Gradient Descent

$$y = x^4 - 3x^3 + 2$$

$$\frac{dy}{dx} = 4x^3 - 9x^2$$

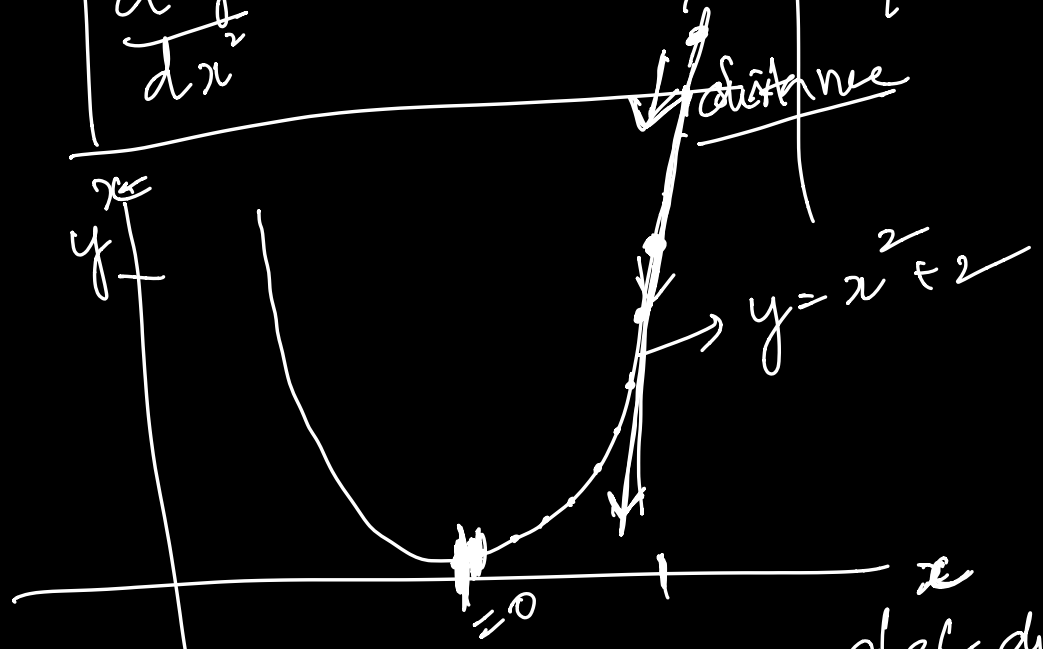
$$\frac{d^2y}{dx^2} = 12x^2 - 9x = 0$$

differential

$$x^n = nx^{n-1}$$

$$4x = 4$$

$$4 = 0$$



$$y = x^4 - 3x^3 + 2$$

$$\frac{dy}{dx} = 4x^3 - 9x^2$$

```
def diff(x):
    return 4*(x**3)-
Learning rate = 0.001
```

1. New_x = Old_x − (Step Size)

Step Size = $\frac{dy}{dx}$ * (learning rate)

$\frac{dy}{dx}$ → direction

learning rate → How much distance

$x = 6$

$\frac{dy}{dx} = 4x^3 - 9x^2$

$= 4(6)^3 - 9(6)^2$

$= 810$

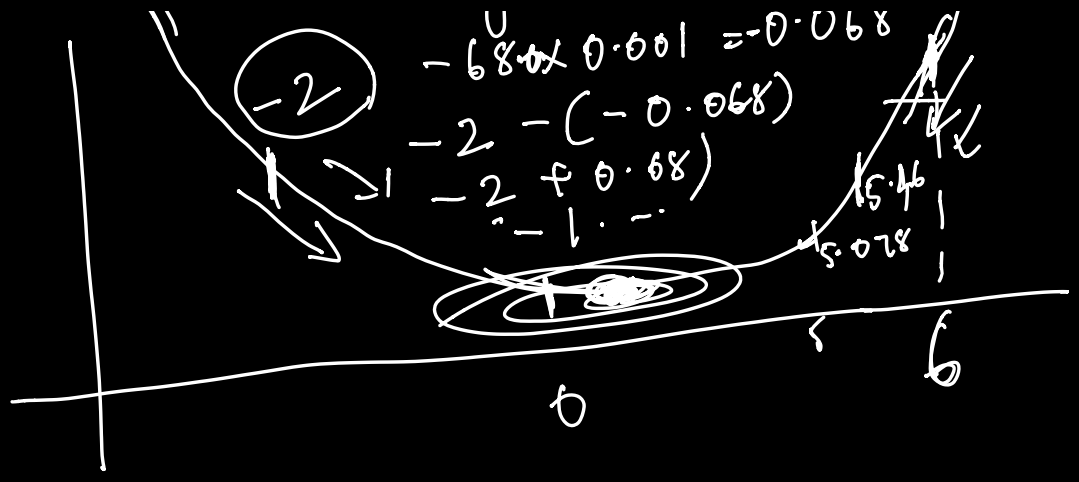
Step size = 810×0.001

$= 0.810$

New_x = $6 - 0.810$

$= 5.146$

$y = x^4 - 3x^3 + 2$ ... 1.



Old x - Step Size

$$5.46 - \left(\left(4x^3 - 9x^2\right) \times 0.001\right)$$

$$\left(4(5.4)^3 - 9(5.4)^2\right)$$

$$382 \times 0.001$$

$$5.46 - 0.382$$

$$= 5.078$$

$$= 5.078 - \left(4(5.078)^3 - 9(5.878)^2\right) \times 0.001)$$

$$\Rightarrow 4.78$$

3.156

101 → 3.15600
102 → 3.15601
103 → 3.15602

11

~~old value = 6~~

```
def doff (x)
    return 4x^3 - 9x^2
```

for loop

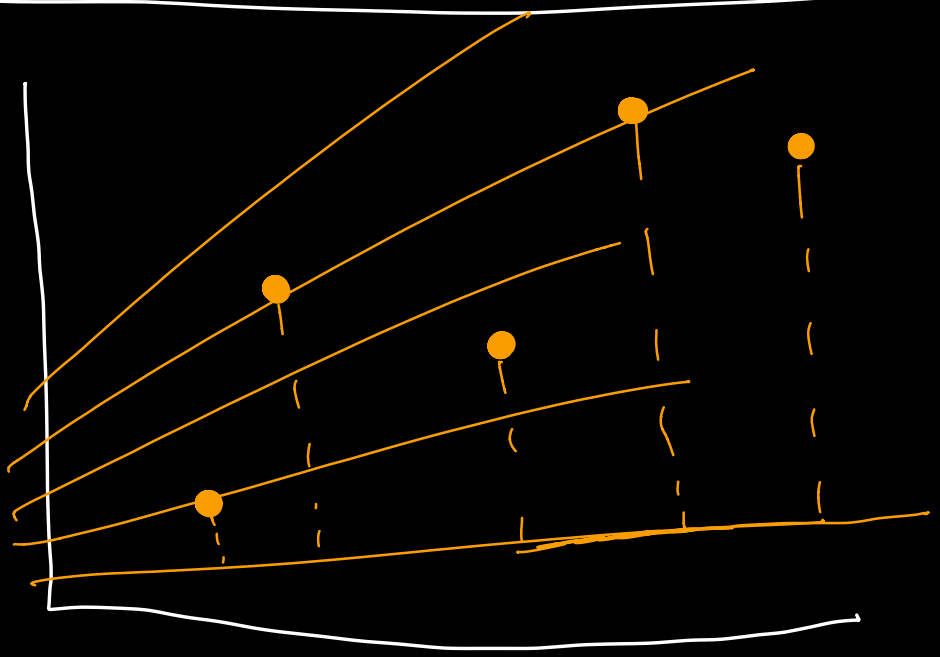
$$New = Old_{value} - diff(x) \times learning\ rate$$

0.001

$$New\ value - Old\ va < 0.0001$$

while loop

Can I break ?



m and b ?

↓ Cost function = $\frac{\sum (y - (mx + b))^2}{n}$

$\frac{\sum (y - (mx + b))^2}{n}$

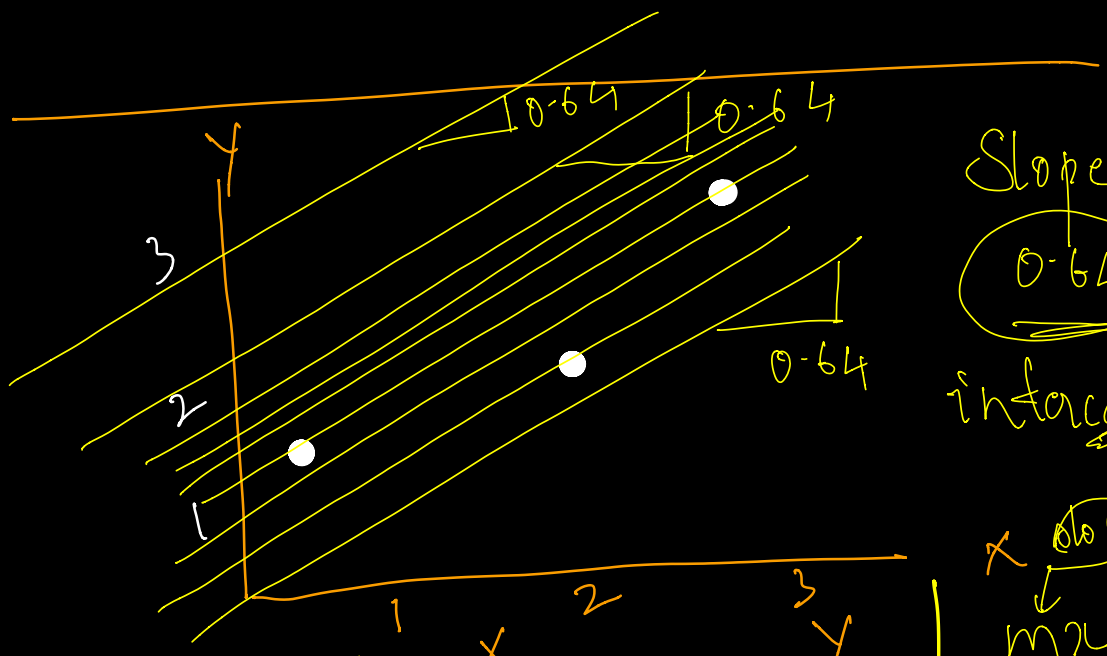
m b

$4x^4 - 3x^3 + 2$

$\frac{d\,m}{d(\text{cost})} = ?$ ⟷ $\frac{d\,b}{d(\text{cost})} = ?$

partial derivative

partial derivative



Slope = 0.64

intercept = ?

$mx + b$

	X	Y
1	0.5	1.4
2	2.3	1.9
3	2.9	3.2

Partial Derivative for intercept

$$\hat{y} = \frac{\sum (y - (mx + b))^2}{n}$$

$$\frac{\partial}{\partial b} = ?$$

$$\frac{\sum (y)^2 + (mx + b)^2 - 2y(mx + b))}{n}$$

$$\frac{\sum [(y^2) + m^2x^2 + b^2 + 2mxb - 2ym\acute{x} - 2yb]}{n}$$

Partial derivative $\frac{\partial}{\partial b}$

$$= \frac{\sum [0 + 0 + 2b + 2mx - 0 - 2y]}{n}$$

$$= \frac{\sum [2b + 2mx - 2y]}{n}$$

$$\frac{2}{n} \sum [-y + mx + b]$$

$$\frac{\partial}{\partial b} \Rightarrow \frac{2}{n} \sum [-(y - (mx + b))]$$

Partial Derivative for slope

$$\sum \left[y^2 + m^2x^2 + b^2 + 2mxb - 2ymx - 2yb \right]$$

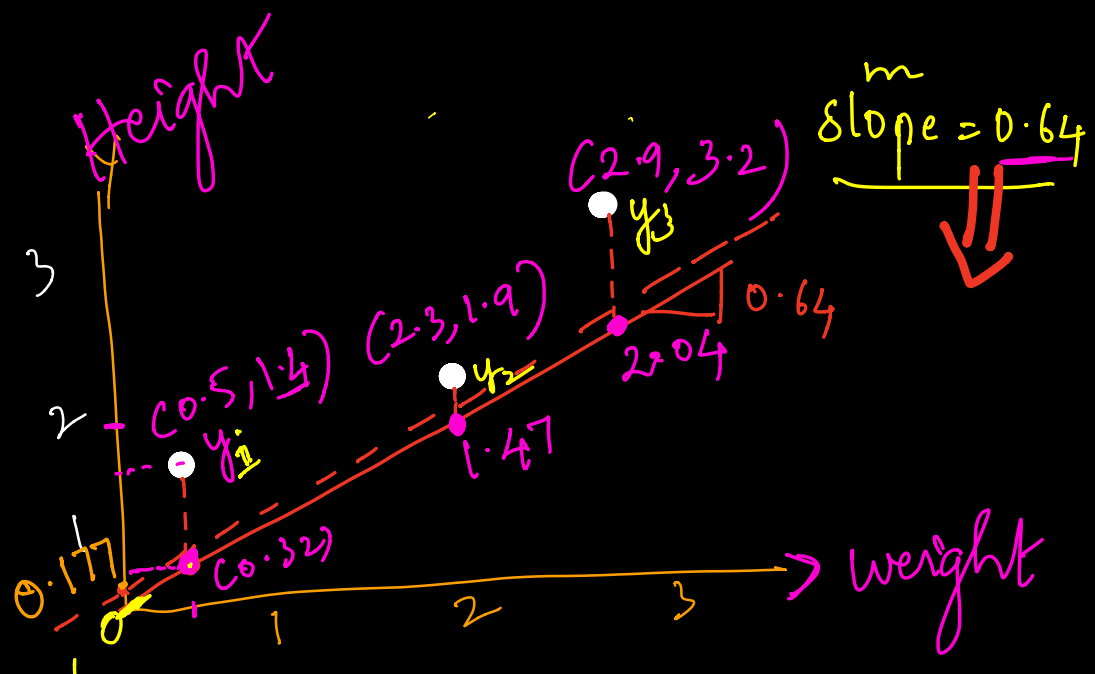
$$\frac{\partial}{\partial m} = \sum \left[0 + 2mx^2 + 0 + 2xb - 2yx - 0 \right]$$

$$\sum \left[2mx^2 + 2xb - 2yx \right]$$

$$\frac{2}{N} \sum \left[mx^2 + xb - yx \right]$$

$$\frac{2}{N} \sum \left[x \left[-y + (mx + b) \right] \right]$$

$$\frac{\partial}{\partial m} = \frac{2}{N} \sum \left[-x(y - (mx + b)) \right]$$



$b = 0.177$

$\hat{y}_1 = mx_1 + b$

$= 0.64(0.5) + 0.177 = 0.32$

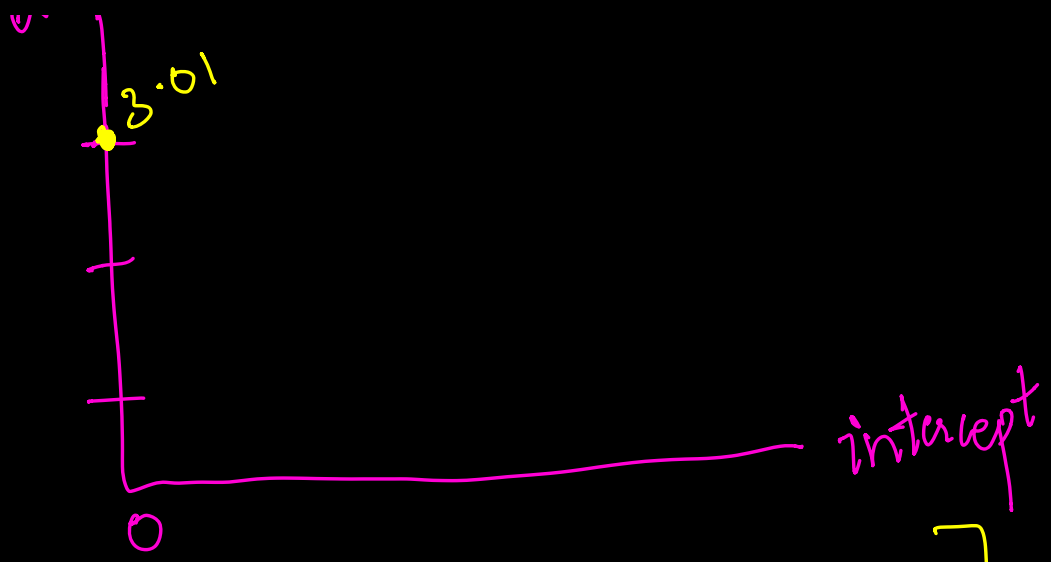
3.2
1.4

$$\text{Error} = (0.32 - 1.4)^2 + (1.47 - 1.9)^2 + (2.04 - 3.2)^2$$

$$(1.08)^2 + (0.4)^2 + (1.3)^2$$

$$= 3.01$$

error



$$\frac{\partial}{\partial b} = \frac{2}{n} \sum_{i=1,2,3} \left[-(y_i - (m x_i + b)) \right]$$

$$New\_b = Old\_b - \left[\frac{\partial}{\partial b}(Old\_b) * \text{learning rate} \right]$$

1st Iteration

$i=1$ $\quad [-(1.4 - 0.32)] = -1.08$

$i=2$ $\quad [-(1.9 - 1.47)] = -0.42$

$i=3$ $\quad [-(3.2 - 2.04)] = -1.16$

$$\frac{2}{3}\left[-1.08 - 0.42 - 1.6\right]$$

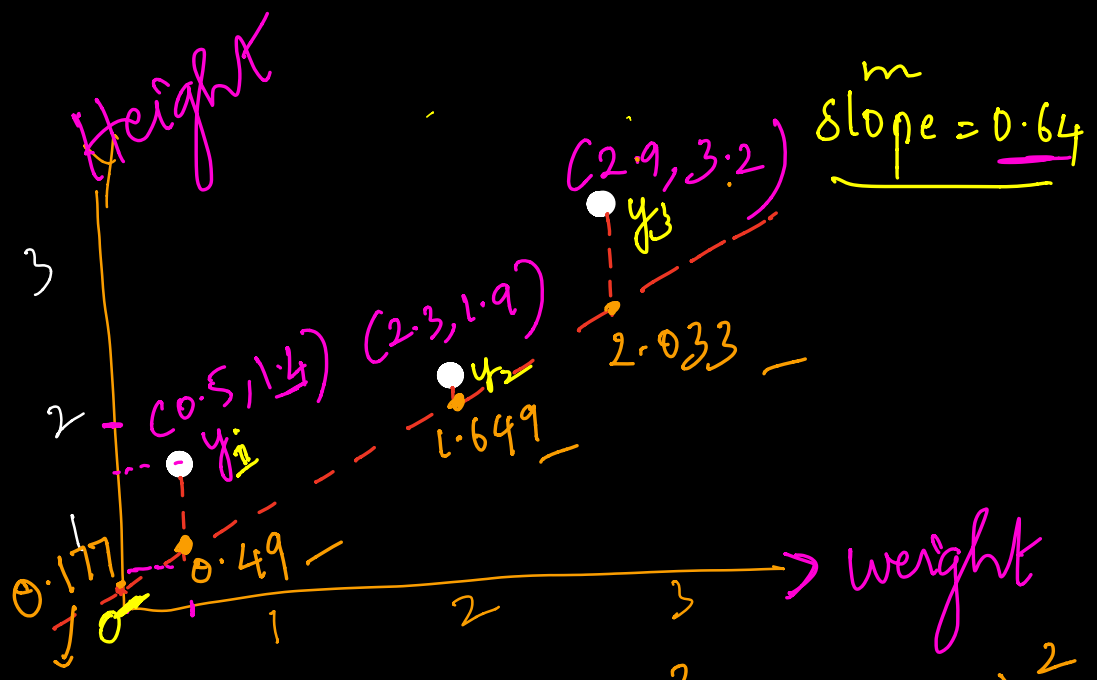
$$\frac{2}{3}\left[\quad\right] = \underline{-1.77}$$

Optimization

$$\text{new\_b} = \text{old\_b} - \left[\left(\frac{\partial}{\partial b}(\text{old\_b})\right) * \text{learning rate}\right]$$

$$= 0 - \left[-1.77 * 0.1\right]$$

$$= \underline{0.177}$$



m

slope = 0.64

$$\text{Error} = (1.4 - 0.49)^2 + (1.9 - 1.6)^2 + (3.2 - 2.03)^2$$

$$= 2.287 \checkmark$$

iteration 2

$b = 0.177$

$$\Rightarrow \frac{\partial}{\partial b} = \frac{2}{n} \sum_{i=1}^{3} \left[-(y_i - (m x_i + b)) \right] \checkmark$$

$$\text{New\_b} = \text{Old\_b} - \left[\frac{\partial}{\partial b} (\text{old\_b}) * \text{learning rate} \right]$$

1st Iteration

Iteration - 2

$$\left[-(1.4 - 0.49\right] = -0.91$$

$$\left[-(1.9 - 1.64)\right] = -0.26$$

$$\left[-(3.2 - 2.03)\right] = -1.16$$

$$\frac{2}{3}\left[-0.91 - 0.26 - 1.16\right]$$

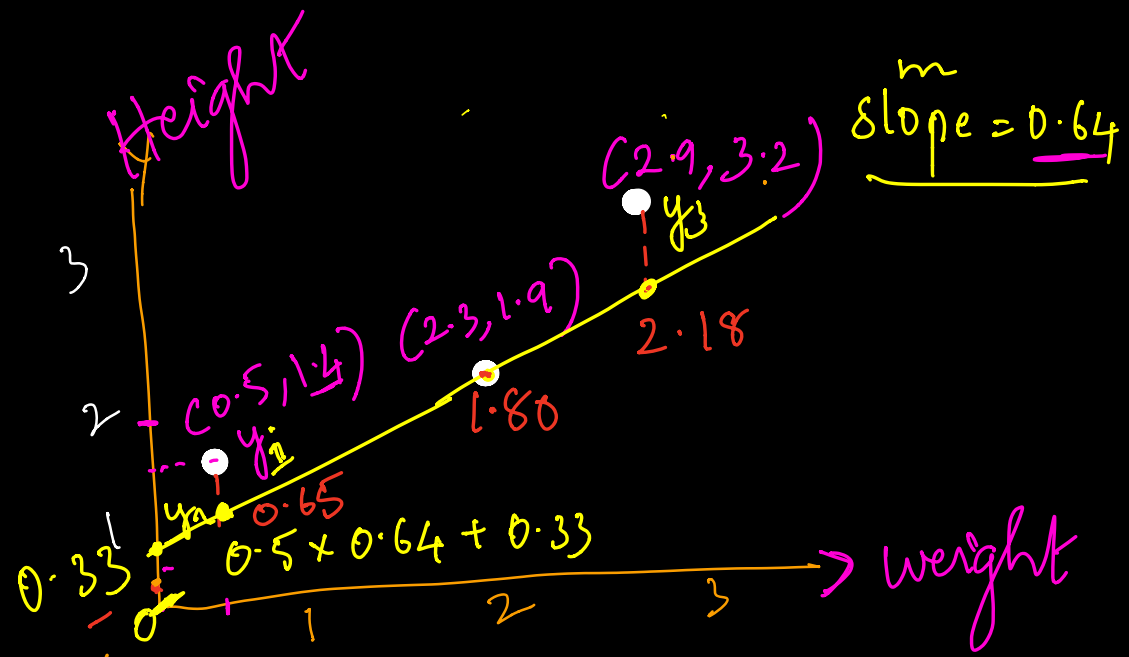
$$= -1.55$$

Optimization

$$new\_b = old\_b - \left[\frac{\partial}{\partial b}(old\_b) * \text{learning rate}\right]$$

$$= 0.177 - \left[-1.55 * 0.1\right]$$

$$= 0.33$$



$$Error = \left[(1.4 - 0.65)^2 + (1.9 - 1.8)^2 + (3.2 - 2.18)^2\right]$$

$= 1.61\dot{5}$ ✓

$0.3\dot{1}\dot{5}\dot{6}\dot{7}$