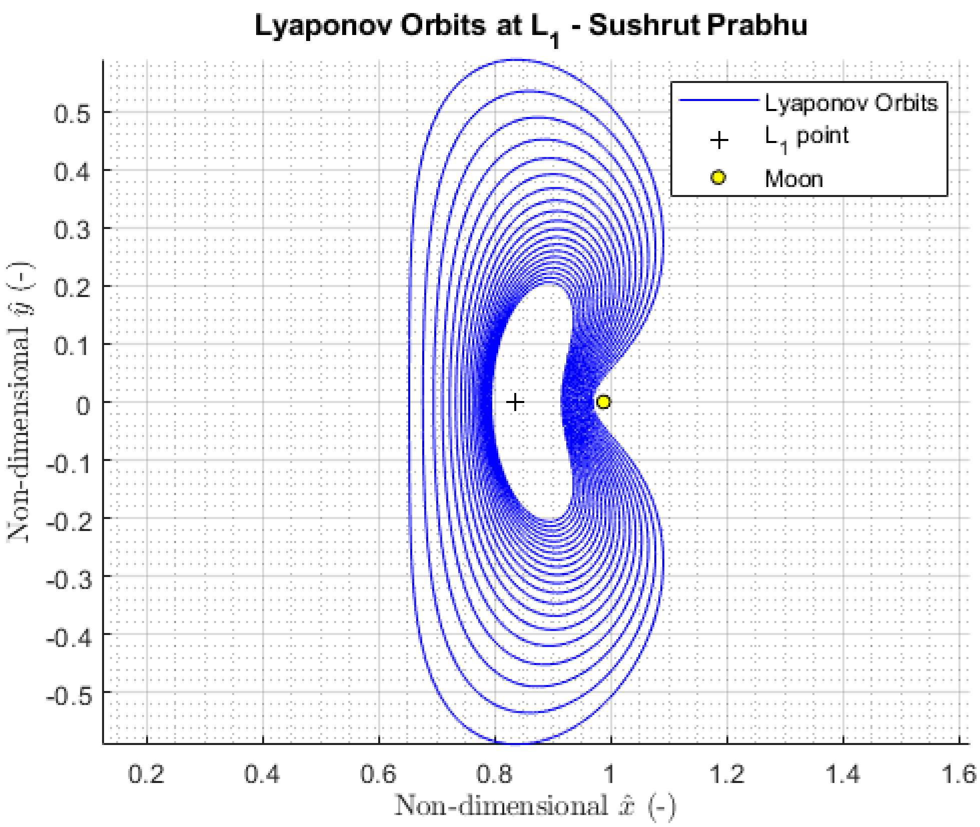
```
y = [.913, 0, 0, 0, -.4835, 0];

x0_step = 0.0028;
k = 1;
t_end = 2.72*.7;
G = [1 0 0 0 0 0; 0 -1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 -1 0 0; 0 0 0 0 1
 0;0 0 0 0 0 -1];
Omega = [0 1 0; -1 0 0; 0 0 0];
figure
hold on

while k < 21
    % Inital Guess
    IC = [y(1,1:3),y(1,4:6)];

    if k > 2
        m = (yd0_vec(k-2) - yd0_vec(k-1)) / (x0_vec(k-2) -
 x0_vec(k-1));
        c = yd0_vec(k-1) - m*x0_vec(k-1);
        yg = m*(x0_step+y(1))+c;
        IC = [y(1,1:3),0,yg,0];
```

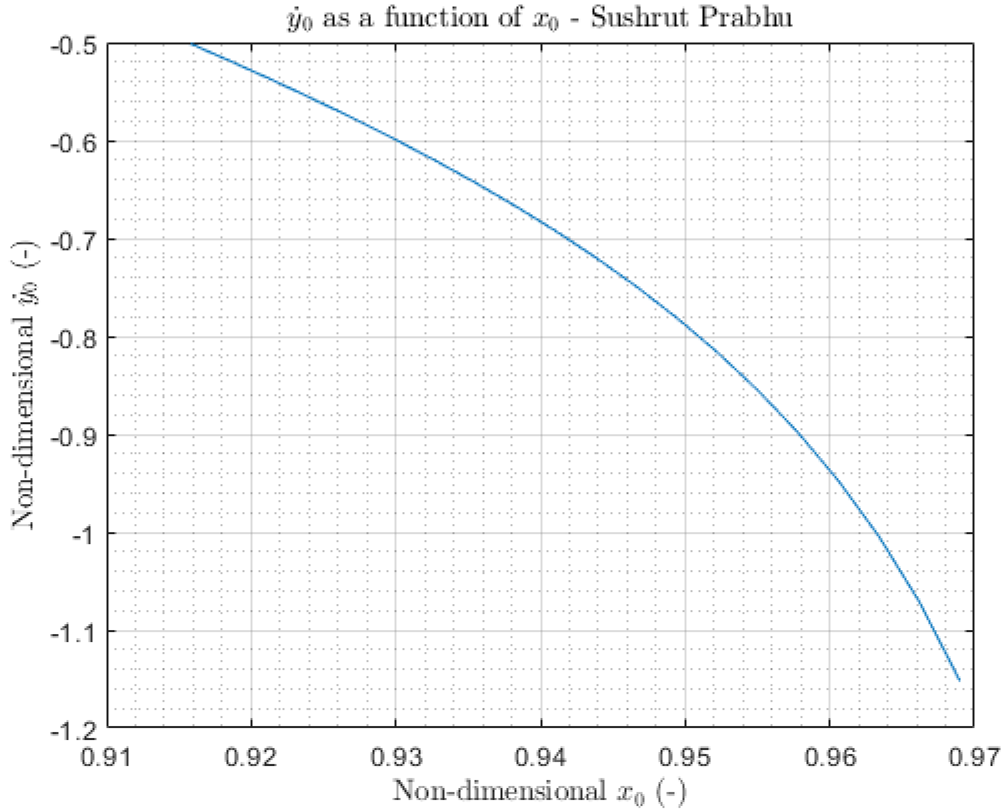
```
    end

    ICm = IC + [x0_step, 0, 0, 0, 0, 0];
    [t,y] = ode45(@cr3bp_df,[0 t_end],ICm,options,dim_vals{4,2});

    t_end = t(find(y(:,2)>0,1));
    clear t

    rv_des = [0 0]';  % y = 0 and xfdot = 0
    [yn,t_end] =
 Target3d_per(rv_des,ICm(1:3),IC(4:6),t_end,dim_vals{4,2},"planar",
 10^-13,"");

    % Final plot and solution
    IC_stm = eye(6);
    IC_stm = IC_stm(:)';
    IC = [yn(1,1:6), IC_stm];
    t_end = t_end;
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

%     monodromy = reshape(y(end,7:end),6,6);

    % Method 2
    stm_12 = reshape(y(end,7:end),6,6);
    monodromy = G* [zeros(3,3), -eye(3);eye(3),
 -2*Omega]*stm_12.'*[-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

    [V,D] = eig(monodromy);
    Dvec(k,:) = diag(D)';

    t_end = 2*t_end;
    [t,y] = ode45(@cr3bp_df,[0
 t_end],yn(1,1:6),options,dim_vals{4,2});


    ly_plt = plot(y(:,1),y(:,2),"-b");
    if k >1

 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
    end
    x0_vec(k) = y(1,1);
    yd0_vec(k) = y(1,5);

    Per_vec(k) = t_end;
    J_vec(k) = Jacobi_C(y(1,1),y(1,2),0,norm(y(1,4:6)),dim_vals{4,2});
    k = k+1;
end
```

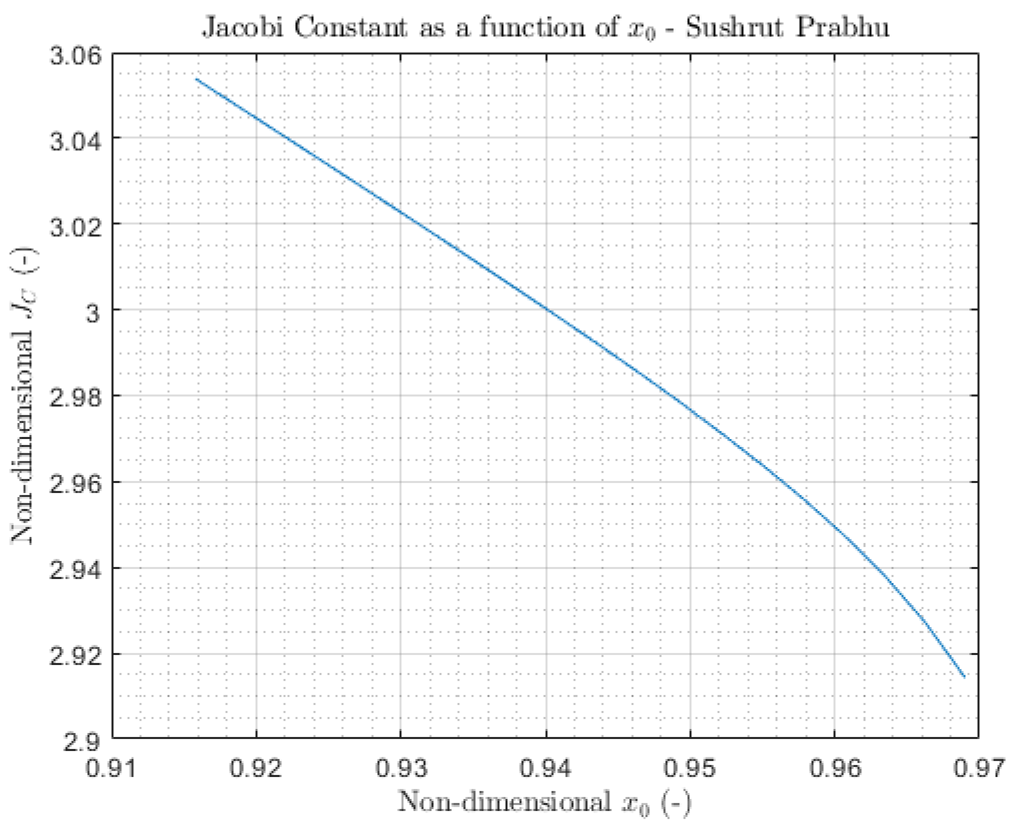



```
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Lyaponov Orbits','L_1 point','Moon')
title('Lyaponov Orbits at L_1 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor

figure
plot(x0_vec,yd0_vec)
grid on
grid minor
title("$$\dot{y}_0$$ as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\dot{y}_0$$ (-)","Interpreter", "latex")

figure
plot(x0_vec,J_vec)
grid on
grid minor
title("Jacobi Constant as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
```

```
ylabel("Non-dimensional $$J_C$$ (-)","Interpreter", "latex")

figure
plot(x0_vec,Per_vec)
grid on
grid minor
title("Period as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional Period (-)")
```




$\dot{y}_0$ as a function of x_0 - Sushrut Prabhu
Non-dimensional $\dot{y}_0$ (-)
Non-dimensional x_0 (-)
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
0.91
0.92
0.93
0.94
0.95
0.96
0.97


Jacobi Constant as a function of x_0 - Sushrut Prabhu
Non-dimensional J_C (-)
Non-dimensional x_0 (-)
3.06
3.04
3.02
3
2.98
2.96
2.94
2.92
2.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97

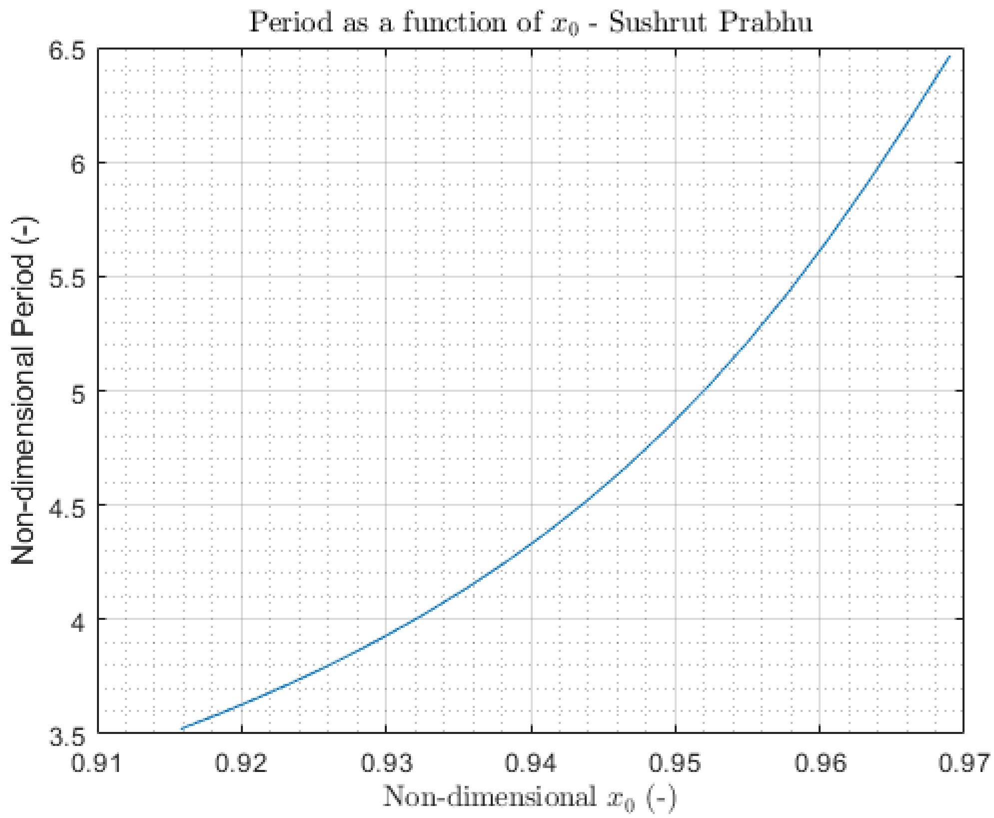


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Table of Contents

PSE1OPT

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L2_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} + dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};

xi0 = 0.01;
eta0 =0;
r = [dim_vals{7,2}+xi0 eta0 0];

[Uxx,Uyy,~,~,~,~]= Unn(r(1),r(2),r(3),dim_vals{4,2});
beta1 = 2 - (Uxx+Uyy)/2;
beta2 = sqrt(-Uxx*Uyy);
s = sqrt(beta1 + sqrt(beta1^2 + beta2^2));
beta3 = (s^2 + Uxx)/2/s;

xi0_dot = eta0*s/beta3;
eta0_dot = -beta3*xi0*s;
v = [xi0_dot eta0_dot 0];

Per = 2*pi/s;
t_end = Per/2 ;
```

```
IC = [r,v*1.2];
[t,y] = ode45(@cr3bp_df,[0 t_end],IC,options,dim_vals{4,2});

t_end = t(find(y(:,2)>0,1));
clear t

rv_des = [0 0]';  % y = 0 and xfdot = 0
[yn,t_end] = Target3d_per(rv_des,r,v,t_end,dim_vals{4,2},"planar",
 10^-8,"");
```

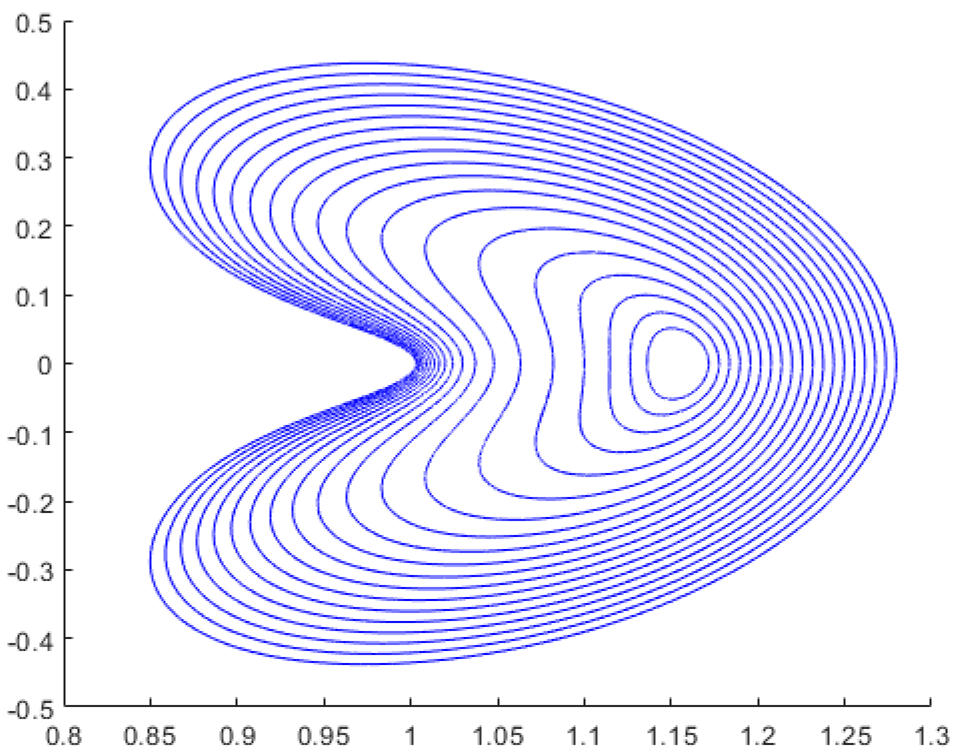
Final Orbit

```
IC_stm = eye(6);
IC_stm = IC_stm(:)';
IC = [yn(1,1:6), IC_stm];
t_end = t_end*2;
[~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});
```

Part b)

```
x0_step = 0.006;
k = 1;
t_end = t_end*.7;
figure
hold on

while k<20
    % Inital Guess
    IC = [y(1,1:3),y(1,4:6)*1.8];

    if k > 3
        m = (yd0_vec(k-2) - yd0_vec(k-1)) / (x0_vec(k-2) -
 x0_vec(k-1));
        c = yd0_vec(k-1) - m*x0_vec(k-1);
        yg = m*(x0_step+y(1))+c;
        IC = [y(1,1:3),0,yg,0];
    end

    ICm = IC + [x0_step, 0, 0, 0, 0, 0];
    [t,y] = ode45(@cr3bp_df,[0 t_end],ICm,options,dim_vals{4,2});

    t_end = t(find(y(:,2)>0,1));
    clear t

    rv_des = [0 0]';  % y = 0 and xfdot = 0
    [yn,t_end] =
 Target3d_per(rv_des,ICm(1:3),IC(4:6),t_end,dim_vals{4,2},"planar",
 10^-8,"");

    % Final plot and solution
    IC_stm = eye(6);
    IC_stm = IC_stm(:)';
```

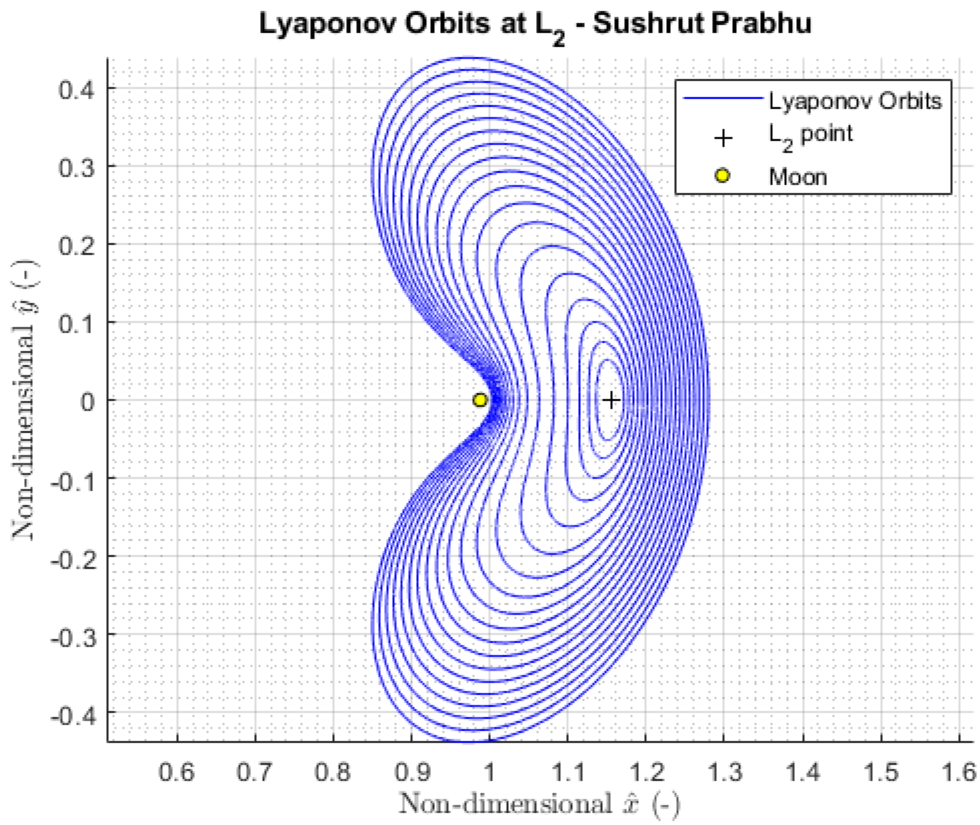
```
    IC = [yn(1,1:6), IC_stm];
    t_end = t_end*2;
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

    monodromy = reshape(y(end,7:end),6,6);
    [V,D] = eig(monodromy);
    D = diag(D);

    ly_plt = plot(y(:,1),y(:,2),'-b');
    if k >1
 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
    end
    x0_vec(k) = y(1,1);
    yd0_vec(k) = y(1,5);

    Per_vec(k) = t_end;
    J_vec(k) = Jacobi_C(y(1,1),y(1,2),0,norm(y(1,4:6)),dim_vals{4,2});
    k = k+1;
end
```

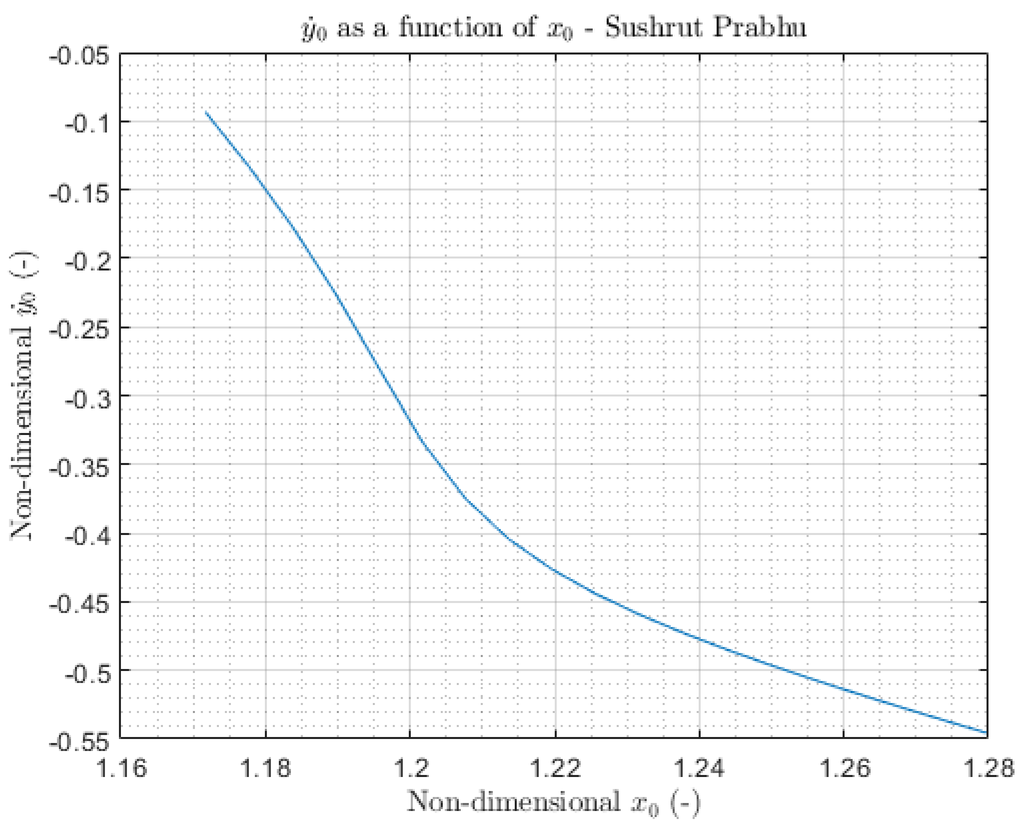


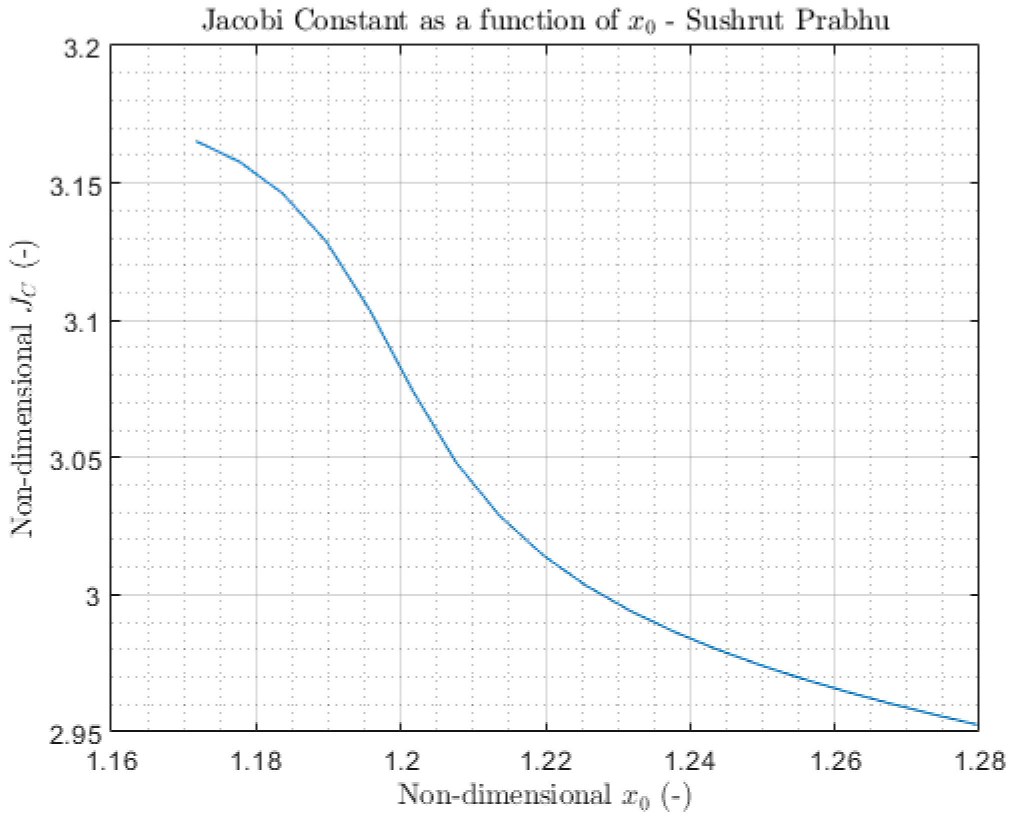
Plot

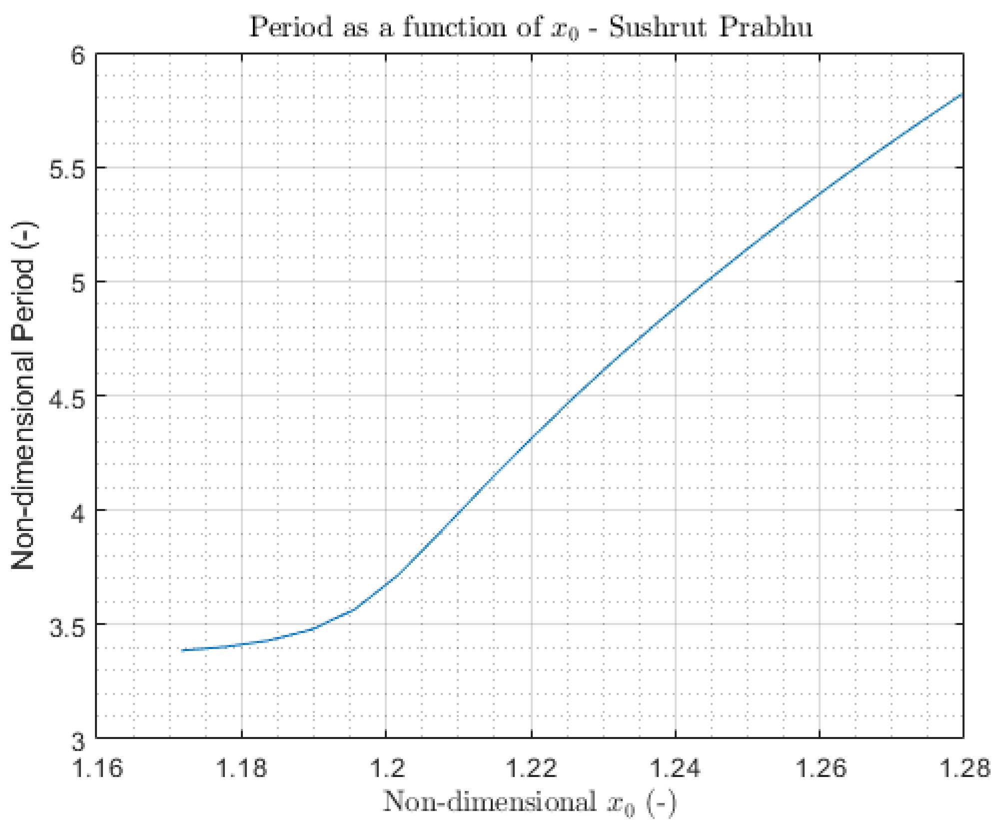
```
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Lyaponov Orbits','L_2 point','Moon')
```

```
title('Lyaponov Orbits at L_2 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor

figure
plot(x0_vec,yd0_vec)
grid on
grid minor
title("$$\dot{y}_0$$ as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\dot{y}_0$$ (-)","Interpreter", "latex")

figure
plot(x0_vec,J_vec)
grid on
grid minor
title("Jacobi Constant as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$J_C$$ (-)","Interpreter", "latex")

figure
plot(x0_vec,Per_vec)
grid on
grid minor
title("Period as a function of $$x_0$$ - Sushrut
 Prabhu","Interpreter", "latex")
xlabel("Non-dimensional $$x_0$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional Period (-)")
```


Lyaponov Orbits at L_2 - Sushrut Prabhu
Lyaponov Orbits
L_2 point
Moon
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6


$\dot{y}_0$ as a function of x_0 - Sushrut Prabhu
Non-dimensional $\dot{y}_0$ (-)
Non-dimensional x_0 (-)
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
1.16
1.18
1.2
1.22
1.24
1.26
1.28


Jacobi Constant as a function of x_0 - Sushrut Prabhu
Non-dimensional J_C (-)
Non-dimensional x_0 (-)
3.2
3.15
3.1
3.05
3
2.95
1.16
1.18
1.2
1.22
1.24
1.26
1.28


Period as a function of x_0 - Sushrut Prabhu
Non-dimensional Period (-)
Non-dimensional x_0 (-)
6
5.5
5
4.5
4
3.5
3
1.16
1.18
1.2
1.22
1.24
1.26
1.28

Published with MATLAB® R2018a

Sushrut Prabhu | AAE632 | PSE2

PSE2

Given. $\bar{r}_0 = [300,000 \quad 0 \quad 0]^T$ km and $\bar{v}_0 = [0 \cdot \quad 0.5 \quad 0.5]$ km/s

we want to reach $\bar{r}_d = [500,000 \quad -10,000 \quad 200000]$ km

Find a) After 10 days how far is $\bar{r}_0, \bar{v}_0 \cdot \|\bar{e}\|$? dim and non-dim

b) Implement $\Delta\bar{V}_0$ for fixed time targeter.

c) Plot result, base then final correction. Make table with $\bar{V}_0$, $\|\Delta\bar{V}_0\|$, $\Delta\bar{Y}_2$, and $\|\bar{F}\|$ for each iteration

d) Is STM baseline? Which should you use? Why? How many iterations? Check n and m. Is solution unique? How do you know? Check Jacobi constant for last history. How accurate is the integration?

e) Try 15 days. Can you use 10 days as initial guess? Do you need continuation? Is one step adequate?

Solution:

a) After propagating the non-linear equation you can get an error

$$\boxed{\begin{aligned} \|\bar{e}\| &= 0.1621 \text{ (non-dim)} \\ \|\bar{e}\| &= 6.2313 \times 10^4 \text{ km} \end{aligned}}$$

b) Since we have used STM as targeter before let us try the least squares method.

$$\bar{F}(\bar{X}) = \bar{r}_d - \bar{r}(v_0) = \bar{e} = \begin{Bmatrix} x_d - x(\bar{v}_0) \\ y_d - y(\bar{v}_0) \\ z_d - z(\bar{v}_0) \end{Bmatrix}$$

continued...

$$D\bar{F}(\bar{x}) = \frac{\partial \bar{F}}{\partial \bar{r}} \frac{\partial \bar{r}}{\partial \bar{v}_0} = \begin{bmatrix} \frac{\partial \bar{F}_1}{\partial x} & \frac{\partial \bar{F}_1}{\partial y} & \frac{\partial \bar{F}_1}{\partial z} \\ \frac{\partial \bar{F}_2}{\partial x} & \frac{\partial \bar{F}_2}{\partial y} & \frac{\partial \bar{F}_2}{\partial z} \\ \frac{\partial \bar{F}_3}{\partial x} & \frac{\partial \bar{F}_3}{\partial y} & \frac{\partial \bar{F}_3}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial x}{\partial \dot{x}_0} & \frac{\partial x}{\partial \dot{y}_0} & \frac{\partial x}{\partial \dot{z}_0} \\ \frac{\partial y}{\partial \dot{x}_0} & \frac{\partial y}{\partial \dot{y}_0} & \frac{\partial y}{\partial \dot{z}_0} \\ \frac{\partial z}{\partial \dot{x}_0} & \frac{\partial z}{\partial \dot{y}_0} & \frac{\partial z}{\partial \dot{z}_0} \end{bmatrix}$$

Identity

$$= -I \begin{bmatrix} \phi_{14} & \phi_{15} & \phi_{16} \\ \phi_{24} & \phi_{25} & \phi_{26} \\ \phi_{34} & \phi_{35} & \phi_{36} \end{bmatrix}$$

Iteration not power

$$\bar{x}^{i+1} = \bar{x}^{i} - D\bar{F}(\bar{x}^{i})^T \left[DF(x^{i}) \cdot D\bar{F}(\bar{x}^{i})^T \right]^{-1} F(x^{i})$$

c) $\boxed{\text{See Figure: E2.1, E2.2, E2.3, and E2.4}}$

Iteration	$\Delta \bar{V}_0$ (km/s)	$\lvert \Delta \bar{V}_0 \rvert$ (km/s)	$\Delta \bar{r}_f$ (km)	$\lVert F(\bar{X}) \rVert$ (km)
1	$0.0429\,\hat{x}$ $-0.0931\,\hat{y}$ $0.1604\,\hat{z}$	0.1904	$-3.034\times10^{4}\,\hat{x}$ $8.9768\times10^{3}\,\hat{y}$ $5.3663\times10^{4}\,\hat{z}$	6.2313×10^{4}
2	$0.0414\,\hat{x}$ $-0.0977\,\hat{y}$ $0.1493\,\hat{z}$	0.1832	$-7.8644\times10^{3}\,\hat{x}$ $1.1107\times10^{4}\,\hat{y}$ $-1.0852\times10^{4}\,\hat{z}$	1.7406×10^{4}
3	$0.0412\,\hat{x}$ $-0.0975\,\hat{y}$ $0.1492\,\hat{z}$	0.1829	$137.8195\,\hat{x}$ $-27.6473\,\hat{y}$ $-10.4601\,\hat{z}$	140.9539
4	$0.0412\,\hat{x}$ $-0.0975\,\hat{y}$ $0.1492\,\hat{z}$	0.1829	$-0.0288\,\hat{x}$ $0.021\,\hat{y}$ $-0.0151\,\hat{z}$	0.0387
5	$0.0412\,\hat{x}$ $-0.0975\,\hat{y}$ $0.1492\,\hat{z}$	0.1829	$-3.4142\times10^{-10}\,\hat{x}$ $-5.3346\times10^{-10}\,\hat{y}$ $-1.2803\times10^{-10}\,\hat{z}$	6.4617×10^{-10}

Shows the sensitivity on non-linear model continued ...

d) The STM is updated with each iteration. This will provide a more accurate correction as the STM represents the new trajectory. You also update your initial condition. Note that in our case only the initial velocity changes.

It takes 5 iterations to converge the previous table gives these iteration to show that the solution converges

In our formulation of the problem we have 3 equations and 3 unknowns. 3 velocities are unknown but we do know 3 equation as shown on previous page; $m = n = 3$. Thus there is a unique solution since equations are independent.

See Figure: E2.5
The Jacobi constant initially moves a lot but then is much lower. The result is of the order of 10^{-13} $J_{error} = \frac{|J_0 - J|}{|J_0|}$

e) The solution doesn't seem to converge for 15 days it seems that it is too far of a guess. See Figure: E2.6 There are 21 iterations attempted in that figure Note: No arrow because it is in disarray

Continuation...

It is better to do this in 2 steps.
The best combination I found was $t = 13.5$ days then $t = 14.5$ days and finally $t = 15$ days.
This implements continuity so that the targeter does not have to search for the local basin we lead it there.

$t = 13.5 \rightarrow$ 5 iterations
$t = 14.5 \rightarrow$ 5 iterations
$t = 15 \rightarrow$ 7 iterations
} Total of 12

Another advantage is that your tolerance for convergence for $t = 13.5$ and $t = 14.5$ can be relatively inaccurate. Since those are not the target an error of $10^{-2} \sim 1\%$ error just the final time needs to be well conditioned.

See Figures: E2.7, E2.8, E2.9, E2.10, E2.12

PSE2

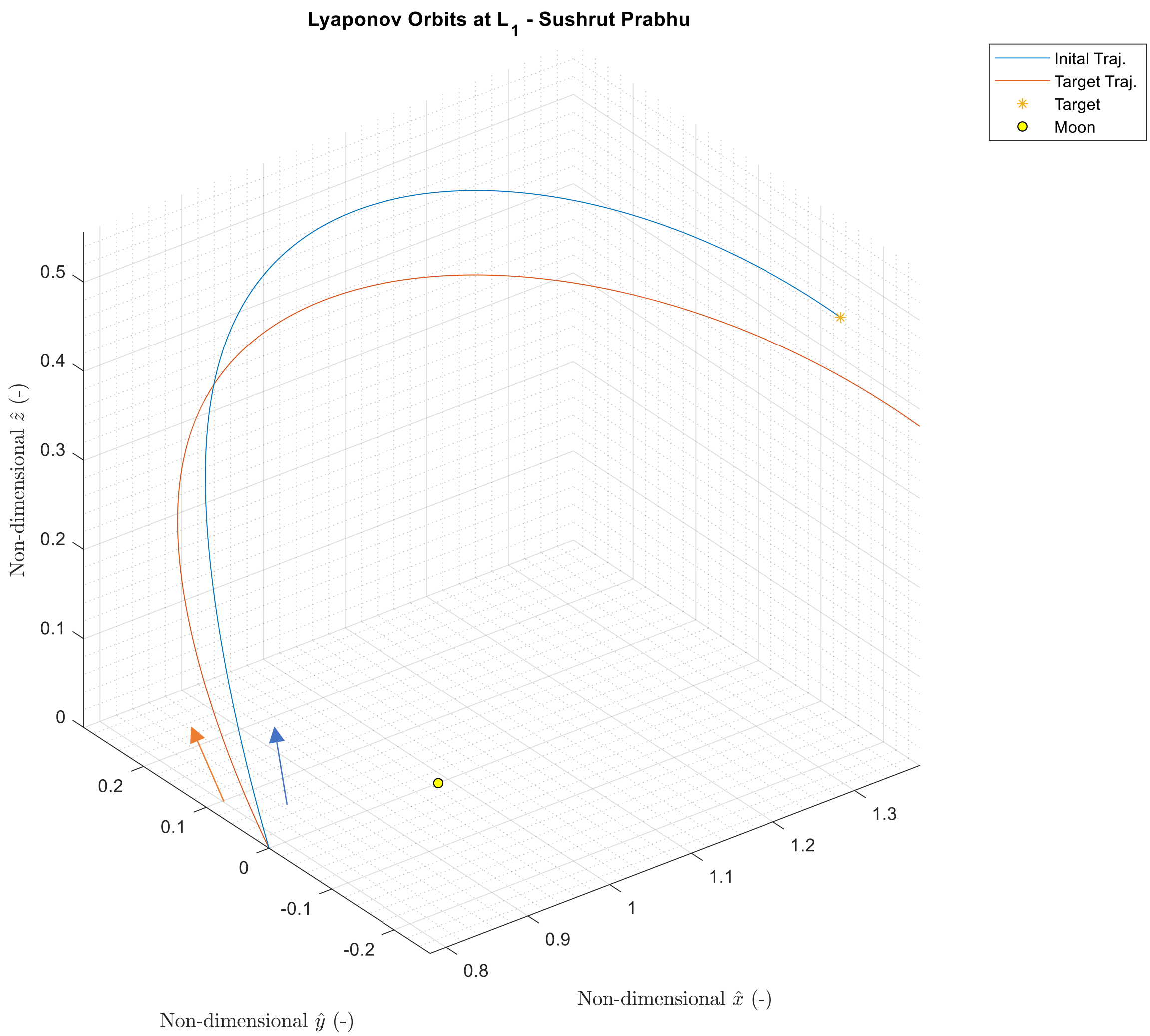


Figure E2.1: Target a 3-dimension position using the least square method.

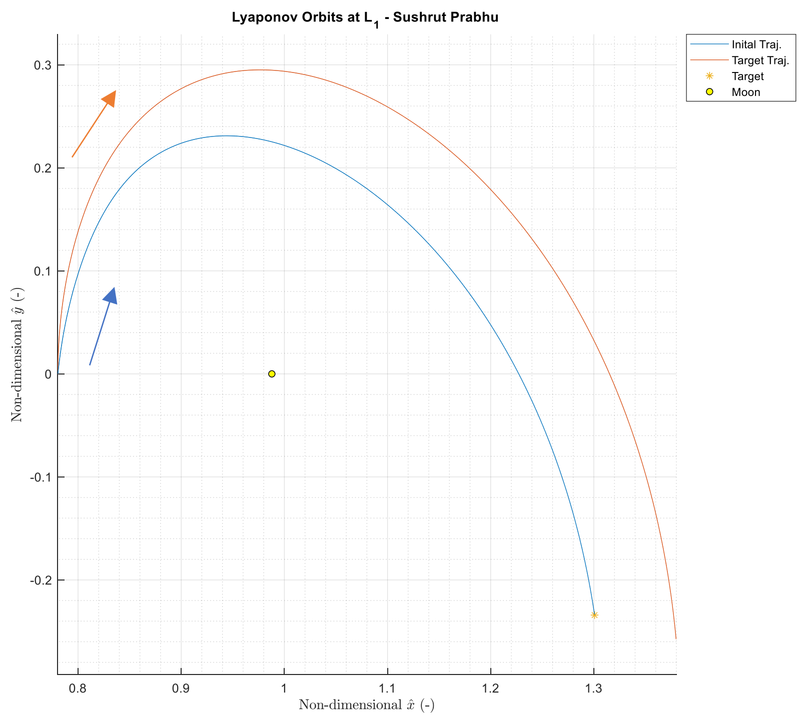


Figure E2.2: Target a 3-dimension position using the least square method in x-y plane.

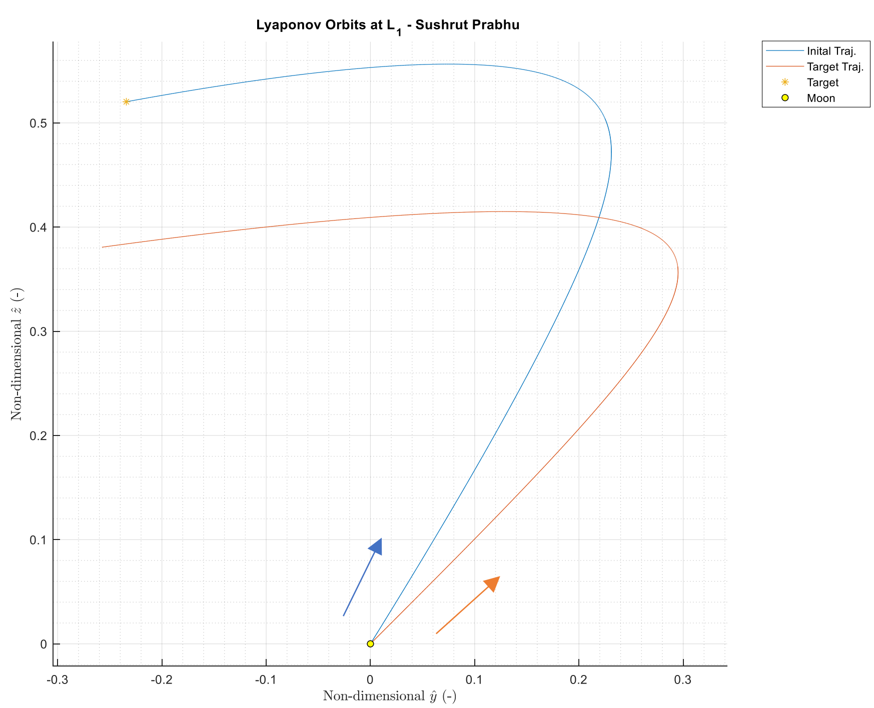


Figure E2.3: Target a 3-dimension position using the least square method in y-z plane.

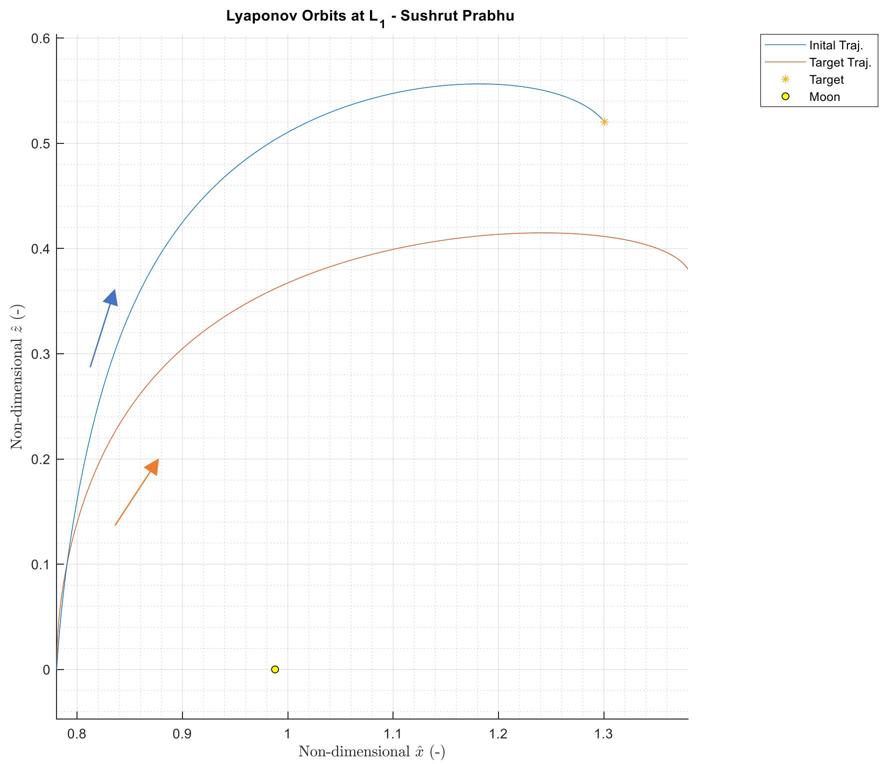


Figure E2.4: Target a 3-dimension position using the least square method in x-z plane.

Part d)

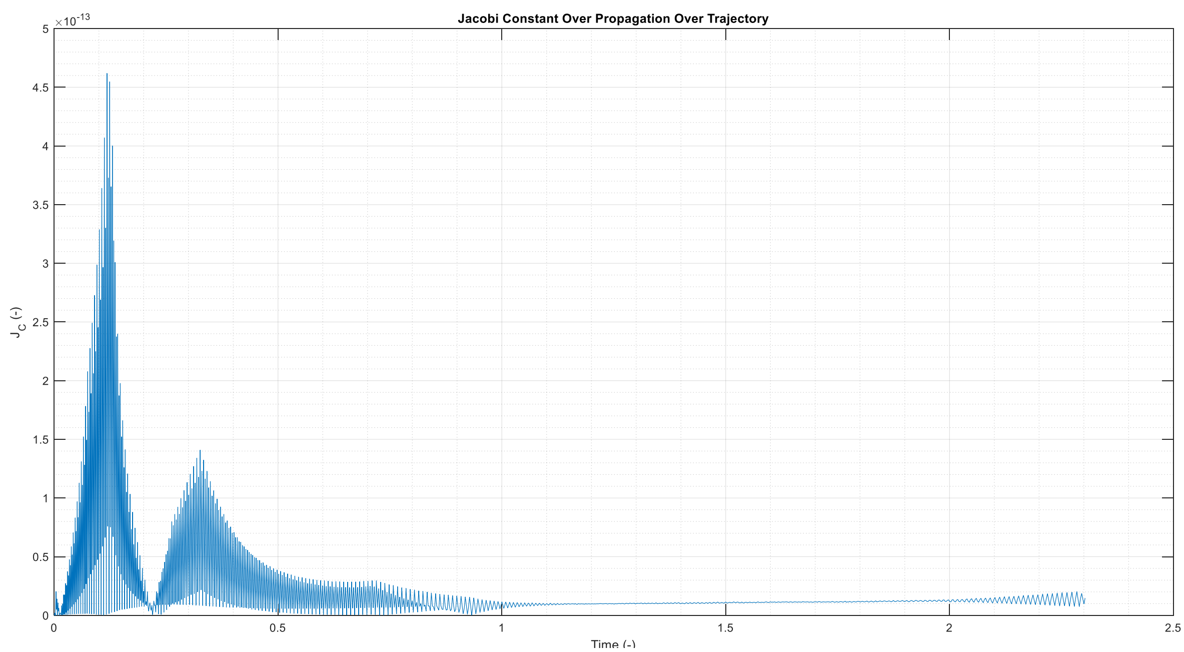


Figure E2.5: Jacobi constant check for the propagation of the orbit.

Part e)

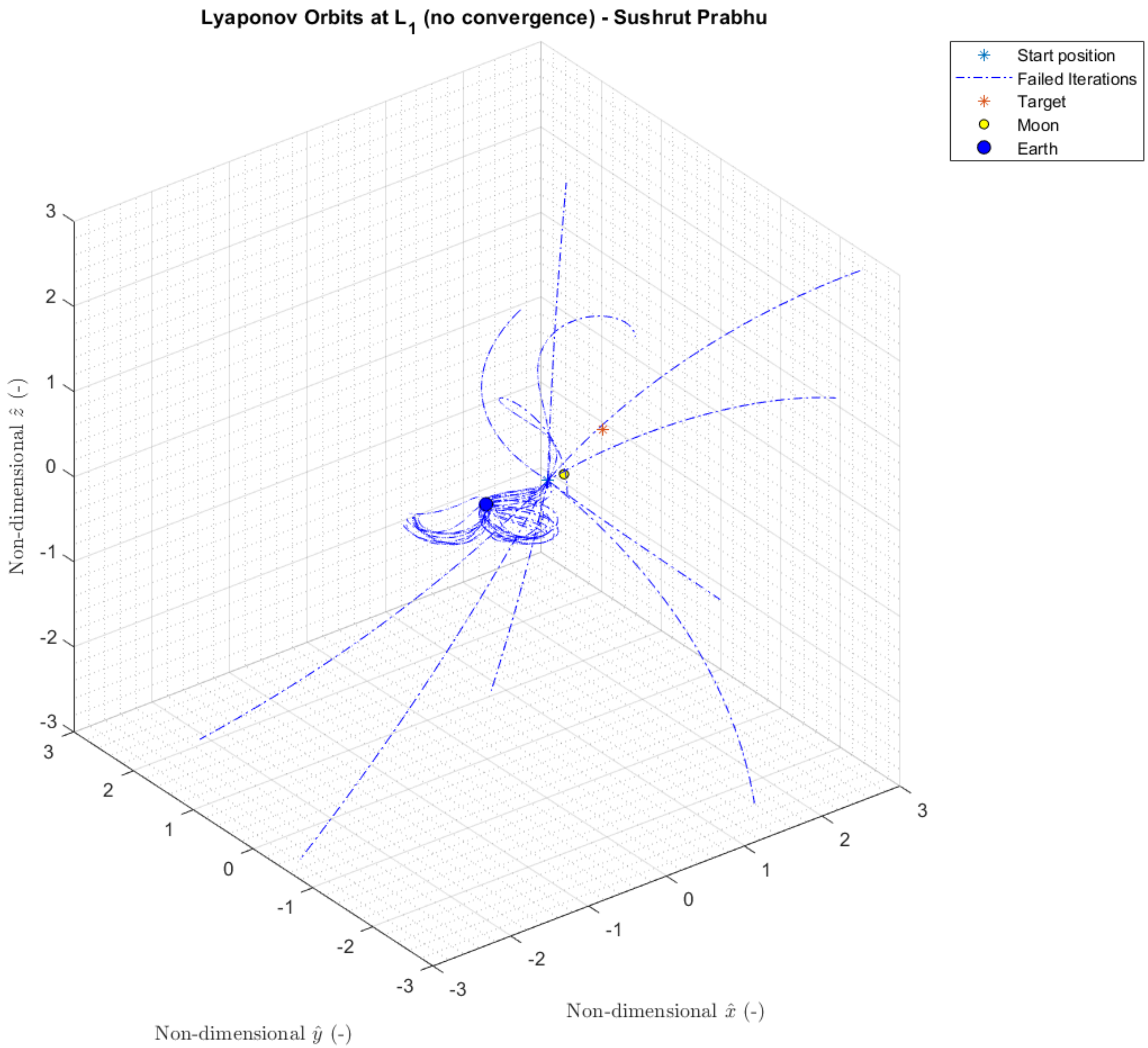


Figure E2.6: Lack of convergence when you try to converge with just 1 step.

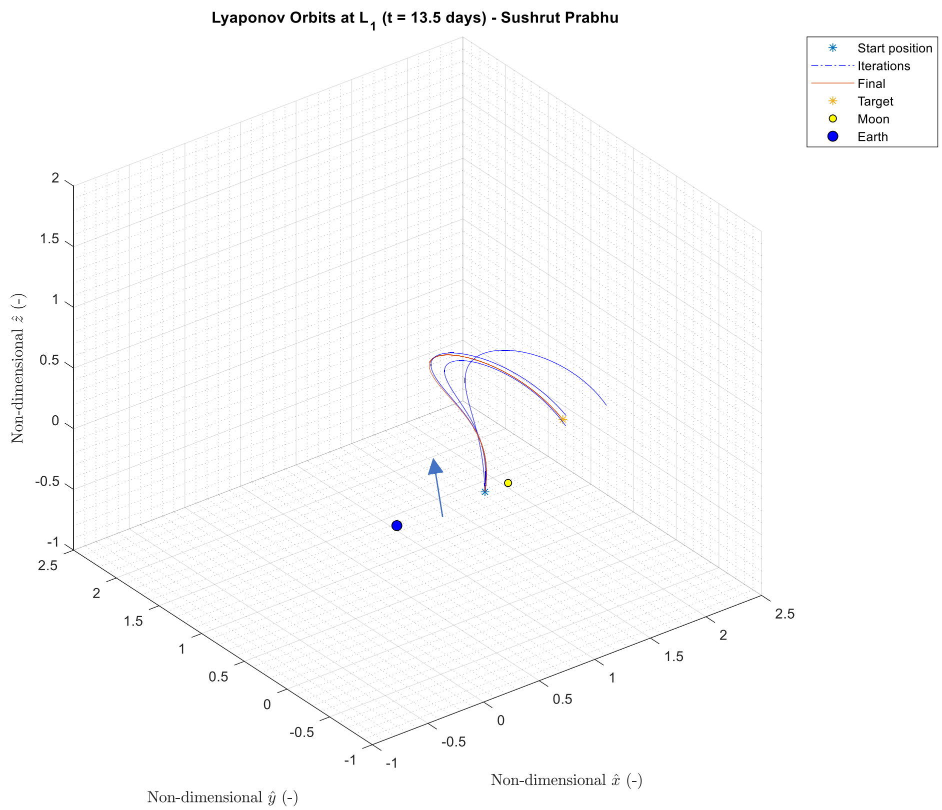


Figure E2.7: Intermediate convergence at t = 13.5 days.

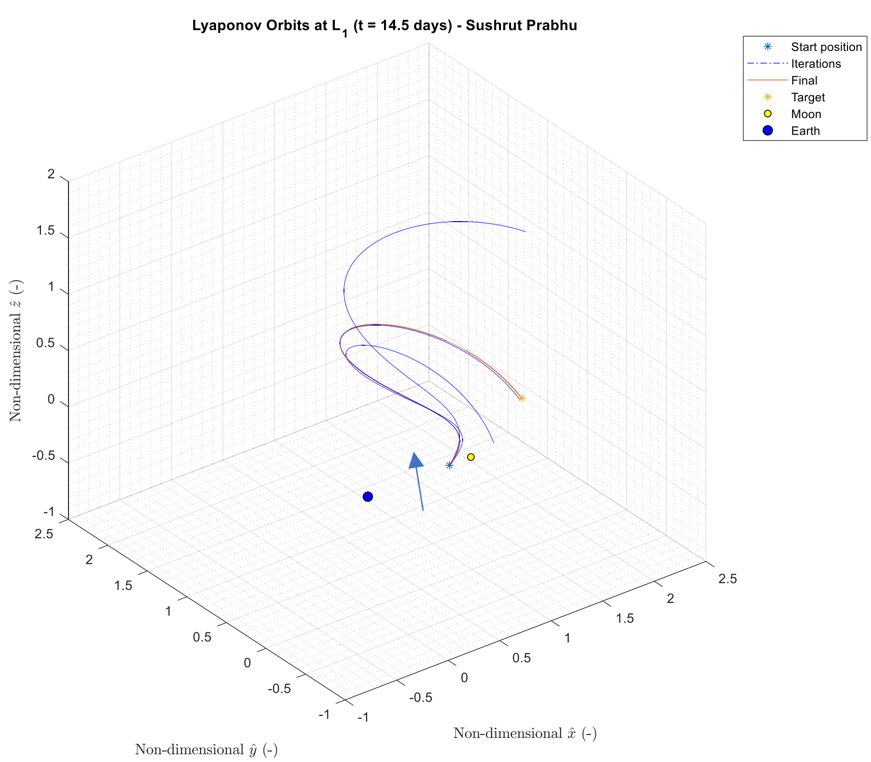


Figure E2.8: Intermediate convergence at t = 14.5 days.

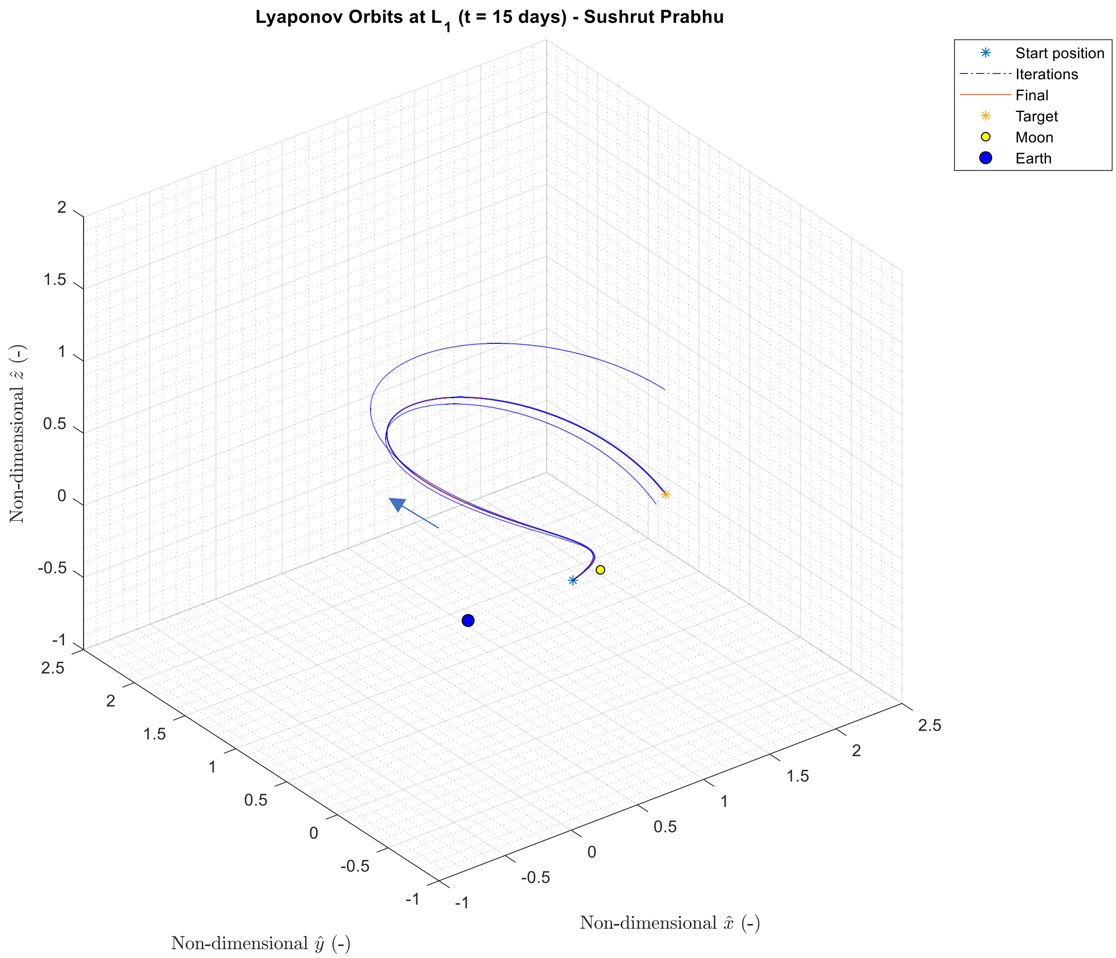


Figure E2.9: Final target and time achieved at t = 15days.

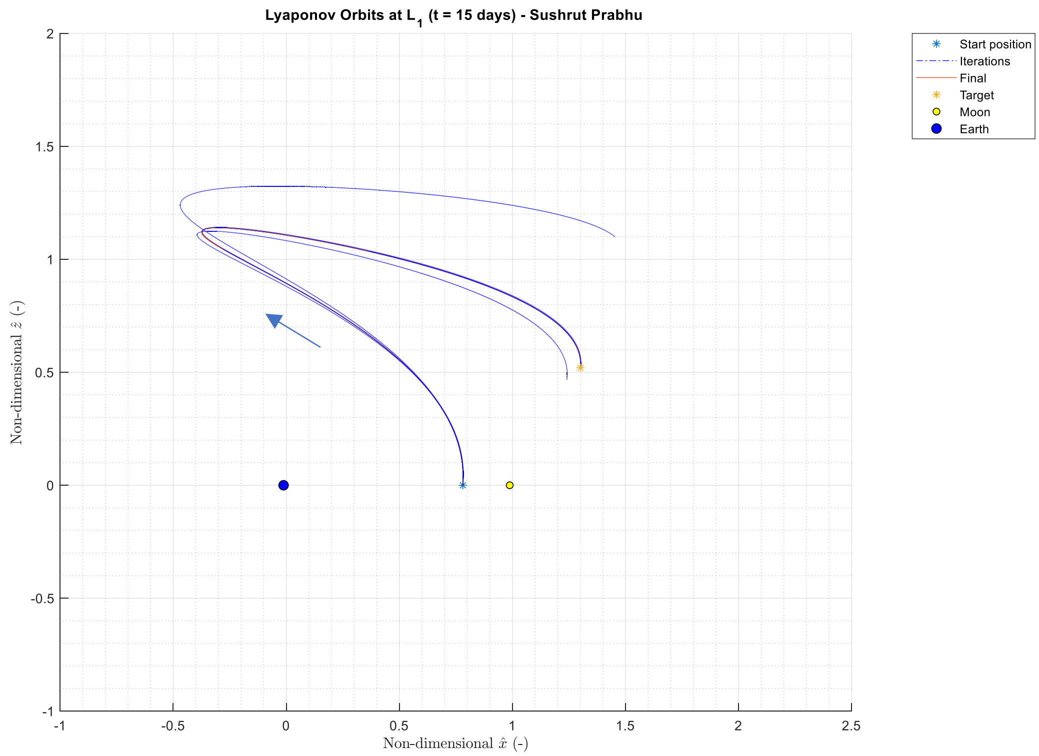


Figure E2.10: Final target and time achieved at t = 15days x-z plane.

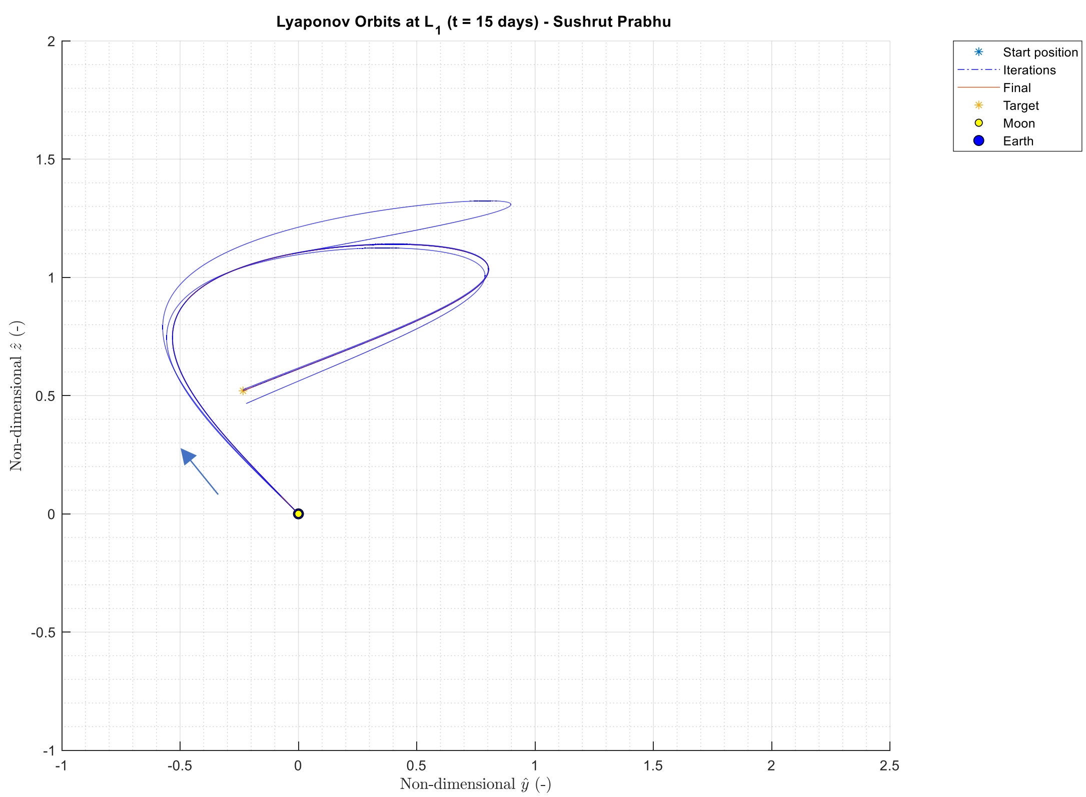


Figure E2.11: Final target and time achieved at t = 15days y-z plane.

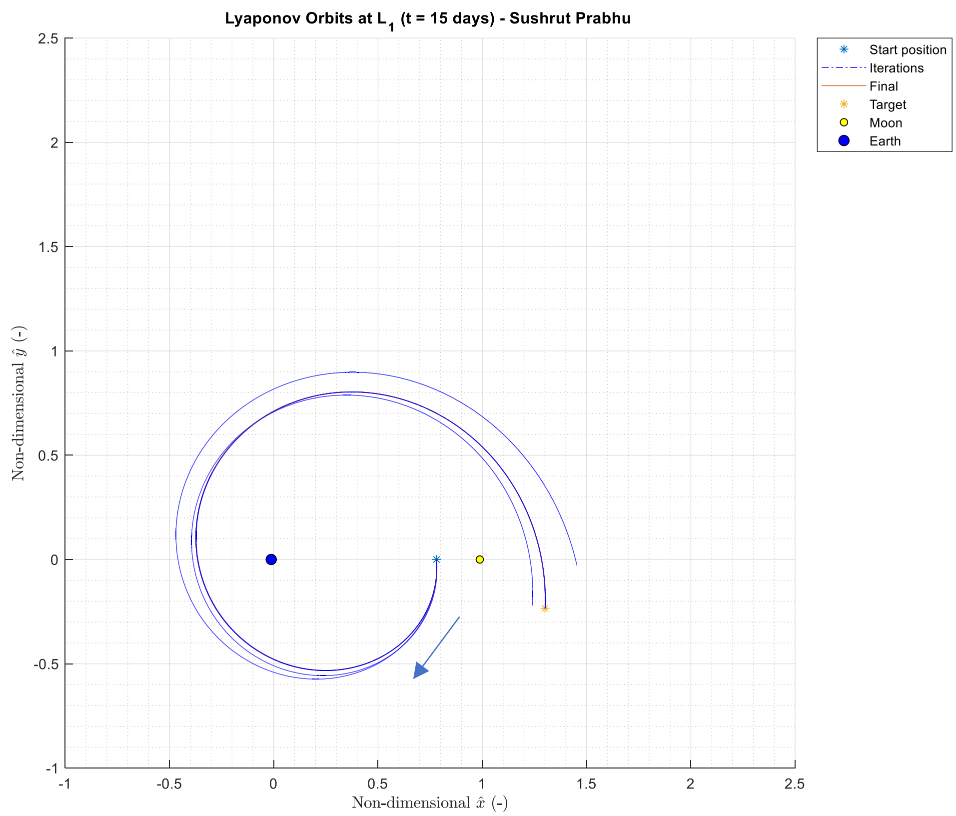


Figure E2.12: Final target and time achieved at t = 15days x-y plane.

Table of Contents

PSE2

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-13); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

r0_dim = [300000 0 0];
v0_dim = [0 0.5 0.5];
rd_dim = [500000 -90000 200000];
```

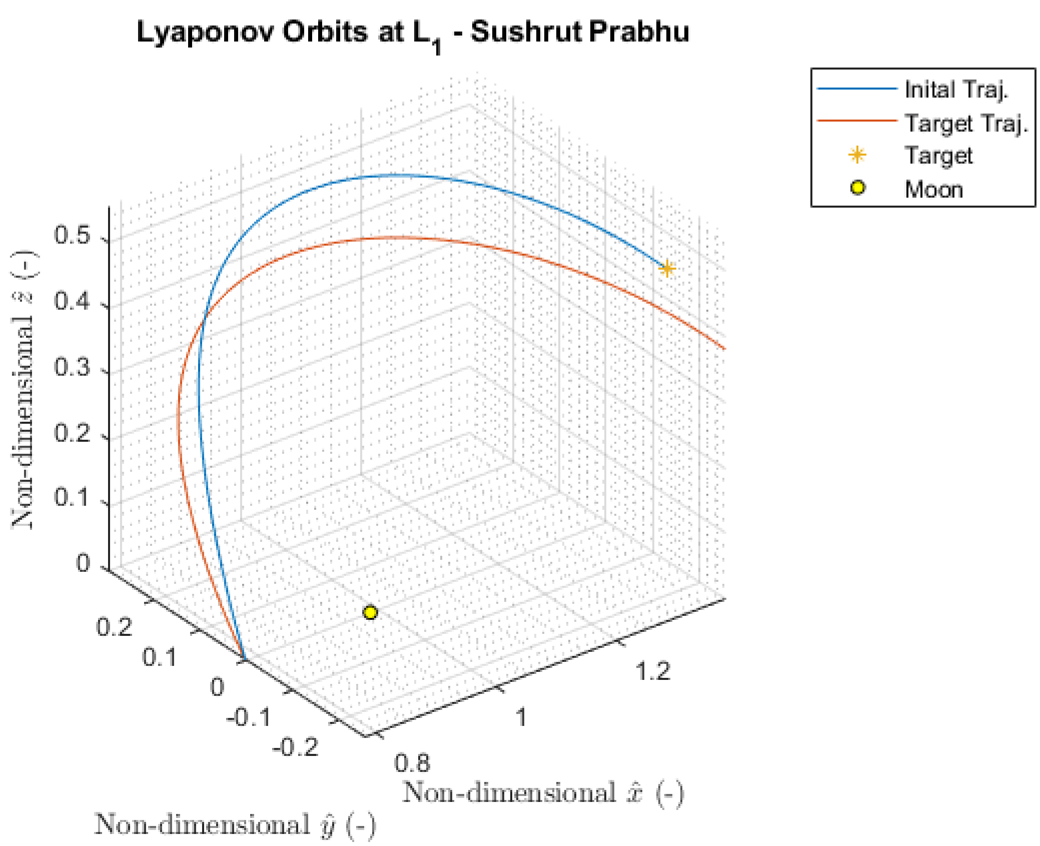
Part a and b)

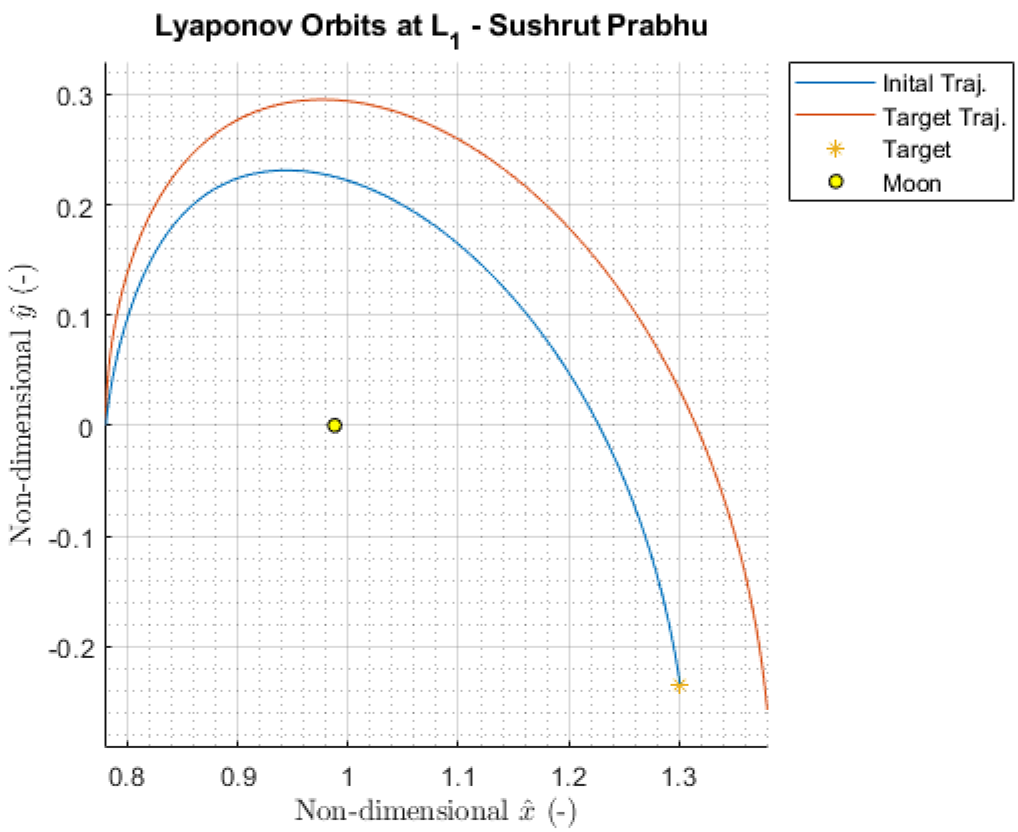
```
r0 = r0_dim/dim_vals{2,2};
v0 = v0_dim/dim_vals{2,2}*dim_vals{5,2};
rd = rd_dim/dim_vals{2,2};

t_end = 10*3600*24/dim_vals{5,2};
[yn,~, tb] = Target3d(rd',r0,v0,t_end,dim_vals{4,2}, .2 * 10^-13,"");

[~,y1] = ode45(@cr3bp_df,[0 t_end],[r0 v0],options,dim_vals{4,2});
[t,yt] = ode45(@cr3bp_df,[0 t_end],yn,options,dim_vals{4,2});

err = norm(y1(end,1:3)-rd);
err_dim = err*dim_vals{2,2};

delv = (yn(4:6)-v0);
```

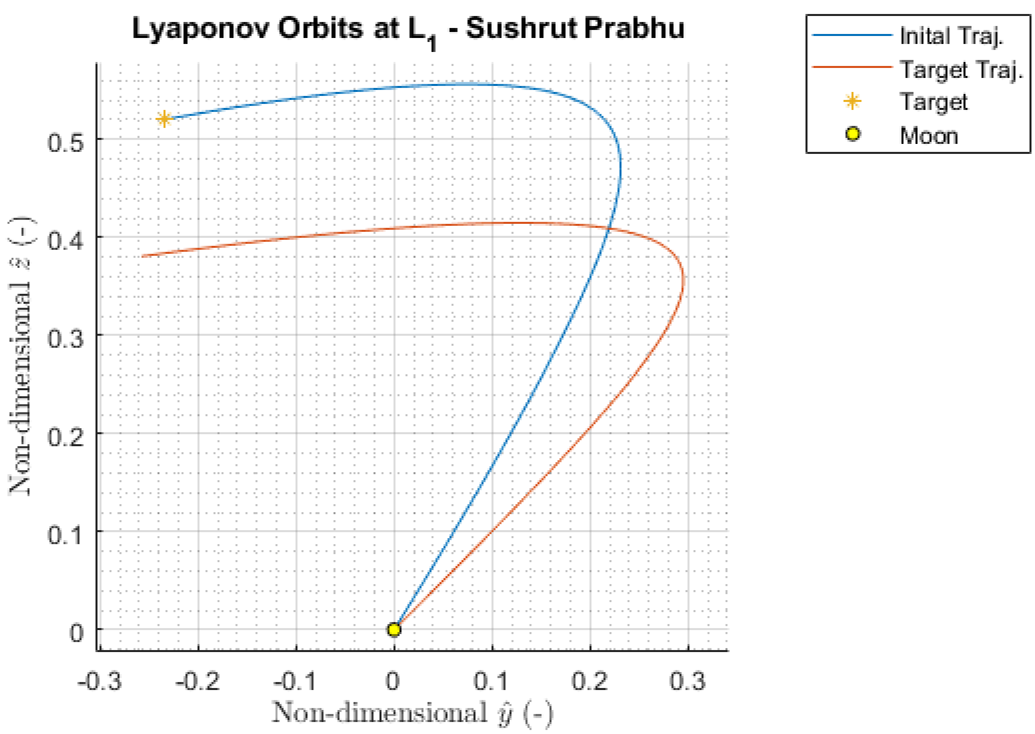
part c

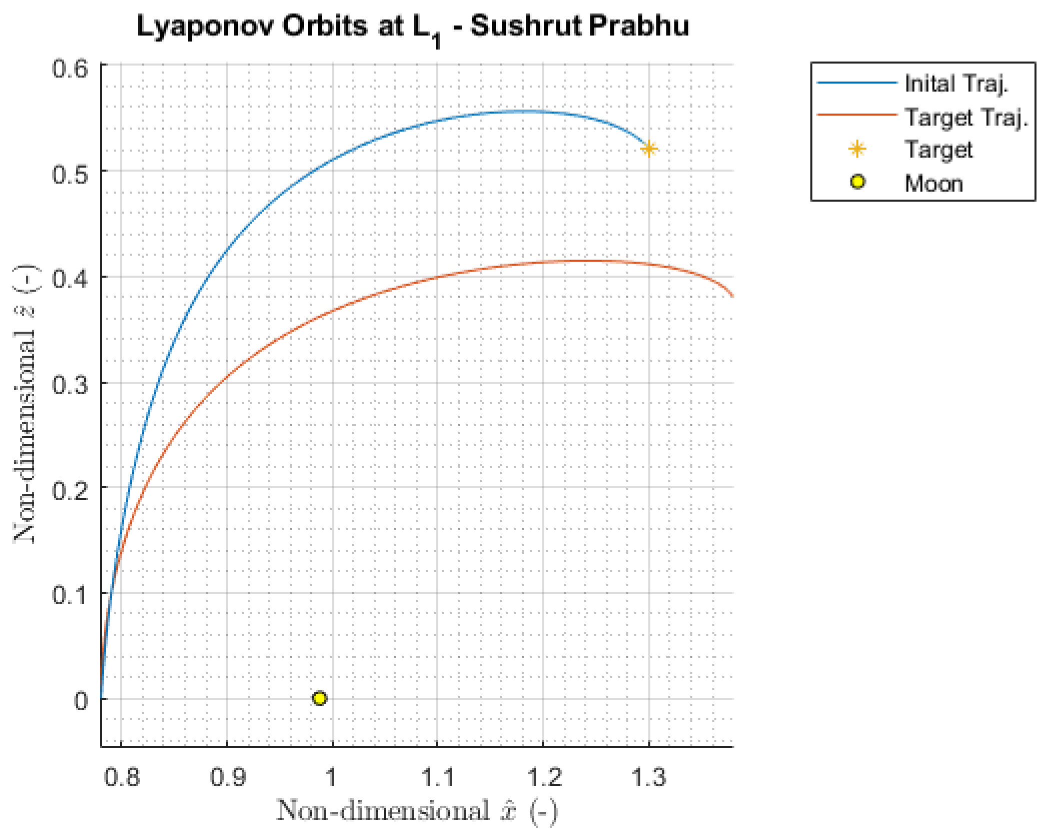
```
ite = {'Del V vec','Del V','r error','F'};
for n = 2:(length(tb)+1)

    ite{n,1} = tb{n-1}{1}*dim_vals{2,2}/dim_vals{5,2};
    ite{n,2} = tb{n-1}{2}*dim_vals{2,2}/dim_vals{5,2};
    ite{n,3} = tb{n-1}{3}*dim_vals{2,2};
    ite{n,4} = tb{n-1}{4}*dim_vals{2,2};
end

figure
plot3(yt(:,1),yt(:,2),yt(:,3))
hold on
plot3(y1(:,1),y1(:,2),y1(:,3))
plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Inital Traj.', 'Target Traj.' , 'Target','Moon')
title('Lyaponov Orbits at L_1 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor

figure
plot3(yt(:,1),yt(:,2),yt(:,3))
hold on
plot3(y1(:,1),y1(:,2),y1(:,3))
plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Inital Traj.', 'Target Traj.' , 'Target','Moon')
title('Lyaponov Orbits at L_1 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
view([0 90])
axis equal
grid on
grid minor

figure
plot3(yt(:,1),yt(:,2),yt(:,3))
hold on
plot3(y1(:,1),y1(:,2),y1(:,3))
plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Inital Traj.', 'Target Traj.' , 'Target','Moon')
title('Lyaponov Orbits at L_1 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
```

```
view([90 0])
axis equal
grid on
grid minor

figure
plot3(yt(:,1),yt(:,2),yt(:,3))
hold on
plot3(y1(:,1),y1(:,2),y1(:,3))
plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Inital Traj.', 'Target Traj.' , 'Target','Moon')
title('Lyaponov Orbits at L_1 - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
view([0 0])
axis equal
grid on
grid minor
```



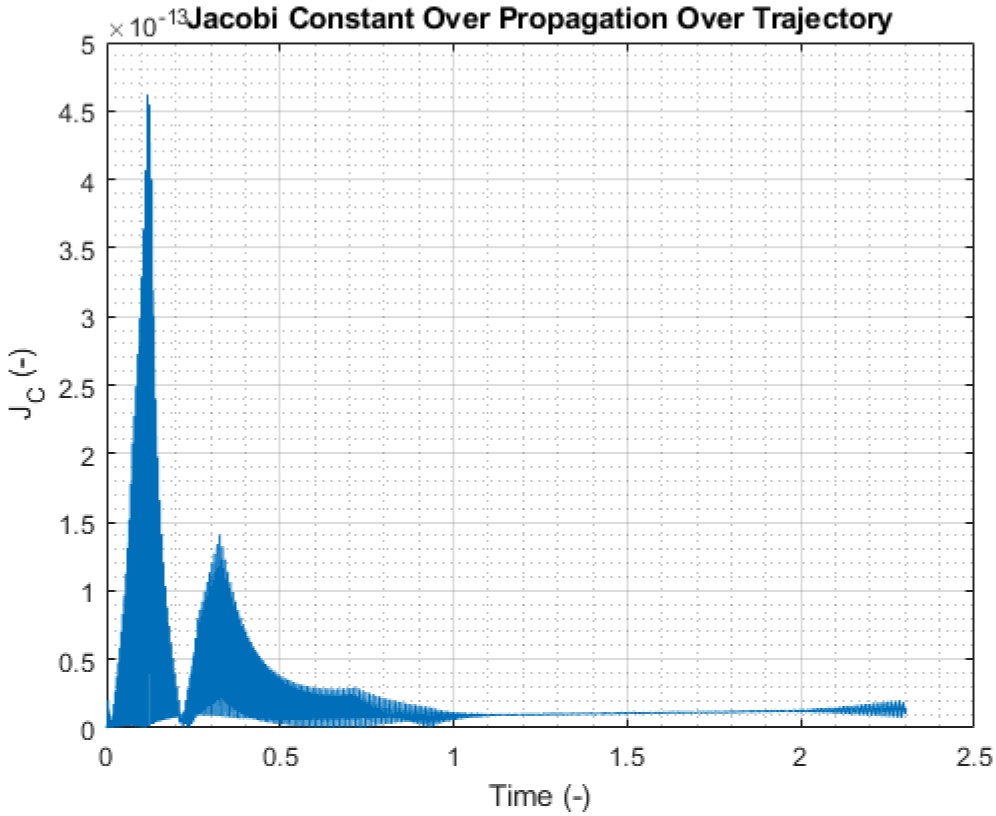

Lyaponov Orbits at L_1 - Sushrut Prabhu
Inital Traj.
Target Traj.
Target
Moon
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)


Lyaponov Orbits at L_1 - Sushrut Prabhu
Inital Traj.
Target Traj.
Target
Moon
Non-dimensional $\hat{z}$ (-)
Non-dimensional $\hat{y}$ (-)



part d)

```
J0 = Jacobi_C(yt(1,1),yt(1,2),yt(1,3),norm(yt(1,4:6)),dim_vals{4,2});
Jvec =
 Jacobi_C(yt(:,1),yt(:,2),yt(:,3),vecnorm(yt(:,4:6),2,2),dim_vals{4,2});

Jerror = abs(J0-Jvec)/abs(J0);

figure
plot(t,Jerror)
grid on
grid minor
title('Jacobi Constant Over Propagation Over Trajectory')
ylabel("J_C (-)")
xlabel("Time (-)")
```



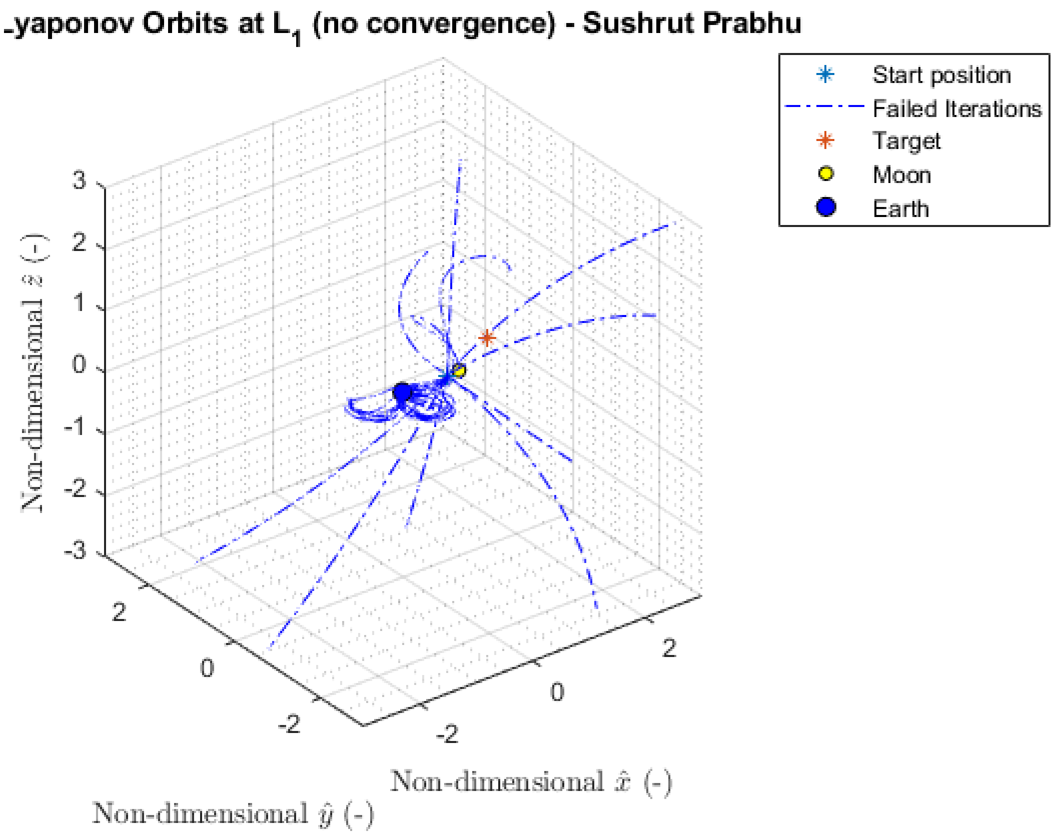
part e)

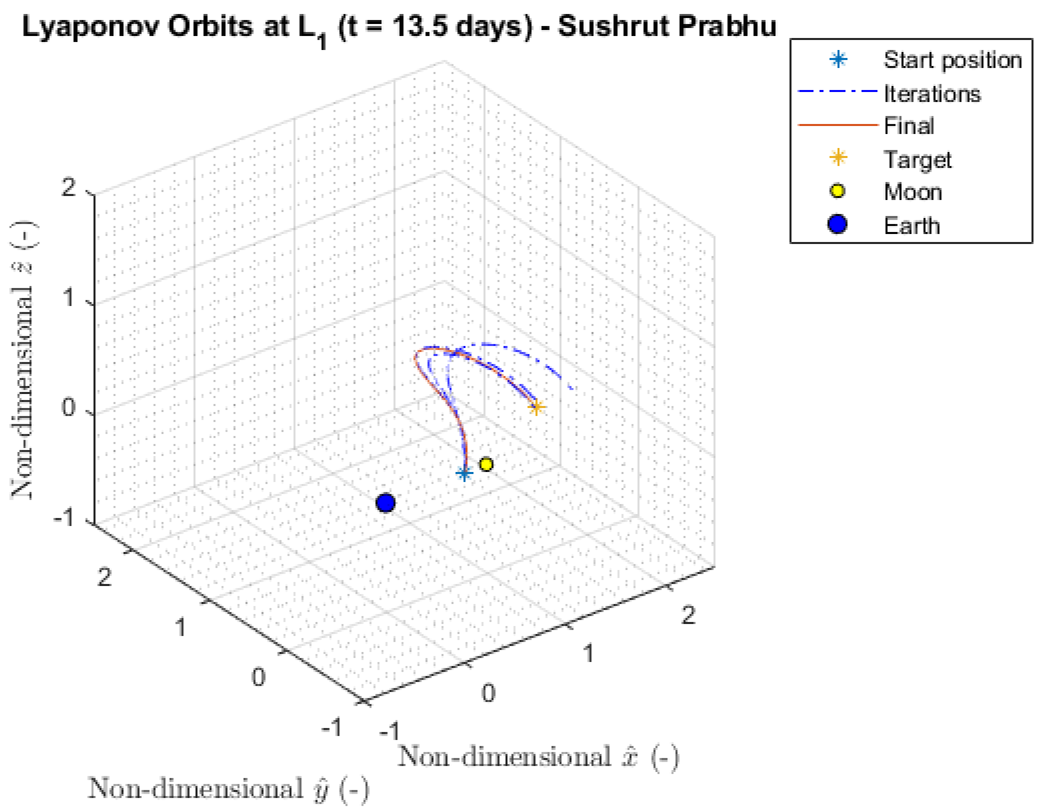
Not Converge

```
t_end = 15*3600*24/dim_vals{5,2};
Target3d(rd',r0,yn(4:6),t_end,dim_vals{4,2}, 10^-8,"plot");

plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot3(-dim_vals{4,2},0,0,'ko','MarkerSize',7,'MarkerFaceColor','b')
legend('Start position', 'Failed Iterations','Target','Moon', "Earth")
title('Lyaponov Orbits at L_1 (no convergence) - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
axis equal
xlim([-3 3])
ylim([-3 3])
zlim([-3 3])
grid on
grid minor


% Converge
t_end = 13.5*3600*24/dim_vals{5,2};
```

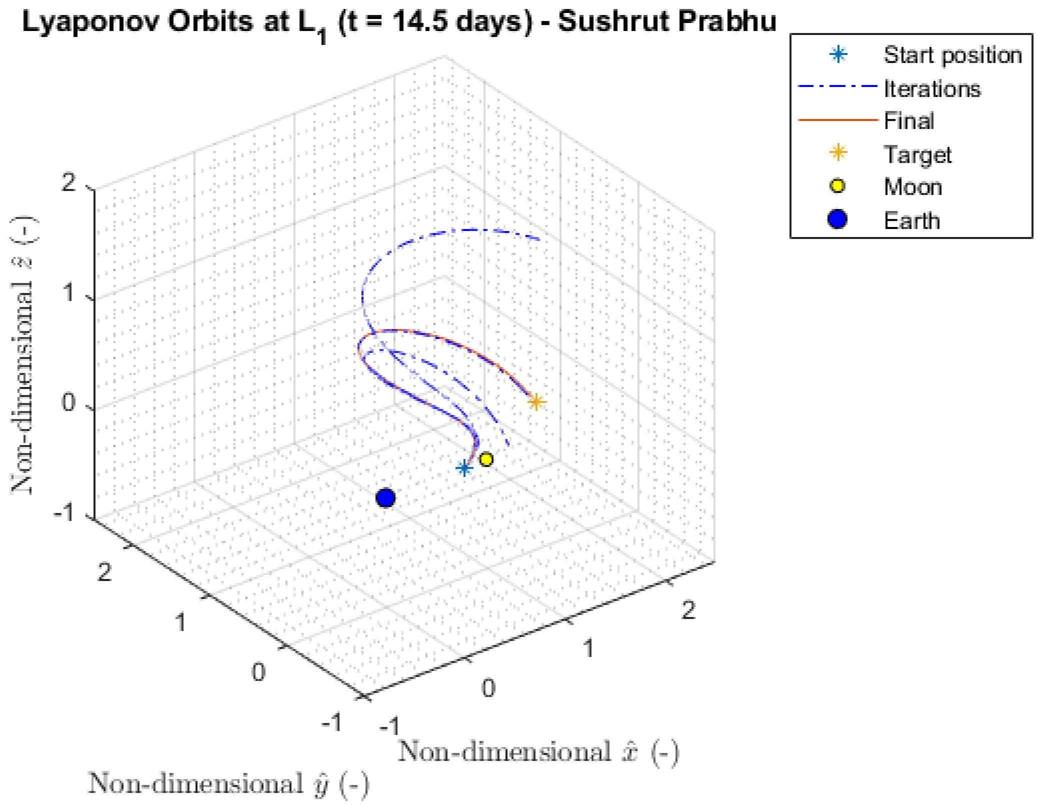
```
[yn,~, ~] = Target3d(rd',r0,yn(4:6),t_end,dim_vals{4,2},
 10^-2,"plot");

plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot3(-dim_vals{4,2},0,0,'ko','MarkerSize',7,'MarkerFaceColor','b')
legend('Start
 position', 'Iterations', 'Final', 'Target','Moon', "Earth")
title('Lyaponov Orbits at L_1 (t = 13.5 days) - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
axis equal
xlim([-1 2.5])
ylim([-1 2.5])
zlim([-1 2])
grid on
grid minor
```

Did not Coverge

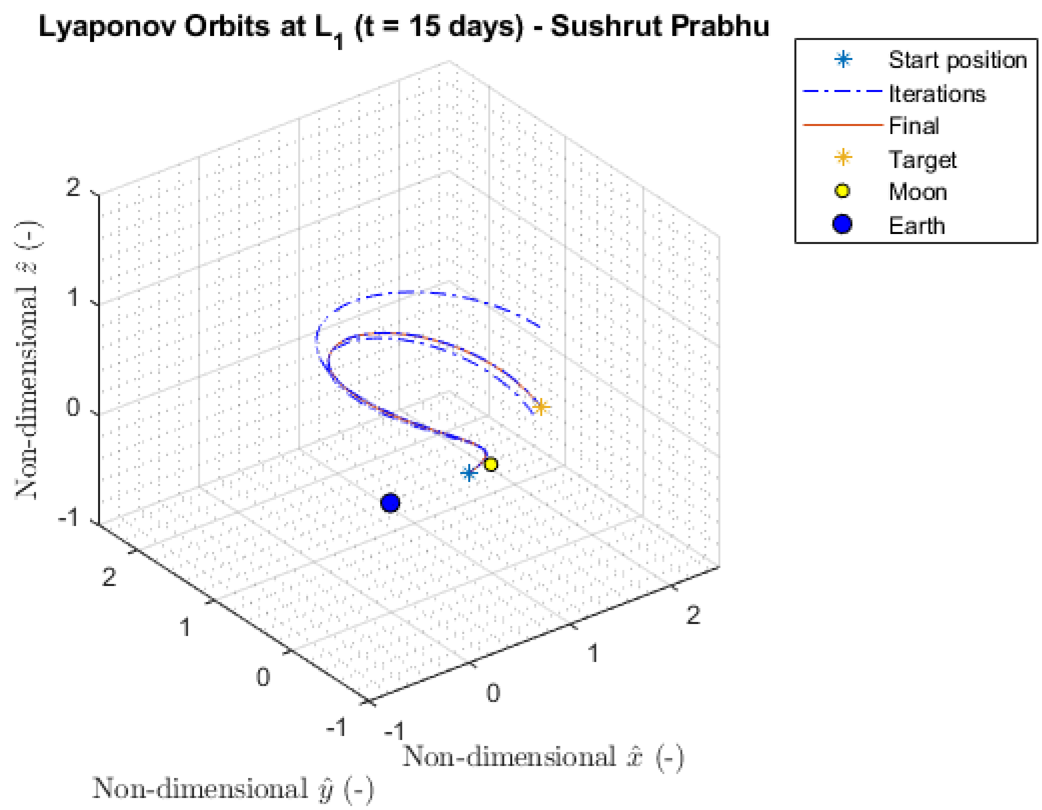





```
t_end = 14.5*3600*24/dim_vals{5,2};
[yn,~, ~] = Target3d(rd',r0,yn(4:6),t_end,dim_vals{4,2},
 10^-2,"plot");

plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot3(-dim_vals{4,2},0,0,'ko','MarkerSize',7,'MarkerFaceColor','b')
legend('Start
 position', 'Iterations', 'Final','Target','Moon', "Earth")
title('Lyaponov Orbits at L_1 (t = 14.5 days) - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
axis equal
xlim([-1 2.5])
ylim([-1 2.5])
zlim([-1 2])
grid on
grid minor
```




```
t_end = 15*3600*24/dim_vals{5,2};
[yn,~, ~] = Target3d(rd',r0,yn(4:6),t_end,dim_vals{4,2},
 10^-13,"plot");

plot3(rd(1),rd(2),rd(3),'*')
plot3(1-dim_vals{4,2},0,0,'ko','MarkerSize',5,'MarkerFaceColor','y')
plot3(-dim_vals{4,2},0,0,'ko','MarkerSize',7,'MarkerFaceColor','b')
legend('Start
 position', 'Iterations', 'Final','Target','Moon', "Earth")
title('Lyaponov Orbits at L_1 (t = 15 days) - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
zlabel("Non-dimensional $$\hat{z}$$ (-)","Interpreter", "latex")
axis equal
xlim([-1 2.5])
ylim([-1 2.5])
zlim([-1 2])
grid on
grid minor
```



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Sushant Prabhu | AAE632 | PSE3

PSE3

Given: Earth - Moon system with $\xi = 0.025$, $\eta = 0$

Find: a) Use STM for perpendicular crossing. Algorithm should be 3D. Compute complete orbit, plot it. Final P. compare with IP of primary system

b) Accuracy of end conditions. dim / non-dim. 2 orbit propagations. How long can you propagate.

c) Compute 6 eigen value. What types of eigenvalues? Six real? 4 complex 2 real?

d) See for E_1 if eigenvalues either 6 of 4. Identify real vs complex. Colour the orbit. What type of change occurs?

Solutions:

From PSE1 we have a relationship for x_0 vs $\dot{y}_0$ so we can start with a reasonable guess for $\xi = 0.025$

$x_0 = 0.025 + L_1$ Lagrange point distance

$\dot{y}_{0,guess} = -0.1752$ (non-dim)

We have designed these trajectories before so I won't repeat it. See Figure. E31

$P = 1.3928$ (non-dim) $= 6.0482$ days

No real comparison to 29.5 days → 5:1 ratio

Continued...

b) The error on 1 period $\bar{x}_f - \bar{x}_0 = 1.556 \times 10^{-12}\,\hat{x} - 1.231 \times 10^{-12}\,\hat{y}$ (non-dim)

$$|\bar{x}_f - \bar{x}_0| = 1.9836 \times 10^{-12}\,\hat{x} \text{ (non-dim)}$$

$$\bar{x}_f - \bar{x}_0 = 5.98 \times 10^{6}\,\hat{x} - 4.731 \times 10^{-6}\,\hat{y} \text{ km}$$

$$|\bar{x}_f - \bar{x}_0| = 7.625 \times 10^{-7} \text{ km}$$

See Figure: E2.2 for 2 period propogation

The maximum number of periods possible is about 3. After the 3, the orbit seems to deviate significantly. See Figure: E2.3 for 4 period propogation

c) All the eigenvalues are real:

$$\lambda = [2.1393 \times 10^{3},\ 4.6743 \times 10^{-4},\ 1,\ 1,\ 1.1708,\ 0.8541]^T$$

d) The combination of eigenvalues are;

i) 6 real

ii) 4 real 2 complex ⇝ The are on unit circle

iii) 2 real 6 complex

See Figure: E3.4 and E3.5

There appears to be a lot of changes. I have provided 2 plots one that indicates the combination of eigenvalues and also where the change occurs

PSE3

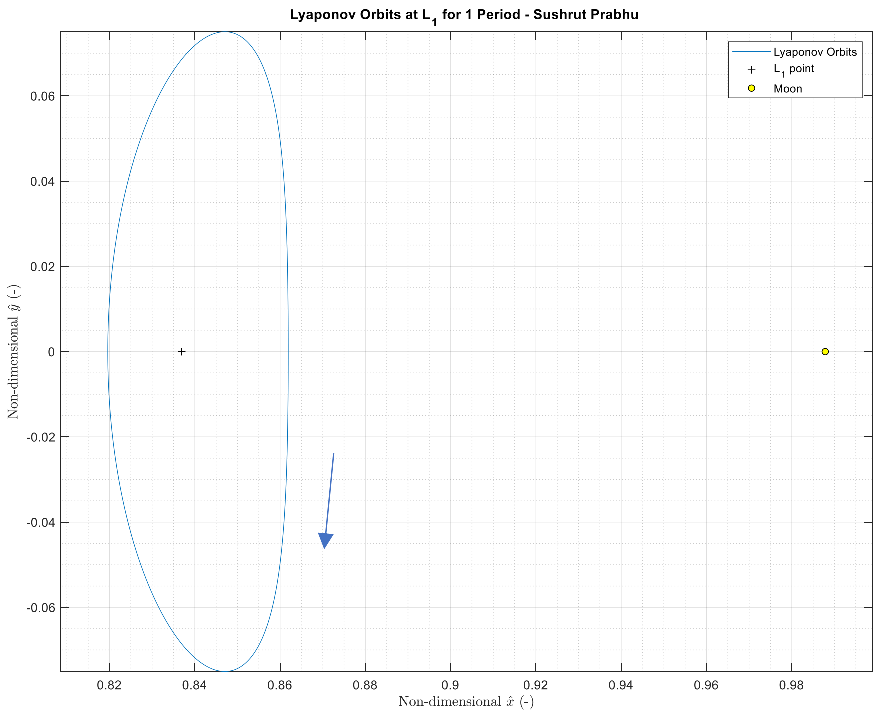


Figure E3.1: Single period around L1 Lagrange point.

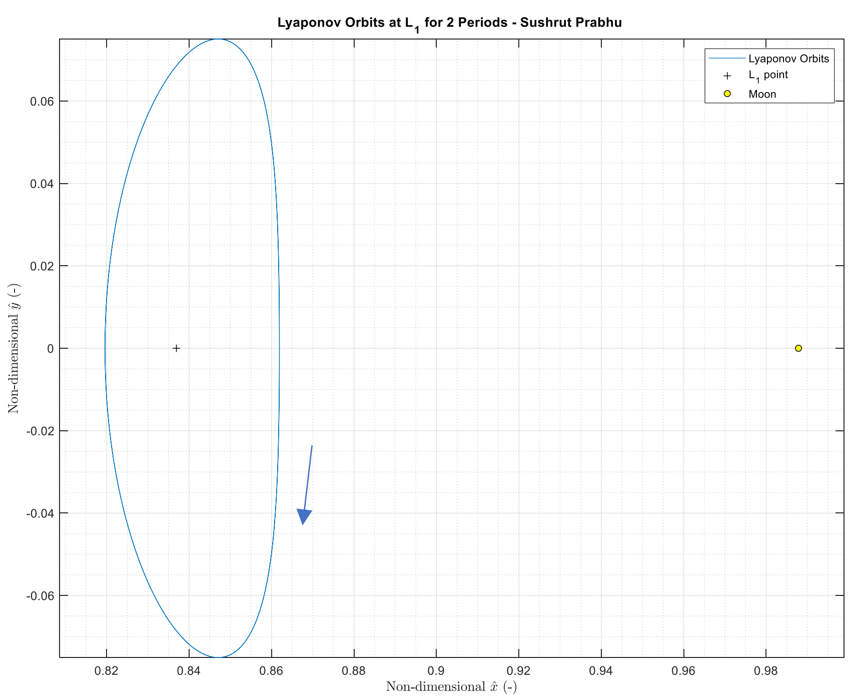


Figure E3.2: Two period around L1 Lagrange point.

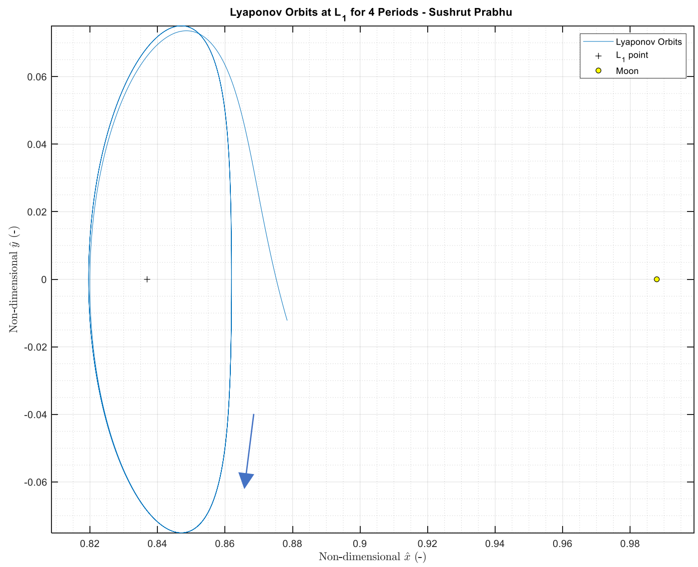


Figure E3.3: Four period around L1 Lagrange point.

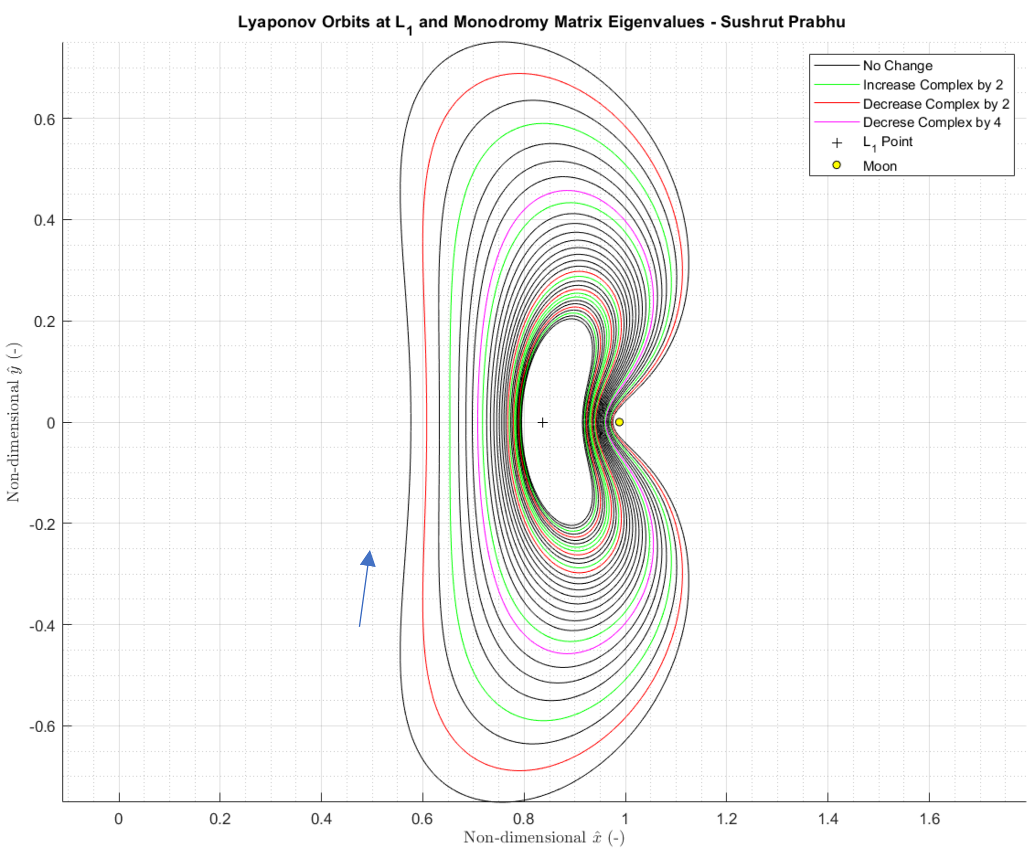


Figure E3.4: Four period around L1 Lagrange point.

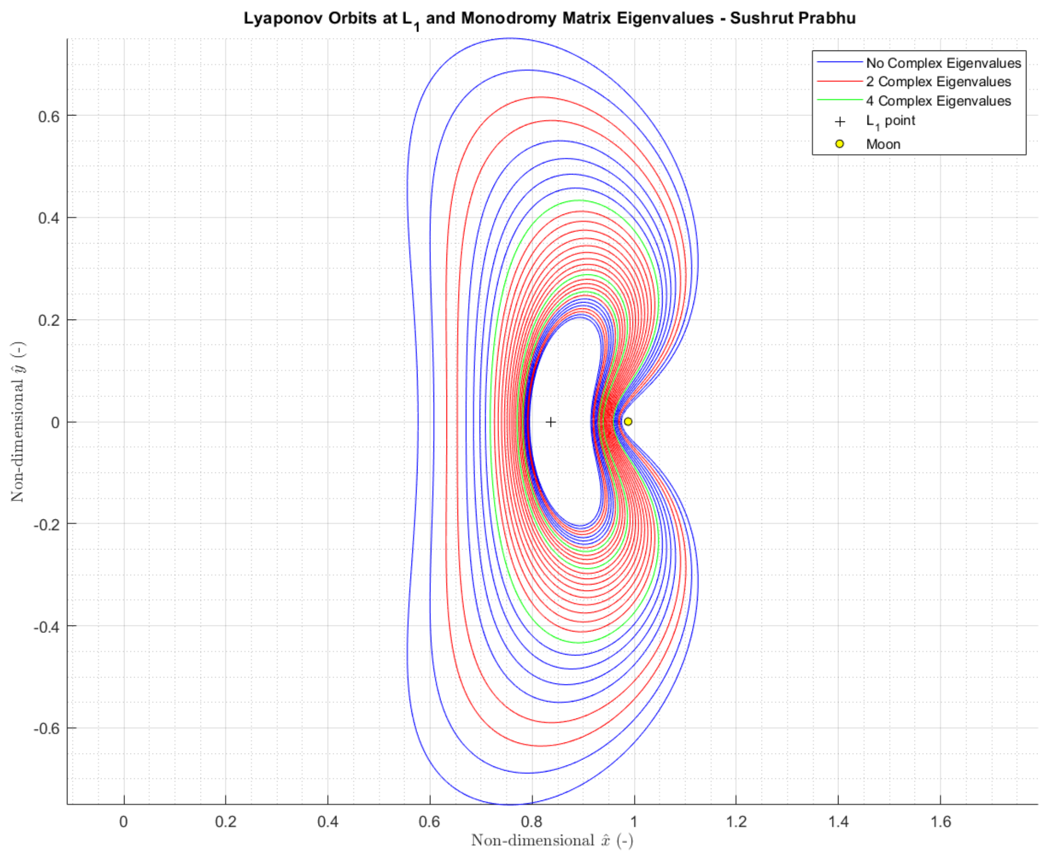


Figure E3.5: Four period around L1 Lagrange point.

Table of Contents

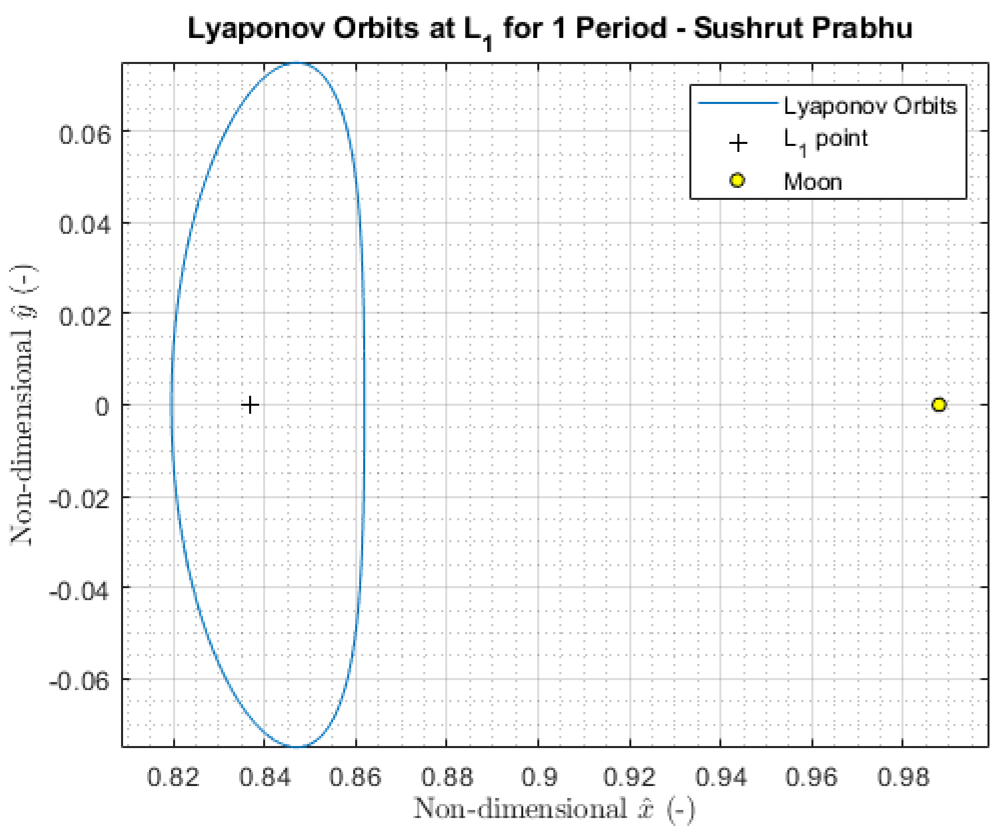
PSE3

```
clear
close all
clc

SS = SolarS;
systems = {'-', 'Earth-Moon'};
param = {'l* (km)','m* (kg)','miu' , 't*', 'gamma_1', 'L_1', 'gamma_1
 (km)', 'L_1 (km)'};
G = 6.6738*10^-20;
options=odeset('RelTol',1e-12, 'AbsTol',1e-15); % Sets integration
 tollerance

dim_vals = num2cell(zeros(length(param),1));
dim_vals = [systems; param',dim_vals];

% System Constants
[dim_vals{2,2}, dim_vals{3,2}, dim_vals{5,2}] =
 charE(SS.dM_E,0,SS.mMoon/G,SS.mEarth/G);        % Earth Moon
dim_vals{4,2} = SS.mMoon/dim_vals{3,2}/G;   % miu

% Lagrange Point 1
dim_vals{6,2} = abs(L1_NRmethod(dim_vals{4,2}*.7,dim_vals{4,2},
 10^-8));
dim_vals{7,2} = 1-dim_vals{4,2} - dim_vals{6,2};
dim_vals{8,2} = dim_vals{6,2}*dim_vals{2,2};
dim_vals{9,2} = dim_vals{7,2}*dim_vals{2,2};
```

Part a)

```
y = [dim_vals{7,2}+.025, 0, 0, 0, -.1752, 0];
x0_step = 0.0028;
k = 1;
t_end = 2.78*.5;

[yn,t_end] = Target3d_per([0
 0],y(1:3),y(4:6),t_end,dim_vals{4,2},"planar", 10^-13,"");
[~,y]=ode45(@cr3bp_df,[0 t_end*2],yn,options,dim_vals{4,2});
```

```
figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Lyaponov Orbits','L_1 point','Moon')
title('Lyaponov Orbits at L_1 for 1 Period - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor
```

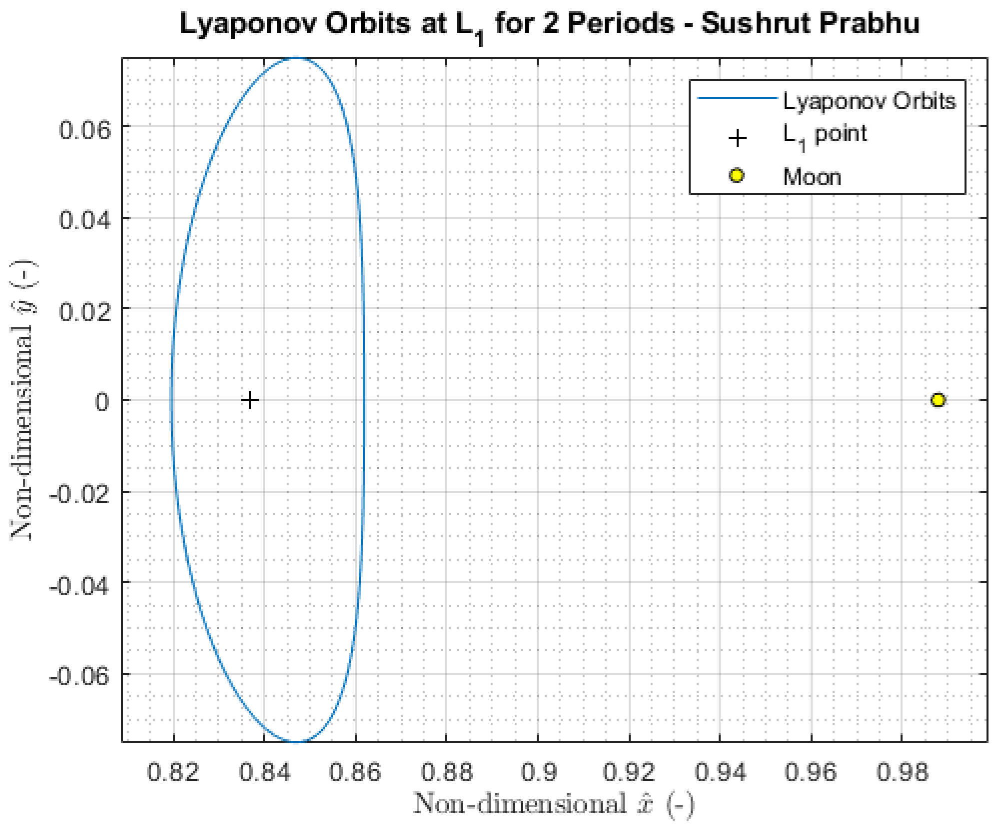


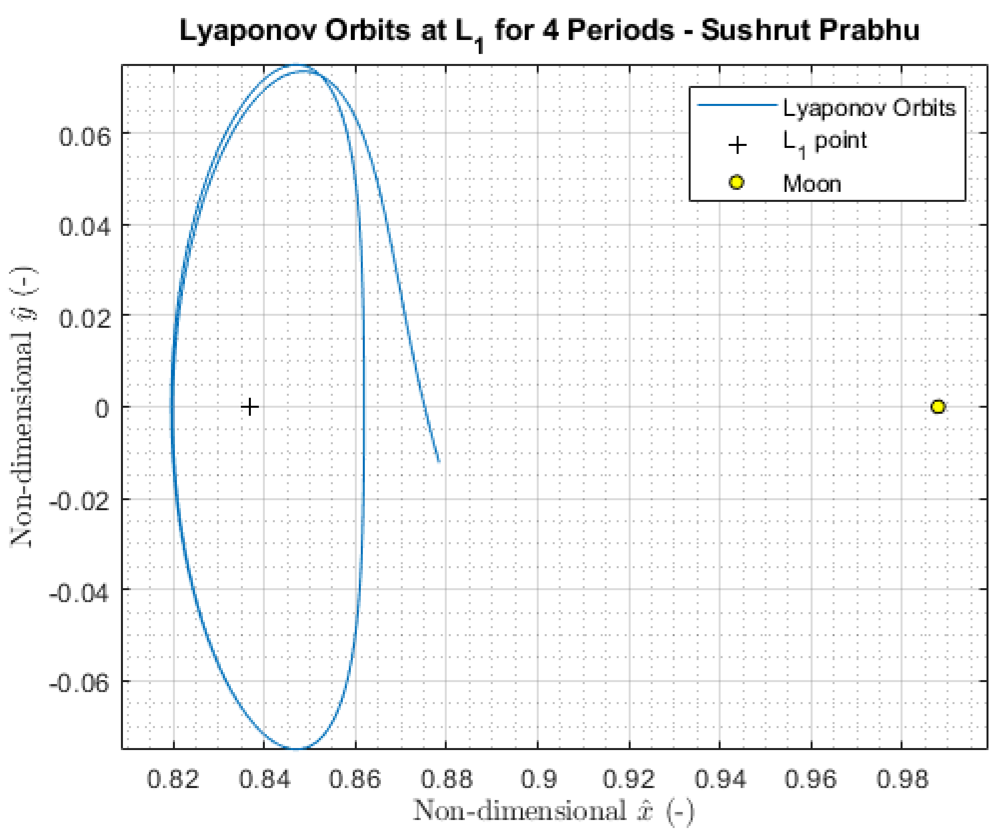
Part b)

```
err = y(end,1:3)-y(1,1:3);
err_dim = err*dim_vals{2,2}

[~,y]=ode45(@cr3bp_df,[0 t_end*4],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Lyaponov Orbits','L_1 point','Moon')
```

```
title('Lyaponov Orbits at L_1 for 2 Periods - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor

[~,y]=ode45(@cr3bp_df,[0 t_end*8],yn,options,dim_vals{4,2});

figure
plot(y(:,1),y(:,2))
hold on
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('Lyaponov Orbits','L_1 point','Moon')
title('Lyaponov Orbits at L_1 for 4 Periods - Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor
```

```
err_dim =

   1.0e-06 *

    0.5980   -0.4731         0
```


Lyaponov Orbits at L_1 for 2 Periods - Sushrut Prabhu
Lyaponov Orbits
L_1 point
Moon
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)


Lyaponov Orbits at L_1 for 4 Periods - Sushrut Prabhu
Lyaponov Orbits
L_1 point
Moon
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
Non-dimensional $\hat{y}$ (-)
Non-dimensional $\hat{x}$ (-)

Part c)

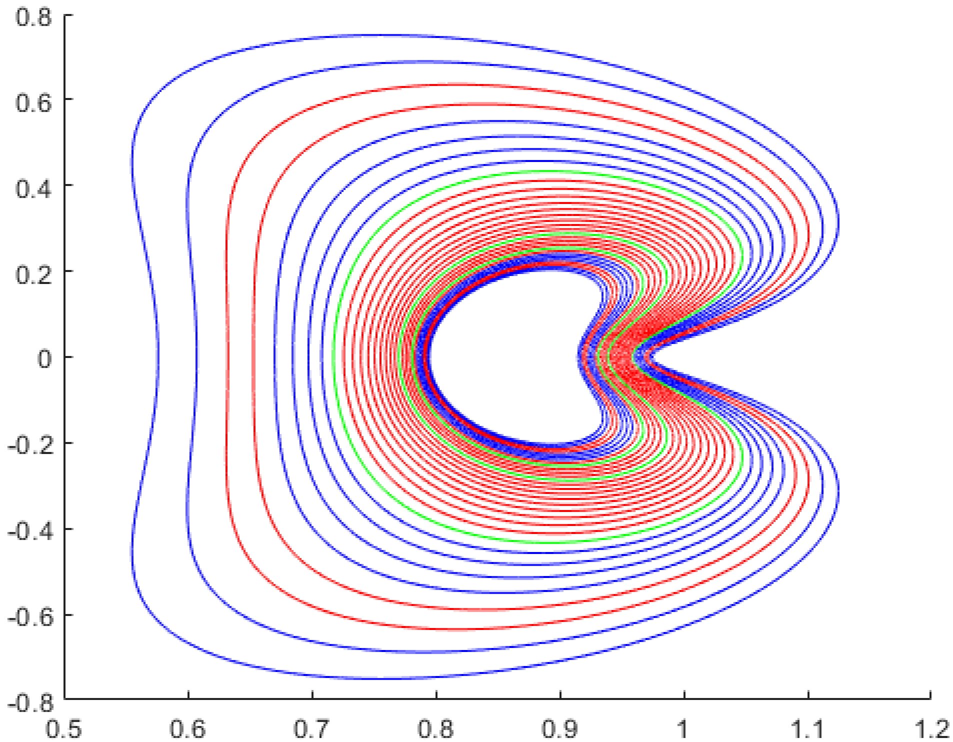
```
STM0 = eye(6);
STM0 = STM0(:)';
IC = [yn, STM0];
G = [1 0 0 0 0 0; 0 -1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 -1 0 0; 0 0 0 0 1
 0;0 0 0 0 0 -1];
Omega = [0 1 0; -1 0 0; 0 0 0];

[~,y]=ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

stm_12 = reshape(y(end,7:end),6,6)';
monodromy = G* [zeros(3,3), -eye(3);eye(3), -2*Omega]*stm_12' *
 [-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

[V,D] = eig(monodromy);
D = diag(D);
```

Part d)

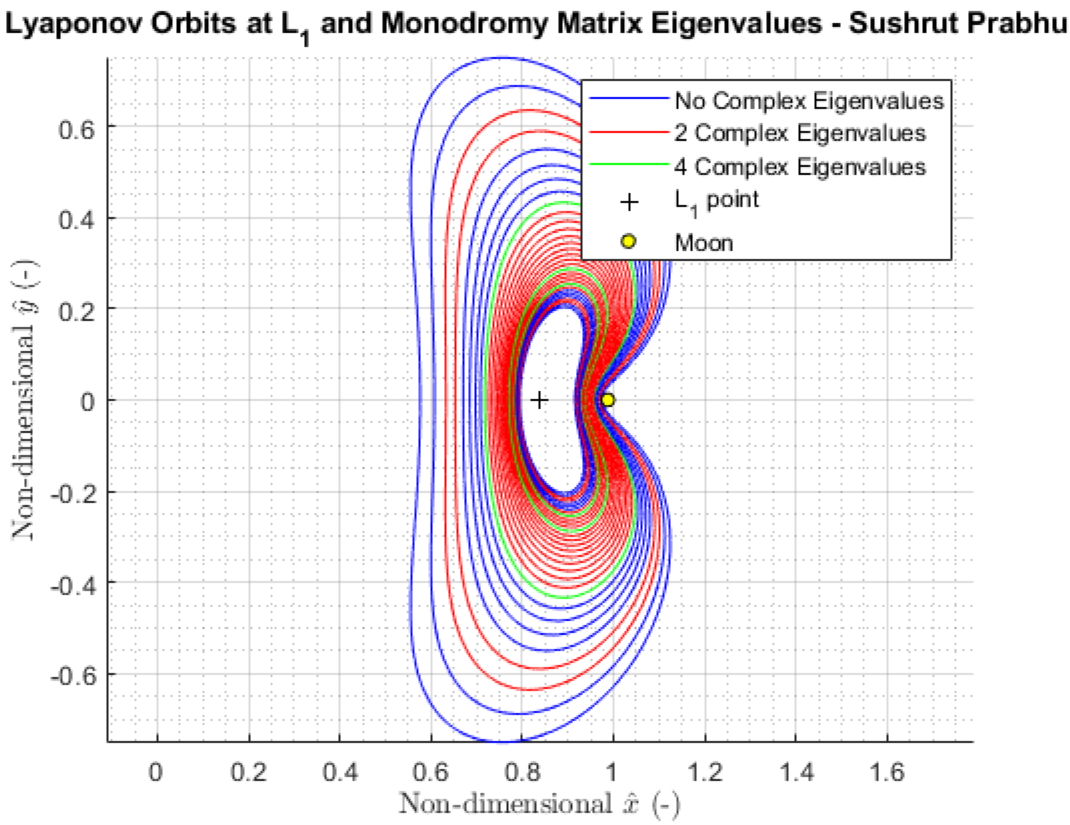
```
y = [.913, 0, 0, 0, -.4835, 0];

x0_step = 0.002;
k = 1;
l = 0;
n = 0;
t_end = 2.72*.7;
figure
hold on

while k < 32
    % Inital Guess
    IC = [y(1,1:3),y(1,4:6)];

    if k > 2
        m = (yd0_vec(k-2) - yd0_vec(k-1)) / (x0_vec(k-2) -
 x0_vec(k-1));
        c = yd0_vec(k-1) - m*x0_vec(k-1);
        yg = m*(x0_step+y(1))+c;
        IC = [y(1,1:3),0,yg,0];
    end

    ICm = IC + [x0_step, 0, 0, 0, 0, 0];
    [t,y] = ode45(@cr3bp_df,[0 t_end],ICm,options,dim_vals{4,2});

    t_end = t(find(y(:,2)>0,1));
    clear t

    rv_des = [0 0]';  % y = 0 and xfdot = 0
    [yn,t_end] =
 Target3d_per(rv_des,ICm(1:3),IC(4:6),t_end,dim_vals{4,2},"planar",
 10^-13,"");
```

```
    % Final plot and solution
    IC_stm = eye(6);
    IC_stm = IC_stm(:)';
    IC = [yn(1,1:6), IC_stm];
%     t_end = t_end;
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

%     monodromy = reshape(y(end,7:end),6,6);

    % Method 2
    stm_12 = reshape(y(end,7:end),6,6)';
    monodromy = G* [zeros(3,3), -eye(3);eye(3), -2*Omega]*stm_12' *
 [-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

    [V,D] = eig(monodromy);
    D = diag(D);
    Dvec(k,:) = D';
    col = "-b";

    t_end = 2*t_end;
    [t,y] = ode45(@cr3bp_df,[0
 t_end],yn(1,1:6),options,dim_vals{4,2});


    c_comp = nnz(find(imag(D)~=0));

    if  c_comp > 2
        col = "-g";
        n = n+1;
    elseif c_comp>0 && c_comp<3
        col = "-r";
        l = l+1;
    end

    ly_plt = plot(y(:,1),y(:,2),col);

    if k > 1

 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
    end

    if l < 2 && l > 0

 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
       l = l+1;
    end

    if n < 2 && n > 0

 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
        n = n+1;
    end
```

```
    x0_vec(k) = y(1,1);
    yd0_vec(k) = y(1,5);

%     Per_vec(k) = t_end;
%     J_vec(k) =
 Jacobi_C(y(1,1),y(1,2),0,norm(y(1,4:6)),dim_vals{4,2});
    k = k+1;
end
```




```
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
legend('No Complex Eigenvalues','2 Complex Eigenvalues','4 Complex
 Eigenvalues','L_1 point','Moon')
title('Lyaponov Orbits at L_1 and Monodromy Matrix Eigenvalues -
 Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor
```

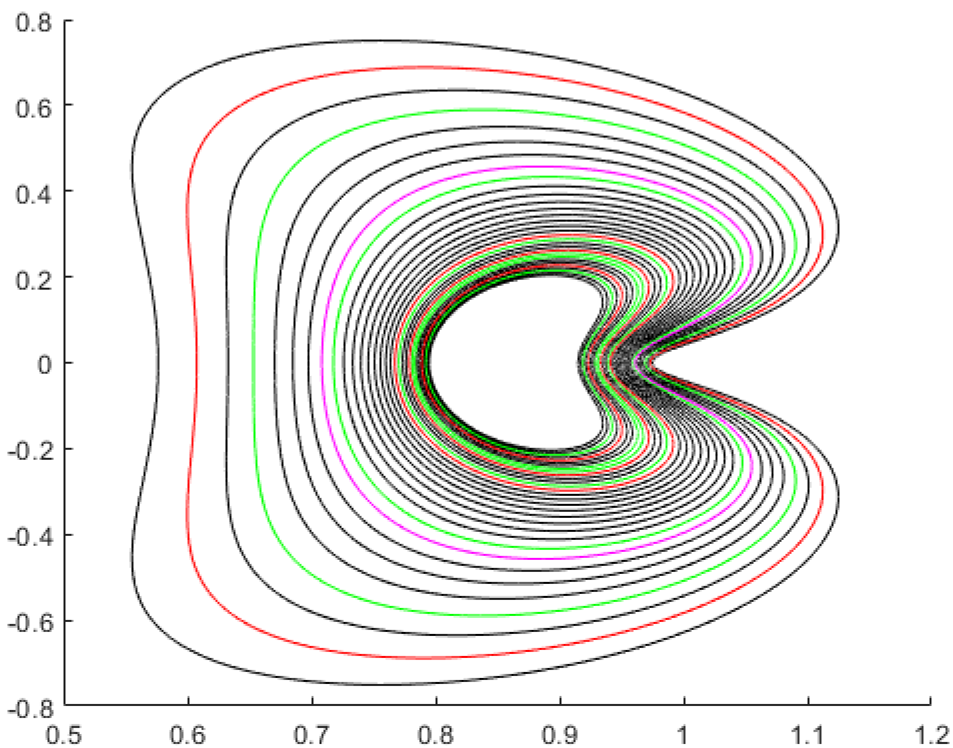


Part d)

```
y = [.913, 0, 0, 0, -.4835, 0];

x0_step = 0.002;
k = 1;
l = 0;
n = 0;
o = 0;
t_end = 2.72*.7;
figure
hold on

while k < 32
    % Inital Guess
    IC = [y(1,1:3),y(1,4:6)];

    if k > 2
        m = (yd0_vec(k-2) - yd0_vec(k-1)) / (x0_vec(k-2) -
 x0_vec(k-1));
        c = yd0_vec(k-1) - m*x0_vec(k-1);
        yg = m*(x0_step+y(1))+c;
        IC = [y(1,1:3),0,yg,0];
    end
```

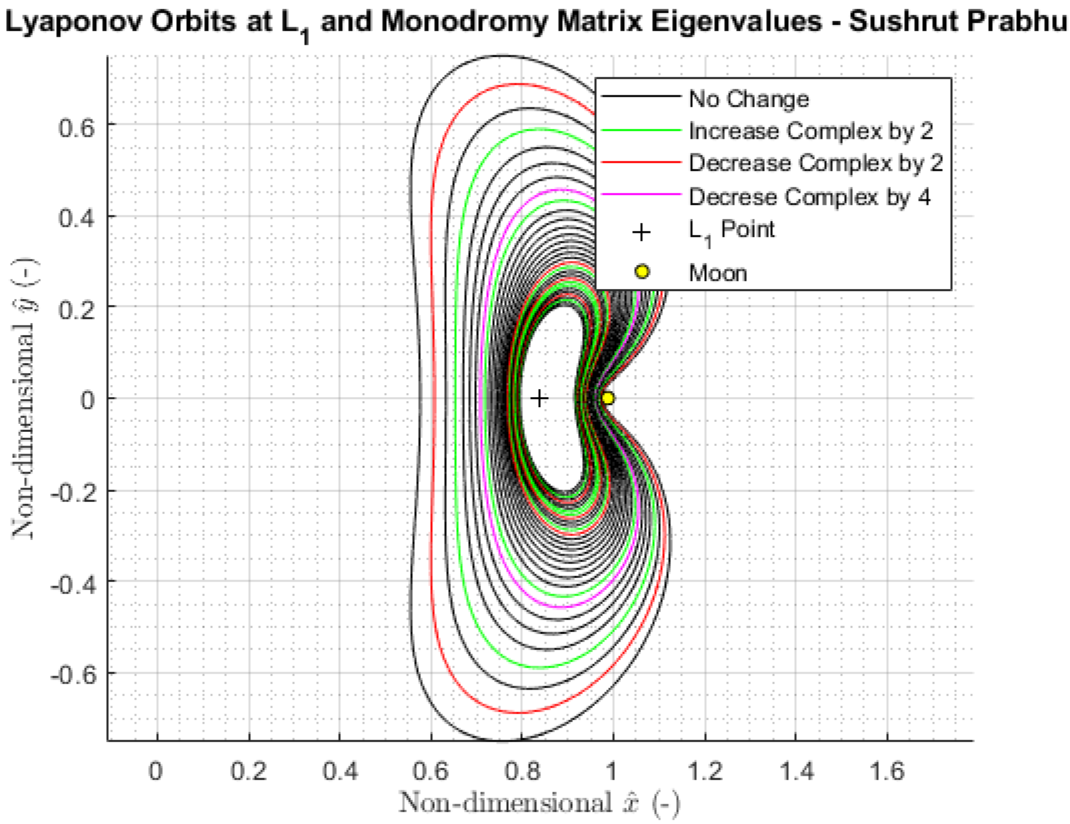
```
   ICm = IC + [x0_step, 0, 0, 0, 0, 0];
   [t,y] = ode45(@cr3bp_df,[0 t_end],ICm,options,dim_vals{4,2});

   t_end = t(find(y(:,2)>0,1));
   clear t

   rv_des = [0 0]';  % y = 0 and xfdot = 0
   [yn,t_end] =
Target3d_per(rv_des,ICm(1:3),IC(4:6),t_end,dim_vals{4,2},"planar",
10^-13,"");

   % Final plot and solution
   IC_stm = eye(6);
   IC_stm = IC_stm(:)';
   IC = [yn(1,1:6), IC_stm];
   [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,dim_vals{4,2});

   % Method 2
   stm_12 = reshape(y(end,7:end),6,6)';
   monodromy = G* [zeros(3,3), -eye(3);eye(3), -2*Omega]*stm_12' *
[-2*Omega, eye(3); -eye(3), zeros(3,3)]*G*stm_12;

   [V,D] = eig(monodromy);
   D = diag(D);
   Dvec(k,:) = D';
   col = "-b";

   t_end = 2*t_end;
   [t,y] = ode45(@cr3bp_df,[0
t_end],yn(1,1:6),options,dim_vals{4,2});

   c_comp(k) = nnz(find(imag(D)~=0));
   col = '-k';

   if k > 1
       if (c_comp(k)- c_comp(k-1)) == 2
           col = 'g';
           l = l+1;
       elseif (c_comp(k)- c_comp(k-1)) == -2
           col = 'r';
           n = n+1;
       elseif (c_comp(k)- c_comp(k-1)) == -4
           col = 'm';
           o = o+1;
       end
   end

   ly_plt = plot(y(:,1),y(:,2),col);

   if k > 1

set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
   end
```

```
    if l == 1
 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
        l = l+1;
    end

    if n == 1
 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
        n = n+1;
    end

    if o == 1
 set(get(get(ly_plt,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
        o = o+1;
    end

    x0_vec(k) = y(1,1);
    yd0_vec(k) = y(1,5);

    k = k+1;
end
```




```
plot(dim_vals{7,2},0,'+k')
plot(1-dim_vals{4,2},0,'ko','MarkerSize',5,'MarkerFaceColor','y')
```

```
legend('No Change','Increase Complex by 2','Decrease Complex by
 2','Decrese Complex by 4','L_1 Point','Moon')
title('Lyaponov Orbits at L_1 and Monodromy Matrix Eigenvalues -
 Sushrut Prabhu ')
xlabel("Non-dimensional $$\hat{x}$$ (-)","Interpreter", "latex")
ylabel("Non-dimensional $$\hat{y}$$ (-)","Interpreter", "latex")
axis equal
grid on
grid minor
```



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Sushant Prabhu | AAE 632 | PSE4

PSE4 Optional

Given: 2 body problem

Find: a) $A(t)$ matrix, use the matrix to find STM at a fixed point

b) Monodromy matrix eigen values and stability

Solution a) $\dot{x} = \begin{pmatrix} \dot{\bar{r}} \\ \dot{\bar{v}} \end{pmatrix} = \begin{pmatrix} \bar{v} \\ -\frac{\mu \bar{r}}{r^3} \end{pmatrix}$ $\quad A(t) = \begin{bmatrix} 0 & I \\ A_{21} & 0 \end{bmatrix}$ $\boxed{H4}$

$$A_{21} = \begin{bmatrix} -\frac{\mu}{r^3} + \frac{3\mu x^2}{r^5} & \frac{3\mu xy}{r^5} & \frac{3\mu xz}{r^5} \\ \frac{3\mu xy}{r^5} & -\frac{\mu}{r^3} + \frac{3\mu y^2}{r^5} & \frac{3\mu yz}{r^5} \\ \frac{3\mu xz}{r^5} & \frac{3\mu yz}{r^5} & -\frac{\mu}{r^3} + \frac{3\mu z^2}{r^5} \end{bmatrix}$$

$r = [10000, 0, 0]$

$v = [0, \sqrt{\frac{\mu}{|r|}}, 0]$

Let us calculate STM at $t = P/4$

$$STM = \begin{bmatrix} 2.7124 & 1 & 0 & 3.4678\times10^{3} & 1.264\times10^{3} & 0 \\ 2 & 1 & 0 & 1.5839\times10^{3} & 3.1678\times10^{3} & 0 \\ 0 & 0 & -1.6146\times10^{-15} & 0 & 0 & 1.5839\times10^{3} \\ 6.3135\times10^{-4} & 6.3135\times10^{-4} & 0 & 1 & 1 & 0 \\ 0.0023 & 6.3135\times10^{-4} & 0 & 2 & 2.7124 & 0 \\ 0 & 0 & -6.3135\times10^{-4} & 0 & 0 & -5.3299\times10^{-16} \end{bmatrix}$$

b) $\lambda = \begin{bmatrix} 0.99999999999953 + i\,9.224\times10^{-7} \\ 0.99999999999953 - 9.224\times10^{-7} \\ 1.0000000000000062 \\ 1.0000000000000181 \\ 1.000000000000012 \\ 0.999999999999992 \end{bmatrix}$

15 decimal places

→ Appears <u>unstable</u> but by a small margin. This could be a numerical error which would make the orbit asymptotically stable

Table of Contents

PSE4

```
clear
close all
clc
```

Setup

```
SS = SolarS;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance

r = [10000 0 0];
v = [0 sqrt(SS.mEarth/norm(r)) 0];
```

part a

```
Per = 2*pi/sqrt(SS.mEarth/norm(r)^3);

STM_0 = eye(6);
STM_0 = STM_0(:)';

[~,y] = ode45(@twobpr,[0 Per/4],[r,v,STM_0],options,SS.mEarth);
STM = reshape(y(end,7:end),6,6)';
```

part b

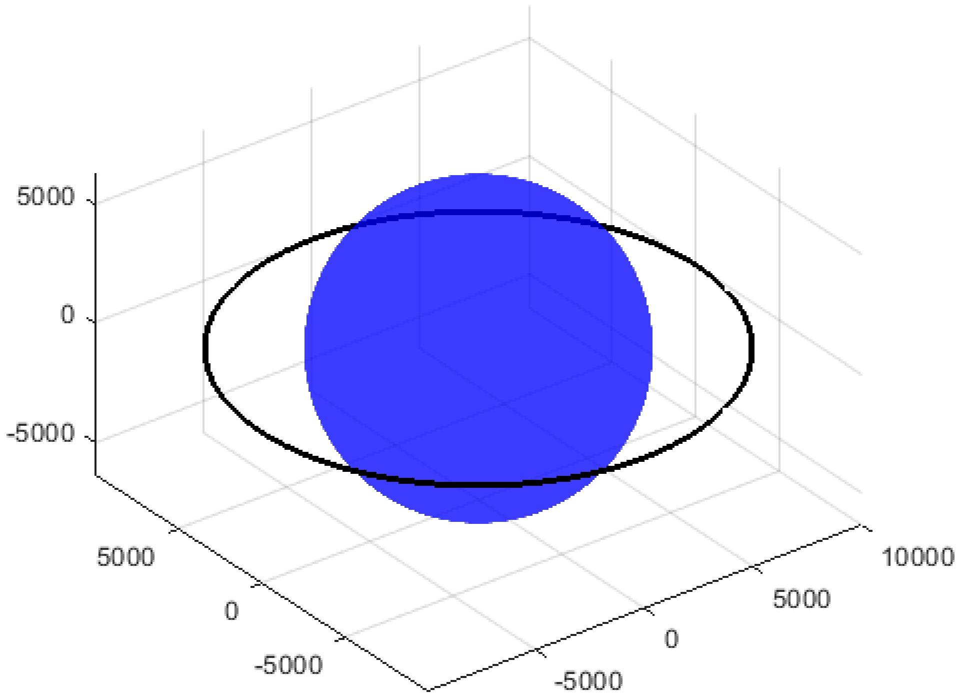
```
[~,y] = ode45(@twobpr,[0 Per],[r,v,STM_0],options,SS.mEarth);
Monodromy = reshape(y(end,7:end),6,6)';

[V,D] = eig(Monodromy);
D = diag(D);


[xx yy zz] = sphere(128);

figure
h = surfl(xx*SS.rEarth, yy*SS.rEarth, zz*SS.rEarth);
hold on
plot3(y(:,1),y(:,2),y(:,3),'-k','LineWidth',2)
set(h,'FaceColor',[0 0
 1],'FaceAlpha',0.5,'FaceLighting','gouraud','EdgeColor','none')
```

```
axis equal
```



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A_kep

```
function A = A_kep(x,y,z,miu)

r = sqrt(x^2 + y^2 + z^2);

A21 = [-miu/r^3 + 3*miu*x^2/r^5, 3*miu*x*y/r^5, 3*miu*x*z/r^5;
    3*miu*x*y/r^5, -miu/r^3+3*miu*y^2/r^5, 3*miu*y*z/r^5;
    3*miu*x*z/r^5, 3*miu*y*z/r^5, -miu/r^3+3*miu*z^2/r^5];

A = [zeros(3,3), eye(3); A21, zeros(3,3)];
end
```

```
Not enough input arguments.

Error in A_kep (line 4)
r = sqrt(x^2 + y^2 + z^2);
```

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Target3d

Periodic orbit targeter

```
function  [IC_final,t_end, tb] = Target3d(r_des,r,v,t_end,miu, tol,
 pl)
% Initialization
% t = 0:.001:t_end;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance
error = 2*tol;
phi_0 = eye(6);
phi_0 = phi_0(:)';
i = 1;
v0 = v;

if pl == "plot"
    figure
    plot3(r(1),r(2),r(3),'*')
    hold on
end

while error > tol
    % Non-linear propgation with phi
    IC = [r,v,phi_0];
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,miu);

    % Phi at final time
    phi_t = y(:,7:end);
    phi_tf = reshape(phi_t(end,:),6,6)';

    K = -phi_tf(1:3,4:6);
    Fx = r_des - y(end,1:3)' ;
    delv1 = K.' * (K*K.')^-1 * Fx;

    IC = [r, v - delv1'];
    [~,yn] = ode45(@cr3bp_df,[0 t_end],IC,options,miu);
    v = yn(1,4:6);

    error = max(abs(yn(end,1:3)-r_des'));

    tb{:,i} = {v-v0, norm(v-v0), Fx, norm(Fx)};
    i = i+1;

    if pl == "plot"
        if error > tol
            pl3 = plot3(yn(:,1),yn(:,2),yn(:,3),'-.b');
            if i > 2
 set(get(get(pl3,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
            end
        else
```

```
            plot3(yn(:,1),yn(:,2),yn(:,3))
        end
    end

    if i > 20
        error = tol/2;
        fprintf("Did not Coverge")
    end
end

IC_final = IC;

end
```

```
Not enough input arguments.

Error in Target3d (line 7)
error = 2*tol;
```

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twobpr

```
function dx = twobpr(t,x,miu)
dx = zeros(42,1);

phi = reshape(x(7:end),6,6)';
r = sqrt(x(1)^2+x(2)^2+x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = -miu*x(1)/r^3;
dx(5) = -miu*x(2)/r^3;
dx(6) = -miu*x(3)/r^3;

phi_dot = [A_kep(x(1),x(2),x(3),miu)* phi]';
dx(7:end) = phi_dot(:);

end
```

```
Not enough input arguments.

Error in twobpr (line 5)
phi = reshape(x(7:end),6,6)';
```

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A_t

Intermediate caluclation for STM

```
function A = A_t(x,y,z,miu)

if isnan(z)
    [Uxx,Uyy,~,Uxy,~,~]= Unn(x,y,0,miu);
    A = [zeros(2,2), eye(2,2); Uxx, Uxy, 0, 2; Uxy, Uyy, -2, 0];
else
    [Uxx,Uyy,Uzz,Uxy,Uxz,Uyz]= Unn(x,y,z,miu);
    A = [zeros(3,3), eye(3,3); Uxx, Uxy,Uxz, 0, 2,0; Uxy, Uyy, Uyz, 0,
 -2, 0; Uxz, Uyz, Uzz, 0, 0, 0];
end

end
```

```
Not enough input arguments.

Error in A_t (line 5)
if isnan(z)
```

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acc_nonlin

Non-linear acceleration for STM

```
function acc = acc_nonlin(r,v,miu)

dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
rr = sqrt((r(1)+miu-1)^2 + r(2)^2 + r(3)^2);

acc(1) = 2*v(2) + r(1) - (1-miu)*(r(1)+miu)/dd^3 - miu*(r(1)-1+miu)/
rr^3;
acc(2) = -2*v(1) + r(2) - (1-miu)*r(2)/dd^3 - miu*r(2)/rr^3;
acc(3) = -(1-miu)*r(3)/dd^3 - miu*r(3)/rr^3;
```

```
Not enough input arguments.

Error in acc_nonlin (line 5)
dd = sqrt((r(1)+miu)^2 + r(2)^2 + r(3)^2);
```

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cr3bp_STM_df3d

3d STM

```
function dx = cr3bp_STM_df3d(t,x,miu)

dx = zeros(42,1);

d = sqrt((x(1)+miu)^2 + x(2)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2);
phi = reshape(x(7:end),6,6)';

d = sqrt((x(1)+miu)^2 + x(2)^2 + x(3)^2);
r = sqrt((x(1)+miu-1)^2 + x(2)^2 + x(3)^2);

dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = 2*x(5) + x(1) - (1-miu)*(x(1)+miu)/d^3 - miu*(x(1)-1+miu)/r^3;
dx(5) = -2*x(4) + x(2) - (1-miu)*x(2)/d^3 - miu*x(2)/r^3;
dx(6) = -(1-miu)*x(3)/d^3 - miu*x(3)/r^3;

phi_dot = [A_t(x(1),x(2),x(3),miu)*phi]';

dx(7:end) = phi_dot(:);
end
```

```
Not enough input arguments.

Error in cr3bp_STM_df3d (line 7)
d = sqrt((x(1)+miu)^2 + x(2)^2);
```

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Target3d_per

Periodic orbit targeter

```
function  [IC_final,t_end] = Target3d_per(r_des,r,v,t_end,miu,fix,
 tol, pl)
% Initialization
% t = 0:.001:t_end;
options=odeset('RelTol',1e-13, 'AbsTol',1e-15); % Sets integration
 tollerance
error = 2*tol;
phi_0 = eye(6);
phi_0 = phi_0(:)';
i = 1;

if pl == "plot"
    figure
    hold on
end

while error > tol
    % Non-linear propgation with phi
    IC = [r,v,phi_0];
    [~,y] = ode45(@cr3bp_STM_df3d,[0 t_end],IC,options,miu);

    % Phi at final time
    phi_t = y(:,7:end);
    phi_tf = reshape(phi_t(end,:),6,6)';
    acc = acc_nonlin(y(end,1:3),y(end,4:6),miu);

    if fix == "planar"
        phi_main_tf = [phi_tf(2,5), y(end,5); phi_tf(4,5), acc(1)];
        err = r_des - [y(end,2), y(end,4)]';
    elseif fix == "x0"
        phi_main_tf = [phi_tf(4,3), phi_tf(4,5); phi_tf(6,3),
 phi_tf(6,5)] + 1/y(end,5)*[acc(1), acc(3)]'*[phi_tf(2,3),
 phi_tf(2,5)];
        err = r_des - [y(end,4), y(end,6)]';
    elseif fix == 2
        phi_main_tf = [phi_tf(1:2,4),y(end,3:4)'];
    elseif dt == 2
        phi_main_tf = [phi_tf(1:2,3),y(end,3:4)'];
    end

    delv1 = phi_main_tf^-1 * err;

    if fix == "planar"
        IC = [r,v+[0, delv1(1), 0]];
        t_end = t_end + delv1(2);
    elseif fix == "x0"
        IC = [r+[0,0,delv1(1)],v+[0,delv1(2),0]];
    elseif fix == 1
```

```
        IC = [r,v+[0, delv1(1), 0]];
        t_end = t_end+delv1(2);
    elseif fix == 2
        IC = [r,v+[delv1(1) 0, 0]];
        t_end = t_end+delv1(2);
    end

    [~,yn] = ode45(@cr3bp_df,[0 t_end],IC,options,miu);

    r = yn(1,1:3);
    v = yn(1,4:6);

    if fix == "planar"
        error = max(abs([yn(end,2),yn(end,4)]-r_des'));
    elseif fix == "x0"
        error = max(abs([yn(end,4),yn(end,6)]-r_des'));
    end

    i = i+1;

    if i > 500
        error = tol/2;
        fprintf("Did not Coverge")
    end
    if pl == "plot"
        if error > tol
            ite = plot(yn(:,1),yn(:,2),'-.b');
            if i > 2

   set(get(get(ite,'Annotation'),'LegendInformation'),'IconDisplayStyle','off');
            end
        else
           plot(yn(:,1),yn(:,2),'r');
        end
    end
end

IC_final = yn(1,:);
end
```

Not enough input arguments.

Error in Target3d_per (line 7)
*error = 2*tol;*

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Characteristic Elements

```
function [lstar, mstar, tstar] = charE(D1,D2,m1,m2)
G = 6.6738*10^-20;

lstar = D1+D2;
mstar = m1 + m2;

tstar = sqrt(lstar^3/G/mstar);
```

```
Not enough input arguments.

Error in charE (line 5)
lstar = D1+D2;
```

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