Machine Learning for Time Series Exercise

Richard Dirauf, M.Sc.
Machine Learning and Data Analytics (MaD) Lab
Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU)
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MLTS Exercise

Fridays, 12:15-13:45 via Zoom

Recordings of last years exercises uploaded to FAU TV

Five topics with two exercise sessions each:

- Session 1: Recap of topic and introduction of coding task
- Session 2: Solution to coding task and questions

Recommended: solve coding task as homework

Slides and tasks uploaded on StudOn

Questions: Exercises or Forum

What is your major?

Python experience?

MLTS Exercises

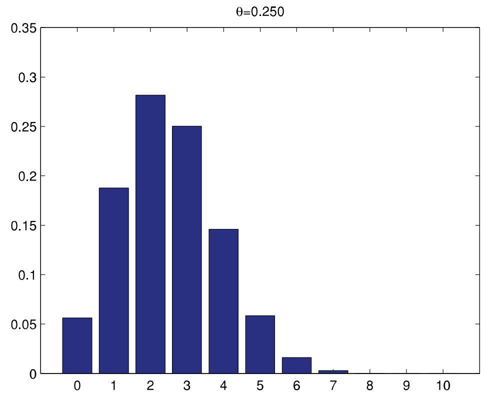
Basics

Probability distributions

MLTS Exercise 01

- Discrete

  probability mass function (pmf)



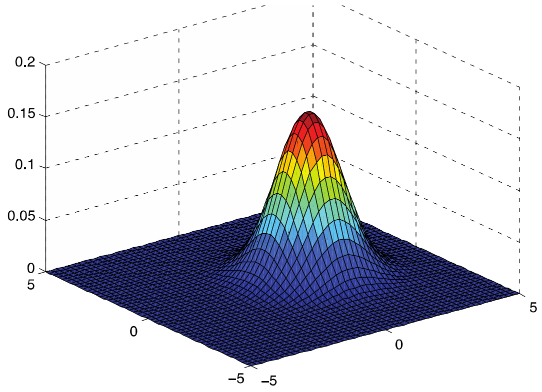
Binomial distribution

- Continuous

  probability density function (pdf)

$$0 \leq p(x) \leq 1$$

$$\int_{-\infty}^{+\infty} p(x)dx = 1$$



Gaussian (normal) distribution

Mean or expected value

MLTS Exercise 01

Mean or **expected value** of a distribution

Discrete distribution $\mathbb{E}[X] \triangleq \sum_{x \in \mathcal{X}} x \; p(x)$

Continuous distribution $\mathbb{E}[X] \triangleq \int_{\mathcal{X}} x \; p(x) dx$

The expected value of function f

Discrete distribution $\mathbb{E}[f] \triangleq \Sigma_{x \in \mathcal{X}} f(x) p(x)$

Continuous distribution $\mathbb{E}[f] \triangleq \int_{\mathcal{X}} f(x) p(x) dx$

Variance

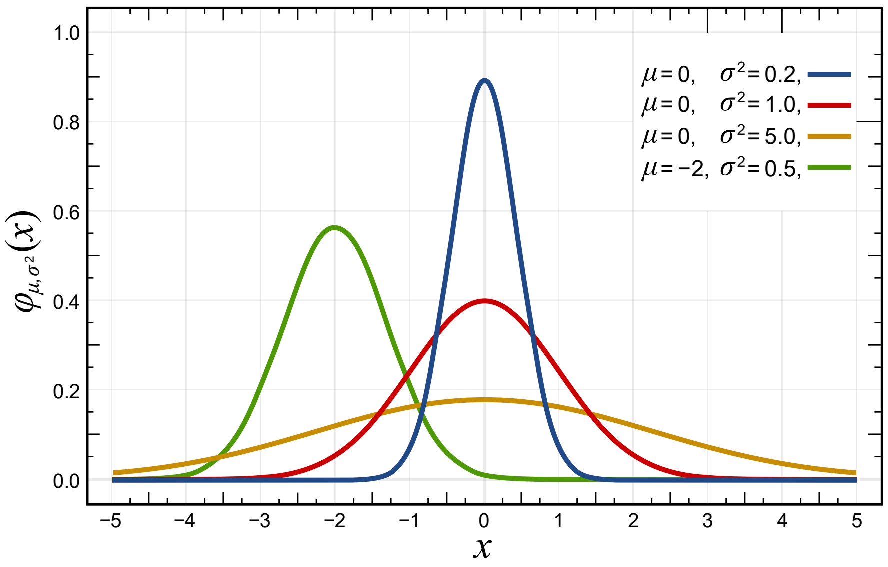
MLTS Exercise 01

The **variance** is a measure of the spread of a distribution

$$\begin{aligned}\operatorname{var}[X] &\triangleq \mathbb{E}\left[(X-\mu)^2\right] = \int (x-\mu)^2 p(x)dx \\ &= \int x^2 p(x)dx + \mu^2 \int p(x)dx - 2\mu \int x p(x)dx = \mathbb{E}\left[X^2\right] - \mu^2\end{aligned}$$

The standard deviation is defined as

$$\operatorname{std}[X] \triangleq \sqrt{\operatorname{var}[X]}$$



https://de.wikipedia.org/wiki/Normalverteilung

Covariance

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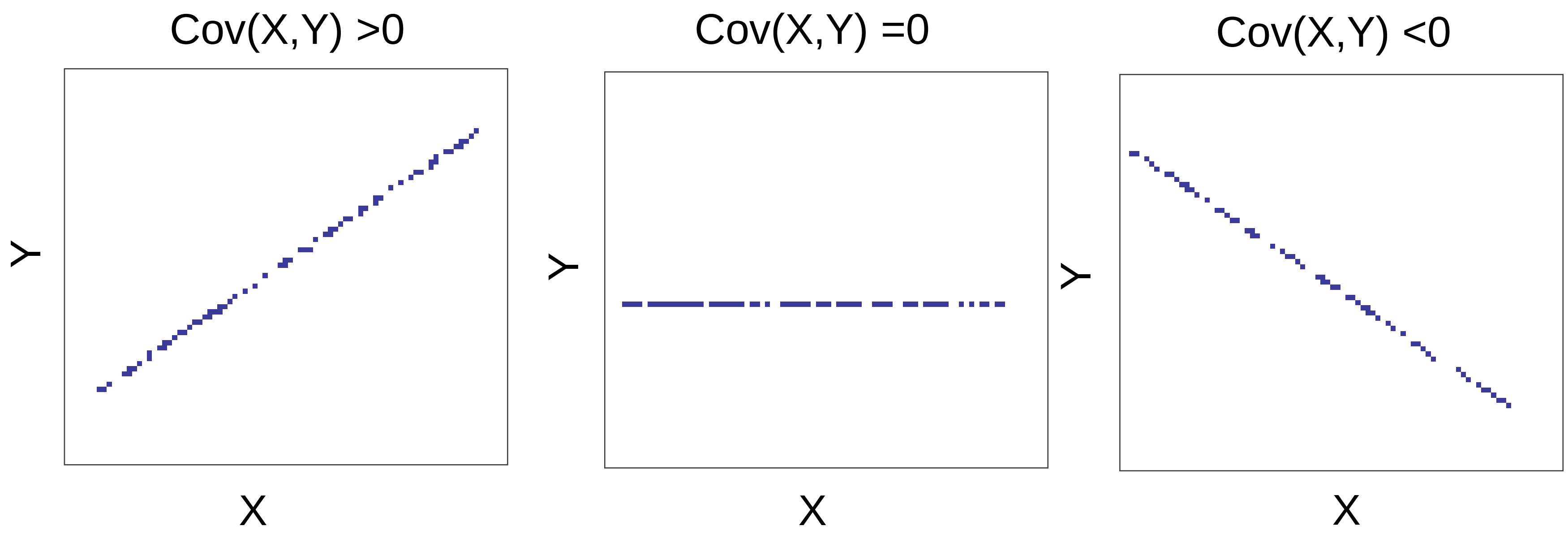
$$\mathrm{cov}\left[X,Y\right] \triangleq \mathbb{E}\left[(X-\mathbb{E}\left[X\right])(Y-\mathbb{E}\left[Y\right])\right] = \mathbb{E}\left[XY\right] - \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right]$$



Basic rules of probability

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- Probability of the **joint distribution** X and Y as follows

$$p(X, Y) = p(X|Y)p(Y)$$

this is sometimes called the **product rule**.

- We define the **marginal distribution** as follows

$$p(X) = \sum_y p(X, Y) = \sum_y p(X|Y = y)p(Y = y)$$

where we are summing over all possible states of Y.

This is sometimes called the **sum rule**.

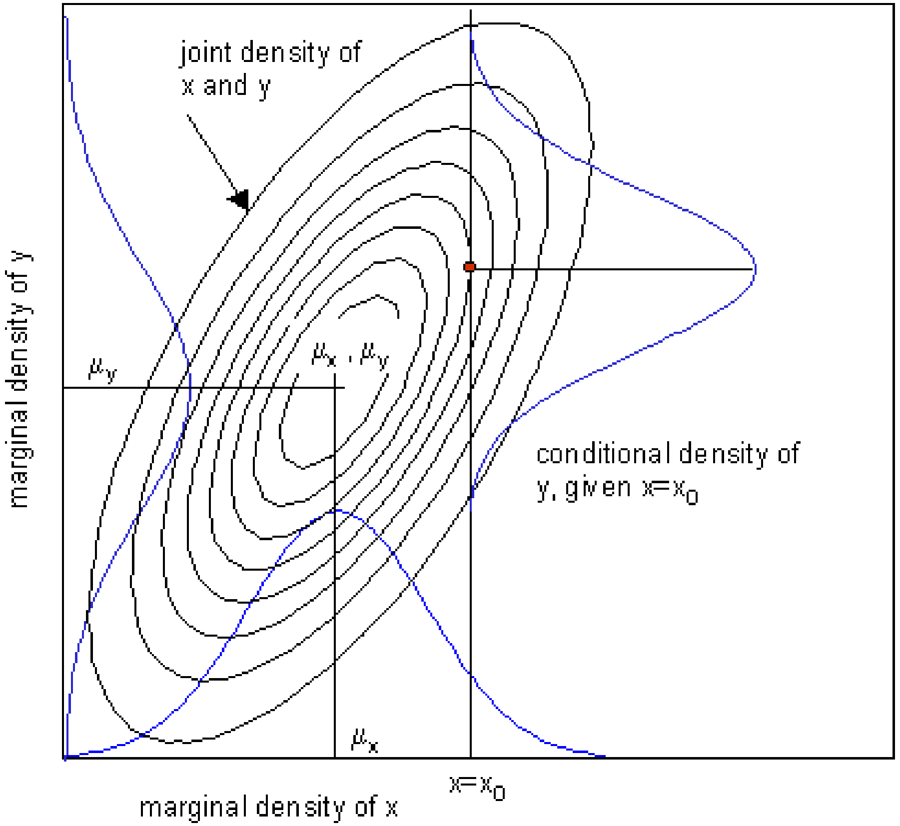
- The product rule can be applied multiple times to yield the chain rule of probability:

$$p(X_{1:D}) = p(X_1)p(X_2|X_1)p(X_3|X_2, X_1)p(X_4|X_1, X_2, X_3)\dots p(X_D|X_{1:D-1})$$

Concepts of joint, marginal, and conditional probabilities

$$x \sim \mathcal{N}(\mu_x, \sigma_x^2)$$

$$y \sim \mathcal{N}(\mu_y, \sigma_y^2)$$



https://statistical-engineering.com/probability-and-statistics/joint-marginal-conditional-distributions/

Bayes rule

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$$\overbrace{\boxed{p(X=x|Y=y)}}^{\text{Posterior probability}} = \frac{p(X=x,Y=y)}{p(Y=y)} = \frac{\overbrace{\boxed{p(X=x)}}^{\text{Prior probability}}\overbrace{\boxed{p(Y=y|X=x)}}^{\text{Likelihood}}}{\underbrace{\boxed{\sum_{x'} p(X=x')p(Y=y|X=x')}}_{\text{Marginal likelihood}}}$$

Thank you for your attention!

Any questions?