GREAT LEARNING

PGP-DSBA Online
Feb'21 Batch
Date: 16/05/2021
References: PCA.pdf, PCA_Multicollinearity.pdf
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PRINCIPAL COMPONENT ANALYSIS(PCA) OF COLLEGE DATA

Problem Statement:

The dataset Education - Post 12th Standard.csv contains information on various colleges. You are expected to do a Principal Component Analysis for this case study according to the instructions given. The data dictionary of the 'Education - Post 12th Standard.csv' can be found in the following file: Data Dictionary.xlsx

1. Perform Exploratory Data Analysis [both univariate and multivariate analysis to be performed]. What insight do you draw from the EDA?

We perform EDA to get a basic information about our dataset, to serve as a base for further analysis to be performed. Let's get a descriptive analysis of the dataset first.

Sample of dataset:

	0	1	2	3	4
Names	Abilene Christian University	Adelphi University	Adrian College	Agnes Scott College	Alaska Pacific University
Apps	1660	2186	1428	417	193
Accept	1232	1924	1097	349	146
Enroll	721	512	336	137	55
Top10perc	23	16	22	60	16
Top25perc	52	29	50	89	44
F.Undergrad	2885	2683	1036	510	249
P.Undergrad	537	1227	99	63	869
Outstate	7440	12280	11250	12960	7560
Room.Board	3300	6450	3750	5450	4120
Books	450	750	400	450	800
Personal	2200	1500	1165	875	1500
PhD	70	29	53	92	76
Terminal	78	30	66	97	72
S.F.Ratio	18.1	12.2	12.9	7.7	11.9
perc.alumni	12	16	30	37	2
Expend	7041	10527	8735	19016	10922
Grad.Rate	60	56	54	59	15

Figure: 1

The College data set has 777 observations and 18 variables in the data set. All the variables except the 'Names' variable are numeric (int64 & float64 types). There are no null or duplicate values in our dataset.

Types of variables:

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 777 entries, 0 to 776
Data columns (total 18 columns):
 #   Column       Non-Null Count  Dtype
---  ------       --------------  -----
 0   Names        777 non-null    object
 1   Apps         777 non-null    int64
 2   Accept       777 non-null    int64
 3   Enroll       777 non-null    int64
 4   Top10perc    777 non-null    int64
 5   Top25perc    777 non-null    int64
 6   F.Undergrad  777 non-null    int64
 7   P.Undergrad  777 non-null    int64
 8   Outstate     777 non-null    int64
 9   Room.Board   777 non-null    int64
 10  Books        777 non-null    int64
 11  Personal     777 non-null    int64
 12  PhD          777 non-null    int64
 13  Terminal     777 non-null    int64
 14  S.F.Ratio    777 non-null    float64
 15  perc.alumni  777 non-null    int64
 16  Expend       777 non-null    int64
 17  Grad.Rate    777 non-null    int64
dtypes: float64(1), int64(16), object(1)
memory usage: 109.4+ KB
```

Figure: 2

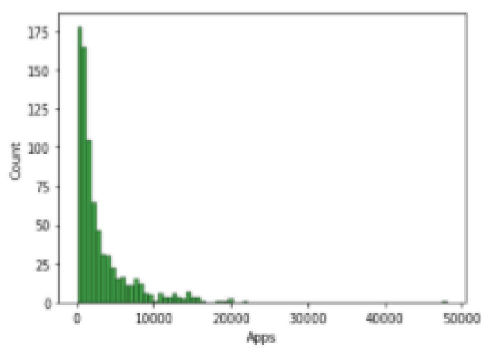
Univariate Analysis: Plotting Histogram, Boxplot & finding Descriptive summary for all 17 numerical columns of dataset, as all columns to be plotted are continuous in nature.

We have eliminated 1st column: 'Names' from the Univariate Analysis, as there are unique mappings for every college in data provided.
So, Count plot, Density plot wouldn't add any value here.

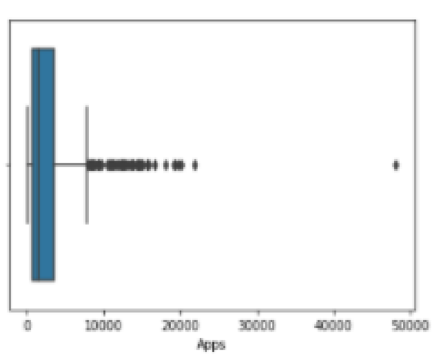
Insights:

- PhD, Terminal, Grad.Rate: **Columns having left-skewed data**
- Expend, perc.alumni, S.F ratio, Personal, P.Undergrad, F.Undergrad, Enroll, Top10perc, Accept, Apps : **Columns having right-skewed data**
- Room.Board, Books, Outstate, Top25perc: **Columns having data following close to normal distribution**
- There are outliers/extreme values present for all columns except 'Top25perc'.
- **'Apps'** data has the maximum range whereas **'S.F.Ratio'** has the minimum range of values.

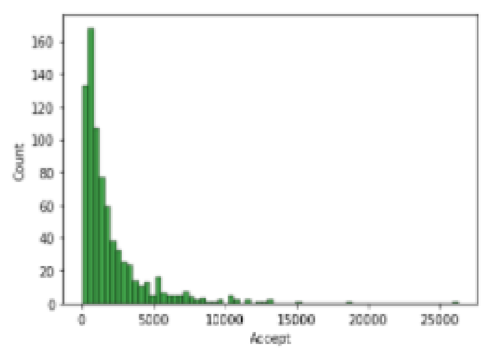
```
Description of Apps
----------------------------------------------------------
count      777.000000
mean      3001.638353
std       3870.201484
min         81.000000
25%        776.000000
50%       1558.000000
75%       3624.000000
max      48094.000000
Name: Apps, dtype: float64 Distribution of Apps
----------------------------------------------------------
```




```
BoxPlot of Apps
----------------------------------------------------------
```




```
Description of Accept
-----------------------------------------------------------------
count      777.000000
mean      2018.804376
std       2451.113971
min         72.000000
25%        604.000000
50%       1110.000000
75%       2424.000000
max      26330.000000
Name: Accept, dtype: float64 Distribution of Accept
-----------------------------------------------------------------
```




```
BoxPlot of Accept
-----------------------------------------------------------------
```

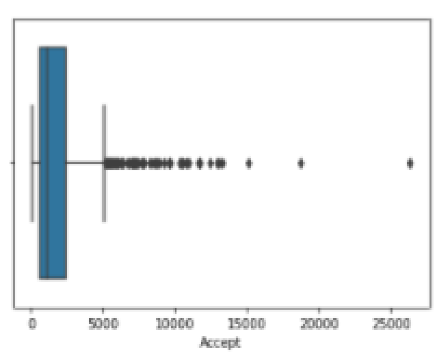
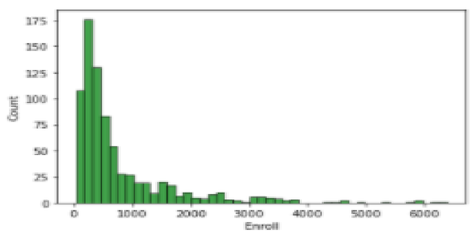


Figure: 3

```
Description of Enroll
----------------------------------------------------------------
count       777.000000
mean        779.972973
std         929.176190
min          35.000000
25%         242.000000
50%         434.000000
75%         902.000000
max        6392.000000
Name: Enroll, dtype: float64 Distribution of Enroll
----------------------------------------------------------------
```




```
BoxPlot of Enroll
----------------------------------------------------------------
```

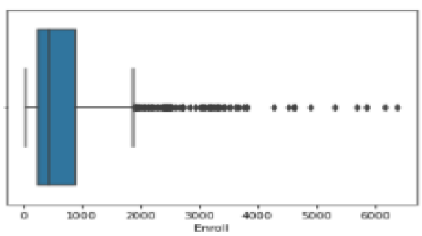
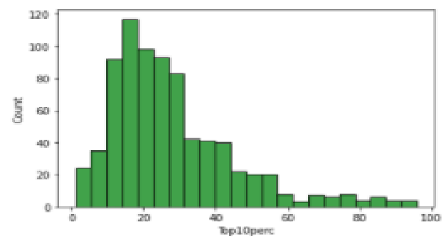


Figure: 5

Figure: 4

```
Description of Top10perc
--------------------------------------------------------------------
count      777.000000
mean        27.558559
std         17.640364
min          1.000000
25%         15.000000
50%         23.000000
75%         35.000000
max         96.000000
Name: Top10perc, dtype: float64 Distribution of Top10perc
--------------------------------------------------------------------
```




```
BoxPlot of Top10perc
--------------------------------------------------------------------
```

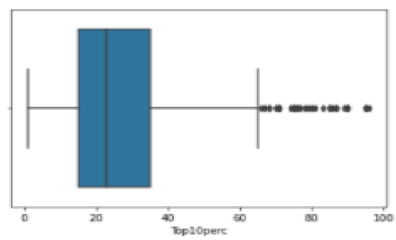
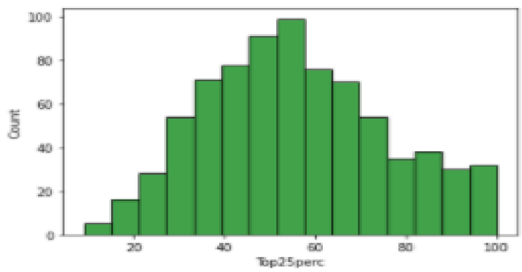


Figure: 6

```
Description of Top25perc
------------------------------------------------------------
count     777.000000
mean       55.796654
std        19.804778
min         9.000000
25%        41.000000
50%        54.000000
75%        69.000000
max       100.000000
Name: Top25perc, dtype: float64 Distribution of Top25perc
------------------------------------------------------------
```




```
BoxPlot of Top25perc
------------------------------------------------------------
```

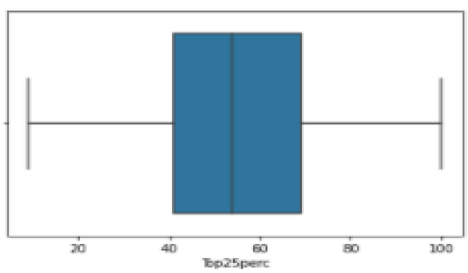
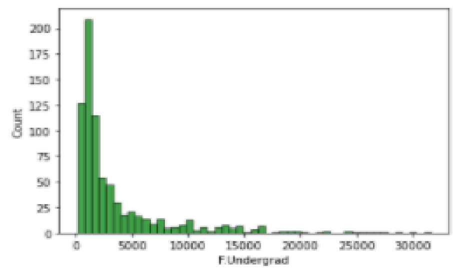


Figure: 7

```
Description of F.Undergrad
------------------------------------------------------------
count       777.000000
mean       3699.907336
std        4850.420531
min         139.000000
25%         992.000000
50%        1707.000000
75%        4005.000000
max       31643.000000
Name: F.Undergrad, dtype: float64 Distribution of F.Undergrad
------------------------------------------------------------
```




```
BoxPlot of F.Undergrad
------------------------------------------------------------
```

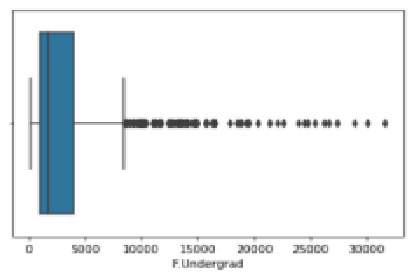
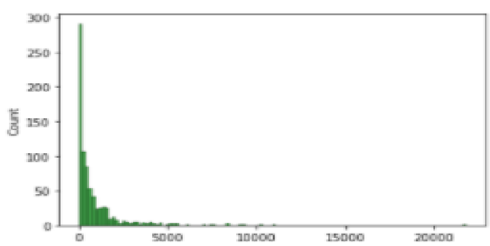


Figure: 8

```
Description of P.Undergrad
------------------------------------------------------------
count       777.000000
mean        855.298584
std        1522.431887
min           1.000000
25%          95.000000
50%         353.000000
75%         967.000000
max       21836.000000
Name: P.Undergrad, dtype: float64 Distribution of P.Undergrad
------------------------------------------------------------
```




```
BoxPlot of P.Undergrad
------------------------------------------------------------
```

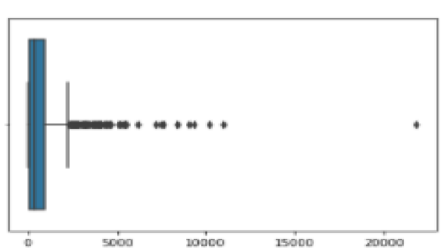
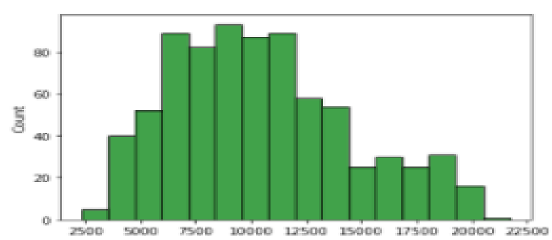


Figure: 9

```
Description of Outstate
------------------------------------------------------------
count       777.000000
mean      10440.669241
std        4023.016484
min        2340.000000
25%        7320.000000
50%        9990.000000
75%       12925.000000
max       21700.000000
Name: Outstate, dtype: float64 Distribution of Outstate
------------------------------------------------------------
```




```
BoxPlot of Outstate
------------------------------------------------------------
```

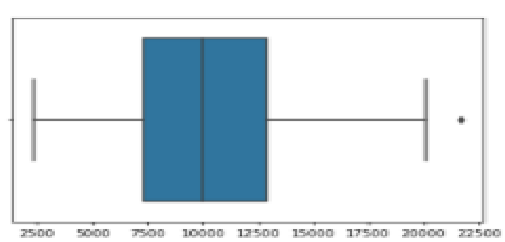
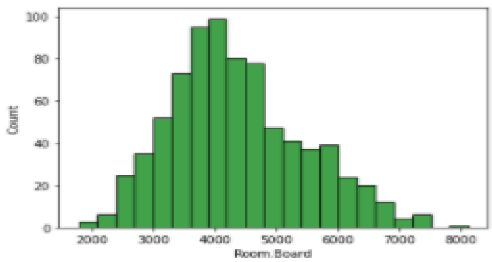


Figure: 10

```
Description of Room.Board
----------------------------------------------------------------
count      777.000000
mean      4357.526384
std       1096.696416
min       1780.000000
25%       3597.000000
50%       4200.000000
75%       5050.000000
max       8124.000000
Name: Room.Board, dtype: float64 Distribution of Room.Board
----------------------------------------------------------------
```




```
BoxPlot of Room.Board
----------------------------------------------------------------
```

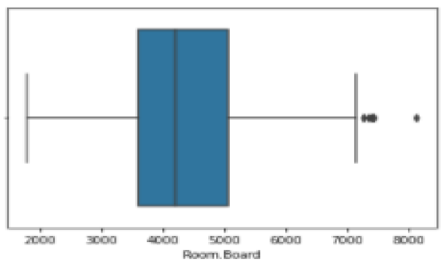
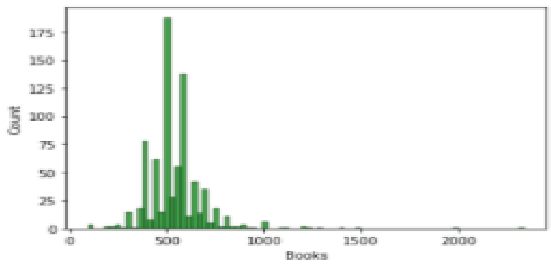


Figure: 12

```
Description of Books
----------------------------------------------------------------
count      777.000000
mean       549.380952
std        165.105360
min         96.000000
25%        470.000000
50%        500.000000
75%        600.000000
max       2340.000000
Name: Books, dtype: float64 Distribution of Books
----------------------------------------------------------------
```




```
BoxPlot of Books
----------------------------------------------------------------
```

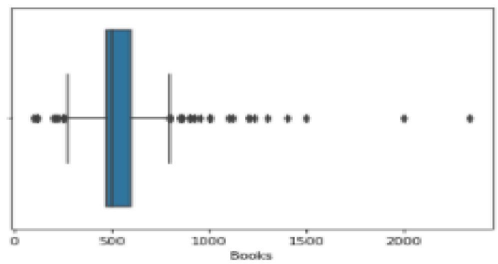
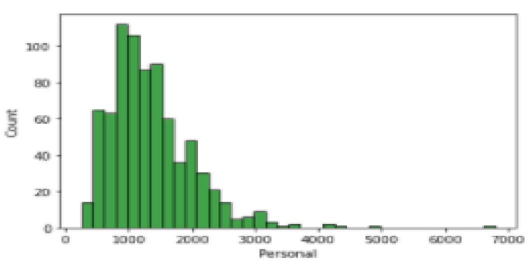


Figure: 12

```
Description of Personal
----------------------------------------------------------------
count      777.000000
mean      1340.642214
std        677.071454
min        250.000000
25%        850.000000
50%       1200.000000
75%       1700.000000
max       6800.000000
Name: Personal, dtype: float64 Distribution of Personal
----------------------------------------------------------------
```




```
BoxPlot of Personal
----------------------------------------------------------------
```

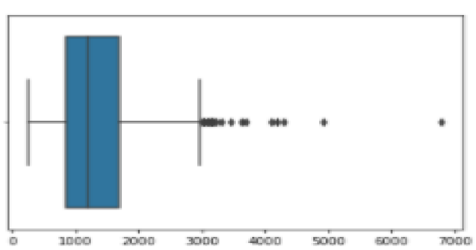
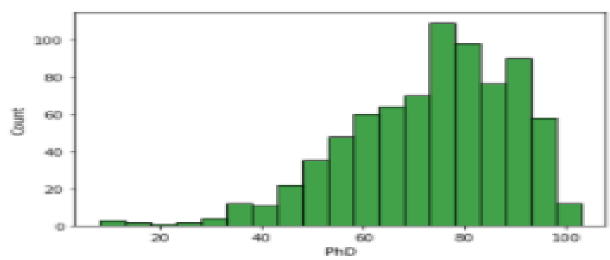


Figure: 13

```
Description of PhD
----------------------------------------------------------------
count      777.000000
mean        72.660232
std         16.328155
min          8.000000
25%         62.000000
50%         75.000000
75%         85.000000
max        103.000000
Name: PhD, dtype: float64 Distribution of PhD
----------------------------------------------------------------
```




```
BoxPlot of PhD
----------------------------------------------------------------
```

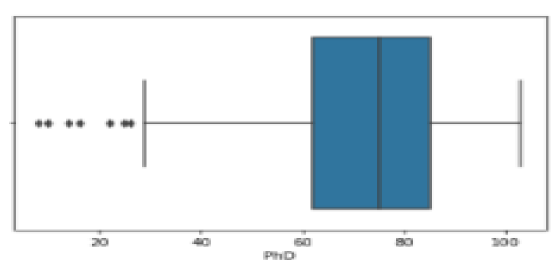
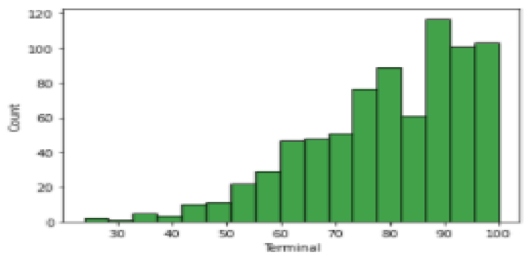


Figure: 14

```
Description of Terminal
------------------------------------------------------------
count     777.000000
mean       79.702703
std        14.722359
min        24.000000
25%        71.000000
50%        82.000000
75%        92.000000
max       100.000000
Name: Terminal, dtype: float64 Distribution of Terminal
------------------------------------------------------------
```




```
BoxPlot of Terminal
------------------------------------------------------------
```

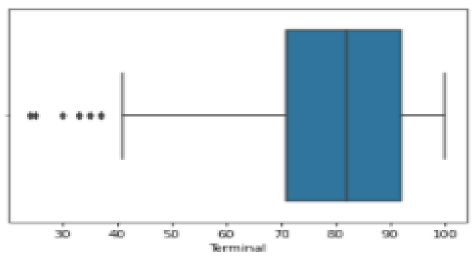
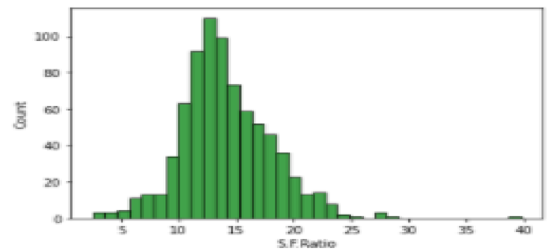


Figure: 15

```
Description of S.F.Ratio
----------------------------------------------------------------
count     777.000000
mean       14.089704
std         3.958349
min         2.500000
25%        11.500000
50%        13.600000
75%        16.500000
max        39.800000
Name: S.F.Ratio, dtype: float64 Distribution of S.F.Ratio
----------------------------------------------------------------
```




```
BoxPlot of S.F.Ratio
----------------------------------------------------------------
```

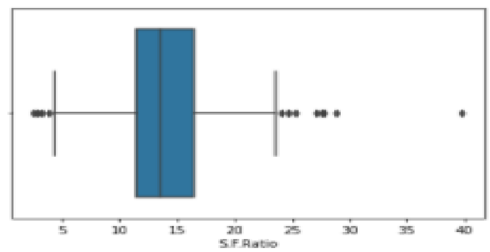
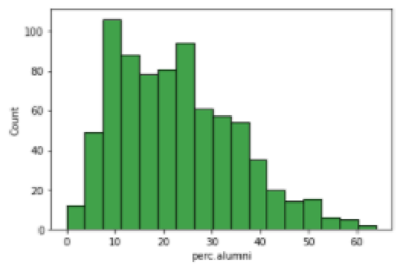


Figure: 16

```
Description of perc.alumni
----------------------------------------------------------------
count     777.000000
mean       22.743887
std        12.391801
min         0.000000
25%        13.000000
50%        21.000000
75%        31.000000
max        64.000000
Name: perc.alumni, dtype: float64 Distribution of perc.alumni
----------------------------------------------------------------
```




```
BoxPlot of perc.alumni
----------------------------------------------------------------
```

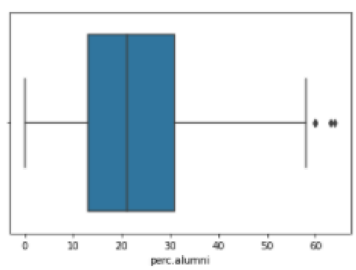
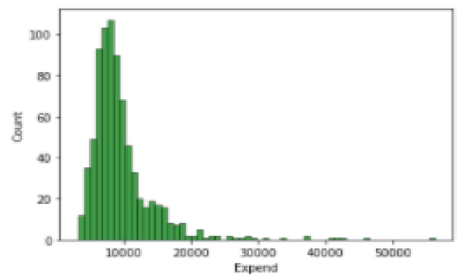


Figure: 17

```
Description of Expend
----------------------------------------------------------------
count       777.000000
mean       9660.171171
std        5221.768440
min        3186.000000
25%        6751.000000
50%        8377.000000
75%       10830.000000
max       56233.000000
Name: Expend, dtype: float64 Distribution of Expend
----------------------------------------------------------------
```




```
BoxPlot of Expend
----------------------------------------------------------------
```

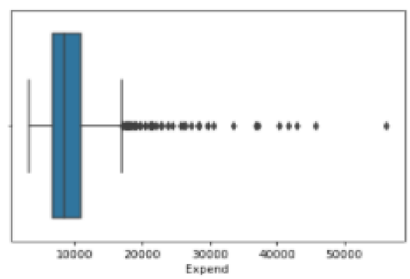
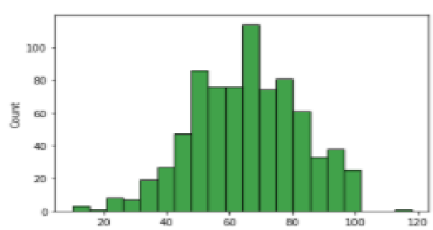


Figure: 18

```
Description of Grad.Rate
----------------------------------------------------------------
count     777.00000
mean       65.46332
std        17.17771
min        10.00000
25%        53.00000
50%        65.00000
75%        78.00000
max       118.00000
Name: Grad.Rate, dtype: float64 Distribution of Grad.Rate
----------------------------------------------------------------
```




```
BoxPlot of Grad.Rate
----------------------------------------------------------------
```

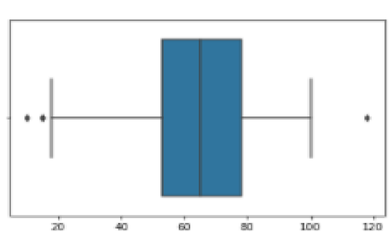


Figure: 19

Bivariate Analysis: Plotting Pair plot (combination of scatterplots), Heat plot is preferred for better readability of all the 17 numerical columns of dataset.

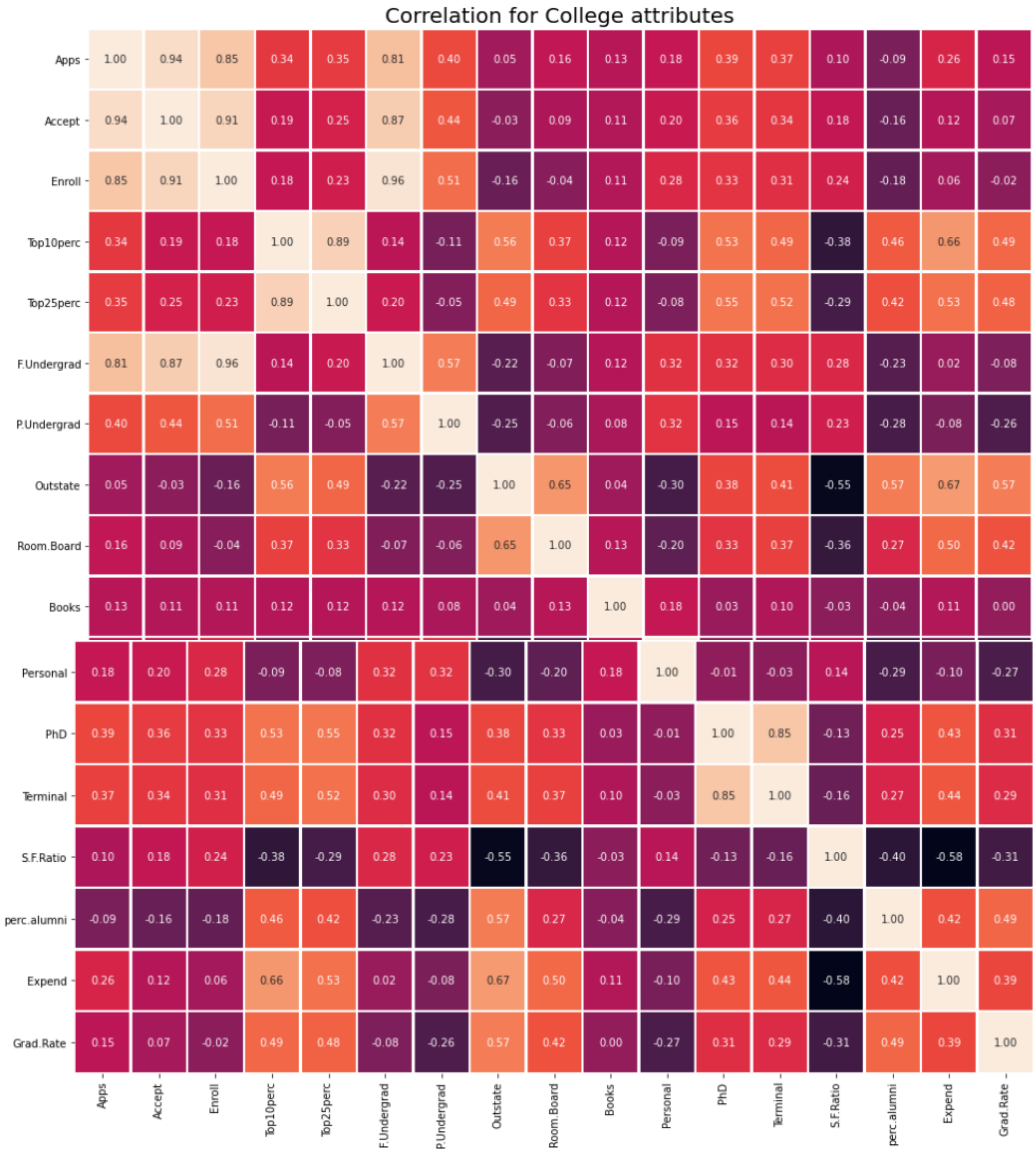


Figure: 20

Insights:

Columns ‘Enroll’ & ‘F.Undergrad’ have the strongest correlation = 0.96
Columns ‘Expend’ & ‘S.F.Ratio’ have the weakest correlation = -0.58

2. Is scaling necessary for PCA in this case? Give justification and perform scaling.

PCA tries to fetch the features with maximum variance, and the variance is high for high magnitude features.

If scaling is not done, the features with high magnitudes will weigh in a lot more than the others with low magnitudes while calculating distance.

In this case, college dataset contains features highly varying in range. (For example, range of 'Apps' = 48013, whereas range of 'Perc. Alumni' = 64).
So, we must bring all features to the same level of magnitudes by performing Scaling.
The scaled data obtained ranges from **-1 to +1** for **Standard Scaler method**, whereas it ranges from **-3 to +3** for **Z-score scaling method.**

```
df_clg.describe().T
```

	count	mean	std	min	25%	50%	75%	max
Apps	777.0	3001.638353	3870.201484	81.0	776.0	1558.0	3624.0	48094.0
Accept	777.0	2018.804376	2451.113971	72.0	604.0	1110.0	2424.0	26330.0
Enroll	777.0	779.972973	929.176190	35.0	242.0	434.0	902.0	6392.0
Top10perc	777.0	27.558559	17.640364	1.0	15.0	23.0	35.0	96.0
Top25perc	777.0	55.796654	19.804778	9.0	41.0	54.0	69.0	100.0
F.Undergrad	777.0	3699.907336	4850.420531	139.0	992.0	1707.0	4005.0	31643.0
P.Undergrad	777.0	855.298584	1522.431887	1.0	95.0	353.0	967.0	21836.0
Outstate	777.0	10440.669241	4023.016484	2340.0	7320.0	9990.0	12925.0	21700.0
Room.Board	777.0	4357.526384	1096.696416	1780.0	3597.0	4200.0	5050.0	8124.0
Books	777.0	549.380952	165.105360	96.0	470.0	500.0	600.0	2340.0
Personal	777.0	1340.642214	677.071454	250.0	850.0	1200.0	1700.0	6800.0
PhD	777.0	72.660232	16.328155	8.0	62.0	75.0	85.0	103.0
Terminal	777.0	79.702703	14.722359	24.0	71.0	82.0	92.0	100.0
S.F.Ratio	777.0	14.089704	3.958349	2.5	11.5	13.6	16.5	39.8
perc.alumni	777.0	22.743887	12.391801	0.0	13.0	21.0	31.0	64.0
Expend	777.0	9660.171171	5221.768440	3186.0	6751.0	8377.0	10830.0	56233.0
Grad.Rate	777.0	65.463320	17.177710	10.0	53.0	65.0	78.0	118.0

Figure: 21

SCALED DATASET USING STANDARD SCALER

	0	1	2	3	4
Names	Abilene Christian University	Adelphi University	Adrian College	Agnes Scott College	Alaska Pacific University
Apps	-0.346882	-0.210884	-0.406866	-0.668261	-0.726176
Accept	-0.321205	-0.038703	-0.376318	-0.681682	-0.764555
Enroll	-0.063509	-0.288584	-0.478121	-0.692427	-0.780735
Top10perc	-0.258583	-0.655656	-0.315307	1.840231	-0.655656
Top25perc	-0.191827	-1.353911	-0.292878	1.677612	-0.596031
F.Undergrad	-0.168116	-0.209788	-0.549565	-0.658079	-0.711924
P.Undergrad	-0.209207	0.244307	-0.49709	-0.520752	0.009005
Outstate	-0.746356	0.457496	0.201305	0.626633	-0.716508
Room.Board	-0.964905	1.909208	-0.554317	0.996791	-0.216723
Books	-0.602312	1.21588	-0.905344	-0.602312	1.518912
Personal	1.270045	0.235515	-0.259582	-0.688173	0.235515
PhD	-0.163028	-2.675646	-1.204845	1.185206	0.204672
Terminal	-0.115729	-3.378176	-0.931341	1.175657	-0.523535
S.F.Ratio	1.013776	-0.477704	-0.300749	-1.615274	-0.553542
perc.alumni	-0.867574	-0.544572	0.585935	1.151188	-1.675079
Expend	-0.50191	0.16611	-0.17729	1.792851	0.241803
Grad.Rate	-0.318252	-0.551262	-0.667767	-0.376504	-2.939613

Figure: 22

SCALED DATASET USING ZSCORE

	0	1	2	3	4
Apps	-0.346882	-0.210884	-0.406866	-0.668261	-0.726176
Accept	-0.321205	-0.038703	-0.376318	-0.681682	-0.764555
Enroll	-0.063509	-0.288584	-0.478121	-0.692427	-0.780735
Top10perc	-0.258583	-0.655656	-0.315307	1.840231	-0.655656
Top25perc	-0.191827	-1.353911	-0.292878	1.677612	-0.596031
F.Undergrad	-0.168116	-0.209788	-0.549565	-0.658079	-0.711924
P.Undergrad	-0.209207	0.244307	-0.497090	-0.520752	0.009005
Outstate	-0.746356	0.457496	0.201305	0.626633	-0.716508
Room.Board	-0.964905	1.909208	-0.554317	0.996791	-0.216723
Books	-0.602312	1.215880	-0.905344	-0.602312	1.518912
Personal	1.270045	0.235515	-0.259582	-0.688173	0.235515
PhD	-0.163028	-2.675646	-1.204845	1.185206	0.204672
Terminal	-0.115729	-3.378176	-0.931341	1.175657	-0.523535
S.F.Ratio	1.013776	-0.477704	-0.300749	-1.615274	-0.553542
perc.alumni	-0.867574	-0.544572	0.585935	1.151188	-1.675079
Expend	-0.501910	0.166110	-0.177290	1.792851	0.241803
Grad.Rate	-0.318252	-0.551262	-0.667767	-0.376504	-2.939613

Figure: 23

If you look at the variables in the scaled dataset, all of them have been normalized, centered on the origins, and scaled in one scale now.

3. Comment on the comparison between the covariance and the correlation matrices from this data [on scaled data].

For the scaled data using z-score, standard deviation is 1. Therefore, the covariance and correlation matrices as per the below relationship must be same, ideally.
Correlation = Covariance (x, y) / {Standard Deviation(x) * Standard Deviation(y)}
There can be some rounding errors while scaling & covariance calculations.

As shown in the Figure: 24 & Figure: 25 below, Covariance & Correlation matrices are approximately same, with an error in the range of 0.001.

```
Covariance matrix :

[[ 1.00128866  0.94466636  0.84791332  0.33927032  0.35209304  0.81554018
   0.3987775   0.05022367  0.16515151  0.13272942  0.17896117  0.39120081
   0.36996762  0.09575627 -0.09034216  0.2599265   0.14694372]
 [ 0.94466636  1.00128866  0.91281145  0.19269493  0.24779465  0.87534985
   0.44183938 -0.02578774  0.09101577  0.11367165  0.20124767  0.35621633
   0.3380184   0.17645611 -0.16019604  0.12487773  0.06739929]
 [ 0.84791332  0.91281145  1.00128866  0.18152715  0.2270373   0.96588274
   0.51372977 -0.1556777  -0.04028353  0.11285614  0.28129148  0.33189629
   0.30867133  0.23757707 -0.18102711  0.06425192 -0.02236983]
 [ 0.33927032  0.19269493  0.18152715  1.00128866  0.89314445  0.1414708
  -0.10549205  0.5630552   0.37195909  0.1190116  -0.09343665  0.53251337
   0.49176793 -0.38537048  0.45607223  0.6617651   0.49562711]
 [ 0.35209304  0.24779465  0.2270373   0.89314445  1.00128866  0.19970167
  -0.05364569  0.49002449  0.33191707  0.115676   -0.08091441  0.54656564
   0.52542506 -0.29500852  0.41840277  0.52812713  0.47789622]
 [ 0.81554018  0.87534985  0.96588274  0.1414708   0.19970167  1.00128866
   0.57124738 -0.21602002 -0.06897917  0.11569867  0.31760831  0.3187472
   0.30040557  0.28006379 -0.22975792  0.01867565 -0.07887464]
 [ 0.3987775   0.44183938  0.51372977 -0.10549205 -0.05364569  0.57124738
   1.00128866 -0.25383901 -0.06140453  0.08130416  0.32029384  0.14930637
   0.14208644  0.23283016 -0.28115421 -0.08367612 -0.25733218]
 [ 0.05022367 -0.02578774 -0.1556777   0.5630552   0.49002449 -0.21602002
  -0.25383901  1.00128866  0.65509951  0.03890494 -0.29947232  0.38347594
   0.40850895 -0.55553625  0.56699214  0.6736456   0.57202613]
 [ 0.16515151  0.09101577 -0.04028353  0.37195909  0.33191707 -0.06897917
  -0.06140453  0.65509951  1.00128866  0.12812787 -0.19968518  0.32962651
   0.3750222  -0.36309504  0.27271444  0.50238599  0.42548915]
 [ 0.13272942  0.11367165  0.11285614  0.1190116   0.115676    0.11569867
   0.08130416  0.03890494  0.12812787  1.00128866  0.17952581  0.0269404
   0.10008351 -0.03197042 -0.04025955  0.11255393  0.00106226]
 [ 0.17896117  0.20124767  0.28129148 -0.09343665 -0.08091441  0.31760831
   0.32029384 -0.29947232 -0.19968518  0.17952581  1.00128866 -0.01094989
  -0.03065256  0.13652054 -0.2863366  -0.09801804 -0.26969106]
 [ 0.39120081  0.35621633  0.33189629  0.53251337  0.54656564  0.3187472
   0.14930637  0.38347594  0.32962651  0.0269404  -0.01094989  1.00128866
   0.85068186 -0.13069832  0.24932955  0.43331936  0.30543094]
 [ 0.36996762  0.3380184   0.30867133  0.49176793  0.52542506  0.30040557
   0.14208644  0.40850895  0.3750222   0.10008351 -0.03065256  0.85068186
   1.00128866 -0.16031027  0.26747453  0.43936469  0.28990033]
 [ 0.09575627  0.17645611  0.23757707 -0.38537048 -0.29500852  0.28006379
   0.23283016 -0.55553625 -0.36309504 -0.03197042  0.13652054 -0.13069832
  -0.16031027  1.00128866 -0.4034484  -0.5845844  -0.30710565]
 [-0.09034216 -0.16019604 -0.18102711  0.45607223  0.41840277 -0.22975792
  -0.28115421  0.56699214  0.27271444 -0.04025955 -0.2863366   0.24932955
   0.26747453 -0.4034484   1.00128866  0.41825001  0.49153016]
 [ 0.2599265   0.12487773  0.06425192  0.6617651   0.52812713  0.01867565
  -0.08367612  0.6736456   0.50238599  0.11255393 -0.09801804  0.43331936
   0.43936469 -0.5845844   0.41825001  1.00128866  0.39084571]
 [ 0.14694372  0.06739929 -0.02236983  0.49562711  0.47789622 -0.07887464
  -0.25733218  0.57202613  0.42548915  0.00106226 -0.26969106  0.30543094
   0.28990033 -0.30710565  0.49153016  0.39084571  1.00128866]]
```

Figure: 24 Covariance Matrix

	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board	Books	Personal	PhD	Terminal	S.F.Ratio	perc.alumni	Expend	Grad.Rate
Apps	1	0.943451	0.846822	0.338834	0.35164	0.814491	0.398264	0.050159	0.164939	0.132559	0.178731	0.390697	0.369491	0.095633	-0.090226	0.259592	0.146755
Accept	0.943451	1	0.911637	0.192447	0.247476	0.874223	0.441271	-0.025755	0.090899	0.113525	0.200989	0.355758	0.337583	0.176229	-0.15999	0.124717	0.067313
Enroll	0.846822	0.911637	1	0.181294	0.226745	0.96464	0.513069	-0.155477	-0.040232	0.112711	0.280929	0.331469	0.308274	0.237271	-0.180794	0.064169	-0.022341
Top10perc	0.338834	0.192447	0.181294	1	0.891995	0.141289	-0.105356	0.562331	0.37148	0.118858	-0.093316	0.531828	0.491135	-0.384875	0.455485	0.660913	0.494989
Top25perc	0.35164	0.247476	0.226745	0.891995	1	0.199445	-0.053577	0.489394	0.33149	0.115527	-0.08081	0.545862	0.524749	-0.294629	0.417864	0.527447	0.477281
F.Undergrad	0.814491	0.874223	0.96464	0.141289	0.199445	1	0.570512	-0.215742	-0.06889	0.11555	0.3172	0.318337	0.300019	0.279703	-0.229462	0.018652	-0.078773
P.Undergrad	0.398264	0.441271	0.513069	-0.105356	-0.053577	0.570512	1	-0.253512	-0.061326	0.0812	0.319882	0.149114	0.141904	0.232531	-0.280792	-0.083568	-0.257001
Outstate	0.050159	-0.025755	-0.155477	0.562331	0.489394	-0.215742	-0.253512	1	0.654256	0.038855	-0.299087	0.382982	0.407983	-0.554821	0.566262	0.672779	0.57129
Room.Board	0.164939	0.090899	-0.040232	0.37148	0.33149	-0.06889	-0.061326	0.654256	1	0.127963	-0.199428	0.329202	0.37454	-0.362628	0.272363	0.501739	0.424942
Books	0.132559	0.113525	0.112711	0.118858	0.115527	0.11555	0.0812	0.038855	0.127963	1	0.179295	0.026906	0.099955	-0.031929	-0.040208	0.112409	0.001061
Personal	0.178731	0.200989	0.280929	-0.093316	-0.08081	0.3172	0.319882	-0.299087	-0.199428	0.179295	1	-0.010936	-0.030613	0.136345	-0.285968	-0.097892	-0.269344
PhD	0.390697	0.355758	0.331469	0.531828	0.545862	0.318337	0.149114	0.382982	0.329202	0.026906	-0.010936	1	0.849587	-0.13053	0.249009	0.432762	0.305038
Terminal	0.369491	0.337583	0.308274	0.491135	0.524749	0.300019	0.141904	0.407983	0.37454	0.099955	-0.030613	0.849587	1	-0.160104	0.26713	0.438799	0.289527
S.F.Ratio	0.095633	0.176229	0.237271	-0.384875	-0.294629	0.279703	0.232531	-0.554821	-0.362628	-0.031929	0.136345	-0.13053	-0.160104	1	-0.402929	-0.583832	-0.30671
perc.alumni	-0.090226	-0.15999	-0.180794	0.455485	0.417864	-0.229462	-0.280792	0.566262	0.272363	-0.040208	-0.285968	0.249009	0.26713	-0.402929	1	0.417712	0.490898
Expend	0.259592	0.124717	0.064169	0.660913	0.527447	0.018652	-0.083568	0.672779	0.501739	0.112409	-0.097892	0.432762	0.438799	-0.583832	0.417712	1	0.390343
Grad.Rate	0.146755	0.067313	-0.022341	0.494989	0.477281	-0.078773	-0.257001	0.57129	0.424942	0.001061	-0.269344	0.305038	0.289527	-0.30671	0.490898	0.390343	1

Figure: 25 Correlation Matrix

4. **Check the dataset for outliers before and after scaling. What insight do you derive here?**

Here are the boxplots of unscaled data **(Figure: 26, Before scaling)** v/s scaled data **(Figure: 27, After scaling).**

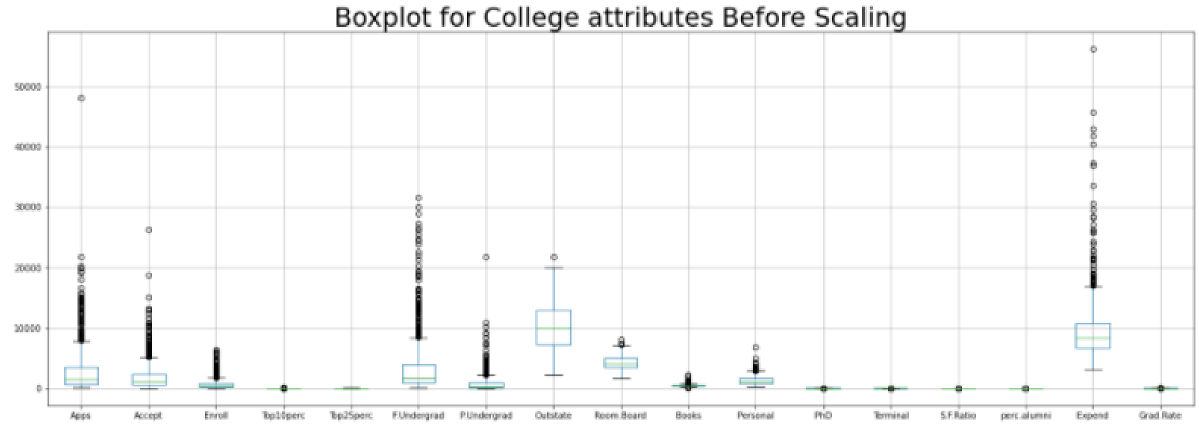


Figure: 26

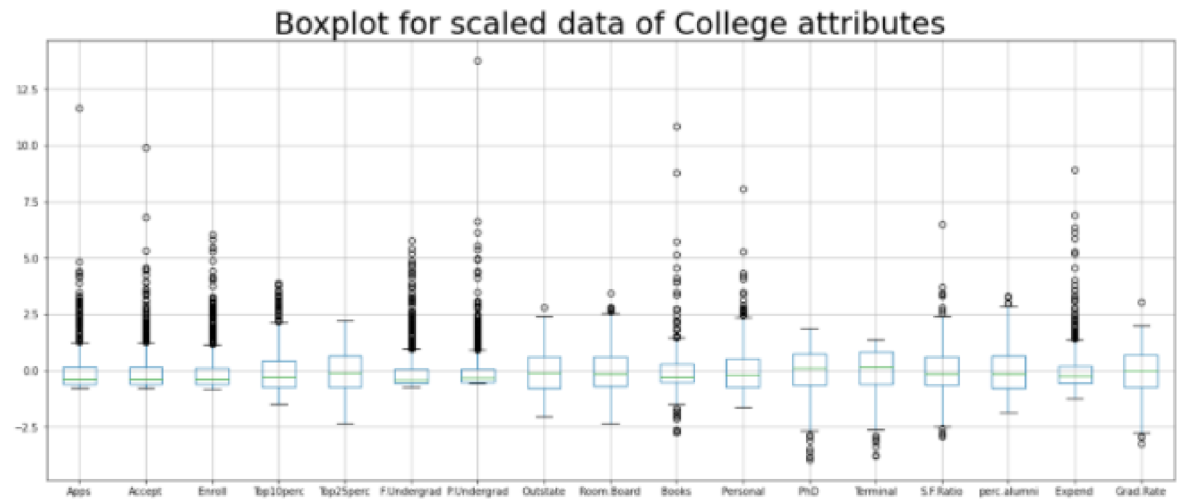


Figure: 27

Insights:

- Here data on all the dimensions are subtracted from their means to shift the data points to the origin. Hence, we get centralized data, mean=0 & Standard deviation=1, post scaling. So, the direct comparison between the data of 2 different columns becomes easier.
- As the range reduces by a considerable factor, the outliers can be visualized easily.
- In order to perform PCA on this dataset, scaled data will provide the variable with highest correlation. If not scaled, PCA will pick variable with highest variance.

5. **Extract the eigenvalues and eigenvectors.**

Steps followed:

- Begin by standardizing the data. Data on all the dimensions are subtracted from their means to shift the data points to the origin. i.e. the data is centered on the origins
- Generate the covariance matrix/correlation matrix for all the dimensions
- Perform eigen-decomposition, that is, compute eigenvectors which are the principal components, and the corresponding eigenvalues which are the magnitudes of variance captured.
- Sort the eigen-pairs in descending order of eigenvalues and select the one with the largest value. This is the first principal component that covers the maximum information from the original data.

EIGEN VALUES: Figure: 28

```
[5.45052162 4.48360686 1.17466761 1.00820573 0.93423123 0.84849117
0.6057878  0.58787222 0.53061262 0.4043029  0.02302787 0.03672545
0.31344588 0.08802464 0.1439785  0.16779415 0.22061096]
```

EIGEN VECTORS: Figure: 29

```
[[-2.48765602e-01  3.31598227e-01  6.30921033e-02 -2.81310530e-01
   5.74140964e-03  1.62374420e-02  4.24863486e-02  1.03090398e-01
   9.02270802e-02 -5.25098025e-02  3.58970400e-01 -4.59139498e-01
   4.30462074e-02 -1.33405806e-01  8.06328039e-02 -5.95830975e-01
   2.40709086e-02]
 [-2.07601502e-01  3.72116750e-01  1.01249056e-01 -2.67817346e-01
   5.57860920e-02 -7.53468452e-03  1.29497196e-02  5.62709623e-02
   1.77864814e-01 -4.11400844e-02 -5.43427250e-01  5.18568789e-01
  -5.84055850e-02  1.45497511e-01  3.34674281e-02 -2.92642398e-01
  -1.45102446e-01]
 [-1.76303592e-01  4.03724252e-01  8.29855709e-02 -1.61826771e-01
  -5.56936353e-02  4.25579803e-02  2.76928937e-02 -5.86623552e-02
   1.28560713e-01 -3.44879147e-02  6.09651110e-01  4.04318439e-01
  -6.93988831e-02 -2.95896092e-02 -8.56967180e-02  4.44638207e-01
   1.11431545e-02]
 [-3.54273947e-01 -8.24118211e-02 -3.50555339e-02  5.15472524e-02
  -3.95434345e-01  5.26927980e-02  1.61332069e-01  1.22678028e-01
  -3.41099863e-01 -6.40257785e-02 -1.44986329e-01  1.48738723e-01
  -8.10481404e-03 -6.97722522e-01 -1.07828189e-01 -1.02303616e-03
   3.85543001e-02]
 [-3.44001279e-01 -4.47786551e-02  2.41479376e-02  1.09766541e-01
  -4.26533594e-01 -3.30915896e-02  1.18485556e-01  1.02491967e-01
  -4.03711989e-01 -1.45492289e-02  8.03478445e-02 -5.18683400e-02
  -2.73128469e-01  6.17274818e-01  1.51742110e-01 -2.18838802e-02
  -8.93515563e-02]
 [-1.54640962e-01  4.17673774e-01  6.13929764e-02 -1.00412335e-01
  -4.34543659e-02  4.34542349e-02  2.50763629e-02 -7.88896442e-02
   5.94419181e-02 -2.08471834e-02 -4.14705279e-01 -5.60363054e-01
  -8.11578181e-02 -9.91640992e-03 -5.63728817e-02  5.23622267e-01
   5.61767721e-02]
 [-2.64425045e-02  3.15087830e-01 -1.39681716e-01  1.58558487e-01
   3.02385408e-01  1.91198583e-01 -6.10423460e-02 -5.70783816e-01
  -5.60672902e-01  2.23105808e-01  9.01788964e-03  5.27313042e-02
   1.00693324e-01 -2.09515982e-02  1.92857500e-02 -1.25997650e-01
  -6.35360730e-02]
 [-2.94736419e-01 -2.49643522e-01 -4.65988731e-02 -1.31291364e-01
   2.22532003e-01  3.00003910e-02 -1.08528966e-01 -9.84599754e-03
   4.57332880e-03 -1.86675363e-01  5.08995918e-02 -1.01594830e-01
   1.43220673e-01 -3.83544794e-02 -3.40115407e-02  1.41856014e-01
  -8.23443779e-01]
 [-2.49030449e-01 -1.37808883e-01 -1.48967389e-01 -1.84995991e-01
   5.60919470e-01 -1.62755446e-01 -2.09744235e-01  2.21453442e-01
  -2.75022548e-01 -2.98324237e-01  1.14639620e-03  2.59293381e-02
  -3.59321731e-01 -3.40197083e-03 -5.84289756e-02  6.97485854e-02
   3.54559731e-01]
 [-6.47575181e-02  5.63418434e-02 -6.77411649e-01 -8.70892205e-02
  -1.27288825e-01 -6.41054950e-01  1.49692034e-01 -2.13293009e-01
   1.33663353e-01  8.20292186e-02  7.72631963e-04 -2.88282896e-03
   3.19400370e-02  9.43887925e-03 -6.68494643e-02 -1.14379958e-02
  -2.81593679e-02]
 [ 4.25285386e-02  2.19929218e-01 -4.99721120e-01  2.30710568e-01
  -2.22311021e-01  3.31398003e-01 -6.33790064e-01  2.32660840e-01
   9.44688900e-02 -1.36027616e-01 -1.11433396e-03  1.28904022e-02
  -1.85784733e-02  3.09001353e-03  2.75286207e-02 -3.94547417e-02
  -3.92640266e-02]
 [-3.18312875e-01  5.83113174e-02  1.27028371e-01  5.34724832e-01
   1.40166326e-01 -9.12555212e-02  1.09641298e-03  7.70400002e-02
   1.85181525e-01  1.23452200e-01  1.38133366e-02 -2.98075465e-02
   4.03723253e-02  1.12055599e-01 -6.91126145e-01 -1.27696382e-01
   2.32224316e-02]
 [-3.17056016e-01  4.64294477e-02  6.60375454e-02  5.19443019e-01
   2.04719730e-01 -1.54927646e-01  2.84770105e-02  1.21613297e-02
   2.54938198e-01  8.85784627e-02  6.20932749e-03  2.70759809e-02
  -5.89734026e-02 -1.58909651e-01  6.71008607e-01  5.83134662e-02
   1.64850420e-02]
 [ 1.76957895e-01  2.46665277e-01  2.89848401e-01  1.61189487e-01
  -7.93882496e-02 -4.87045875e-01 -2.19259358e-01  8.36048735e-02
  -2.74544380e-01 -4.72045249e-01 -2.22215182e-03  2.12476294e-02
   4.45000727e-01  2.08991284e-02  4.13740967e-02  1.77152700e-02
  -1.10262122e-02]
 [-2.05082369e-01 -2.46595274e-01  1.46989274e-01 -1.73142230e-02
  -2.16297411e-01  4.73400144e-02 -2.43321156e-01 -6.78523654e-01
   2.55334907e-01 -4.22999706e-01 -1.91869743e-02 -3.33406243e-03
  -1.30727978e-01  8.41789410e-03 -2.71542091e-02 -1.04088008e-01
   1.82660654e-01]
 [-3.18908750e-01 -1.31689865e-01 -2.26743985e-01 -7.92734946e-02
   7.59581203e-02  2.98118619e-01  2.26584481e-01  5.41593771e-02
   4.91388809e-02 -1.32286331e-01 -3.53098218e-02  4.38803230e-02
   6.92088870e-01  2.27742017e-01  7.31225166e-02  9.37464497e-02
   3.25982295e-01]
 [-2.52315654e-01 -1.69240532e-01  2.08064649e-01 -2.69129066e-01
  -1.09267913e-01 -2.16163313e-01 -5.59943937e-01  5.33553891e-03
  -4.19043052e-02  5.90271067e-01 -1.30710024e-02  5.00844705e-03
   2.19839000e-01  3.39433604e-03  3.64767385e-02  6.91969778e-02
   1.22106697e-01]]
```

6. Perform PCA and export the data of the Principal Component (eigenvectors) into a data frame with the original features

Following steps are followed to fetch principal components:
Step 1- Create the covariance Matrix **(Figure: 24)**
Step 2- Get eigen values and eigen vector (**(Figure: 28 & 29)**
Steps1 & 2 are already shown in the previous questions

Step 3- View Scree Plot **(Figure: 30)** to identify the number of components to be built
By analyzing the scree plot, we conclude that we can proceed with considering only 6 components out of 17, as a part of dimension reduction.

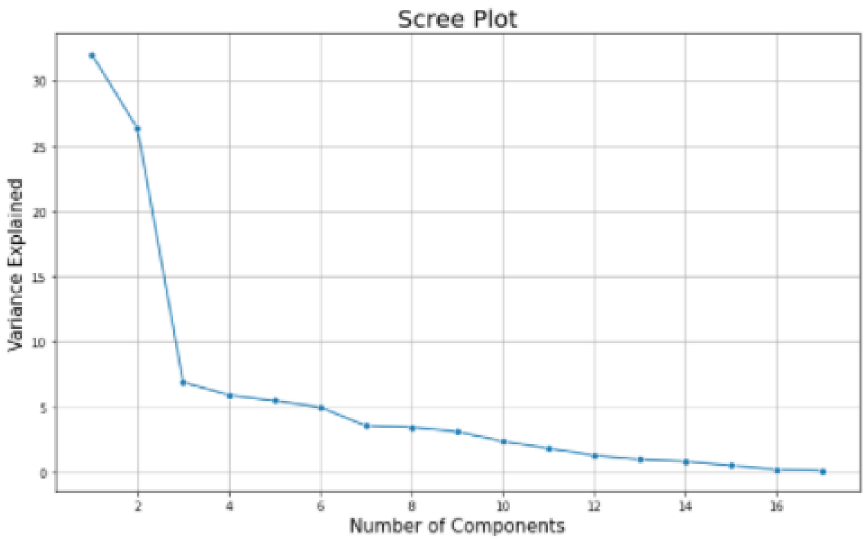


Figure: 30

Step 4- Apply PCA for the number of decided components to get the loadings and component output **(Figure: 31)**

Eigen vectors obtained post dimension reduction:

```
array([[ 0.2487656 ,  0.2076015 ,  0.17630359,  0.35427395,  0.34400128,
         0.15464096,  0.0264425 ,  0.29473642,  0.24903045,  0.06475752,
        -0.04252854,  0.31831287,  0.31705602, -0.17695789,  0.20508237,
         0.31890875,  0.25231565],
       [ 0.33159823,  0.37211675,  0.40372425, -0.08241182, -0.04477866,
         0.41767377,  0.31508783, -0.24964352, -0.13780888,  0.05634184,
         0.21992922,  0.05831132,  0.04642945,  0.24666528, -0.24659527,
        -0.13168986, -0.16924053],
       [-0.06309209, -0.10124907, -0.08298558,  0.03505553, -0.02414794,
        -0.06139296,  0.13968171,  0.04659888,  0.14896739,  0.67741165,
         0.49972112, -0.12702837, -0.06603755, -0.2898484 , -0.14698927,
         0.22674398, -0.20806465],
       [ 0.28131052,  0.26781736,  0.16182679, -0.05154725, -0.10976654,
         0.10041231, -0.15855849,  0.13129136,  0.18499599,  0.08708922,
        -0.23071057, -0.53472483, -0.51944302, -0.16118949,  0.01731422,
         0.0792735 ,  0.26912907],
       [ 0.00574142,  0.05578609, -0.05569364, -0.39543435, -0.42653359,
        -0.04345436,  0.30238541,  0.222532  ,  0.56091947, -0.12728883,
        -0.22231102,  0.14016633,  0.20471973, -0.07938825, -0.21629741,
         0.07595812, -0.10926791],
       [-0.01623744,  0.00753468, -0.04255798, -0.0526928 ,  0.03309159,
        -0.04345423, -0.19119858, -0.03000039,  0.16275545,  0.64105495,
        -0.331398  ,  0.09125552,  0.15492765,  0.48704587, -0.04734001,
        -0.29811862,  0.21616331]])
```

Figure: 31

Below is the exported data of the Principal Component (eigenvectors) into a data frame with the original features (Figure: 32)

```
df_pca_loading.T
```

	0	1	2	3	4	5
Apps	0.248766	0.331598	-0.063092	0.281311	0.005741	-0.016237
Accept	0.207602	0.372117	-0.101249	0.267817	0.055786	0.007535
Enroll	0.176304	0.403724	-0.082986	0.161827	-0.055694	-0.042558
Top10perc	0.354274	-0.082412	0.035056	-0.051547	-0.395434	-0.052693
Top25perc	0.344001	-0.044779	-0.024148	-0.109767	-0.426534	0.033092
F.Undergrad	0.154641	0.417674	-0.061393	0.100412	-0.043454	-0.043454
P.Undergrad	0.026443	0.315088	0.139682	-0.158558	0.302385	-0.191199
Outstate	0.294736	-0.249644	0.046599	0.131291	0.222532	-0.030000
Room.Board	0.249030	-0.137809	0.148967	0.184996	0.560919	0.162755
Books	0.064758	0.056342	0.677412	0.087089	-0.127289	0.641055
Personal	-0.042529	0.219929	0.499721	-0.230711	-0.222311	-0.331398
PhD	0.318313	0.058311	-0.127028	-0.534725	0.140166	0.091256
Terminal	0.317056	0.046429	-0.066038	-0.519443	0.204720	0.154928
S.F.Ratio	-0.176958	0.246665	-0.289848	-0.161189	-0.079388	0.487046
perc.alumni	0.205082	-0.246595	-0.146989	0.017314	-0.216297	-0.047340
Expend	0.318909	-0.131690	0.226744	0.079273	0.075958	-0.298119
Grad.Rate	0.252316	-0.169241	-0.208065	0.269129	-0.109268	0.216163

Figure: 32

7. **Write down the explicit form of the first PC (in terms of the eigenvectors. Use values with two places of decimals only).**

All principal components obtained are linear combination of original features.
The coefficients of linearity are determined using the Eigen vectors shown in the Figure: 31
Each row in that array corresponds to a principal component.

Thus, the first principal component, substituting the Original features is represented as:
(Considering two places of decimals, Eigen vector components are rounded off here)

PC0 = 0.25*Apps + 0.21*Accept + 0.18*Enroll + 0.35* Top10perc + 0.34* Top25perc + 0.15* F.Undergrad + 0.03* P.Undergrad + 0.29*Outstate + 0.25*Room.Board + 0.06*Books - 0.04*Personal + 0.32*PhD + 0.32*Terminal - 0.18*S.F.Ratio + 0.21*perc.alumni + 0.32*Expend + 0.25*Grad.Rate

We can fetch the values of PC0 by substituting the values (1st row of all columns of the dataset in this case, refer **Figure:** 23 for scaled dataset) in the equation shown above.

Post calculations, we get the first principal component, PC0 = -1.592855
Similarly, other values can be calculated. Below data frame shows the values of PCs obtained.
This dataset can be used for any further analysis

NEW DATASET - POST DIMENSION REDUCTION

	0	1	2	3	4
Names	Abilene Christian University	Adelphi University	Adrian College	Agnes Scott College	Alaska Pacific University
PC_Expenses_Of_Outstates	-1.592855	-2.192402	-1.430964	2.855557	-2.212008
PC_College_Funds_Collected_From_Students	0.767334	-0.57883	-1.092819	-2.630612	0.021631
PC_Student_Expenses	-0.101074	2.278798	-0.438093	0.141722	2.38703
PC_Graduates_Or_More	-0.921749	3.588918	0.677241	-1.295486	-1.114538
PC_Accomodation_Expenses_Incurred_On_Toppers	-0.743975	1.059997	-0.369613	-0.183837	0.684451
PC_Student_By_Faculty_Ratio	-0.298306	-0.177137	-0.960592	-1.059508	0.004918

8. **Consider the cumulative values of the eigenvalues. How does it help you to decide on the optimum number of principal components? What do the eigenvector s indicate?**

Following are the cumulative values calculated for the Eigenvalues (**Figure: 33)**

```
Cumulative variable explained : [ 32.0206282   58.36084263  65.26175919  71.18474841  76.67315352
  81.65785448  85.21672597  88.67034731  91.78758099  94.16277251
  96.00419883  97.30024023  98.28599436  99.13183669  99.64896227
  99.86471628 100.        ]
```

Figure: 33

We consider only the principal components which constitute around 80% coverage of the total variance generally. As seen in **Figure: 34**, first 6 components contribute around 81% of total variance of the College dataset.

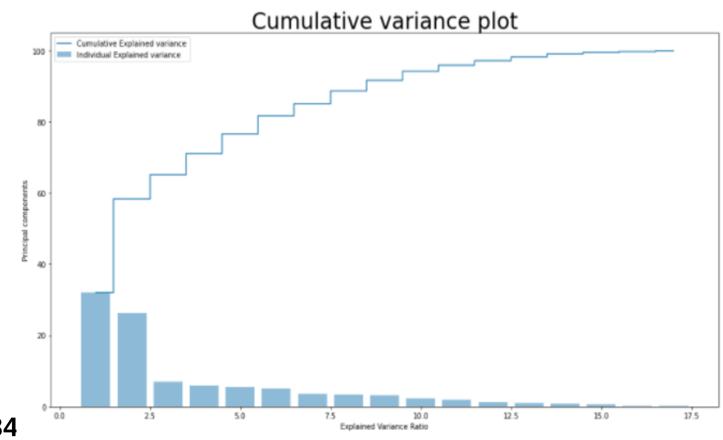


Figure: 34

The eigenvectors in this case indicate the direction of data variance of a particular column. Ideally, the number of Eigen vectors will be equal to the number of features present. However, there will be some directions along which the variance will be predominant.

The strength of variance along various Eigen vectors or directions is determined from Eigen values.
In this particular case, Top 6 Eigen values, when sorted in descending order, as shown in Figure: 33
Contribute to 81% of the variance.
Therefore, the Eigen vectors corresponding to these 6 Eigen values represent the Principal components.

9. **Explain the business implication of using the Principal Component Analysis for this case study. How may PCs help in the further analysis?**

Business Implications of using PCA:

- The number of dimensions have been reduced significantly from 17 to 6, which contributes to approximately 81% of total variance. This helps improve the processing speed, time & performance for data wrangling algorithms.
- Some features from the dataset can have strong correlation with each other, For example., ‘Enroll’ variable has high correlation with ‘Apps’ & ‘Accept’. PCA helps reduce problem of multicollinearity, as the PCs obtained are independent of each other.
- The lower dimensional dataset thus obtained facilitates visualization & memory requirements during further analysis of data.

The interpretation of Principal components has been done by:

- Plotting a heatmap for the data frame having component loading against each field.
- Picking up the variables having highest correlation coefficients across each field.
- Each principal component can be renamed as per the dominant properties of the fields.

- The renamed new features are given as:

    1. PC_Expenses_Of_Outstates: as the PC0(First component) has 'Outstate'(0.29) and 'Expend'(0.32) as dominant features. Similarly, others have been renamed
    2. PC_College_Funds_Collected_From_Students
    3. PC_Student_Expenses
    4. PC_Graduates_Or_More
    5. PC_Accomodation_Expenses_Incurred_On_Toppers
    6. PC_Student_By_Faculty_Ratio

HEATMAP SHOWING DIMENSION REDUCTION

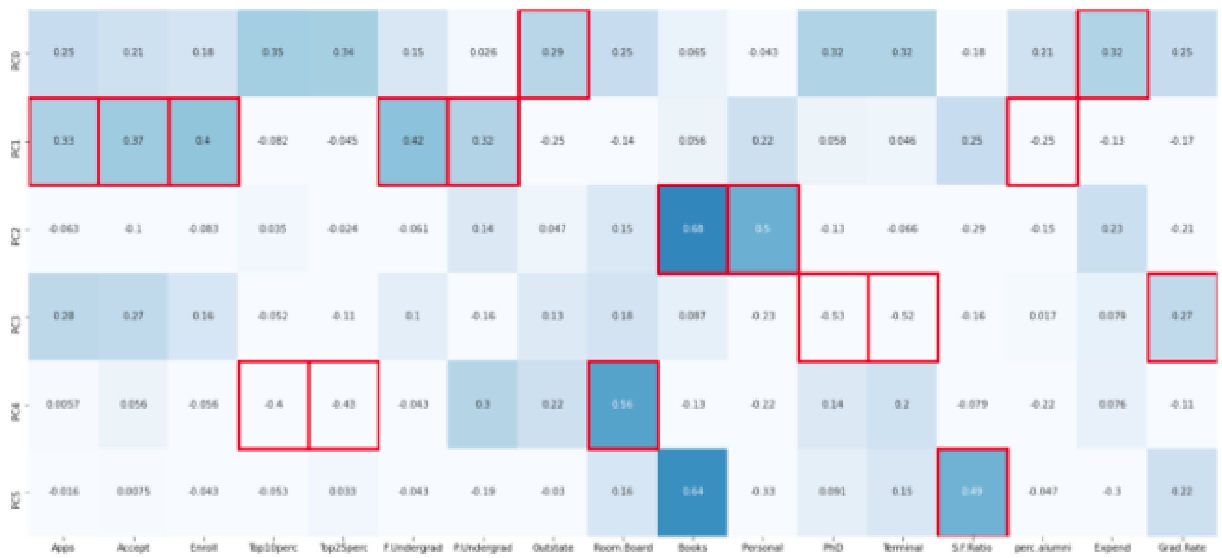


Figure: 35

The 6 reduced variables (considering the ones having highest correlation) have been renamed, as per their business significance, and the new dataset thus formed shows in **Figure 35**.

NEW DATASET - POST DIMENSION REDUCTION

	0	1	2	3	4
Names	Abilene Christian University	Adelphi University	Adrian College	Agnes Scott College	Alaska Pacific University
PC_Expenses_Of_Outstates	-1.592855	-2.192402	-1.430964	2.855557	-2.212008
PC_College_Funds_Collected_From_Students	0.767334	-0.57883	-1.092819	-2.630612	0.021631
PC_Student_Expenses	-0.101074	2.278798	-0.438093	0.141722	2.38703
PC_Graduates_Or_More	-0.921749	3.588918	0.677241	-1.295486	-1.114538
PC_Accomodation_Expenses_Incurred_On_Toppers	-0.743975	1.059997	-0.369613	-0.183837	0.684451
PC_Student_By_Faculty_Ratio	-0.298306	-0.177137	-0.960592	-1.059508	0.004918

Figure: 36

DESCRIPTIVE SUMMARY OF NEW DATASET

	count	mean	std	min	25%	50%	75%	max
PC_Expenses_Of_Outstates	777.0	4.693800e-17	2.334635	-5.662905	-1.731200	-0.299457	1.339533	8.047182
PC_College_Funds_Collected_From_Students	777.0	5.701145e-17	2.117453	-3.590891	-1.348075	-0.626833	0.692447	12.002374
PC_Student_Expenses	777.0	2.286174e-18	1.083821	-2.941286	-0.666305	-0.101074	0.494371	9.006415
PC_Graduates_Or_More	777.0	-9.267933e-17	1.004094	-2.943103	-0.655832	-0.058428	0.598776	5.177647
PC_Accomodation_Expenses_Incurred_On_Toppers	777.0	8.573151e-18	0.966556	-2.690124	-0.699850	-0.051124	0.631348	4.248195
PC_Student_By_Faculty_Ratio	777.0	-1.100221e-17	0.921136	-3.822954	-0.522954	-0.002959	0.455386	5.991244

Figure: 37

BOXPLOT FOR NEW DATASET WITH REDUCED DIMENSIONS

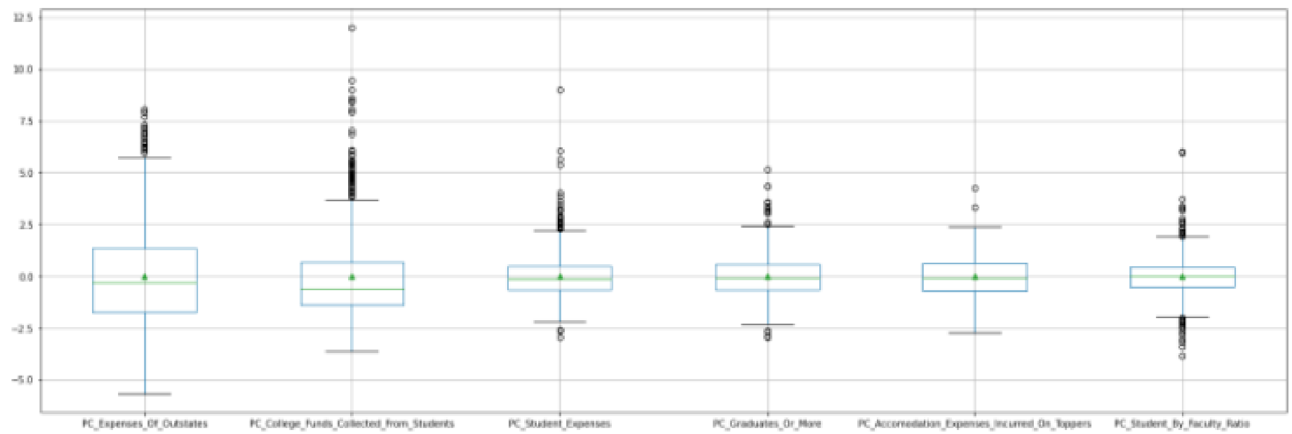


Figure: 38

Conclusion:

- With help of PCA we have been able to reduce 17 numeric features into 6 components which is able to explain 81% of variance in the data.

- With help of reduced components we have been able to observe some patterns. Using some rules around business context **Figure: 38**, we observe that :
  the means of columns: **'PC_Expenses_Of_Outstates'** &
  '**PC_College_Funds_Collected_From_Students'** are relatively higher than the means of other columns.
  Using the components additional rules can be derived and analyzed.

- Unsupervised learning like clustering can further be applied on the data to segment the customers based on the components created and further analyzed.