# INTRODUCTION TO QUANTITATIVE TECHNIQUES

Quantitative technique is a very powerful tool, by using this we can augment our production, maximize profits, minimize costs, and production methods can be oriented for the accomplishment of certain pre – determined objectives. Quantitative techniques used to solve many of the problems that arise in a business or industrial area. A large number of business problems, in the relatively recent past, have been given a quantitative representation with considerable degree of success. All this has attracted the students, business executives, public administrators alike towards the study of these techniques more and more in the present times.

Scientific methods have beenman’s outstanding asset to pursue an ample number of activities. It is analyzed that whenever some national crisis, emerges due to the impact of political, social, economic or cultural factors the talents from all walks of life amalgamate together to overcome the situation and rectify the problem. In this chapter we will see how the quantitative techniques had facilitated the organization in solving complex problems on time with greater accuracy. The historical development will facilitate in managerial decision-making & resource allocation, The methodology helps us in studying the scientific methods with respect to phenomenon connected with human behaviour like formulating the problem, defining decision variable and constraints, developing a suitable model, acquiring the input data, solving the model, validating the model, implementing the results. The major advantage of mathematical model is that it facilitates in taking decision faster and more accurately.

Managerial activities have become complex and it is necessary to make right decisions to avoid heavy losses. Whether it is a manufacturing unit, or a service organization, the resources have to be utilized to its maximum in an efficient manner. The future is clouded with uncertainty and fast changing, and decision- making – a crucial activity – cannot be made on a trial-and-error basis or by using a thumb rule approach. In such situations, there is a greater need for applying scientific methods to decision-making to increase the probability of coming up with good decisions. Quantitative Technique is a scientific approach to managerial decision-making. The successful use of Quantitative Technique for management would help the organization in solving complex problems on time, with greater accuracy and in the most economical way. Today, several scientific management techniques are available to solve managerial problems and use of these techniques helps managers become explicit about their objectives and provides additional information to select an optimal decision. This study material is presented with variety of these techniques with real life problem areas.

# MEANING OF QUANTITATIVE TECHNIQUES

Quantitative techniques are those statistical and operations research programming techniques which help in the decision making process especially concerning business and industry. These techniques involve the introduction of the element of quantities i.e., they involve the use of numbers, symbols and other mathematical expressions. The quantitative techniques are essentially helpful supplement to judgement and intuition. These techniques evaluate planning factors and alternative as and when they arise rather than prescribe courses of action. As such, quantitative techniques may be defined as those techniques which provide the decision maker with a systematic and powerful means of analysis and help, based on quantitative data, in exploring policies for achieving pre – determined goals. These techniques are particularly relevant to problems of complex business enterprises.

# CLASSIFICATION OF QUANTITATIVE TECHNIQUES

We have many quantitative techniques in modern times. They can broadly be put under two categories:

1. Statistical Techniques (b) Programming Techniques

**Statistical Techniques:**

These techniques are used in conducting the statistical inquiry concerning a certain phenomenon. They are including all the statistical methods beginning from the collection of data till the task of interpretation of the collected data. More clearly, the methods of collection of statistical data, the technique of classification and tabulation of the collected data, the calculation of various statistical measures such as mean, standard deviation, coefficient of correlation etc, the techniques of analysis and interpretation and finally the task of deriving inference and judging their reliability are some of the important statistical techniques.

Statistical Techniques also help in Correlation too. Correlation is a statistical technique that can show whether and how strongly pairs of variables are related. For example, height and weight are related; taller people tend to be heavier than shorter people. The relationship isn’t perfect. People of the same height vary in weight, and you can easily think of two people you know where the shorter one is heavier than the taller one. Nonetheless, the average weight of people 5’5'’ is less than the average weight of people 5’6'’, and their average weight is less than that of people 5’7'’, etc. Correlation can tell you just how much of the variation in peoples’ weights is related to their heights.

Although this correlation is fairly obvious your data may contain unsuspected correlations. You may also suspect there are correlations, but don’t know which are the strongest. An intelligent correlation analysis can lead to a greater understanding of your data.

There are several different correlation techniques. The Survey System’s optional Statistics Module includes the most common type, called the Pearson or product-moment correlation. The module also includes a variation on this type called partial correlation. The latter is useful when you want to look at the relationship between two variables while removing the effect of one or two other variables.

Like all statistical techniques, correlation is only appropriate for certain kinds of data. Correlation works for quantifiable data in which numbers are meaningful, usually quantities of some sort. It cannot be used for purely categorical data, such as gender, brands purchased, or favourite colour.

**Programming Techniques:**

It can be defined as operational research or simply (O.R.) are the model building techniques used by decision maker in modern times. They include wide variety of techniques such as linear programming, theory of games, simulation, network analysis, queuing theory and many other similar techniques.

# ROLE/IMPORTANCE OF QUANTITATIVE TECHNIQUES

These techniques are especially increasing since World War II in the technology of business administration. These techniques help in solving complex and intricate problems of business and industry. Quantitative techniques for decision making are, in fact, examples of the use of scientific method of management. Their role can be well understood under the following heads:

1. **Provide a tool for scientific analysis:** These techniques provides executives with a more precise description of the cause and effect relationship and risks underlying the business operations in measurable terms and this eliminates the conventional intuitive and subjective basis on which managements used to formulate their decisions decades ago. In fact, these techniques replace the intuitive and subjective approach of decision making by an analytical and objective approach. The use of these techniques has transformed the conventional techniques of operational and investment problems in business and industry. Quantitative techniques thus encourage and enforce disciplined thinking about organisational problems.
2. **Provide solution for various business problems:** These techniques are being used in the field of production, procurement, marketing, finance and allied fields. Problems like, how best can the managers and executives allocate the available resources to various products so that in a given time the profits are maximum or the cost is minimum? Is it possible for an industrial enterprise to arrange the time and quantityof orders of its stock such that the overall profit withgiven resources is maximum? How far is it within the competence of a business manager to determine the number of men and machines to be employed and used in such a manner that neither remains idle and at the same time the customer or the public has not to wait undulylong for service? And similar other problems can be solved with the help of quantitative techniques.
3. **Enable proper deployment of resources:** It render valuable help in proper deployment of resources. For example, PERT enables us to determine the earliest and the latest times for each of the events and activities and there by helps in identification of the critical path. All this helps in the deployment of the resources from one activity to another to enable the project completion on time. This techniques, thus, provides for determining the probability of completing an event or project itself by a specified date.
4. **Helps in minimizing waiting and servicing costs:** This theory helps the management in minimizing the total waiting and servicing costs. This technique also analyses the feasibility of adding facilities and thereby helps the business people to take a correct and profitable decision.
5. **Assists in choosing an optimum strategy:** Game theory is especially used to determine the optimum strategy in a competitive situation and enables the businessmen to maximise profits or minimize losses by adopting the optimum strategy.
6. **They render great help in optimum resource allocation:** Linear programming technique is used to allocate scarce resources in an optimum manner in problem of scheduling, product – mix and so on.
7. **Enable the management to decide when to buy and how much to buy:** The techniques of inventory planning enables the management to decide when to buy and how much to buy.
8. **They facilitate the process of decision making:** Decision theory enables the businessmen to select the best course of action when information is given to probabilistic form. Through decision tree techniques executive’s judgement can systematically be brought into the analysis of the problems. Simulation is an other important technique used to imitate an operation or process prior to actual performance. The significance of simulation lies in the fact that it enables in finding out the effect of alternative courses of action in situation involving uncertainty where mathematical formulation is not possible. Even complex groups of variables can be handled through this technique.
9. **Through various quantitative techniques management can know the reactions of the integrated business systems:** The Integrated Production Models techniques are used to minimise cost with respect to work force, production and inventory. This technique is quite complex and is usually used by companies having detailed information concerning their sales and costs statistics over a long period. Besides, various other O.R. techniques also help in management people taking decisions concerning various problems of business and industry. The techniques are designed to investigate how the integrated business system would react to variations in its component elements and/or external factors.

# LIMITATION OF QUANTITATIVE TECHNIQUES

Quantitative techniques though are a great aid to management but still they cannot be substitute for decision making. The choice of criterion as to what is actually best for the business enterprise is still that of an executive who has to fall back upon his experience and judgement. This is so because of the several limitations of quantitative techniques. Important limitations of these techniques are as given below:

1. **The inherent limitation concerning mathematical expressions:** Quantitative techniques involve the use of mathematical models, equations and similar other mathematical expressions. Assumptions are always incorporated in the derivation of an equation and such an equation maybe correctly used for the solution of the business problems when the underlying assumptions and variables in the model are present in the concerning problem. IF this caution is not given due care then there always remains the possibility of wrong application of the quantitative techniques. Quite often the operations researchers have been accused of having many solutions without being able to find problems that fit.
2. **High costs are involved in the use of quantitative techniques:** Quantitative techniques usually prove very expensive. Services of specialised persons are invariably called for while using quantitative techniques. Even in big business organisations we can expect that quantitative techniques will continue to be of limited use simply because they are not in many cases worth their cost. As opposed to this a typical manager, exercising intuition and judgement, maybe able to make a decision very inexpensively. Thus, the use of quantitative techniques is a costlier affair and this in fact constitutes a big and important limitation of such techniques.
3. **Quantitative techniques do not take into consideration the intangible factors i.e., non measurable human factors**: Quantitative techniques make no allowances for intangible factors such as skill, attitude, vigour of the management people in taking decisions but in many instances success or failure hinges upon the consideration of such non-measurable intangible factors. There cannot be any magic formula for getting an answer to management problems; much depends upon proper managerial attitudes and policies.
4. **Quantitative techniques are just the tools of analysis and not the complete decision making process:** It should always be kept in mind that quantitative techniques, whatsoever it may be, alone cannot make the final decision. They are just tools and simply suggest best alternatives but in final analysis many business decisions will involve human element. Thus, quantitative analysis is at best a supplement rather than, a substitute for management; subjective judgement is likely to remain a principal approach to decision making.

**UNIT – 2 AND 3 – PROBABILITY AND THEORITICAL DISTRIBUTIONS**

## Introduction

Probability has a very old history, it was originated in the games of chance related to gambling. For instance, throwing of dice or coin and drawing cards from a pack. Jerome Cardan (1501~1576), an Italian mathematician was the first man to write a book on the subject “Book on Games of chance” which was published in 1663 after his death. The probability formulae and techniques were developed by Jacob Bernoulli(1654-1705), De Moivre (1667-1754), Thomas Bayes(1702-1761) and Joseph Lagrange(1736- 1813). Pierre Simon, Laplace in the nineteenth century unified all these early ideas and compiled the first general theory of probability.

In the beginning, the probability theory was successfully applied at the gambling tables. But after some time, it was applied in the solution of social, political, economic and business problems. In fact, it has become a part of our everyday lives. We face uncertainty in personal and management decisions and use probability theory. Probability constitutes the foundation of statistical theory.

## Approaches

There are mainly three approaches to probability

1. Classical approach
2. Empirical approach
3. Axiomatic approach

Few terms can be defined / explained with reference to simple experiments relating to tossing of coins, throwing of a diee or drawing cards from a pack of cards.

Random Experiment

An experiment can be considered as a random experiment if all the possible outcomes are known in advance and none of the outcomes can be predicted with certainty. e.g. throwing a dice, tossing a coin etc.

Trial & Event

When a random experiment is performed, it is called a trial and outcome or combinations of outcomes are termed as events. For example

1. When a coin is tossed repeatedly, the result is not unique. We may get any of the two faces; head or tail. Thus, throwing a coin is a random experiment and getting of a head or tail is an event.
2. In the similar manner, when a dice is thrown, it is called a random experiment. Getting anyof the faces 1, 2, 3, 4, 5 or 6 is an event. Getting an odd no. or an even no., getting no. greater than 3 or lower than five, these are called events.
3. Similarly, drawing of two balls from an urn containing ‘a’ red balls and ‘b’ white balls is a trial and getting of both red balls, or both white balls, or one red and one white ball are events.

Exhaustive Cases

When a random experiment is done, there are some outcomes; the total numbers of possible outcomes are called exhaustive cases for the experiment. For e.g. when a coin is tossed, we can get head (H) or tail (T). Hence exhaustive no. of cases is 2 (i.e. H,T) If two coins are tossed, the various possibilities areHH, HT, TH, TT (number of exhaustive cases are four) where HT means, Head on first coin and Tail on second coin, and TH means, Tail on first coin and Head on second coin and so on.

In case of toss of three coins, number of outcomes is

= (H,T) x (H,T) x(H,T)

= (HH,HT,TH,TT) x (H,T)

= (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

No. of possible outcomes is 8 = 23. In general, in a throw of n coins, the exhaustive no. of cases is 2n.

In a throw of a die, exhaustive number of cases is 6, since we can get any one of the six faces marked 1, 2, 3, 4, 5 and 6. If two dice are thrown, the possible outcomes are

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (5,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (6,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |

* 1. total no. is 36, where (i,j) means number i on the first die and j on the second die, i and j both taking the values from 1 to 6. In the case of throw of two dice, no. of possible outcome = 62 = 36 and in the case of throw of three dice, no. of possible outcome = 63 = 216; in the case of throw of n dice, no. of possible outcome = 6n

Favourable cases or events

The number of outcomes of a random experiment which result in the happening of anevent are named as the cases favourable to the event.

* + 1. When a toss of two coins takes place, the no. of cases favourable to the event ‘exactly one head is two (i.e. TH or HT) and for getting ‘two heads’ is one (i.e. HH)
    2. When a card is drawn from a pack of cards, the no. of cases favourable to the event ‘getting a diamond’ are 13 and getting ‘an ace of spade’ is one.

Mutually Exclusive events or cases

Two or more events are considered as mutually exclusive if the happening of any one of them excludes the happening of all others in the same experiment. For example, in toss of a coin, the events ‘head’ and ‘tail’ are mutually exclusive because if head comes, we can’t get tail and if tail comes we can’t get head. Similarly, in the throw of a die, the six faces numbered 1, 2, 3, 4, 5 and 6 are mutually exclusive. Thus, events are said to be mutually exclusive of no two or more of them can happen simultaneously.

Classical/priori Probability

It is the oldest and simplest approach. Under this approach, there is no need to physically perform the experiment. The basic assumption is that the outcomes of a random experiment are equally likely. e.g. in a throw of a dice, occurrence of 1,2,3,4,5,6 are equally likely event.

If a random experiment results in N exhaustive, mutually exclusive and equally likely outcomes out of which mare favourable to the happening of an event X then the probability of occurrence of X i.e. P(X) is given by

P(X) =

*Favourable Exhaustive*

*cases cases*

*m*

= *N*

**Example 1.** Abag containing 10 green and 20 red balls. Aball is drawn at random. What is the probability that it is green.

**Sol.** Total number of balls in the bag = 10+20 = 30 Number of green balls = 10

Probabilityof getting a green ball =

10 1

= 30 = 3

Empirical Probability

The classical definition is difficult to apply as soon as we move from the field of coins, cards, dice and other games of chance. It may not explain the actual results in certain cases e.g if a coin is tossed 20 times , we may get 14 heads and 6 tails. The probability of head is thus 0.7 and tail is 0.3. However, if experiment is carried out large number of times, we should expect approximately equal number of heads and tails.

If an experiment is performed **repeatedly** under essentially homogeneous and identical conditions then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials become indefinitely large is known as the probability of happening of the event.

*m*

P(X) = lim *N*

N  

AxiomaticApproach

The axiomatic Probability theory is an attempt at constructing a theory of probability which is free from inadequacies of both the classical and empirical approaches. It plays an important role in rendering a reasonable amount of comprehensibility and tractability to the understanding of chance phenomenon at least in the initial stages of any scientific inquiry into their structure and composition where other approaches are less comprehensible and tractable.

## Addition Law

The probability of occurrence of either event A or event B of two mutually exclusive events is equal to the sum of their individual probability.

Mathematically, we can represent as P(A  B) = P(A) + P (B)

**Proof :-** If an event Acan happen in a ways and B in a ways then

1 2

The number of ways in which either event can happen in a + a ways.

1 2

Total number of possibilities is n.

Then by definition , the probability of either the first or second event happening is

a1 + a 2 =

n

a1 + a 2

n n

a1

But

n

= P(A)

And a 2

n

= P(B)

Hence P(A  B) = P(A) + P (B)

The theorem can be extended to three or more mutually exclusive events, thus P(A B C) = P(A) + P (B) + P(C)

**Example 2.** A deck of 52 cards, one card is drawn. What is the probability that it is either a king or a queen?

**Sol.** There is four kings and four queens in a pack of 52 cards.

The probability of drawing a card that is king = 4

52

The probability of drawing a card that is queen = 4

52

Since the events are mutually exclusive, the probability that the card drawn is either a king or

a queen = 4 +

52

4 = 2

52 13

If two events A & B are not **mutually exclusive** (joint events) then the addition law can be stated as follows

The probability of the occurrence of either event A or event B or both is equal to the probability that event Aoccurs, plus the probability that event B occurs minus the probability that both events occur. I can be shown as

P(A  B) = P(A) + P (B) – P (A  B)

**Example 3.** The managing committee of Residents Welfare Association formed a sub-committee of 5 persons to look into electricity problem. Profiles of the 5 persons are

|  |  |
| --- | --- |
| Male age | 40 |
| Male age | 43 |
| Female age | 38 |
| Female age | 27 |
| Male age | 65 |

If a chairperson has to be selected from this, what is the probability that he would be either female or over 32 years.

**Sol.** P (female or over 32) = P(female) + P(over 32) – P(female and over 32)

2 4 1

= + - = 1 5 5 5

**Example 4.** What is the probability of picking a card that was a heart or a spade.

**Sol.** Using the addition rule,

P(heart or spade) = P(heart) + P(spade) – P(heart and spade)

= 13 + 13 - 1 = 1

52 52 2 2

## Multiplication Law

It states that if two events A and B are independent , the probability that they both will occur is equal to the product of their individual probability.

P(A and B) = P(A) X P(B)

It can be extended to three or more independent events. P(A,B and C) = P(A) X P(B) X P(C)

**Proof :-** If an event A can happen in n1ways of which a1 are successful and B in n2ways of which a2 are successful then

The number of successful happening in both cases is a1x a2 . Total number of possibilities is n1 x n2.

Then by definition , the probability of the occurrence of both events is

a1 x a2 =

n1 x n2

a1 x a2 n n

But

and

a1 = P(A)/n

a2 = P(B)/n

Hence P(A and B) = P(A) x P (B)

**Example 5.** In order to marry with a girl, a man wants these qualities White complexion – the probability of getting this is one in fifty.

Etiquettes – the probability is one in hundred.

Dowry – the probability of getting this is one in Twenty.

Calculate the probability of his getting married to such a girl when the possession of these three attributes is independent.

**Sol.** Probability of a girl with white complexion =

Probability of a girl with handsome dowry =

1 = 0.05

20

1 = 0.02

50

Probability of a girl with etiquettes =

1 = 0.01

100

The probability of simultaneously occurrence of all these qualities = 0.05 x 0.02 x 0.01

= 0.00001

## Conditional Probability

The multiplication theorem described above is not applicable in case of dependent events. Two events A and B are said to be dependent when B can occur only when A is known to have occurred. The probability attached to such an event is called conditional probability and is denoted by P(A/B) i.e. probability of A given that B has occurred.

*Similary*,

*P*( *A* / *B*)  *P*( *A*  *B*) / *P*(*B*);

*P*(*B* / *A*)  *P*( *A*  *B*) / *P*( *A*);

*P*(*B*)  0

*P*( *A*)  0

*Symbolically*, *we write*

*P*( *A*  *B*)  *P*( *A* / *B*)  *P*(*B*)

*or P*(*B*  *A*)  *P*(*B* / *A*)  *P*( *A*)

## Probability Function

If the function permits us to compute the probability for any event that is defined in terms of value of the random variable, then the function is called a probability function. Just as there are discrete and continuous random variables, so there are discrete and continuous probability functions. Different types of probability functions are shown:

PROBABILTY FUNCTION

DISCRETE PROBABILTY FUNCTION

CONTINUOUS PROBABILTY FUNCTION

Discrete Probability Function

PROBABILTY MASS FUNCTION

CUMULATIVE MASS FUNCTION

PROBABILTY DENSITY FUNCTION

CUMULATIVE DENSITY FUNCTION

A probability function for a discrete random variable is called a discrete probability function since the domain of the function is discrete.

Probability Mass Function

A probability function that specifies the probability that any single value of discrete random variable will occur is called a probability mass function. If f(x) is the probability mass function of the random variable X, then f(x) = P(X=x) has the following properties

1. f(x) ≥ 0 for all values of X; and
2. Σf(x) = 1

Cumulative Mass Function

If X is a discrete random variable with p.m.f f(x), its cumulative mass function (c.m.f.) specifies the probability that an observed value of X will be no greater than X. i.e. if F(x) is a c.m.f. and f(x) is a p.m.f. then F(x) = P( X ≤ x ).

Continuous Probability Function

A probability function for a continuous random variable is called a continuous probability function since the domain of the function is continuous.

Probability Density function

For a continuous random variable, the corresponding function f(x) is called a probability density function (p.d.f.). Unlike a p.m.f., a p.d.f. doesn’t specify probabilities for specific individual value of the random variable.

Cumulative Density function

Corresponding to the cumulative mass function of a discrete random variable, the cumulative density function (c.d.f.) of a continuous random variable specifies the probability that an observed value of X will be no greater than x.

## Bernoulli Distribution

**BERNOULLI TRIALS** A random variable X which takes two values 0 & 1 with probabilities p and q respectively

It was discovered by James Bernoulli (1654 – 1705). Consider a set of n Bernoulli trials. Following are conditions for Bernoulli trials

1. An experiment is performed for a fixed number of trials.
2. In each trial, there are two possible outcomes of the experiment i.e. success or failure.
3. The probability of a success(p) remains constant from trial to trial. If the probability of success is not the same in each trial, we will not have binomial distribution.
4. The trials are statistically independents i.e. the outcomes of any trial do not affect the outcomes of subsequent trials.

A random variable X is said to be followed binomial probability distribution if it assumes only nonnegative values and its probability distribution is given by

P (X = x) = p (x) = { nc px qn-x , x= 0, , 2 \_ \_ \_ \_ , n

x

A random variable which satisfies conditions of Bernoulli trials, can be represented by Binomial Probability Distribution.

Binomial Distribution is denoted as

X ~ B (n,p) where q = 1 – p

n,p are the parameters of binomial distribution and n is the degree of binomial distribution. Some formulas are

Mean of Binomial Distribution = np

**Variance of Binomial distribution = npq Standard Deviation of Binomial Distribution =**

*npq*

**Example 6.** Find n,p and q when the mean of a binomial distribution is 40 and standard deviation is 6.

**Sol.** Mean of a binomial distribution = np Standard deviation of a binomial distribution =

*npq*

= 6 (given) ..........(1)

*npq*

np = 40 (given) ..........(2)

**=** 6 gives

*npq*

npq = 36; .............(3)

Putting (2) in (3)

40. q = 36, q = 36/40

q = 0.9, putting value in p = 1 – q, gives p = 0.1

np = 40

= 40

0.1

= 400

Ans. n=400, p = 0.1, q = 0.9

**Example 7.** Suppose that the half of the population of town are consumers of rice. One hundred investigators are given the duty to search for the truth. Each investor investigates 10 individuals. How many investigators do you expect to report that three or less of the people interviewed in there sample are consumers of rice?

**Sol.** Given n = 10

1

P =

2

Probability that three people or less consume rice P (X  3)

= P [X = 0] + P [X = 1] + P [X = 2] + P [X = 3]

= 10C0 p0 q10 + 10C1 p1q9 + 10C2 p2q8 + 10C3 p3 q7

= ( 1 )10 + 10 ( 1 )10 + 45 ( 1 )10 + 120 ( 1 )10

2 2 2 2

**=** ( 1 )10 (1 + 10 + 45 + 120)

2

= 176

1024

Therefore the no. of investigators to report that three people or less consume rice is given by NP where N is total number of trials.

= 176 x 100

1024

= 17.2 = 17 (approx.)

**Example 8.** In an industry, the workers have a 20% chance of suffering from an occupational disease. What is the probability that out of six workers, 4 or more will contract the disease?

**Sol.** The probability that a worker is suffering from the disease (p) = 20 / 100, The probability that a worker is not suffering from the disease

q = 1-p =1- 1/5 = 4/5

Probability that 4 people or more; i.e. 4, 5 or 6 will contract disease P (X ≥ 4)

= P [X = 4] + P [X = 5] + P [X = 6]

= 6C4 (

1

5

)4 ( 4

5

)2 + 6C5 (

5

1

)5 ( 4

5

)1 + 6C6 (

5

1

)6 ( 4 )0

5

= 15( 1 )4 ( 4 )2+ 6 ( 1 )5 ( 4 )1 + ( 1 )6 ( 4 )0

5

= 15 x

5 5 5 5 5

16 + 6 x 4 + 1

15625

15625

15625

**=** 0.01696

## Poisson Distribution

Poisson distribution was developed by the French mathematician and physicist Simeon Denis Poisson (1781 – 1840). This distribution is frequently used in context of Operation Research. This distribution plays an important role in Inventory control problems, Queuing theory and also in Risk models. Unlike binomial distribution, Poisson distribution can not be deduced on purely theoretical grounds based on the experiment conditions. In fact, it must be based on empirical results of past experiments relating to the problem under study.

Poisson distribution is a limiting case of a binomial distribution under the following conditions

1. n, the no. of trials is indefinitely large i.e. n ∞ [ n is positive integer]
2. p, the constant probability of success for each trial is indefinitely small i.e. p 0
3. np = m is finite where m is a positive real number

−x

e

Therefore, lim. b(x; n, p) = mx

x!

n ∞

Definition

A random variable X is said to follow a Poisson distribution if it assumes only non- negative values and its probability distribution is given by

−x

e

P(x,m) = P (X=x) =

x!

= 0; otherwise

mx ; x = 0, 1, 2, -------------, ∞

Here, m is known as the parameter of the distribution and is equal to np.

Following are conditions for Poisson Distribution

1. It is a discrete probability distribution with

i) Mean = m

i) Variance = m

1. Total number of terms are “.
2. Probability of success is constant from trial to trial for any given specific interval size.

**Example 9.** The standard deviation of Poisson variable X is “2, Find the probability that X is strictly positive.

**Sol.** We know that for Poisson distribution with parameter m,

Variance = m = (√2)2 = 2 [Because S.D. = √2, given]

−m

e

Therefore, P(X = r) = mr

r!

−2

e

= 2r ;r = 0,1,2.......

r!

The probability that X is strictly positive is given by P( X > 0 ) = 1 – P ( X = 0 ) = 1 - e-2

Answer is 1 - e-2

**Example 10.** On the basis of past data and experience, it was found that in a plant, there are on an average four accidents per month. Find the probability that in a particular month, there will be less than four accidents. Assuming Poisson distribution (e-4 = 0.0183)

**Sol.** We suppose that random variable X indicates the no. of accidents in the plant per month, (m is given as 4 in the usual notation) then by Poisson’s probability law

−m

e

P(X = r) = mr

r!

= −4 4**n**

e

r!

The probability that no. of accidents in a particular month in the plant is less than 4; is given by P (X< 4) = P (X = 0) + P (X = 1) + P (X = 2) + P (X = 3)

= e-4 [1 + 4 + 42 / 2! + 43 / 3!]

= e-4 [1 + 4 + 8 + 10.67]

= e-4 x 23.67 = 0.0183 x 23.67 = 0.4332

**Example 11.** A manufactured product has 2 defects per unit of product inspected. Using Poisson distribution, calculate the probabilities of finding a product

1. without any defect
2. three defects
3. four defects Given e-2 = 0.135

**Sol.** Average number of defects m = 2

P(0) = e-2 = 0.135 (Given) P(1) = P(0) x m

= 0.135 x 2

= 0.27

P(2) = P(1) x *m*

2

= 0.27 x 2

2

= 0.27

P(3) = P(2) x *m*

3

= 0.27 x 2

3

= 0.18

P(4) = P(3) x *m*

4

= 0.18 x 2

4

= 0.09

Hence the probability that a product has no defect is 0.135, product has 3 defects is 0.18 and product has 4 defects is 0.09.

## Normal Distribution

The normal distribution was discovered in 1733 by English mathematician De-Moivre who obtained this continuous distribution as a limiting case of the binomial distribution and applied it to problems arising in the game of chance throughout the eighteenth and nineteenth centuries, various efforts were made to establish the normal model as the underlying law ruling all continuous random variables. Thus the name ‘normal’. These efforts however failed because of false premises. The normal model has nevertheless become the most important probability model in statistical analysis.

Note

1. A random variable x with mean µ variance σ2 and following the normal law is expressed by X ~ N (µ, σ2)
2. If z = x - µ / σ

Ø(z) = 1/ σ √2π . exp.[ -z2/2]

Mean of z = E (z) = E [(x- µ) / σ]

= E(x) - µ / σ

= µ - µ / σ = 0 Variance of z = 1

Z is known as standard normal variate

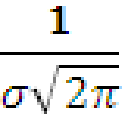
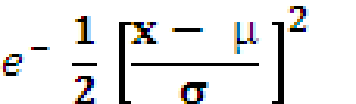
-∞ < z < +∞

## Main Characteristics of Normal Distribution

**Normal probability curve**

x = µ

Fig. 6.1

The normal probability curve with mean µ and standard deviation σ is given by the equation F(x) =   -∞ < x < +∞

and has the following characteristics

1. The curve is bell shaped,unimodal and symmetrical about the line x = µ.
2. Mean, median and mode of the distribution coincide.
3. Since f(x) being the probability, can never be negative, no portion of the curve lies below the x-axis.
4. Linear combination of independent normal

ƒ Ø(z) dz is known as normal probability integral and gives the

area under standard normal curve between the coordinates at z = 0, and z = z

1

In particular, the probability that a random value of x lies in the interval (µ­σ, µ+σ) is given by

µ+σ

P (µ­σ < x < µ+σ) = ƒf(x) dx

µ­σ

1

P (-1 < z < 1) = ƒ Ø(z) dz

-1

1

= 2 ƒ Ø(z) dz 0

1

 1

= 2/ σ “2π ƒ e 2

0

dz = 2 x 0.3413

= 0.6826

Similarly,

P (µ­2σ < x < µ+2σ)

= P (-2 < z < 2)

2

= 2 ƒ Ø(z) dz

0

and

= 2 x 0.4772 = 0.9544

3

P (µ­3σ < x < µ+3σ) = P (­3 < z < 3) = ƒ Ø(z) dz

-3

= 2 x 0.49865 = 0.9973

Thus, the probability that a normal variate x lies outside the range µ±3σ is given by P [(x- µ) > 3σ] = P (ǀzǀ > 3)

= 1 – P (-3 ≤ z ≤ 3)

= 1 -0.9973

= 0.0027

## Importance of Normal Distribution

Normal distribution plays a very important role in statistical theory because of the following reasons i

1. Most of the distribution occurring in practice e.g. Binomial, Poisson etc. can be approximated by

Normal distribution.

1. Even if a variable is not normally distributed it can sometimes be brought to normal form by simple transformation of variable.

i. P (µ-3σ < x < µ+3σ) = 0.9973 P (-3 < z < 3) = 0.9973

P (ǀzǀ < 3) = 0.9973

P (ǀzǀ > 3) = 0.0027

This property of the normal distribution forms the basis of entire large sample theory.

1. Theory of normal curves can be applied to the graduation of the curves which are not normal.
2. Normal distribution finds large applications in Statistical Quality Control in industry for setting control limits
3. Many of the distributions of sample statistics (e.g. the distribution of sample mean, sample, variance etc.) tends to normality for large samples and as such they can best be studied with the help of normal curves.
4. The entire theory of small sample tests is based on the fundamental assumption that the parent populations from which the samples have been drawn follow normal distribution.

**Example 12.** The annual demand for one type of fertilizer is normally distributed witha mean of 120 tonnes and standard deviation of 16 tonnes. If a wholesale distributor orders only once a year, what quantity should be ordered to ensure that there is only 5% chances of running short of quantity?

**Sol.** We suppose that the annual demand (in tonnes) be denoted by the random variable X. Therefore, z = (X-120) / 16

The desired area of 0.05 can be shown in the figure below. Since the area between the mean and the given value of X is 0.45, therefore from the table we get this area of 0.45 corresponding to z = 1.64



0.45

120 Shaded Area

By substituting value of z = 1.64 in standardized normal variate, we will get 1.64 = (X – 120) / 16

X = 120 + (1.64) (16) = 146.24

We can take the order value 146.24 as 147 by rounding if the ordered value is a whole unit.

**Example 13.** The income distribution of officers of a certain company was normally distributed with a mean income per month Rs. 15,000 and standard deviation Rs. 5,000. If a there were 242 officers, drawing salary above Rs. 18,500, how many officers were there in the company?

[The area under standard normal curve between 0 and 0.7 is 0.2580] **Given**

**Sol.** We suppose that the income of the officers (in Rs.) be denoted by the random variable X. Therefore, X – N(µ ,σ) where µ = Rs. 15000, and σ = Rs. 5000

Let N be the no. of officers in the company, then the no. of officers in the company drawing salary above Rs. 18500 is

N. P (X > 18500) = 242 (given) ———————————————————— (1) When X = 18,500, Z = (X – µ) / σ = (18500 – 15000) / 5000 = 0.7

Therefore, P(X > 18500) = P (Z> 0.7)

= 0.5 – P(0 ≤ Z ≤0.7)

= 0.5 – 0.2580

= 0.242

Substituting value of P(X > 18500) in equation (1) We get from eq. N. P (X > 18500) = 242

N. 0.242 = 242

N =1000

**Example 14.** There are 1000 soldiers in a regiment, it is assumed that the average height of soldiers is

68.22 inches, variance being 10.8 inch2. How many soldiers in the regiment would you expect to be

1. Over six feet tall i) less than 5.5 feet. It is assumed that height is normally distributed.

**Sol.** We suppose that the height of soldiers (in inches) be denoted by the random variable X. Then, we are given

Mean = µ = 68.22 and σ =

10.8

A soldier will be six feet if X is greater than 72 (because X is height in inches).

When X = 72, Z = (X ­ µ) / σ = (72 – 68.22) / 10.8 = 3.78 / 3.286 = 1.15

* 1. The probability that a soldier is over six feet (= 72"), is given by

P(X > 72) = (Z> 1.15) = 0.5 – P(0  Z  1.15)

= 0.5 – 0.3749 = 0.1251 (From Normal tables) Hence, in a regiment of 1000 soldiers, the no. of soldiers over six feet is

1000 x 0.1251 = 125.1 = 125 (rounded)

i) The probability that a soldier is below 5.5’(= 66"), is given by P(X < 66) = P(Z < (66 – 68.22) / 10.8 ) = P(Z < ( – 2.22) / 3.286 )

= P(Z< -0.6756) = P( Z > 0.6756)

= 0.5 – P(0 < Z< 0.6756) = 0.5 - 0.2501 (From Normal tables)

= 0.2499 (approx.)

Hence, the no. of soldiers over 5.5’ in a regiment of 1000 soldiers is 1000 x 0.2499 = 249.9 = 250 (rounded)

## Summary

This unit provides you a detailed description about probability, various probability laws and probability distribution. Under given conditions, observed frequency distributions can be approximated by well known theoretical distributions. The theoretical distributions indicate as what to expect if a random variable behaves as we assume it does. Probability distributions fall in two categories:

* Discrete
* Continuous

Most important discrete distributions are Binomial and Poisson. Similarly important continuous distribution is Normal distribution.

## Key Words

* + **Random Experiment** An experiment can be considered as a random experiment if when conducted rapidly under essentially homogeneous conditions, the result is not unique but maybe anyone of the various possible outcomes.
  + **Trial & event** When a random experiment is performed, it is called a trial and outcome or combinations of outcomes are termed as events.
  + **Exhaustive Cases** When a random experiment is done, there are some outcomes; the total numbers of possible outcomes are called exhaustive cases for the experiment.
  + **Mutually Exclusive events or cases** Two or more events are considered as mutually exclusive if the happening of anyone of them excludes the happening of all others in the same experiment

## Self Assessment Test

1. Explain in detail
   * Binomial Distribution
   * Normal Distribution
   * Poisson Distribution
2. Explain Probability Function in detail with diagram.
3. Out of 800 families with 4 children each, what percentage would be expected to have
4. At least one boy
5. No girls
6. Two boys and two girls
7. At most 2 girls

Assume equal probability for boys and girls.

(Ans. (a) 93.75% (b) 6.25% (c).37.5% (d) 68.75%)

1. There are 1000 employees, mean of their wages is Rs. 70. Daily wages are distributed normally distributed around the mean and their Standard deviation is Rs. 5. Find out ‘Number of workers’ whose daily wages will be
2. Between Rs. 70 and 72
3. Between Rs. 69 & 72
4. More than Rs. 75
5. Also find out the lowest daily wages of the 100 highest paid workers (Ans. i. 155, ii. 235, iii. 159, iv. 76.40)
6. A company is manufacturing electric bulbs, if 5% of the electric bulbs manufactured by the company are considered to be defective, use Poisson’s distribution to find the probability that in a sample of 100 bulbs
7. None is defective
8. 5 bulbs will be defective (e-5 = 0.007) (Ans. i. 0.007, ii. 0.1823)
9. If on an average 8 ships out of 10 arrive safely at a port, find the mean and standard deviation of the number of ships arriving safely out of a total of 1600 ships.

(Ans. 1280,16)

1. A machine is processing screws and screws are being checked for quality by examining number of defectives in a sample of 6. The following table shows the distribution of 128 samples according to the number of defective items they contained

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. of defectives | 0 | 1 | 2 | 3 | 4 | 5 | 6 | total |
| No. of samples | 7 | 6 | 19 | 35 | 30 | 23 | 7 | 128 |

i) Fit a binomial distribution and find the expected frequencies if the chance of screw being defective is ½.

i) Find the mean and standard deviation of the fitted distribution.

1. Ten unbiased coins are tossed concurrently. Find the probability of obtaining
2. Exactly six heads iii At least 8 heads
3. No head iv At least one head

(Ans. i. 105/512, ii. 1/1024, iii. 7/128, iv. 1023/1024)

1. Find the probability that at most 5 defective bolts will be found in a box of 200 bolts if it is known that 2% of such bolts are expected to be defective. (Given e-4 = 0.0183 )
2. There are 600 business students in the post-graduate department of a university. The probability for any student to need a copy of a particular book from the library on any day is 0.05. How many copies of the book should be kept in library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed.

(Ans. 37)