SC 651: Estimation on Lie Groups

Spring 2024

Assignment-2

1. Estimate the rotation matrix M that transforms v_i to w_i for the following data:

v_i	w_i
(0.63, 0.45, 0.87)	(0.17, 0.03, 0.50)
(0.03, 0.40, 0.86)	(0.26, 0.37, 0.15)
(0.86, 0.17, 0.16)	(0.78, 0.65, 0.67)
(0.29, 0.79, 0.04)	(0.83, 0.96, 0.82)
(0.60, 0.38, 0.15)	(0.69, 0.72, 0.04)
(0.99, 0.43, 0.76)	(0.64, 0.77, 0.34)

   using Wahba's algorithm.

2. Find the expression for:

   (a) $exp(\hat{x})$ where

$$\begin{aligned} \hat{\cdot}: \quad \mathbf{R}^3 \quad &\longrightarrow \quad \mathfrak{so}(3) \\ x = (x_1, x_2, x_3) \longmapsto \hat{x} &= \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}. \end{aligned}$$

   (b) $exp(\tilde{x})$ where

$$\begin{aligned} \tilde{\cdot}: \quad \mathbf{R}^3 \quad &\longrightarrow \quad \mathfrak{su}(2) \\ x = (x_1, x_2, x_3) \longmapsto \tilde{x} &= \begin{pmatrix} x_3 & x_1 - ix_2 \\ x_1 + ix_2 & -x_3 \end{pmatrix}. \end{aligned}$$

3. Show that XZY and YZY set of Euler angles have a singular point, hence, verifying that all 12 sets of Euler angles have a singularity.
   Note: Let $\{e_1, e_2, e_3\}$ be the standard orthonormal basis for $\mathbf{R}^3$, then, the rotation matrix corresponding to XYZ sequence of Euler angles can be expressed as

$$exp(\psi\hat{e}_3)\, exp(\phi\hat{e}_2)\, exp(\theta\hat{e}_1)$$

   where θ, ϕ and ψ are angles of rotation about the X, Y and Z axes, respectively.