Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

Feedback Generation in Rendezvous Problems Using Synthetic Dynamics

Raktim Bhattacharya
Department of Aerospace Engineering
Dwight Look College of Engineering
Texas A&M University
College Station, TX, 77843-3141

Abstract— **Rendezvous problems belong to the class of consensus problems where it is desired that vehicles arrive at a desired position simultaneously. Recently, the problem of rendezvous has been addressed considerably in the graph theoretic framework, which is strongly based on the communication aspects of the problem. Stability is analyzed with respect to communication topology. The current methods do not characterize the behavior of the collective system, which is necessary to generate feedback between the vehicles. In this paper we focus on generating feedback between multiple agents such that rendezvous is achieved. We treat the rendezvous problem in the set invariance setting and define rendezvous on the positions of the agents.**

In the proposed framework, the problem of rendezvous is cast as a stabilization problem, with a set of constraints on the trajectories of the agents defined on the phase plane. We pose the n-agent rendezvous problem as an ellipsoidal cone invariance problem in the n dimensional phase space. The necessary and sufficient conditions for rendezvous of linear systems are presented in form of linear matrix inequalities. These conditions are also interpreted in the Lyapunov framework using multiple Lyapunov functions. Numerical examples that demonstrate application are also presented.

I. INTRODUCTION

Recently there has been considerable interest in multi-agent coordination or cooperative control [1]. This has led to the emergence of several interesting control problems. One such problem is the *rendezvous problem*. In a rendezvous problem, one desires to have several agents arrive at predefined destination points *simultaneously*. Cooperative strike or cooperative jamming are two examples of the rendezvous problem. In this paper we assume that the agents arrive at the predefined locations *only once*.

Rendezvous has been recently addressed as a consensus problem in the graph theoretic framework. Lin *et al.* [2] apply consensus seeking to a rendezvous problem for a group of mobile autonomous agents, where both the synchronous case and the asynchronous case are considered. The algorithm presented is provable correct, however does not address uncertainty in communication or dynamics. Cortes *et al.* [3] proposed an iterative algorithm with guaranteed convergence and is robust with respect to communication failures. Jadbabaie *et al.* [4] developed a coordination algorithm based on nearest neighbor rules. Smith *et al.* [5] solves the rendezvous problem with fixed communication topology based on Euclidian curve shortening methods and is restricted to planar rendezvous. Ren *et al.* [6] provides a survey of multi-agent coordination problems based on graph theoretic framework. The strength of the graph theoretic framework is its ability to analyze the communication aspect of the rendezvous problem. It however does not characterize the behavior of the collective system, which is necessary to generate feedback between the vehicles. This is the prime difference between the state-of-the-art in this area and the work presented in this paper.

In the dynamical systems literature the problem of cooperation and competition have been addressed in the context of cone invariance. The cone is used to define a partial order on the system trajectories, which results in the cooperative or competitive behavior of the system. In the seminal work by Hirsch [7], [8], [9], [10], [11], [12] on systems of differential equations that are competitive or cooperative, he developed what is known as *monotone dynamical systems* theory [13]. He demonstrated that the generic solution of a cooperative and irreducible system of differential equations converges to a set of equilibria. Furthermore, the flow on a compact limit set of an n-dimensional cooperative or competitive system of differential equations is shown to be topologically conjugate to the flow of an $n-1$ dimensional system of differential equations, restricted to a compact invariant set.

Invariant sets play an important role in many situations when the behavior of the closed-loop system is constrained in some way. Blanchini [14] provides an excellent survey of set invariant control. Invariant sets that are cones have found application in problems related to areas as diverse as industrial growth [15], ecological systems and symbiotic species [16], arms race [17] and compartmental system analysis [18], [19]. In general, cone invariance is an essential component in problems involving competition or cooperation.

In our earlier work, we formulated rendezvous problem as cone invariance problem [20], [21], [22]. The nature of the cone depends on the norm used to define the metric for rendezvous. For one, two and infinity norms the corresponding cones are: a polyhedral cone with n-sides, an ellipsoidal cone, and a polyhedral cone with 2^n-1-sides, respectively; where n is the number of vehicles. In our previous work, the problem of rendezvous is shown to be equivalent to cone invariance problems and will not be discussed here. In this paper, we address rendezvous in the framework of ellipsoidal cones.

In this paper we present a method to generate position reference trajectories that guarantee rendezvous of n-agents. The reference trajectories are determined from the present location of the agents, thus enabling feedback between them. The feedback mechanism enables the collective system to react to uncertainties in the behavior of the individual vehicles. The reference trajectories are generated using a *synthetic* system defined in terms of the position of the agents. The synthetic system characterizes the collective dynamics of the n-agent system and is *constructed* to satisfy the necessary and sufficient conditions for rendezvous.

The paper is organized as follows. We first present mathematical preliminaries on ellipsoidal cones and cone invariance. Necessary and sufficient conditions for rendezvous are presented next. This is followed interpretation of these results in the Lyapunov framework. The paper concludes with numerical examples that demonstrates application.

II. Mathematical Preliminaries

A. Ellipsoidal Cones

An ellipsoidal cone in $\mathbb{R}^n$ is the following,

$$\Gamma_n = \{\xi \in \mathbb{R}^n : K_n(\xi, Q) \leq 0,\ \xi^T u_n \geq 0\}, \qquad (1)$$

where $K_n(\xi, Q) = \xi^T Q \xi$, $Q \in \mathbb{R}^{n,n}$ is a symmetric nonsingular matrix with a *single* negative eigen-value λ_n and u_n is the eigen-vector associated with λ_n.

The boundary of the cone Γ_n is denoted by $\partial\Gamma_n$ and is defined by

$$0 \neq \xi \in \partial\Gamma_n \equiv \{\xi \in \Gamma_n : K_n(\xi, Q) = 0\}.$$

The outward pointing normal is the vector $Q\xi$ for $\xi \in \partial\Gamma_n$.

Theorem 1 (2.7 in [23]): If Γ_n is an ellipsoidal cone, then there exists a nonsingular transformation matrix $M \in \mathbb{R}^{n,n}$ such that

$$(M^{-1})^T Q M^{-1} = \begin{bmatrix} P & 0 \\ 0 & -1 \end{bmatrix} = Q_n$$

where $P \in \mathbb{R}^{n-1,n-1}$, $P > 0$ and $P = P^T$.
Let the transformed state be $x = M\xi$. The ellipsoidal cone in x is therefore,

$$\Gamma_n = \{x : \begin{pmatrix} w \\ z \end{pmatrix}^T \begin{bmatrix} P & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} w \\ z \end{pmatrix} \leq 0\} \qquad (2)$$

where $x = (w\ \ z)^T$, $w \in \mathbb{R}^{n-1}, z \in \mathbb{R}$.

An ellipsoidal cone in three dimension is shown in Fig.(1). The axis of the cone is the eigen-vector associated with the z axis.

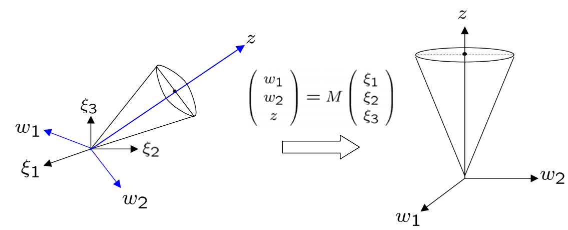


Fig. 1. Ellipsoidal cone in 3-dimension.

B. Ellipsoidal Cone Invariance

Consider a linear autonomous system

$$\dot{\xi} = A\xi. \qquad (3)$$

A cone Γ_n is said to be invariant with respect to the dynamics in eqn.(3) if $\xi(t_0) \in \Gamma_n \Rightarrow \xi(t) \in \Gamma_n, \forall t \geq t_0$, i.e. if the system starts inside the cone, it stays in the cone for all future time. Such a condition is also known as *exponential non-negativity*, i.e. $e^{At}\Gamma_n \in \Gamma_n$.

It is well known that certain structure in the matrix A imposes constraints on e^{At} [24]. The most well known result is the condition of non-negativity on A which states that if $A_{ij} \geq 0$ for $i \neq j$, then non-negative initial conditions yield non-negative solutions. Schneider and Vidyasagar [25] introduced the notion of *cross-positivity* of A on Γ_n which was shown to be equivalent to exponential non-negativity. Meyer *et al.* [26] extended cross-positivity to nonlinear fields.

Let us characterize $p(\Gamma_n)$ to be the set of matrices $A \in \mathbb{R}^{n,n}$ which are exponentially non-negative on Γ_n. It is defined by the following theorem.
Theorem 2 (3.1 in [23]): Let Γ_n be an ellipsoidal cone as in eqn.(2). Then,

$$p(\Gamma_n) = \{A \in \mathbb{R}^{n,n} : < A\xi, Q\xi > \leq 0,\ \forall \xi \in \Gamma_n\}. \qquad (4)$$

Theorem 2 states that A is such that the flow of the associated vector field is directed towards the interior of Γ_n, i.e. the dot product of the outward normal of Γ_n and the field is negative at the boundary of the cone. This leads to the result on the necessary and sufficient condition for exponential non-negativity of ellipsoidal cones.

Theorem 3 (3.5 in [23]): A necessary and sufficient condition for $A \in p(\Gamma_n)$ is that there exists $\gamma \in \mathbb{R}$ such that,

$$Q_n\hat{A} + \hat{A}^T Q_n - \gamma Q_n \leq 0.$$

where Q_n is defined in Theorem 1 and $\hat{A} = MAM^{-1}$. **Proof**

Please refer to pg.162 of [23].

III. Rendezvous in One Dimension

Given a cone Γ_n, as in eqn.(2) and dynamics as in eqn.(3), we present conditions for stability and invariance. We transform dynamics as

$$x = M\xi \Rightarrow \dot{x} = MAM^{-1}x = \hat{A}x.$$

With respect to the partition $x = (w\ \ z)^T$, the dynamics can be written as

$$\left(\begin{array}{c} \dot{w} \\ \dot{z} \end{array}\right) = \left[\begin{array}{c|c} A_{ww} & A_{wz} \\ \hline A_{zw} & a_{zz} \end{array}\right]\left(\begin{array}{c} w \\ z \end{array}\right), \tag{5}$$

where a_{zz} is written in small case to emphasize that it is a scalar.

For cone invariance, theorem 3 implies $\exists\gamma \in \mathbb{R}$ such that

$$\left[\begin{array}{cc} A_{ww}^T P + PA_{ww} - \gamma P & PA_{wz} - A_{zw}^T \\ A_{wz}^T P - A_{zw} & \gamma - 2a_{zz} \end{array}\right] \leq 0. \tag{6}$$

For stability, necessary and sufficient condition is $\exists Q = Q^T > 0$ such that $A^T Q + QA < 0$. Now consider the Lyapunov function $V(w,z) = w^T Pw + z^2$. It is a valid Lyapunov function since $P > 0$. Therefore, for stability $\dot{V}(w,z) < 0$, which implies the following sufficient condition

$$\left[\begin{array}{cc} A_{ww}^T P + PA_{ww} & PA_{wz} + A_{zw}^T \\ A_{wz}^T P + A_{zw} & 2a_{zz} \end{array}\right] < 0. \tag{7}$$

Therefore, for stability and cone invariance sufficient conditions is given by equations (6) and (7). Note that the sufficient conditions will become necessary and sufficient if a generalized Lyapunov function is chosen instead of the one that depends on P.

This leads to the following corollary.

Corollary 1: Trajectories originating outside the cone will enter the cone in finite time.

Proof

The cone $K_n(\xi, Q)$ can be written as $K_n(x, Q_n)$. Condition for cone invariance implies

$$\dot{K}_n(x, Q_n) \leq \gamma K_n(x, Q_n).$$

For x outside the cone, $K_n(x, Q_n) > 0$. Stability and cone invariance implies $\gamma < 2a_{zz} < 0$, which implies $\dot{K}_n(x, Q_n) < 0$ outside the cone. Hence proved.

IV. Rendezvous in Two Dimensions

Here we consider rendezvous of n agents in two dimensions. Let the state of each agent be $(\xi_{x_i}, \xi_{y_i}), i = 1, \cdots, n$. Collectively their dynamics can be written as

$$\left(\begin{array}{c} \dot{\xi}_x \\ \dot{\xi}_y \end{array}\right) = \left[\begin{array}{cc} A_{\xi_{xx}} & A_{\xi_{xy}} \\ A_{\xi_{yx}} & A_{\xi_{yy}} \end{array}\right]\left(\begin{array}{c} \xi_x \\ \xi_y \end{array}\right), \tag{8}$$

where $\xi_x = (\xi_{x_1} \cdots \xi_{x_n})^T$ and $\xi_y = (\xi_{y_1} \cdots \xi_{y_n})^T$ and $A_{\xi_{xx}}, A_{\xi_{xy}}, A_{\xi_{yx}}, A_{\xi_{yy}} \in \mathbb{R}^{n\times n}$.

In this work we solve the rendezvous problem in two dimensions as two separate rendezvous problems in one dimension. We assume that cones $\xi_x^T Q_{\xi_x}\xi_x \leq 0$ and $\xi_y^T Q_{\xi_y}\xi_y \leq 0$, each satisfying eqn.(1), are given. We are interested in determining necessary and sufficient conditions for cone invariance and stability.

For ellipsoidal cones $\xi_x^T Q_{\xi_x}\xi_x \leq 0$ and $\xi_y^T Q_{\xi_y}\xi_y \leq 0$, there exists transformation R_x and R_y respectively such that

$$\begin{array}{rcccl} Q_x^c & = & (R_x^{-1})^T Q_{\xi_x} R_x^{-1} & = & \left[\begin{array}{cc} P_x & 0 \\ 0 & -1 \end{array}\right], \\ Q_y^c & = & (R_y^{-1})^T Q_{\xi_y} R_y^{-1} & = & \left[\begin{array}{cc} P_y & 0 \\ 0 & -1 \end{array}\right], \end{array}$$

where $P_x, P_y > 0 \in \mathbb{R}^{(n-1)\times(n-1)}$ and the superscript "c" on Q_x and Q_y denotes cones.

Let the transformed states be $x = R_x\xi_x, y = R_y\xi_y$. The system dynamics with respect to the transformed states (x,y) can be written as

$$\begin{array}{rcl} \left(\begin{array}{c} \dot{x} \\ \dot{y} \end{array}\right) & = & R\left[\begin{array}{cc} A_{\xi_{xx}} & A_{\xi_{xy}} \\ A_{\xi_{yx}} & A_{\xi_{yy}} \end{array}\right]R^{-1}\left(\begin{array}{c} x \\ y \end{array}\right) \\ & = & \left[\begin{array}{cc} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{array}\right]\left(\begin{array}{c} x \\ y \end{array}\right) \end{array}$$

where

$$R = \left[\begin{array}{cc} R_x & 0 \\ 0 & R_y \end{array}\right].$$

Using theorem (3), the necessary and sufficient conditions for cone invariance with respect to trajectories $x(t, t_0, x_0)$ and $y(t, t_0, y_0)$ are

$$\left[\begin{array}{cc} A_{xx}^T Q_x^c + Q_x^c A_{xx} - \gamma_x Q_x^c & Q_x^c A_{xy} \\ A_{xy}^T Q_x^c & 0 \end{array}\right] \leq 0, \tag{9}$$

$$\left[\begin{array}{cc} 0 & A_{yx}^T Q_y^c \\ Q_y^c A_{yx} & A_{yy}^T Q_y^c + Q_y^c A_{yy} - \gamma_y Q_y^c \end{array}\right] \leq 0, \tag{10}$$

for some $\gamma_x \in \mathbb{R}$ and $\gamma_y \in \mathbb{R}$.

For stability, define

$$Q_x^s = \left[\begin{array}{cc} P_x & 0 \\ 0 & 1 \end{array}\right],\ Q_y^s = \left[\begin{array}{cc} P_y & 0 \\ 0 & 1 \end{array}\right],$$

where the superscript "s" on Q_x and Q_y denotes stability.

Therefore $V(x,y) = x^T Q_x^s x + y^T Q_y^s y$ is a valid Lyapunov function. Stability with respect to $V(x,y)$ implies

$$\left[\begin{array}{cc} A_{xx}^T Q_x^s + Q_x^s A_{xx} & Q_x^s A_{xy} + A_{yx}^T Q_y^s \\ Q_y^s A_{yx} + A_{xy}^T Q_x^s & A_{yy}^T Q_y^s + Q_y^s A_{yy} \end{array}\right] < 0. \tag{11}$$

Therefore, equations (9, 10, 11) are the sufficient conditions for rendezvous in two dimensions. If the dynamics of ξ_x and ξ_y are decoupled, then the conditions simplify to the following,

$$\begin{array}{rcl} A_{xx}^T Q_x^c + Q_x^c A_{xx} - \gamma_x Q_x^c & \leq & 0, \\ A_{yy}^T Q_y^c + Q_y^c A_{yy} - \gamma_y Q_y^c & \leq & 0, \\ A_{xx}^T Q_x^s + Q_x^s A_{xx} & < & 0, \\ A_{yy}^T Q_y^s + Q_y^s A_{yy} & < & 0. \end{array} \tag{12}$$

Note that the sufficient conditions will become necessary and sufficient if Q^s_x and Q^s_y are not defined with respect to P_x and P_y, but are any positive definite matrices.

Following the treatment presented in this section, these results can be easily extended to define necessary and sufficient conditions for rendezvous in higher dimensions. Note that the approach presented, solves higher dimensional rendezvous problems as separate rendezvous problems in each dimension, which is restrictive.

V. Rendezvous in Lyapunov Framework

In this section we derive necessary and sufficient conditions for rendezvous in the Lyapunov framework. Interpretation in the Lyapunov framework is important as these results will be useful in the qualitative analysis of rendezvous problems for nonlinear systems. We first consider rendezvous in one dimension, followed by rendezvous in two dimensions.

A. Rendezvous in One Dimension

Consider two Lyapunov functions $V_w(w) = w^T P w, P > 0$ and $V_z(z) = z^T z$. The cone Γ_n can then be represented as

$$\Gamma_n = \{ \begin{pmatrix} w \\ z \end{pmatrix} : V_w(w) \leq V_z(z)\}.$$

Conditions for rendezvous in the Lyapunov framework is then given by the following theorem.

Theorem 4: Sufficient conditions for rendezvous in terms of Lyapunov functions V_w and V_z are

$$\textit{Cone Invariance: } \dot{V}_w - \dot{V}_z \leq \gamma(V_w - V_z),\ \gamma \in \mathcal{R} \quad (13)$$

and

$$\textit{Stability: } \dot{V}_w + \dot{V}_z < 0. \quad (14)$$

Proof: These conditions are obtained by rewriting equations (6) and (7) in terms of the Lyapunov functions and their derivatives. As before, the sufficient conditions can be extended to necessary and sufficient conditions if a generalized Lyapunov function is assumed for stability purposes.

B. Rendezvous in Two Dimensions

To analyze rendezvous in two dimensions in the Lyapunov framework, we first partition the states as $x = (w_x\, z_x)$ and $y = (w_y\, z_y)$. Define Lyapunov functions

$$\begin{aligned} V_{w_x} &= w_x^T P_x w_x, \\ V_{z_x} &= z_x^2, \\ V_{w_y} &= w_y^T P_y w_y, \\ V_{z_y} &= z_y^2. \end{aligned}$$

Theorem 5: Sufficient conditions for rendezvous in two dimensions can be written in terms of these four Lyapunov functions as follows,

$$\begin{aligned} \dot{V}_{w_x} - \dot{V}_{z_x} &\leq \gamma_x(V_{w_x} - V_{z_x}),\ \gamma_x \in \mathbb{R}, \\ \dot{V}_{w_y} - \dot{V}_{z_y} &\leq \gamma_y(V_{w_y} - V_{z_y}),\ \gamma_y \in \mathbb{R}, \\ \dot{V}_{w_x} + \dot{V}_{z_x} &< 0, \\ \dot{V}_{w_y} + \dot{V}_{z_y} &< 0. \end{aligned} \quad (15)$$

Proof: These conditions are obtained by rewriting the equations (9, 10, 11) in terms of the Lyapunov functions and their derivatives. Once again, as in the 1D case, the sufficient conditions can be extended to necessary and sufficient conditions if a generalized Lyapunov function is assumed for stability purposes.

We would like to highlight the role of multiple Lyapunov functions in the necessary and sufficient conditions for nonlinear rendezvous problems.

VI. Design of Dynamics

Let us assume that there are n agents for which rendezvous is desired. In the trivial setting, let us also assume that the agents are modeled as *first order* LTI systems. Collectively, they can be written as

$$\dot{\xi} = A\xi + Bu, \quad (16)$$

where (A, B) is controllable. If we consider a *full state feedback* control framework, then

$$u = F\xi = FM^{-1}x$$

and the closed-loop system is therefore

$$\dot{x} = M(A + BF)M^{-1}x. \quad (17)$$

which can be represented in the form as in eqn.(5). The desired dynamics is achieved by determining F such that the LMI constraints in eqn.(7,6) are feasible.

In the non-trivial setting, when the system is described by higher order dynamics, the controller synthesis problem is not straight forward. For systems of the form,

$$\begin{pmatrix} \dot{\xi} \\ \dot{\eta} \end{pmatrix} = \begin{bmatrix} 0 & I_N \\ A_{\eta\xi} & A_{\eta\eta} \end{bmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} + \begin{bmatrix} 0 \\ B_\eta \end{bmatrix} u, \quad (18)$$

the cone Γ_n defined on position states ξ is not closed-loop *holdable* (pg.65 [27]). A cone Γ_n is said to be closed-loop holdable if there exists control $u(t)$ such that the condition of exponential non-negativity can be enforced, i.e.

$$\exists\, u(t) : \dot{K}_n(\xi, Q) < 0,\ \forall \xi \in \partial\Gamma.$$

For the system in eqn.(18) and the cone in eqn.(1),

$$\begin{aligned} \dot{K}_n(\xi, Q) &= \dot{\xi}^T Q \xi + \xi^T Q \dot{\xi} \\ &= \eta^T Q \xi + \xi^T Q \eta, \end{aligned}$$

which is *independent* of u. Therefore, the condition of exponential non-negativity cannot be enforced by any choice of u.

In such a case, one has to adopt a two-degree of freedom design problem. In such a setting, the desired position trajectories of the n-agents are determined from their present locations, using a model $\mathcal{M}_R$ that is closed-loop holdable. These trajectories are then tracked by individual inner loop tracking controllers. Since, the reference trajectories are determined from present locations, this enables a feedback mechanism between the n-agents. This feedback will ensure that the collective system reacts to anomalies in the behavior of the individual agents.

The reference generating model $\mathcal{M}_R$ characterizes the collective behavior of the n-agent system. Therefore, it is possible to embed desired collective dynamics by synthesizing $\mathcal{M}_R$ appropriately. The synthetic system $\mathcal{M}_R$ can be used to *design* the global dynamics of the collective system and should involve variables that capture long time/length scale phenomena. This is illustrated in the following examples.

VII. Example

In this section we consider rendezvous of three agents in the (x, y) plane. The open loop dynamics of the x and y positions of each agent are modeled as second order systems, i.e.

$$\begin{pmatrix} \dot{x}_i \\ \dot{v}_{x_i} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{pmatrix} x_i \\ v_{x_i} \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{x_i}, \quad (19)$$

$$\begin{pmatrix} \dot{y}_i \\ \dot{v}_{y_i} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{pmatrix} y_i \\ v_{y_i} \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{y_i}. \quad (20)$$

Since the x and y dynamics of the agents are second order systems, we first solve the rendezvous problem using the first order dynamics and eqn.(6),(7) to generate reference trajectories $x_i^r(t), y_i^r(t)$, for each agent. Full state feedback is assumed in determining the reference trajectories, i.e. every agent has position information of all the agents. The feedback structure of the outer-loop (reference generation) structure is shown in Fig.2. Observe that the reference trajectory is generated in a decentralized manner.

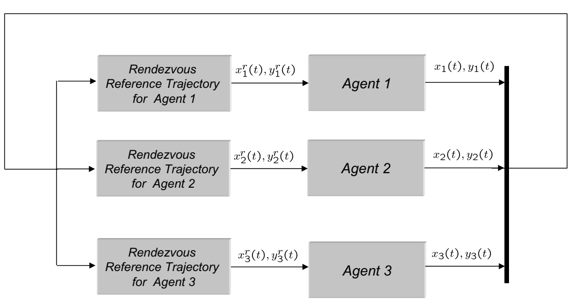


Fig. 2. Feedback structure of the outer-loop.

The reference trajectories are then tracked using a separately designed tracking controller. The inner-loop (tracking) structure is shown in fig.(3) for tracking of reference $x^r(t)$. The tracking controller is identical for both $x^r(t)$ and $y^r(t)$ and also for every agent.

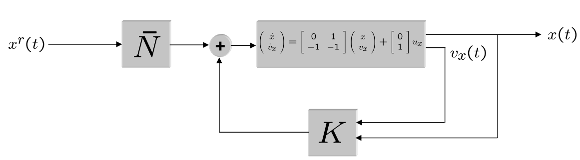
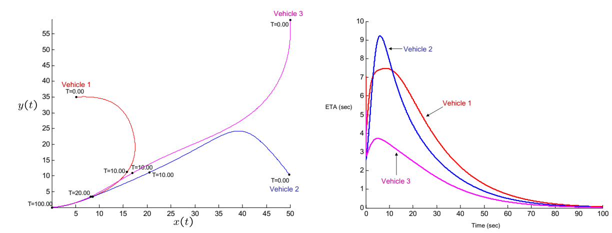


Fig. 3. Feedback structure of the inner-loop.

A. Simulation with Tracking Controller

Figure 4(a) shows the trajectories of the three agents. The initial conditions for position of the three agents are $(5, 35)$, $(50, 10)$ and $(50, 60)$ respectively. The initial velocities of the three agents along x, y are $(10, 1)$, $(-10, 20)$ and $(1, -30)$ respectively. We observe that the agents achieve rendezvous with a reasonably good position tracking controller. The expected arrival times of the agents are shown in fig. 4(b). We observe that the ETA of all the vehicles increase initially. This is due to the mismatch in the velocity of the system and the required velocity for rendezvous. The ETAs become identical as the vehicles approach the origin. This is also visible from the plots in fig.4(a). We observe that the trajectories are close to each other at $T = 10s$ and become identical at $T = 20s$.



(a) Initial conditions (x, y): $(5, 35), (50, 10), (50, 60)$. Initial conditions for (v_x, v_y): $(10, 1), (-10, 20)$ and $(1, -30)$.

(b) Estimated arrival times.

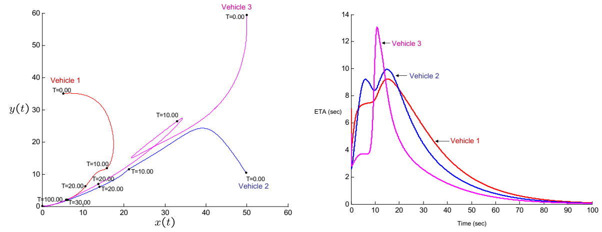
Fig. 4. Rendezvous of three agents with second order dynamics in (x, y) plane. Reference position trajectories are generated using first order dynamics. Position tracking controller is then used to track the reference.

B. Simulation with Tracking Controller & Uncertainty in Vehicle Behavior

Figure 5(a) shows the same simulation as the previous example, but with vehicle 3 making an unexpected loop in the time interval of $T = [5, 15]$ seconds. We observe that the other vehicles modify their trajectories accordingly to achieve rendezvous. This is particularly visible in the ETA plots as shown in fig.5(b). Due to the diversion of vehicle 3, its ETA increases considerably. ETA of the other vehicles also increase appropriately so that they achieve rendezvous. Note that the first peak in the ETA of vehicles 1 and 2 are due to the mismatch in the velocity as in the previous example. The second peak is due to the deviation of vehicle 3 from the reference trajectory. Once again the ETAs become identical as the vehicles approach the origin. Figure 5(a) shows that the vehicle trajectories come close to each other by $T = 20s$ and become identical at $T = 30s$.

VIII. Summary

In this paper we presented a framework to generate feedback between multiple vehicles achieving rendezvous. The feedback is used to compute the desired trajectories that will guarantee rendezvous. The desired trajectories are determined by means of synthetic dynamical system which



(a) Initial conditions (x, y): $(5, 35), (50, 10), (50, 60)$. Initial conditions (v_x, v_y): $(10, 1), (-10, 20)$ and $(1, -30)$.

(b) Estimated arrival times.

Fig. 5. Rendezvous of three agents with second order dynamics in (x, y) plane and robustness with respect to uncertainty in vehicle behavior.

characterizes the collective dynamics of the multiple agents. This enables us to embed any desirable dynamics on the collective system.

Necessary and sufficient conditions for rendezvous were presented. These results were also interpreted in the Lyapunov framework to facilitate analysis of nonlinear rendezvous problems. Numerical examples demonstrating application of this method to rendezvous in two dimensions were presented. The effectiveness of the feedback mechanism to guarantee robustness with respect to uncertainty in vehicle behavior was also demonstrated.

In the proposed method we have assumed full state feedback for controller synthesis. In reality, the communication topology may not allow such a luxury. In such cases, state estimations are required. Recent developments on multi-agent consensus can be applied to estimate the positions of the agents. This will be addressed in future work.

Future work along this theme is focused on multiple directions including formal analysis of multi-agent rendezvous with higher order dynamics, addressing state estimation and consensus and extension of this framework to nonlinear systems using multiple Lyapunov functions.

References

[1] T. McLain. Coordinated control of unmanned air vehicles. Technical Report ASC-99-2426, Air Vehicles Directorate of the Air Force Research Laboratory, 1999.

[2] J. Lin, A.S. Morse, and B.D.O Anderson. The Multi-Agent Rendezvous Problem. *42nd IEEE Conference on Decision and Control*, 2:1508–1513, 2003.

[3] J. Cortes, S. Martinez, and F. Bullo. Robust Rendezvous for Mobile Autonomous Agents via Proximity Graphs in Arbitrary Dimensions. 51(8), 2006.

[4] A. Jadbabaie, J. Lin, and A.S. Morse. Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbour Rules. *IEEE Transactions on Automatic Control*, 48(6):988–1001, 2003.

[5] S.L. Smith, M.E. Broucke, and B. A. Francis. Curve Shortening and its Application to Multi-Agent Systems. *European Control Conference, 2005, Spain.*

[6] W. Ren, R.W. Beard, and E.M. Atkins. A Survey of Consensus Problems in Multi-Agent Coordination. *American Control Conference, Portland Oregon, June, 2005.*

[7] M. Hirsch. Systems of differential equations which are competitive and cooperative I: limit sets. *Siam Journal on Applied Math*, 13:167–179, 1982.

[8] M. Hirsch. Systems of differential equations which are competitive and cooperative II: convergence almost everywhere. *Siam Journal on Mathematical Analysis*, 16:423–439, 1985.

[9] M. Hirsch. Systems of differential equations which are competitive and cooperative III: Competing species. *Nonlinearity*, 1:51–71, 1988.

[10] M. Hirsch. Systems of differential equations which are competitive and cooperative IV: Structural stability in three dimensional systems. *Siam Journal on Mathematical Analysis*, 21:1225–1234, 1990.

[11] M. Hirsch. Systems of differential equations which are competitive and cooperative V: Convergence in three dimensional systems. *Journal of Differential Equations*, 80:94–106, 1989.

[12] M. Hirsch. Systems of differential equations which are competitive and cooperative VI: A local C^r closing lemma for 3-dimensional systems. *Ergodic Theory Dynamical Systems*, 11:443–454, 1991.

[13] H. L. Smith. *Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems*, volume 41 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, Rhode Island, 1995.

[14] F. Blanchini. Set Invariance in Control. *Automatica*, 35:1747–1767, 1999.

[15] J. Stiglitz, editor. *A Collected Scientific Papers of Paul A. Samuelson*. MIT Press, Cambridge, Massachusetts, 1966.

[16] D. G. Luenberger. *Introduction to Dynamic Systems*, chapter 6. Wiley, New York, 1979.

[17] L. F. Richardson. *Arms and Insecurity*. Boxwood Press, Pittsburgh Quadrangle Books, Pittsburgh,Pennsylvania, 1960.

[18] J. A. Jacquez. *Compartmental Analysis in Biology and Medicine*, volume 50. Elsevier, New York, 1972.

[19] D. H. Anderson. Compartmental Modeling and Tracer Kinetics. *Lecture Notes in Biomathematics*, 1983.

[20] A. Tiwari, J. Fung, J. M. Carson III, R. Bhattacharya, and R. M. Murray. A Framework for Lyapunov Certificates for Multi-Vehicle Rendezvous Problems. In *Proceedings of the American Control Conference*, Boston, 2004.

[21] A. Tiwari, J. Fung, R. Bhattacharya, and R. M. Murray. Polyhedral Cone Invariance Applied to Rendezvous of Multiple Agents. In *IEEE Conference on Decision and Control*, Bahamas, 2004.

[22] R. Bhattacharya, J. Fung, A. Tiwari, and R. M. Murray. Ellipsoidal Cones and Rendezvous of Multiple Agents. In *IEEE Conference on Decision and Control*, Bahamas, 2004.

[23] R. J. Stern and H. Wolkowicz. Exponential Nonnegativity on the Ice Cream Cone. *Siam J. Matrix Anal. Appl.*, 12:160–165, January 1991.

[24] J. A. Yorke. Invariance for Ordinary Differential Equations. *Math Sys. Theory*, 1:353–372, 1967.

[25] H. Schneider and M. Vidyasagar. Cross-Positive Matrices. *SIAM J. Numer. Anal.*, 7(4):508–519, 1970.

[26] D. G. Meyer, T. L. Piatt, H. N. G. Wadley, and R. Vancheeswaran. Nonlinear Invariance: Cross-Positive Vector Fields. In *Proceedings of the American Control Conference*, Philadelphia, 1998.

[27] A. Berman, M. Neumann, and R. J. Stern. *Nonnegative Matrices in Dynamic Systems*. A Wiley-Interscience Publication, New York, 1989.