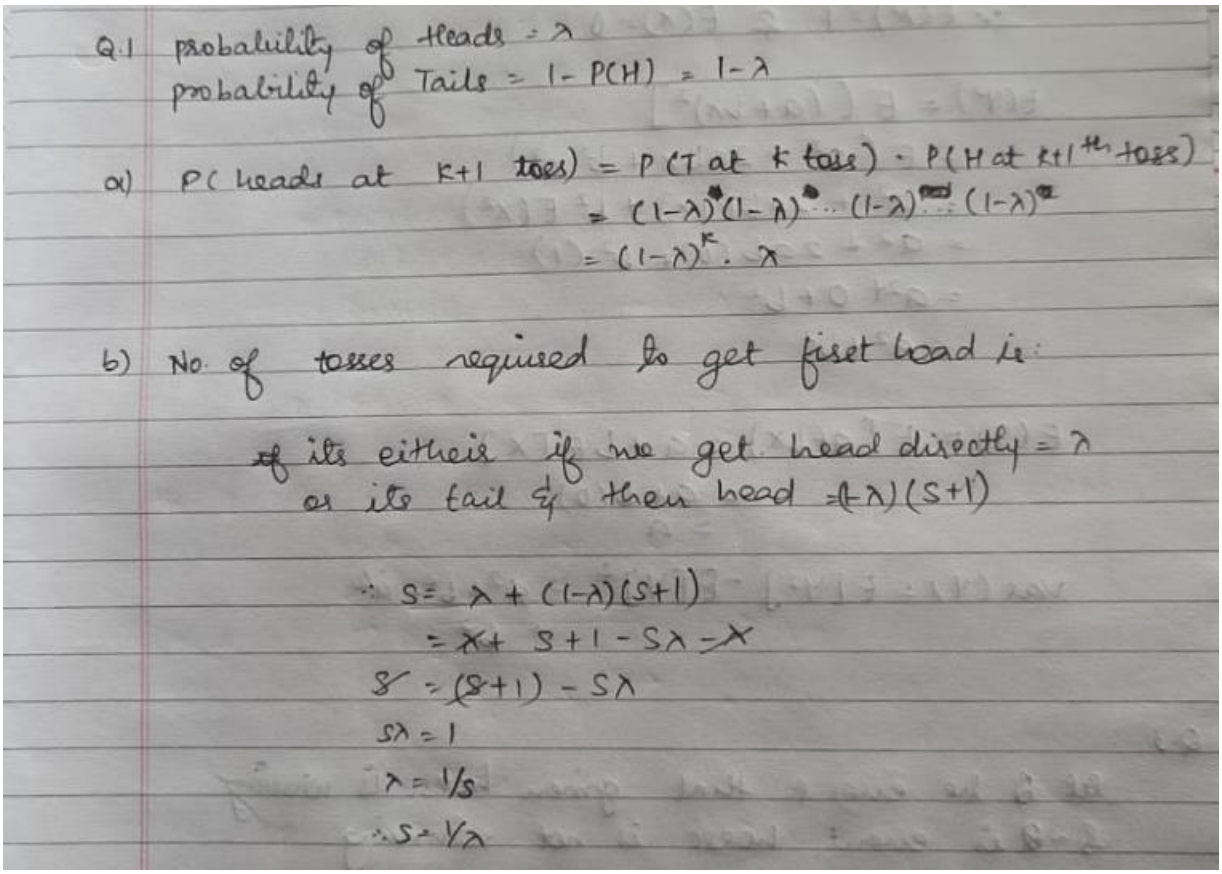
Data Science, 2022
Tut 6: Machine Learning 1

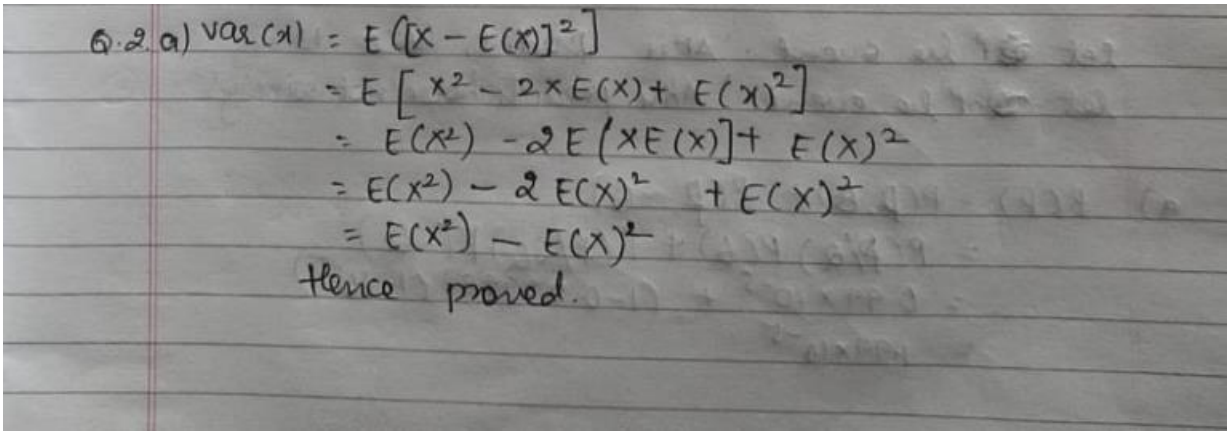
Name: Prachi Mehta

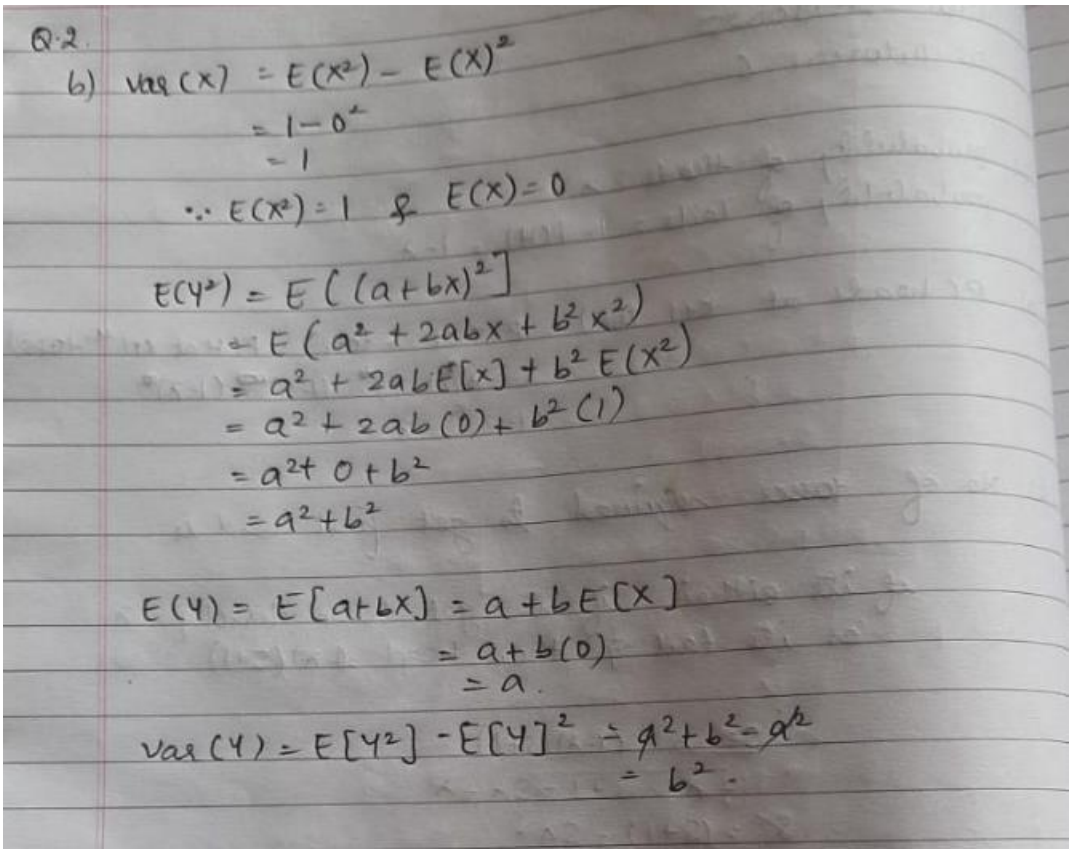
UID: 2018130025

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ.
   a. What is the probability of obtaining the first head at the (k + 1)-th toss?
   b. What is the expected number of tosses needed to get the first head?



2. [Probability] Assume X is a random variable.
   a. We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.
   b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?





3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3)
   With probability 10^{-5}, Aku predicts that a black beauty is a winning horse. This also means that with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.
   a. Given a horse, what is the probability that it wins?
   b. What is the probability that Aku correctly predicts a black beauty is winning ?

Q.3

Let Q be event that given horse is winning & ~Q is event horse is not winning.

Let P be event 'Aku predicts horse winning'
Let ~P be event 'Aku predicts horse not winning'

a) $P(P) = P(P, Q) + P(P, \sim Q)$
$= P(P/Q)\,P(Q) + P(P/\sim Q)\,P(\sim Q)$
$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$
$= 1.99 \times 10^{-5}$.

Q.3

b) $P(P/Q) = \dfrac{P(P, Q)}{P(Q)} = \dfrac{P(P/Q) \cdot P(Q)}{P(Q)}$

$= \dfrac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$

$= 0.49$