Friedrich-Alexander-Universität
Technische Fakultät

Machine Learning
Data Analytics

Machine Learning for Time Series

(MLTS or MLTS-Deluxe Lectures)

Dr. Dario Zanca

Machine Learning and Data Analytics (MaD) Lab

Friedrich-Alexander-Universität Erlangen-Nürnberg

18.10.2022

Organisational Information

Machine Learning for time series

- 5 ECTS
- Lectures + Exercises

~~**Machine Learning for Time Series (Deluxe)**~~

- ~~7.5 ECTS~~
- ~~Lectures + Exercises + Project~~

Topics overview

- Time series fundamentals and definitions (2 lectures) ←
- Bayesian Inference (1 lecture)
- Gaussian processes (2 lectures)
- State space models (2 lectures)
- Autoregressive models (1 lecture)
- Data mining on time series (1 lecture)
- Deep learning on time series (4 lectures)
- Domain adaptation (1 lecture)

Classical

DL based

Course times

Lectures (online)

A new lecture recording is generally released every Thursday on FAU.TV

Consultation hours by appointment, write to dario.zanca@fau.de

Exercises (online)

Live Zoom Session starting on November 3rd

Recordings from previous editions are available at https://www.fau.tv/course/id/3178

StudOn 2023-2024:
https://www.studon.fau.de/crs5276833.html

Exams and evaluation

Written Exam (5 ECTS)

- 70% from lectures, 30% from exercises
- **On-campus**

Both Important

May have additional topics.

Course organizers
Lecturers

Machine Learning and Data Analytics (MaD) Lab

- Dr. Dario Zanca, dario.zanca@fau.de *
- Prof. Dr. Björn Eskofier, bjoern.eskofier@fau.de

* Please, address all your correspondence about the course to Dr. Dario Zanca

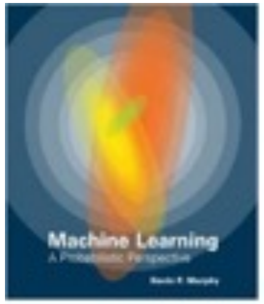
Exercises, responsibles:

- Richard Dirauf (M.Sc.), richard.dirauf@fau.de
- Philipp Schlieper (M.Sc.), philipp.schlieper@fau.de

References

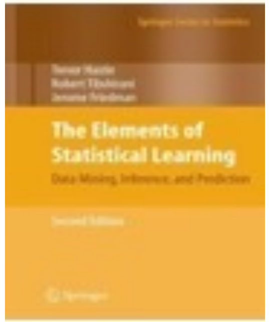
Machine learning: A Probabilistic Perspective,

by Kevin Murphy (2012)



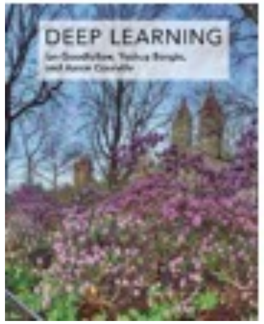
The Elements of Statistical Learning: Data Mining, Inference, and Prediction

by Trevor Hastie, Robert Tibshirani, and Jerome Friedman (2009)



Deep Learning

by Ian Goodfellow, Yoshua Bengio, and Aaron Courville (2016)



Time series fundamentals

Motivations

FAU

An old history of time series analysis: Babylonian astronomical diaries

Noting down astronomical events on rocks & predicting them correctly.

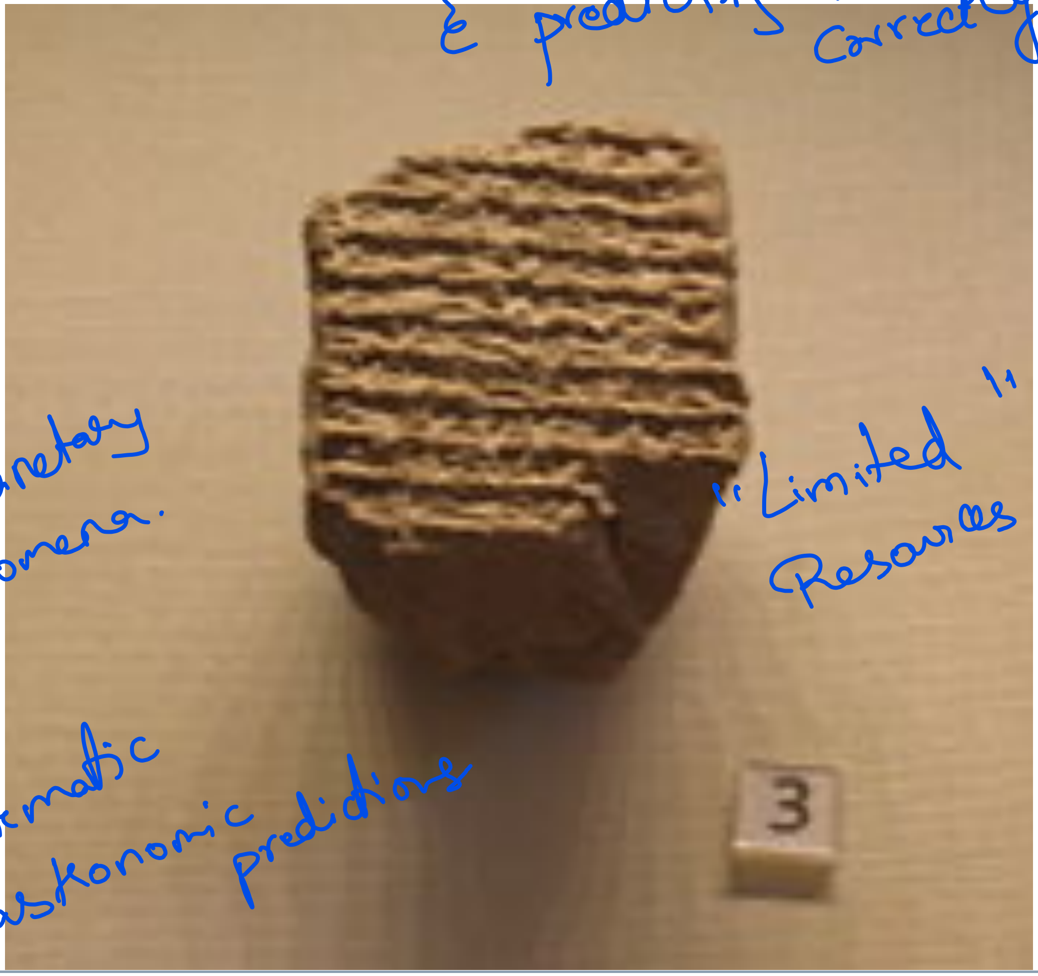
VII century B.C.

"[…] Night of the 5th, beginning of the night, the moon was 2 ½ cubits behind Leonis […] Night of the 17th, last part of the night, the moon stood 1 ½ cubits behind Mars, Venus was below."

- Babylonians collected the earliest evidence of periodic planetary phenomena
- Applied their mathematics for systematic astronomic predictions

"periodic" planetary phenomena.

systematic astronomic predictions



"Limited" Resources

An old history of time series analysis: Babylonian astronomical diaries

Ground based + Space based telescopes

⇒ New knowledge every night.

Nowadays, thousands of ground-based and space-based telescopes[a] generate new knowledge every night.

Still math tools & statistic tools are important.

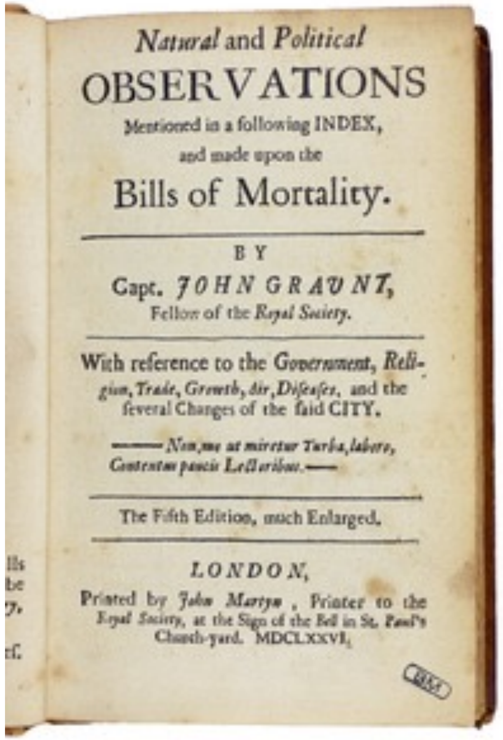
- The Vera C. Rubin Observatory in Chile is geared up to collect 20 terabytes per night from 2022[b].
- The Square Kilometre Array, the world's largest radio telescope, will generate up to 2 petabytes daily, starting in 2028.
- The Very Large Array (ngVLA) will generate hundreds of petabytes annually.

far more than what humans can process.

(a) https://research.arizona.edu/stories/space-versus-ground-telescopes
(b) https://www.nature.com/articles/d41586-020-02284-7

An old history of time series analysis: The Birth of Epidemiology

Observations.


Natural and Political
OBSERVATIONS
Mentioned in a following INDEX,
and made upon the
Bills of Mortality.
BY
Capt. JOHN GRAUNT,
Fellow of the Royal Society.
With reference to the Government, Religion, Trade, Growth, Air, Diseases, and the several Changes of the said CITY.
—— Non, me ut miretur Turba, laboro,
Contentus paucis Lectoribus. ——
The Fifth Edition, much Enlarged.
LONDON,
Printed by John Martyn, Printer to the Royal Society, at the Sign of the Bell in St. Paul's Church-yard. MDCLXXVI.

1662, John Graunt describes the data collection:

"When anyone dies, […] the same is known to the Searchers, corresponding with the said Sexton. The Searchers hereupon...examine by what Disease, or Casualty the corps died. Hereupon they make their Report to the Parish-Clerk, and he, every Tuesday night, carries in an Accompt of all the Burials, and Christnings, hapning that Week, to the Clerk of the Hall."

Collection in a systematic way.

Allows repeatability of Experiment

An old history of time series analysis: The Birth of Epidemiology

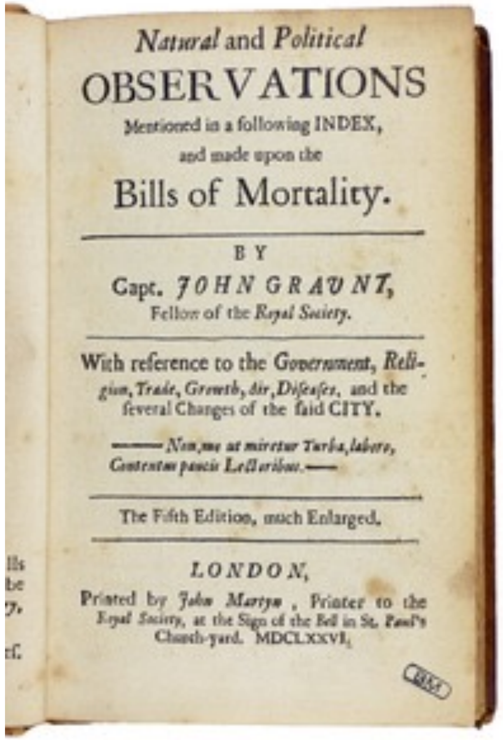
1662, John Graunt describes the data collection:

"When anyone dies, […] the same is known to the Searchers, corresponding with the said Sexton. The Searchers hereupon...examine by what Disease, or Casualty the corps died. Hereupon they make their Report to the Parish-Clerk, and he, every Tuesday night, carries in an Accompt of all the Burials, and Christnings, hapning that Week, to the Clerk of the Hall."

Basic / Baseless Conclusions ?

- Rudimentary conclusions about the mortality and morbidity of certain diseases
- Graunt's work is still used today to study population trends and mortality

Still in usage.



Importance of time series

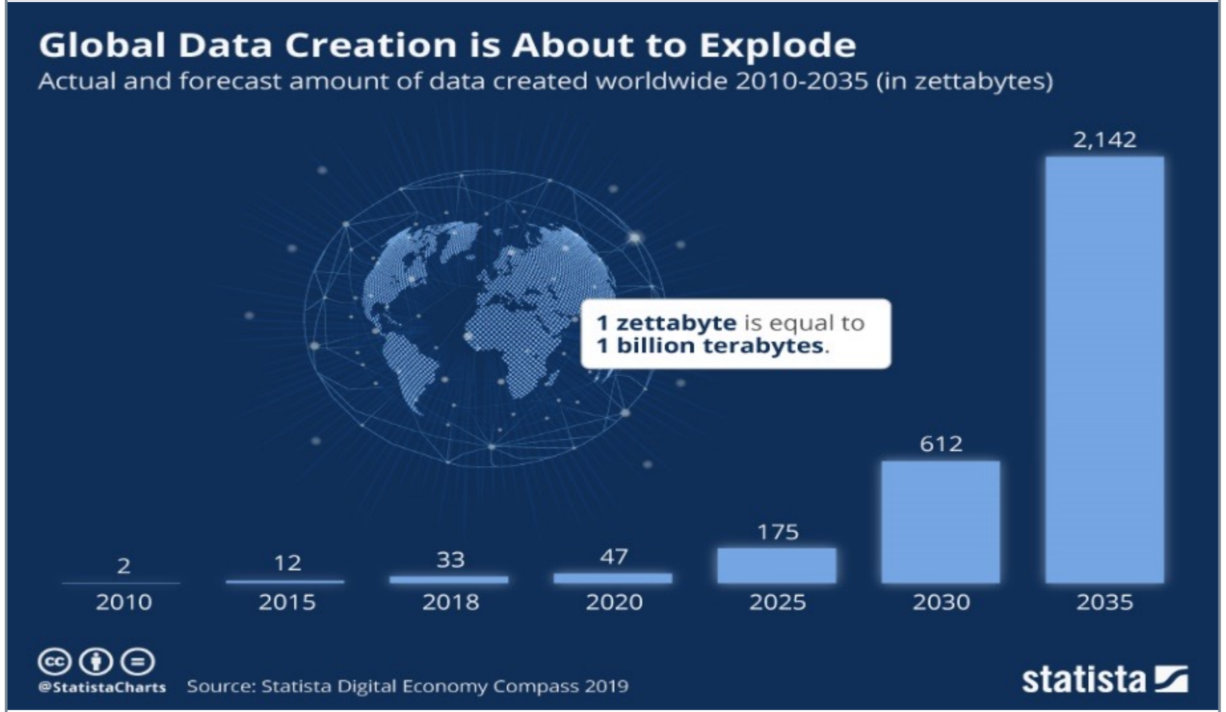
Massive production of data.

Machine learning on time series is becoming increasingly important because of the massive production of time series data from diverse sources, e.g.,

- Digitalization in healthcare
- Internet of things
- Smart cities
- Process monitoring

Explosion of time series data.

The amount of created data increased from two zettabytes in 2010 to 47 zettabytes in 2020



https://www.statista.com

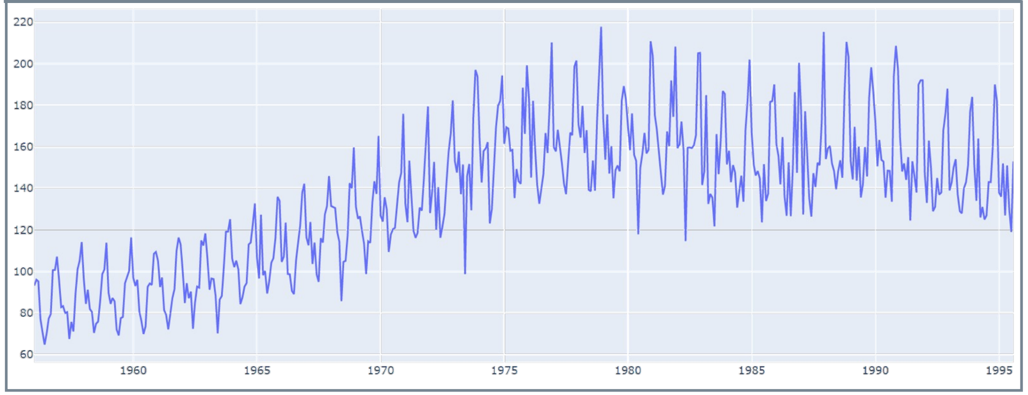
Example: Predicting demand of amazon products

Amazon sells 400 million products in over 185 countries[a].

- ➢ Maintaining surplus inventory levels for every product is cost-prohibitive.
- ➢ Predict future demand of products

Amazon researchers tried to predict "predictive Shipping"

Some products are sold depending on the season



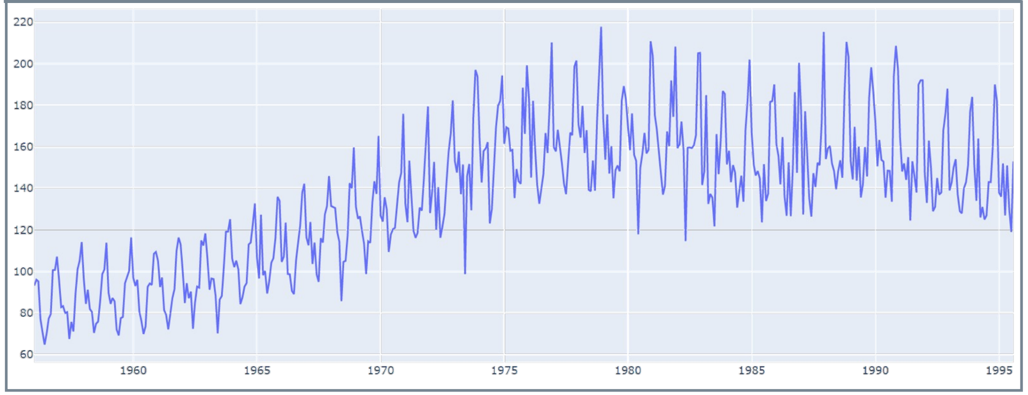
Example: Predicting demand of amazon products

Seasonal products ?

Amazon sells 400 million products in over 185 countries[a].

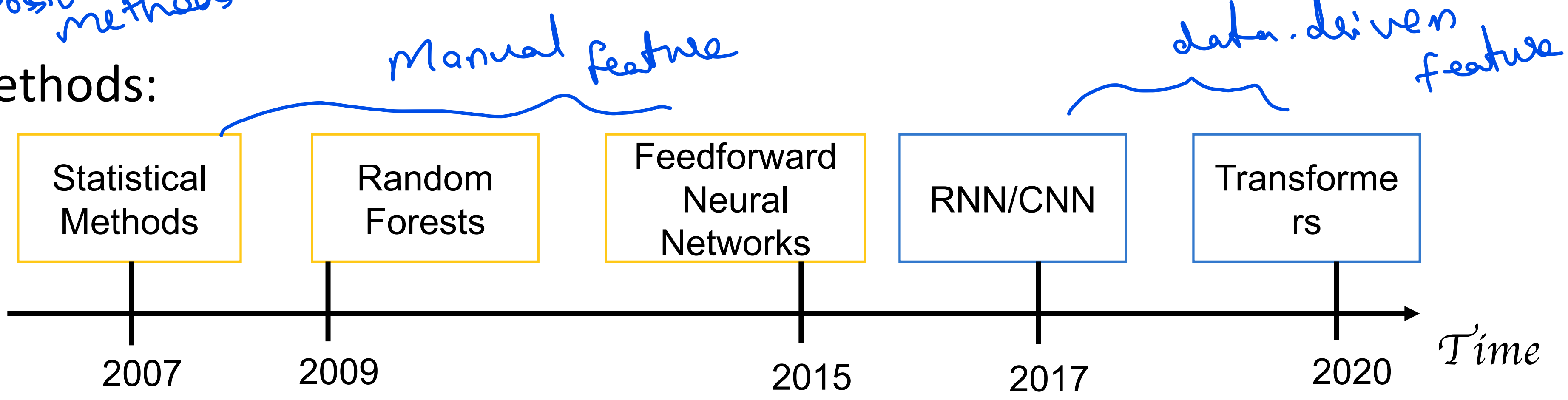
- Maintaining surplus inventory levels for every product is cost-prohibitive.
- Predict future demand of products

Some products are sold depending on the season



All possible methods.

Methods:



- □ First models required manual feature engineering
- □ New methods are fully data-driven

Example: Google Duplex makes tedious phone calls

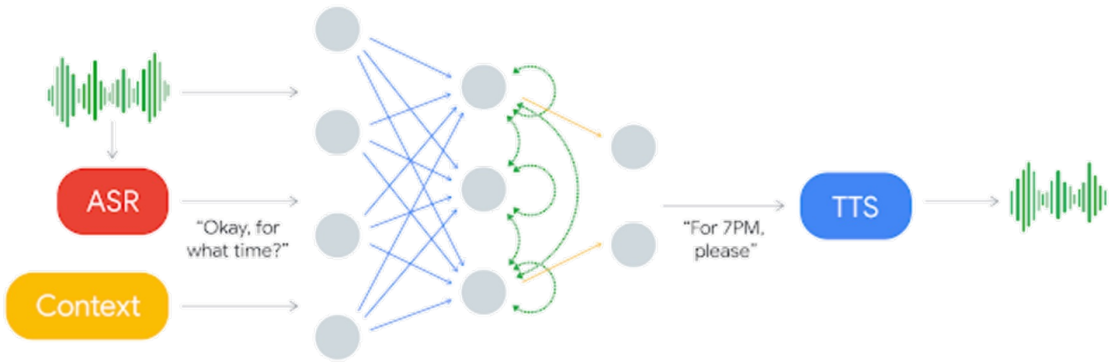
Long standing goal of making humans having a natural conversation with machines,as they would with each other.

➢ Carry out real-world tasks over the phone

real world

Conversations.

Natural Conversations.



■ Additional audio features
■ Automatic speech recognition
■ Desired service, time/day

Language-Generation
+ Voice Synthesizer,

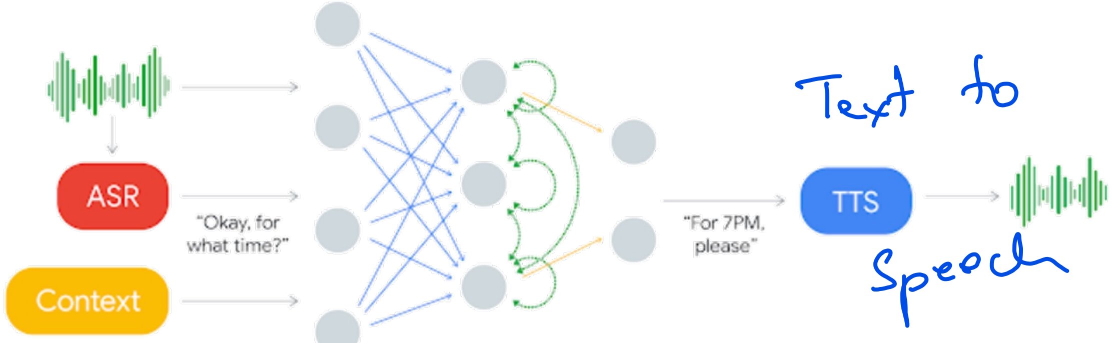
E.g., Duplex calling a restaurant.

https://ai.googleblog.com/2018/05/duplex-ai-system-for-natural-conversation.html

Example: Google Duplex makes tedious phone calls

Method: An RNNs with several features. We use a combination of text to speech (TTS) engine and a synthesis TTS engine to control intonation (e.g., “hmm”s and “uh”s).

(natural way)



Text to Speech Engine.

- Additional audio features
- Automatic speech recognition
- Desired service, time/day

Limitations: trained on specific tasks. Cannot deal general conversations.

Problem Specific
NOT Generic.

E.g., Duplex calling a restaurant.

https://ai.googleblog.com/2018/05/duplex-ai-system-for-natural-conversation.html

Own laboratory.

Example: Activity recognition in sports (FAU Erlangen)

Many injuries in sports are caused by overuse.

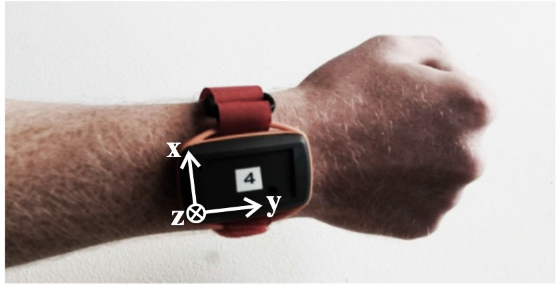
- These injuries are a major cause for reduced performance of professional and non-professional beach volleyball players.
- Monitoring of player actions could help identifying and understanding risk factors and prevent such injuries.

— Professional

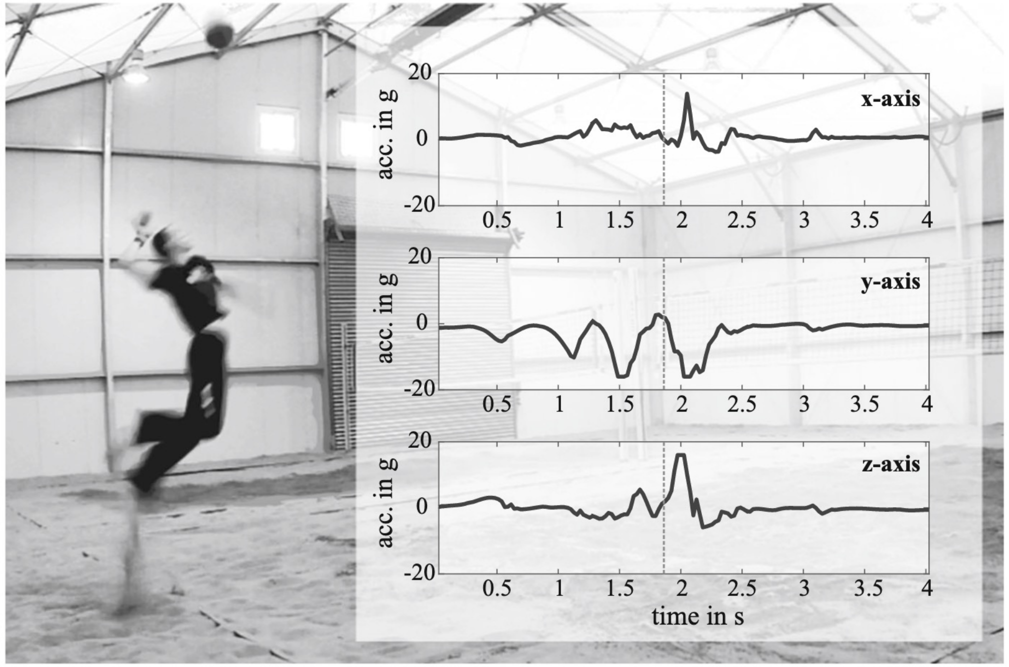
— Non-professional.

Injuries

Causes.



Sensor attachment at thewrist of the dominant hand witha soft, thin wristband



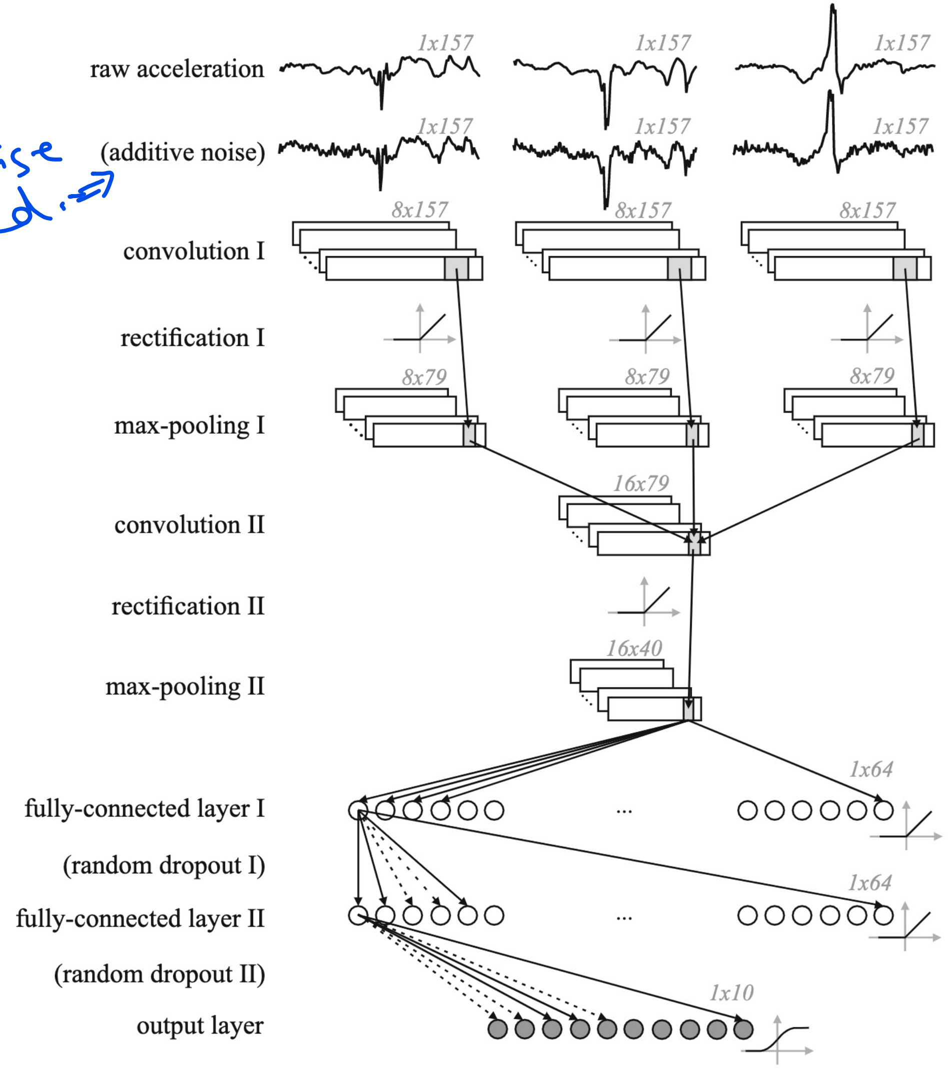
https://doi.org/10.1007/s10618-017-0495-0

Example: Activity recognition in sports (FAU Erlangen)

Method: A CNN is used to classify players' activities. Classifications allow to create players' profiles.

CNN - classify.

Noise added.



Actions:

- Underhand serve
- Overhand serve
- Jump serve
- Underarm set
- Overhead set
- Shot attack
- Spike
- Block
- Dig
- Null class.

https://doi.org/10.1007/s10618-017-0495-0

Time series fundamentals

Definitions and basic properties

formal definitions.

FAU

What is a time series?

set of observations taken sequentially in time.

A time series can be described as a set of observations, taken sequentially in time,

If independent observations => no prediction.

$$S = \{s_1, \dots, s_T\}$$

'd' dimensional vector space.

where $s_i \in \mathbb{R}^d$ is the measured state of the observed process at time t_i.

measured state of observed process at time ti

Typically, observations are generally $\boxed{\textit{dependent}}$ - typical - dependent observations.

- Studying the nature of this dependency is of particular interest
- Time series analysis is concerned with techniques for the analysis of these dependencies

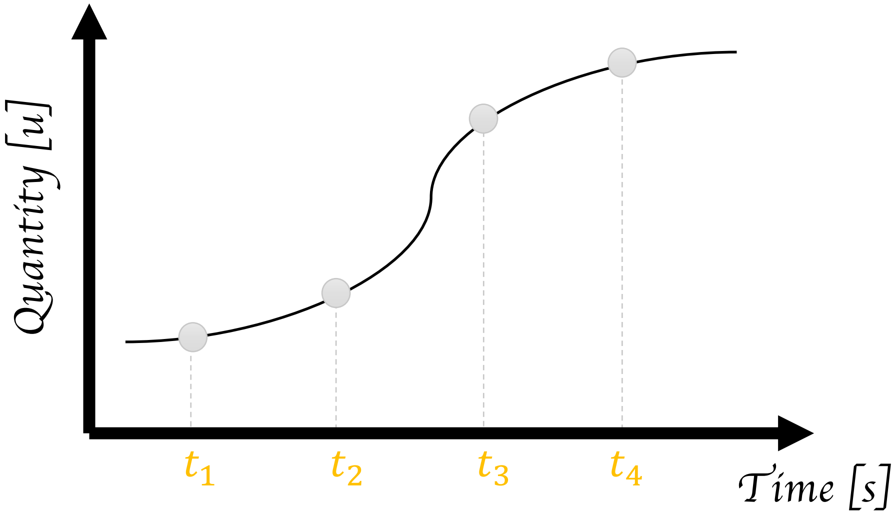
Analyse dependencies using techniques.

Terminology: Regularly Sampled vs Irregularly Sampled

Discrete time series are **regularly sampled** if their observations are euqally spaced in time.

$$\forall i \in \{1, \dots, T-1\},$$

$$\Delta_{t_i} = t_{i+1} - t_i = const.$$



Terminology: Regularly Sampled vs Irregularly Sampled

Equal time intervals.

system with precise clock.

Discrete time series are regularly sampled if their observations are euqally spaced in time.

(all times)

$$\forall i \in \{1, \ldots, T-1\},$$

$$\Delta_{t_i} = t_{i+1} - t_i = const.$$

Non equal times.

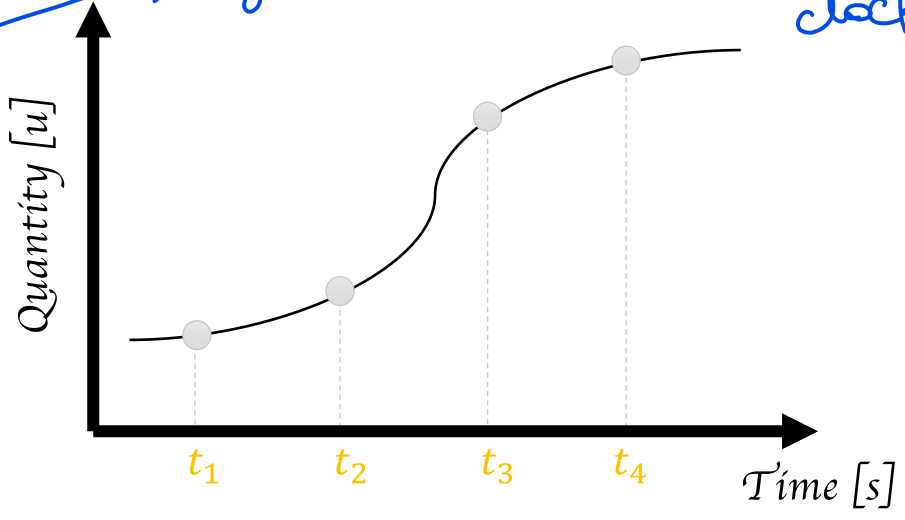
In contrast, for irregularly sampled time sequences, the observations are not equally spaced.

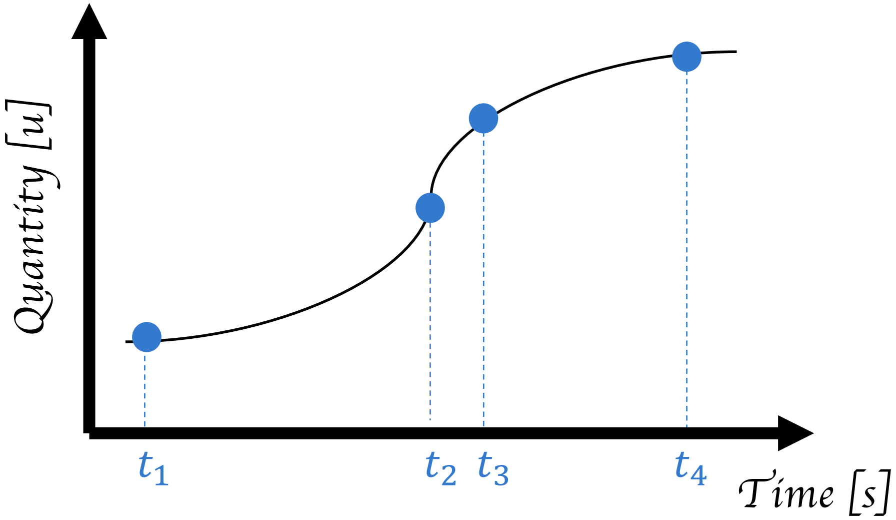
collection of pairs

- They are generally defined as a collection of pairs

sample, time

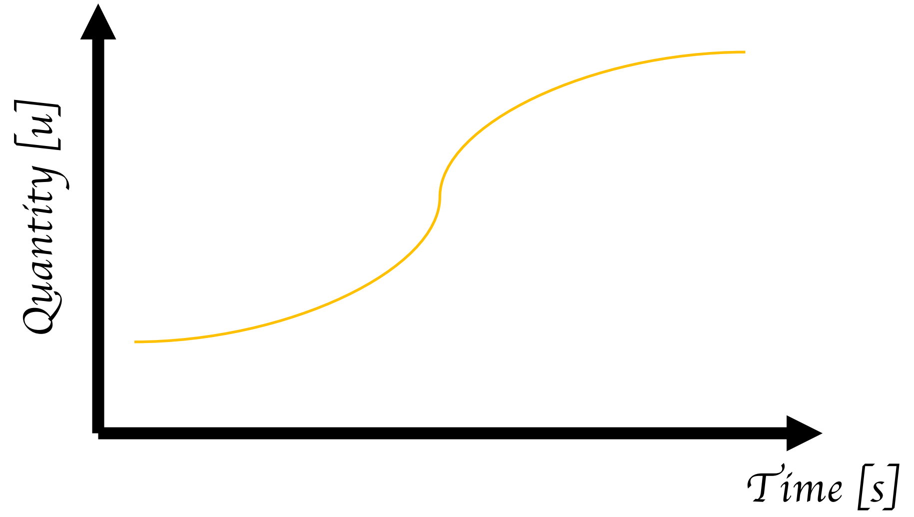
$$S = \{(s_1, t_1), \ldots, (s_T, t_T)\}$$





Terminology: Univariate vs Multivariate

Let $S = (s_1, \dots, s_T)$ be a time series,
where $s_i \in \mathbb{R}^d, \forall i \in \{1, \dots, T\}$.



If $d = 1$, S is said **univariate**.

- Only one variable is varying over time.

Time [s]

Terminology: Univariate vs Multivariate

Let $S = (s_1, \ldots, s_T)$ be a time series,
where $s_i \in \mathbb{R}^d, \forall i \in \{1, \ldots, T\}$.

One variable varying over time.

If $d = 1$, S is said univariate.

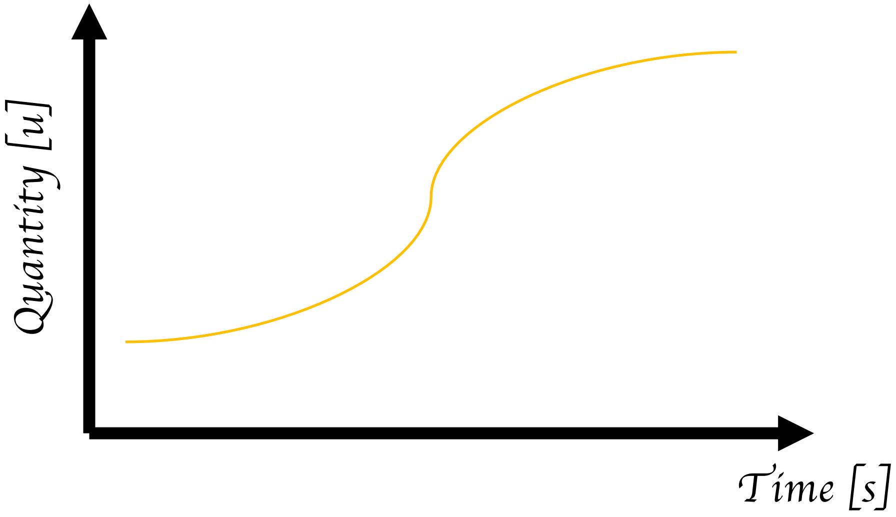
- Only one variable is varying over time.

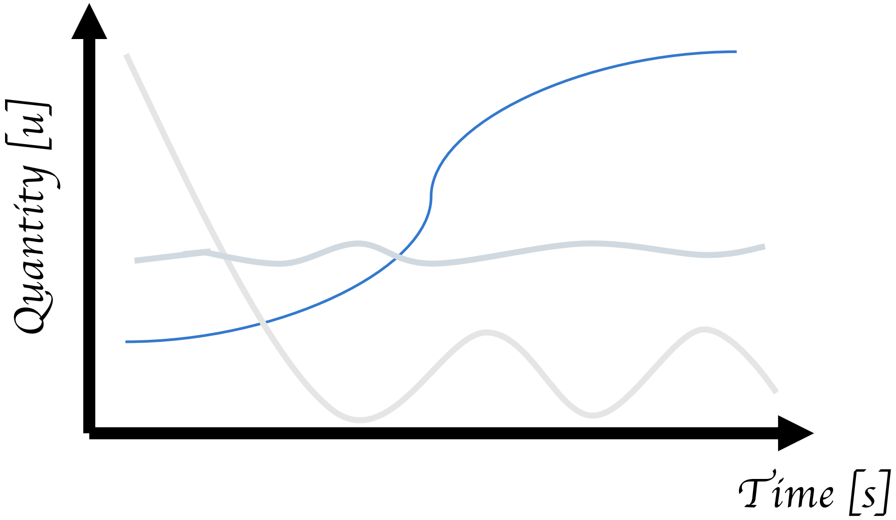
many variables vary over time

If $d > 1$, S is said multivariate.

- Multiple variables are varying over time
  - E.g., tri-axial accelerometer measurements

tri-axial





Terminology: Discrete vs Continuous

each instant of time.

A time series is said to be **continuous** if observations are made at each instant of time, even when its measurements consist only of a discrete set of values.

Irrespective of y 'x' continous.

- E.g., the number of people in a room.

Specific times / intervals.

A time series is said to be **discrete** if observations are taken at specific times. Discrete time series can arise in different ways:

Sample

Aggregate.

- Sampled (e.g., daily rainfall) – Going down
- Aggregated (e.g., monthly reports of daily rainfalls)

Going Up.

Terminology: Discrete vs Continuous

$d > 1$ (multiple variables are varying at a time)

We will denote as **mixed-type** a multivariate time series consisting of both continuous and discrete observations

- E.g., a time series consisting of continuous sensor values and discrete event log for the monitoring of an industrial machine

Sensor – continuous
logs – discrete.

Terminology: Periodic

period exists.

A time series is said **periodic** if there exists a number $\tau \in \mathbb{R}$, called *period*, such that

period.

If after some time; back to initial state. & repeat.

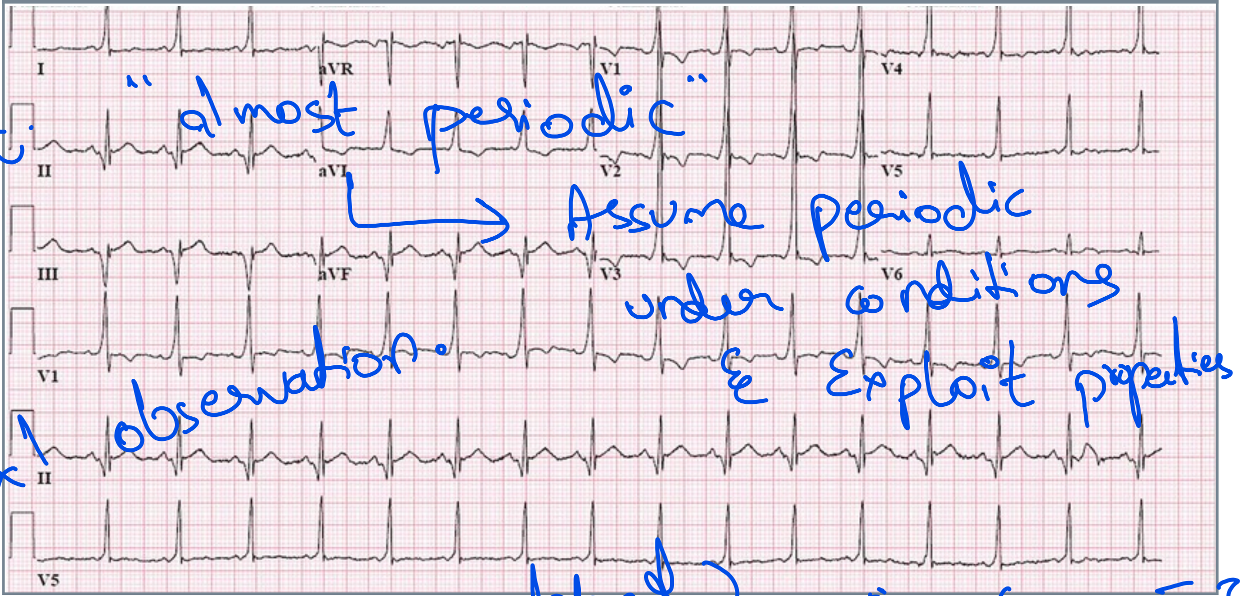
$$s_i = s_{i+\tau}\,, \forall i \in \{1, \ldots, T-\tau\}$$

Every τ observations; value gets repeated; not just initial observation.

E.g., the continuous time series defined by the trigonometric function $f(x) = \sin(x)$

Not exactly changing periodically.

Is the biological signal of an heartbeat a periodic function?



"almost periodic"

Assume periodic under conditions & Exploit properties.

(indexes; matched)

$\forall i \in \{1, \ldots T-\tau\}$

$S_{T-\tau} = S_{T-\tau+\tau}$

Terminology: Deterministic vs Non-Deterministic

A **deterministic** time series is one that could be expressed explicitly by an analytical expression.

System of Equations without randomness.

- Observations are generated from a system with no randomness.

In contrast, a **non-deterministic** time series can not be described by an analytic expression. A time series may be non-deterministic because :

↳ Arises in real world because of

- The information necessary to describe the process is not fully observable, or
- The process generating the time series is inherently random

NOT fully observable process.

↳ Random process generation.

Non-determinism understood as stochastic process

↳ Defined set of random variables.

Stochastic Process

Non-deterministic time series can be regarded as manifestations (equiv., realization) of a **stochastic process**, which is defined as a set of random variables $\{X_t\}_{t\in\{1,\ldots,T\}}$

Even if we were to imagine having observed the process for an infinite period T of time, the infinite sequence

old to New.

$$S = \{\ldots, s_{t-1}, s_t, s_{t+1,\ldots}\} = \{s_t\}_{t=-\infty}^{+\infty}$$

Stochastic process
mathematical
relization.

would still be a single **realization** from that process.

Stochastic process → ∞ Time period ???

Stochastic Process

Still, if we had a battery of N computers generating series $S^{(1)}, \ldots, S^{(N)}$, and considering selecting the observation at time t from each series,

{series of series}

$$\left\{s_t^{(1)}, \ldots, s_t^{(N)}\right\}$$

this would be described as a sample of N realizations of the random variable X_t

Drawing relationship b/w X_t (random variable)
and S_t (time series)

Stochastic Process

Still, if we had a battery of N computers generating series $S^{(1)}$, …, $S^{(N)}$, and considering selecting the observation at time t from each series,

$$\left\{s_t^{(1)}, \dots, s_t^{(N)}\right\}$$

this would be described as a sample of N realizations of the random variable X_t

This random variable X_t is associated with an **unconditional density**, denoted by

function of time series.

$$f_{X_t}(s_t) \longrightarrow \text{unconditional density.}$$

- E.g., for the Gaussian white noise process $f_{X_t}(s_t) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-s_t^2}{2\sigma^2}}$

Stochastic Process

It exists.

The **unconditional mean** is the expectation, provided it exists, of the t-th observation, i.e.,

$$E(X_t) = \int_{-\infty}^{+\infty} s_t \, f_{X_t}(s_t) ds_t = \mu_t$$

Expectation of $(X_t - \mu_t)^2$

Similarly, the **variance** of the random variable X_t is defined as

$$E(X_t - \mu_t)^2 = \int_{-\infty}^{+\infty} (s_t - \mu_t)^2 \, f_{X_t}(s_t) ds_t$$

Superscript is for ith computer.

Stochastic Process

Given any particular realization $S^{(i)}$ of a stochastic process (i.e., a time series), we can define the vector of the $j + 1$ most recent observations

old → New.

Randomvariable at time t

from ith computer

$$= \underbrace{\left[s^{(i)}_{t-j} \; \ldots \; s^{(i)}_{t} \right]}_{j+1 \text{ recent observations}}$$

$$x_t^{\mathrm{i}} = [s_{t-j}^{(i)}, \ldots, s_t^{(i)}]$$

We want to know the probability distribution of this vector x_t^{i} across realizations. We can calculate the ***j*-th autocovariance**

j-th Auto covariance

$$\gamma_{jt} = E(X_t - \mu_t)(X_{t-j} - \mu_{t-j})$$

Concept.

If neither μ_t or γ_{jt} ; t

Stationarity

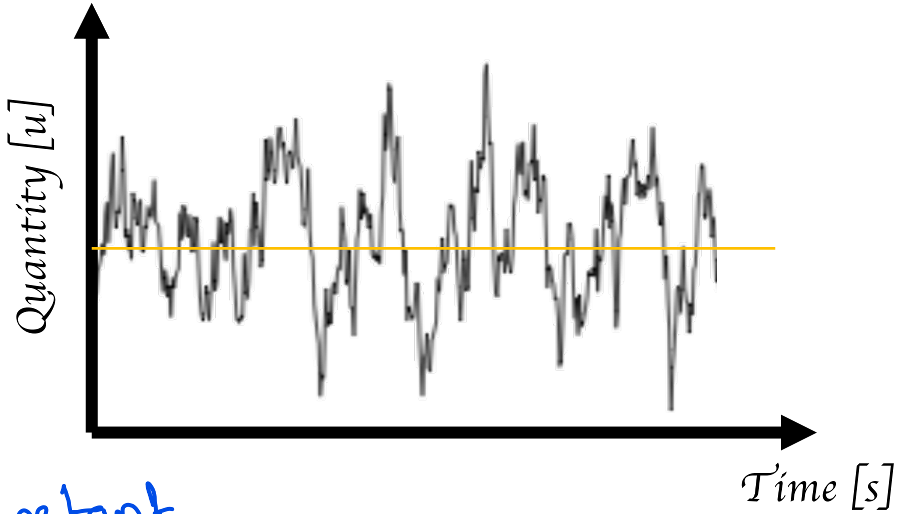
If neither the mean μ_t or the autocovariance γ_{jt} depend on the temporal variable t, then the process is said to be $\boxed{\text{(weakly) } \textbf{stationary}.}$

weakly stationary.

Sum of const ; Gaussian.

E.g., let the stochastic process $\{X_t\}_{t=-\infty}^{+\infty}$ represent the sum of a constant μ with a Gaussian white noise process $\{\epsilon_t\}_{t=-\infty}^{+\infty}$, such that

$$X_t = \mu + \epsilon_t$$



zero mean

Then, its mean is constant: $E(X_t) = \mu + E(\epsilon_t) = \mu$

and its j-th autocovariance: $E(X_t - \mu)(X_{t-j} - \mu) = \gamma_j$

Auto Covariance

of Gaussian noise.

constant variance.

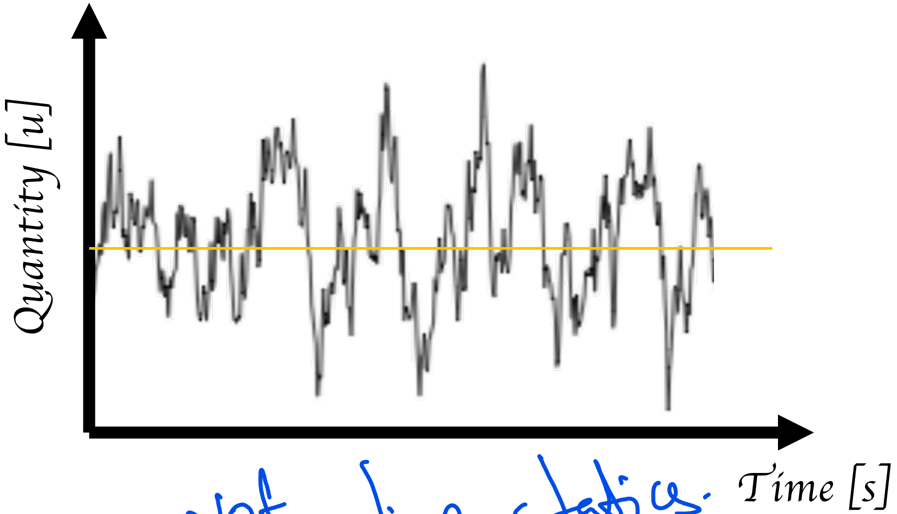
Stationarity

Stationary process.

If neither the mean μ_t or the autocovariance γ_{jt} depend on the temporal variable t, then the process is said to be (weakly) **stationary**.

E.g., let the stochastic process $\{X_t\}_{t=-\infty}^{+\infty}$ represent the sum of a constant μ with a Gaussian white noise process $\{\epsilon_t\}_{t=-\infty}^{+\infty}$, such that

$$X_t = \mu + \epsilon_t$$



Then, its mean is constant: $E(X_t) = \mu + E(\epsilon_t) = \mu$

and its j-th autocovariance: $E(X_t - \mu)(X_{t-j} - \mu) = \gamma_j$

Not time statics.

In other words: A process is said to be stationary if the process statistics do not depend on time.

Ergodicity

Given a time series, denoted by $S^{(i)} = \left\{s_1^{(i)}, \dots, s_T^{(i)}\right\}$, we can compute the sample temporal average as

Average over time.

$$\bar{s} = \frac{1}{T}\sum_{t=1}^{T} s_t^{(i)}$$

The ergodicity of a time series bind the concept of the process mean with that of temporal sample mean:

- A process is said to be ergodic if $\bar{s}$ converges to μ_t as $T \rightarrow \infty$

Ergodicity

Given a time series, denoted by $S^{(i)} = \left\{s_1^{(i)}, \ldots, s_T^{(i)}\right\}$, we can compute the sample temporal average as

Average over time period.

$$\bar{s} = \frac{1}{T}\sum_{t=1}^{T} s_t^{(i)}$$

The ergodicity of a time series bind the concept of the process mean with that of temporal sample mean:

temporal sample mean ⇒ process mean.

- A process is said to be ergodic if $\bar{s}$ converges to μ_t as $T \to \infty$

Unconditional mean of density.

"Process repeats itself ???" after time.

In other words: A process is said to be ergodic if its time statistics equals the process statistic, provided that the process is observed long enough.

observed over long time.

Example: Stationarity and Ergodicity

To clarify the concept, we give an example of stationary but not ergodic process. Suppose the mean $\mu^{(i)}$ of the i-th realization of $\{X_t\}_{t=-\infty}^{+\infty}$ is sampled from the normal distribution $U(0, \lambda^2)$ and, similarly to the previous example, $X_t^{(i)} = \mu^{(i)} + \epsilon_t$.

Stationary not ergodic

We have that the process is stationary because:

process statistics

$-\mu_t, \gamma_{jt}$

Normal process

+ Normal noise

$$\mu_t = E\left(\mu^{(i)}\right) + E(\epsilon_t) = 0$$

$$\gamma_{jt} = E\left(\mu^{(i)} + \epsilon_t\right)\left(\mu^{(i)} + \epsilon_{t-j}\right) = \lambda^2$$

independent of time t.

Example: Stationarity and Ergodicity

However, its sample temporal mean, converges to a different value than the process mean, i.e.,

$$\bar{s} = (1/T)\sum(\mu^{(i)}+\epsilon_t) = \mu^{(i)}$$

Mean of temporal samples computed by i^{th} Computer.

Time series fundamentals

i.i.d. observations and central limit theorems

Time series and i.i.d. data

Observations collected in a time series $S = (s_1, \ldots, s_T)$ are **generally not i.i.d.**

- Observation s_i could be **dependent** on previous observations s_j, with $j < i$
- The distribution of the underlying data generation process could change over time, i.e. it is **not identically distributed**

independent identically distributed.

Time series and i.i.d. data

Observations collected in a time series $S = (s_1, \dots, s_T)$ are **generally not i.i.d.**

past observations.

- Observation s_i could be **dependent** on previous observations s_j, with $j < i$
- The distribution of the underlying data generation process could change over time, i.e. it is **not identically distributed**

could change overtime

Some previous data one process other latest data other process.

For example:

dependency ; change in underlying

- The price of a stock today depends on its price yesterday **(dependence)**
- and the volatility of the stock, i.e., its dispersion of returns, might change over time **(change on the underlying distribution)**

Time series and i.i.d. data

The structure of this dependence imposes challenges on the statistical data analysis of time series.

- Many tools for statistical inference are valid only for i.i.d. data

i.i.D → statistical inference

Time series and i.i.d. data

It might be useful to be able to assess the structure of the dependence between random variables. For this reason we make use of their correlation.

- Generally, we measure the correlation between two variables X_i and X_j with their **covariance** $Cov(X_i, X_j)$.

Covariance = 0 → uncorrelated.

  - $Cov(X_i, X_j) = 0 \rightarrow \text{uncorrelated}$

- We measure dependence of an entire time series with a similar concept, the **long-run variance**

long-run variance.

= Σ all covariances.

  - $\sigma_i^2 = \sum_{\mathbb{Z}} Cov(X_i, X_{i+h})$

The Central Limit Theorem

The **Central Limit Theorem (CLT)** suggests that the sum of random variables converges to a normal distribution, under precise conditions.

More precisely, for a sequence of i.i.d. random variables $\{X_t\}_{t \in \{1,\ldots,\mathrm{T}\}}$ with $\mu = E(X_t)$ and $\sigma^2 = E(X_t - \mu)^2$, by the CLT it holds:

$$\sqrt{T}\left(\frac{1}{T}\sum_{1}^{T} X_i - \mu\right) \to \mathcal{N}(0, \sigma^2)$$

The Central Limit Theorem

The **Central Limit Theorem (CLT)** suggests that the sum of random variables converges to a normal distribution, under precise conditions.

↳ what condition?

More precisely, for a sequence of i.i.d. random variables $\{X_t\}_{t\in\{1,\ldots,T\}}$ with $\mu = E(X_t)$ and $\sigma^2 = E(X_t - \mu)^2$, by the CLT it holds:

$$\sqrt{T}\left(\frac{1}{T}\sum_{1}^{T} X_i - \mu\right) \to \mathcal{N}(0, \sigma^2)$$

long-run variance

left converges to Normal distribution.

process statistics do not depend on time

For stationary time series with mean μ and long-run variance σ^2 the CLT holds as before.

Why is the CLT important?

If the CLT holds for a time series, we can draw from a larger range of methods.

- Statistical inference depends on the possibility to take a broad view of results from a sample to the population.
- The CLT legitimizes the assumption of normality of the error terms in linear regression.

However,

- Many time series we encounter in the real world satisfy CLT assumption of independence and stationarity
- Or can be transformed into stationary time series, e.g., by differentiations or other transformations

Why is the CLT important?

CLT satisfied? → many doors open

If the CLT holds for a time series, we can draw from a larger range of methods.

- Statistical inference depends on the possibility to take a broad view of results from a sample to the population.
- The CLT legitimizes the assumption of normality of the error terms in linear regression.

However,

Some satisfy | Some do not

- Many time series we encounter in the real world satisfy CLT assumption of independence and stationarity
- Or can be transformed into stationary time series, e.g., by differentiations or other transformations

check process statistics

It is a good idea to start by checking whether the data is independent or stationary.

Insight: CLT for dependent random variables

Different version of the CLT exist for dependent random variables. For example, under the assumption of a M-dependent random process[a], we have that the following limit theorem holds:

Let $\{X_t\}_{t\in\{1,\dots,T\}}$ be M-dependent stationary process with mean μ, covariance γ_j, and denoted with V_M the variance of the mean of n observations,

Variance of mean

$$V_M := \sum_{j=-M}^{M} \gamma_j$$

M dependent stationary process.

depdt k independt M+1

'Converges to N'

If $V_M > 0$, then,

$$\sqrt{n}(X_i - \mu) \to N(0, V_M).$$

independt $\geq M+1$

[a] A stochastic process $\{X_t\}_{t\in\{1,\dots,T\}}$ is said to be M-dependent if $\{X_t\}_{t\leq k}$ are independent of the stochastic variables $\{X_t\}_{t\geq k+M+1}$

conditions on data lost ?

statistical tools lost

sense of time dependencies

Make

Time series fundamentals
Recap

Recap

Time series have long been studied in history

- Recent digitalization increases the importance of time series analysis

Recap

Time series have long been studied in history

- Recent digitalization increases the importance of time series analysis

Properties of time series

- Regularly vs irregularly sampled
- Univariate vs multivariate
- Discrete vs continuous
- Periodic
- Deterministic vs non-deterministic
- Stationarity
- Ergodicity

Recap

CLT only for stationary.

Time series have long been studied in history

- Recent digitalization increases the importance of time series analysis

Properties of time series

- Regularly vs irregularly sampled
- Univariate vs multivariate
- Discrete vs continuous
- Periodic
- Deterministic vs non-deterministic
- Stationarity
- Ergodicity

Central limit theorem only holds for stationary time series

- Less restrictive CLT versions exist
- Need to properly learn dependences