

Data Science, 2022

Tut 6: Machine Learning 1

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1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ .
 - a. What is the probability of obtaining the first head at the $(k + 1)$ -th toss?
 - b. What is the expected number of tosses needed to get the first head?

Q.1 probability of Heads = λ
probability of Tails = $1 - P(H) = 1 - \lambda$

a) $P(\text{heads at } k+1 \text{ toss}) = P(T \text{ at } k \text{ toss}) \cdot P(H \text{ at } k+1^{\text{th}} \text{ toss})$
 $= (1-\lambda)^k (1-\lambda)^0 \dots (1-\lambda)^0 (1-\lambda)^1$
 $= (1-\lambda)^k \cdot \lambda$

b) No. of tosses required to get first head is:
if its either if we get head directly = λ
or its tail & then head = $(1-\lambda)(S+1)$

$$\therefore S = \lambda + (1-\lambda)(S+1)$$
$$= \lambda + S + 1 - S\lambda - \lambda$$
$$S = (S+1) - S\lambda$$
$$S\lambda = 1$$
$$\lambda = 1/S$$
$$\therefore S = 1/\lambda$$

2. [Probability] Assume X is a random variable.
 - a. We define the variance of X as: $\text{Var}(X) = E[(X - E[X])^2]$. Prove that $\text{Var}(X) = E[X^2] - E[X]^2$.
 - b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X ? If $Y = a + bX$, what is the variance of Y ?

$$\begin{aligned}
 \text{Q.2 a) } \text{var}(x) &= E[(x - E(x))^2] \\
 &= E[x^2 - 2xE(x) + E(x)^2] \\
 &= E(x^2) - 2E[xE(x)] + E(x)^2 \\
 &= E(x^2) - 2E(x)^2 + E(x)^2 \\
 &= E(x^2) - E(x)^2 \\
 &\text{hence proved.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q.2.} \\
 \text{b) } \text{var}(x) &= E(x^2) - E(x)^2 \\
 &= 1 - 0^2 \\
 &= 1 \\
 \therefore E(x^2) &= 1 \text{ \& } E(x) = 0 \\
 E(y^2) &= E[(a+bx)^2] \\
 &= E(a^2 + 2abx + b^2x^2) \\
 &= a^2 + 2abE(x) + b^2E(x^2) \\
 &= a^2 + 2ab(0) + b^2(1) \\
 &= a^2 + 0 + b^2 \\
 &= a^2 + b^2
 \end{aligned}$$

$$\begin{aligned}
 E(y) &= E[a+bx] = a + bE(x) \\
 &= a + b(0) \\
 &= a
 \end{aligned}$$

$$\begin{aligned}
 \text{var}(y) &= E[y^2] - E[y]^2 = a^2 + b^2 - a^2 \\
 &= b^2
 \end{aligned}$$

3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5} , Aku predicts that a black beauty is a winning horse. This also means that with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.
- Given a horse, what is the probability that it wins?
 - What is the probability that Aku correctly predicts a black beauty is winning ?

Q-3

let A be event that given horse is winning
& $\sim A$ is event horse is not winning

let P be event 'Aku predicts horse winning'

let $\sim P$ be event 'Aku predicts horse not winning'

$$\begin{aligned} \text{a) } P(P) &= P(P, A) + P(P, \sim A) \\ &= P(P/A)P(A) + P(P/\sim A)P(\sim A) \\ &= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5}) \\ &= 1.99 \times 10^{-5} \end{aligned}$$

Q-3

$$\begin{aligned} \text{b) } P(P/Q) &= \frac{P(P, Q)}{P(Q)} = \frac{P(P/Q) \cdot P(Q)}{P(Q)} \\ &= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}} \\ &= \underline{\underline{0.49}} \end{aligned}$$