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SCHOOL OF ENGINEERING

Activation Functions and Convolutional Neural Networks

A. Maier, V. Christlein, K. Breininger, Z. Yang, L. Rist, M. Nau, S. Jaganathan, C. Liu, N. Maul, L. Folle, K. Packhäuser, M. Zinnen
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
April 24, 2023

Outline

Activation Functions

Convolutional Neural Networks

Convolutional Layers

Pooling Layers

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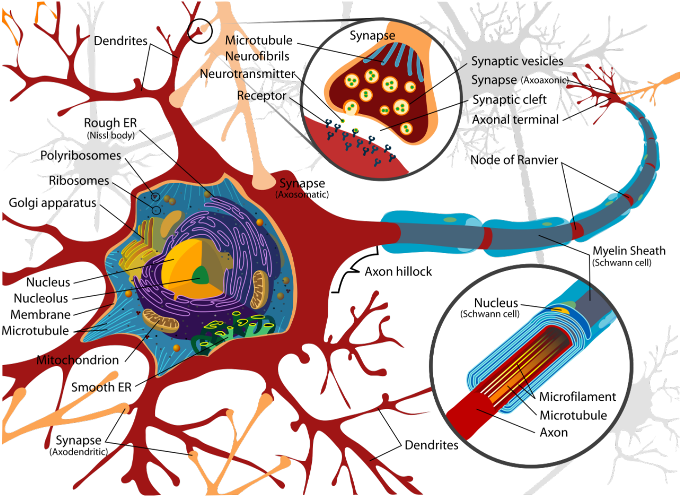
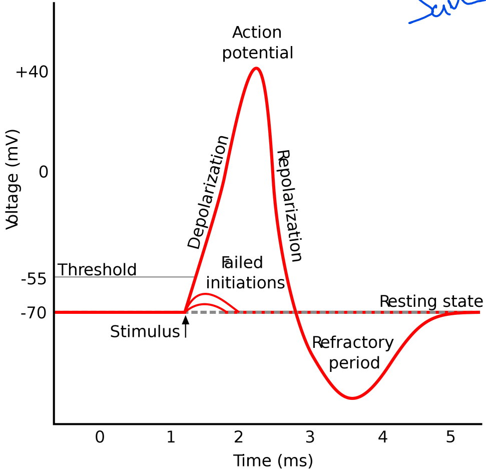
Activation Functions

Biological Activation



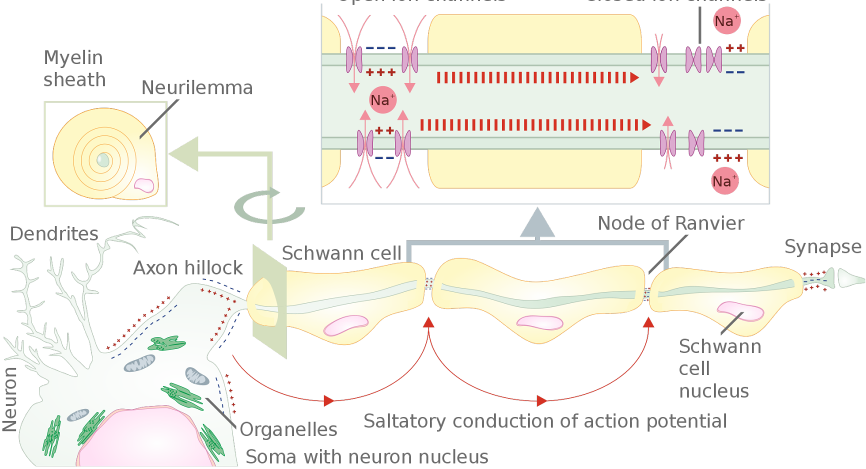
Biological Activation

Always
Same Action
potential



Biological Activation

stabilise



Summary - Activations in Biological Neurons

- The knowledge lies in the **connections**
- Both **inhibitory** and **excitatory** connections exist
- The synapses anatomically enforce **feed forward** processing
- However, the connections can be in any direction
- The **sum** of activations is crucial $\rightarrow a_i\ \Sigma > \Theta$ Threshold
- Activations are electric spikes
  - With $\boxed{\textbf{specified intensity}}$
  - Which also encode **information over time**

whole process Runs at a frequency.

Activations in Artificial Neural Networks so far

- Non-linear activation functions enable **universal function approximation**:
  ➔ Highly important for a powerful network

Activations in Artificial Neural Networks so far

without non-linearity → just matrix multiplication

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- Compared to biology:
  - ✓ Heaviside step (sign) function can model **all or nothing response**
  - ✗ Artificial activations have **no time component** (model by activation strength?)

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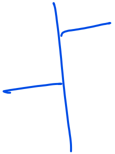
$$f'(x) = \begin{cases} \infty & \text{for } x = 0 \\ 0 & \text{else} \end{cases}$$



$\lightning$ back-propagation

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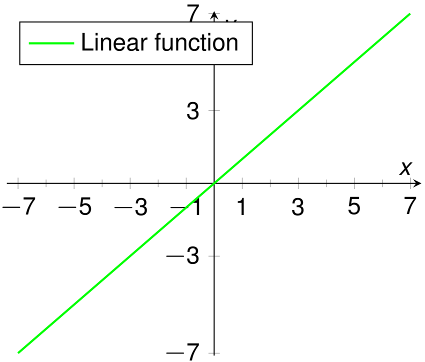
- So far: Sigmoid-function $f(x) = \frac{1}{1+exp(-x)}$

(Analytic Derivative is Easy)

- Can we do better?

$f(x)(1-f(x))$

Linear Activation Function



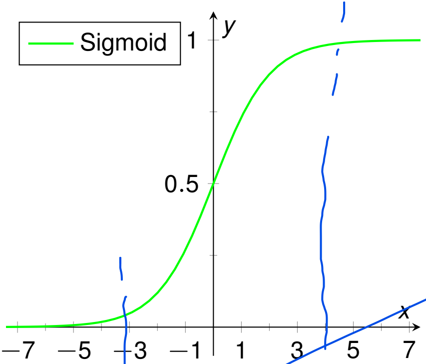
Linear function with parameter α:

$$f(x) = \alpha x$$
$$f'(x) = \alpha$$

Doesn't allow for DNN

- + Simple
- − Does not introduce non-linearity
- + Therefore, it renders the optimization problem convex
- • Listed here mainly for completeness

Sigmoid Activation Function



Sigmoid (logistic function)

$$f(x) = \frac{1}{1 + exp(-x)}$$

$$f'(x) = f(x)(1 - f(x))$$

- Close to biological model, but differentiable

\+ Probabilistic output

− Saturates for $x \ll 0$ and $x \gg 0$

− Not zero-centered

(farther from (-3, 3))
slow (ω)
saturation

Zero-Centering

- Sigmoid: $f : \mathbb{R} \mapsto]0, 1[$

Zero-Centering

- Sigmoid: $f : \mathbb{R} \mapsto]0, 1[$
- Output of activation always $+$
  - ➔ $\nabla_{\mathbf{w}}$ will either be all $+$ or all $-$

(Pushing to zero.)

Zero-Centering

Problem → always values b/w [0,1] positive

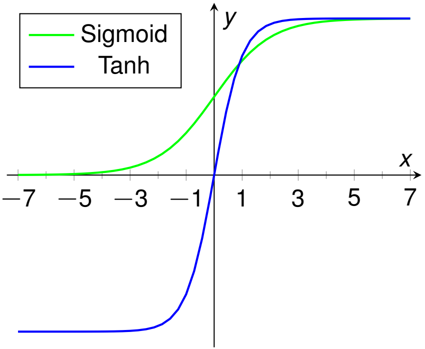
- Sigmoid: $f : \mathbb{R} \mapsto]0, 1[$
- Output of activation always $+$
  - ➔ $\nabla_{\mathbf{w}}$ will either be all $+$ or all $-$
- A mean $\mu = 0$ of the input distribution will always be shifted to $\mu > 0$
  - ➔ **co-variate shift** of successive layers
  - ➔ layers **constantly** have to **adapt** to the shifting distribution
- Batch learning reduces the variance σ of the updates

centered around mean

Cat light
Classify Example

Handled.

Tanh Activation Function



Tanh

$$f(x) = tanh(x)$$
$$f'(x) = 1 - f(x)^2$$

relationship.

(CS)

- \+ Zero-centered (LeCun '91)
- − Shifted version of sigmoid σ: $\mathtt{tanh}(x) = 2\sigma(2x) - 1$
  - ➔ Still saturates
  - ➔ Still causes vanishing gradients

(2, −2) − still exists

Vanishing and Exploding Gradients

- Essence of learning: How does **x** affect **y**?
- Sigmoid/tanh map **large regions** of **X** to a **small range** in **Y**

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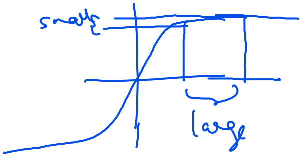

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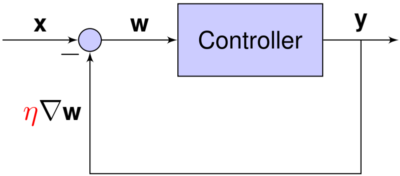
  - ➔ **gradient vanishes**

→ Deeper layers ; faster vanishing.

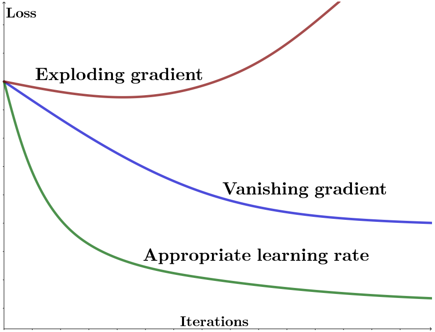
- Problem is amplified by back-propagation: Multiplication of small gradients
- Related problem: Exploding gradients

[-1,1] ~ y.

Recap: Feedback Loop - Vanishing and Exploding Gradients



Analogy to control theory



- If η is to high $\mapsto$ **positive feedback** $\mapsto$ loss grows **without bounds**
- If η is to small $\mapsto$ **negative feedback** $\mapsto$ **gradient vanishes**
- Choice of η is **critical** for learning

Can be done by activation functions as well

NEXT TIME

ON DEEP LEARNING

Activation Functions and Convolutional Neural Networks - Part 2

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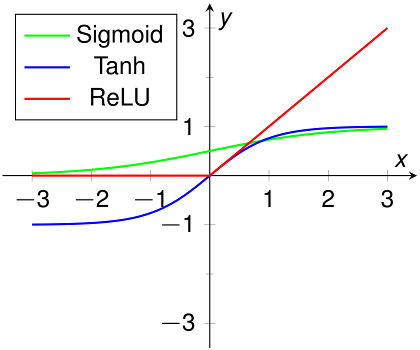
Most Common

Rectified Linear Units (ReLU)



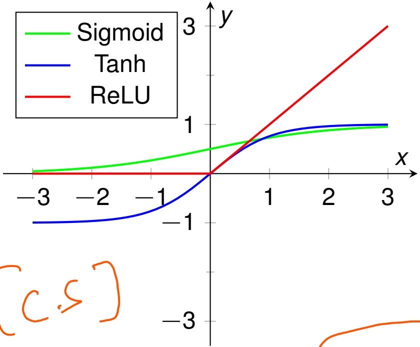
Rectified Linear Unit (ReLU)

$$f(x) = \max(0, x)$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{else} \end{cases}$$

- fast
- No vanishing gradient
- piecewise linear

Rectified Linear Units (ReLU)



[C.S]

Rectified Linear Unit (ReLU)

$$f(x) = \max(0, x)$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{else} \end{cases}$$

+ Good generalization due to piece-wise linearity
+ Speed up during learning (≈ 6x (Krizhevsky '12))
+ No vanishing gradient problem
− Not zero-centered

More on ReLUs

Earlier - more than 3 layers; Gradient vanishes

- ReLUs were a **big step forward**!
- ReLUs enable training of **deep** supervised neural networks **without unsupervised pre-training**
- First derivative is 1 if the unit is active, second derivative is 0 almost anywhere
  - → No second-order effects

Dying ReLUs

- Weight/biases trained to yield negative values for **any x**

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  - ➔ Our precious ReLU is "dead"

Dying ReLUs

Stops at this point ; b/c no updates

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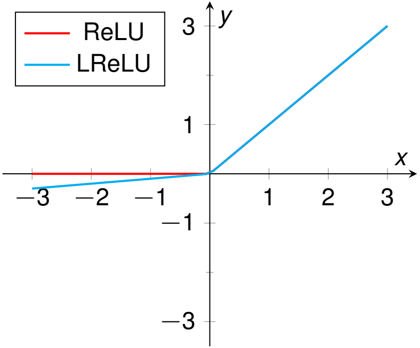
- Often related to a (too) high learning rate

Activation Function



Leaky ReLU / Parametric ReLU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha x & \text{else} \end{cases}$$

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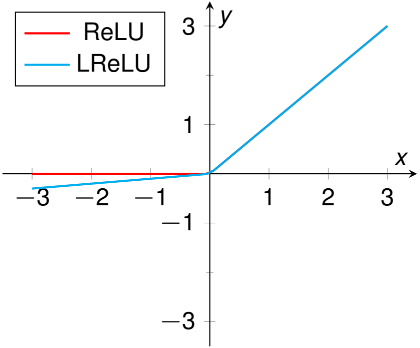
+ Fixes dying ReLU problem
- Leaky ReLU: $\alpha = 0.01$ [5]

Activation Function



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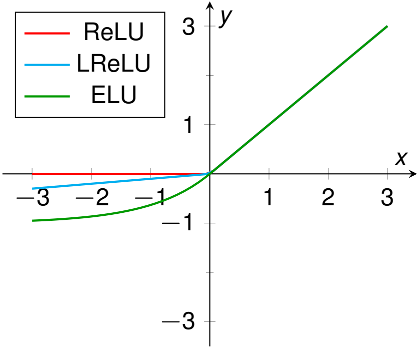
+ Fixes dying ReLU problem
- Leaky ReLU: $\alpha = 0.01$ [5]
- Parametric ReLU (PReLU): learn α [2]

→ trainable

→ Error reduce better

Exponential Linear Units (ELU)



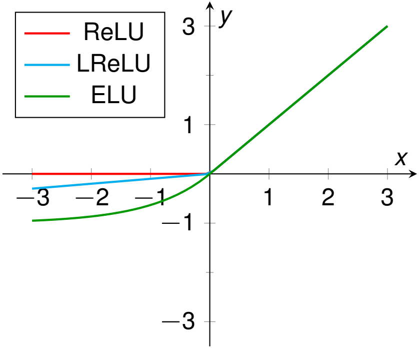
Exponential Linear Unit (ELU)

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha(\exp(x) - 1) & \text{else} \end{cases}$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ \alpha \exp(x) & \text{else} \end{cases}$$

→ Smooth Function on negative Halfspace

Exponential Linear Units (ELU)



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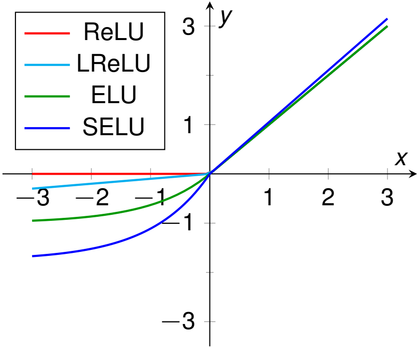
- + Also no vanishing gradient
- + Reduces shift in activations

also negative o/p's can be present

[RLES]

Scaled ELU (SELU)



Scaled Exponential Linear Unit

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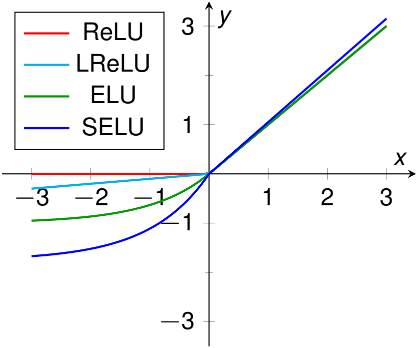
$$\lambda_{01} = 1.0507$$

$$\alpha_{01} = 1.6733$$

≙ BN

Scaled ELU (SELU)

$\mu = 0; \sigma = 1$



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$$\lambda_{01} = 1.0507$$
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use this if 0 mean & 1 variance

- Alternative variant of ReLU [3]
- Idea: Self-normalizing - $\mu = 0, \sigma = 1 \mapsto \lambda_{01}, \alpha_{01}$
- Alternative to Batch Normalization? (see next lecture)

↳ No problem with internal Covariate shift

Other Activation Functions

Other Activation Functions

- Maxout: Learns the activation function [1]

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This is getting ridiculous - what should we use?

Finding the Optimal Activation Function

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- **Reinforcement learning/search** problem

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  - ➔ If you have a **cloud/supercomputer** you can do something like this
- “Searching for Activation Functions” [6] published by Google on Oct 16, 2017

Finding the Optimal Activation Function

later

- **Reinforcement learning/search** problem
  - ➔ Unfortunately a single "**step**" means **training a network** from scratch
  - ➔ If you have a **cloud/supercomputer** you can do something like this
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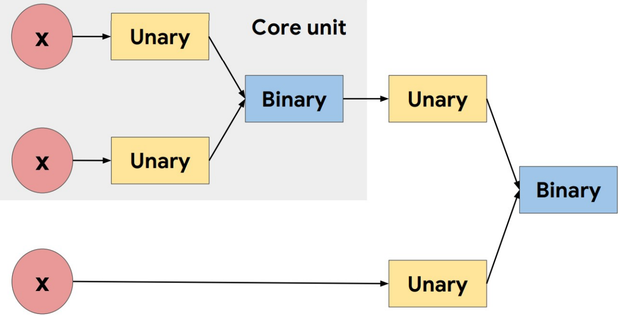
Strategy

1. Define a search-space
2. Perform the search using a RNN with reinforcement learning
3. Use the best result

Searching for Activation Functions [6]

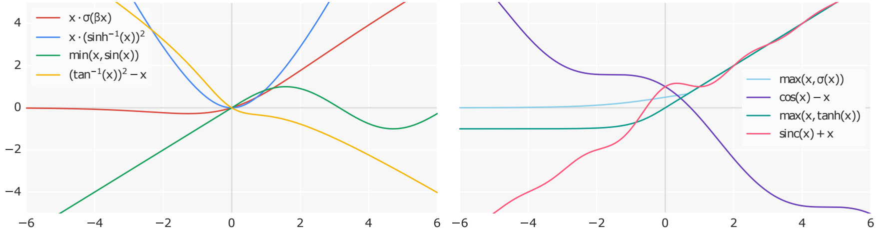


Search space for activation functions

Search space

Source: https://arxiv.org/pdf/1710.05941.pdf

Searching for Activation Functions [6]



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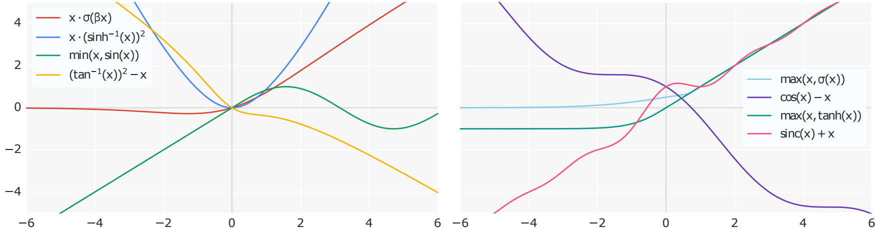

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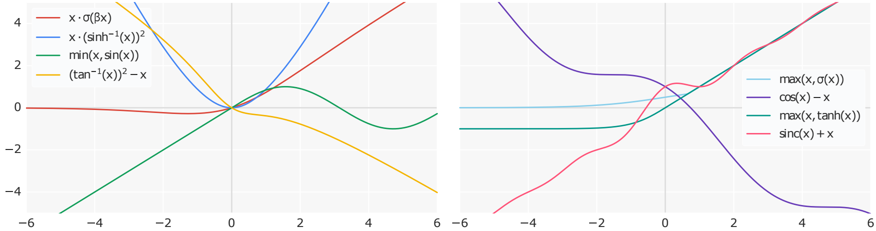

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- They now call $x \cdot \sigma(\beta x)$ the "Swish" function

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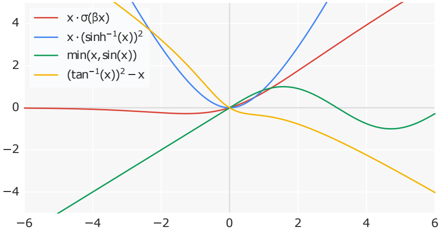

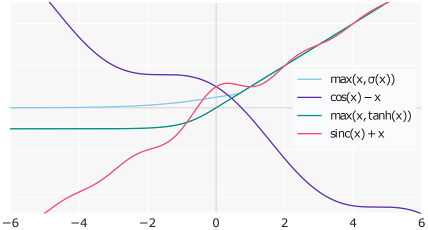

- We don't have a **cloud**, but we can use those
- **Complicated** activation functions **didn't perform well** Quite well
- They now call $\boxed{x \cdot \sigma(\beta x)}$ the "Swish" function
- Has been proposed before as "Sigmoid-weighted Linear Unit" [7]

Source: https://arxiv.org/pdf/1710.05941.pdf

Let’s look into their results in detail

Disclaimer

Never show tables in you slides. Try hard to find a better representation.

Source: https://arxiv.org/pdf/1710.05941.pdf

Let's look into their results in detail

Model	Top-1 Acc. (%)		
LReLU	79.5	79.5	79.6
PReLU	79.7	79.8	80.1
Softplus	80.1	80.2	80.4
ELU	75.8	79.9	80.0
SELU	79.0	79.2	79.2
GELU	79.6	79.6	79.9
ReLU	79.5	79.6	79.8
Swish-1	80.2	80.3	80.4
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Inception-Resnet-V2 architecture trained on ImageNet.

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Inception-Resnet-V2 architecture trained on ImageNet.

- Are any of these differences significant?

Not much

is observation a result of radomness

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Summary Activation Functions

- Go-to solution: **ReLU**

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Summary Activation Functions

use it; if problem? ↳ Batch Normalize

- Go-to solution: **ReLU**
- SELUs have **attractive** theoretical properties.

self adaptation property.

- ... but try ReLU first.
- Best activation function is a **difficult, expensive optimization problem**.

↳ no better alternative

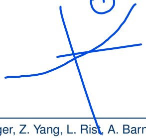
What we know about good activation functions:

[S]

- They have almost linear areas to prevent **vanishing gradients**.
- They have **saturating areas** to provide **non-linearity**.
- They should be **monotonic**.

linearity
⇓
no vanishing gradient

where gradient = 0



NEXT TIME

ON DEEP LEARNING

Activation Functions and Convolutional Neural Networks - Part 3

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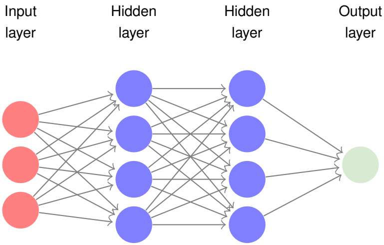
Convolutional Neural Networks

↳ Powerful & building blocks

Motivation

- So far: Fully connected layers - each input is connected to each node
- Very powerful: Can represent any kind of (linear) relationship between inputs



Matrix multiplications

Motivation

- So far: Fully connected layers - each input is connected to each node
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- Large part of machine learning: images/videos/sounds

Motivation

- So far: Fully connected layers - each input is connected to each node
- Very powerful: Can represent any kind of (linear) relationship between inputs

- Large part of machine learning: images/videos/sounds
- Assume we have:
  - An image with size 512×512 pixels
  - One hidden layer with 8 neurons

    ➔ $(512^2 + 1) \cdot 8 >$ 2 million trainable weights!

bias

512

WSD

512

Motivation (cont.)

So the size is a problem. Is there something else?

Source: https://news.nationalgeographic.com

Motivation (cont.)

not the only problem

So the size is a problem. Is there something else?

- Example: Classify between cat and dog

Source: https://news.nationalgeographic.com

Motivation (cont.)

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Motivation (cont.)

So the size is a problem. Is there something else?

- Example: Classify between cat and dog
- Pixels are **bad** features!
  - Highly correlated
  - Scale dependent
  - Intensity variations
  - ...

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Motivation (cont.)

So the size is a problem. Is there something else?

- Example: Classify between cat and dog
- Pixels are **bad** features!
  - Highly correlated
  - Scale dependent
  - Intensity variations
  - ...
- Pixels are a bad representation from a machine learning point of view

light, camera

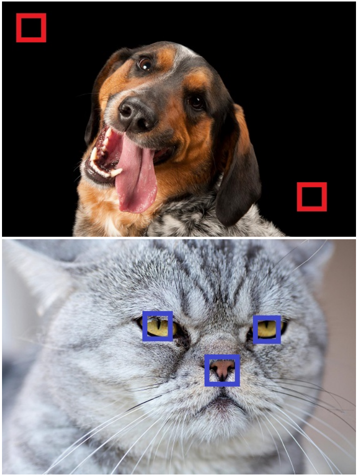
above reasons

shadow, reflections
noise vs feature

Source: https://news.nationalgeographic.com

Motivation (cont.)

Can we find a better representation?



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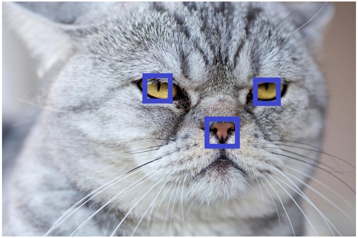
Motivation (cont.)

Can we find a better representation?

- We have a certain degree of the locality in an image



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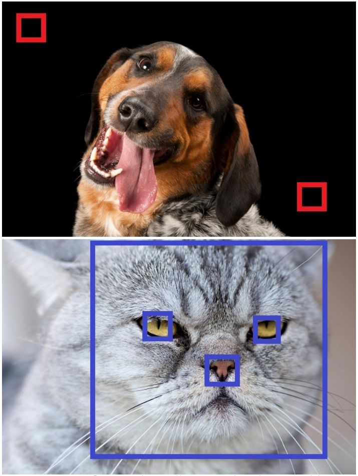
Motivation (cont.)

Can we find a better representation?

- We have a certain degree of the locality in an image
- We can find the same “macro features” at different locations
- Hierarchy of features:
  - edges + corners $\mapsto$ eyes
  - eyes + nose + ears $\mapsto$ face
  - face + body + legs $\mapsto$ animal



Source: https://news.nationalgeographic.com

Motivation (cont.)

[C.S]

Can we find a better representation?

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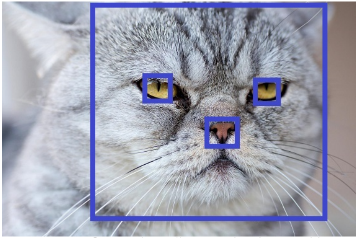
- We can find the same "macro features" at different locations
- Hierarchy of features:
  - edges + corners $\mapsto$ eyes
  - eyes + nose + ears $\mapsto$ face
  - face + body + legs $\mapsto$ animal
- Composition matters!

low to High

➔ Learn better representation, then classify!



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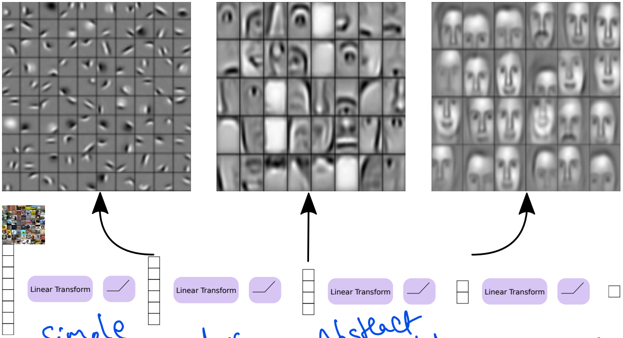
Convolutional Neural Networks



[CS]

simple descriptors

Abstract Representations

line, eyes, face

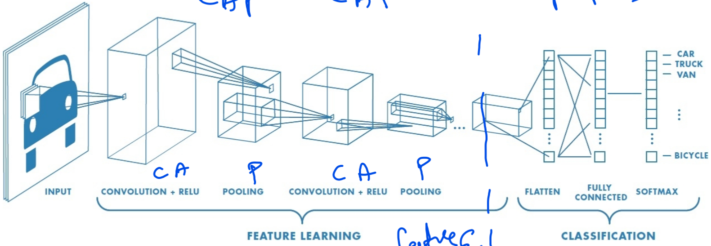
- **Local** connectivity ➔ filters
- Use **same filters** over the whole image ➔ translational equivariance
- Hierarchy of filters working on **different scales**
- + **learning** = **Convolutional Neural Networks**

Nose, move nose; filtered move nose, still nose

CAPL

Convolutional Neural Networks - Architecture

CAP CAP FFS



feature, reduce Dimensions

FFS

Four essential building blocks:

[CS]

- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress and aggregate information, save parameters
- Last layer: Fully-connected for classification

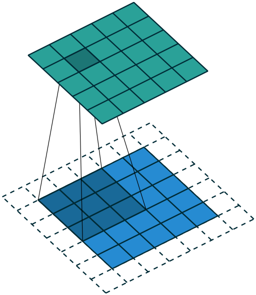
Remove BS (L.P filter)

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Convolutional Layers

Convolutional Layer - Local Connectivity

- Exploit spacial structure by only connecting pixels in a neighborhood
- Can be expressed as fully connected layer: **Except** for local connections, each entry in **W** is 0



Source: https://github.com/vdumoulin/conv_arithmetic

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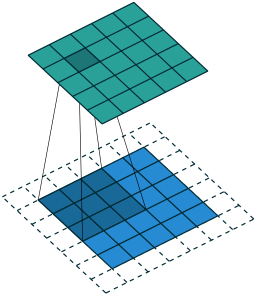
- Effective weights: Filter of size 3×3, 5×5, 7×7, ...



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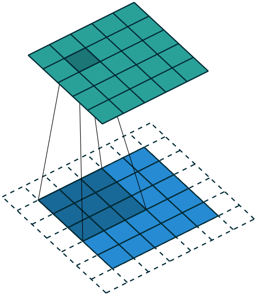
  - ➔ Use the same weights all over: **tied weights** (also **shared weights**).



Source: https://github.com/vdumoulin/conv_arithmetic

FC = Conv + '0' Outside Receptive field.

Convolutional Layer - Local Connectivity

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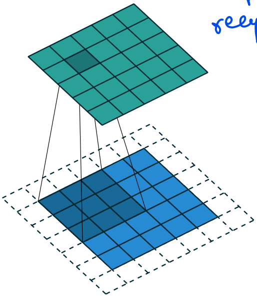
➔ **Convolution with trainable filters**

finding best filters

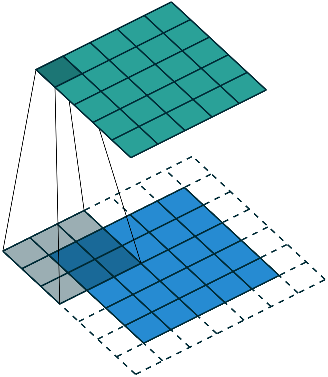
FC with '0' everywhere except receptive field



(Modelling Convolution)

Source: https://github.com/vdumoulin/conv_arithmetic

Recap: Convolution



Convolution

Source: https://github.com/vdumoulin/conv_arithmetic

Recap: Convolution

- Convolution:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(\tau)g(x-\tau)\mathrm{d}\tau$$

Recap: Convolution

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$$(f * g)(x) = \int_{-\infty}^{\infty} f(\tau)g(x-\tau)\mathrm{d}\tau$$

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$$(f \star g)(x) = \int_{-\infty}^{\infty} f(\tau)g(x+\tau)\mathrm{d}\tau$$

Recap: Convolution

- Convolution:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(\tau) g(x - \tau) \mathrm{d}\tau$$

- Cross-correlation:

$$(f \star g)(x) = \int_{-\infty}^{\infty} f(\tau) g(x + \tau) \mathrm{d}\tau$$

- Cross-correlation is convolution with a flipped kernel g – and vice versa!

C =

Recap: Convolution

- Convolution:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(\tau) g(x - \tau) \mathrm{d}\tau$$

different domain (move negative)

- Cross-correlation:

(move positive)

$$(f \star g)(x) = \int_{-\infty}^{\infty} f(\tau) g(x + \tau) \mathrm{d}\tau$$

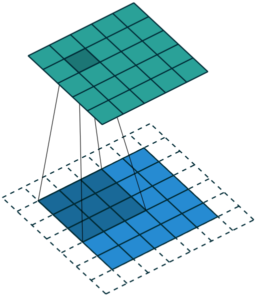
- Cross-correlation is convolution with a flipped kernel g – and vice versa!
- Implementation: Cross-correlation is frequently used in the forward pass - the weights are initialized randomly anyway

Doesn't matter in training; it's just a sign

$$\left\lfloor \frac{w-k+2p}{s} + 1 \right\rfloor$$

Padding

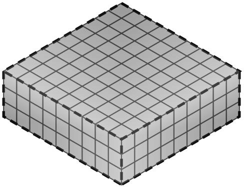
- Convolution reduces image size by $2 \cdot \left\lfloor \frac{n}{2} \right\rfloor$ pixels (n: kernel size).
- Necessary to pay attention to the borders:
- 'Same' padding (usually zero padding):
  - ➔ Input and output have the same size
- 'Valid'/no padding:
  - ➔ The output is smaller than the input



Source: https://github.com/vdumoulin/conv_arithmetic

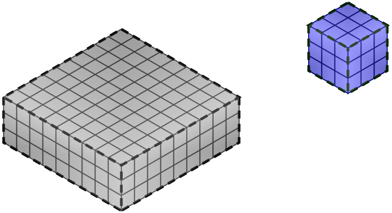
Forward Pass: Multi-channel convolution

- Input of size $X \times Y \times S$, where S is the number of input channels



Forward Pass: Multi-channel convolution

- Input of size $X \times Y \times S$, where S is the number of input channels
- H Filters with size $M \times N \times S \mapsto$ **fully connected** across channels



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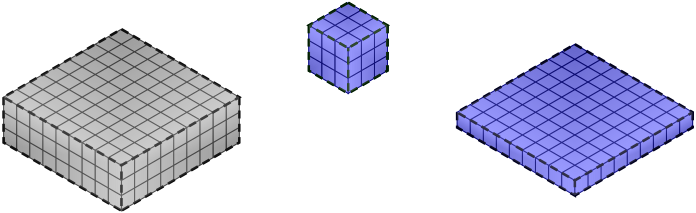

Forward Pass: Multi-channel convolution

No. of Colors

RGB.

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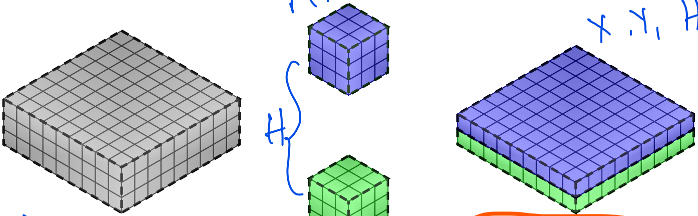
#Masks



$$\left\lfloor \frac{X-M+2P}{S} + 1 \right\rfloor$$

$$\begin{matrix} a & b & c & d \\ b & a & b & c \\ c & b & a & b \\ d & c & b & a \end{matrix}$$

Backward pass: Multi-channel convolution

- Convolution can be expressed as matrix multiplication with matrix **W** using a **Toeplitz matrix**

Circular Matrix (weight sharing)

- We can use the **same formulas** as for the fully connected layer!

$\hat{y} = W^T x$

$\nabla W = E_l x^T$

$E_{l-1} = W^T \cdot E$

$$\left.\begin{aligned} \mathbf{E}_{l-1} &= \mathbf{W}^T \mathbf{E}_l \\ \nabla \mathbf{W} &= \mathbf{E}_l \mathbf{X}^T \end{aligned}\right\} \text{Same update formulas}$$

where

- $\mathbf{X}$ is the input and
- $\mathbf{E}_l / \mathbf{E}_{l-1}$ is the error in layer $l/l-1$

[CS]

$E_l = \frac{\partial L}{\partial \hat{y}}$

Backward pass: Multi-channel convolution

- Convolution can be expressed as matrix multiplication with matrix **W**: using a **Toeplitz matrix**
- We can use the **same formulas** as for the fully connected layer!

$$\mathbf{E}_{l-1} = \mathbf{W}^T \mathbf{E}_l$$
$$\nabla \mathbf{W} = \mathbf{E}_l \mathbf{X}^T$$

  where

  - $\mathbf{X}$ is the input and
  - $\mathbf{E}_l / \mathbf{E}_{l-1}$ is the error in layer $l / l-1$
- Backward pass can also be expressed as convolutions $\mapsto$ exercise

Convolutional Layer - What have we gained?

- Stack multiple filters to a get a trainable filter bank.
- Layer with 8 filters (nodes) with 5×5 neighborhood
  - ➔ $5^2 \cdot 8 = \underline{200}$ weights

Convolutional Layer - What have we gained?

M x N x H
5 x 5 x 8

- Stack multiple filters to a get a trainable filter bank.
- Layer with 8 filters (nodes) with 5×5 neighborhood
  - ➔ $5^2 \cdot 8 = \underline{200}$ weights

diff from 2M earlier

- Convolution: **Independent** of image size!
- Much more training data for one weight!

Strided Convolutions

- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
- **Stride** s describes the offset

Source: https://en.wikipedia.org, Monty Python's Flying Circus episode, Dinsdale

Strided Convolutions

- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
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- **Reduces** the **size** of the output by a factor of s

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Strided Convolutions

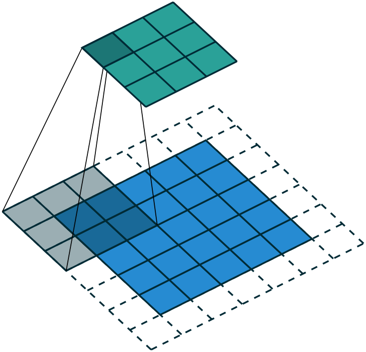
- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
- **Stride** *s* describes the offset
- **Reduces** the **size** of the output by a factor of *s*
- **Mathematically: Convolution + subsampling**

stride = Conv + Subsampling

(C · S)

Source: https://en.wikipedia.org, Monty Python's Flying Circus episode, Dinsdale

Strided Convolutions



Strided Convolution with stride $s = 2$

Source: https://github.com/vdumoulin/conv_arithmetic

Dilated/Atrous Convolutions

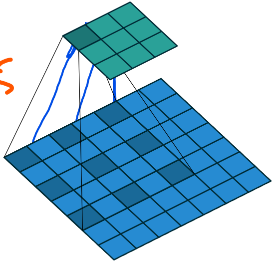
- Additional variant of convolution in neural networks
- Dilate convolution kernel: Skip certain pixels → i/p
- Goal: Wider receptive field with less parameters/weights

9w

9

5x5

3x3



receptive field is not connected

Dilated Convolution

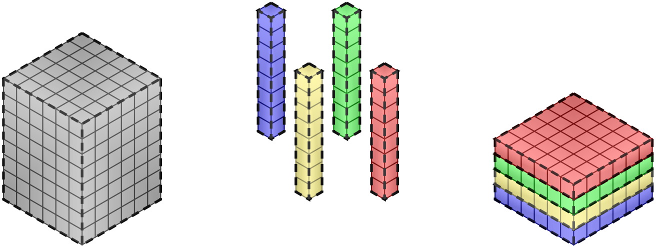
Source: https://github.com/vdumoulin/conv_arithmetic

1×1 Convolution Concept

- So far: H filters with neighborhood $3 \times 3, 5 \times 5, \ldots$ and ‘depth’ S
- Filters are fully connected in ‘depth’ direction

1×1 Convolution Concept

- So far: H filters with neighborhood $3 \times 3, 5 \times 5, \ldots$ and 'depth' S
- Filters are fully connected in 'depth' direction
- We can decrease the neighborhood to 1×1
- And **just** use the fully connected property in the depth dimension



1×1 Convolution Concept

i/p channel

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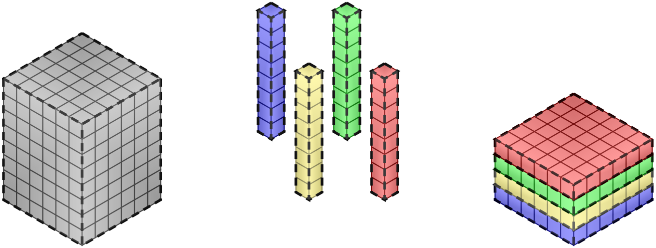

→ Dimensionality reduction/expansion from S channels to H channels

→ **If we flatten the input, 1×1 convolutions are fully connected layer!**

1×1 Convolution Concept (cont.)

- First described in “Network in Network” by Lin et al. [4]
- 1×1 convolutions simply calculate **inner products** at each position
- Simple and efficient method to **decrease** the **size** of a network
- **Learns** dimensionality reduction, e.g., can reduce redundancy in your feature maps

1×1 Convolution Concept (cont.)

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- 1×1 convolutions simply calculate **inner products** at each position
- Simple and efficient method to **decrease** the **size** of a network
- **Learns** dimensionality reduction, e.g., can reduce redundancy in your feature maps
- Equivalent but more flexible: $N \times N$ convolution

NEXT TIME

ON DEEP LEARNING

Activation Functions and Convolutional Neural Networks - Part 4

A. Maier, V. Christlein, K. Breininger, Z. Yang, L. Rist, M. Nau, S. Jaganathan, C. Liu, N. Maul, L. Folle, K. Packhäuser, M. Zinnen
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
April 24, 2023

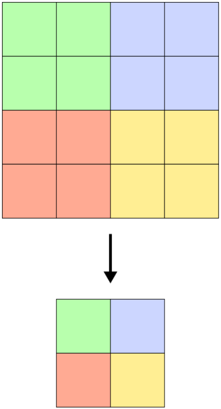
Pooling Layers

Idea behind Pooling Layers

adder info

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting

Introduces → Hierarchy

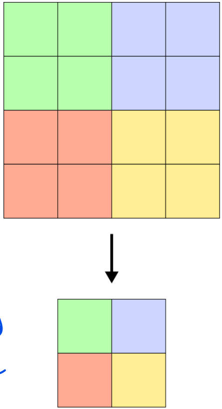


Idea behind Pooling Layers

[c.5]

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting
- Assumptions:
  - Features are hierarchically structured
  - Creates “summaries” of regions
  - Provides translational invariance
  - Exact location of a feature is not important

Nose, Right shifted nose



Max Pooling – Forward Pass

- Propagate maximum value in a neighborhood to next layer
- Typical choices: 2×2 or 3×3 neighborhood
- "Stride" of pooling usually equals the neighborhood size
- Maximum propagation adds additional non-linearity Yes

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

3.0	3.0	3.0
3.0	3.0	3.0
3.0	2.0	3.0

Max pooling concept. Note that usually a stride > 1 is used for pooling.

Max Pooling – Backward Pass

Max Pooling – Backward Pass

- → Only one value contributes to error
- → Error is propagated only along the path of the maximum value

Rest all get 0

Average Pooling

- Propagate average of the neighborhood
- Does not consistently perform better than max pooling
- Backward pass: Error is shared to equal parts

Additional Pooling Strategies

- Fractional max pooling
- L_p pooling
- Stochastic pooling
- Spacial pyramid pooling
- Generalized pooling
- ...

Many strategies

Alternative: Strided Convolution

- Historically, max pooling was the most frequently used pooling strategies due to additional non-linearity

Max ⇒ Non-linearity.

- More recently, convolution with stride $s > 1$ has become more common

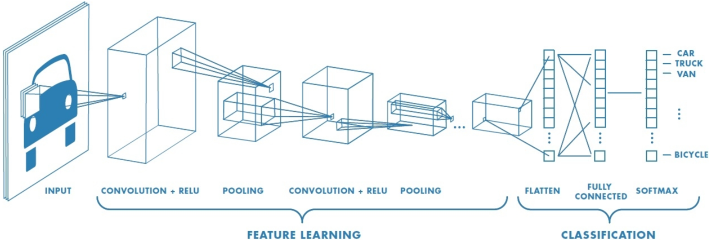
➔ Allows for trainable downsampling strategy

since $s > 1$

Conv + pooling
≈ strided Conv

Don't need to
Encode
max pooling

Recap: Convolutional Neural Networks - Architecture

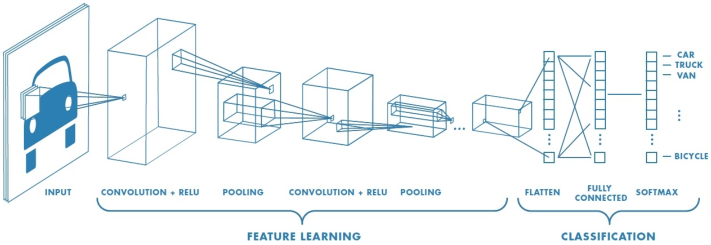


Four essential building blocks:

- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress and aggregate information, save parameters
- Last layer: Fully-connected for classification

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Recap: Convolutional Neural Networks - Architecture

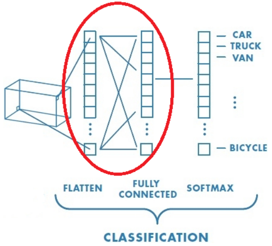


~~Four~~Three essential building blocks:

- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress and aggregate information, save parameters
- ~~Last layer: Fully-connected for classification~~**We can replace this layer!**

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Replacing the Fully Connected Layer



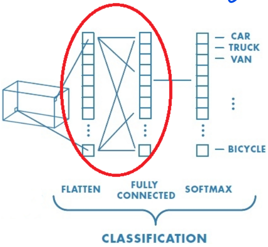
- Conv and pooling layers generate better representation ➔ better features
- Fully connected layers for classification

flatten + 1x1 = full

Replacing the Fully Connected Layer

Entire chain just by Conv + Pooling

1x1 + Global Avg Pool → Any i/p size



- Conv and pooling layers generate better representation → better features
- Fully connected layers for classification
- **Alternatively and equivalently**: Use flatten & 1 × 1 convolution or $N \times N$ convolution

  Any size.
- Enables **arbitrary input sizes** in combination with global average pooling!

Inception model

- Szegedy et al. (2014): Going Deeper With Convolutions [8]
  - Very influencial publication: > 17000 citations (Nov. ’19)
  - Won ImageNet Large-Scale Visual Recognition Challenge 2014
  - GoogLeNet as one incarnation
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- Self-stated motto:

Dream Deep

Inception model

Going Deeper

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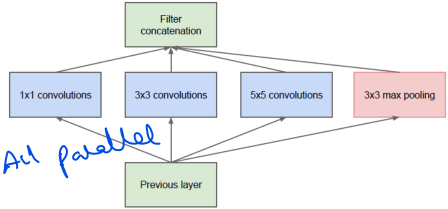
Source: http://knowyourmeme.com/photos/531557-we-need-to-go-deeper

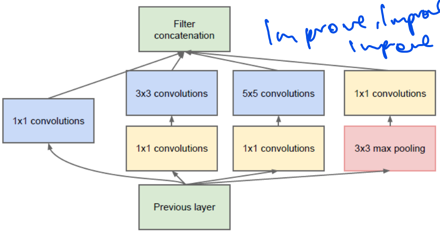
Inception model

- **Idea**: Why use only one type of filter in one layer?
  Why not combine different neighborhoods/pooling/etc.?
- Construction of 'inception modules' that are stacked to form a large network.

Improve, Improve, Improve



Naïve inception module



Inception module with dimensionality reduction

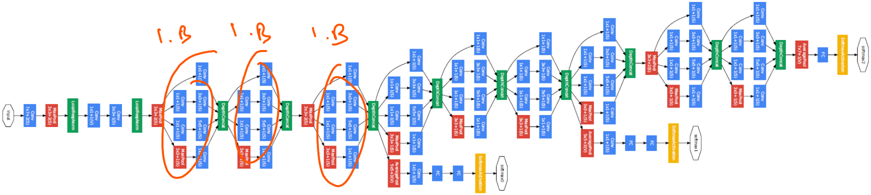
let it learn when it wants to pool; when to convolve

Source: [8]

Inception model – GoogLeNet

Error

I.B I.B I.B



B.P.

GoogLeNet architecture with inception modules.

Pretty deep

Source: [8]

NEXT TIME

ON DEEP LEARNING

Coming up

- How to prevent networks from just memorizing the training data?
- Is there a way to force features to be independent?
- How can we make sure our network also recognizes cats in different poses?
- Can we fix the covariate shift problem?

Changes in distribution of train & test data

Poor Generalization.

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Comprehensive Questions

[C.S] — tanh, sign, Relu, sign

- Name five activation functions.
- Discuss those 5 activation functions.
- What is the zero-centering problem? → Action, internal covariate shift
- [C.S] Why does ReLU as activation function perform much better than sigmoid/tanh in a large number of tasks? → Saturation NO; hierarchy
- Why are convolutional networks well suited for image and audio processing? receptive field
- Write down a mathematical description of strided convolution.
- What is the connection between 1 × 1 convolutions and fully connected layers? fla + 1x1 = f.c
- How would you implement a classifier which operates on image patches?
- What is a pooling layer? [S] C, A, P,
- Why do we use pooling layers?
- On what data would CNNs probably perform bad? → Insufficient Data, D shift.

Further Reading

- Link - [3] for a paper about Self Normalizing Networks
- Link - [4] for a creative Network in Network paper
- Link - [6] for details on learned activation functions
- Link - [8] if everything so far was not deep enough for you

Questions?

References

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References I

[1] I. J. Goodfellow, D. Warde-Farley, M. Mirza, et al. “Maxout Networks”. In: ArXiv e-prints (Feb. 2013). arXiv: 1302.4389 [stat.ML].

[2] Kaiming He, Xiangyu Zhang, Shaoqing Ren, et al. “Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification”. In: CoRR abs/1502.01852 (2015). arXiv: 1502.01852.

[3] Günter Klambauer, Thomas Unterthiner, Andreas Mayr, et al. “Self-Normalizing Neural Networks”. In: Advances in Neural Information Processing Systems (NIPS). Vol. abs/1706.02515. 2017. arXiv: 1706.02515.

[4] Min Lin, Qiang Chen, and Shuicheng Yan. “Network In Network”. In: CoRR abs/1312.4400 (2013). arXiv: 1312.4400.

[5] Andrew L. Maas, Awni Y. Hannun, and Andrew Y. Ng. “Rectifier Nonlinearities Improve Neural Network Acoustic Models”. In: Proc. ICML. Vol. 30. 1. 2013.

References II

[6] Prajit Ramachandran, Barret Zoph, and Quoc V. Le. "Searching for Activation Functions". In: CoRR abs/1710.05941 (2017). arXiv: 1710.05941.

[7] Stefan Elfwing, Eiji Uchibe, and Kenji Doya. "Sigmoid-weighted linear units for neural network function approximation in reinforcement learning". In: arXiv preprint arXiv:1702.03118 (2017).

[8] Christian Szegedy, Wei Liu, Yangqing Jia, et al. "Going Deeper with Convolutions". In: CoRR abs/1409.4842 (2014). arXiv: 1409.4842.