The Minimum Cut Problem For An Undirected Graph

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***Abstract* –In this project, the goal is to count the number of occurrences of letters in text files and, for instance, identify the most common ones. Three types of counters were analyzed. The *Exact Counter*; which provides the exact count or frequency of each letter present in the text, the *Fixed Probability Counter*, which approximates the frequency or number of counts of each letter in the text using a fixed probability value of $\frac{1}{2}$. The last counting method considered, being the *Decreasing Probability Counter* approximates the count of each letter in the text with each future encounter of the letter having a decreased probability of being counter, with probability ($\frac{1}{\sqrt{2}^k}$) where k is the number of occurrence of the letter of interest.**

I. Notation and Problem Definition

II. Outline Of Implementation

III. EXACT COUNTER

IV. FIXED PROBABILITY COUNTER

TABLE I

Number of Events	$\mathbb{E}[S]$	Expected Counter Value
1	1	1
5	$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	2
13	$\mathbb{E}[S]_{10}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	3
27	$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	4
51	$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	4

We define an expression for the sum function as

$$\begin{aligned}\sum_{k=1}^{n}\frac{1}{2} &= \sum_{k=0}^{n-1}\frac{1}{2}\\ &= n\cdot\frac{1}{2}\\ &= \frac{n}{2}\end{aligned}$$

The complexity of this computation completes in $\mathcal{O}\left(\frac{n}{2}\right) = \mathcal{O}(n)$ time.

V. DECREASING PROBABILITY COUNTER

The above summation could be expressed as

$$\sum_{k=1}^{n}\frac{1}{\left(\sqrt{2}\right)^{k-1}} = \sum_{k=0}^{n-1}\frac{1}{\left(\sqrt{2}\right)^{k}}$$

To derive an upperbound, we derive an expression for the sum function as

$$\sum_{k=1}^{n}\frac{1}{\left(\sqrt{2}\right)^{k-1}} = \frac{2^{(1-\frac{n}{2})}-2}{\sqrt{2}-2}$$

We obtain an upperbound for the expected counts by evaluating the limit of this function as n becomes bigger and bigger

Let $\phi(S) \geq \mathbb{E}[s], \forall s$

$$\begin{aligned}\phi(S) &= \lim_{n\to\infty}\frac{2^{(1-\frac{n}{2})}-2}{\sqrt{2}-2}\\ &= \lim_{n\to\infty}\frac{2^{(1-\frac{n}{2})}-2}{\sqrt{2}-2}\cdot\frac{\sqrt{2}+2}{\sqrt{2}+2}\\ &= \lim_{n\to\infty} 2^{\frac{1}{2}}+2-2^{\frac{1-n}{2}}-2^{\frac{2-n}{2}}\end{aligned}$$

As $k\to\infty$, $2^{-k}\to 0$

$$\begin{aligned}\therefore \phi(S) &= \sqrt{2}+2\\ &\approx 3.14142\end{aligned}$$

VI. Auxiliary Functions

References

[1] Paul E Black. greedy algorithm, dictionary of algorithms and data structures. *US Nat. Inst. Std. & Tech Report*, 88:95, 2012.

[2] Nykamp DQ. Adjacency matrix definition.

[3] Anthony Kim. Min cut and karger's algorithm : Min cut and karger's algorithm, 2016.