```
import numpy as np
from tensorflow import keras
from tensorflow.keras import layers
from sklearn.metrics import accuracy_score
from keras.optimizers import Adam
from sklearn.ensemble import RandomForestClassifier
     WARNING:tensorflow:From C:\ProgramData\Anaconda3\lib\site-packages\keras\src\losses.py:2976: The name tf.losses.sparse softmax cross entropy is deprecated.
mnist = keras.datasets.mnist
(x_train, y_train), (x_test, y_test) = mnist.load_data()
x_train.shape, y_train.shape, x_test.shape, y_test.shape
     ((60000, 28, 28), (60000,), (10000, 28, 28), (10000,))
x_train, x_test = x_train / 255.0, x_test / 255.0
X_{\text{train}} = x_{\text{train.reshape}}((-1, 28, 28, 1))
x_{\text{test}} = x_{\text{test.reshape}}((-1, 28, 28, 1))
```

a) Model 1

```
build_model_1 = keras.Sequential([
layers.Conv2D(32, (3, 3), strides=(2, 2), activation='relu', input_shape=(28, 28, 1)),
layers.MaxPooling2D((2, 2), strides=(1, 1)),
layers.Conv2D(16, (4, 4), strides=(2, 2), activation='relu'),
layers.MaxPooling2D((4, 4), strides=(2, 2)),
layers.Flatten(),
layers.Dense(8, activation='relu'),
layers.Dense(10, activation='softmax')
])
learning_rate = 0.001 # Adjust this value as needed
optimizer = Adam(learning_rate=learning_rate)
build_model_1.compile(optimizer=optimizer,loss='sparse_categorical_crossentropy',metrics=['accuracy'])
build_model_1.summary()
```

 $WARNING: tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.com tensorflow: From C: \ProgramData\Anaconda3\lib\site-packages\keras\src\backend.py: 873: The name tf.get_default_graph is deprecated. Please use tf.get_default_graph is deprecated.$

WARNING:tensorflow:From C:\ProgramData\Anaconda3\lib\site-packages\keras\src\layers\pooling\max_pooling2d.py:161: The name tf.nn.max_pool is deprecated. Pl

Model: "sequential"

Layer (type)	Output Shape	Param #
conv2d (Conv2D)	(None, 13, 13, 32)	320
<pre>max_pooling2d (MaxPooling2 D)</pre>	(None, 12, 12, 32)	0
conv2d_1 (Conv2D)	(None, 5, 5, 16)	8208
<pre>max_pooling2d_1 (MaxPoolin g2D)</pre>	(None, 1, 1, 16)	0
flatten (Flatten)	(None, 16)	0
dense (Dense)	(None, 8)	136
dense_1 (Dense)	(None, 10)	90
	=======================================	=======

```
Total params: 8754 (34.20 KB)
Trainable params: 8754 (34.20 KB)
Non-trainable params: 0 (0.00 Byte)
```

Training model1 3...

```
model1_accuracies = []
for i in range(5):

    print(f"Training model1 {i+1}...")
    build_model_1.fit(x_train, y_train, epochs=5,batch_size=128)

    print(f"Evaluating model1 {i+1}...")
    test_loss_m1, test_accuracy_m1 = build_model_1.evaluate(x_test, y_test, verbose=2)
    model1_accuracies.append(test_accuracy_m1)

mean_test_accuracy_m1 = np.mean(model1_accuracies)
print("Mean test accuracy of Model 1:", mean_test_accuracy_m1)
Training model1 1...
```

```
Epoch 1/5
469/469 [============= ] - 3s 6ms/step - loss: 0.1055 - accuracy: 0.9672
Epoch 2/5
469/469 [============ ] - 3s 6ms/step - loss: 0.1010 - accuracy: 0.9689
Epoch 3/5
469/469 [============ ] - 3s 6ms/step - loss: 0.1010 - accuracy: 0.9681
Epoch 4/5
469/469 [============= ] - 3s 6ms/step - loss: 0.0985 - accuracy: 0.9686
Epoch 5/5
Evaluating model1 1...
313/313 - 0s - loss: 0.1294 - accuracy: 0.9627 - 396ms/epoch - 1ms/step
Training model1 2...
Epoch 1/5
469/469 [============ ] - 3s 6ms/step - loss: 0.0947 - accuracy: 0.9706
Epoch 2/5
469/469 [============= ] - 3s 6ms/step - loss: 0.0934 - accuracy: 0.9702
Epoch 3/5
Epoch 4/5
Epoch 5/5
Evaluating model1 2...
313/313 - 0s - loss: 0.1363 - accuracy: 0.9599 - 393ms/epoch - 1ms/step
```

```
Epoch 1/5
Epoch 2/5
469/469 [===========] - 3s 6ms/step - loss: 0.0860 - accuracy: 0.9735
Epoch 3/5
469/469 [============= ] - 3s 6ms/step - loss: 0.0859 - accuracy: 0.9735
Epoch 4/5
469/469 [===========] - 3s 6ms/step - loss: 0.0848 - accuracy: 0.9732
Epoch 5/5
469/469 [===========] - 3s 6ms/step - loss: 0.0825 - accuracy: 0.9740
Evaluating model1 3...
313/313 - 0s - loss: 0.1681 - accuracy: 0.9515 - 404ms/epoch - 1ms/step
Training model1 4...
Epoch 1/5
Epoch 2/5
469/469 [=========== ] - 3s 6ms/step - loss: 0.0819 - accuracy: 0.9746
Epoch 3/5
469/469 [===========] - 3s 6ms/step - loss: 0.0791 - accuracy: 0.9753
Epoch 4/5
469/469 [===========] - 3s 6ms/step - loss: 0.0776 - accuracy: 0.9760
Epoch 5/5
469/469 [============] - 3s 6ms/step - loss: 0.0766 - accuracy: 0.9760
Evaluating model1 4...
313/313 - 0s - loss: 0.1243 - accuracy: 0.9637 - 457ms/epoch - 1ms/step
Training model1 5...
Epoch 1/5
469/469 [===========] - 3s 6ms/step - loss: 0.0757 - accuracy: 0.9763
Epoch 2/5
469/469 [============ ] - 3s 7ms/step - loss: 0.0754 - accuracy: 0.9761
Enach 3/5
```

```
# Extract the features from second Last fully connected Layer (having 8 neurons)
feature extractor m1 = keras.Model(inputs=build model 1.inputs, outputs=build model 1.layers[-2].output)
#predict the feature train and test
train features m1 = feature extractor m1.predict(x train)
test features m1 = feature extractor m1.predict(x test)
# Model features using Random Forest classifier
rf classifier m1 = RandomForestClassifier(n estimators=100, random state=42)
rf_classifier_m1.fit(train_features_m1, y_train)
# Predict using the Random Forest classifier
predictions_m1 = rf_classifier_m1.predict(test_features_m1)
# Calculate accuracy
accuracy m1 = accuracy score(y test, predictions m1)
print("Accuracy of Random Forest classifier on extracted features model 1:", accuracy m1)
    313/313 [=========== ] - Os 1ms/step
    Accuracy of Random Forest classifier on extracted features model 1: 0.9608
```

b) Model 2

```
build_model_2 = keras.Sequential([
layers.Conv2D(32, (3, 3), strides=(2, 2), activation='relu', input_shape=(28, 28, 1)),
layers .AveragePooling2D((2, 2), strides=(1, 1)),
layers.Conv2D(16, (4, 4), strides=(2, 2), activation='relu'),
layers .AveragePooling2D((4, 4), strides=(2, 2)),
layers.Flatten(),
layers.Dense(8, activation='relu'),
layers.Dense(10, activation='softmax')
])
learning_rate = 0.001 # Adjust this value as needed

optimizer = Adam(learning_rate=learning_rate)
build_model_2.compile(optimizer=optimizer,loss='sparse_categorical_crossentropy',metrics=['accuracy'])
build_model_2.summary()
```

Model: "sequential_1"

Layer (type)	Output Shape	Param #
conv2d_2 (Conv2D)	(None, 13, 13, 32)	320
<pre>average_pooling2d (Average Pooling2D)</pre>	(None, 12, 12, 32)	0
conv2d_3 (Conv2D)	(None, 5, 5, 16)	8208
<pre>average_pooling2d_1 (Avera gePooling2D)</pre>	(None, 1, 1, 16)	0
flatten_1 (Flatten)	(None, 16)	0
dense_2 (Dense)	(None, 8)	136
dense_3 (Dense)	(None, 10)	90
Total params: 8754 (34.20 KB Trainable params: 8754 (34.2	, 0 КВ)	=======

Non-trainable params: 0 (0.00 Byte)

```
model2_accuracies = []
for i in range(5):
    print(f"Training model1 {i+1}...")
    build_model_2.fit(x_train, y_train, epochs=5,batch_size=128)
    print(f"Evaluating model1 {i+1}...")
    test_loss_m2, test_accuracy_m2 = build_model_2.evaluate(x_test, y_test, verbose=2)
    model2_accuracies.append(test_accuracy_m2)
mean_test_accuracy_m2 = np.mean(model2_accuracies)
print("Mean test accuracy of Model 1:", mean_test_accuracy_m2)
```

```
Epoch 2/5
469/469 [============ ] - 3s 6ms/step - loss: 0.5118 - accuracy: 0.8368
Epoch 3/5
Epoch 4/5
Epoch 5/5
469/469 [============== ] - 3s 6ms/step - loss: 0.4292 - accuracy: 0.8665
Evaluating model1 2...
313/313 - 0s - loss: 0.3957 - accuracy: 0.8771 - 439ms/epoch - 1ms/step
Training model1 3...
Epoch 1/5
Epoch 2/5
469/469 [============== ] - 3s 6ms/step - loss: 0.3818 - accuracy: 0.8827
Epoch 3/5
Epoch 4/5
469/469 [===========] - 3s 6ms/step - loss: 0.3448 - accuracy: 0.8959
Epoch 5/5
469/469 [===========] - 3s 6ms/step - loss: 0.3286 - accuracy: 0.9000
Evaluating model1 3...
313/313 - 0s - loss: 0.3142 - accuracy: 0.9052 - 482ms/epoch - 2ms/step
Training model1 4...
Epoch 1/5
469/469 [============= ] - 3s 6ms/step - loss: 0.3147 - accuracy: 0.9052
Epoch 2/5
Epoch 3/5
Epoch 4/5
469/469 [============ ] - 3s 6ms/step - loss: 0.2810 - accuracy: 0.9154
Epoch 5/5
469/469 [============ ] - 3s 6ms/step - loss: 0.2740 - accuracy: 0.9171
Evaluating model1 4...
313/313 - 0s - loss: 0.2565 - accuracy: 0.9227 - 361ms/epoch - 1ms/step
Training model1 5...
Epoch 1/5
469/469 [============== ] - 3s 6ms/step - loss: 0.2662 - accuracy: 0.9203
Epoch 2/5
```

```
# Extract the features from second Last fully connected Layer (having 8 neurons)
feature_extractor_m2 = keras.Model(inputs=build_model_2.inputs, outputs=build_model_2.layers[-2].output)
#predict the feature train and test
train features m2 = feature extractor m2.predict(x train)
test_features_m2 = feature_extractor_m2.predict(x_test)
# Model features using Random Forest classifier
rf classifier m2 = RandomForestClassifier(n estimators=100, random state=42)
rf_classifier_m2.fit(train_features_m2, y_train)
# Predict using the Random Forest classifier
predictions_m2 = rf_classifier_m2.predict(test_features_m2)
# Calculate accuracy
accuracy m2 = accuracy score(y test, predictions m2)
print("Accuracy of Random Forest classifier on extracted features model 1:", accuracy m2)
     313/313 [============ ] - 0s 1ms/step
     Accuracy of Random Forest classifier on extracted features model 1: 0.9405
v c) Model 3
train stacked features = np.hstack((train features m1, train features m2))
test stacked features = np.hstack((test features m1, test features m2))
# Model stacked features using Random Forest classifier
rf classifier stack = RandomForestClassifier(n estimators=100, random state=42)
rf classifier stack.fit(train stacked features, y train)
# Predict using the Random Forest classifier
predictions m3 = rf_classifier_stack.predict(test_stacked_features)
# Calculate accuracy
accuracy m3 = accuracy score(y test, predictions m3)
print("Accuracy of Random Forest classifier on Stacked Features:", accuracy m3)
```

Accuracy of Random Forest classifier on Stacked Features: 0.9722 $\,$

mean test accuracy of model 1: 0.8934599995613098

d) Model 4

```
# Extract the deep features from Model-1 and Model-2 stack the features horizontally, reduce the
#dimension to either 8, 10 or 12 using principal component analysis (PCA) and model the reduced features
#using a Random Forest classifier.
from sklearn.decomposition import PCA
pca = PCA(n components=8) # use 8,18 or 12
pca.fit(train_stacked_features)
features train reduced = pca.transform(train stacked features)
features_test_reduced = pca.transform(test_stacked_features)
features train reduced.shape, features test reduced.shape
     ((60000, 8), (10000, 8))
 # Model stacked features using Random Forest classifier
rf classifier pca = RandomForestClassifier(n estimators=100, random state=42)
rf_classifier_pca.fit(features_train_reduced, y train)
# Predict using the Random Forest classifier
predictions m4 = rf classifier pca.predict(features test reduced)
# Calculate accuracy
accuracy m4 = accuracy score(y test, predictions m4)
print("Accuracy of Random Forest classifier After feature reduction:", accuracy m4)
```

e) Analysis

Accuracy of Random Forest classifier After feature reduction: 0.9675

Based on the provided specifications, Model-1 and Model-2 are similar in architecture but differ in the type of pooling layers used (maxpooling vs. average pooling). Model-3 combines the features extracted from both Model-1 and Model-2, while Model-4 further reduces the dimensionality using PCA before using a Random Forest classifier.

Model-4 might have an edge over the others if the dimensionality reduction via PCA effectively captures the most discriminative information while reducing noise.

Therefore, the best model would likely be Model-4 because it combines the features from both Model-1 and Model-2, reduces dimensionality using PCA, and then employs a Random Forest classifier for classification

Model 3 and model 4 has only slight varience in the accuracy hence both can be the best models precisely model 4 has best accuracy with 8,10,12 componets which has mostly same accuracy of 0.97.